Modeling and Forecasting Serially Dependent Yield Curves

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Abstract

Considering that yield curves usually are serially dependent, this paper proposes a new method to estimate and forecast yield curves based on factors driving serial dependence of yield curves. Gathering information at different lags of yield curves, the dimensionality and the lag order of yield curves are jointly determined. Applying this method to monthly U.S. government bond yields from January 1985 through December 2009, I find that the dynamic structure of yield curves reduces to a vector process lying in a 3-dimensional space, with 1-month lag information. Yield curve residuals from this new model over time exhibit zero mean and less autocorrelation. Moreover, this new model's 1-month ahead forecasts outperform those of all competitors including the dynamic Nelson–Siegel and random walk forecasts at all maturities.

Motivation

Yield curves, or the term structure of interest rates, are usually serially dependent.

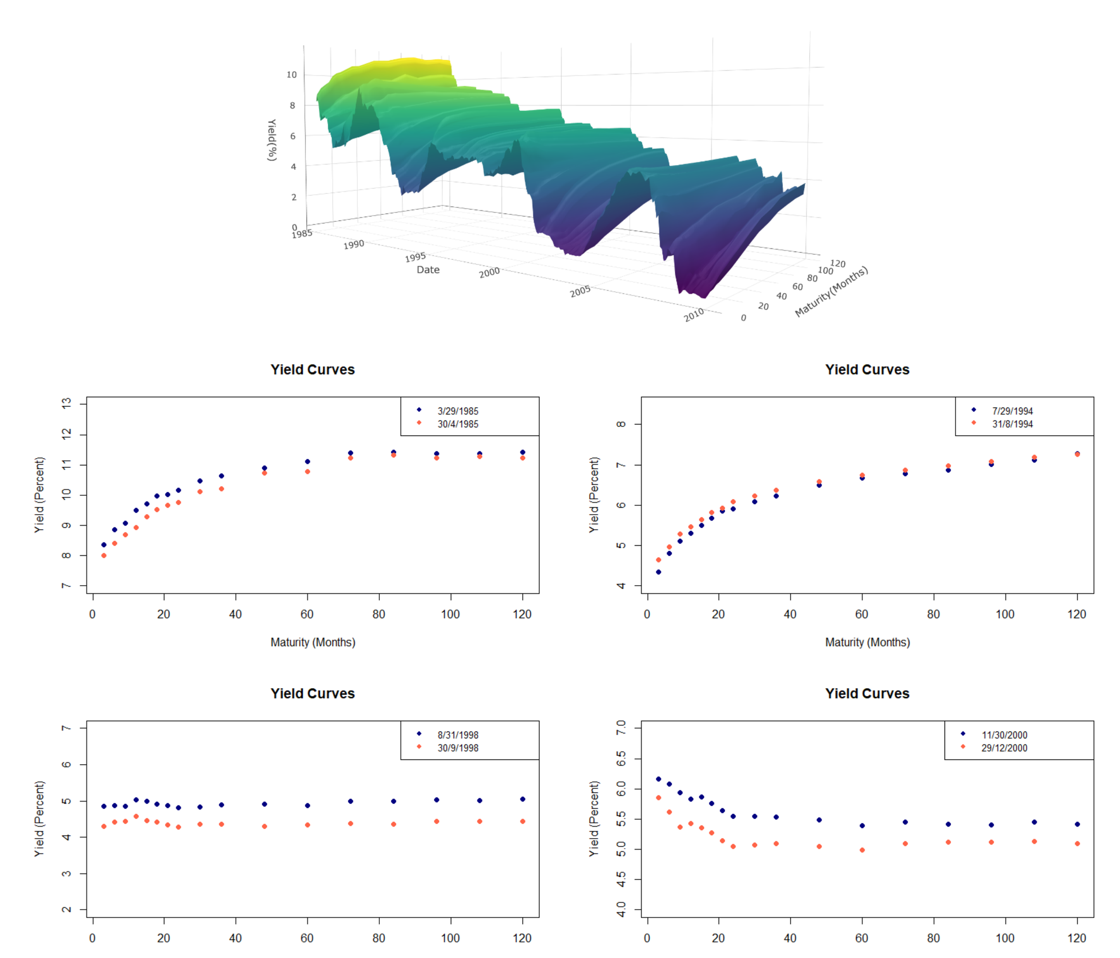


Figure 1:Autocorrelated yield curves (monthly unsmoothed Fama-Bliss zero-coupon yields of U.S. Treasuries) and examples of two consecutive yield curves, January 1985 to December 2009.

