A new approach to dating the reference cycle

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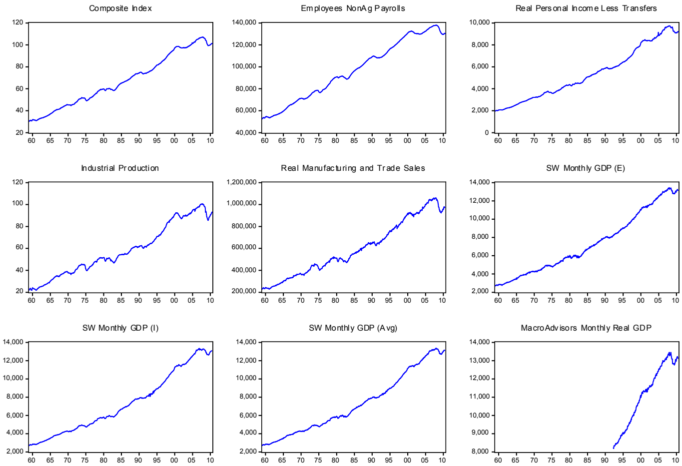
The reference cycle

- Business cycles versus growth cycles
- Dates (troughs and peaks) of *overall economy*
- Determining these dates is useful
  - Design and evaluate fiscal and monetary policies
  - Compute business cycle characteristics
  - International comparative assessments
- Burns and Mitchell (1946)
  - Definition: dates of wave-like movements occurring
    - at about the same time
    - in many economic activities
  - Computation
    - Step 1: Collect a number of coincident indicators
    - Step 2: Determine *global* change points

The reference cycle

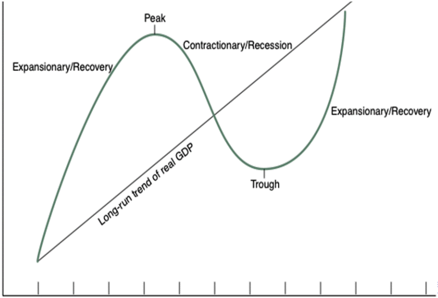
Step 1: NBER use 10 coincident indicators in dating 2009.06 trough



The reference cycle

Step 2: Determine *global* change points

- Dating turning points from a time series
  - We take the dating procedure as given
  - There are many parametric (Hamilton, 1989) and non-parametric alternatives (Bry and Boschan, 1971)
  - More recently, marching learning techniques (Piger, 2019 for a survey)
  - We focus on the simplest: Bry-Boschan locate local max and min subject to restrictions (distance, alternate)



The reference cycle

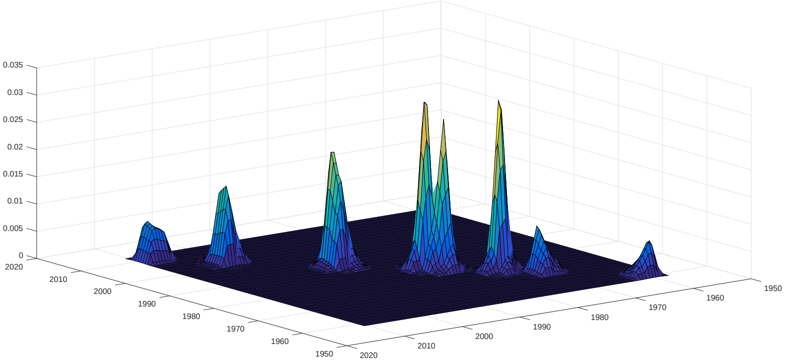
Step 2: Determine *global* change points

Two alternatives of Burns and Mitchell (1946)

- Average-then-date
  - Aggregate coincident indicators
  - Date turning points in the aggregate index
  - There is a vast literature
- Date-then-aggregate
  - Date specific peaks and troughs in individual indicators
  - Central tendency in each cluster
  - They follow this alternative and so do we

Date-then-average approaches

- Burns and Mitchell (1946): specific clusters about reference dates



- How can these specific dates be *averaged*?

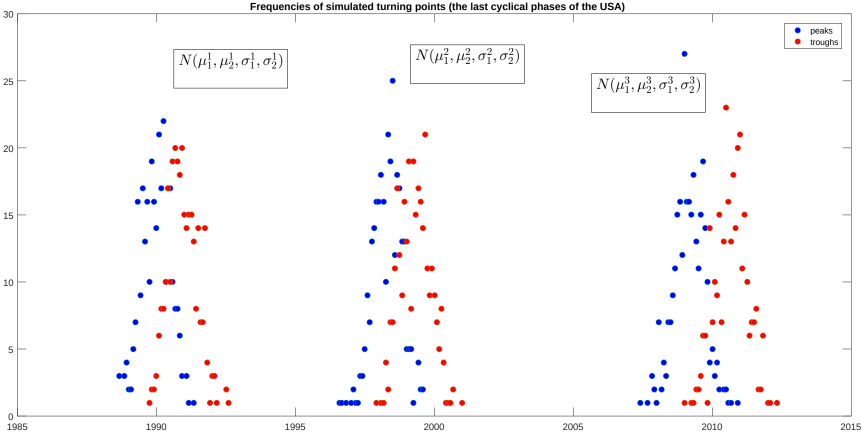
Date-then-average approaches

- Burns and Mitchell (1946)
  - Visual inspection of clusters
  - Drawback: use expert judgment
- Harding and Pagan (2006): First modern contribution
  - Algorithm minimizes distances reference and specific dates
  - Automatic but application-specific and without inference
- Stock and Watson (2010): $\tau_{is} = \mu_s + \epsilon_{is}$
  - Specific dates are observations of panel data with missing data
  - Allows inference but requires a given sequence of business cycles
- Stock and Watson (2014): $\hat{\mu}_s = mode(\tau_{is} \in (-12 + NBER_s + 12))$
  - Reference date is mode of kernel density of specific dates at NBER date $\pm$ 12 months
  - Allows inference but requires a given sequence of business cycles

