# **A Macro-Finance model with Realistic Crisis Dynamics**

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#### Abstract

What causes deep recessions and slow recovery? I revisit this question and develop a macro-finance model that quantitatively matches the salient empirical features of financial crises such as a large drop in the output, a high risk premium, reduced financial intermediation, and a long duration of economic distress. The model has leveraged intermediaries featuring stochastic productivity and regime-dependent exit rate that governs the transition in and out of crises. A model without these two features suffers from a trade-off between the amplification and persistence of crises. I show that my model resolves this tension and generates realistic crisis dynamics.

### Introduction

A Macro-finance model with financial amplification to explain deep and persistent financial crises . Two sector model with households, and experts facing a) stochastic productivity and b) regime-

## **Solution Method**

- Two state variables: wealth share of experts  $z_t$  (endogenous), productivity of experts  $a_{e,t}(exogenous)$
- Solution boils down to solving coupled system of PDEs in  $J_h$  and  $J_e$



- dependent exit rate
- 2. Multi-dimensional model  $\rightarrow$  Active deep learning that encodes economic information as regularizers (Gopalakrishna (2021))
- **B** Quantification of a simpler model: shut-off time variation in productivity and remove exit
- 1. Trade-off between unconditional risk premium and probability of crisis
- 2. Trade off between conditional risk premium (amplification) and duration of crisis (persistence)
- 3. My model resolves these tensions and provides a better match to data

## **Economic Mechanisms**

#### Setup:

- Two classes of agents: **Households**, and **Experts** (financially constrained, leveraged).
- Normal times: More productive experts sufficiently capitalized, hold all capital

#### **Crisis dynamics**:

• Capital and Productivity shock: negative shock  $\rightarrow \downarrow$  leveraged expert net worth  $\rightarrow$  amplification (large risk premium, GDP falls, investment falters, and return volatility increases)

#### • Regime-dependent exit

- 1. Larger exit in crisis pushes economy deeper into recession
- 2. only way to come out of crisis is by increased expert productivity. Slow mean reversion in productivity  $\implies$  delayed recovery (persistence)

#### Value functions (PDEs): Finite Difference Neural Network

S : H

#### Figure 2: Figure caption

• Neural network approach (ALIENs) developed in Gopalakrishna (2021)

## **Quantitative Analysis**

	Data			Ben	chmark Model RA=1	Benchmark Model (RA=20)	
	All	Recession	Crisis	All	Crisis	All	Crisis
E(Risk premium) Std(Risk premium)	7.5 5.1	16.6 6.5	25.0 7.4	1.7 2.8	13.4 1.3	7.3 0	-
Probability of Crisis	7			7.8		0	

#### **Table 1:** Empirical vs Model moments

#### • Trade-off 1: Risk premium and Prob. of crisis

• Trade-off 2: Conditional Risk premium and Duration. of crisis



## Model



#### Figure 1: Balance sheet

AK technology  $y_{j,t} = a_{j,t}k_t, \quad j \in e, h$ 

$$\frac{dk_t}{k_t} = (\Phi(\mathbf{l}_t) - \delta)dt + \sigma dZ_t^k$$

. Productivity of experts is time-varying and follows the process

 $da_{e,t} = \pi(\hat{a}_e - a_{e,t})dt + \nu(\overline{a}_e - a_{e,t})(a_{e,t} - \underline{a}_e)dZ_t^a$ 

with  $d\langle Z_t^k, Z_t^a \rangle = \varphi dt > 0$  and  $a_h < \underline{a}_e < \hat{a}_e < \overline{a}_e \rightarrow$  Reflects bank economies of scale

2. Experts exit at rate  $\tau_t \in \{\tau_{normal}, \tau_{crisis}\}$ , with  $\tau_{crisis} = \mathbf{9} \times \tau_{normal}$ .  $\rightarrow$  Reflects bank runs during crises



