Capital Investment and Asset Returns in Dynamic Oligopolies*

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Abstract

We develop a dynamic multi-consumption good, production-based general equilibrium model with an oligopolistic sector to examine the effects of market power on product and asset markets. Equilibrium investment and production strategically moderate effects of aggregate and sectoral shocks on collusive product price with attendant effects on industry equity risk-premium and Sharpe ratio. The model is calibrated with U.S. aggregate and manufacturing industry data. The oligopoly model provides a better fit to product and asset markets' data compared with the benchmark competitive industry. We find support for theoretical predictions on the link between industry competition and product and asset market outcomes.

Keywords: Oligopoly; capital investment; markups; equity risk premium; Sharpe ratio

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1 Introduction

The effects of product market power on asset markets draws increasing interest. In particular, oligopolies are ubiquitous, and the strategic interaction of oligopolistic firms with attendant effects on production capacity and product prices are considered by long-standing theoretical and empirical literatures.[1] In this paper, we argue that the interaction of aggregate and industry production shocks with dynamic strategic behavior of oligopolistic firms can help explain observed product and asset markets phenomena, including some existing empirical puzzles. We examine the effects of oligopolistic collusion on firm-level capital investment and industry product and asset prices in a dynamic production-based, multi-consumption good general equilibrium model with an oligopolistic sector. Fitting the model to U.S. aggregate and manufacturing industry data, we find support for the model's predictions on the relation of industry market structure with the *volatilities* of capital investment, output, and equity returns, as well as the cyclical behavior of markups. In addition, because of higher equilibrium volatility of the multi–good consumption bundle, the model fits asset markets related variables—such as the industry equity risk premium (ERP), Sharpe ratio, and volatility of returns—better than comparable single-good equilibrium models.

We develop an infinite-horizon, two-sector general equilibrium model in an economy with two consumption goods. One of the goods is "produced" in a large competitive sector through an exogenous Markov process (similar to Lucas (1978)). The second good is produced by an oligopolistic sector using capital and materials with a decreasing returns to scale technology. The competitive good can be used for consumption or utilized for productive inputs by the oligopolistic sector, which is also exposed to sector-specific Markov productivity shocks. The empirical distinction between aggregate and sectoral shocks is emphasized by the real business cycle literature (Long and Plosser (1987), Foerster et al. (2011)).

The representative consumer has time separable expected utility of the constant elasticity of substitution (CES) form (Dixit and Stiglitz (1977)). We assume time-additive expected utility to help identify the effects of extending the classical model (Mehra and Prescott (1985)) to include imperfect competition in a multi-consumption good setting. We analyze symmetric subgame perfect equilibrium (SPE) oligopolistic paths with simultaneous clearing of product and asset markets. As

[1] Observations on strategic interaction among oligopolistic firms occur at least as far back as Adam Smith (1776). While the early literature (Cournot (1838)) analyzed static intreactions, the literature in the past few decades focuses on tacit collusion in dynamic oligopolies. Friedman (1983), Bresnahan (1989), Feuerstein (2005), and Green et al. (2013) provide useful surveys of these literatures.

in the dynamic oligopoly literature (Abreu (1986), Rotemberg and Saloner (1986)), industry firms implicitly collude through credibly threatening to punish deviations. We fit the model to production and asset returns data from 456 U.S. manufacturing industries in the NBER-CES database (for 1958-2011) and with the corresponding U.S. aggregate data. To account for heterogeneous industry concentrations in the data, the model is analyzed separately—both theoretically and empirically—for highly and moderately concentrated industries. Based on our theoretical analysis, we simulate dynamic monopoly and dynamic Cournot equilibrium paths for 31 highly concentrated and 425 moderately concentrated industries, respectively. We derive quantitative implications of the model using both log-linear techniques and global solutions that take into account the nonlinearities of the model. To clarify the implications of product market power, these computations are compared with those derived from a benchmark competitive industry.[2] Both solution approaches generate qualitatively similar conclusions.

We find—theoretically and empirically—that the volatilities of capital investment, material inputs, and industry ERP are negatively related to product market power. Intuitively, oligopolistic firms strategically adapt investment and material input demand in response to aggregate or sectoral shocks to moderate their effects on the general equilibrium industry price. Thus, product market power tends to "smooth out" the effects of aggregate and industry shocks on optimal investment, material inputs, and hence dividend payouts compared with competitive firms in identical settings. Consequently, the volatility of industry ERP is also ceteris paribus negatively related to market power. In contrast, the effect of competition on expected ERP—and hence the Sharpe ratio—is theoretically ambiguous because of the indeterminate relation of the stochastic discount factor (SDF) with industry returns. However, our empirical analysis indicates a positive relation of expected ERP and competition, consistent with the literature (Hou and Robinson (2006)). In light of the positive effect of competition on the volatility of ERP, we thus find that competition significantly degrades the industry Sharpe ratio.

As is well known, the canonical equilibrium single-good asset pricing model leads to several empirical puzzles under the "classical" assumptions of time-additive power utility and Markov

[2]In our theoretical model the extent of subgame perfect collusion is determined by industry concentration. But, from an empirical perspective, one can not generally view moderately concentrated industries as more "competitive" than highly concentrated industries. Since these are very different industries, they face different consumer preferences and utilize different production functions. The conceptually valid analysis of the effects of product market power is, therefore, a comparison of equilibrium oligopoly outcomes with those in the *benchmark* perfectly competitive industry, with the same production and consumer preference parameterization.

output shocks, principally because the observed volatility of per capita consumption (and hence the volatility of the SDF) in the U.S. is too low (e.g., Campbell (2000)). However, in our multi-good model, the variability of the SDF is determined by the volatility of the (CES) consumption *bundle*. While computing this volatility empirically for multiple goods is generally challenging—for example, data on per capita consumption on individual products is relatively sparse—we exploit the equilibrium condition in our model that relates the optimal consumption bundle to the real income of the CI (that is, aggregate income divided by the price index). Our calibrated equilibrium computations indicate that the volatility of the consumption bundle, and hence the volatility of SDF and its covariance with asset returns, is significantly higher compared with the benchmark standard consumption CAPM. Consequently, the industry ERP and its volatility, as well as the maximal Sharpe ratios (Hansen and Jagannathan (1991)) are higher—while the equilibrium riskfree rate (Weil (1989)) is lower—than the benchmark model.[3] Moreover, oligopolistic collusion impacts aggregate asset market outcomes because equilibrium consumption is affected by the production and dividend payouts in the oligopolistic sector.

Standing outside the specific assumptions of the model, the more general point made by our study is the empirical importance of consumer preference parameters related to product variety, such as the intra-period elasticity of substitution (ES) amongst consumption goods and oligopolistic collusion on asset market outcomes. The empirical heterogeneity of ERP at the industry level is well known (Fama and French (1997)) and presumably reflects differences in market structure across industries. Our framework distinguishes between aggregate and industry equity returns and helps clarify the relation of industry competition to asset market outcomes.

Meanwhile, in the product market space, the model fits reasonably well the volatilities of investment, material inputs, and output in the data. The cyclical properties of equilibrium investment and material inputs (with respect to aggregate and industry shocks) are also qualitatively consistent with the data. In particular, the model generates procylical investment and material input use with respect to aggregate and sectoral productivity shocks. Because of decreasing returns to scale, short-run marginal costs are procyclical in our model. In a competitive setting, this would result in countercyclical price-marginal cost (or markup) ratios (Bils (1987)). Meanwhile, in their analysis of price-collusion in oligopolistic supergames, Rotemberg and Saloner (1986) argue that subgame perfect collusion requires countercyclical markups because the gains from defection are

[3] The role of heterogeneous consumption goods in helping explain the equity premium puzzle is also noted in the literature in a different setting (Ait-Sahalia et al. (2004)).

higher in booms. But due to persistent shocks and endogenous investment, gains from defection are not necessarily procylical in our model. Our theoretical framework implies, however, that the procyclicalty of markups is negatively related to industry competition. Consistent with this prediction, the empirical analysis finds countercyclical markups in highly concentrated oligopolies, but procyclical markups in moderately concentrated industries. These results shed light on the mixed empirical results in the literature with respect to the cyclicality of markups (Nekarda and Ramey (2010)).

Overall, the oligopoly model is generally a better fit than the competitive benchmark industry in both the product market space—where the latter generates excessive investment volatility and counterfactual time-invariant markup ratios—and the asset markets space—in terms of industry Sharpe ratios. In sum, modeling industry market structures in multi-good general equilibrium models may help explain important product and asset markets phenomena.

Our study is related to, but distinct from, several strands of the literature. For example, the macroeconomic literature develops dynamic equilibrium models to examine the effects of industry level imperfect competition on transmitting the effects of aggregate shocks on real variables (Rotemberg and Woodford (1992)). In a related vein, the 'New Keynesian' literature incorporates imperfect competition and frictions (such as nominal rigidities) to analyze the effects of monetary policy (Gali (2008)). This literature largely abstracts from equilibrium asset pricing implications, however. And papers on price collusion in repeated oligopoly (Abreu (1986), Rotemberg and Saloner (1986)) do not consider dynamic investment and asset pricing. Meanwhile, Opp et al. (2014) examine price-cost markups in a general equilibrium model but do not focus on asset pricing moments.

Our paper is also linked to the growing recent finance literature that examines the relation of product market competition and equity returns. One strand of this literature analyzes general equilibrium models with product market power modeled through monopolistic competition to help explain priced risk factors in the cross-section (Carlson et al. (2004), Van Binsbergen (2016), Dou et al. (2019), Barrot et al. (2019), Loualiche (2019)). Garlappi and Song (2017) show that flexible capital utilization and market power in monopolistically competitive industries with persistent technological growth and recursive preferences can help explain the observed market ERP. There is also a recent literature that examines the relation of Cournot competition with expected returns (Bustamante and Donangelo (2017), Corhay et al. (2020)). But, to our knowledge, the empirical links between endogenous market power through sustainable oligopolistic collusion and the volatility of returns and Sharpe ratios in dynamic general equilibrium settings highlighted in our analysis are novel. Our framework also differs from the literature in highlighting the theoretically ambiguous relation of industry competition with expected industry ERP and Sharpe ratios.

In the rest of the paper, Sections 2 through 4 describe the model and characterize equilibrium investment, production, and pricing paths. Section 5 specifies the methodology for numerical computations of equilibrium paths. Section 6 describes the data and empirical measures. Sections 7 and 8 present the empirical results, and Section 9 concludes.[4]

2 The Model

2.1 Firms and Industry Structure

There are two sectors in the economy, specializing in the production of non-storable goods x and y.[5] For simplicity, output in sector x is modeled as an exogenous stochastic process $\{X_t\}_{t=0}^{\infty}$ that is sold competitively. This good also serves as the numeraire and its price (p^x) is normalized to unity each period. It is convenient to consider a representative firm that sells X_t at unit price each period. Finally, good x can be either consumed or used to facilitate production in the other sector that is described next.

The second sector is an oligopoly with N firms (labeled $i = 1, ..., N$), who produce an identical good y. All firms utilize an identical production technology that stochastically converts their beginning-of-the-period capital (K_{it}) and material input chosen during the period (H_{it}) to output Y_{it} through the production function

$$F(K_{it}, H_{it}, \theta_t) = \theta_t (K_{it})^{\psi_K} (H_{it})^{\psi_H}, i = 1, ...N. \tag{1}$$

Here, θ_t represents the stochastically evolving industry-wide productivity level and $0 < \psi_K < 1$, $0 < \psi_H < 1$ are the output elasticities of capital and material inputs, respectively; that is, there are decreasing returns to scale with respect to each productive input. The industry output at t is given by $\tilde{Y}_t = \sum_{i=1}^{N} Y_{it}$. Firms use x for their material input. For notational ease, let x be directly converted to material input so that H_{it} also represents the total material cost at t.

[4] Proofs and computational details are collected in an Appendix.

[5] We will identify these as sectors x and y, respectively, and use capital letters to denote their outputs.

To introduce general equilibrium effects of sectoral investment in a tractable way, we assume that production in sector y uses x for investment. There is a cost of converting x to investment, however. Letting I_{it} denote the investment by firm i at t, the investment cost function is[6]

$$A(I_{it}, K_{it}) = I_{it} + 0.5\upsilon \left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}. \tag{2}$$

Conditional on I_{it}, the firms capital accumulation process is given by

$$K_{it+1} = (1-\delta)K_{it} + I_{it}, K_{i0} = \bar{K}_{i0}, \tag{3}$$

where δ is the per-period depreciation rate (that is common for all firms in the sector) and the initial capital stocks are pre-specified.

The output in sectors x and y evolve according to correlated and persistent lognormal processes

$$\log X_t = \rho_x \log X_{t-1} + \varepsilon_t^x; \log \theta_t = \rho_\theta \log \theta_{t-1} + \varepsilon_t^\theta, \tag{4}$$

where, for $j \in \{X, \theta\}$, $0 \leq \rho_j \leq 1$ are the autocorrelation parameters and, ε_t^j are conditionally bivariate normal mean zero variables with the variance-covariance matrix $\Lambda = [\lambda_{ij}]$.

All firms in the model are unlevered and publicly owned, with their equity being traded in frictionless security markets. The number of shares outstanding at the beginning of t is denoted by Q_t^x and $Q_{it}^y, i = 1, ...N$. Because the net revenue of sector x at t is X_t, its dividend payout is $D_t^x = X_t$. Given the "Lucas tree" structure of this sector, we fix the number of outstanding shares to unity (that is, $Q_t^x \equiv 1$). Meanwhile, the dividends of firms in sector y are

$$D_{it}^y = p_t^y Y_{it} - H_{it} - A(I_{it}, K_{it}), i = 1, ..., N. \tag{5}$$

Dividends can be negative, financed by equity issuance. In the absence of taxes and transactions costs, negative dividends are equivalent to the market value of new equity share issuance. Thus, if $D_{it}^y < 0$, then $D_{it}^y = S_{it}^y(Q_{it+1}^y - Q_{it}^y)$ is the implied cash inflow from new equity issuance.

[6] This investment cost function can also be interpreted in terms of capital adjustment costs (see Abel and Eberly (1994)). Here, the quadratic parameterization conforms to strictly convex adjustment costs (Summers (1981), Cooper and Haltiwanger (1996)) and is useful for interior optima used in the numerical simulations.

2.2 Consumers

There is a continuum of identical consumers in the economy. The representative consumer-investor (CI) maximizes expected discounted time-additive utility of random consumption streams of the two goods subject to period-by-period budget constraints. The CI also has access every period to a (one-period) risk-free security (f) that pays a unit of the numeraire good next period. The mass of risk-free securities is also fixed at unity. The profile of securities outstanding at t is thus $\mathbf{Q}_t = (Q_t^x, Q_1^y, .., Q_N^y, Q_t^f)$.

Thus, in each t, the CI chooses the consumption vector $\mathbf{c}_t = (c_t^x, c_t^y)$ taking as given product prices $\mathbf{p}_t = (1, p_t^y)$. The portfolio of asset holdings at the beginning of the period is $\mathbf{q}_t = (q_t^x, q_1^y, .., q_N^y, q_t^f)$. Along with consumption, the CI simultaneously chooses the new asset holdings $\mathbf{q}_{t+1}$, taking as given the corresponding asset prices $\mathbf{S}_t = (S_t^x, S_1^y, .., S_N^y, S_t^f)$. For simplicity, there is no other endowment or labor income. Hence, the CI is subject to a wealth constraint determined by the dividend payouts $\mathbf{D}_t = (X_t, D_1^y, .., D_N^y, 1)$. More precisely, let Z_t be the wealth net of new asset purchases during the period—that is, the consumers disposable income available for consumption. Then, the CI's optimization problem is

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma} \right], \gamma \quad \geq \quad 0, \beta < 1, \tag{6}$$

$$\text{s.t., } \mathbf{p}_t \cdot \mathbf{c}_t \quad \leq \quad \mathbf{q}_t \cdot (\mathbf{D}_t + \mathbf{S}_t) - \mathbf{q}_{t+1} \cdot \mathbf{S}_t \equiv Z_t, \;\; \mathbf{c}_t \geq 0. \tag{7}$$

In (6), γ determines the representative CI's degree of risk aversion; β is the subjective discount factor; and $C_t \equiv C(\mathbf{c}_t)$ is an aggregated consumption index with constant elasticity of substitution (CES) between the consumption of the two goods:

$$C(\mathbf{c}_t) = \left[(1-\phi)(c_t^x)^{(\sigma-1)/\sigma} + \phi(c_t^y)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}. \tag{8}$$

Here, $\sigma > 1$ is the ES and $0 < \phi < 1$ is a pre-specified consumption weight for good y. Because preferences are strictly increasing, the budget constraint (7) will be binding in any optimum and hence Z_t also represents the total consumption expenditure at t. It follows from (7) that, as long as the asset markets clear, the disposable income is $Z_t = D_t^x + \sum_{i=1}^N D_{it}^y + 1$.

The optimal consumption demand functions (superscripted by '*') for the optimization problem

(6)-(7) are multiplicatively separable in Z_t and $\mathbf{p}_t$ (see Appendix A)

$$c_t^{j*}(\mathbf{p}_t, Z_t) = \frac{Z_t}{P_t}\left[\frac{P_t\phi^j}{p_t^j}\right]^\sigma, \; j = x, y, \tag{9}$$

where $p_t^x = 1$, $\phi^x \equiv (1-\phi), \phi^y \equiv \phi$, and $P_t \equiv P(\mathbf{p}_t)$ is the aggregate price index

$$P(\mathbf{p}_t) = \left[(1-\phi)^\sigma + (\phi)^\sigma (p_t^y)^{1-\sigma}\right]^{1/(1-\sigma)}. \tag{10}$$

At the optimum, the aggregate real consumption $C_t^* \equiv C(\mathbf{c}_t^*) = \frac{Z_t}{P_t}$, which is the real income.

2.3 Asset Markets

The equilibrium asset price vector can be derived from the representative CI's optimal portfolio condition (see Appendix A), namely,

$$\mathbf{S}_t = \mathbb{E}_t\left[\beta\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}(\mathbf{D}_{t+1}+\mathbf{S}_{t+1})\right]. \tag{11}$$

As is standard, the pricing kernel (or the SDF) for future equity payoffs is defined in terms of the intertemporal marginal rate of substitution of real consumption (IMRS). Since $C_t = \frac{Z_t}{P_t}$, the SDF is given by $M_{t+1} \equiv \beta\left(\frac{Z_{t+1}}{Z_t}\right)^{-\gamma}\left(\frac{P_{t+1}}{P_t}\right)^{\gamma-1}$. Equation (11) thus becomes $\mathbf{S}_t = \mathbb{E}_t\left[M_{t+1}(\mathbf{D}_{t+1}+\mathbf{S}_{t+1})\right]$. In terms of the gross returns $R_{t+1}^j = (D_{t+1}^j + S_{t+1}^j)/S_t^j$ (with $R_{t+1}^f = 1/S_t^f$), the asset market equilibrium condition can be written in the standard form as $\mathbf{1} = \mathbb{E}_t\left[M_{t+1}\mathbf{R}_{t+1}\right]$, where $\mathbf{1}$ is the four-dimensional unit column vector and $\mathbf{R}_t = (R_t^x, R_1^y, .., R_N^y, R_t^f)'$.

2.4 Information and Timing Assumptions

At the beginning of t, the aggregate and sectoral shocks (X_t, θ_t) are realized and are commonly observable by all agents. Firms then simultaneously choose their *actions:* prices $\mathbf{p}_t^y = (p_{1t}^y, .., p_{Nt}^y)$, investments $\mathbf{I}_t = (I_{1t}, .., I_{Nt})$, and material inputs $\mathbf{H}_t = (H_{1t}, .., H_{Nt})$. These actions will be denoted by $\boldsymbol{\mu}_t^y = (\mathbf{p}_t^y, \mathbf{I}_t, \mathbf{H}_t)$.

Conditional on observed history, (X_t, θ_t), and current actions by firms $\boldsymbol{\mu}_t^y$, the CI determines its consumption and portfolio choices according to (6)-(7) based on anticipated dividends. Since all firms produce an identical good, the lowest-price firm sells as much of the quantity demanded (see (9)) at its quoted price that is allowed by its production capacity $F(\theta_t, K_{it}, H_{it})$. The firm with the next higher price then sells as much as it can at its quoted price and so forth.[7] Firms' actions, along with the quantities sold, then determine their dividends according to (5). At the end of the period, the actions of all agents become commonly observable. In particular, the prices and quantities sold by each firm, along with the new capital stock distribution (given by (3)) are commonly observable. The asset markets variables, namely, dividends, security prices, and any new equity issuances are also observable. At each t, the history $\boldsymbol{\omega}^t$ is the profile of all actions up to the end of $t-1$ and is common knowledge.