Our contribution

- We view the reference cycle as a multiple change-point mixture model
- There is a double approach: The two faces of the same coin
  - Multiple change-point model: date the changes
    - The reference cycle is a collection of increasing change points: peak-trough dates
    - They segment the time span into K non-overlapping episodes
  - Mixture model: classify the dates
    - About the change points the specific turning points cluster around the K reference turning points
    - Homogeneity within clusters: At each episode k the specific turning point dates are generated from $N(\mu_k, \Sigma_k)$
    - Heterogeneity across clusters: At episode k', they are generated from $N(\mu_{k'}, \Sigma_{k'})$
    - Reference dates are not observed: each specific date is a realization of a mixture of K Gaussian distributions

Multiple change-point model: Chib (1998)



Our contribution

- Advantages over existing methods
  - The number of historical change points are data-driven: the estimates are not conditioned by the known occurrence of a phase shift
  - Simple estimation: Bayesian techniques of finite Markov mixture models under very minimal distributional assumptions
  - Reference dates are population concepts: perform inference (credible intervals)
  - Missing data for some coincident indicators is not a problem
  - Detection of phase changes in real time is a simple classification problem

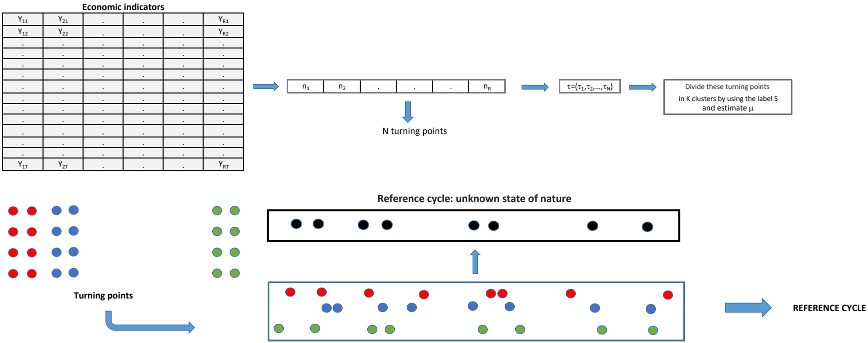
Our contribution

- Monte Carlo analysis: our proposal is robust to
  - Long recessions as the Great Recession
  - Short expansions as the double-dip recessions (80s in US; 00s in Europe)
  - Large variances in some clusters
  - Few observations in one cluster
  - Very few observations in all clusters
  - Asymmetric distributions
- Empirical application: reliability to assess the NBER dates
  - In 1959-2010: $K = 8$ clusters as in the NBER chronology
  - Our reference dates differ from the NBER dates by less than one quarter
  - The estimated probability perform a segmentation of the time span into non-overlapping episodes that agrees with the NBER classification
  - Dating analysis in real time
    - Accuracy: close agreement to those determined by the NBER
    - Speed: peaks 4.4 months, troughs 11.2 months earlier

Some notation

- The reference cycle: $\mu_k, k = 1, \ldots, K$
  - Is a time-partition determined by K **pairs** of peaks and troughs or change points $\mu = (\mu_1, \ldots, \mu_K)$
  - May be labeled through s that takes values $\{1, 2, \ldots, K\}$
- The specific dates: $\tau_i, i = 1, \ldots, N$
  - From R coincident indicators obtain $N = n_1 + \ldots + n_R$ specific dates
  - Collect them in $\tau = (\tau_1, \ldots, \tau_N)$
  - Let $S = (s_1, \ldots, s_N)$ discrete random variable
  - If $s_i = k$ then τ_i is drawn from $N(\mu_k, \Sigma_k)$
- Main purpose
  - Perform inference on μ and S from the sample of specific dates τ

Multiple change-point model: An naive illustration



Multiple change-point model: Chib (1998)

- Transitions across the K episodes should agree with a time-partition
  - Assume s follows a first-order K-state Markov chain

$$\Pr(s_i = k | s_{i-1} = l, ..., s_1 = w, \tau^{i-1}) = \Pr(s_i = k | s_{i-1} = l) = p_{lk}$$

  - Turning points are stacked in ascending order ($s_i \leq s_{i+1}$), then

$$P = \begin{pmatrix} p_{11} & 1 - p_{11} & 0 & \cdots & 0 \\ 0 & p_{22} & 1 - p_{22} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & 0 & p_{K-1K-1} & 1 - p_{K-1K-1} \\ 0 & \cdots & & 0 & 1 \end{pmatrix},$$

- Collect $\theta_P = (p_{11}, ..., p_{K-1K-1})$

Finite mixture Markov chain model

- Collect all the parameters in $\theta = (\theta_1, ..., \theta_K, \theta_P)$
- Each τ_i is a realization of a mixture of K separate bivariate Gaussian distributions

$$p\left(\tau_i|\theta, \tau^{i-1}\right) = \sum_{k=1}^{K} \Pr\left(s_i = k|\theta_k, \tau^{i-1}\right) N\left(\mu_k, \Sigma_k\right)$$

- The means of the mixture μ_k are the reference dates
- Inference on θ and s Bayesian MCMC with Gibbs sampler
  - Starts with a k-means: $S^{(0)} = \left(s_1^{(0)}, ..., s_N^{(0)}\right)$, $N_k(S^{(0)})$, $\mu_k^{(0)}$
  - Iterate the following steps for $m = 1, ..., M_0, M_0 + 1, ..., M_0 + M$