Contributions

1. This is the first work that considers factors driving serial dependence across yield curves into the modeling, estimation and forecasting of the term structure of interest rates.
2. Second, a data-driven method is proposed to determine the lag order and dimensionality of yield curves simultaneously.

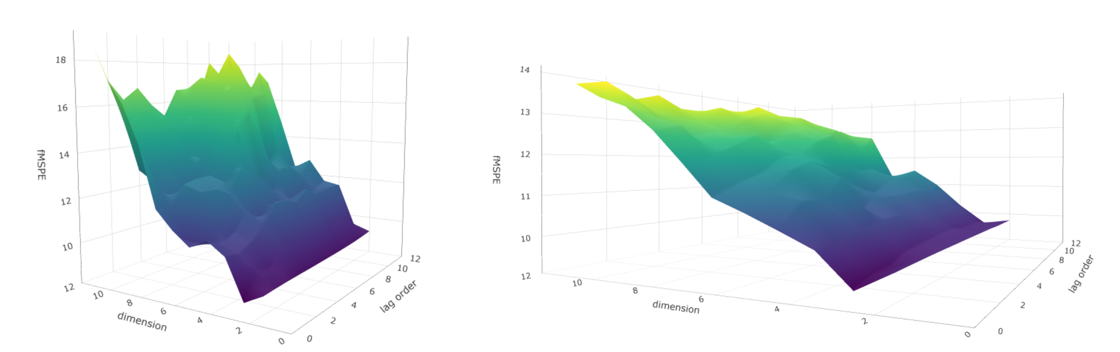
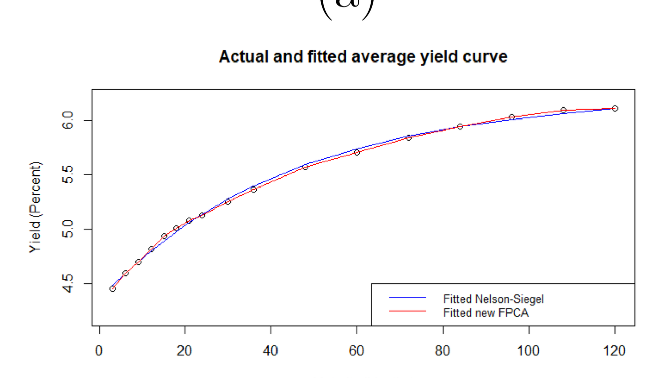


Figure 2:Three-dimensional surface plots of functional Mean Squared Prediction Errors (fMSPEs) depending on different values of dimension and lag order.

3. This new method has favorable in-sample and out-of-sample properties.

Fitting yield curves

(a)



(b)

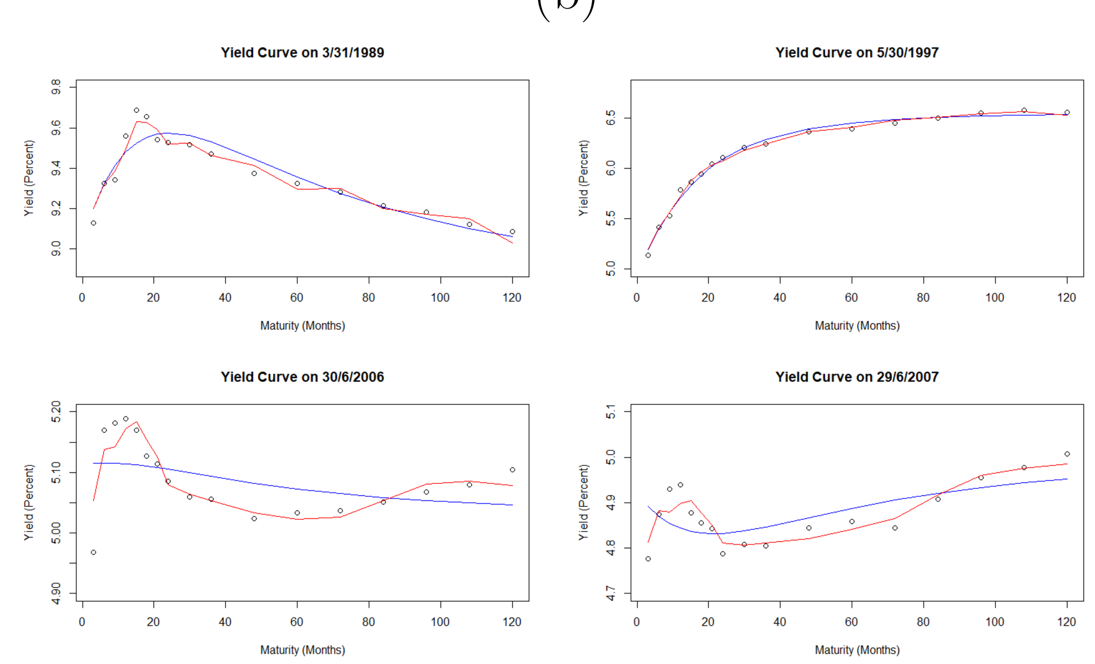


Figure 5:Actual and fitted yield curves.