#### Figure 3: Trade-offs in benchmark model

- Benchmark: Only one shock: i.i.d Brownian.
- 1. In steady state, capital shock to risk averse experts is not enough to generate sufficient crises periods (trade-off 1)
- 2. Once in crisis, amplification happens but experts repair their balance sheet faster  $\implies$  quick recovery (trade-off 2)
- My model: Two correlated Brownian shocks plus higher exit in crisis.
- 1. In steady state, capital shock to risk averse experts also lowers productivity and generates crisis (trade-off 1)

2. Once in crisis, amplification happens but experts exit economy at higher rate 3. Productivity shoots up slowly  $\implies$  sluggish recovery (trade-off 2)

		My mode	el	Benchmark model			
	All	Crisis	Normal	All	Crisis	Normal	
E[leverage]	2.80	4.79	2.62	3.23	5.50	3.10	
E[inv. rate]	7.70%	2.80%	8.20%	6.00%	5.00%	6.00%	
E[risk free rate]	0.90%	-7.20%	1.70%	4.80%	0.00%	5.00%	
E[risk premia]	6.70%	17.50%	5.70%	1.70%	13.40%	1.00%	
E[GDP growth rate]	1.20%	-8.00%	1.90%	2.30%	-7.90%	2.70%	
Std[inv. rate]	3.18%	1.31%	2.91%	0.36%	1.09%	0.11%	
Std[risk premia]	5.35%	1.57%	4.45%	2.82%	1.31%	0.18%	
Std[risk free rate]	3.98%	1.64%	3.21%	1.19%	0.42%	0.28%	
Corr(leverage,shock)	-0.25	-0.17	-0.30	-0.28	-0.05	-0.25	
Probability of crisis	7.0%			7.80%			
Duration of crisis (months)	18.5			6			

#### Table 2: Comparison of moments

Experts solve

$$\begin{split} U_{e,t} &= \sup_{\substack{C_{e,t}, K_{e,t}, \chi_{e,t}}} \quad E_t \bigg[ \int_t^{\tau'} f(C_{e,s}, U_{e,s}) ds + U_{h,\tau'} \bigg] \\ \text{s.t.} \quad \frac{dW_{e,t}}{W_{e,t}} &= \big( r_t - \frac{C_{e,t}}{W_{e,t}} + \frac{q_t K_{e,t}}{W_{e,t}} (\mu_{e,t}^R - r_t - (1 - \chi_{e,t})\epsilon_{h,t}) - \lambda_d + \frac{\bar{z}}{z_t} \lambda_d - \tau_t \big) dt \\ &+ \sigma_{w_e,t} \big( (\sigma + \sigma_t^{q,k}) dZ_t^k + \sigma_t^{q,a} dZ_t^a \big) \end{split}$$

• Transition time  $\tau'$  is exponentially distributed with rate  $\tau_t \in \{\tau_{normal}, \tau_{crisis}\}$ •  $\frac{q_t K_{e,t}}{W_{e,t}}$ : fraction of capital invested

•  $\chi_{e,t}$ : fraction of equity retained in balance sheet

• Preferences follow Duffie-Epstein utility

$$f(c_{j,t}, U_{j,t}) = (1 - \gamma)\rho U_{j,t} \left( \log(c_{j,t}) - \frac{1}{1 - \gamma} \log \left( (1 - \gamma) U_{j,t} \right) \right)$$

## Conclusions

- Wealth share of intermediaries alone cannot jointly match asset pricing, output, and crisis moments 1. Trade-off between unconditional risk premium and probability of crisis 2. Trade-off between conditional risk premium (amplification) and duration of crisis (persistence) • A model of stochastic productivity and regime-dependent exit generates realistic crisis dynamics, and a better match to data • Active machine learning opens new avenues for future research 1. 'Brunnermeier-Sannikov meets Bansal-Yaron' economy (Gopalakrishna (2021)) 2. Heterogeneous intermediaries
- 3. Main street vs Wall street disconnect, good booms vs bad booms
- 4. Sunspot equilibria
- 5. ....and more