Estimation. Gibbs sampling. STEP 1: sample $\theta^{(m)}$ given $S^{(m-1)}$

- Sample $p_{ii}|S,\tau \sim Beta\left(e_{i1}\left(S\right), e_{i2}\left(S\right)\right)$
  - Prior: $p_{ii} \sim Beta\left(e_{i1}, e_{i2}\right)$, where $e_{i1} = 6$ and $e_{i2} = 0.1$
  - Posterior: $e_{i1}\left(S\right) = e_{i1} + N_{ii}(S)$; $e_{i2}\left(S\right) = e_{i2} + N_{ii+1}(S)$
- Sample $\mu_k|S,\tau$, $\Sigma_k|S,\tau$ from independent Normal-Wishart
  - $\Sigma_k^{-1}|\mu_k, S, \tau \sim W\left(c_k\left(S\right), C_k\left(S\right)^{-1}\right)$
    - Prior: $\Sigma_k^{-1} \sim W\left(c_0(S), C_0(S)^{-1}\right)$, where $c_0(S) = 0$, $C_0(S) = I_2$
    - Posterior: $c_k\left(S\right) = c_0(S) + N_k(S)$;
      $C_k\left(S\right) = C_0(S) + \sum_{i:s_i=k}\left(\tau_i - \mu_k\right)\left(\tau_i - \mu_k\right)'$
  - $\mu_k|\Sigma_k, S, \tau \sim N\left(b_k\left(S\right), B_k\left(S\right)\right)$
    - Prior: $\mu_k \sim N\left(b_0(S), B_0(S)\right)$, where $b_0(S) = 0$, $B_0(S) = 1000I_2$
    - Posterior: $B_k\left(S\right) = \left(B_0(S)^{-1} + N_k(S)\Sigma_k^{-1}\right)^{-1}$;
      $b_k\left(S\right) = B_k\left(S\right)\left(B_0^{-1}b_0 + \Sigma_k^{-1}\sum_{i:s_i=k}\tau_i\right)$

Estimation. Gibbs sampling. STEP 2: Multi-move sampling $S^{(m)}$ given $\theta^{(m)}$

- Because s follows a first-order K-state Markov chain

$$\Pr\left(S|\theta,\tau\right) = \Pr\left(s_N|\theta,\tau^N\right)\prod_{i=1}^{N-1}\Pr\left(s_i|s_{i+1}\theta,\tau^i\right),$$

- Hamilton (1989): Compute filtered probabilities $\Pr\left(s_i = k|\theta,\tau^i\right)$
  0. Starting from an initial value $\Pr\left(s_0 = k|\theta,\tau^0\right)$
  1. the one-step ahead prediction is computed

$$\Pr\left(s_i = k|\theta,\tau^{i-1}\right) = \sum_{l=1}^{K} p_{lk}\Pr\left(s_{i-1} = l|\theta,\tau^{i-1}\right)$$

  2. when the i-th turning point is added, the filtered probability is updated as follows

$$\Pr\left(s_i = j|s_{i+1}^{(m)} = l,\theta,\tau^i\right) = \frac{p_{jl}\Pr\left(s_i = j|\theta,\tau^{i-1}\right)}{\sum_{k=1}^{K} p_{kl}\Pr\left(s_i = k|\theta,\tau^{i-1}\right)}$$

Model identification and model selection

- Identification
  - The posterior of the mixture model could have $K!$ different modes
  - Frühwirth-Schnatter, 2001: obtain draws from a unique labeling subspace
  - Rejection sampling
    - Draws must imply a segmentation of the time span according with K
    - Discard simulations that do not satisfy $\mu_k^{P(m)} < \mu_k^{T(m)} < \mu_{k+1}^{P(m)}$ for all $k = 1, \ldots, K$
- Model selection
  1. from $K = 1$ to $Kmax$: Akaike (AIC), Swchartz (BIC), BIC corrected with Entropy
  2. from $K = 2/1$, $K = 3/2$,...,$K + 1/K$ sequential selection through Bayes factor
  3. endogenize K adapting Koop and Potter (2007): the duration of a regime k, d_k, is modeled using a Poisson distribution with mean λ_k

KP TP dating

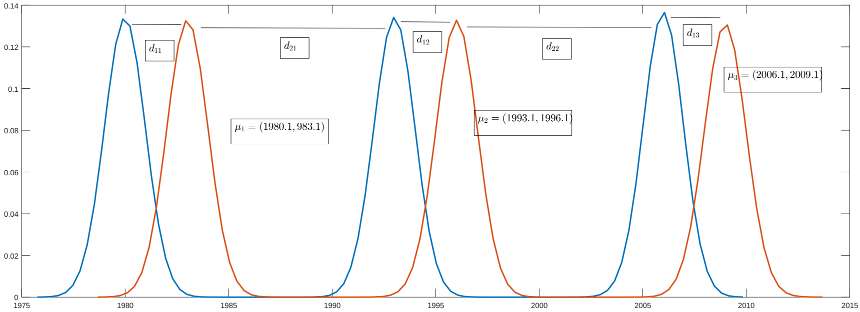
Monte Carlo

- Baseline framework (case 0)
  - Generate 1000 mixtures of K=3 Gaussian densities
  - μ_k: fix P_1 and use NBER to define P-T=11.1; T-P=58.4
  - Σ_k: averaged (10 coincident) σ_1^2 and σ_2^2; $\sigma_{12} = 0$
  - $N = 900$; $n_k = 300$ specific dates in each cluster

- Robustness
  1. Long recession such as the Great Recession P_3-T_3=18
  2. Short expansion such as double-dip recession in 80s T_2-P_3=12
  3. Large variances in one cluster σ_{13}^2 and σ_{23}^2 twice the average
  4. Few observations in one cluster $N_{k=3}$=50
  5. Very few observations n_k=7 $\forall K$
  6. Asymmetry distributions: left-skewed (case 6a), right-skewed (case 6b)