Factor loadings

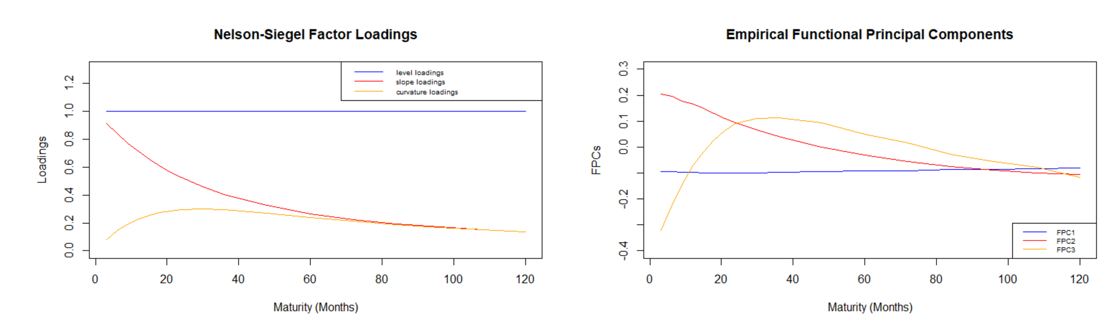


Figure 3:Nelson-Siegel factor loadings and the empirical functional principal components that account for serial dependence (lag order $p = 1$) across yield curves.

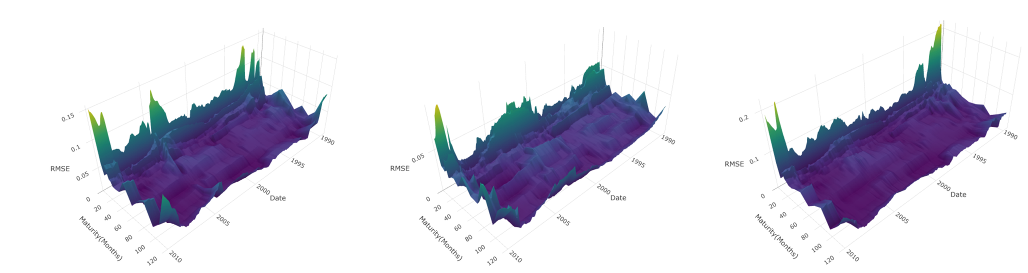


Figure 4:Five years rolling RMSEs, ($p = 1$, $d = 3$), ($p = 1$, $d = 4$), and ($p = 2$, $d = 3$).

Forecasting yield curves

Maturity	Mean	SD	RMSPE	$\hat{\rho}(1)$	$\hat{\rho}(12)$
3	**-0.052**	0.240	0.245	0.202	0.008
6	-0.004	0.217	0.217	0.107	-0.041
9	0.014	0.235	0.235	0.136	-0.016
12	0.016	0.252	0.252	0.079	-0.025
15	0.008	0.263	0.263	0.100	0.009
18	0.009	0.273	0.272	0.081	0.004
21	0.011	0.284	0.283	0.095	-0.002
24	0.013	0.292	0.292	0.101	-0.006
30	0.004	0.295	0.294	0.066	0.007
36	-0.005	0.294	0.294	0.055	0.004
48	-0.007	0.306	0.305	0.066	0.034
60	-0.011	0.295	0.294	0.060	-0.015
72	-0.007	0.295	0.294	0.053	-0.002
84	0.0001	0.281	0.280	0.001	-0.029
96	-0.016	0.281	0.280	-0.006	-0.028
108	-0.006	0.278	0.277	0.015	-0.006
120	-0.001	0.280	0.279	0.023	0.040

Table 1:Out-of-sample 1-month-ahead forecasting results, new functional factor model ($p = 1$, $d = 3$). Bold number indicates a residual mean significantly different from zero at the 5% level of significance.

