Forecasting Risk Measures Using Intraday Data in a GAS Framework

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Main Objectives

1. This study extends the set of semiparametric GAS models of [2] to investigate whether realized measures can improve the predictive accuracy of GAS models;
2. We shed light on the potential improvement in risk forecasting from adding intraday information in the GAS framework for four stock indices using a long forecasting period (that includes the financial crisis period);
3. Robust empirical evidence that semiparametric models enhanced with realized volatility measures outperform other benchmark models via various backtesting methods;
4. We compare four different types of realized measures with regard to their forecasting ability for risk measures, when added to GAS models.

Introduction

A new framework for the joint estimation and forecasting of dynamic Value-at-Risk (VaR) and Expected Shortfall (ES) is proposed by incorporating intraday information into a generalized autoregressive score (GAS) model to estimate risk measures. We consider four intraday measures: the realized volatility at 5-min and 10-min frequencies, and the overnight return incorporated into realized volatilities. In a forecasting study, the set of newly proposed models is applied to 4 stock market indices, and is compared with a range of parametric, nonparametric and semiparametric models. VaR and ES forecasts are backtested individually, and the joint loss function is used for comparisons. Our results show that GAS models, enhanced with the realized volatility measures, outperform the benchmark models consistently across all indices and various probability levels.

Models

Models proposed in this study can be estimated by minimizing the loss function of [1] called FZ0:

$$L_{FZ0}(Y,v,e;\alpha) = -\frac{1}{\alpha e}\mathbf{1}\{Y \le v\}(v-Y) + \frac{v}{e} + \log(-e) - 1.$$

(A) One-factor GAS model (GAS-1F-Re):

$$\begin{aligned} v_t &= a\exp\{\kappa_t\}, \\ e_t &= b\exp\{\kappa_t\}, \quad b<a<0, \\ \kappa_t &= \omega + \beta\kappa_{t-1} + \gamma H_{t-1}^{-1}s_{t-1} + c\log(RM_{t-1}), \end{aligned}$$

where $s_t \equiv \frac{\partial L_{FZ0}(Y_t, a\exp\{\kappa_t\}, b\exp\{\kappa_t\};\alpha)}{\partial\kappa} = -\frac{1}{e_t}\left(\frac{1}{\alpha}\mathbf{1}\{Y_t \le v_t\}Y_t - e_t\right);$

(B) Two-factor GAS model with realized measures (GAS-2F-Re):

$$\begin{bmatrix} v_t \\ e_t \end{bmatrix} = \mathbf{w} + \mathbf{B}\begin{bmatrix} v_{t-1} \\ e_{t-1} \end{bmatrix} + \mathbf{A}\begin{bmatrix} \lambda_{v,t-1} \\ \lambda_{e,t-1} \end{bmatrix} + \mathbf{C}\, RM_{t-1},$$

where

$$\begin{aligned} \lambda_{v,t} &\equiv -v_t(\mathbf{1}\{Y_t \le v_t\} - \alpha), \\ \lambda_{e,t} &\equiv \frac{1}{\alpha}\mathbf{1}\{Y_t \le v_t\}Y_t - e_t; \end{aligned}$$

(C) GARCH-FZ model with realized measures (GARCH-FZ-Re):

$$\begin{aligned} v_t &= a\cdot\sigma_t, \\ e_t &= b\cdot\sigma_t, \quad b<a<0, \\ \sigma_t^2 &= \omega + \beta\sigma_{t-1}^2 + cRM_{t-1}^2, \end{aligned}$$

(2.D) A hybrid GAS/GARCH model with realized measures (Hybrid-Re):

$$\begin{aligned} v_t &= a\exp\{\kappa_t\}, \\ e_t &= b\exp\{\kappa_t\}, \quad b<a<0, \\ \kappa_t &= \omega + \beta\kappa_{t-1} + \gamma\left(-\frac{1}{e_{t-1}}\left(\frac{1}{\alpha}\mathbf{1}\{Y_t \le v_t\}Y_{t-1} - e_{t-1}\right)\right) + \delta\log|Y_{t-1}| + c\log(RM_{t-1}), \end{aligned}$$

The parameters of our proposed GAS-Realized models are estimated by minimizing the FZ0 loss function. This table presents the estimated parameters of GAS models for the S&P 500 for $\alpha = 5\%$.

