The Financial Structure Implicit in the Sraffa Pasinetti Framework

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Main argument

1. Monetary value of financial assets and their cash flows

2. Monetary values and flows of a given (existing or expected) structure of production

- Compatibility between 1 and 2. If not present, re-stablish.

- Financial structure and production commitments

- Cantillo (2016 and 2019):

Main Concept and Ideas

- Finance drives production
- Production and technical input-output
- a) Inter-industry (Sraffa-Pasinetti) and b) inter-sectoral (Keynes)
- Connection between a) and b): Production Commitments
- Production commitments: the most basic form of finance
- Vertical integration 1973=> Pasinetti. Bortis: No expectations. Finance also excluded. However, they can be included.
- Keynes: C,I,S (inter-sectoral). Currency-Bonds
- Sraffa: Input-output (Inter-industry). Industry specific assets: Stocks
- Sraffa+Keynes: Complete understanding finance and production
- Givens Sraffa and Keynes.

Main Concept and Ideas

- Theory is expectation Cantillo (2004) and Cantillo (2016)
- Analytical period of production
- Multiple industries, specialization, production commitments.
- Production commitments—finance (credit/debit relation)
- Paper notes and monetization of commitments and complex instruments
- Value of financial assets only backed by production commitments. Cantillo (2019)
- Money value of assets and their cash flows compared to changes in money value of assets and monetary flows in the productive structure
- Multiple interpretations of Leontief matrix: multiple arrangements of commitments

Main Concept and Ideas

- Vertical integration and multiple interpretations: Not retrospectively but prospectively.

Appendix: The Model of Production Commodities by means of commodities

- Pasinetti, L. (1973). The notion of vertical integration in economic analysis. *Metroeconomica, 25* (1) 1-29.

- $(I - A^{\Theta})X_{(t)} = Y_{(t)}$ **(A.1)**

- $a_{[n]}X_{(t)} = L_{(t)}$ **(A.2)**

- $AX_{(t)} = S_{(t)}$ **(A.3)**

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Appendix: The Model of Production Commodities by means of commodities

- $(I - A^{\Theta})X_{(t)} = Y_{(t)}$ **(A.1)**

$X_{(t)}$: $m \times 1$ vector; total quantities produced of each commodity x_i

$i = (1, 2, \ldots m)$

m = number of commodities

I: $m \times m$ Identity matrix

A^{Θ}: $m \times m$ matrix; components $\left[a_{ij}^{\Theta}\right]$ fixed and circulating capital spent by each industry j for each unit of total output x_i

$Y_{(t)}$: $m \times 1$ column vector; net output of each commodity y_i. Final commodities.

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Appendix: The Model of Production Commodities by means of commodities

- $\boldsymbol{a}_{[n]}\boldsymbol{X}_{(t)} = \boldsymbol{L}_{(t)}$ **(A.2)**

$\boldsymbol{a}_{[n]}$**:** $1 \times m$ row vector; Components $[a_i]$: quantities of labor (# workers in men-hours per production period) required per unit of total output.

$\boldsymbol{L}_{(t)}$: Scalar, total amount of labor required for the production of the total output $X_{(t)}$.

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Appendix: The Model of Production Commodities by means of commodities

- $AX_{(t)} = S_{(t)}$ **(A.3)**

A: $m \times m$ matrix; components $[a_{ij}]$: Total stock of produced means of production required at the beginning of the production period for each unit of total output $X_{(t)}$.

$S_{(t)}$: $m \times 1$ vector Total produced means of production required at the beginning of the period.

