Linear Cross-Sectional Model Comparisons

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Abstract

This paper evaluates a specification for a conditional beta model following Fama and French (2019, WP). Using a linear-beta model, I show;

- We can reject the Fama and French model that assumes characteristics are conditional betas in favor of a linear conditional beta model following Shanken (1990).
- Model-implied zero-beta rates are particularly sensitive to the specification, and that the linear conditional beta model provides a noticeably lower rate.
- Out-of-sample tests find the linear-beta model has a significantly lower bias, and Clark and West (2007) adjusted-MSPE, but it may come at the cost of a larger variance than the Fama and French model.

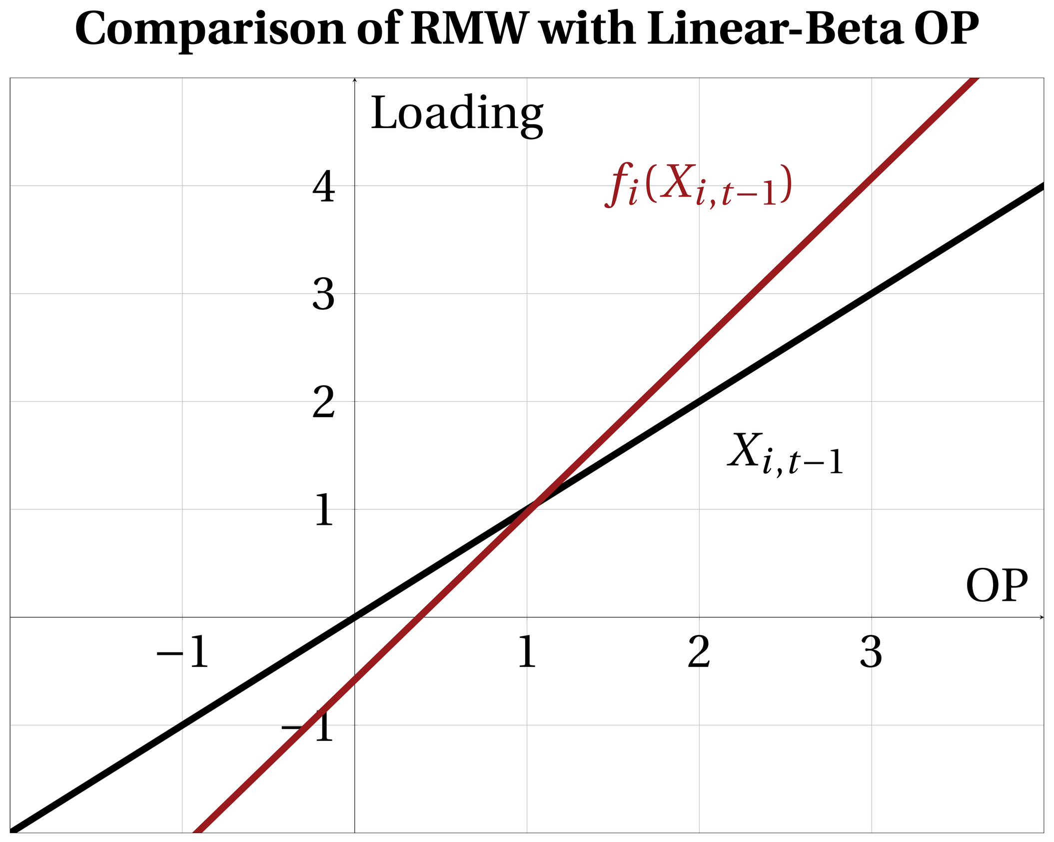
Introduction

Fama and French (FF, 2019) show that stacked Fama-Macbeth (1973) cross-sectional regressions of returns on characteristics price assets well compared to tried-and-true standards (such as the FF5). By using characteristics as factor loadings, this method provides a few benefits:

- Only one round of estimation (no times-series regression is necessary)
- No manual risk-factor calculation (such as replicating the FF5 or FF3 factors)
- Natural time-varying beta (the characteristics)

This paper tests a more flexible Shanken (1990) style linear-beta, $f_i(X_{i,t-1}) = b_{i,0} + b_{i,1}X_{i,t-1}$.