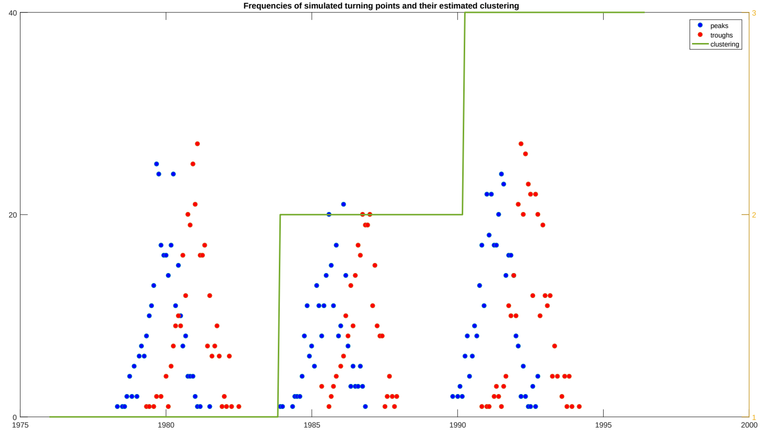
Monte Carlo

Scheme of the Monte Carlo simulations



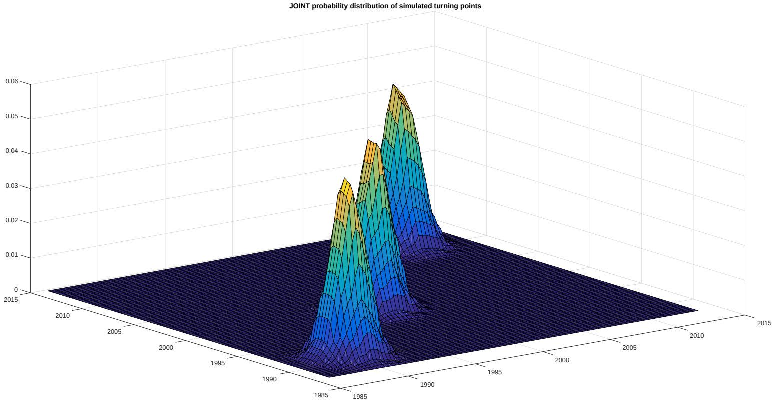
Monte Carlo

Baseline simulation (case 0)



Monte Carlo

Baseline simulation (case 0)



Monte Carlo

- Model selection
  - Based on likelihood: ML, AIC, BIC
  - Quality of partitions: Entropy, BIC+Entropy
  - Bayes factor
  - Adapted Koop and Potter (2007) method
- Model classification
  - Classification probabilities $\Pr(s_i = k|\theta) = \frac{1}{M} \neq \left\{ s_i^{(m)} = k \right\}$
  - Construct $S^K = (s_1^K, \ldots, s_N^K)$, where $s_i^K = k$ if Max$Pr(s_i = k|\theta)$
  - Compute classification rate $CR(S^K, S^*) = \frac{1}{N} \sum_{i=1}^N I_{\{s_i^K = s_i^*\}}$
- Model estimation: MSE (over 1000 MC) true and estimated
  - Means μ
  - Variances σ and $tr(\Sigma)$

Model selection and model classification

	Case 0				Case 1			
	Panel A. Based on log-likelihood							
K	1	2	3	4	1	2	3	4
LogLik	-3803.77	-2793.37	-1640.50	-1642.03	-3911.22	-2811.41	-1627.97	-1630.75
	(0.00)	(0.00)	(0.82)	(0.16)	(0.00)	(0.00)	(1.00)	(0.00)
AIC	7617.54	5608.74	3315.00	3330.07	7832.43	5644.82	3289.94	3307.50
	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)	(1.00)	(0.00)
BIC	7641.55	5661.57	3396.65	3440.52	7856.44	5697.65	3371.58	3417.96
	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)	(1.00)	(0.00)
Ent	-	0.39	0.00	6.26	-	0.15	0.00	9.67
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
BIC+Ent	-	5662.35	3396.65	3453.04	-	5697.95	3371.58	3437.31
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
	Panel B. Based on Bayes factor							
$K-1/K$	-	1979.98	2264.92	-43.88	-	2158.80	2326.06	-46.38
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
	Panel C. Model classification							
$CR(S^K, S^*)$	-	0.46	1.00	0.50	-	0.33	1.00	0.67
	Case 2				Case 3			
	Panel A. Based on log-likelihood							
K	1	2	3	4	1	2	3	4
LogLik	-3476.39	-2181.26	-1643.67	-1645.12	-4121.47	-3217.42	-2267.99	-2267.99
	(0.00)	(0.00)	(0.80)	(0.20)	(0.00)	(0.00)	(0.58)	(0.42)
AIC	6962.77	4384.52	3321.34	3336.25	8252.94	6456.83	4568.99	4581.97
	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)	(1.00)	(0.00)
BIC	6986.78	4437.35	3402.34	3446.70	8276.95	6509.66	4650.63	4692.43
	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)	(1.00)	(0.00)
Ent	-	0.00	0.17	8.25	-	0.46	0.00	9.65
		(1.00)	(0.00)	(0.00)		(0.00)	(0.99)	(0.01)
BIC+Ent	-	4437.35	3405.33	3463.20	-	6510.58	4650.63	4711.73
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
	Panel B. Based on Bayes factor							
$K-1/K$	-	2549.43	1034.37	-43.66	-	1767.29	1859.02	-41.72
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
	Panel C. Model classification							
$CR(S^K, S^*)$	-	0.67	1.00	0.34	-	0.49	1.00	0.49

Model selection and model classification

	Case 4				Case 5			
	Panel A. Based on log-likelihood							
K	1	2	3	4	1	2	3	4
LogLik	-3963.62	-2328.01	-1639.66	-1640.47	-619.63	-192.83	-118.34	-119.62
	(0.00)	(0.00)	(0.68)	(0.32)	(0.00)	(0.00)	(1.00)	(0.00)
AIC	7937.24	4678.02	3313.32	3326.93	1249.27	407.67	270.69	285.24
	(0.00)	(0.00)	(0.99)	(0.01)	(0.00)	(0.00)	(1.00)	(0.00)
BIC	7961.25	4730.85	3394.96	3437.39	1259.74	430.71	306.29	333.41
	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)	(1.00)	(0.00)
Ent	-	0.00	0.00	5.77	-	0.04	0.00	0.00
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
BIC+Ent	-	4730.85	3394.96	3448.93	-	430.79	306.29	333.41
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
	Panel B. Based on Bayes factor							
$K-1/K$	-	3230.40	1335.89	-42.43	-	829.03	124.42	-27.12
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
	Panel C. Model classification							
$CR(S^K,S^*)$	-	0.50	1.00	0.50	-	0.67	1.00	1.00
	Case 6a				Case 6b			
	Panel A. Based on log-likelihood							
K	1	2	3	4	1	2	3	4
LogLik	-3777.81	-2760.87	-1648.09	-1649.57	-4395.32	-3780.27	-2778.76	-1672.24
	(0.00)	(0.00)	(0.81)	(0.19)	(0.00)	(0.00)	(0.88)	(0.12)
AIC	75645.61	5543.74	3330.17	3345.13	7570.54	5579.53	3377.80	3390.48
	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)	(1.00)	(0.00)
BIC	7589.62	5596.56	3411.81	3455.59	7594.55	5632.35	3459.44	3500.94
	(0.00)	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)	(1.00)	(0.00)
Ent	-	0.22	0.00	7.41	-	0.29	0.00	7.52
		(0.00)	(1.00)	(0.00)		(0.00)	(0.99)	(0.01)
BIC+Ent	-	5597.01	3411.81	3470.41	-	5632.94	3459.44	3515.97
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
	Panel B. Based on Bayes factor							
$K-1/K$	-	1993.06	2184.75	-43.77	-	1962.19	2172.92	-43.92
		(0.00)	(1.00)	(0.00)		(0.00)	(1.00)	(0.00)
	Panel C. Model classification							
$CR(S^K,S^*)$	-	0.63	1.00	0.46	-	0.57	1.00	0.32