Forecasting performance

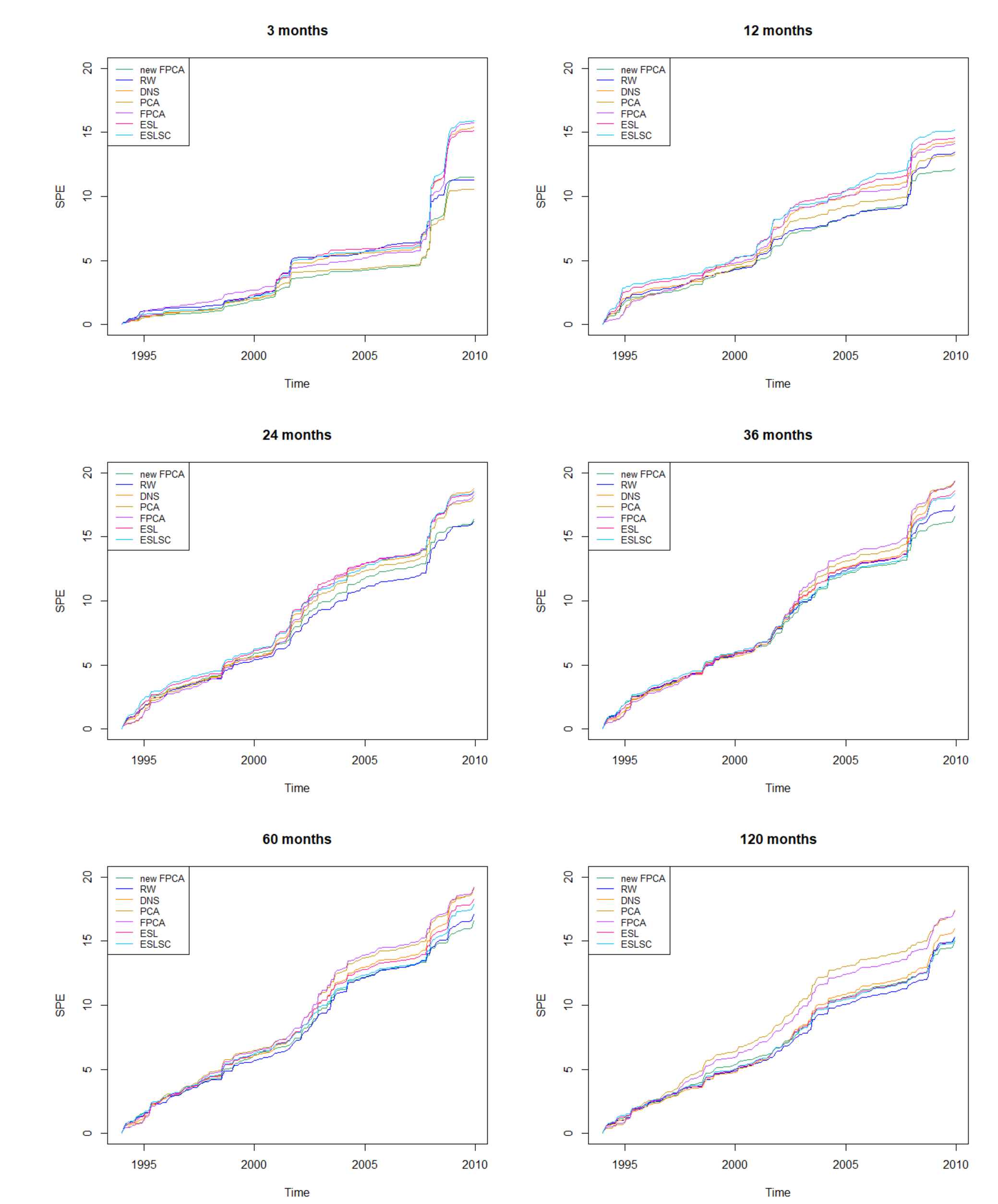


Figure 6:Cumulative sum of Squared Prediction Errors (SPE) of monthly yield data at maturities of 3, 12, 24, 36, 60, 120 months from January 1994 to December 2009.

Conclusions

- This method produces adequate dimension reduction for the serially dependent yield data. (2 PCs here VS 16 PCs from the traditional FPCA).
- Yield curve residuals from this new model's fit exhibit less autocorrelation and have zero mean.
- The forecasts of this new model have superiority:
  (i) less non-zero mean in prediction errors,
  (ii) less autocorrelated prediction errors at different maturities,
  (iii) smaller root mean squared prediction errors (RMSPE)
  over time and across term structure of interest rates at the 1-month-ahead horizon.