3 Subgame Perfect Equilibrium Paths

3.1 Equilibrium Definition

The model specified above defines a multi-stage game with perfectly observed actions (Fudenberg and Tirole (1991)). The information set at beginning of each t is the history of prior moves and the current realization of the shocks, denoted by the *state* $\Gamma_t = (\boldsymbol{\omega}^t, X_t, \theta_t)$. Under our informational assumptions, each Γ_t defines a subgame. A subgame perfect equilibrium (SPE) specifies state-contingent strategies for each firm $\mu_{it}^{y*}(\Gamma_t)$ that maximize the present discounted value of real dividends $(\frac{D_{it}^y}{P_t}, ...)$. In general, there will not exist complete contingent markets in this model; hence, the discount rate is given by the representative consumer's marginal utility of real consumption (Brock (1982), Horvath (2000)). Thus, in every subgame Γ_τ, $\tau \geq 0$, firm i chooses $\mu_{i\tau}^y$ to maximize the conditional present value of real dividends given by

$$\mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} \beta^{t-\tau} \left(\frac{Z_t}{P_t} \right)^{-\gamma} \left(\frac{p_{it}^y Y_{it} - H_{it} - A(I_{it}, K_{it})}{P_t} \right) \mid \Gamma_\tau \right], \text{ s.t., (1)–(3).} \tag{12}$$

As is typical in the oligopoly literature, we will focus on symmetric equilibrium paths where all firms adopt the same strategies, that is, $\mu_{it}^{y*}(\Gamma_t) = \mu_t^{y*}(\Gamma_t)$. It follows from (1)-(3) that all firms also have symmetric capital stocks investments K_t and output Y_t and hence the industry capacity and output are $\tilde{K}_t = NK_t$ and $\tilde{Y}_t = NY_t$, respectively. The dividends per firm are also symmetric and the industry dividends are $\tilde{D}_t^y = ND_t^y$. Furthermore, we will assume realistically that the effects of actions by firms in sector y on the aggregate consumption C and the aggregate price index P are of order small. This implies that firms take the pricing kernel (M) as a given, which is a reasonable

[7] These rules of demand allocation are intuitive and consistent with those adopted by the oligopolistic supergame literature (Abreu (1986), Rotemberg and Saloner (1986)).

assumption. This convention also allows one to treat X_t as a proxy for aggregate output, which will be useful for the empirical interpretation of the results.

Conditional on the state and actions (Γ_t, μ_t^{y*}), the CI can perfectly anticipate the equilibrium industry dividends $\tilde{D}_t^{*y}$ and hence the disposable income Z_t^*.[8] The CI then chooses the consumption and asset demand vectors $(\mathbf{c}_t^*, \mathbf{q}_{t+1}^*)$ to solve the constrained optimization problem (6)-(7) such that the product and asset price vectors $(\mathbf{p}_t^*, \mathbf{S}_t^*)$ clear both the asset and product markets, that is $\mathbf{q}_{t+1}^* = \mathbf{Q}_{t+1}$ and

$$c_t^{x*}(\mathbf{p}_t^*, Z_t^*) + N[A(I_t^*, K_t) + H_t^*)] \quad = \quad X_t, \tag{13}$$

$$c_t^{y*}(1, p_t^{y*}, Z_t^*) \quad = \quad \tilde{Y}_t. \tag{14}$$

A time-profile of firms' strategies $\left\{\mu_t^{y*}\right\}_{t=0}^{\infty}$ and CI's consumption and portfolio policies $\{\mathbf{c}_t^*, \mathbf{q}_t^*\}_{t=0}^{\infty}$ is an SPE if and only if for every Γ_t (1) they satisfy the conditions annunciated above *and* (2) if no firm i can gain from deviating from μ_t^{y*} to an alternative $(\hat{p}_{it}^y, \hat{H}_{it}, \hat{I}_{it})$ and choosing $\mu_{t+1}^{y*}(\Gamma_{t+1})$ thereafter. The latter requirement is the well known "one-stage-deviation principle" of checking for subgame perfection (Fudenberg and Tirole, page 109).[9] Using the Bellman representation of (12), it follows that along the symmetric SPE path, firm value can be recursively computed as

$$V_t^*(\Gamma_t) = \left(\frac{Z_t}{P_t}\right)^{-\gamma} \left(\frac{1}{P_t}\right) [p_t^{y*} F(K_t, H_t^*, \theta_t) - (H_t^* + A(I_t^*, K_t)] + \beta \mathbb{E}_t \left[V_{t+1}^*(\Gamma_{t+1}^*)\right], \tag{15}$$

where $\Gamma_{t+1}^* = ((\boldsymbol{\omega}^t, \mu_t^{y*}), K_{t+1}^*, X_{t+1}, \theta_{t+1})$ and $K_{t+1}^* = (1-\delta)K_t + I_t^*$. Because quantities sold by firms are determined by the demand function (9)), there are consistency constraints on firms' actions. In particular, firms may choose prices and investment and the optimal input demand will then be determined by the output sold (since (θ_t, K_{it}) are pre-determined). Alternatively, firms may choose quantities and choose their inputs and investment and the equilibrium industry price p_t^{y*} will be determined by (13)-(14).

[8] Note that there is no uncertainty *within* any period on firms' outputs, investments, and input choices conditional on (Γ_t, μ_t^{y*}).

[9] Although this principle applies generally only to finite games, we can show that it also holds for our infinite-horizon game because of discounting and continuity of agents' payoffs in the limit.

3.2 Benchmark Competitive Equilibrium

Prior to the analysis of oligopolistic equilibria, it is useful to set up a benchmark where sector y is competitive, which facilitates intuition on the effect of product market power. Along the competitive equilibrium path, all firms take prices as given and equate them to marginal cost. For notational ease, we will write $\eta \equiv \phi/(1-\phi)$, the partial derivatives of the investment cost function as $A_I(I,K) \equiv 1 + \upsilon(I/K)$, $A_K(I,K) \equiv -0.5\upsilon(I/K)^2$, and the net market supply of good x as $W_t \equiv X_t - N[A(I_t,K_t) + H_t]$. We will denote the (symmetric) competitive equilibrium strategies by $\breve{\mu}_t^y = (\breve{p}_t^y, \breve{H}_t, \breve{I}_t)$, with $\breve{Y}_t = F(K_t, \breve{H}_t, \theta_t)$ and $\breve{K}_{t+1} = (1-\delta)K_t + \breve{I}_t$.

Proposition 1 *Along a symmetric competitive equilibrium path, for any Γ_t, $\breve{\mu}_t^y$ is characterized by*

$$\breve{p}_t^y = \left(\frac{\breve{W}_t}{N\breve{Y}_t}\right)^{1/\sigma} \eta = [F_H(K_t, \breve{H}_t, \theta_t)]^{-1}, \tag{16}$$

$$\begin{aligned} A_I(\breve{I}_t, K_t) = \mathbb{E}_t \Big[\breve{M}_{t+1} \Big(\breve{p}_{t+1}^y F_K(\breve{K}_{t+1}, \breve{H}_{t+1}, \theta_{t+1}) - A_K(\breve{I}_{t+1}, \breve{K}_{t+1}) \; + \\ (1-\delta) A_I(\breve{I}_{t+1}, \breve{K}_{t+1})\Big)\Big]. \end{aligned} \tag{17}$$

And the asset market equilibrium satisfies (11).

Equation (16) reflects the competitive equilibrium pricing condition where prices clear markets in both sectors and industry price equals the marginal cost. In a general equilibrium, the relative price of y (in terms of the numeraire), p^y, should be decreasing with the supply of y relative to that of x. And the sensitivity of p^y to this relative supply should be increasing (in algebraic terms) with the ES. Furthermore, ceteris paribus, p^y should be positively related to the weight of good y in the consumer's utility function, ϕ. These properties are satisfied by the equilibrium price function in the left hand side (LHS) of (16) since the net supply of goods y and x are $N\breve{Y}_t$ and $\breve{W}_t$, respectively. Furthermore, η is increasing with the consumer's utility weight of y, namely, ϕ. Meanwhile, the right hand side (RHS) of (16) represents the marginal cost since in our model the only variable input in any period is H. Hence, standard cost minimization implies that the marginal cost is the inverse of the marginal productivity of material inputs. Finally, (17) is the Euler condition with respect to investment that trades of current marginal cost of investment—represented by the LHS—with the discounted expected marginal value of current investment. We now turn to characterization of SPE paths when sector y is an oligopoly.

4 Equilibrium Characterization

Kreps and Scheinkman (1983) show that in the two stage game, with capacity choices chosen in first stage and price competition in the second stage, there is a unique pure strategy symmetric equilibrium with identical capacity choices in the first stage and identical prices in the second stage. Of course, in an infinite horizon model there is a possibility of multiple non-stationary equilibria. Indeed, the folk theorem of dynamic oligopoly (Friedman (1971), Fudenberg and Maskin (1986)) implies that various levels of collusion are possible for sufficiently high discount factors of shareholders through subgame perfect threats of "punishment phases" of low prices. In general, the discounted expected profits of firms along the equilibrium path are positively related to profit losses that can be credibly inflicted in "off equilibrium path" punishment phases. The *optimal* (or extremal) SPE achieves the maximum discounted expected utility along the equilibrium path through this mechanism (Abreu (1986), Rotemberg and Woodford (1992)).

But it is also well known that enforceable collusion is negatively related to the number of firms in the industry (Friedman (1971), Rotemberg and Saloner (1986)). Hence, for the purposes of empirical analysis, it is useful to distinguish between *highly concentrated* oligopolies—where a small number of firms account for the preponderant share of the industry output—and the remaining oligopolies, which for expositional convenience will be labeled as *moderately concentrated* oligopolies. This distinction allows us to focus on empirically identified monopoly and quantity-setting (or Cournot) SPE outcome paths for the two market structures.

4.1 Highly Concentrated Oligopolies

As we mentioned above, the optimal SPE path in the oligopolistic market structure will be enforced by maximal punishment "off-equilibrium" paths following deviation by any firm. Clearly, the maximal punishment path can not involve negative profits. We now establish that in every subgame (Γ_t), a symmetric strategy where all firms do average cost pricing that clears both sectoral markets with zero investment is a SPE.

Lemma 1 *In any subgame Γ_τ ($\tau \geq 0$), the symmetric action profile $\{\mu_t^{y\prime} = (\underline{p}_t^y, \underline{H}_t, \underline{I}_t)\}_{t\geq\tau}$ with*

$\underline{I}_t = 0$ *and* $(\underline{p}^y_t, \underline{H}_t)$ *determined by*

$$\eta X_t = N[\underline{H}_t + (F(K_t, \underline{H}_t, \theta_t))^{1-\sigma}(\underline{H}_t)^{\sigma}], \tag{18}$$

$$\underline{p}^y_t = \left(\frac{X_t - N\underline{H}_t}{NF(K_t, \underline{H}_t, \theta_t)}\right)^{\frac{1}{\sigma}} \eta, \tag{19}$$

is a SPE.

By construction, $\mu^{y\prime}_t$ implies that all firms have zero cash flows in t because revenues just cover input costs by construction. Note that no firm gains from deviation either in terms of prices and/or input and investment choices. If a firm raises only its price, it makes negative profits. But if the firm unilaterally cuts its price, it can only serve the additional market share by increasing inputs. But this additional cost cannot be recovered because marginal input costs are increasing due to decreasing returns to scale (see Abbink and Brandts (2008)).

With the credibility of zero firm value for all $\tau \geq t+1$ following a defection from a prescribed strategy in hand, we can focus on *collusive* SPE paths in highly concentrated oligopolies where the (state-contingent) optimal monopoly policies are followed. Now, it is well known that in the static monopoly model, the price and quantity choice solutions are identical. The following result establishes that this is true as well in our dynamic general equilibrium setting. We will denote by $\tilde{\mu}^{y*}_{t,p}$ the strategy whereby the monopolist chooses $(\tilde{p}^{y*}_t, \tilde{I}^*_t)$ to solve (12) and then $\tilde{H}^*_t$ is determined by inverting the production function based on the equilibrium product demand $c^{y*}_t(\tilde{\mathbf{p}}^*_t, Z^*_t)$ (where $\tilde{\mathbf{p}}^*_t = (1, \tilde{p}^{y*}_t)$. Analogously, $\tilde{\mu}^{y*}_{t,H}$ denotes the strategy whereby the monopolist chooses $(\tilde{H}^*_t, \tilde{I}^*_t)$ and $\tilde{p}^{y*}_t$ is then determined by (13)-(14).

Lemma 2 *For each* t, *and any* Γ_t, $\tilde{\mu}^{y*}_{t,p}$ *and* $\tilde{\mu}^{y*}_{t,H}$ *result in the same firm value* $V^*_t(\Gamma_t)$.

Using Lemma 1 and 2, we now characterize the symmetric optimal SPE that enforces the monopoly (or "collusive") outcome when the number of firms is sufficiently small.[10]

Proposition 2 *For any* t *and given any state* Γ_t, *the optimal policies of a monopolist* $\tilde{\mu}^{y*}_{t,H}(\Gamma_t)$ *with a capital stock* $\tilde{K}_t$ *are determined by the following set of conditions*

[10] In the following, we recall the notation $\eta \equiv \phi/(1-\phi)$, $A_I(I,K) \equiv 1 + \upsilon(I/K)$, $A_K(I,K) \equiv -0.5\upsilon(I/K)^2$.

$$\tilde{p}_t^{y*} = \left[\frac{X_t - [A(\tilde{I}_t^*, \tilde{K}_t) + \tilde{H}_t^*]}{F(\tilde{K}_t, \tilde{H}_t^*, \theta_t)}\right]^{1/\sigma} \eta, \tag{20}$$

$$\frac{\sigma + (\tilde{p}_t^{y*})^{1-\sigma}\eta^\sigma}{(\sigma-1)} = \tilde{p}_t^{y*} F_H(\tilde{K}_t, \tilde{H}_t^*, \theta_t), \tag{21}$$

$$-\frac{\partial \tilde{D}_t^{y*}}{\partial I_t} = \mathbb{E}_t\left[M_{t+1}^*\left(\frac{\partial \tilde{D}_{t+1}^{y*}}{\partial K_{t+1}} - (1-\delta)\frac{\partial \tilde{D}_{t+1}^{y*}}{\partial I_{t+1}}\right)\right] \tag{22}$$

where $\tilde{D}_t^{y*} = \tilde{p}_t^{y*} F(\tilde{K}_t, \tilde{H}_t^*, \theta_t) - [A(\tilde{I}_t^*, \tilde{K}_t) + \tilde{H}_t^*]$ *and*

$$\frac{\partial \tilde{D}_t^{y*}}{\partial I_t} = -A_I(\tilde{I}_t^*, \tilde{K}_t)\left[1 + \frac{(\tilde{p}_t^{y*})^{1-\sigma}\eta^\sigma}{\sigma}\right], \tag{23}$$

$$\frac{\partial \tilde{D}_t^{y*}}{\partial K_t} = \tilde{p}_t^{y*} F_K(\tilde{K}_t, \tilde{H}_t^*, \theta_t)\left(\frac{\sigma-1}{\sigma}\right) - A_K(\tilde{I}_t^*, \tilde{K}_t). \tag{24}$$

Because of the joint market clearing requirement (see (13)-(14)), the general equilibrium price (20) has the same form as the competitive price function (16). However, Equations (21)-(24) reflect the impact of product market power on firms' input and investment choices. Specifically, (21) is the optimal input choice of a monopolist that incorporates the marginal effect of increased output (due to higher input choice) on the price. Compared with the optimal input condition in the competitive market (see (16)), we see the deviation from marginal-cost "pricing" since the price-to-marginal cost ratio (*pmcr*)—that is, $p_t^y F_H(K_t, H_t, \theta_t)$—in (21) deviates significantly from 1.

Similarly, (22) is the Euler condition for optimal investment in monopoly that takes into account its effect on the price. In particular, at the margin, investment has two effects on dividends, D_t^y. The LHS is the marginal cost of current investment, which is specified in (23). There is a direct marginal cost, given by the first term, because of the one-to-one reduction in the dividend for a given increase in investment costs. This is familiar from the existing literature (Love (2003)). But, because of market power, the monopoly investment policy recognizes the negative effect of its investment on price, given by the second term in (23).[11] Meanwhile, the RHS of (22) represents the discounted expected marginal value of current investment in a monopoly. The first term represents the discounted expected marginal effect of higher capital on dividends next period, which is recur-

[11] We note that this investment Euler condition modifies the corresponding optimality condition in the competitive benchmark (see (17)) to incorporate the price effect of investment due to market power. In particular, the left hand side of this condition is equivalent to $A_I(\tilde{I}_t^*, \tilde{K}_t)\left[1 + \frac{(\tilde{p}_t^{y*})^{1-\sigma}\eta^\sigma}{\sigma}\right]$, which reflects both the direct investment marginal cost and indirect marginal cost from the effect on product price.

sively given by (24). Since $\left(\frac{\sigma-1}{\sigma}\right) < 1$, a comparison with (17) indicates that market power ceteris paribus reduces future gain from investment because of its negative impact on *next* period's price (through the marginal productivity of capital). Hence, optimal investment is lower with imperfect competition, other things being equal, and this difference reaches the maximum along the collusive policy. The second and third terms in (22) are qualitatively similar to that in (17).

Propositions 1 and 2 suggest that product market power will also affect the second moments of equilibrium investment and material input choice. Along the collusive equilibrium path, oligopolistic firms strategically incorporate the effects of investment and input use on prices, which tends to "smooth out" the effects of aggregate and industry shocks on optimal factor demands, as mentioned already. We now show that the collusive policies in Proposition 2 can be implemented as a symmetric SPE for sufficiently high industry concentration.

Proposition 3 *There exists $\bar{N}$ such that if the number of firms $N \leq \bar{N}$, then along the optimal symmetric SPE, at every Γ_t, each firm chooses actions $\bar{\mu}_t^{y*}(\Gamma_t) = (\bar{p}_t^{y*}, \bar{H}_t^*, \bar{I}_t^*)$, which have the following relation to the optimal monopoly policies actions specified in Proposition 2: $\bar{p}_t^{y*} = \tilde{p}_t^{y*}$, $\bar{H}_t^* = \left(N^{\frac{\psi_K - 1}{\psi_H}}\right)\tilde{H}_t^*$, $\bar{I}_t^* = N^{-1}\tilde{I}_t^*$. Furthermore, if at any t, any firm deviates from $\bar{\mu}_t^{y*}(\Gamma_t)$, then all firms play $\mu_\tau^{y\prime}(\Gamma_\tau), \tau \geq t+1$.*

With the symmetric firm-level investment and input choices specified in Proposition 3, the industry output at each Γ_t is the optimal monopoly output $\tilde{Y}_t^* = F(\tilde{K}_t, \tilde{H}_t^*, \theta_t)$, and the industry capital stock next period $\tilde{K}_{t+1} = \tilde{K}_t(1-\delta) + \tilde{I}_t^*$ is also the optimal choice by a monopolist. But for $\left\{\bar{\mu}_t^{y*}\right\}_{t=0}^{\infty}$ to be an SPE, it must be true that for each t and any Γ_t, no firm should strictly benefit from deviating. While the details are provided in the Appendix, it is useful to present the basic argument here. Because $\bar{\mu}_t^{y*}(\Gamma_t)$ generates the maximal or monopoly profits at t, the optimal deviation strategy is to charge a price $\bar{p}_t^{y*} - \varepsilon$ for some ε arbitrarily small, serve the entire market by using the requisite inputs. Since the capital stock of each firm is K_t, the inputs required to produce the equilibrium market output $\tilde{Y}_t^*$ are (using a straightforward inversion of the production function)

$$\bar{H}_t^{\prime}(\tilde{Y}_t^*; (K_t, \theta_t)) = \left(\frac{\tilde{Y}_t^*}{\theta_t (K_t)^{\psi_K}}\right)^{\frac{1}{\psi_H}}. \tag{25}$$

Because it is common knowledge that deviation will be followed by zero economic profits in the future, optimal deviation requires investment to be set to zero. Thus, the current payoffs (or dividends) from the optimal deviation strategy are no greater than $\bar{D}_t^{y^\prime} = \bar{p}_t^{y*}\tilde{Y}_t^* - \bar{H}_t^\prime$. Therefore, no firm gains from deviation if

$$\left(\frac{Z_t}{P_t}\right)^{-\gamma}\left(\frac{1}{P_t}\right)\bar{D}_t^{y'} \leq \bar{V}_t^*(\Gamma_t), \tag{26}$$

where the RHS of (26) represents the value loss from punishment following the deviation. Clearly, (26) holds strictly when $N = 1$. But $\bar{D}_t^{y'}$ can be shown to be increasing in N and hence there exists some $N(\Gamma_t)$ such that deviations are not optimal for $N \leq N(\Gamma_t)$. The statement of the Proposition then follows by defining $\bar{N}$ as the infimum of $N(\Gamma_t)$ over the space of feasible states. Conversely, the collusive path is not a SPE for all states if N is exceeds $\bar{N}$. We will identify this case with moderately concentrated oligopolies, and consider it next.