Model estimation

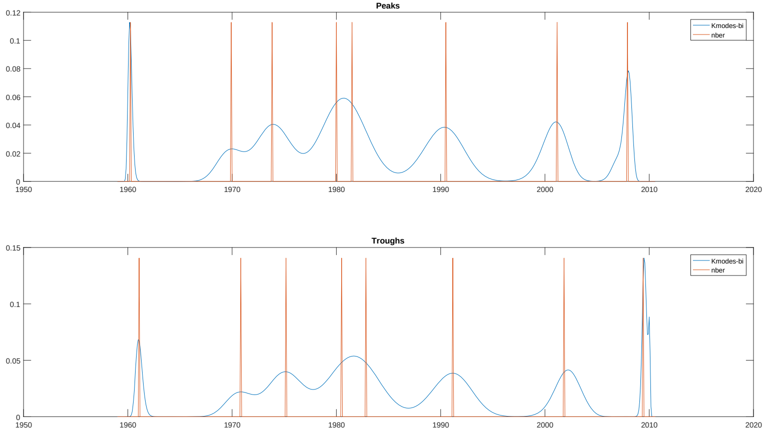
Measure	Case 0	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6a	Case 6b
Panel A. Estimation of means								
MSE_{μ}	2.17	1.50	2.17	4.33	2.00	1.50	10.17	9.00
Panel B. Estimation of variances								
MSE_{σ}	0.67	0.33	0.72	0.78	1.44	0.56	0.56	0.44
MSE_{trace}	0.67	0.67	0.67	1.67	6.00	0.34	0.33	0.67
Panel C. Model classification								
$CR(S^K, S^*)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Sample of specific reference dates

- Evaluate the empirical performance to assess the NBER *reference date*
- 10 coincident indicators used by NBER to determine June 2009 trough
- Sample: 1959.01-2010.08
  - Some indicators are not updated
  - But no further turning points since then $\Rightarrow$ valid up to historical dates
- Compute $\tau = (\tau_1, ..., \tau_N)$
  - Bry-Boschan (Watson, 1994) to each indicator
  - Specific turning points cluster around the NBER *reference date*

Sample of specific reference dates

Kernel density of specific peaks and troughs

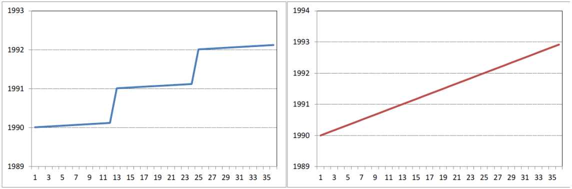


Data problems

- Discontinuities
  - Monthly calendar dates *YYYY.mm* generates jumps
  - Transform turning points to dates *YYYY.d*, where $d = 1/12(mm - 1)$

1990.01	1990	0.01	1	1990.00	1990.00	1990.01	1990	0.01	1990.01
1990.02	1990	0.02	2	1990.08	1990.08	1990.02	1990	0.02	1990.02
1990.03	1990	0.03	3	1990.17	1990.17	1990.03	1990	0.03	1990.03
1990.04	1990	0.04	4	1990.25	1990.25	1990.04	1990	0.04	1990.04
1990.05	1990	0.05	5	1990.33	1990.33	1990.05	1990	0.05	1990.05
1990.06	1990	0.06	6	1990.42	1990.42	1990.06	1990	0.06	1990.06
1990.07	1990	0.07	7	1990.50	1990.50	1990.07	1990	0.07	1990.07
1990.08	1990	0.08	8	1990.58	1990.58	1990.08	1990	0.08	1990.08
1990.09	1990	0.09	9	1990.67	1990.67	1990.09	1990	0.09	1990.09
1990.10	1990	0.10	10	1990.75	1990.75	1990.10	1990	0.10	1990.10
1990.11	1990	0.11	11	1990.83	1990.83	1990.11	1990	0.11	1990.11
1990.12	1990	0.12	12	1990.92	1990.92	1990.12	1990	0.12	1990.12

months_to_decimals　　decimals_to_months



- Some indicators are not available for the full span
  - It is not a problem in practice
  - Estimate some reference dates from smaller samples

Determine the number of clusters

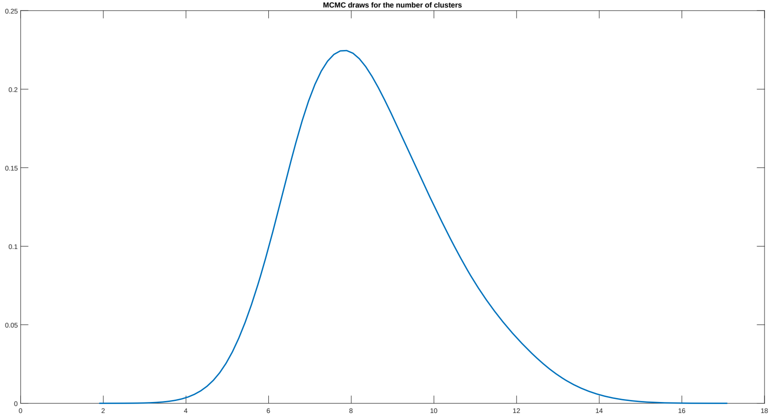
$$p\left(\tau_i|\theta,\tau^{i-1}\right)=\sum_{k=1}^{K}\Pr\left(s_i=k|\theta_k,\tau^{i-1}\right)N\left(\mu_k,\Sigma_k\right)$$