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$$\checkmark (I - A^{\Theta})X_{(t)} = Y_{(t)} \qquad\qquad \textbf{(A.1)}$$

$$\checkmark a_{[n]}X_{(t)} = L_{(t)} \qquad\qquad\qquad \textbf{(A.2)}$$

$$\checkmark AX_{(t)} = S_{(t)} \qquad\qquad\qquad \textbf{(A.3)}$$

Sub-system of each commodity i in the net output $\boldsymbol{Y}_{(t)}$

- Pasinetti, L. (1973). The notion of vertical integration in economic analysis. *Metroeconomica, 25* (1) 1-29.

$$\boldsymbol{X}^{(i)}_{(t)} = (\boldsymbol{I} - \boldsymbol{A^{\Theta}})^{-1}\boldsymbol{Y}_{i(t)} \qquad \textbf{(A.4)}$$

$$\boldsymbol{L}^{(i)}_{(t)} = \boldsymbol{a}_{[n]}(\boldsymbol{I} - \boldsymbol{A^{\Theta}})^{-1}\boldsymbol{Y}_{i(t)} \qquad \textbf{(A.5)}$$

$$\boldsymbol{S}^{(i)}_{(t)} = \boldsymbol{A}(\boldsymbol{I} - \boldsymbol{A^{\Theta}})^{-1}\boldsymbol{Y}_{i(t)} \qquad \textbf{(A.6)}$$

Sub-system of each commodity i in the net output $Y_{(t)}$

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$$X_{(t)}^{(i)} = (I - A^{\theta})^{-1} Y_{i(t)} \tag{A.4}$$

$$L_{(t)}^{(i)} = a_{[n]} (I - A^{\theta})^{-1} Y_{i(t)} \tag{A.5}$$

$$S_{(t)}^{(i)} = A (I - A^{\theta})^{-1} Y_{i(t)} \tag{A.6}$$

Sub-system of each commodity i in the net output $Y_{(t)}$

- $X_{(t)}^{(i)} = \boxed{(I - A^{\Theta})^{-1}} Y_{i(t)}$ **(A.4)**

$Y_{i(t)}$: Vector of zeroes, except for commodity i.

$\boxed{(I - A^{\Theta})^{-1}}$: Leontief inverse matrix. Each component: Quantities of commodities directly and indirectly required for the production of each unit of $Y_{i(t)}$

Sub-system of each commodity i in the net output $Y_{(t)}$

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- $X_{(t)}^{(i)} = (I - A^{\Theta})^{-1} Y_{i(t)}$ **(A.4)**

- $L_{(t)}^{(i)} = a_{[n]}(I - A^{\Theta})^{-1} Y_{i(t)}$ **(A.5)**

- $S_{(t)}^{(i)} = A(I - A^{\Theta})^{-1} Y_{i(t)}$ **(A.6)**

Sub-system of each commodity i in the net output $Y_{(t)}$

- $L_{(t)}^{(i)} = \boxed{a_{[n]}(I - A^{\Theta})^{-1}} Y_{i(t)}$ **(A.5)**

$$a_{[n]}(I - A^{\Theta})^{-1} \equiv V \qquad\qquad\qquad \textbf{(A.7)}$$

Each one of the components of vector V, $[v_i]$, represents the direct and indirect quantities of labor required in the whole economic system in order to produce commodity i as a final output while restoring the initial production capabilities.

Sub-system of each commodity i in the net output $Y_{(t)}$

- Pasinetti, L. (1973). The notion of vertical integration in economic analysis. *Metroeconomica, 25* (1) 1-29.

- $X_{(t)}^{(i)} = (I - A^{\Theta})^{-1} Y_{i(t)}$ **(A.4)**

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- $S_{(t)}^{(i)} = A(I - A^{\Theta})^{-1} Y_{i(t)}$ **(A.6)**

Sub-system of each commodity i in the net output $\mathbf{Y}_{(t)}$

- $\mathbf{S}_{(t)}^{(i)} = \boxed{\mathbf{A}(\mathbf{I} - \mathbf{A}^{\Theta})^{-1}}\mathbf{Y}_{i(t)}$　　　　　　　　　**(A.6)**

$\mathbf{A}(\mathbf{I} - \mathbf{A}^{\Theta})^{-1} \equiv \mathbf{H}$　　　　　　　　　**(A.8)**

Each one of the column vectors that form matrix H, $[h_i]$, represents the heterogeneous collection of commodities required directly and indirectly in the form of capital stocks, for the production of commodity i as a final product.