4.2 Moderately Concentrated Oligopolies

For empirical testing, we need a focal SPE path for low concentration oligopolies that can be checked against deviations. A natural candidate is a symmetric dynamic Cournot-type equilibrium path where firms optimally choose their material inputs and investment, taking as given the input and investment choices of rival firms. Our next result specifies this SPE path.

Proposition 4 *There exists $N^* > \bar{N}$ such that if the number of firms $\bar{N} < N \leq N^*$, then along the optimal symmetric SPE, at each Γ_t, all firms in the industry choose actions $\mu_t^{y*} = (p_t^{y*}, H_t^*, I_t^*)$ determined by the following set of conditions.*

$$\begin{aligned}
p_t^{y*} &= \left(\frac{W_t^*}{NY_t^*}\right)^{1/\sigma}\eta, && (27)\\
\frac{N\sigma + (p_t^{y*})^{1-\sigma}\eta^{\sigma}}{(N\sigma - 1)} &= p_t^{y*}F_H(K_t, H_t^*, \theta_t), && (28)\\
-\frac{\partial D_t^{y*}}{\partial I_t} &= \mathbb{E}_t\left[M_{t+1}^*\left(\frac{\partial D_{t+1}^{y*}}{\partial K_{t+1}} - (1-\delta)\frac{\partial D_{t+1}^{y*}}{\partial I_{t+1}}\right)\right], && (29)
\end{aligned}$$

where $D_t^{y*} = p_t^{y*}F(K_t, H_t^*, \theta_t) - (H_t^* + A(I_t^*, K_t))$ and

$$\begin{aligned}
\frac{\partial D_t^{y*}}{\partial I_t} &= -A_I(I_t^*, K_t)\left[1 + \frac{(p_t^{y*})^{1-\sigma}\eta^{\sigma}}{N\sigma}\right], && (30)\\
\frac{\partial D_t^{y*}}{\partial K_t} &= p_t^{y*}F_K(K_t, H_t^*, \theta_t)\frac{(N\sigma - 1)}{N\sigma} - A_K(I_t^*, K_t). && (31)
\end{aligned}$$

Finally, if at any t, any firm deviates from $\mu_t^{y*}(\Gamma_t)$, then all firms utilize $\mu_\tau^{y'}(\Gamma_\tau), \tau \geq t+1$.

Since the monopoly outcome is also characterized through the optimal quantity choice problem, the Cournot equilibrium path bears functional similarity to the collusive equilibrium path characterized in Proposition 3. Intuitively, the strategic price effect of input and investment choices should moderate with multiple firms and this is evident in (28)-(31). Indeed, the strategic effect dissipates asymptotically to zero as the number of firms gets unboundedly large.

4.3 Procyclical and Countercyclical Markup Ratios

We examine cyclical properties of equilibrium price-cost markup in the oligopolistic equilibria described above because this issue attracts much attention in the literature. It follows from (21) or (28) that in equilibrium $pmcr_t$ is positively related to $(\bar{p}_t^{y*})^{1-\sigma}$ (or $(p_t^{y*})^{1-\sigma}$).[12] Since $\sigma > 1$, $pmcr_t$ and p_t^y have an opposite sign in terms of their cyclical properties. More precisely, we can show that

$$\frac{\partial pmcr_t}{\partial X_t} \propto W_t^* F_H(K_t, H_t^*, \theta_t) - Y_t \left[1 - N \frac{\partial(A(I_t^*, K_t) + H_t^*)}{\partial X_t}\right]. \tag{32}$$

Since the first term in (32) is positive, markups are procyclical if the industry investment and input demands are sufficiently sensitive to the aggregate shock, that is, if the term in the square parenthesis is negative.

More generally, marginal productivity of inputs will be countercyclical if optimal input demand is procyclical, which is the empirical observation. Hence, the markup ratio will be countercylical if the industry factor investment and input demands are relatively insensitive to the aggregate shocks. And, as we noted above, along the collusive equilibrium path investment and input demand will be relatively insensitive to aggregate shocks because of the strategic price effect. Hence, the prediction from our model is that countercyclical markups will be more likely in highly concentrated oligopolies. Conversely, markup ratios are more likely to be procyclical in moderately concentrated industries.

4.4 Asset Returns

As mentioned already, the SDF in this model is more complex compared with the benchmark consumption CAPM (CCAPM) because it involves the growth rate of the optimal consumption bundle $C(\mathbf{c}_t^*)$, which in equilibrium equals the product of the growth of aggregate income $\frac{Z_{t+1}}{Z_t}$

[12] Because the collusive and Cournot equilibrium paths have a similar functional structure, it is notationally convenient to genericall represent the equilibrium with the superscript "$*$."

(raised to the power $-\gamma$) and the growth of the aggregate price index $\frac{P_{t+1}}{P_t}$ (raised to the power $(\gamma-1)$). As seen in (10) and Proposition 3, these quantities are affected by both the aggregate shock (X_t) and the industry shock (θ_t), along with the ES (σ), the production function parameters, and the market structure (N). We now examine the implications of the model on the equity risk premia and the maximal Sharpe ratio.

5 Equilibrium Computations

Even though X and θ are conditionally lognormal, the dividend D^{y*} and the pricing kernel M^* are *not* lognormal in equilibrium. Note that (from (27))

$$D_t^{y*} = N^{-1/\sigma}\eta\,(W_t^*)^{1/\sigma}\,[F(K_t^*,H_t^*,\theta_t)]^{-1/\sigma} - H_t^* - A(I_t^*,K_t), \tag{33}$$

which is generally not lognormally distributed (conditional on Γ_t). It follows that CI's income Z^* and the aggregate price index P^* are also not conditionally lognormal, and hence neither is the (pricing kernel) M^*. This complicates substantially the analysis of the equilibrium. For tractability, we follow first the standard approach (Woodford (1986)) and analyze the equilibrium by computing its log-linearized approximation around the steady state where (i) the production in sector x and the technology levels in sector y are non-stochastic with $X_t = \mathbb{E}[X](\equiv \bar{X})$ and $\theta_t = \mathbb{E}[\theta]$ $(\equiv \bar{\theta})$ (for each t) and (ii) the equilibrium quantities in sector y are time-invariant. The details of the computation procedure outlined below are given in Appendix B, and the existence and characterization of the steady state (for the collusive and Cournot equilibria) are given in Appendix C. Subsequently, we also check the robustness of our results by computing global solutions using projection methods.

5.1 Real Variables

Note that for any time index τ, we can write the firm's investment I_τ as the first-order forward equation in capital stocks,

$$I_\tau = K_{\tau+1} - (1-\delta)K_\tau. \tag{34}$$

Hence, by replacing I_t, I_{t+1} with the appropriate forward equation we can represent the system of equilibrium conditions in Propositions 2 or 4 in terms of the vector of costate and state variables $\Omega_t = (K_{t+2}, K_{t+1}, K_t, H_{t+1}, H_t, X_{t+1}, X_t, \theta_{t+1}, \theta_t)$. For example, the equilibrium investment condition (29) in Proposition 4 can be written in terms of Ω_t as $\mathbb{E}_t[\Phi_I(\Omega_t)] = 0$ where

$$\Phi_I(\Omega_t) \equiv -Z_t^{-\gamma} P_t^{\gamma-1} \frac{\partial D_t^y}{\partial I_t} + \beta \left[Z_{t+1}^{-\gamma} P_{t+1}^{\gamma-1} \left(\frac{\partial D_{t+1}^y}{\partial K_{t+1}} - (1-\delta) \frac{\partial D_{t+1}^y}{\partial I_{t+1}} \right) \right], \tag{35}$$

(and $I_{t+1} = K_{t+2} - (1-\delta)K_{t+1}$ etc.). Similarly the optimality condition for material inputs (28) can be written $\mathbb{E}_t[\Phi_H(\Omega_t)] = 0$ where

$$\Phi_H(\Omega_t) \equiv -\left(1 + \frac{(p_t^y)^{1-\sigma}\eta^\sigma}{N\sigma}\right) + p_t^y \psi_H \theta_{t+1} (K_{t+1})^{\psi_K} (H_{t+1})^{\psi_H - 1} \left[1 - \frac{1}{N\sigma}\right].$$

And (35) and (36) use

$$p_t^y(\Omega_t) = \left(\frac{X_t - N(A(I_t, K_t), H_t)}{N\theta_t (K_t)^{\psi_K} (H_t)^{\psi_H}} \right)^{1/\sigma} \eta. \tag{36}$$

One then solves for the equilibrium policies by using (36) and log-linearizing $\Phi_I(\Omega_t)$, $\Phi_H(\Omega_t)$ around their steady state values of Ω (denoted $\bar{\Omega}$) with a first-order Taylor Series expansion. Using the standard notation, the log deviation around the steady state quantity for any variable w_t is denoted by $\hat{w}_t \equiv \ln(\frac{w_t}{\bar{w}}) \simeq \frac{w_t - \bar{w}}{\bar{w}}$ (for small deviations), and the log-deviation form of Ω will be labeled $\hat{\Omega}$.[13] Then let $\pi_t = [\hat{K}_{t+1} \ \hat{H}_t]$. The solution to the log-linearized version of the model takes the form

$$\pi_t = U\pi_{t-1} + U_X \hat{X}_t + U_\theta \hat{\theta}_t, \tag{37}$$

where the square matrix $U = [v_{js}]$, $j = K, H, s = 1, 2$, and the vectors U_X, U_θ (with elements u_{jX} and $u_{j\theta}$, $j = K, H$) are determined by the solution to log-linearized versions of the Euler conditions.

5.2 Asset Returns

Denoting the logarithms of variables by small letters, the equilibrium asset return condition $\mathbf{1} = \mathbb{E}_t [M_{t+1}\mathbf{R}_{t+1}]$ can be written as

$$1 = \mathbb{E}_t \left[\exp\left(m_{t+1} + r_{t+1}^j \right) \right], j \in \{x, y, f\}. \tag{38}$$

[13] We adopt the usual notation in the real business cycle literature for steady state variables (for example, $\bar{K}$) for expositional convenience. The discussion will be explicit in distinguishing this from the notation for the collusive equilibrium path ((for example, $\bar{K}^*$).

In this model, the log of the pricing kernel is

$$m_{t+1} = -\gamma g_{z,t+1} + (\gamma - 1) g_{p,t+1}, \tag{39}$$

where $g_{z,t+1}$ and $g_{p,t+1}$ are the log changes in the aggregate income and the price index, respectively, between t and $t+1$. As mentioned already, m_{t+1} and equity returns r^j_{t+1} $(j = x, y)$ are not generally conditionally joint normal. Since m_{t+1} and r^j_{t+1} are also functions of Ω_t, we take the first-order Taylor series approximation around their steady state values. Because log-linearized approximations of the pricing kernel and the equity returns will be derived around the steady state general equilibrium in the real economy and, hence, will also be functions of Ω_t. Nevertheless, the resultant approximations are joint normal and hence the expected ERP in the two sectors (x and y) can be computed in the standard fashion.

In the steady state, $\bar{M} = \beta$. Hence, log-linearization of the pricing kernel gives $m_{t+1} \simeq \log\beta + \hat{m}_{t+1}$, for

$$\hat{m}_{t+1} = \mathbf{a} \cdot \hat{\Omega}_t + \omega_{mx}\varepsilon^X_{t+1} + \omega_{m\theta}\varepsilon^\theta_{t+1}, \tag{40}$$

where the coefficient vector $\mathbf{a}$ is determined by taking the first-order Taylor approximation of $m_{t+1}(\Omega_{t+1})$ around the steady state $\bar{\Omega}$. Note that the coefficient for the shocks $\boldsymbol{\omega}_m = (\omega_{mX}, \omega_{m\theta})$ are time-invariant because shocks to the $\log X_t$ and $\log\theta_t$ process are additive (see (4)) with a stationary variance-covariance matrix Λ. In fact, we can use the equilibrium solution (37) to express $\mathbf{a}\cdot\hat{\Omega}_t$ in (40) in terms of the log-deviation form of the state variable vector $\hat{\Gamma}_t = (\hat{K}_t, \hat{X}_t, \hat{\theta}_t)$, namely, $\tilde{\mathbf{a}}\cdot\hat{\Gamma}_t$(see Appendix B). Then m_{t+1} is conditionally normal with the mean $(\log\beta + \tilde{\mathbf{a}}\cdot\hat{\Gamma}_t)$ and variance $\boldsymbol{\omega}_m \Lambda \boldsymbol{\omega}_m$. It follows immediately that the equilibrium risk-free rate is

$$r^f_{t+1} = -(\log\beta + \tilde{\mathbf{a}} \cdot \hat{\Gamma}_t) - \frac{\boldsymbol{\omega}_m \Lambda \boldsymbol{\omega}_m}{2}. \tag{41}$$

To calculate the equilibrium equity returns, we utilize the standard log-linearization of returns in the literature (Campbell and Shiller (1988)). In the situation at hand, the steady state dividend-price ratio for equities (in both sectors) is $\frac{\bar{D}^j}{\bar{S}^j} = \frac{1-\beta}{\beta}$ and log-linearization yields the return approximation (see Appendix B)

$$r^j_{t+1} \simeq -[\beta\log\beta + (1-\beta)\log(1-\beta)] + \beta\xi^j_{t+1} - \xi^j_t + g^j_{d,t+1}, \tag{42}$$

where $g^j_{d,t+1} \equiv d^j_{t+1} - d^j_t$ is the log growth rate of dividends between t and $t+1$ and ξ^j_t is the log *stock* price-dividend ratio (that is, $\log(\bar{S}^j_t/\bar{D}^j_t)$) at t of equity $j = x, y$. But here—unlike Campbell and Shiller (1988)—the evolution of the log price-dividend ratio and dividend growth is determined in general equilibrium. As noted above, the equilibrium dividends in sector y (D^{y*}) will not generally be conditionally lognormal,[14] but $\hat{g}^j_{d,t+1}$ will be a function of the costate and state variables. Hence, log-linearization yields

$$\hat{g}^j_{d,t+1} \simeq \mathbf{b}\cdot\hat{\Gamma}_t + \omega^y_{dx}\varepsilon^X_{t+1} + \omega^y_{m\theta}\varepsilon^\theta_{t+1}, \tag{43}$$

and it follows from (38) that log-linearization of ξ^j_t takes the form $\hat{\xi}^j_t \simeq e^j_0 + \mathbf{e}^j\cdot\hat{\Gamma}_t$. The coefficients of $\hat{g}^j_{d,t+1}$ and $\hat{\xi}^j_t$ are computed through the equilibrium condition (38). Inserting these relationships in (42) yields

$$r^j_{t+1} = v^j_0 - \boldsymbol{\nu}^j \cdot \hat{\Gamma}_t + \omega^j_{rx}\varepsilon^x_{t+1} + \omega^j_{r\theta}\varepsilon^\theta_{t+1}, j = x, y, \tag{44}$$

With these linearized relationships in hand, m_{t+1} and r^j_{t+1} are jointly normal, conditional on $\hat{\Gamma}_t$. Hence, using the properties of exponential functions of joint normal variables, we obtain in the usual fashion (for $j = x, y$) :

$$\mathbb{E}_t[r^j_{t+1} - r^f_{t+1}] = -Cov_t(m_{t+1}, r^j_{t+1}) - \frac{\mathrm{Var}(r^j_{t+1})}{2}. \tag{45}$$

Note that $\mathrm{Cov}_t(m_{t+1}, r^j_{t+1}) = \frac{\boldsymbol{\omega}_m\Lambda\boldsymbol{\omega}^j_r}{2}$ and $\frac{\mathrm{Var}(r^j_{t+1})}{2} = -\frac{\boldsymbol{\omega}^j_r\Lambda\boldsymbol{\omega}^j_r}{2}$ (where $\boldsymbol{\omega}^j_r = (\omega^j_{rx}, \omega^j_{r\theta})$ and $\boldsymbol{\omega}_m = (\omega_{mX}, \omega_{m\theta})$) are time-invariant, *conditional* on $\hat{\Gamma}_t$, Hence, the unconditional ERP are time-invariant and analogously the conditional volatility of the ERP and the Sharpe Ratio are also time-invariant. The time-invariance of the conditional ERP and the Sharpe Ratio arises here because of stationary oligopolistic policies along the equilibrium path (see Propositions 2 and 4) and the assumption of additive output shocks (in sector x) and technology shocks (in sector y) with time-invariant moments.

In a log-linear framework, with the joint (conditional) log-normality of m_t and r^j_t, one can compare the equilibrium equity premia and Sharpe ratios in this model with the standard single good asset pricing models that have been extensively studied with similar distributional assumptions. This analysis will also be useful in clarifying the relation of industry competition with the mean and volatility of the ERP. In the standard way, by using (39) in (45) we can also write the equilibrium

[14] Of course, $d^x_t = \log X_t$, and is conditionally normal.

equity premium as

$$\mathbb{E}_t[r^j_{t+1} - r^f_{t+1}] = \gamma \mathrm{Cov}_t(g_{z,t+1}, r^j_{t+1}) - (\gamma - 1)\mathrm{Cov}_t(g_{p,t+1}, r^j_{t+1}) - \frac{\mathrm{Var}(r^j_{t+1})}{2}, j = x, y. \tag{46}$$

Equation (46) indicates that the risk premium is positively related to the covariance of the asset return with log change in aggregate income Z, and negatively related (for $\gamma > 1$) to the covariance of asset return with the log change in the aggregate price index P. In terms of empirical magnitudes, $g_{z,t+1}$ will be largely driven by shocks to the aggregate output (x), which is similar to single good models. Nevertheless, the percentage change in industry dividends $g^y_{d,t+1}$ will affect $g_{z,t+1}$ (as long as the industry is not infinitesimal compared with the aggregate output). Since industry productivity shocks have a first order impact on $g^y_{d,t+1}$, it follows that the θ_t process will also influence the first term. Turning to the second term, from the definition of P (see (10)), $g_{p,t+1}$ is determined by log changes in the industry price, which is driven by shocks to both aggregate output and industry productivity, along with industry concentration.

We can also derive the Hansen-Jagannathan (1991) upper bound on the Sharpe ratios for assets in the model. Using the fact that $R_f = 1/\mathbb{E}[m]$ is close to 1, we have

$$\mathrm{SR}_{\max} = \frac{\sqrt{\mathrm{Var}(m)}}{\mathbb{E}[m]} \propto \sqrt{\gamma^2 \mathrm{Var}(g_z) + (\gamma - 1)^2 \mathrm{Var}(g_p) - 2\gamma(\gamma - 1)\mathrm{Cov}(g_z, g_p)}. \tag{47}$$

Hence, the maximal Sharpe ratio depends on the variance of (non-linear functions of) log changes in X and θ and the covariance between them. In comparison, the maximal Sharpe ratio in the CCAPM is approximately $\gamma \mathrm{Vol}(g_C)$. As is well known, the low variability in per capita consumption growth in the data restricts $\mathrm{SR}_{\max}$ to be quite low for the reasonable range of risk aversion. But as we mentioned already, in our multi-good model $\mathrm{SR}_{\max}$ is based on the volatility of the (CES) consumption bundle and, in equilibrium, this volatility is driven by the volatilities of aggregate income and, industry productivity, mediated by the industry market structure. This equilibrium relation will be very useful in our empirical analysis below. In sum, (47) clarifies conceptually the difference in the maximal Sharpe ratio between our model and the CCAPM even when assuming time-additive power expected utility.

We can summarize the model's theoretical predictions on effects of industry competition on asset returns as follows. Based on the strategic smoothing of optimal investment and output, and hence equity payouts, along oligopolistic equilibrium paths noted in Propositions 2 and 4, it follows from

(42)-(44) that Var(r^y) is positively related to industry competition. From Equation (46), ceteris paribus, this channel will lead to a negative relation of expected ERP and industry competition. However, the net effects of industry competition on the first two terms in (46)—that represent the interaction of imperfect competition with the SDF—are ambiguous. Thus, the overall relation of competition with expected ERP (and hence industry Sharpe ratio) is ambiguous in our model.

6 Empirical Tests

We analyze the model empirically through calibrated, numerical simulations of the log-linearized approximation of the equilibrium. Subsequently, we compute global solutions for robustness.