K	LogLik	AIC	BIC	Entropy	BIC-Entropy	Bayes factor $K-1$ / K
1	-530.40	1070.80	1080.84	-	-	-
2	-216.92	455.83	477.91	1.64	481.20	602.92
3	-179.98	393.95	428.08	0.98	430.03	49.84
4	-144.96	335.92	382.09	0.99	384.06	45.99
5	-117.94	293.87	352.09	0.66	353.41	30.00
6	-103.45	276.90	347.16	0.29	347.73	4.93
7	-77.30	236.61	318.91	0.00	318.91	28.25
8	-54.33	202.66	297.01	0.00	297.01	21.90
9	-53.48	212.96	319.35	0.00	319.35	-22.35

This agrees with the number of NBER pairs of turning points

Determine the number of clusters

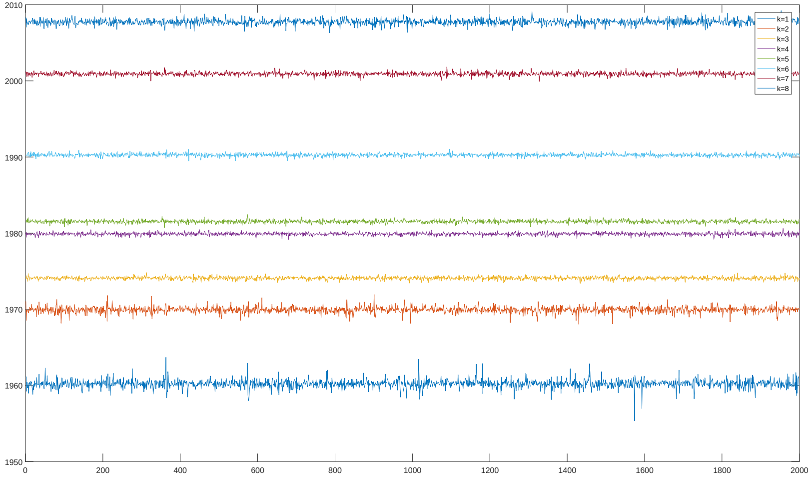
Figure: Classification probabilities



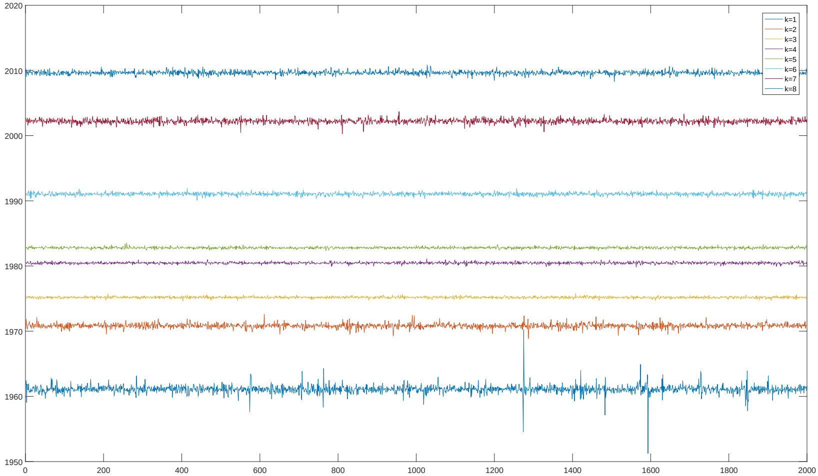
In-sample estimation results

NBER		MSMM		Deviation (in months)	
Peaks	Troughs	Peaks	Troughs	Peaks	Troughs
1960.04	1961.02	1960.02 (1959.04,1960.12)	1961.02 (1960.05,1962.03)	2	0
1969.12	1970.11	1969.11 (1969.03,1970.08)	1970.11 (1970.04,1971.07)	1	0
1973.11	1975.03	1974.02 (1973.09,1974.06)	1975.03 (1974.12,1975.06)	−3	0
1980.01	1980.07	1979.11 (1979.07,1980.04)	1980.06 (1980.03,1980.10)	2	1
1981.07	1982.11	1981.07 (1981.03,1981.11)	1982.10 (1982.07,1983.01)	0	1
1990.07	1991.03	1990.04 (1989.12,1990.08)	1991.01 (1990.08,1991.06)	3	2
2001.03	2001.11	2000.12 (2000.07,2001.05)	2002.03 (2001.08,2002.11)	3	−4
2007.12	2009.06	2007.10 (2007.02,2008.06)	2009.08 (2009.01,2010.03)	2	−2
sum				10	−2
average				1.25	−0.125

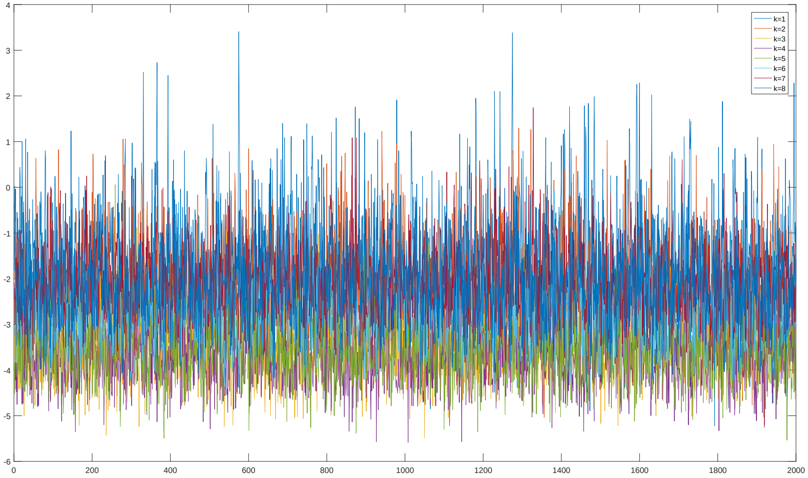
MCMC draws: the rejection sampler is free of label switching



