Sequential Learning, Asset Allocation, and Bitcoin Returns

with James Yae

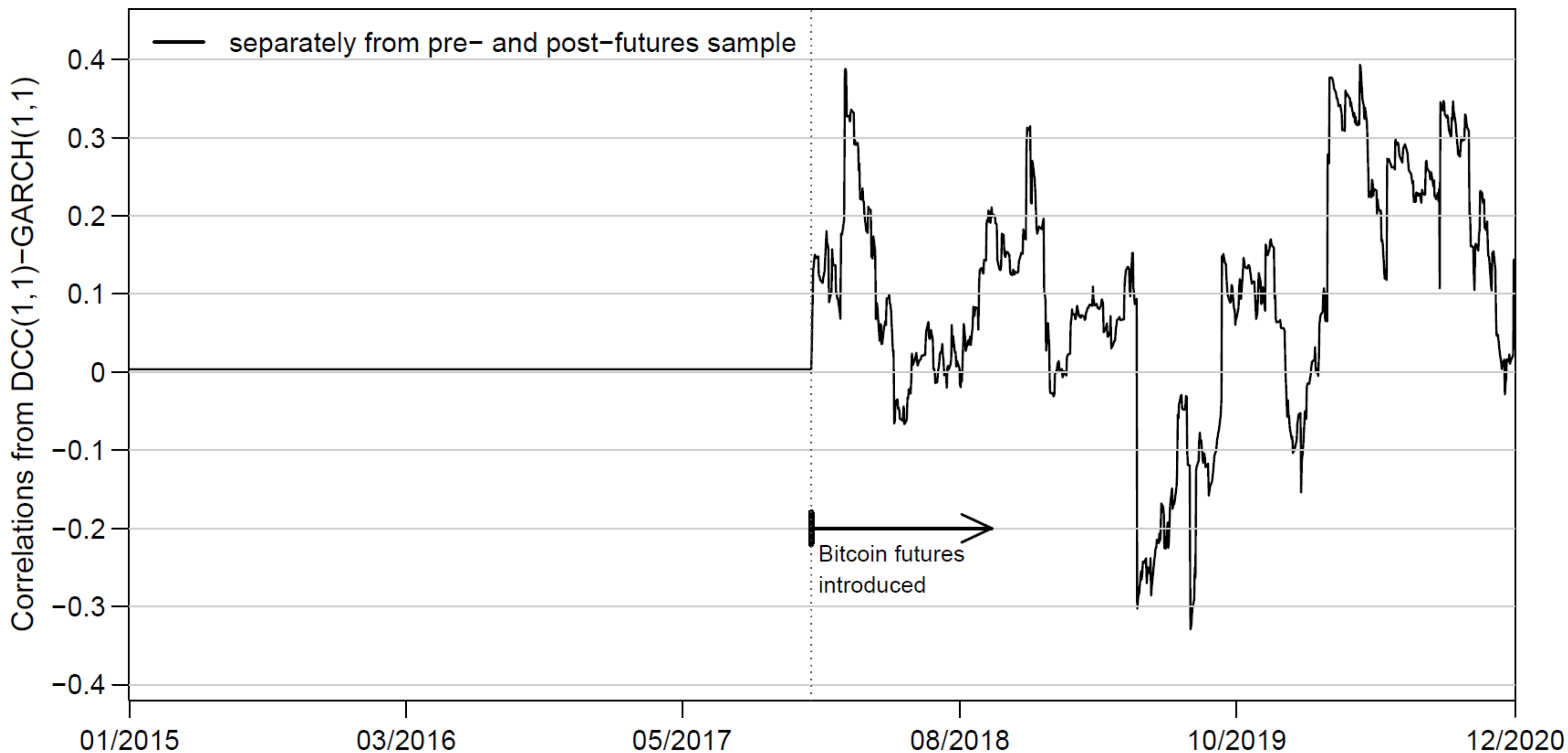
AFA 2022 Annual Meeting

George Tian
University of Houston

contact: george.zhe.tian@gmail.com

Structural Break in Correlation

Dynamic Correlation between Bitcoin and S&P500 returns: Pre- and Post-futures



Correlations are time varying only after the inception of Bitcoin futures!

How does it affect investors' behavior who seek portfolio optimization?

Bitcoin Demand Proxy

Optimal weight on Bitcoin (in a BTC-stock market portfolio)

$$w_{b,t} = \frac{\mu_t^* - \rho_t \sigma_t^*}{(\mu_t^* - \rho_t \sigma_t^*) + (\sigma_t^* - \rho_t \mu_t^*)\sigma_t^*}$$

Max Sharpe Ratio!