	GAS-2F		GAS-2F-RV5		GAS-2F-RN5	
	VaR	ES	VaR	ES	VaR	ES
w	-0.009	-0.012	-0.009	-0.016	-0.011	-0.023
b	0.995	0.995	0.833	0.810	0.814	0.849
a_v	-0.129	-0.140	-0.125	-0.066	-0.114	-0.118
a_e	0.002	0.003	0.002	0.001	0.001	0.001
c	-	-	-0.323	-0.477	-0.353	-0.360
Avg loss	0.756		0.735		0.733	

The b parameters are statistically significantly different from zero at both 1% and 5% significance levels for both VaR and ES, which can be explained by the volatility clustering effect. The four columns on the right side of this panel show the parameters of GAS-2F extended with the 5-minute realized measures. The parameters of the one-day lagged realized measures RM_{t-1}, c, are statistically significantly negative for both VaR and ES, indicating that larger values of these realized variables will result in a lower estimated quantile or ES.

Results

We backtest the VaR and ES forecasts of the proposed models and compare their performance with that of benchmark models via different tests:

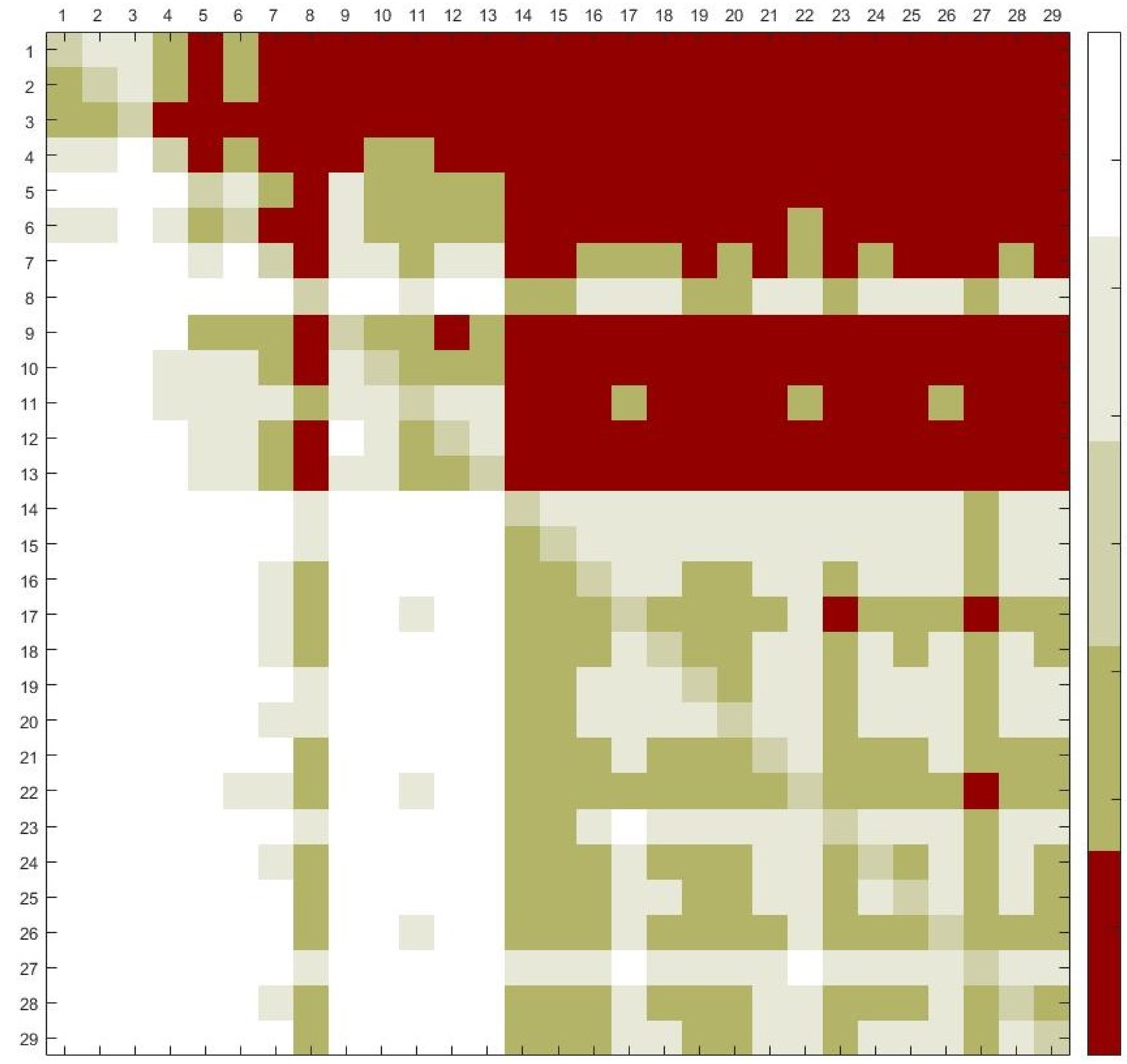
- Unconditional coverage test for VaR;
- Conditional coverage test (dynamic quantile regression) for VaR;
- Dynamic Expected Shortfall (DES) regression;
- Rankings based on FZ0 loss function;
- Diebold-Mariano (DM) test;
- Model Confidence Set (MCS) test.