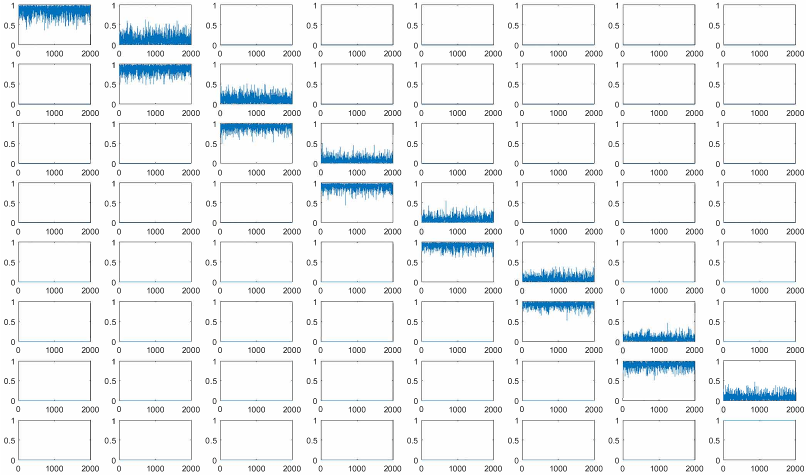
MCMC draws: the rejection sampler is free of label switching



MCMC draws: the rejection sampler is free of label switching

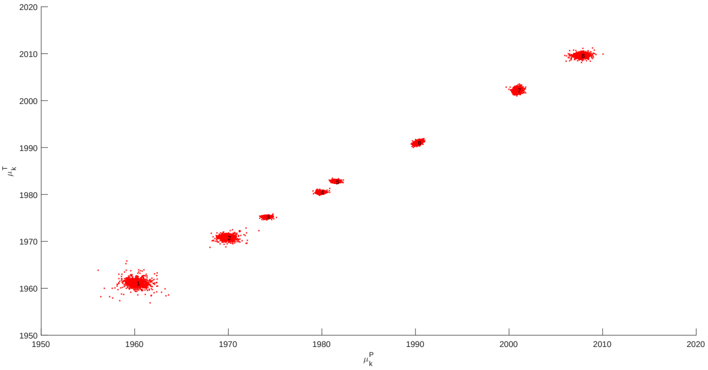


MCMC draws: the rejection sampler is free of label switching



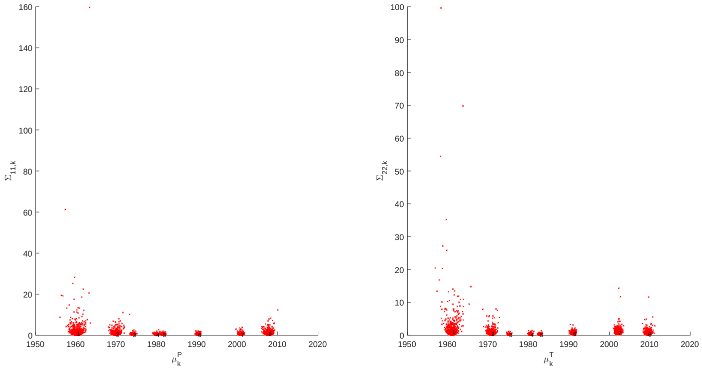
Two-dimensional scatter plots of the MCMC draws

Scatter-plot of the MCMC draws $\left(\mu_i^{(m)}, \mu_i^{(m)}\right)$, $m = 1, \ldots, 8$

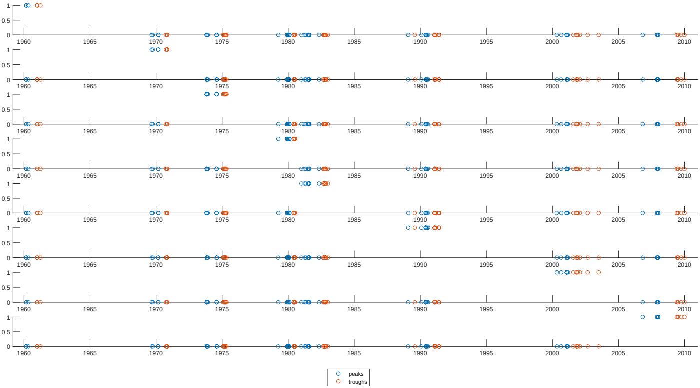


Two-dimensional scatter plots of the MCMC draws

Scatter-plot of the MCMC draws $\left(\mu_i^{(m)}, \Sigma_{ii}^{(m)}\right)$, $m = 1, ..., 8$



Inference: classification probabilities $\Pr(s_i = k|\theta)$



Pseudo real-time results

- Unfeasible real-time vintages: historical records of some of the ten indicators are not available
- Produce vintages as successive one-month enlargements of last available data
- Iterative procedure
  1. Begin with 1959.01-1966.01
  2. Bayes factor to confirm $K = 1$ and date μ_1^P, μ_2^T
  3. Enlarge the data by one month and implement Bry-Boschan
  4. If new no new τ_i^P: go to (3)
  5. If new peaks: artificial $\hat{\tau}_i^T = \tau_i^P + \overline{rece}$ until Bry-Boschan detect τ_i^T
  6. Apply Bayes factor:
     - if $K = 1$ go to (3)
     - if $K = 2$ wait 3 months (Chauvet and Piger, 2018) and warning the new peak. Date peak when $1/3$ of indicators show τ_i^P
  7. Go to (3) and follow a symmetric procedure to date the following trough
  8. Repeat the procedure until 2010.08

Pseudo real-time results

NBER						MSMM					
Peak	Announce	Lag	Trough	Announce	Lag	Peak	Identify	Lag	Trough	Identify	Lag
1960.04	-	-	1961.02	-	-	1960.03 (1959.04,1961.01)	1966.01	-	1961.02 (1960.01,1962.01)	1966.01	-
1969.12	-	-	1970.11	-	-	1969.11 (1969.02,1970.07)	1971.06	18	1970.11 (1970.02,1971.12)	1971.06	7
1973.11	-	-	1975.03	-	-	1973.09 (1972.02,1974.11)	1974.08	9	1975.06 (1974.08,1976.12)	1975.11	8
1980.01	1980.06	5	1980.07	1981.07	12	1980.01 (1979.05,1980.08)	1980.07	6	1980.07 (1980.04,1981.05)	1981.01	6
1981.07	1982.01	6	1982.11	1983.07	20	1981.05 (1980.11,1981.10)	1982.03	8	1982.10 (1982.01,1983.09)	1983.05	6
1990.07	1991.04	9	1991.03	1992.12	21	1989.12 (1988.08,1991.04)	1990.03	-4	1990.11 (1990.01,1992.08)	1991.03	0
2001.03	2001.11	8	2001.11	2003.12	25	2000.12 (1999.10,2001.10)	2001.04	1	2001.11 (2000.10,2002.12)	2002.06	7
2007.12	2008.12	12	2009.06	2010.09	15	2007.10 (2005.10,2009.01)	2008.07	7	2009.06 (2008.07,2010.06)	2010.02	8