6.1 Empirical Measures and Data

For empirical tests of the model, we need industry data on capital, investment, material inputs, sales, and productivity. We take these data from the NBER-CES manufacturing database. The latest data available are for 1958-2011 (annually). We empirically identify highly concentrated industries as those where more than seventy percent of the output is generated by the largest four firms. The data on the proportion of industry output accounted for by the largest four and eight firms are available through the US Census Bureau—at the six-digit NAICS level—for 1997, 2002, and 2007. Hence, we have to hold concentration at the 1997 levels for years prior to 1997. To maintain a uniform classification of industries for the entire sample period, we use the industry concentration data from 1997.[15]

We have a total of 473 unique six-digit industries. We require 20 firms in a given industry, which drops the number of industries to 456. Of these, 31 industries (6.8% of the total) satisfy our definition of highly concentrated oligopolies—that is, where the top 4 firms generate more than 70% of the output. Table 1 provides the industry codes and names of these oligopolistic industries. This procedure identifies moderately concentrated industries as the remaining group of 425 unique industries (out of the total admissible sample of 456 industries). We then measure the output of the two industry groups—say, Y_h for highly concentrated industries and Y_m for moderately concentrated industries—as the sum of value of shipments of component industries, obtained from

[15] Changing the industry identification in 2002 and 2007 leads to significant in-sample data "discontinuities" and therefore muddles inference. Namely, are the time-variations in results due to changes in the competitive environment of the oligopolistic sector or are they due to changes in the composition of the sector?.

the NBER-CES database. In a similar fashion, for both industry groups, we obtain the time-series of investments (I), material costs (H), capital (K), and total 4-factor productivity (θ) from the database.[16]

Consistent with our theoretical framework, for each concentration group, we measure the output (X) of the non-oligopolistic "aggregate" sector (x) as the difference between the *aggregate* output of all sectors obtained from the US Bureau of Economic Affairs (BEA) and the output of the group.[17] This procedure thus results in two time series for X, namely, $X_h =$ (Aggregate output $-Y_h$) and $X_m =$ (Aggregate output $-Y_m$). For all of these quantities, the data also provides information about the relevant price deflator in 1997 dollars. We use these deflators to convert the values in real terms.

To compute the financial variables of the model, we use annual CRSP value-weighted returns and the annual risk-free rate obtained from Kenneth French's website (the inflation data to derive the real rate is obtained from the US Bureau of Labor Statistics). We compute the sectoral financial variables as follows. We first map the 1997 NAICS codes to 1987 Standard Industry Classification (SIC) codes. We then use four-digit SIC codes to compute the portfolio returns. Following the standard procedure in the literature, we compute the value-weighted index monthly returns of all firms in all industries classified as oligopolies. Using these returns, we obtain the annualized ERP, annualized equity premium volatility, and the Sharpe ratio for the oligopolistic sector (y). In a similar fashion, we obtain the financial variables for the aggregate sector (x) using the annual CRSP value-weighted index returns as the proxy.

6.2 Parameterization

Using the data described above, we compute the log-linearized version of the model (described in Section 5 and Appendix B) by calibrating the parameters separately for the moderately and highly concentration industry groups. We now describe the calibration procedures for the product and asset markets related variables.

[16] The four factors are capital, production worker hours, non-prduction worker hours, and material inputs.

[17] To be consistent with the definition of sector x in our model,we use the aggregate output in all sectors rather than the Gross Domestic Product (GDP).

6.2.1 Product Market Variables

For each concentration industry group, the estimates from the logarithmic regression (based on the production function (1)) provide the values for the production parameters ψ_K and ψ_H :

$$\ln Y = \psi_K \ln K + \psi_H \ln H + \ln \theta + \varepsilon. \tag{48}$$

The elements of the variance-covariance matrix of the shocks X_t and θ_t are obtained from the data.[18] The values for the autocorrelation coefficients ρ_j, $j = X, \theta$ are also estimated from the data using the first-order autocorrelations.

It is well known that because different types of capital—equipment, structures, and intellectual property—depreciate at different rates, estimating the empirical depreciation rates is challenging. The literature notes that depreciation rates have been trending upwards because of the increased use of computer equipment and software since this lowers the useful life of capital stock (Oliner (1989)). Moreover, the depreciation rates on such equipment themselves have been rising. For example, Gomme and Rupert (2007) mention that the annual depreciation rates of computer equipment have risen from 15% in 1960-1980 to 40% in 1990s. And they give estimates for depreciation rates of software in the range of 50%. Meanwhile, Epstein and Denny (1980) estimate the depreciation rate of physical capital (in the first part of our sample-period) to be about 13%. We use an annual depreciation rate of 25%. Untabulated results show that the results are quite robust to variations in the value of depreciation rate parameter (δ). Meanwhile, there is a wide range of estimates available in the literature regarding the capital investment (or adjustment) cost parameter υ. Using US plant level data, Cooper and Haltiwanger (2006) report υ of around 10% when estimating a strictly convex adjustment cost function, as used in our model. Hence, we use $\upsilon = 0.1$ for our simulations.

To calibrate the equilibrium markup ratio *pmcr*, we turn to the long-standing empirical literature that estimates markups in manufacturing industries.[19] Domowitz et al. (1988) estimate *pmcr* for 19 manufacturing industries, with the range being 1.3 to 1.7 for 17 of these industries (the estimates for the other two are larger). Morrison (1993) estimates the markup ratio to be between 1.2 and 1.4 for 16 out 18 manufacturing industries. Meanwhile, De Loecker et al. (2018) provide

[18] The covariance between percent variability in X and θ for both industry groups is very low in the data and is hence set to zero in the simulations.

[19] In theory, we could estimate *pmcr* by multiplying the coefficient ψ_H from (48) with the average ratio of revenues to material costs input costs (see De Leocker et al. (2018)). But the empirical literature on markups uses data at the firm level on prices and costs in addition to detailed productivity data (Hall (2018)). Hence, we calibrate the markup ratio based on the literature.

evidence of rising markup ratios over time and present estimates of 2.3 for the 90th percentile in manufacturing industries during 1980-2016. On the other hand, Hall (2018) puts an upper bound of 1.5 on markup ratios (with the average in manufacturing being 1.4). Similar to Rotemberg and Woodford (1992), we adopt a conservative approach to the selection of the markup ratios to show the significant influence of product market power on real and financial variables. Specifically, for the highly concentrated group we use a markup ratio of 1.5, while for the moderately concentrated group we use the ratio 1.2.

6.2.2 Consumer Preferences Parameters

We set the discount rate β as $(1.03)^{-1} = 0.97$, which implies a three percent annual discount, which is consistent with the literature (Horvath (1999)). The ES σ and the consumption weight of the oligopolistic sector ϕ are parameters of unobservable utility function of the representative consumer and are, therefore, calibrated internally by matching the target *pmcr* values discussed above. There is still no consensus on the appropriate parameterization of the risk aversion coefficient γ. The literature (Mehra and Prescott (1985), Bansal and Yaron (2004)) considers a reasonable upper bound on RRA to be about 10. We take $\gamma = 10$ for our calculations, which facilitates comparison with some of the existing literature. But for sensitivity analysis, we also analyze the model for $\gamma = 7.5$.

7 Results

Table 2 summarizes the parameters utilized in our analysis.[20] For the reasons mentioned earlier, we compare the equilibrium oligopoly outcomes with those in the benchmark perfectly competitive industry.[21] We present results for moderately concentrated industries first because they serve as a useful comparison benchmark for the collusive outcomes in the highly concentrated industries.

[20] The first two moments of the aggregate output ($\bar{X}$) are different between the two industry groups because, as mentioned above, the theoretically consistent notion of X in our analysis is the net aggregate output. Meanwhile, the first two moments of sectoral productivity (θ) are slightly different, but the autocorrelation estimated coefficient ρ_θ here is significantly higher than in the case of the moderately concentrated industries.

[21] For computing the benchmark industry competitive outcomes, we retain the parameterization used for the oligopolistic industry but force firms to follow the competitive equilibrium policies of Proposition 1 in the steady state and along the simulation path. It turns out that the number of competitive firms does not materially affect the second moments of product market variables, and the ERP and its volatility—variables that are the focus of our analysis. The competitive benchmark results reported below are based on 15 firms.

7.1 Moderately Concentrated Industries

We compute the equilibrium on an annual frequency (to match the data). The following results are based on 5000 replications of the equilibrium paths of a 54 year model economy (1958-2011).

7.1.1 Product Market Variables

Table 3 shows the equilibrium computations for product market variables for the dynamic Cournot analysis when $N = 15$.[22] The volatility of annual log changes in X in the data (when sector y is represented by moderately concentrated manufacturing industries) is 3.12%, which is slightly lower than the calibration given in recent macroeconomic models (He and Krishnamurthy (2019)).[23] As seen in Table 2, the volatility of the aggregate output and industry productivity shocks in the simulations are close to the data. The mean *pmcr* (in Column 2) matches the target markup ratios of 1.2, while the endogenous *pmcr* of the competitive benchmark is fixed at 1.

Because the model does not have a growth component in productivity and computations are in terms of deviation from the steady state, we report the second moments of the control variables. We focus attention first on the baseline risk aversion of 10. The investment, material input and output volatilities along the oligopolistic equilibrium path are a reasonable fit with the data. But compared to the oligopolistic equilibrium path, all three variables are significantly more volatile in the competitive benchmark, especially investment. This is consistent with the intuition from the model that the strategic price effects of market power on optimal input and investment demands will smooth out the effects of aggregate output and industry productivity shocks. Overall, with respect to volatilities of endogenous real variables, the fit of the oligopolistic model is better than the competitive benchmark.

Consistent with the data, the model generates pro-cyclical investment and material input demands with respect to aggregate output and industry productivity. And the correlation of log changes in investment with the percent changes net aggregate output (X) matches the data quite closely. However, the model significantly overstates the correlation of log changes in investment with respect to log changes in sectoral productivity shocks. But the corresponding correlation with respect to percent changes in material inputs is a better match with the data compared with

[22] The steady state values of capital stock ($\bar{K}$) and material inputs ($\bar{H}$) are computed from the steady state optimality conditions (see Appendix C), and based on the parameterization in Table 2 the collusive path is not a SPE in the stady state if the number of firms is 15.

[23] In He and Krishnamurthy (2019), the simulated volatility of annual output growth is 3.6%.

the competitive benchmark. Meanwhile, using the De Loecker et al. (2018) measure of *pmcr*, the markup ratio is procyclical in our sample of moderately concentrated industries, which is also consistent with our theoretical framework (as noted in Section 4.3), but is much lower than in the data.[24]

Finally, lower risk aversion ($\gamma = 7.5$) does not have significant effects on the product market variables. But risk aversion will significantly affect the asset market variables that we discuss next.

7.1.2 Asset Markets Variables

In Table 4, we present the equilibrium computations with respect to the asset markets variables. As above, we focus discussion first to the baseline value of risk aversion (that is, $\gamma = 10$). Consistent with the theoretical prediction, the volatility of excess returns in oligopoly is significantly lower compared to the competitive benchmark. This supports the strategic smoothing effect of product market power noted in the foregoing analysis. And while the effect of industry competition on expected ERP is theoretically ambiguous, the results indicate a positive relationship between competition and expected returns.

We also note that the unconditional expected ERP generated by the model for oligopoly (2.7%)—and a fortiori for the competitive benchmark (4%)—is lower than the data but significantly higher than the market ERP. Indeed, the expected ERP from the parameterization at hand is relatively high compared to benchmark consumption CAPM models with similar "classical" assumptions on power utility, Markov shocks, and absence of security market frictions.[25] In a related vein, both the industry and market Sharpe ratios are also higher than benchmark models with time-additive power expected utility (Lettau and Uhlig (2002)). We also note that the unconditional volatility of the industry ERP is 11.4%, which is much closer to the data compared with benchmark single-good models (as indicated by volatility of the market excess returns), and this is related to the excess return volatility puzzle (Shiller (1981)).

The economic mechanism underlying the relatively high industry ERP, Sharpe ratios, and return volatility (seen in Table 4) is indicated in Section 5.2. In the multi-good model at hand, the volatility

[24] Nekarda and Ramey (2010) also report procyclical markup ratios. We will return to this issue after examining the results for highly concentrated industries in the next section.

[25] As a benchmark, Jermann (1998) reports an aggregate ERP of 0.7% without habit formation, but including capital adjustment costs (as is also the case in our model). As another benchmark, Bansal and Yaron (2004) obtain an (aggregate) ERP of about 1.2% when $\gamma = 10$, the intertemporal elasticity of substitution (IES) is 0.5, and there are non-fluctuating (or homoscedastic) long-run risks.

of the SDF is driven by the percent variability of the equilibrium consumption bundle $C_t^* = C(\mathbf{c}_t^*)$ (see (8)). But in equilibrium $C_t^* = \frac{Z_t}{P_t}$ and hence, as mentioned already, the normalized variability of the SDF (see (47)) is positively related to the volatility and comovements of aggregate income and the price index. In terms of empirical magnitudes, the annual volatility of real per capita consumption in the US during our sample period is about 1.3%, while the volatility of X—the principal driver of Z—is 3.1%. Of course, the volatility of the price index P is complex, but this is computed in our simulations. Indeed, the mean volatility of the equilibrium C_t^* (for the moderately concentrated sample of industries) is 3.2%. Consistent with this, the model-generated risk-free rate is about 3%, which is significantly lower than those generated by benchmark single-consumption good models (Weil (1989)).

Stepping outside the model, the general point here is that the volatility of consumption bundles in multi-good models can be significantly higher than the per capital overall consumption. As noted above, in our sample period the volatility of annual log changes in real per capita consumption (using the BEA data) is 1.3%. But during the same period, BEA data indicate that the volatility of annual log changes in real per capita durable goods consumption is over 5% while that of goods is 3%.[26] Of course, this intuition applies independent of the industry market structure; our analysis indicates that effective industry competition significantly impacts the effect of this volatility on asset prices.

Finally, the computations for the case $\gamma = 7.5$ show lower ERP and its volatility for both the oligopoly and the market (compared with the baseline risk aversion). This is consistent with the standard intuition due to reduced value of consumption smoothing by the representative CI. And in a similar vein, the Sharpe ratios are also lower.

7.2 Highly Concentrated Industries

As earlier, the following results are based on 5000 replications of the equilibrium paths of a 54 year model economy (1958-2011).

[26] As we mentioned above, the role of product differentiation in helping explain the equity premium puzzle is also noted in the literature in a different setting. Ait-Sahalia et al. (2004) differentiate between the consumption of luxury and basic goods. They find that luxury good consumption covaries much more with equity returns compared with aggregate (or basic) goods consumption so that the risk aversion required to match the data is lower than in single-goods settings.

7.2.1 Product Market Variables

In Table 5, we present the equilibrium computations for product market variables for the dynamic collusive outcome path depicted in Proposition 3. The equilibrium investment, material input, and output along the oligopolistic equilibrium path are smoother than in the benchmark competitive outcome path.[27] This is similar to the results in Table 3 and is another manifestation of the strategic effects of product market power on optimal input and investment policies. The investment volatility from the model is actually close to the data, albeit somewhat higher. The volatilities of material input and output are lower than that in the data, as in Table 3. Consistent with the data, the model generates procyclical investment and input policies with respect to both aggregate and sectoral business cycles. In particular, the correlation of percent changes in inputs with the aggregate and sectorial shocks generated by the model are relatively close to the data.

Notably, the collusive outcome path results in countercyclical price-cost margins with respect to the aggregate business cycle, although the magnitude is lower than in the data. Along with the procyclical markups observed for moderately concentrated industries above, this result is consistent with the theoretical predictions of our model (see Section 4.3). The countercyclical collusive markup ratios are also consistent with subgame perfect collusion considered by Rotemberg and Saloner (1986) and Rotemberg and Woodford (1992).

7.2.2 Asset Market Variables

Table 6 shows equilibrium asset markets' variables for the collusive equilibrium path. The volatility of the industry ERP along the oligopolistic equilibrium path is significantly lower compared with the competitive benchmark industry, similar to the results for moderately concentrated industries (Table 4). Meanwhile, the expected ERP in the highly concentrated oligopoly is lower than the competitive benchmark industry. Along with the results in Table 4, these observations indicate a *positive* relation of competition and ERP.

But the equilibrium ERP for highly concentrated oligopolies is still higher than those reported for analogous risk aversion and power utility preferences (without habit formation) in single good production-based asset pricing model (Jermann (1998)).[28] And since the industry and market

[27] The benchmark competitive outcomes here are computed based on the assumption of an industry with 8 (competitive) firms, to maintain consistency with a treasonable number of firms in highly concentrated industries.

[28] With a risk aversion of 10, no habit formation, but in the presence of adjustment costs, Jermann (1998) reports an expected ERP of about 0.7%, which is less than one-half of the 1.49% generated by our model for the sample of

expected ERP are close in this case, the same observation applies for the aggregate equity premium as well.

Notably, the industry Sharpe ratio (0.27) almost exactly matches the (industry) Sharpe ratio in the data; it is also similar to the Sharpe ratio of the market (which is lower than that in the data). In contrast, the industry Sharpe ratio for moderately concentrated industries (in Table 4) was significantly lower than the market Sharpe ratio. Thus, there is a positive relation of equilibrium Sharpe ratios to product market power. This view is reinforced by the relatively low Sharpe ratio of the competitive benchmark (0.20), which is higher than the Sharpe ratio for the competitive benchmark in moderately concentrated industries. We also continue to find that the equilibrium risk free rate is lower than that in benchmark CCAPM models.

7.3 Impulse Response Functions

The theoretical and empirical analysis above indicates that oligopolistic firms tend to "smooth out" responses to the aggregate wealth and industry productivity shocks. To examine this further, Figures 1 and 2 present impulse response functions (IRFs) of log investment, log dividends, log sales, and log SDF of one standard deviation shocks to X_t and θ_t for highly concentrated oligopolies.[29] For comparison, we also display the IRFs for the competitive benchmark. Specifically, starting from the empirical means of these shocks $\bar{X}$ and $\bar{\theta}$ (the proxies for their steady state values) at the initial point, we generate two simulations that differ only by one standard deviation of the respective shocks at $t = 1$ (see Petrosky-Nadeau and Zhang (2017) and Garlappi and Song (2017)).

Turning, first, to capital investment, exogenous increases in aggregate wealth and industry productivity generate an immediate positive response, which is consistent with average procyclical behavior of investment seen in Table 6. This effect declines over time, but it is clearly persistent. However, for both kinds of shocks, the immediate impulse response and its subsequent decline is much sharper in the competitive benchmark industry relative to the oligopolistic market structure. This supports the view that oligopolies tend to "smooth out" the effects of aggregate and industry shocks on investment.

Meanwhile, the effect of the positive shocks on sales ($p^y Y$) follows a hump-shaped pattern, that is, the difference between the sales level in the "shocked" economy and the baseline economy

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highly concentrated industries.

[29] To economize on space, we do not present the analogous analysis for the sample of moderately concentrated industries, which is qualitatively similar to the one presented in Figures 1 and 2.

continues to rise up to a maximum (roughly around 4 or 5 years after the initial shock) and then declines in a persistent manner. But here again, the slope of the initial ascent and the subsequent decline is much sharper in competitive industry relative to oligopoly. We also notice that aggregate wealth has a greater effect in magnitude, relative to industry shocks, which is not surprising given the relative calibration of these shocks. Turning to dividends, recall that $D_t^y = p_t^y Y_t - H_t - A(I_t, K_t)$; hence, we expect their IRF to reflect the features of the IRFs of both investment and sales. And this is what we observe in Figures 1 and 2: The IRF for dividends in oligopolies tend to follow the IRF pattern of investment, but for the competitive benchmark firms we see the hump-shaped IRF pattern of sales. This difference (between oligopolies and competitive firms) possibly reflects the closer link between strategic investment and dividends in oligopolies relative to competitive firms.

Finally, we analyze the IRFs of log SDF to aggregate wealth and industry productivity shocks. Theoretically, the effect of aggregate wealth shocks on the SDF is ambiguous, as can be seen from (39), because both the growth rate of aggregate income and the price index are positively related to X. Empirically, we see in Figure 1 that increases in aggregate wealth generate a sharp increase in log SDF and this effect then declines almost linearly over time. Consistent with our theoretical assumption, market structure (or industry market power) do not have a significant impact on the SDF and similarly for the effects of industry productivity shocks.

8 Global Solutions

As we pointed out already, because of the endogenous industry demand function and asset pricing kernel, the general equilibrium consumption, dividends and asset returns in our model are not conditionally lognormal. While the log-linear solution approach used above is convenient and allows a ready comparison with existing macrofinance literature, there is a concern that it may generate errors in light of the nonlinearities in the model, the parameterization of $\gamma = 10$, and high persistence in the aggregate output (X) series (e.g., Pohl et al. (2018)). For robustness we, therefore, also compute global solutions of the model's equilibrium using projection methods, that is, polynomial approximations of unknown policy and asset pricing functions (e.g., Judd (1996)).

Because of the Cobb-Douglas production function and the CES utility function, the equilibrium price function (27) is smooth, that is, belongs to the class of infinitely continuously differentiable functions C^∞. The Theorem of the Maximum then implies that optimal (interior) equilibrium firm policies are also smooth. In particular, with the parameterization on ψ_K, ψ_H and σ used above, errors from quadratic polynomial approximations around critical points of our model have tight upper bounds. We exploit this fact and iterate on multi-variable (with the three states $\Gamma = (K, X, \theta)$) quadratic polynomial approximations to the optimal policy functions (I^*, H^*), as characterized in Propositions 2 and 4.[30] We use the same parameterization as we employed for log-linear solutions discussed above.