Vertically integrated sectors

- $a_{[n]}(I - A^{\Theta})^{-1} \equiv V$ **(A.7)**
- $A(I - A^{\Theta})^{-1} \equiv H$ **(A.8)**

The Leontief Inverse Matrix

- $X_{(t)} = (I - A^{\Theta})^{-1} Y_{(t)}$ (1)
- Two kinds of interpretation: Intra-periodic and inter-periodic
- Goals:
  - Self-replacement requires effective fulfillment of commitments
  - Unveiling of commitments uncovers the expectational character
  - Multiple interpretations of the Leontief inverse matrix=> multiple commitment structures compatible with a single structure of production

Leontief Inverse Matrix: First interpretation

- Pasinetti in (1973 and 1977):
  - Pasinetti, L. (1973). The notion of vertical integration in economic analysis. *Metroeconomica, 25* (1) 1-29.
  - Pasinetti, L. (1977). *Lectures on the theory of production.* Columbia University Press.

- Each component α_{ij}^{Θ} of the Leontief inverse matrix represents the direct and indirect quantities of commodity i required, *in the economic system as a whole,* for the net production of one unit of commodity j.

Leontief Inverse Matrix: First interpretation

$$X_{(t)} = \boxed{(I - A^\Theta)^{-1}} Y_{(t)} \qquad\qquad \textbf{(1)}$$

- Commodity i is required directly as input in industry j and indirectly as input in other industries that provide inputs to industry j.

- Refers to a single period of production.

- Each row i: Total quantity of commodity i required in all industries.

- Each column j: Heterogeneous commodities needed directly and indirectly to produce commodity j.

Leontief Inverse Matrix: Second interpretation

- Pasinetti, L. (1977). *Lectures on the theory of production.* Columbia University Press.

- Taylor expansion:

$$(I - A_\Theta)^{-1} = I + A^\Theta + A^{\Theta^2} + A^{\Theta^3} + \ldots \quad (2)$$

By replacing (2) in (1) the following expression is obtained:

$$X_t = (I + A^\Theta + A^{\Theta^2} + A^{\Theta^3} \ldots)Y_t = IY_t + A^\Theta Y_t + A^{\Theta^2}Y_t + A^{\Theta^3}Y_t \ldots \quad (3)$$

- Stages of rounds of production
- Inter-period interpretation

Leontief Inverse Matrix: Third interpretation

- Taylor expansion
- Stages of production within a single period
- Intra-periodic interpretation

Leontief Inverse Matrix: Fourth interpretation

- Taylor expansion
- Gradual increments of production
- Each stage: % completion of production goal
- Intra-periodic interpretation

Leontief Inverse Matrix: Fifth interpretation

- Taylor expansion
- Rate at which inputs and outputs are supplied
- Continuous stream supply of inputs and outputs

Result: Multiple interpretations

- Multiple interpretations imply that there are multiple ways in which production can be committed.

- This provides the possibility of multiple configurations of financial assets that a re compatible with a given structure of production

Specialization, commitments and interpretations of the Leontief matrix

- Specialization requires production commitments.

- Production commitments are expectational

- First interpretation of Leontief matrix: each industry must deliver the quantities of its product directly and indirectly required by the net output.

- Second interpretation of Leontief matrix: industries commit to offer their products progressively in stages in accordance with the Taylor expansion.

- Third interpretation: each industry advances one stage per period

- Fourth interpretation: Industries commit to advance a percentage

- Fifth : Industries commit to continuous replacement

Vertical Integration and Production Commitments

- Vertical integration: re-classification of heterogeneous commodities in their vertically integrated units of labor and productive capacity:

$$a_{[n]}(I - A^{\Theta})^{-1} \equiv v \qquad\qquad \textbf{(A.7)}$$

$$A(I - A^{\Theta})^{-1} \equiv H \qquad\qquad \textbf{(A.8)}$$

- Using the multiple interpretations of the Leontief inverse matrix, vector V represents the quantities of labor that must be committed in order to produce a given net output.

Production commitments of capital and labor and multiple interpretations of the Leontief inverse matrix

- The 5 interpretations of the Leontief Inverse Matrix apply to the vertically integrated units of labor V and productive capacity or capital H.