Comparison of RMW with Linear-Beta OP



Model

FF (2019) with a Shanken (1990) style Linear-Beta;

$$R_{i,t} = R_{z,t} + f_i(MC_{i,t-1})R_{MC,t} + f_i(BM_{i,t-1})R_{BM,t} + f_i(OP_{i,t-1})R_{OP,t} + f_i(INV_{i,t-1})R_{INV,t} + \epsilon_{i,t}$$

$$f_i(X_{i,t-1}) = b_{i,0} + b_{i,1}X_{i,t-1}$$

- Market Capitalization: $MC_{i,t-1}$
- Book-to-Market: $BM_{i,t-1}$
- Operating Profitability: $OP_{i,t-1}$
- Investment: $INV_{i,t-1}$

Estimating a stacked cross-sectional regression for each characteristic $X_{i,t-1}$, zero-beta rate $R_{z,t}$, and risk-premiums $R_{MC,t}$, $R_{BM,t}$, $R_{OP,t}$, and $R_{INV,t}$. Fama and French (2019) take $b_{i,0} = 0$ and $b_{i,1} = 1$.

Natural Questions;

- Does $\epsilon_{i,t}$ satisfy time-series requirements?
- Do the characteristics want $b_{i,0} = 0$ and $b_{i,1} = 1$?
- If not, is the difference meaningful?

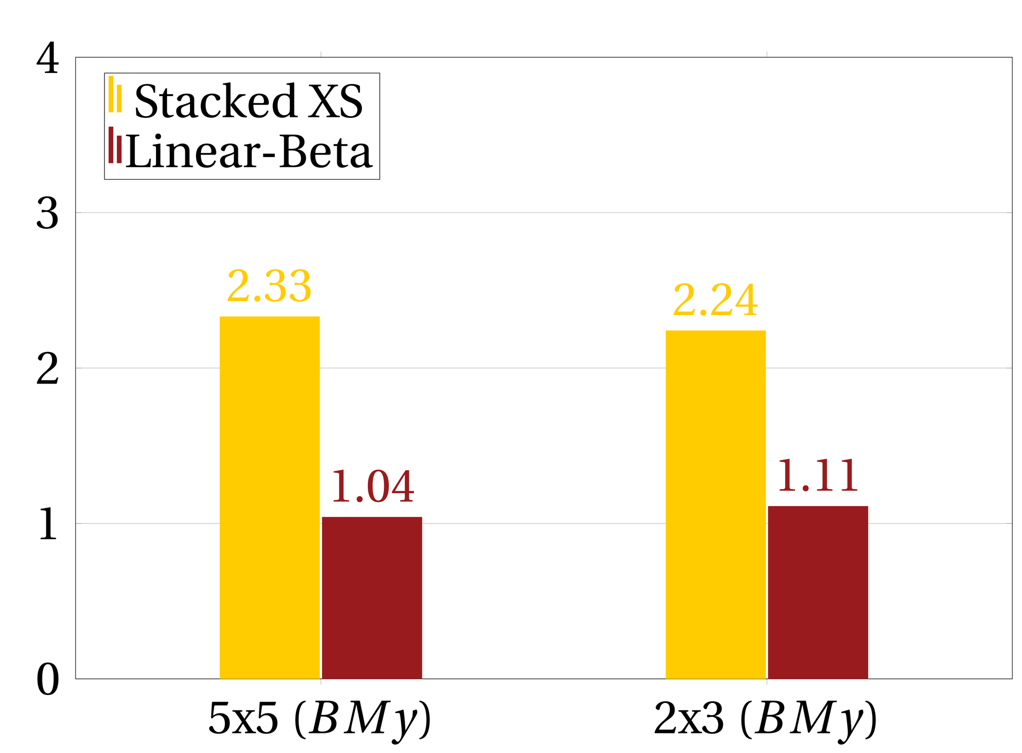
Data

The data are from CRSP, COMPUSTAT; breakpoints from Ken French's Website, 1963-2018

- Characteristics
  - Asset Weighted for MC, OP, INV, and BM
  - Include Annual (BMy) and Monthly (BMm) Updated BM for Characteristics and 5x5 Portfolios
- Portfolios
  - 18 2x3 Portfolios (ME with OP, INV, BMy)
  - 100 5x5 Portfolios (ME with OP, INV, BMy, BMm)

Repeat the model with the above datasets, with and without cross-sectionally studentizing the right-hand side characteristics.