With the optimal investment and material input policy functions in hand, we solve for unconditional expected returns and volatilities of equities (in both sectors x and y) utilizing the fact that log returns r_t^j, $j = x, y$, can be written in terms of log price-dividend ratios ξ_t^j (see section 5.2) as

$$r_{t+1}^j = \ln(\exp(\xi_{t+1}^j) + 1) - \xi_t^j + g_{d,t+1}^j. \tag{49}$$

We approximate the unknown ξ_t^j as quadratic polynomial functions of Γ_t (denoted by $\tilde{\xi}_t^j$) by solving the asset return Euler conditions $1 = \mathbb{E}_t \left[\exp\left(m_{t+1} + \tilde{r}_{t+1}^j\right)\right]$, $j = x, y$. This process allows us to compute the unconditional first and second moments of equity returns. Similarly, the equilibrium expected risk-free return is computed by approximating the risk-free bond price, S^f with a quadratic polynomial and solving $1 = \mathbb{E}_t \left[M_{t+1}(1/S_t^f)\right]$.[31] The model is then simulated using these solutions, starting at the steady state, with cubic spline interpolation.

We display the results in Table 7 for the sample of highly concentrated industries analyzed in Tables 5 and 6.[32] Panel A indicates that, in terms of fitting the data, and when compared to the log-linear solutions, the global solution for the oligopolistic equilibrium performs better in some dimensions, but less well in other dimensions. For example, the global solution performs better with respect to the volatilities of material inputs and outputs and the correlation of the price-cost margin with the aggregate shock. But it (the global solution) performs less well than the log-linear solution with respect to the investment volatility and the correlations of the investment and inputs with the aggregate and industry productivity shocks. In particular, there is a tendency

[30] Because of the three state variables, approximations using oscillating polynomials such as Chebyshev polynomials of the first kind $T_n(\cdot)$ are quite computationally intensive. For example, using $n = 0, ..6$, which is a typical choice in the literature, results in computing 343 coefficients for investment and material inputs alone, and the computational needs for the three asset returns are even higher. The use of value- or policy-iteration methods to compute global solutions for multi-sectoral production-based asset pricing models is consitent with the literature (see, e.g., Garlappi and Song (2017)).

[31] The three (state) variable quadratic polynomial has ten coefficients. We use a $30 \times 30 \times 30$ point grid for (K, X, θ). We discretize the continuous shock space with a 10×10 grid for $(\varepsilon^x, \varepsilon^\theta)$ using Gauss-Hermite quadrature. We simulate the product market variables using cubic spline interpolation and the mean Euler equation errors in the simulations are less than 0.1%.

[32] Untabulated results indicate that results using global solutions for moderately concentrated industries (and the competitive benchmark) are qualitatively similar to those in Tables 3 and 4.

for the investment volatility and the correlations of investment and inputs in the global solution to understate the data; in contrast, these quantities have a tendency to over-state the data in the log-linear solution (Table 5). We continue to find a negative relation of industry competition and volatilities of investment and material inputs, consistent with the previous results. Overall, results from the oligopoly model fit the data better than the competitive benchmark when using the global solution approach.

Panel B indicates that the empirical fit of the global solution relative to the log-linear solution (Table 6) is also mixed. The global solution has a higher ERP and its volatility for the equity of the aggregate sector, compared with the log-linear solution. Therefore, the fit with observed equity return moments of this sector improves with the global solution; however, the Sharpe ratio is a little lower with the global solution, implying a greater deviation from the data. Meanwhile, the risk free rate is lower than that in the log-linear case, and this also improves the fit with the data. But the ERP and its volatility of the oligopolistic sector are lower than in Table 6, and hence the fit with the data is relatively worse along these dimensions compared to the log-linear solution. Meanwhile, the relation of industry competition to the ERP, its volatility, and the Sharpe ratio with the global solution is similar to that observed with log-linear solution in Table 6.

9 Summary and Conclusions

The role of imperfect competition—in particular, oligopolistic collusion—in transmitting the effects of aggregate and industry shocks on industry and aggregate real and financial outcomes is of substantial interest. We develop a dynamic production-based general equilibrium multi-consumption good model with an oligopolistic industry and fit it to U.S. aggregate and manufacturing industry data. At the industry level, separate analysis of highly and moderately concentrated oligopolistic industries shows that the model conforms reasonably well with the cyclical properties of capital investment, material inputs, and price-cost markups. And, consistent with the theoretical predictions from the model, the volatilities of ERP, investment, inputs, and excess equity returns, as well as the procylicality of markups are negatively related to industry competition. In equilibrium, the SDF is driven by aggregate output and sectoral productivity shocks as well as industry competition. Compared with the benchmark single-consumption good model, the interaction of aggregate and sectoral shocks can increase the volatility of the SDF and covariance of asset returns with the SDF, thereby raising the expected ERP and the maximal Sharpe ratio. These conclusions are robust to using both log-linear and global solution approaches.

Overall, the oligopoly model provides a better fit overall to product and asset markets' data compared to the competitive benchmark industry. We conclude that modeling industry market structures in multi-good general equilibrium models may help explain important product and asset markets phenomena at the industry level.

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Table 1. List of Highly Concentrated Manufacturing Industries

This table lists the 6-digit NAICS Codes and names of industries in our sample of oligopolies, that is, industries where the largest 4 firms account for more than 70% of industry output in 1997.

NAICS Code	Industry Name
311221	Wet corn milling
311222	Soybean processing
311230	Breakfast cereal mfg.
311320	Beans
311919	Other snack food mfg.
311930	Flavoring syrup & concentrate mfg.
312120	Breweries
316212	House slipper mfg.
321213	Engineered wood member
325181	Alkalies & chlorine mfg.
325191	Gum & wood chemical mfg.
325312	Phosphatic fertilizer mfg.
326192	Resilient floor covering mfg.
326211	Tire mfg (except retreading).
331528	Other nonferrous foundries (except die-casting).
332992	Small arms ammunition mfg.
332995	Other ordnance & accessories mfg.
333315	Photographic & photocopying equipment mfg.
333611	Turbine & turbine generator set unit mfg.
335110	Electric lamp bulb & part mfg.
335222	Household refrigerator & home freezer mfg.
335912	Primary battery mfg.
336111	Automobile mfg.
336112	Light truck & utility vehicle mfg.
336120	Heavy duty truck mfg.
336391	Motor vehicle air-conditioning mfg.
336411	Aircraft mfg.
336412	Aircraft engine & engine parts mfg.
336419	Auxiliary equip mfg.
336992	Military armored vehicle, tank and tank component mfg.
339995	Burial casket mfg.

Table 2. Parameter Assumptions

This table displays the parameterization of the model for a sample of 425 manufacturing industries (based on the NBER-CES sample of manufacturing industries (1958-2011) where the largest four firms account for less than 70% of industry output. The notation is as in the text, but the various parameters are defined for convenience.

Global Parameters			
Parameter	Highly Conc. Industries	Moderately Conc. Industries	Description
β	0.97	0.97	Discount factor
δ	0.25	0.25	Depreciation rate
γ	10/7.5	10/7.5	Relative risk aversion
υ	0.10	0.10	Capital adjustment cost
1958-2011			
$\bar{X}$ ($ billion)	7603.13	9983.71	Mean X
$\lambda_X^{0.5} \times 100$	3.12	3.32	Annual volatility of ε^X
ρ_X	0.998	0.998	Autocorrelation of X
ψ_K	0.16	0.43	Output elasticity: capital
ψ_H	0.88	0.63	Output elasticity: materials
$\bar{\theta}$	1.19	1.20	Mean θ
$\lambda_\theta^{0.5} \times 100$	2.07	1.96	Volatility of ε^θ
$\lambda_{X\theta}$	0.0	0.0	Covariance of shocks
ρ_θ	0.948	0.97	Autocorrelation of θ

Table 3. Moderately Concentrated Industries: Product Market Variables

This table presents salient statistics on equilibrium capital investment, material input demand, output and price-cost margins for a sample of 425 manufacturing industries (based on the NBER-CES sample of manufacturing industries (1958-2011)) where the largest four firms account for less than 70% of output. The internally calibrated values for the elasticity of substitution (σ) and the consumption weight in the utility function of the good produced in the oligopolistic sector (ϕ) are 2.92 and 0.11, respectively. The other parameters of the model are specified in Table 2. The statistics are derived from numerical simulations involving 5000 replications of the equilibrium paths of a 54-year model economy. For any variable w, g_w denotes the log change in adjacent periods. The p-values of the correlations are given in the parentheses.

	Data	Oligopoly ($\gamma = 10$)	Competitive ($\gamma = 10$)	Oligopoly ($\gamma = 7.5$)
Vol(ε^X)	3.12%	3.20%	3.24%	3.20%
Vol(ε^θ)	2.07%	2.11%	2.12%	2.12%
Mean($pmcr$)	1.2	1.2	1.0	1.2
Vol(g_I)	9.72%	14.29%	29.64%	14.29%
Vol(g_H)	4.26%	4.28%	4.40%	4.28%
Vol(g_Y)	4.26%	5.66%	5.86%	5.65%
Corr(g_I, g_X)	0.62	0.68 (0.0)	0.51 (0.0)	0.66 (0.0)
Corr(g_I, g_θ)	0.31	0.66 (0.0)	0.62 (0.0)	0.67 (0.0)
Corr(g_H, g_X)	0.82	0.62 (0.0)	0.60 (0.0)	0.62 (0.0)
Corr(g_H, g_θ)	0.63	0.77 (0.0)	0.75 (0.0)	0.77 (0.0)
Corr(g_{pmcr}, g_X)	0.83	0.09 (0.0)	0.0 (0.0)	0.08 (0.0)

Table 4. Moderately Concentrated Industries: Asset Markets Variables

This table presents salient statistics on equilibrium asset markets variables for the sample of industries described in Tables 2 and 3. The calibration for σ and ϕ are given in Table 3, while the other parameters are specified in Table 2. The statistics are derived from numerical simulations involving 5000 replications of the equilibrium paths of a 54-year model economy.

	Data	Oligopoly ($\gamma = 10$)	Competitive ($\gamma = 10$)	Oligopoly ($\gamma = 7.5$)
$\mathrm{Vol}(\varepsilon^X)$	3.12%	3.20%	3.20%	3.20%
$\mathrm{Vol}(\varepsilon^\theta)$	2.07%	2.12%	2.12%	2.12%
$\mathrm{Mean}(pmcr)$	1.2	1.2	1.0	1.2
$\mathbb{E}(r^y - r^f)$	5.93%	2.65%	3.97%	1.75%
$\mathbb{E}(r^x - r^f)$	5.55%	1.41%	1.44%	0.94%
$\mathrm{Vol}^u(r^y - r^f)$	16.10%	11.40%	18.73%	10.82%
$\mathrm{Vol}^u(r^x - r^f)$	15.69%	5.02%	5.10%	4.58%
$\frac{\mathbb{E}(r^y - r^f)}{\mathrm{Vol}^u(r^y - r^f)}$	0.37	0.23	0.21	0.16
$\frac{\mathbb{E}(r^x - r^f)}{\mathrm{Vol}^u(r^x - r^f)}$	0.35	0.28	0.28	0.21
$\mathbb{E}(r^f)$	1.36%	3.00%	2.99%	3.00%

Table 5. Highly Concentrated Industries: Product Market Variables

This table presents salient statistics on equilibrium capital investment, material input demand, output and price-cost margins for a sample of 31 manufacturing industries (based on the NBER-CES sample of manufacturing industries (1958-2011)) where the largest four firms account for at least 70% of industry output. The internally calibrated values for the elasticity of substitution (σ) and the consumption weight in the utility function of the good produced in the oligopolistic sector (ϕ) are 3 and 0.05, respectively. The other parameters of the model are specified in Table 5. The statistics are derived from numerical simulations involving 5000 replications of the equilibrium paths of a 54-year model economy. For any variable w, g_w denotes the log change in adjacent periods. The p-values of the correlations are given in the parentheses.

	Data	Oligopoly ($\gamma = 10$)	Competitive ($\gamma = 10$)	Oligopoly ($\gamma = 7.5$)
Vol(ε^X)	3.20%	3.24%	3.30%	3.24%
Vol(ε^θ)	1.90%	1.94%	1.90%	1.94%
Mean($pmcr$)	1.5	1.5	1.00	1.5
Vol(g_I)	17.69%	19.50%	26.07%	19.22%
Vol(g_H)	7.84%	3.49%	3.67%	3.48%
Vol(g_Y)	7.12%	4.57%	4.85%	4.55%
Corr(g_I, g_X)	0.45	0.61 (0.0)	0.55 (0.0)	0.60 (0.0)
Corr(g_I, g_θ)	0.13	0.65 (0.0)	0.60 (0.0)	0.66 (0.0)
Corr(g_H, g_X)	0.68	0.54 (0.0)	0.52 (0.0)	0.54 (0.0)
Corr(g_H, g_θ)	0.62	0.63 (0.0)	0.59 (0.0)	0.63 (0.0)
Corr(g_{pmcr}, g_X)	-0.41	-0.07 (0.0)	0.0 (0.0)	-0.07 (0.0)

Table 6. Highly Concentrated Industries: Asset Markets Variables

This table presents salient statistics on equilibrium asset markets variables for the sample of industries described in Tables 5 and 6. The calibration for σ and ϕ are given in Table 6, while the other parameters are specified in Table 5. The statistics are derived from numerical simulations involving 5000 replications of the equilibrium paths of a 54-year model economy.

	Data	Oligopoly ($\gamma = 10$)	Competitive ($\gamma = 10$)	Oligopoly ($\gamma = 7.5$)
$\text{Vol}(\varepsilon^X)$	3.20%	3.24%	3.24%	3.24%
$\text{Vol}(\varepsilon^\theta)$	1.90%	1.94%	1.94%	1.94%
Mean($pmcr$)	1.5	1.5	1.00	1.5
$\mathbb{E}(r^y - r^f)$	5.09%	1.49%	3.86%	0.99%
$\mathbb{E}(r^x - r^f)$	5.55%	1.49%	1.55%	0.99%
$\text{Vol}^u(r^y - r^f)$	18.28%	5.46%	19.20%	5.00%
$\text{Vol}^u(r^x - r^f)$	15.69%	5.10%	5.19%	4.61%
$\frac{\mathbb{E}(r^y - r^f)}{\text{Vol}^u(r^y - r^f)}$	0.28	0.27	0.20	0.20
$\frac{\mathbb{E}(r^x - r^f)}{\text{Vol}^u(r^x - r^f)}$	0.35	0.29	0.30	0.21
$\mathbb{E}(r^f)$	1.36%	2.99%	2.99%	3.0%

Table 7. Global Solutions: Highly Concentrated Industries

This table presents salient statistics on equilibrium product and asset markets variables for the sample of highly concentrated industries (described in Tables 5 and 6) computed using the projection method described in Section 8 . The calibration for σ and ϕ are given in Table 6, while the other parameters are specified in Table 5. The statistics are derived from numerical simulations involving 5000 replications of the equilibrium paths of a 54-year model economy.

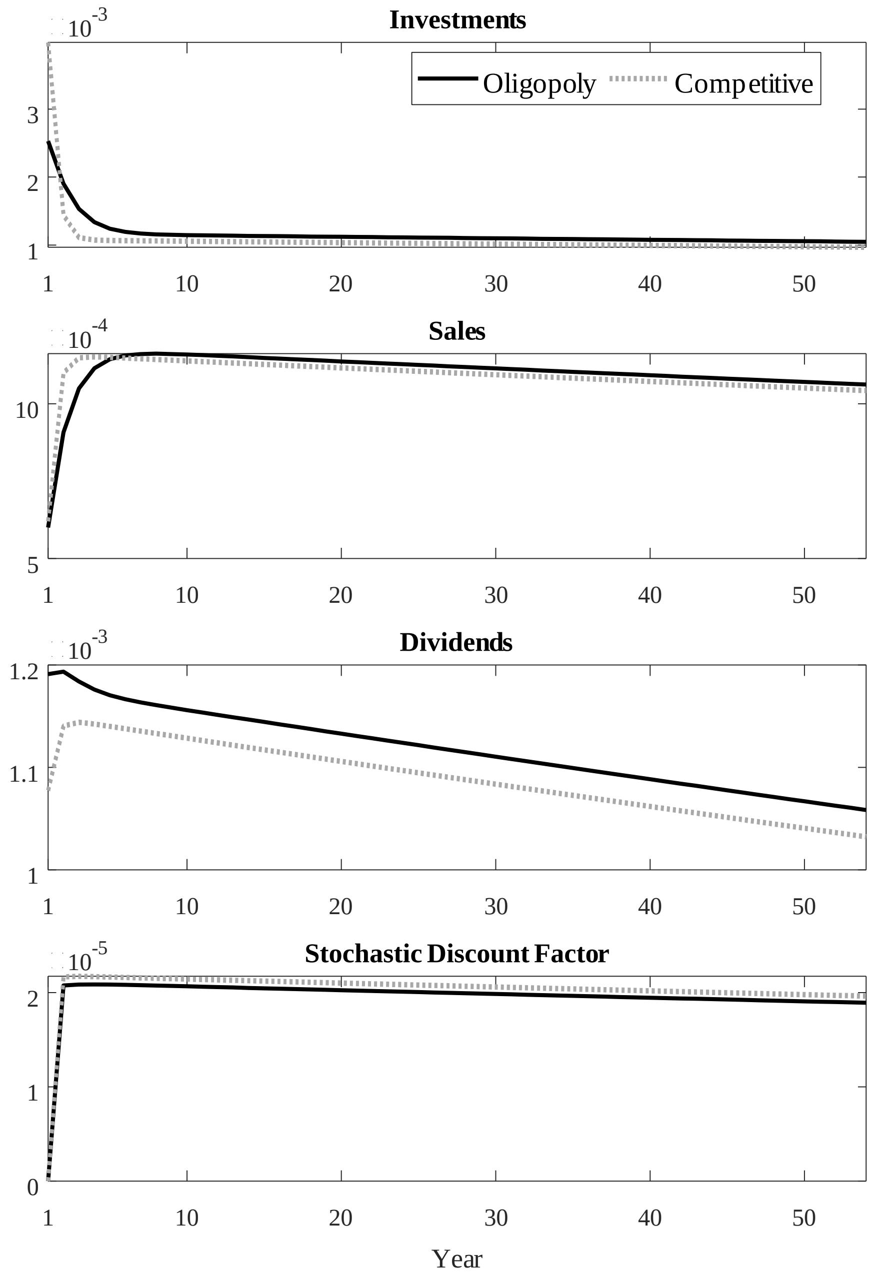
Panel A: Product Market Variables

	Data	Oligopoly ($\gamma = 10$)	Competitive ($\gamma = 10$)
Vol(ε^X)	3.20%	3.22%	3.14%
Vol(ε^θ)	1.90%	1.94%	1.94%
Mean($pmcr$)	1.5	1.5	1.00
Vol(g_I)	17.69%	8.84%	13.48%
Vol(g_H)	7.84%	9.62%	40.27%
Vol(g_Y)	7.12%	9.60%	40.18%
Corr($g_I,\ g_X$)	0.45	0.15 (0.0)	-0.07 (0.0)
Corr($g_I,\ g_\theta$)	0.13	0.05 (0.0)	-0.01 (0.0)
Corr($g_H,\ g_X$)	0.68	0.20 (0.0)	0.00 (0.0)
Corr($g_H,\ g_\theta$)	0.62	0.21 (0.0)	0.07 (0.0)
Corr($g_{pmcr},\ g_X$)	-0.41	-0.23 (0.0)	0.0 (0.0)