- Conditional risk premium ratio $\mu_t^* = \mu_{b,t} / \mu_{m,t}$
- Conditional volatility ratio $\sigma_t^* = \sigma_{b,t} / \sigma_{m,t}$

Bitcoin Demand Decomposition

non-speculative + speculative

$$\Delta w_{b,(t-1):t} \triangleq w_{b,t} - w_{b,t-1} = \boxed{\Delta w^{(cor)}_{b,(t-1):t} + \Delta w^{(vol)}_{b,(t-1):t}} + \boxed{\Delta w^{(mean)}_{b,(t-1):t}} + e_t$$

DCC(1,1)-GARCH(1,1) for estimation

$w^{(mean)}_{b,t} \triangleq w_b(\mu_t^*, \overline{\sigma^*}, \bar{\rho})$ investors only learn μ_t^*

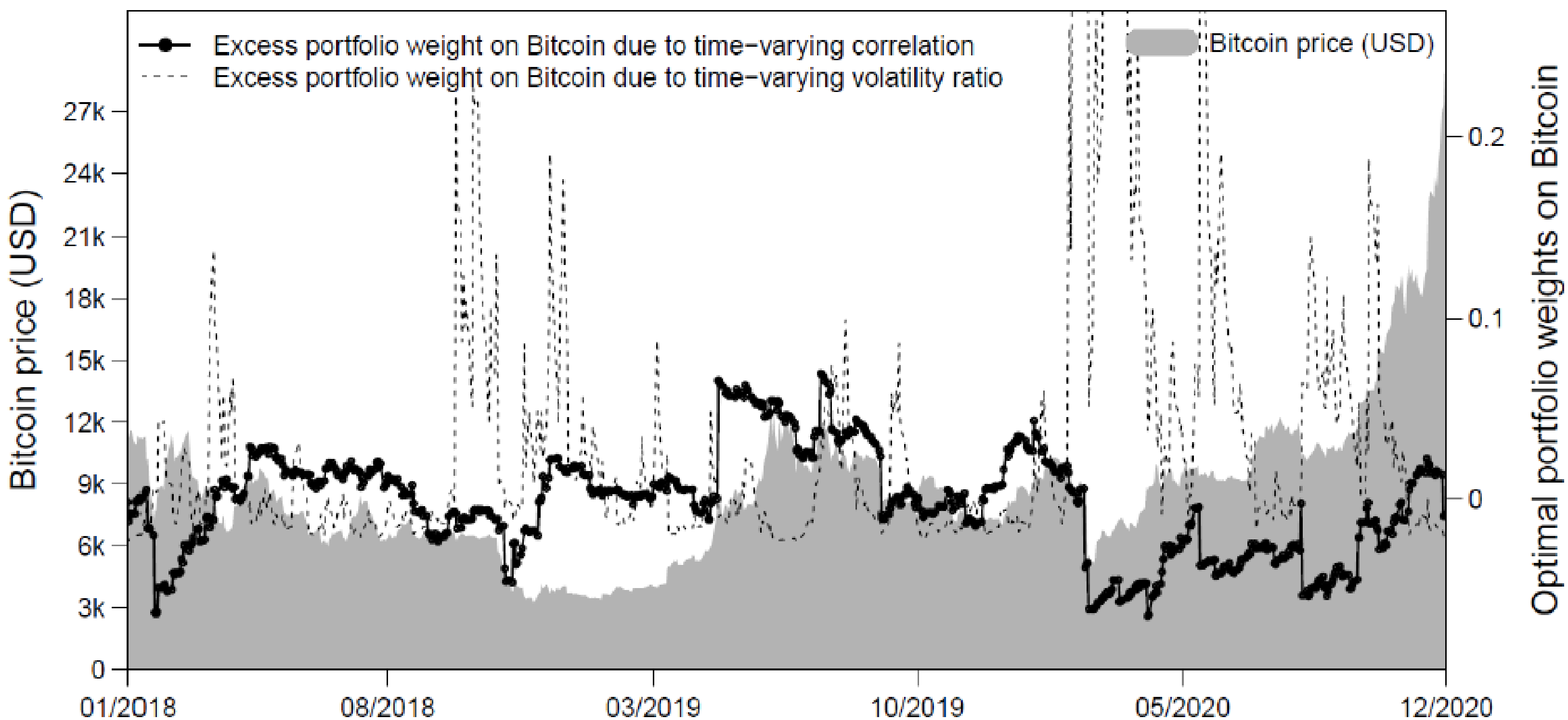
$w^{(cor)}_{b,t} \triangleq w_b(\overline{\mu^*}, \overline{\sigma^*}, \rho_t)$ investors only learn (ρ_t)

$w^{(vol)}_{b,t} \triangleq w_b(\overline{\mu^*}, \sigma_t^*, \bar{\rho})$ investors only learn (σ_t^*)

Bitcoin Demand Proxy

Bitcoin demand variation = Due to **Corr.** change + Due to **Vol.** change

Which effect is dominant?