Reduced Zero-Beta Rate



Error Correlations Test

Does stacking cross-sectionally estimated risk-premiums translate into a time-series regression? Time-series regression should have error term $\epsilon_{i,t}$ orthogonal to the explanatory variables (stacked risk-premiums for characteristic k, λ_t^k). Specifically I test,

$$z_{i,t}^k = \epsilon_{i,t}\lambda_t^k X_{i,t-1}$$
$$g^k = \frac{1}{T}\sum_t z_{i,t}^k$$

This setup allows us to test each characteristic to see if the portfolio error terms are, in fact, orthogonal.

Multivariate Regression Test

Do the characteristics *want* more flexibility? I use a multivariate regression test for potential linear-beta style factor loadings. The linear-beta time-series model used for this test includes both characteristics and factor terms;

$$R_{i,t} = \alpha_i + b_{0,i}'F_t + b_{1,i}'(F_t \cdot X_{i,t-1}) + \epsilon_{i,t}$$

$X_{i,t-1}$ denotes the various characteristics (BM, MC, INV, OP), and F_t denotes the risk-premiums or factors.

Test Results

Error Correlation Test demonstrates that the errors interacted with the risk-premiums and characteristics are statistically different from zero (only 2x3 pricing 5x5).

	R_{zt}	OP	MC	INV	BM
$g^{k\prime}Wg^k$	0.79	4.51	19.65	1.35	1.88

We can reject the null of orthogonal errors.

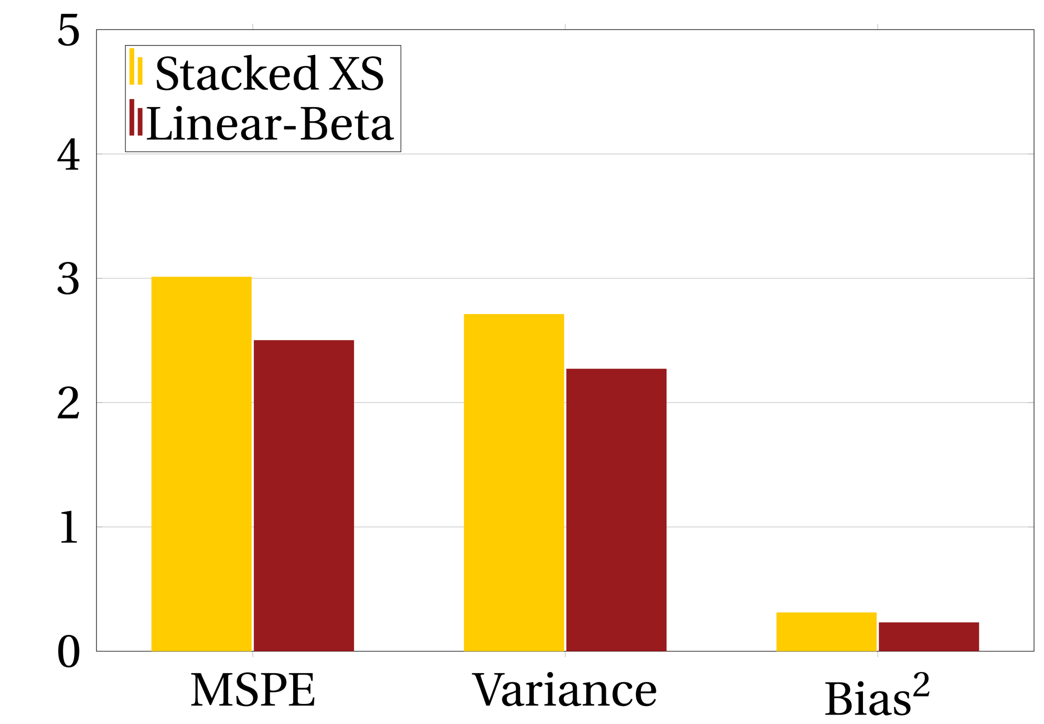
Multivariate Regression Test shows that many of the characteristics would benefit from a non-zero intercept and/or a non-unit slope.