Panel B: Asset Markets Variables

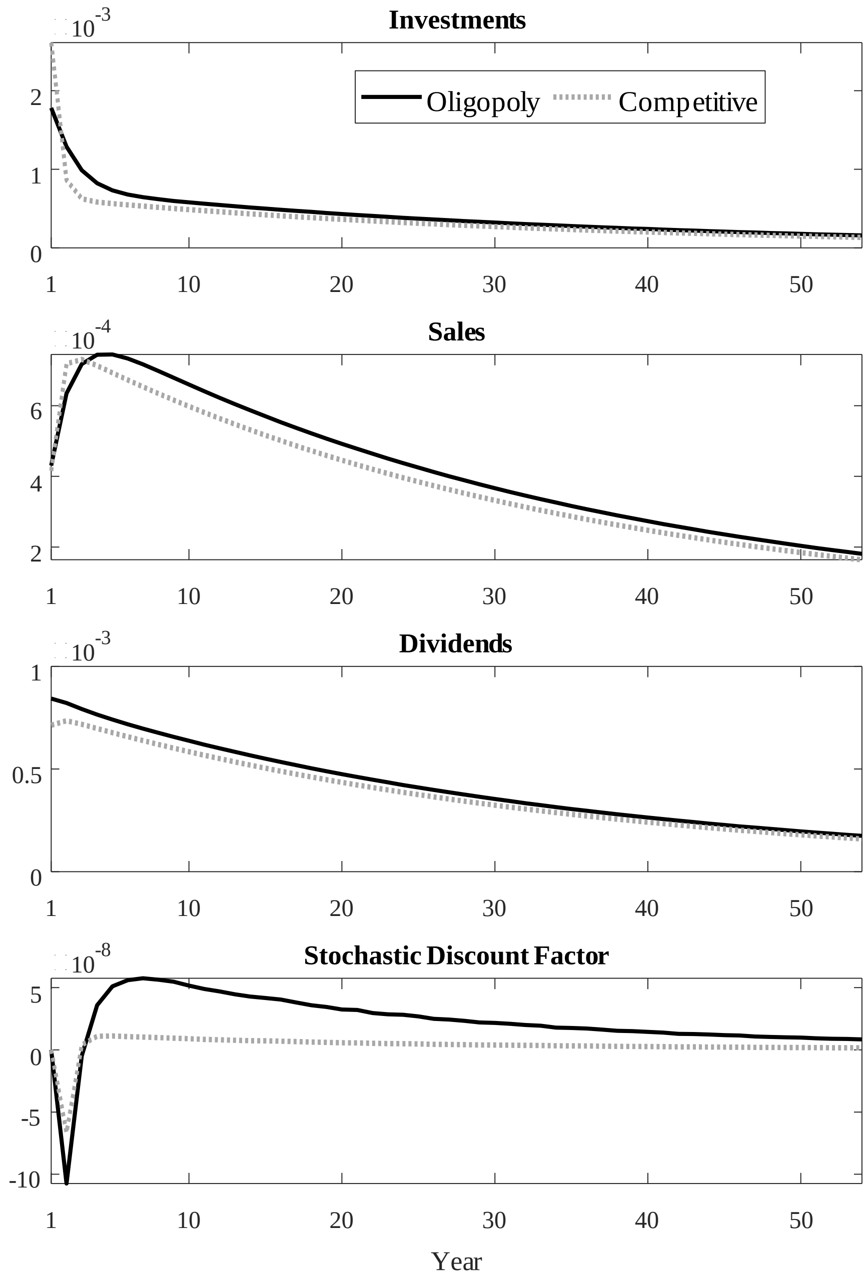
	Data	Oligopoly ($\gamma = 10$)	Competitive ($\gamma = 10$)
Vol(ε^X)	3.20%	3.24%	3.24%
Vol(ε^θ)	1.90%	1.94%	1.94%
Mean($pmcr$)	1.5	1.5	1.00
$E(r^y - r^f)$	5.09%	1.32%	2.02%
$E(r^x - r^f)$	5.55%	2.70%	3.96%
$\mathrm{Vol}^u(r^y - r^f)$	18.28%	4.30%	17.31%
$\mathrm{Vol}^u(r^x - r^f)$	15.69%	9.61%	4.21%
$\frac{\mathbb{E}(r^y - r^f)}{\mathrm{Vol}^u(r^y - r^f)}$	0.28	0.31	0.25
$\frac{\mathbb{E}(r^x - r^f)}{\mathrm{Vol}^u(r^x - r^f)}$	0.35	0.28	0.29
$E(r^f)$	1.36%	2.62%	2.61%

Figure 1: Highly Concentrated Industries: Impulse Response Function (X)



This figure plots the impulse response functions (IRFs) of logarithms of salient product and asset markets variables that are subjected to one standard-deviation shock to the (aggregate) output in sector x (X).

Figure 2: **Highly Concentrated Industries: Impulse Response Function** (θ)



This figure plots the impulse response functions (IRFs) of logarithms of salient product and asset markets variables that are subjected to one standard-deviation shock to the productivity shock in sector y (θ).

Online Appendix

Appendix A: Derivations and Proofs

A.1 Derivation of Optimal Consumption and Portfolio Policies

Since the objective function is strictly increasing and concave, the optimal consumption and portfolio policies can be characterized through a two-step process, where optimal consumption c_t is determined as a function of available consumption expenditure Z_t, and the optimal portfolio is then determined taking as given the optimal consumption policy. Using the dynamic programming principle (DP), at any t, the representative consumers optimization problem (6)-(7) can be written as

$$\max_{\mathbf{c}_t,\mathbf{q}_{t+1}} E_t\left[\sum_{\tau=t}^{\infty}\beta^{\tau-t}\frac{C_\tau^{1-\gamma}-1}{1-\gamma}\right]+\chi_t\left[Z_t-\mathbf{p}_t\cdot\mathbf{c}_t\right]. \tag{A1.1}$$

Here, χ_t is the Lagrange multiplier for the budget constraint (7). Since preferences are strictly increasing, the budget constraint is binding and $\chi_t > 0$. Next, using the definition of aggregate consumption (8), the first order optimality conditions for $c_t^j, j = x, y$, can be written

$$(C_t)^{\frac{1-\gamma\sigma}{\sigma}}(c_t^j)^{-\frac{1}{\sigma}}\phi^j = \chi_t p_t^j, \tag{A1.2}$$

where $p_t^x = 1$, $\phi^x \equiv (1-\phi), \phi^y \equiv \phi$. It follows from (A1.2) that

$$p_t^j c_t^j = \chi_t^{-\sigma}(p_t^j)^{1-\sigma}(C_t)^{-(1-\gamma\sigma)}(\phi^j)^\sigma \tag{A1.3}$$

Then recognizing that $Z_t = p_t \cdot c_t$, and using (A1.3), and the definition of the aggregate price index P_t (see (10)) allows one to solve for the Lagrange multiplier as

$$\chi_t = \left(\frac{Z_t}{P_t}\right)^{-\frac{1}{\sigma}} P_t^{-1}(C_t)^{\frac{1-\gamma\sigma}{\sigma}}. \tag{A1.4}$$

Substituting this in (A1.2) and rearranging terms then gives the optimal consumption functions given in (9).

Next, for any $\tau \geq t$, let $U_\tau \equiv \beta^{\tau-t}\frac{C_\tau^{*1-\gamma}-1}{1-\gamma}$ denote the indirect period utility function with the optimal consumption functions given in (9). The envelope theorem then yields $\chi_\tau = \frac{\partial U_\tau}{\partial Z_\tau}$. Using the fact that

$$Z_\tau = q_\tau\cdot(D_\tau+S_\tau) - q_{\tau+1}\cdot S_\tau,$$

then yields the optimality conditions for q_{t+1}

$$\chi_t \mathbf{S}_t = \mathbb{E}_t \left[\beta \chi_{t+1} (\mathbf{D}_{t+1} + \mathbf{S}_{t+1}) \right]. \tag{A1.5}$$

But using $C_t^* = \frac{Z_t}{P_t}$ and substituting in (A1.4) gives $\chi_t = (C_t^*)^{-\gamma} P_t^{-1}$. Since this holds for any τ, inserting in (A1.5) yields Eq. (11). ∎

A.2 Proofs

Proof of Proposition 1: Substituting the optimal consumption functions (9) in the market clearing conditions (13)-(14) in a symmetric equilibrium yield

$$\frac{Z_t}{P_t} \left[P_t (1-\phi) \right]^{\sigma} = X_t - \sum_{i=1}^{N} [A(I_{it}, K_{it}) + H_{it})] \tag{A2.1}$$

$$\frac{Z_t}{P_t} \left[\frac{P_t \phi}{p_t^y} \right]^{\sigma} = \tilde{Y}_t \tag{A2.2}$$

Dividing (A2.1) by (A2.2) and rearranging terms yields $\breve{p}_t^y = \left(\frac{\breve{W}_t}{N \breve{Y}_t} \right)^{1/\sigma} \eta_t$ in a symmetric equilibrium. Since competitive firms equate marginal costs with any given price, in equilibrium the marginal cost $[F_H(K_t, \breve{H}_t, \theta_t)]^{-1}$ is equated with the price as given in (16). Next, using the Bellman-representation (15), along any competitive equilibrium path, at any t, conditional on Γ_t, the optimization problem for the typical competitive firm is

$$V_t(\Gamma_t) = \max_{I, H_t \geq 0} \left(\frac{Z_t}{P_t} \right)^{-\gamma} \left(\frac{\breve{p}_t^y Y_t - H_t - A(I_t, K_t)}{P_t} \right) + \beta \mathbb{E}_t \left[V_{t+1}(\Gamma_{t+1}) \right], \text{ s.t., (1)–(3).} \tag{A2.3}$$

Then, subject to (1)–(3), the optimal (interior) investment input path satisfies

$$0 = -\frac{\partial V_t(\Gamma_t)}{\partial K_t} + \left(\frac{Z_t}{P_t} \right)^{-\gamma} \left(\frac{1}{P_t} \right) \frac{\partial D_t^y}{\partial K_t} + \beta (1-\delta) \frac{\partial \mathbb{E}_t \left[V_t(\Gamma_{t+1}) \right]}{\partial K_{t+1}}, \tag{A2.4}$$

$$0 = \left(\frac{Z_t}{P_t} \right)^{-\gamma} \left(\frac{1}{P_t} \right) \frac{\partial D_t^y}{\partial I_t} + \beta \frac{\partial \mathbb{E}_t \left[V_{t+1}(\Gamma_{t+1}) \right]}{\partial I_t}. \tag{A2.5}$$

Now, $D_t^y = p_t^y Y_t - H_t - A(I_t, K_t)$. Hence, $\frac{\partial D_t^y}{\partial I_t} = -A_I(I_t, K_t)$. Furthermore, $\frac{\partial K_{t+1}}{\partial I_t} = 1$ and thus

$\frac{\partial \mathbb{E}_t[V_{t+1}(\Gamma_{t+1})]}{\partial I_t} = \frac{\partial \mathbb{E}_t[V_{t+1}(\Gamma_{t+1})]}{\partial K_{t+1}}$. Recalling that the SDF is

$$M_{t+1} \equiv \beta \left(\frac{Z_{t+1}}{Z_t}\right)^{-\gamma} \left(\frac{P_{t+1}}{P_t}\right)^{\gamma-1},$$

(A2.4) and (A2.5) then together imply that the Euler condition characterizing the equilibrium investment path is

$$-\frac{\partial D_t^y}{\partial I_t} = \mathbb{E}_t \left[M_{t+1} \left(\frac{\partial D_{t+1}^y}{\partial K_{t+1}} - (1-\delta) \frac{\partial D_{t+1}^y}{\partial I_{t+1}} \right), \right] \tag{A2.6}$$

where in (A2.6) we have used iterated expectations and recursively substituted the optimality condition for I_{t+1}. Now, using the envelope theorem (that sets the indirect effects of ∂K_{t+1} on the optimally chosen I_{t+1} and H_{t+1} to zero), in a symmetric competitive (price-taking) equilibrium with $Y_{it+1} = Y_{t+1}$, we have

$$\frac{\partial D_{t+1}^y}{\partial K_{t+1}} = p_{t+1}^y F_K(K_{t+1}, H_{t+1}, \theta_{t+1}) - A_K(I_{t+1}, K_{t+1}). \tag{A2.7}$$

(A2.6)-(A2.7) and $\frac{\partial D_{t+1}^y}{\partial I_{t+1}} = -A_I(I_{t+1}, K_{t+1})$ then together characterize the equilibrium path for investment in a symmetric competitive equilibrium viz.,

$$\begin{aligned} A_I(\breve{I}_t, K_t) \quad = \quad & \mathbb{E}_t \Big[\breve{M}_{t+1} \{ \breve{p}_{t+1}^y F_K(\breve{K}_{t+1}, \breve{H}_{t+1}, \theta_{t+1}) - A_K(\breve{I}_{t+1}, \breve{K}_{t+1}) \} + \\ & \qquad (1-\delta) A_I(\breve{I}_{t+1}, \breve{K}_{t+1}) \Big]. \end{aligned} \tag{A2.8}$$

■

Proof of Lemma 1: By construction, for each $t \geq \tau$ and any Γ_t, ($\underline{\mathrm{p}}_t^y$,$\underline{\mathrm{H}}_t$) determined by (18)-(19) satisfy the general equilibrium market clearing conditions (13)-(14) and just cover the average input cost. Note that the average cost with material input $\underline{\mathrm{H}}_t$ is

$$\frac{\underline{\mathrm{H}}_t}{F(K_t, \underline{\mathrm{H}}_t, \theta_t)}, \tag{A.2.9}$$

and hence (18) implies that $\underline{\mathrm{p}}_t^y$ given in (19) equals (A.2.9). Furthermore, because of decreasing returns to scale in H, the marginal cost $[F_H(K, H, \theta)]^{-1}$ exceeds the average cost for every $Y > 0$. Hence, any deviation to a price below $\underline{\mathrm{p}}_t^y$ yields negative profits while any deviation to a price above $\underline{\mathrm{p}}_t^y$ clearly generates zero sales. Hence, with $\underline{\mathrm{I}}_t = 0$, ($\underline{\mathrm{p}}_t^y$,$\underline{\mathrm{H}}_t$) yields zero economic profits. The action profiles $\mu_t^{y\prime} = (\underline{\mathrm{p}}_t^y, \underline{\mathrm{H}}_t, \underline{\mathrm{I}}_t)$ is (weakly) optimal for each firm and hence a Nash equilibrium for any Γ_t.

But we have also established that no firm can improve expected profits by deviating from $\mu_t^{y\prime}$ at t and returning to this strategy at $t+1$. Hence, Hence, $\{\mu_t^{y\prime} = (\underline{p}_t^y, \underline{H}_t, \underline{I}_t)\}_{t \geq \tau}$ is a SPE along subgame Γ_τ.■

Proof of Lemma 2: Fix any $\Gamma_t = (\cdot, \tilde{K}_t, (X_t, \theta_t))$. Note from (15) that

$$\begin{aligned}\tilde{\mu}_{t,p}^{y*} \quad \in \quad \arg\max_{p_t^y, I_t} \left(\frac{Z_t}{P_t}\right)^{-\gamma} \left(\frac{1}{P_t}\right) \Big[p_t^y c_t^{y*}(p_t, Z_t) - H^{-1}(c_t^{y*}; (\tilde{K}_t, \theta_t)) - \\ - A(I_t, \tilde{K}_t)\Big] + \beta E_t \left[V_{t+1}(\Gamma_{t+1})\right], \qquad \text{(A.2.10)}\end{aligned}$$

where, from (9),

$$c_t^{y*}(p_t, Z_t) = \phi^\sigma Z_t (P_t)^{1-\sigma} (p_t^y)^{-\sigma}, \qquad \text{(A.2.11)}$$

and $H^{-1}(c_t^{y*}; (\tilde{K}_t, \theta_t))$ is the material input demand function for output conditional on $(\tilde{K}_t, \theta_t)$, that is

$$H^{-1}(c_t^{y*}; (\tilde{K}_t, \theta_t)) = \left(\frac{c_t^{y*}}{\theta_t (\tilde{K}_t)^{\psi_K}}\right)^{\frac{1}{\psi_H}}. \qquad \text{(A.2.12)}$$

But market clearing in sector y requires (see (14)) that for any chosen $\tilde{p}_t^{y*}$, $c_t^{y*}((p_t^x, \tilde{p}_t^{y*}), Z_t) = \tilde{Y}_t$ and hence from (A.2.11) we must have

$$\tilde{p}_t^{y*} = \phi \left(\frac{Z_t P_t^{1-\sigma}}{\tilde{Y}_t}\right)^{\frac{1}{\sigma}}. \qquad \text{(A.2.13)}$$

Meanwhile, the market clearing price for sector x, say $\tilde{p}_t^{x*}$, must satisfy (see (13))

$$\tilde{p}_t^{x*} = (1-\phi) \left(\frac{Z_t P_t^{1-\sigma}}{X_t - (H_t + A(I_t, \tilde{K}_t))}\right)^{\frac{1}{\sigma}}. \qquad \text{(A.2.14)}$$

Hence, normalizing $\tilde{p}_t^{x*} = 1$ and dividing (A.2.13) by (A.2.14) gives the sector y price that must hold along any (general) equilibrium path, viz.,

$$\tilde{p}_t^{y*}(H_t, I_t) = \left(\frac{X_t - (H_t + A(I_t, \tilde{K}_t))}{\tilde{Y}_t}\right)^{\frac{1}{\sigma}}. \qquad \text{(A.2.15)}$$

It also follows that in equilibrium

$$H^{-1}(c_t^{y*}; (\tilde{K}_t, \theta_t)) = \left(\frac{\tilde{Y}_t}{\theta_t (\tilde{K}_t)^{\psi_K}}\right)^{\frac{1}{\psi_H}} = H_t. \qquad \text{(A.2.16)}$$

Then substituting (A.2.15)-(A.2.16) in (A.2.10) yields the equivalent optimization problem

$$\begin{aligned}\tilde{\mu}^{y*}_{t,H} \quad &\in \quad \arg\max_{H_t \geq 0, I_t} \left(\frac{Z_t}{P_t}\right)^{-\gamma} \left(\frac{1}{P_t}\right) \left[\tilde{p}^{y*}_t(H_t, I_t) F(\tilde{K}_t, H_t, \theta_t) - (H_t + A(I_t, \tilde{K}_t))\right. \\ &\qquad\qquad\qquad\qquad\qquad \left. + \beta E_t \left[V_{t+1}(\Gamma_{t+1})\right]\right. . \qquad \text{(A.2.17)}\end{aligned}$$

∎

Proof of Proposition 2: As shown in Lemma 2 (see (A.2.15)) along the equilibrium path the relative price of good y as a function of (H_t, I_t) is, viz.,

$$\tilde{p}^{y*}_t(H_t, I_t) = \left[\frac{X_t - [A(I_t, \tilde{K}_t) + H_t]}{F(\tilde{K}_t, H_t, \theta_t)}\right]^{1/\sigma} \eta. \qquad \text{(A.2.18)}$$

It is convenient to set $\tilde{W}_t \equiv X_t - [A(I_t, \tilde{K}_t) + H_t]$. Since H_t does not directly affect expected future profits, the optimal material input demand in a monopoly is determined by $\frac{\partial(\tilde{p}^{y*}_t \tilde{Y}_t)}{\partial \tilde{H}_t} = 1$. We compute (using (A.2.18))

$$\tilde{p}^{y*}_t \tilde{Y}_t = \eta(\tilde{W}_t)^{\frac{1}{\sigma}} (\tilde{Y}_t)^{\frac{\sigma-1}{\sigma}}, \qquad \text{(A.2.19)}$$

and, hence, at the optimum,

$$\frac{\partial(\tilde{p}^{y*}_t \tilde{Y}_t)}{\partial \tilde{H}_t} = \eta\sigma^{-1}(\tilde{W}_t)^{\frac{1-\sigma}{\sigma}} (\tilde{Y}_t)^{-\frac{1}{\sigma}} \left[(\sigma-1)\tilde{W}_t F_H(\tilde{K}_t, H_t, \theta_t) - \tilde{Y}_t\right] = 1. \qquad \text{(A.2.20)}$$

Using (A.2.18), (A.2.20) can be written (denoting the optimal material input as $\tilde{H}^*_t$),

$$\frac{(\sigma-1)}{\sigma} \tilde{p}^{y*}_t F_H(\tilde{K}_t,\ \tilde{H}^*_t, \theta_t) - \frac{(\tilde{p}^{y*}_t)^{1-\sigma}\eta^{\sigma}}{\sigma} = 1. \qquad \text{(A.2.21)}$$

Rearranging terms in (A.2.21) then gives (21).