Daily Bitcoin Return Predictability

$$r_{b,t+1} = b_0 + \boxed{b_1}\Delta w^{(cor)}_{b,(t-1):t} + \boxed{b_2}\Delta w^{(vol)}_{b,(t-1):t} + Z_t\gamma + \varepsilon_{t+1}$$

0.5% higher returns with 1 std demand increase

30 times higher est. uncertainty (noisy signal)

Predictor	Post-futures (12/18/2017 to 12/31/2020)					Post-futures before COVID-19 (12/18/2017 to 02/29/2020)				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\Delta w^{(cor)}_{b,(t-1):t}$	0.51		0.47	0.47	0.47	0.50		0.48	0.47	0.48
	(4.15)		(2.89)	(2.91)	(2.80)	(3.16)		(2.78)	(2.63)	(2.37)
$\Delta w^{(vol)}_{b,(t-1):t}$		0.06	0.03	0.04	0.03		-0.19	-0.23	-0.22	-0.20
		(0.42)	(0.28)	(0.30)	(0.20)		(-1.27)	(-1.66)	(-1.67)	(-1.56)
$r_{b,t}$			-0.42	-0.42	-0.42			-0.22	-0.23	-0.22
			(-1.79)	(-1.76)	(-1.74)			(-0.83)	(-0.83)	(-0.82)
β_t			0.04	0.06	0.07			0.06	0.07	0.09
			(0.22)	(0.31)	(0.31)			(0.28)	(0.32)	(0.40)
$Volume_{b,t}$			-0.58	-0.55	-0.55			-0.74	-0.91	-0.87
			(-2.59)	(-1.35)	(-1.39)			(-3.52)	(-1.93)	(-1.78)
$EPU_{b,t}$			0.63	0.64	0.63			0.21	0.20	0.21
			(1.71)	(1.68)	(1.76)			(0.98)	(0.93)	(0.96)
Controls			M	MB	MBL			M	MB	MBL
R^2 (%)	1.15	0.01	3.94	3.99	4.00	1.17	0.16	4.04	4.14	4.38
$Adj.R^2$	1.02	-0.12	2.92	2.71	2.47	0.99	-0.02	2.62	2.36	2.24

Data are at *daily* frequency. Newey-West t statistics are reported in parenthesis.

Equilibrium Model: Intuition

BTC Investors

Speculative Investors
$(\mu_t^*, \overline{\sigma^*}, \bar{\rho})$

Non-speculative Investors
$(\overline{\mu^*}, \sigma_t^*, \rho_t)$

"asynchronous portfolio rebalancing"

links increase in BTC demand due to ρ_t (or decrease in ρ_t) to higher subsequent BTC returns!

t — t+1

SI trade based on μ_t^* (or $P_{b,t}$) → trade based on μ_{t+1}^* (or $P_{b,t+1}$)

NI buys BTC from SI

Equilibrium

NI estimate ρ_t (then set $w_{b,t}^{(cor)}$) $\xrightarrow[\text{asynchronous}]{\text{delay in order executions}}$ rebalance portfolio at t+1

Highlights

1) **Bitcoin return predictability**:
   Increase in daily Bitcoin demand change due to dynamic correlation predicts higher subsequent Bitcoin returns
2) **Rational asset allocation**:
   The empirical pattern is consistent with investors' learning on time-varying correlations and practice on rational portfolio optimization
3) **Asynchronous portfolio rebalancing**:
   We use an equilibrium model to explain how Bitcoin return predictability emerges from asynchronous portfolio rebalancing

The paper also explains:
- Why predictability from $\Delta w_{b,(t-1):t}^{(cor)}$ not $\Delta w_{b,(t-1):t}^{(vol)}$
- Is there out-of-sample predictability?
- other Bitcoin demand proxies?
- Does the evidence show up in other cryptos and other equity markets?

…