	OP	MC	INV	BM
Mean $\lvert t(0)\rvert$, $b_{0,i} = 0$	2.49	1.20	2.08	2.15
Mean $\lvert t(1)\rvert$, $b_{1,i} = 1$	1.76	4.30	1.91	4.90

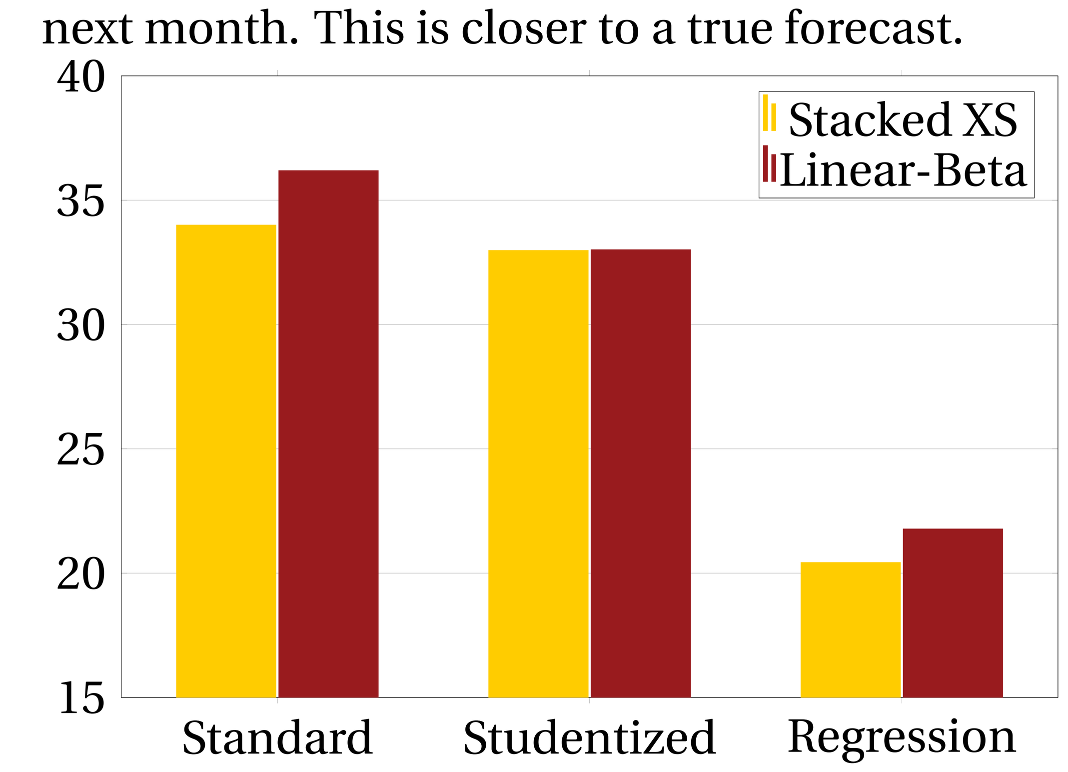
Overall, the original Fama and French (2019) model does not appear to minimize the sum of squares with this data set. There may be potential for improvement.

Out-of-Sample Predictions

Conditional: Average of one-month prediction assuming the RHS risk-premiums are available.



60-Month Average MSPE: Average of portfolio MSPEs with 60-month average of risk-premiums and 60-month calculated $f_i(X)$ used to predict the next month. This is closer to a true forecast.



Conclusion

In this paper, I use Shanken (1990) style linear-betas as loadings on cross-sectionally estimated risk-premiums. The result is a reduction in conditional out-of-sample MSPE, similar or slightly higher 60-Month Average out-of-sample MSPE, and a large reduction in the model-implied zero-beta rate. Overall, it appears that the stacked XS model used by Fama and French (2019, WP) is better suited for forecasting, but the proposed linear-beta model is better for return attribution.

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