Next, the optimal (interior) investment chosen by the monopolist satisfies the following system of equations

$$\begin{aligned}\frac{\partial V_t(\Gamma_t)}{\partial K_t} \quad &= \quad \left(\frac{Z_t}{P_t}\right)^{-\gamma} \left(\frac{1}{P_t}\right) \frac{\partial D^y_t}{\partial K_t} + \beta(1-\delta) \frac{\partial \mathbb{E}_t \left[V_{t+1}(\Gamma_{t+1})\right]}{\partial K_{t+1}}, \qquad \text{(A2.22)} \\ -\left(\frac{Z_t}{P_t}\right)^{-\gamma} \left(\frac{1}{P_t}\right) \frac{\partial D^y_t}{\partial I_t} \quad &= \quad \beta \frac{\partial \mathbb{E}_t \left[V_{t+1}(\Gamma_{t+1})\right]}{\partial I_t}. \qquad \text{(A2.23)}\end{aligned}$$

where $D_t^y = \tilde{p}_t^{y*} F(\tilde{K}_t, H_t, \theta_t) - (H_t + A(I_t, \tilde{K}_t))$. Then it follows from (A.2.18) and (A.2.19) that

$$\begin{aligned} \frac{\partial D_t^y}{\partial I_t} &= -A_I(I_t, \tilde{K}_t) \left[1 + \frac{(\tilde{p}_t^{y*})^{1-\sigma} \eta^\sigma}{\sigma} \right], && \text{(A2.24)} \\ \frac{\partial D_t^y}{\partial K_t} &= \tilde{p}_t^{y*} F_K(\tilde{K}_t, H_t, \theta_t) \left(\frac{\sigma - 1}{\sigma} \right) - A_K(I_t, \tilde{K}_t). && \text{(A2.25)} \end{aligned}$$

Furthermore, $\frac{\partial K_{t+1}}{\partial I_t} = 1$ and hence $\frac{\partial \mathbb{E}_t[V_{t+1}(\Gamma_{t+1})]}{\partial I_t} = \frac{\partial \mathbb{E}_t[V_{t+1}(\Gamma_{t+1})]}{\partial K_{t+1}}$. Then (A2.24) and (A2.25) together imply that the Euler condition characterizing the equilibrium investment path is given by

$$-\frac{\partial \tilde{D}_t^{y*}}{\partial I_t} = \mathbb{E}_t \left[M_{t+1}^* \left(\frac{\partial \tilde{D}_t^{y*}}{\partial K_{t+1}} - (1 - \delta) \frac{\partial \tilde{D}_t^{y*}}{\partial I_{t+1}} \right) \right], \tag{A2.26}$$

where $\tilde{D}_t^{y*} = \tilde{p}_t^{y*} F(\tilde{K}_t, \tilde{H}_t^*, \theta_t) - [A(\tilde{I}_t^*, \tilde{K}_t) + \tilde{H}_t^*]$. In (A2.26) we have used iterated expectations and recursively substituted the optimality condition for I_{t+1}^* (using A(2.25)),

$$-\left(\frac{Z_{t+1}}{P_{t+1}} \right)^{-\gamma} \left(\frac{1}{P_{t+1}} \right) \frac{\partial \tilde{D}_{t+1}^{y*}}{\partial I_{t+1}} = \beta \frac{\partial \mathbb{E}_{t+1} [V_{t+2}(\Gamma_{t+2})]}{\partial K_{t+2}}. \tag{A2.27}$$

Now, using the envelope theorem (that sets the indirect effects of ∂K_{t+1} on the optimally chosen $\tilde{I}_{t+1}^*$ and $\tilde{H}_{t+1}^*$ to zero), (A2.27) implies that

$$\frac{\partial \bar{D}_{t+1}^{y*}}{\partial K_{t+1}} = p_{t+1}^{y*} \frac{\partial Y_{t+1}}{\partial K_{t+1}} \left[\frac{\sigma - 1}{\sigma} \right] - A_K(\bar{I}_{t+1}^*, K_{t+1}). \tag{A2.28}$$

■

Proof of Proposition 3: Along the symmetric collusive path, each firm produces $1/N$ of the monopoly output and hence each firm chooses $\bar{H}_t^*$ such that

$$N \left(\theta_t (K)^{\psi_K} (\bar{H}_t^*)^{\psi_H} \right) = \theta_t (N K_t)^{\psi_K} (\tilde{H}_t^*)^{\psi_H}, \tag{A.2.29}$$

which yields $\bar{H}_t^* = \left(N^{\frac{\psi_K - 1}{\psi_H}} \right) \tilde{H}_t^*$. And because of the linearity of the law of motion of capital stock

$$\begin{aligned} \tilde{K}_{t+1} &= N(K_t(1-\delta) + N^{-1} \tilde{I}_t^*) \\ &= \tilde{K}_t(1-\delta) + \tilde{I}_t^* && \text{(A.2.30)} \end{aligned}$$

so that the equilibrium evolution of the industry capital stock is the same as that in the monopoly.

Now $\{\bar{\mu}_t^{y*}(\Gamma_t)\}_t$ specifies a symmetric SPE if and only the following two incentive constraints must be satisfied. First, at every Γ_t such that there has been no deviation by an firm up to $t-1$, there should be no gain to any firm from deviating at t (compared to continuing with the collusive policies). Using (25)-(26) and noting that $\tilde{Y}_t^* = NY_t^*$ and $\bar{H}_t'(\tilde{Y}_t^*;(K_t,\theta_t)) = (N)^{\frac{1}{\psi_H}}\bar{H}_t^*$, this incentive constraint can be written

$$\left(\frac{Z_t}{P_t}\right)^{-\gamma}\left(\frac{1}{P_t}\right)\left[(\bar{p}_t^{y*}(NY_t^*) - N^{\frac{1}{\psi_H}}\bar{H}_t^* - A(\bar{I}_t^*,K_t)) - \bar{D}_t^{y*}\right] \leq \beta\mathbb{E}_t\left[\bar{V}_{t+1}^*(\bar{\Gamma}_{t+1}^*)\right], \tag{A.2.31}$$

where the second term in the RHS of (A.2.31) is the expected value of the firm next period conditional on following the collusive policies. But then using the fact that $\bar{D}_t^{y*} = \bar{p}_t^{y*}F(K_t,\bar{H}_t^*,\theta_t) - (\bar{H}_t^* + A(\bar{I}_t^*,K_t))$, (A.2.31) can be written

$$\left(\frac{Z_t}{P_t}\right)^{-\gamma}\left(\frac{1}{P_t}\right)\left[\bar{p}_t^{y*}\bar{Y}_t^*(N-1) - (N^{\frac{1}{\psi_H}} - 1)\bar{H}_t^*\right] \leq \beta\mathbb{E}_t\left[\bar{V}_{t+1}^*(\bar{\Gamma}_{t+1}^*)\right]. \tag{A.2.32}$$

Hence, the incentive constraint will be satisfied strictly if

$$\left(\frac{Z_t}{P_t}\right)^{-\gamma}\left(\frac{1}{P_t}\right)\left[\bar{p}_t^{y*}\bar{Y}_t^*(N-1)\right] \leq \beta\mathbb{E}_t\left[\bar{V}_{t+1}^*(\bar{\Gamma}_{t+1}^*)\right]. \tag{A.2.33}$$

But the LHS of (A.2.33) is increasing in N while the RHS is non-increasing in N. Hence, there exists some $\bar{N}$ such that (A.2.33) is satisfied if the number of firms $N \leq \bar{N}$. The second incentive constraint requires that there be no gains from one-stage deviation during the "punishment" phase. But this has been established in Lemma 1.

It now remains to show that if (A.2.33) holds, then given Lemma 1, there exist no infinite sequence of deviations that are value-improving for any firm. Suppose to the contrary that there exists some Γ_t such that (for some firm) deviations for $\tau \geq t$ are value-improving. Now let

$$V_t^-(\Gamma_t) = \sup_{\{\mu_\tau^{y-}\}_{\tau\geq t}} \mathbb{E}_t\left[\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left(\frac{Z_\tau}{P_\tau}\right)^{-\gamma}\left(\frac{D_\tau^y}{P_\tau}\right) \mid \Gamma_t\right], \text{ s.t., (1)–(3).} \tag{A.2.34}$$

i.e., the supremum of expected payoffs along all deviating strategies $\{\mu_\tau^{y-}\}_{\tau\geq t}$ along the continuation game defined by Γ_t. But given Lemma 1 and (A.2.32)-(A.2.33), we have (using the recursive representation of $V_t^-(\Gamma_t)$)

$$V_t^-(\Gamma_t) < \left(\frac{Z_t}{P_t}\right)^{-\gamma}\left(\frac{\bar{p}_t^{y*}\bar{Y}_t^*(N-1)}{P_t}\right) + \beta\mathbb{E}_t\left[V_{t+1}^-(\Gamma_{t+1})\right]. \tag{A.2.35}$$

But by Lemma 1, $E_t\left[V_{t+1}^-(\Gamma_{t+1})\right] \leq E_t\left[V_{t+1}^{'}(\Gamma_t)\right]$, which is the expected value along the punishment strategies $\{\mu_\tau^{y'}\}_{\tau\geq t+1}$ (see Lemma 1) played following any defection at t. Since $E_t\left[V_{t+1}^{'}(\Gamma_t)\right] = 0$,

$$V_t^-(\Gamma_t) < \left(\frac{Z_t}{P_t}\right)^{-\gamma}\left(\frac{\bar{p}_t^{y*}\bar{Y}_t^*(N-1)}{P_t}\right). \tag{A.2.36}$$

Because this is true for all t and Γ_t, it follows from (A.2.33) that for $N \leq \bar{N}$, there exists no state Γ_t such that $V_t^-(\Gamma_t) \geq \bar{V}_t^*(\Gamma_t)$, which contradicts the hypothesis that an infinite sequence of deviations exists that is value-improving. ■

Proof of Proposition 4: Fix any t and any state Γ_t. As above, substituting the optimal consumption functions (9) in the market clearing conditions (13)-(14) in a symmetric equilibrium yield

$$\frac{Z_t}{P_t}\left[P_t(1-\phi)\right]^\sigma = X_t - \sum_{i=1}^{N}[A(I_{it}^*, K_{it}) + H_{it}^*)] \tag{A2.37}$$

$$\frac{Z_t}{P_t}\left[\frac{P_t\phi}{p_t^y}\right]^\sigma = \tilde{Y}_t \tag{A2.38}$$

Dividing (A.2.38) by (A.2.37) and rearranging terms yields p_t^{y*} given in (27) for a symmetric equilibrium. Next, the constrained optimization problem for firm i is (for $I_i = (I_{i0}, ...), H_i = (H_{i0}, ...)$),

$$\max_{\mathbf{I}_i, \mathbf{H}_i \geq 0} E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{Z_t}{P_t}\right)^{-\gamma}\left(\frac{p_t^{y*}Y_{it} - H_{it} - A(I_{it}, K_{it})}{P_t}\right)\right], \text{ s.t., (1)–(3).} \tag{A2.39}$$

Substituting the constraints in the objective function, and using the assumption that firms take the pricing kernel as exogenous, the dynamic programming (DP) principle implies that along any equilibrium path, at any t and conditional on the state $\Gamma_t = (K_t, X_t, \theta_t)$, $K_t = (K_{1t}, ..., K_{Nt})$, if the firm takes as given the rival firms' investment profile $\{I_{j\tau}, H_{j\tau}\}_{\tau\geq t}$ $(j = 1, ...i-1, i+1, ..N)$ and its own future optimal investment $\{I_{i\tau}^*\}_{\tau\geq t+1}$, then the firms indirect value function is given by

$$V_{it}(\Gamma_t) = \max_{I_{it}, H_{it}\geq 0}\left(\frac{Z_t}{P_t}\right)^{-\gamma}\left(\frac{p_t^{y*}Y_{it} - H_{it} - A(I_{it}, K_{it})}{P_t}\right) + \beta E_t\left[V_{it+1}(\Gamma_{t+1})\right], \tag{A2.40}$$

where, for $\tau \geq t$, p_τ^{y*} is given in (27) and $Y_{i\tau} = \theta_\tau (K_{i\tau})^{\psi_K} (H_{i\tau})^{\psi_H}$, and $D_{i\tau}^{y*} = p_\tau^{y*} Y_{i\tau} - H_{i\tau} - A(I_{i\tau}, K_{i\tau})$. Then the optimal (interior) investment and material input path satisfies the following system of equations

$$\begin{aligned}
\frac{\partial V_{it}(\Gamma_t)}{\partial K_{it}} &= \left(\frac{Z_t}{P_t}\right)^{-\gamma} \left(\frac{1}{P_t}\right) \frac{\partial D_{it}^{y*}}{\partial K_{it}} + \beta(1-\delta) \frac{\partial \mathbb{E}_t \left[V_{it+1}(\Gamma_{t+1})\right]}{\partial K_{it+1}}, && \text{(A2.41)} \\
\left(\frac{Z_t}{P_t}\right)^{-\gamma} \left(\frac{1}{P_t}\right) \frac{\partial D_{it}^{y*}}{\partial I_{it}} &= \beta \frac{\partial \mathbb{E}_t \left[V_{it+1}(\Gamma_{t+1})\right]}{\partial I_{it}}, && \text{(A2.42)} \\
\frac{\partial (p_t^{y*} Y_{it})}{\partial H_{it}} &= 1. && \text{(A2.43)}
\end{aligned}$$

Furthermore, in a symmetric equilibrium with $K_{i\tau} = K_\tau$, $Y_{i\tau} = Y_\tau$, and $I_{i\tau} = I_\tau, H_{i\tau} = H_\tau$ for $i = 1, ...N$ and each $\tau = 1, 2...$ Hence, $\sum_{j=1, i\neq j}^{N} [A(I_{i\tau}, K_{i\tau}) + H_{i\tau})] = (N-1)[A(I_\tau, K_\tau) + H_\tau]$. Now let

$$\tilde{W}_{i\tau} \equiv X_\tau - (N-1)(A(I_\tau, K_\tau) + H_\tau) - (A(I_{i\tau}, K_\tau) + H_{i\tau}). \qquad \text{(A2.44)}$$

Note that in a symmetric equilibrium, $W_\tau = \tilde{W}_{i\tau}, i = 1, ...N$, $\left(\frac{W_{i\tau}}{Y_\tau}\right)^{-1} = \frac{(p_\tau^{y*})^{-\sigma} \eta^\sigma}{N}$ and

$$p_\tau^{y*} Y_{i\tau} = \eta (W_{i\tau})^{1/\sigma} ((N-1)Y_\tau + Y_{i\tau})^{-\frac{1}{\sigma}} Y_{i\tau}. \qquad \text{(A2.45)}$$

Therefore, recognizing that $\frac{\partial Y_{i\tau}}{\partial I_{i\tau}} = 0$, we have in a symmetric equilibrium,

$$\frac{\partial (p_t^{y*} Y_{it})}{\partial I_{it}} = -\frac{(p_t^{y*})^{1-\sigma} A_I(I_{it}, K_t) \eta^\sigma}{N\sigma}. \qquad \text{(A.2.46)}$$

Hence, in any symmetric equilibrium, for $I_{it}^* = I_t^*$,

$$\frac{\partial D_{it}^{y*}}{\partial I_{it}} = -\left[A_I(I_t^*, K_t)\left(1 + \frac{(p_t^{y*})^{1-\sigma} \eta^\sigma}{N\sigma}\right)\right]. \qquad \text{(A2.47)}$$

Furthermore, $\frac{\partial K_{t+1}}{\partial I_t} = 1$ and hence $\frac{\partial \mathbb{E}_t [V_{t+1}(\Gamma_{t+1})]}{\partial I_t} = \frac{\partial \mathbb{E}_t [V_{t+1}(\Gamma_{t+1})]}{\partial K_{t+1}}$. Then (A2.41) and (A2.42) together imply that the Euler condition characterizing the equilibrium investment path is given by

$$-\frac{\partial D_{it}^{y*}}{\partial I_{it}} = E_t \left[M_{t+1}^* \left(\frac{\partial D_{t+1}^{y*}}{\partial K_{t+1}} - (1-\delta) \frac{\partial D_{t+1}^{y*}}{\partial I_{t+1}}\right), \right] \qquad \text{(A2.48)}$$

where in (A2.48), we have used iterated expectations and recursively substituted the optimality condition for I^*_{t+1} (using A(2.41)),

$$-\left(\frac{Z_{t+1}}{P_{t+1}}\right)^{-\gamma}\left(\frac{1}{P_{t+1}}\right)\frac{\partial D^{y*}_{t+1}}{\partial I_{t+1}}=\beta\frac{\partial \mathbb{E}_{t+1}\left[V_{t+2}(\Gamma_{t+2})\right]}{\partial K_{t+2}}. \tag{A2.49}$$

Now, using the envelope theorem (that sets the indirect effects of ∂K_{t+1} on the optimally chosen I^*_{t+1} and H^*_{t+1} to zero), (A2.45) implies that in a symmetric equilibrium with $Y_{it+1}=Y_{t+1}$, we have

$$\frac{\partial D^{y*}_{t+1}}{\partial K_{t+1}}=p^{y*}_{t+1}F_K(K_{t+1},H^*_{t+1},\theta_t)\left[\frac{N\sigma-1}{N\sigma}\right]-A_K(I^*_{t+1},K_{t+1}). \tag{A2.50}$$

(A2.47)-(A2.50) then together characterize the equilibrium path for investment in a symmetric equilibrium.

Next, to determine H^*_{it}, using (A2.43), we have

$$\frac{\partial(p^{y*}_t Y_t)}{\partial H_{it}}=-\frac{(p^{y*}_t)^{1-\sigma}\eta^\sigma}{N\sigma}+p^{y*}_t F_H(K_t,H^*_t,\theta_t)\left[\frac{N\sigma-1}{N\sigma}\right]. \tag{A2.51}$$

Inserting this in (A2.43) and rearranging terms gives,

$$\frac{N\sigma+(p^{y*}_t)^{1-\sigma}\eta^\sigma}{(N\sigma-1)}=p^{y*}_t F_H(K_t,H^*_t,\theta_t). \tag{A2.52}$$

Finally, if $N\le\bar{N}$, then by Proposition 2 the monopoly or collusive outcome path is a symmetric SPE. Hence, the Cournot path is not an optimal symmetric SPE. So consider the case where $N>\bar{N}$. If deviation from $\mu^{y*}_t(\Gamma_t)$ by any firm at any Γ_t is followed by all firms playing the symmetric SPE strategies $\mu^{y\prime}_\tau(\Gamma_\tau),\tau\ge t+1$, then defection is suboptimal if Equation (26) (in the text) holds. Since $\bar{D}^{y\prime}_t$ is increasing in N, there must exist some uniform upper bound N^* such that defection is optimal for some $\Gamma_t, t\ge 0$, if $N>N^*$. ∎

Appendix B: Equilibrium Computations

B.1 Capital Investment and Material Input Policies

Log-linearization allows one to write $\Phi_I(\Omega_t)$

$$\begin{aligned}\Phi_I(\Omega_t) \quad &\simeq \quad \alpha_{K1}\hat{K}_{t+2}+\alpha_{K2}\hat{K}_{t+1}+\alpha_{K3}\hat{K}_t+\alpha_{K4}\hat{H}_{t+1}+\alpha_{K5}\hat{H}_t+\\ &\qquad \alpha_{K6}\hat{X}_{t+1}+\alpha_{K7}\hat{X}_t+\alpha_{K8}\hat{\theta}_{t+1}+\alpha_{K9}\hat{\theta}_t,\end{aligned} \tag{B1}$$

where $\alpha_{K1}=\bar{K}\frac{\partial\Phi_I}{\partial K_{t+2}},...,\alpha_{K9}=\bar{\theta}\frac{\partial\Phi_I}{\partial\theta_t}$. In particular, the steady state endogenous variables $(\bar{K},\bar{H})$

are derived from specializing the optimality conditions in Theorem 1 to the steady state with $K_t = \bar{K}$, $I_t = \delta\bar{K}$ $(\equiv \bar{I})$, and $H_t = \bar{H}$. In a similar fashion, we have

$$\Phi_H(\Omega_t) \simeq \alpha_{H1}\hat{K}_{t+1} + \alpha_{H2}\hat{K}_t + \alpha_{H3}\hat{H}_t + \alpha_{H4}\hat{X}_t + \alpha_{H5}\hat{\theta}_t, \tag{B2}$$

where $\alpha_{H1} = \bar{K}\frac{\partial\Phi_H}{\partial K_{t+1}}, ..., \alpha_{H5} = \bar{\theta}\frac{\partial\Phi_I}{\partial\theta_t}$. Then the linearized Euler condition for $\pi_t = [\hat{K}_{t+1}\ \hat{H}_t]$ is

$$\mathbb{E}_t\left[\varsigma_0\pi_{t+1} + \varsigma_1\pi_t + \varsigma_2\pi_{t-1} + \nu_{X0}\hat{X}_{t+1} + \nu_{X1}\hat{X}_t + \nu_{\theta0}\hat{\theta}_{t+1} + \nu_{\theta1}\hat{\theta}_t\right] = 0, \tag{B3}$$

where $\varsigma_0 = \begin{bmatrix} \alpha_{K1} & \alpha_{K4} \\ 0 & 0 \end{bmatrix}, \varsigma_1 = \begin{bmatrix} \alpha_{K2} & \alpha_{K5} \\ \alpha_{H1} & \alpha_{H3} \end{bmatrix}, \varsigma_2 = \begin{bmatrix} \alpha_{K3} & 0 \\ \alpha_{H2} & 0 \end{bmatrix}, \nu_{X0} = \begin{bmatrix} \alpha_{K6} \\ 0 \end{bmatrix},$ $\nu_{X1} = \begin{bmatrix} \alpha_{K7} \\ \alpha_{H4} \end{bmatrix}, \nu_{\theta0} = \begin{bmatrix} \alpha_{K8} \\ 0 \end{bmatrix}, \nu_{\theta1} = \begin{bmatrix} \alpha_{K9} \\ \alpha_{H5} \end{bmatrix}$. Therefore, if $\pi_t = V\pi_{t-1} + U_X\hat{X}_t + U_\theta\hat{\theta}_t$, then the Euler condition (B3) imposes the restriction,

$$\begin{aligned} \varsigma_0 V^2 + \varsigma_1 V + \varsigma_2\mathcal{I} &= 0, & \text{(B4)} \\ \rho_X(\nu_{X0} + \varsigma_0 U_X) + ((\varsigma_1 + \varsigma_0 V)U_X + \nu_{X1}) &= 0, & \text{(B5)} \\ \rho_\theta(\nu_{\theta0} + \varsigma_0 U_\theta) + (\varsigma_1 U_\theta + \nu_{\theta1}) &= 0. & \text{(B6)} \end{aligned}$$

(where I is the identity matrix). Writing $V = \begin{bmatrix} V_{K1} & V_{K2} \\ V_{H1} & V_{H2} \end{bmatrix}$, a solution to (B4) is found by $V_{K2} = V_{H2} = 0$ and V_{K1}, V_{H1} that satisfy

$$\begin{aligned} \alpha_{K1}(V_{K1})^2 + \alpha_{K2}V_{K1} + \alpha_{K4}V_{H1}V_{K1} + \alpha_{K3} &= 0, & \text{(B7)} \\ \alpha_{H1}V_{K1} + \alpha_{H3}V_{H1} &= 0. & \text{(B8)} \end{aligned}$$

The condition for saddlepoint stability requires that there should be one non-explosive (that is, with modulus less than 1) and two explosive roots of (B4). The non-explosive root, say V_{K1}^*, is chosen.