- Commitments of labor and capital can be so translated into the wages that must be committed to labor and to capital.

- These are commitments in the form of consumption (for wages committed) and capital (for savings in the for of bonds)

Vertically integrated units of productive capacity

- Final, net output Y_t can be classified in consumption (wage goods) and Investment (luxury goods in a non-growing system).

- Vertically integrated quantities of labor and capital: capital and labor that must be committed.

- j^{th} component of vector V quantity of labor that must be committed

- j^{th} column of vector H, quantity of capital goods that must be committed.

- Commitments organized in multiple ways according to the interpretations of the Leontief inverse matrix.

Full employment condition

- Viability of the system

- Pasinetti, L. (1981). *Structural change and economic growth.* Cambridge: Cambridge University Press:

"Each sector i must be endowed with that stock of productive capacity which is necessary to produce the amount of commodity i which is demanded." (Pasinetti, 1981, p. 47)

Full employment condition

- Breach between micro decisions and macro outcomes.
- Decisions to commit: individual, outcomes: Macroeconomic
- Self-replacement is an expectation and not an actual occurrence.

Financial aspect of commitments

1. Production commitments at the beginning of the period, fulfillment at the end and flows of commodities between industries and sectors.

2. Cashflows of financial assets between the beginning of the period and the end of the period.

- Compatibility between 1 and 2.

- Use national accounts and financial accounts: Monetary value of assets Vs. monetary value of inputs. Monetary value of future cash flows versus monetary value of product flows.

Connection between Keynes and Sraffa

- Bortis, H. (2002). Piero Sraffa and the revival of classical political economy. *Journal of Economic Studies 29* (1), 74-89:

    - Use vertical integration for the connection. Dispose of Keynes' MEK. Replace with natural rate of profits. Equilibrium and institutions.
    - Mark-up pricing and Marxian schemes of reproduction
    - BUT Earnest Mendel in introduction to Capital Vol II: This is a misuse. Marx: Evolution of the structure of production.
    - Marxian schemes not tied to Mark up but to production commitments.

Wages and committed labor

- Wages are paid post-factum.
- Promise of payment of wages are a commitment by the suppliers of wage goods to supply labor to the producers of capital goods and vice versa.

Measurement of labor: Sraffa

- The quantity of labour employed in each industry has now to be represented explicitly, taking the place of the corresponding quantities of subsistence. We suppose labour to be uniform in quality or, what amounts to the same thing, we assume any differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labour receives the same wage.p10 (Sraffa, 1963, p.10)

Measurement of labor: Keynes

- For, in so far as different grades and kinds of labour and salaried assistance enjoy a more or less fixed relative remuneration, the quantity of employment can be sufficiently defined for our purpose by taking an hour's employment of ordinary labor as our unit and weighting an hour's employment of special labor in proportion to its remuneration; i.e. an hour of special labour remunerated at double ordinary rates will count as two units. We shall call the unit in which the quantity of employment is measured the labour unit; and the money-wage of a labour unit we shall call the wage unit. Thus, if E is the wages (and salaries) bill, W the wage-unit, and N the quantity of employment, $E = N \cdot W$. (Keynes, 1936, p. 41)

Keynes' inter-sectoral analysis Vs. Sraffa's inter-industry analysis

- Keynes: Changes in aggregate levels of employment (vertical analysis). Determination of profits by MEK and interest rates. Expectations and uncertainty.

- Sraffa: Level of employment given. Determination of prices (inter-industry). Effects of changes in distribution. Expectations are given.

Conclusions

- The existence of production commitments imply that there is a financial structure in the Sraffa-Pasinetti framework. The flows of payments in production can be compared with the flows of payments of financial assets to determine over/under leverage.

- Multiple interpretations of Leontief inverse allow for multiple possible payment structures associated to a given structure of production. This allows for multiple financial assets.

- Intersectoral and inter-industry analysis are compatible. The *GT* has production commitments. Quantities of labor and capital are analogous in Sraffa and Keynes.

Conclusions

∞Theory is expectation.∞