Table 1: The 75% model confidence set for the R and SQ methods across the four stock indices

	Summed absolute values (R method)					*Summed squares (SQ method)*				
	1%	**2.5%**	**5%**	**10%**	**TOTAL**	**1%**	**2.5%**	**5%**	**10%**	**TOTAL**
GARCH-Skt	0	0	0	0	0	0	1	0	0	1
HEAVY-Skt-RV5	3	**3**	3	2	**11**	3	**3**	3	**2**	11
GAS-2F	0	0	0	0	0	0	0	0	0	0
GAS-1F	0	0	0	0	0	0	0	0	0	0
GARCH-FZ	2	1	0	0	3	2	1	0	0	3
Hybrid	1	1	1	0	3	1	1	1	0	3
GAS-2F-RV5	**4**	1	3	**3**	**11**	**4**	2	3	**2**	11
GAS-1F-RV5	**4**	2	3	2	**11**	**4**	3	3	**2**	**12**
GARCH-FZ-RV5	2	3	3	**3**	**11**	3	3	3	**2**	11
Hybrid-RV5	1	1	2	1	5	2	2	2	1	7

Note: The highest value (in bold) means that the model is the most favored one across four stock indices and for different probability levels.

Figure 1: Color map based on the Diebold-Mariano (DM) test comparing the average losses using the FZ0 loss function



Note: Models 14-29 are our proposed models. White blocks mean that the row model has lower average loss than the column model at 5% significance level; light green (below white in the color bar) blocks mean that the row model has lower average loss than the column model, but not significantly different from it, and so on. Darker color blocks mean that the row model has higher average loss than the column model.

Conclusions

- This study provides an extension of GAS framework, using exogenous information from high-frequency data, in order to improve on the prediction of VaR and ES.
- Throughout a set of individual and joint backtests for VaR and ES forecasts, we find that forecasts generated from the GAS-FZ-Realized models outperform forecasts based on GARCH models or historical simulations, even these based on the original GAS-FZ models.
- The 5-min realized variance incorporated in the GAS model can provide more accurate risk measures across different stock indices and probability levels compared to other realized measures.

References

[1] Tobias Fissler and Johanna F Ziegel. Higher order elicitability and osbands principle. *The Annals of Statistics*, 44(4):1680–1707, 2016.

[2] Andrew J Patton, Johanna F Ziegel, and Rui Chen. Dynamic semiparametric models for expected shortfall (and value-at-risk). *Journal of Econometrics*, 211(2):388–413, 2019.