- Accurate dates
- Speed: peaks 4.4 months and troughs 11.2 months earlier than NBER

Conclusion

- We are the first in providing a complete formal framework to determine the reference cycle
- Easy-to-implement in any region
  1. Choose a set of coincident indicators
  2. Determine the specific turning points
  3. Model them as realizations of a Markov-switching mixture model with non-ergodic transition probabilities (multiple change point of Chib, 1998)
  4. The means of the Normal distributions are the reference dates
  5. Estimation and inference by rejection sampler: draws must imply a segmentation of the time span and ensure the sequence of turning points in ascending order
- Monte Carlo simulations: very robust to data irregularities
- Empirical application: very accurate to determine the NBER *reference dates*

Conclusion

- Further research
  1. Extend the model to examine lag-lead behavior in particular episodes
     - Business cycle indicators
     - Countries
  2. Empirical applications
     - Other countries
     - Regional data
     - World business cycle reference dates
     - (very) Large databases

Unknown number of change points (Koop and Potter, 2007)

- A change-point model with an unknown number of breaks, which is taken as random and estimated using the data.
- The duration of a regime k, d_k, is modeled using a Poisson distribution with mean λ_k

$$d_k - 1 \sim Po(\lambda_k).$$

- The non-constant transition probability from regime k to regime $k+1$ depends on the current duration of regime.

$$\Pr\left[s_{t+1} = k+1 | s_t = k, d_k\right] = \frac{\exp(-\lambda_k)\lambda_k^{d_k - 1}}{(d_k - 1)!\left(1 - \sum_{j=0}^{d_k - 2} \frac{\exp(-\lambda_k)\lambda_k^j}{-j!}\right)},$$

where the sum in the denominator is defined to be 0 when $d_k = 1$.

back

Unknown number of change points (Koop and Potter, 2007)

These authors propose a hierarchical prior for λ_k such that $p(\lambda_1, \ldots, \lambda_T) = p(\lambda_1) \cdots, p(\lambda_T)$ with the prior

$$\lambda_k | \beta_\lambda \sim G(\alpha_\lambda, \beta_\lambda),$$

where β_λ, which reflects the degree of dissimilarity of the durations, is an unknown parameter with prior

$$\beta_\lambda^{-1} \sim G(\xi_1, 1/\xi_1),$$

Then, the posterior conditional for λ_k is

$$\lambda_k | \beta_\lambda \sim G(\alpha_k, \beta_k)$$

where $\alpha_k = \alpha_\lambda + d_m$ and $\beta_k = \left[\beta_\lambda^{-1} + 1\right]^{-1}$.

- As discussed in Koop and Potter (2007), Chib's algorithm can still be applied in the case of a non-constant homogeneous transition matrix.
- We have adapted this algorithm to the characteristics of our data because it is not directly applicable in the case of turning point dating.

back

Koop and Potter (2007) for turning point dating

Although this method is extremely useful in our context to propose a tentative number of clusters, let us show that this is not directly applicable in the case of turning point dating for several reasons.

- They apply this change-point model to US GDP growth and inflation, which, using our notation, we denote here as τ_i, $i = 1, \ldots, N$. In their model, the parameter likelihood within a break segment k, $p(\tau_i|\theta_k, \tau^{i-1})$, is obtained from

$$\tau_i = c^0_{s_i} + c^1_{s_i}\tau_{i-1} + c^2_{s_i}\tau_{i-2} + \exp(\sigma_{s_i}/2)\epsilon_i,$$

where $\epsilon_i \sim N(0,1)$, $c_{s_i} = \{c^0_{s_i}, c^1_{s_i}, c^2_{s_i}\}$ and for a given regime k, $\phi_{s_i} = \{c_{s_i}, \sigma_{s_i}\}$ satisfies

$$c_k = c_{k-1} + U_k$$
$$\sigma_k = \sigma_{k-1} + u_k,$$

where $U_k \sim N(0, V)$ and $u_k \sim N(0, \eta)$.

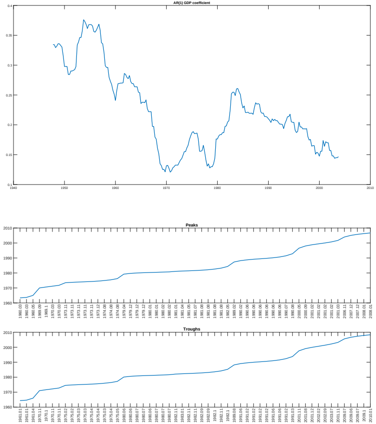
back

Koop and Potter (2007) for turning point dating

1. The first reason that precluded us from using this method directly in our approach is that our model's parameters do not depend on their previous values. Focusing on the means, the pair of reference cycle turning points that determine the business cycle k, (μ_k^P, μ_k^T), cannot be viewed as the pair that determine the previous reference cycle $(\mu_{k-1}^P, \mu_{k-1}^T)$ plus a noise. This is not difficult to solve because we can consider the within-cycle likelihoods $p(\tau_i|\theta_k, \tau^{i-1})$ as $N(\mu_k, \Sigma_k)$, which implies that, conditional to the k-th reference cycle, $\tau_i = \mu_k + u_i$, where $u_i \sim N(0, \Sigma_k)$.
2. The second reason is that, while the evolution of model's parameter is gradual in Koop and Potter (2007), the reference cycle turning points are constant within each cycle and change sharply in each cycle change.

back

Koop and Potter (2007) for turning point dating



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