Given V, the elements of $U_X = [U_{XK}\ U_{XH}]$ and $U_\theta = [U_{\theta K}\ U_{\theta H}]$ are then derived from (B5)-(B6).

B.2 Financial Asset Returns

Given the pricing kernel $M_{t+1} \equiv \beta \left(\frac{Z_{t+1}}{Z_t}\right)^{-\gamma} \left(\frac{P_{t+1}}{P_t}\right)^{\gamma-1}$, $m_{t+1} = \log M_{t+1}$ is

$$m_{t+1} = \log \beta - \gamma g_{z,t+1} + (\gamma - 1) g_{p,t+1}, \tag{B10}$$

where $g_{z,t+1} \equiv \ln(Z_{t+1}) - \ln(Z_t)$ and $g_{p,t+1} \equiv \ln(P_{t+1}) - \ln(P_t)$. Now, from the definitions of the aggregate income Z_t and price index P_t , it follows that m_{t+1} is a function of the state and costate vector Ω_t. Then using the relation $m_{t+1} = \beta \exp(\hat{m}_{t+1})$, the first order Taylor series expansion of m_{t+1} around steady state values gives

$$\begin{aligned} \hat{m}_{t+1} \quad = \quad & \varphi_{m1} \hat{K}_t + \varphi_{m2} \hat{K}_{t+1} + \varphi_{m3} \hat{K}_{t+2} + \varphi_{m4} \hat{H}_t + \varphi_{m5} \hat{H}_{t+1} + \\ & \varphi_{m6} \hat{X}_t + \varphi_{m7} \hat{X}_{t+1} + \varphi_{m8} \hat{\theta}_t + \varphi_{m9} \hat{\theta}_{t+1}, \end{aligned} \tag{B11}$$

where $\varphi_{m1} = \bar{K} \frac{\partial m_{t+1}}{\partial K_t}, \varphi_{m2} = \bar{K} \frac{\partial m_{t+1}}{\partial K_{t+1}}, ..., \varphi_{m9} = \bar{\theta} \frac{\partial m_{t+1}}{\partial \theta_{t+1}}$ (when these derivatives are evaluated at the steady state). But using the relation $\pi_t = V \pi_{t-1} + U_X \hat{X}_t + U_\theta \hat{\theta}_t$ (where V, U_X and U_θ have been determined as specified previously) and the facts that $\hat{\theta}_{t+1} = \rho_\theta \hat{\theta}_t + \varepsilon^\theta_{t+1}$ and $\hat{X}_{t+1} = \rho_x \hat{X}_t + \varepsilon^x_{t+1}$ in (B11) yields the following coefficients for $\hat{m}_{t+1} \simeq \tilde{a} \cdot \hat{\Gamma}_t + \omega_{mx} \varepsilon^X_{t+1} + \omega_{m\theta} \varepsilon^\theta_{t+1}$

$$\begin{aligned} \tilde{a}_1 &= \varphi_{m1} + \varphi_{m2} v_{K1} + \varphi_{m3} (v_{K1})^2 + \varphi_{m4} v_{H1} + \varphi_{m5} v_{H1} v_{K1}, \\ \tilde{a}_2 &= \varphi_{m6} + u_{HX} (\varphi_{m4} + \varphi_{m5} v_{H1}) + u_{KX} (\varphi_{m3} v_{K1} + \varphi_{m2}) + \rho_x (\varphi_{m7} + u_{HX} \varphi_{m5} + u_{KX} \varphi_{m3}), \\ \tilde{a}_3 &= \varphi_{m8} + u_{H\theta} (\varphi_{m4} + \varphi_{m5} v_{H1}) + u_{K\theta} (\varphi_{m3} v_{K1} + \varphi_{m2}) + \rho_\theta (\varphi_{m9} + u_{H\theta} \varphi_{m5} + u_{K\theta} \varphi_{m3}), \\ \omega_{mx} &= \varphi_{m7} + \varphi_{m3} u_{KX} + \varphi_{m5} u_{HX}, \\ \omega_{m\theta} &= \varphi_{m9} + \varphi_{m3} u_{K\theta} + \varphi_{m5} u_{H\theta}. \end{aligned} \tag{B12}$$

Then, following a procedure similar to that for approximating m_{t+1}, the first order Taylor series expansion of g^y_{dt+1} yields the appropriate coefficients b, ω_{dx}, and $\omega_{d\theta}$, so that $\hat{g}^y_{d,t+1} = b \cdot \hat{\Gamma}_t + \omega^y_{dx} \varepsilon^X_{t+1} + \omega^y_{m\theta} \varepsilon^\theta_{t+1}$. Next, the log equity return (for $j = x, y$) can be written

$$r^j_{t+1} = s^j_{t+1} - s^j_t + \log(1 + \exp(d^j_{t+1} - s^j_{t+1})). \tag{B13}$$

Treating this as a function of $d^j_{t+1} - s^j_{t+1}$, taking the first order Taylor approximation around the steady state $\bar{r}^j = -\log \beta$, recalling that $\bar{d}^j - \bar{s}^j = \log(1 - \beta) - \log \beta$, and adding and subtracting

d_t^j, yields the approximation $r_{t+1}^j \simeq -\log\beta + \hat{r}_{t+1}^j$, where

$$\hat{r}_{t+1}^j = \beta\hat{\xi}_{t+1}^j - \hat{\xi}_t^j + \hat{g}_{d,t+1}^y. \tag{B14}$$

Furthermore, letting $\hat{\xi}_t^j \simeq e_0^j + e^j \cdot \hat{\Gamma}_t$, the coefficients e_n^j $(n = 0, 1, 2, 3)$ are determined as follows. In log-linearized form, the Equilibrium return condition (38) can be written

$$1 = \mathbb{E}_t \left[\exp\left(\kappa_0^j + \kappa_1^j \hat{K}_t + \kappa_2^j \hat{X}_t + \kappa_3^j \hat{\theta}_t + \kappa_4^j \varepsilon_{t+1}^x + \kappa_5^j \varepsilon_{t+1}^\theta\right)\right], \tag{B15}$$

where κ_n^j $(n = 0, 1, 2, 3)$ are linear functions of the coefficients of $V, U_X, U_\theta, \tilde{a}_n, b_n$ (when $j = y$), and e_n^j. Now let $j = y$. Collecting together the coefficients for $\hat{K}_t$ from the first-order Taylor series expansions of m_{t+1} and r_{t+1}^y around the steady state, one gets

$$\kappa_1^y = \tilde{a}_1 + e_1^y(\beta v_{K1} - 1) + b_1. \tag{B16}$$

But since (B15) must hold for all realizations of $\hat{\Gamma}_t$, $\kappa_1^j = 0$ $(n = 1, 2, 3)$. Hence, from (B16), it follows that $e_1^y = \frac{b_1 + \tilde{a}_1}{(1 - \beta v_{K1})}$. In a similar fashion, we can compute,

$$\kappa_2^y = \beta u_{XK} e_1^y + e_2^y(\rho_x\beta - 1) + b_2 + \tilde{a}_2. \tag{B17}$$

Since e_1^y is already determined from (B16), it follows that $e_2^y = \frac{\beta u_{XK} e_1^y + b_2 + \tilde{a}_2}{(1 - \rho_x\beta)}$, and following an analogous computation, $e_3^y = \frac{\beta u_{X\theta} e_1^y + b_3 + \tilde{a}_3}{(1 - \rho_\theta\beta)}$. Finally, κ_0^y is obtained as follows. Since e_n^j $(n = 1, 2, 3)$ are chosen to set $\kappa_n^j = 0$ $(n = 1, 2, 3)$, the equilibrium condition (B15) must satisfy

$$1 = \mathbb{E}_t \left[\exp\left(\kappa_0^y + \kappa_4^y \varepsilon_{t+1}^x + \kappa_5^y \varepsilon_{t+1}^\theta\right)\right]. \tag{B18}$$

Then, by collecting the appropriate terms, we can compute

$$\kappa_0^y = (1 - \beta)[\log\beta - \log(1 - \beta)] + (\beta - 1)e_0^y \tag{B19}$$

$$\kappa_4^y = \omega_{mx} + \omega_{dx} + \beta e_2^y; \kappa_5^y = \omega_{m\theta} + \omega_{d\theta} + \beta e_3^y. \tag{B20}$$

Now let $\tilde{r}^y \equiv m + r^y$ denote logarithm of the discounted return MR^y. Hence, using the foregoing, $\mathrm{Var}(\tilde{r}^y) \equiv (\kappa_4^y)^2\lambda_x^2 + (\kappa_5^y)^2\lambda_\theta^2 + 2\lambda_{x\theta}\kappa_4^y\kappa_5^y$. Then, exploiting the bivariate normality of $(\varepsilon_{t+1}^X, \varepsilon_{t+1}^\theta)$

and taking the logarithm of both sides of (B18) gives $\kappa_0^y + 0.5 \text{ Var}(\tilde{r}^y) = 0$, which implies that

$$e_0^y = \log\left(\frac{\beta}{1-\beta}\right) + \frac{0.5 \text{ Var}(\tilde{r}^y)}{1-\beta}. \tag{B21}$$

Using (42) and collecting together the relevant terms from above, one can write

$$r_{t+1}^y = v_0^y + v_1^y \hat{K}_t + v_2^y \hat{X}_t + v_3^y \hat{\theta}_t + \omega_{rx}^y \varepsilon_{t+1}^x + \omega_{r\theta}^y \varepsilon_{t+1}^\theta, \tag{B22}$$

with the following coefficients:

$$\begin{aligned} v_0^y &= -[\beta \log \beta + (1-\beta)\log(1-\beta)] + e_0^y(\beta - 1) = -\log\beta - 0.5 \text{ Var}(\tilde{r}^y); \\ v_1^y &= e_1^y(\beta v_{K1} - 1) + b_1 = -\tilde{a}_1; v_2^y = \beta e_1^y \tilde{a}_2 + e_2^y(\rho_x \beta - 1) + b_2 = -\tilde{a}_2; \\ v_3^y &= \beta e_1^y \tilde{a}_3 + e_3^y(\rho_\theta \beta - 1) + \tilde{a}_3 = -\tilde{a}_3; \omega_{rx}^y = \beta e_2^y + b_2;\ \omega_{r\theta}^y = \beta e_3^y + b_3. \end{aligned} \tag{B23}$$

Turning to security x,let

$$r_{t+1}^x = v_0^x + v_1^x \hat{K}_t + v_2^x \hat{X}_t + v_3^x \hat{\theta}_t + \omega_{rx}^x \varepsilon_{t+1}^x + \omega_{r\theta}^x \varepsilon_{t+1}^\theta.$$

Note that the coefficients e_n^x $(n = 0, 1, 2, 3)$ are similarly obtained, except that in this case the log of dividends is directly obtained as $d_t^x = \log X_t \equiv x_t$. Note that $x_{t+1} - x_t = \hat{X}_{t+1} - \hat{X}_t$ (by subtracting $\log \bar{X}$ from both x_{t+1} and x_t). Then repeating the foregoing procedure (allowing for the difference in the log dividend growth) leads to the following:

$$e_1^x = \frac{b_1}{(1-\beta v_{K1})}; e_2^x = \frac{\beta u_{XK} e_1^x + b_2 + (\rho_x - 1)}{(1-\rho_x\beta)}; e_3^x = \frac{\beta u_{X\theta} e_1^x + b_3}{(1-\rho_\theta\beta)}. \tag{B24}$$

Since the equilibrium condition (B15) must hold given (B24) (for $j = x$),

$$\begin{aligned} \kappa_0^x &= (1-\beta)[\log\beta - \log(1-\beta)] + (\beta - 1)e_0^x, \\ \kappa_4^x &= \omega_{mx} + 1 + \beta e_2^x; \kappa_5^x = \omega_{m\theta} + \beta e_3^x, \end{aligned} \tag{B25}$$

Then, since $\text{Var}(\tilde{r}^x) \equiv (\kappa_4^x)^2\lambda_x^2 + (\kappa_5^x)^2\lambda_\theta^2 + 2\lambda_{x\theta}\kappa_4^x\kappa_5^x)$, $e_0^x = \log\left(\frac{\beta}{1-\beta}\right) + \frac{0.5 \text{ Var}(\tilde{r}^x)}{1-\beta}$. Hence, $\hat{r}_{t+1}^x$ has the following coefficients:

$$
\begin{aligned}
v_0^x &= -\log\beta - 0.5\ \mathrm{Var}(\tilde{r}^x); v_1^x = -\tilde{a}_1; \\
v_2^x &= --\tilde{a}_2; v_3^x = --\tilde{a}_3; \omega_{rx}^x = \beta e_2^x + 1;\ \omega_{r\theta}^x = \beta e_3^x.
\end{aligned} \tag{B26}
$$

Appendix C: Steady State

In the steady state, exogenous competitive sector output and oligopolistic sector productivity are constant over time, as are the endogenous variables of the model. Thus, $X_t = \bar{X} > 0$, $\theta_t = \bar{\theta} > 0$, and $K_t = \bar{K}$, $H_t = \bar{H}$, $\forall t$. It follows that the steady state investment per period is $\bar{I} = \delta\bar{K}$, so that $\frac{\partial \bar{I}}{\partial \bar{K}} = \delta$. We characterize the steady state of the Cournot dynamic equilibrium (Proposition 4) for general N, with the collusive industry equilibrium being the special case for $N = 1$. We recall that the maintained assumptions on the parameters are $\sigma > 1$ and $(\beta, \delta, \phi, \psi_H, \psi_K) \in (0,1)^5$.

Let $\bar{W}(N) = \bar{X} - N(A(\bar{I}, \bar{K}) + \bar{H})$. Assuming, for the moment, that $\bar{W}(N) > 0$, the steady state equilibrium product price is

$$
\bar{p}^y = \left(\frac{\bar{W}(N)}{NF(\bar{K}, \bar{H}, \bar{\theta})}\right)^{1/\sigma}\eta, \tag{C1}
$$

when $\bar{K} > 0, \bar{H} > 0$, and is undefined else. Meanwhile, the optimality condition for H in steady state is

$$
\frac{N\sigma + (\bar{p}^y)^{1-\sigma}\eta^\sigma}{(N\sigma - 1)} = \bar{p}^y F_H(\bar{K}, \bar{H}, \bar{\theta}). \tag{C2}
$$

Since $F_H(\bar{K}, \bar{H}, \bar{\theta}) = \bar{\theta}\psi_H \bar{K}^{\psi_K}\bar{H}^{\psi_H - 1}$, it follows from (C1)-(C2) that $\bar{K} > 0, \bar{H} > 0$ if $\bar{W}(N) > 0$ (since $\bar{\theta} > 0$ and $\sigma > 1$). Next, put $\bar{D}^y = \bar{p}^y F(\bar{K}, \bar{H}, \bar{\theta}) - (A(\bar{I}, \bar{K}) + \bar{H})$, so that

$$
\begin{aligned}
\frac{\partial \bar{D}^y}{\partial I} &= -A_I(\bar{I}, \bar{K})\left[1 + \frac{(\bar{p}^y)^{1-\sigma}\eta^\sigma}{N\sigma}\right], &\text{(C3)}\\
\frac{\partial \bar{D}^y}{\partial K} &= \bar{p}^y F_H(\bar{K}, \bar{H}, \bar{\theta})\frac{(N\sigma - 1)}{N\sigma} - A_K(\bar{I}, \bar{K}). &\text{(C4)}
\end{aligned}
$$

It follows that in the steady state, for each t, $Z_t = \bar{Z} = \bar{X} + \bar{D}^y + 1$ and the aggregate price index $P_t = \bar{P}$. Hence, the SDF is $M_t = \beta\ \forall t$. Then the Euler condition for optimal investment is

$$
-\frac{\partial \bar{D}^y}{\partial I} = \beta\left(\frac{\partial \bar{D}^y}{\partial K} - (1-\delta)\frac{\partial \bar{D}^y}{\partial I}\right), \tag{C5}
$$

which can be rearranged as

$$(1+\delta\upsilon)\left(\frac{N\sigma+(\bar{p}^{y})^{1-\sigma}\eta^{\sigma}}{N\sigma}\right)[(1-\beta(1-\delta)]=\beta\left(\bar{p}^{y}F_{K}(\bar{K},\bar{H},\bar{\theta})\frac{(N\sigma-1)}{N\sigma}-0.5\upsilon\delta^{2}\right). \quad \text{(C6)}$$

Using the notation of Proposition 2, the steady state collusive industry prices ($\tilde{p}^{y*}$), material inputs ($\tilde{H}^{*}$), and capital stock ($\tilde{K}^{*}$) are derived from (C1), C2), and (C6) for case $N=1$. Using Proposition 3, the per firm material inputs are $\bar{H}^{*}=\left(N^{\frac{\psi_{K}-1}{\psi_{H}}}\right)\tilde{H}^{*}$, while the per firm capital stocks and investment are $\bar{K}^{*}=\frac{\tilde{K}^{*}}{N}$ and $\bar{I}^{*}=\delta\bar{K}$, respectively. Hence, the steady state dividends for each firm in the symmetric collusive equilibrium are

$$\bar{D}^{y*}=\bar{p}^{y*}F(\bar{K}^{*},\bar{H}^{*},\bar{\theta})-\bar{H}^{*}-\delta\bar{K}^{*}(1+0.5\delta\upsilon), \quad \text{(C7)}$$

where the last term in $(C7)$ is the total investment cost $A(\bar{I}^{*},\bar{K}^{*})$. Therefore, at any date, the steady state firm value is $\bar{V}^{*}=\frac{\bar{D}^{y*}}{\bar{P}^{*}(1-\beta)}$, where $\bar{P}^{*}$ is the steady state aggregate price index. Then, from (A.2.33), the upper bound ($\bar{N}$) for the number of firms that strictly sustain collusion along a symmetric SPE is determined implicitly by the condition

$$\bar{p}^{y*}F(\bar{K}^{*},\bar{H}^{*},\bar{\theta})\left(\frac{N-1}{N}\right)+\delta\bar{K}^{*}(1+0.5\delta\upsilon)=\left(\frac{\beta}{1-\beta}\right)\bar{D}^{y*}. \quad \text{(C8)}$$

so that $\bar{D}^{y*}>0$ if $\bar{p}^{y*}>0,\bar{H}^{*}>0,\bar{K}^{*}>0,\bar{\theta}>0$. Finally, the equilibrium asset returns are given by $\bar{R}^{j}=\frac{1}{\beta}$, $j=x,y,f$.

We now show that $\bar{X}>N(A(\delta\bar{K}^{*},\bar{K}^{*})+\bar{H}^{*})$. Fix any $N\leq\bar{N}$ and $\bar{X}$. Note that $\bar{W}^{*}(N)\geq 0$ (else equilibrium prices are undefined). So suppose to the contrary that $\bar{W}^{*}(N)=0$. Hence, from (C7), $\bar{D}^{y*}<0$ and from (C2) $\frac{\partial\bar{D}^{y*}}{\partial H}<0$. Hence, perturbations $\bar{H}^{*}-\varepsilon$ (for $\varepsilon>0$) are strictly improving as long as (C2) is satisfied, so that $\bar{W}^{*}(N)=0$ cannot be an optimum.