Discordant Relaxations of Misspecified Models

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Introduction

- Misspecification in partially identified models can lead to spuriously informative bounds
- Problem of Inference: misleading CI.
  Andrews and Kwon (2019), Molinari (2020)

This paper:

- Problem of Identification: misleading outer set

Outer Set in Practice

Often in practice, outer set $\widetilde{\Theta}$ instead of Θ_I is estimated:

- Blundell, Gosling, Ichimura and Meghir(2007)
- Ciliberto and Tamer(2009), Ciliberto, Murry and Tamer(2018)
- Aucejo, Bugni and Hotz(2017)
- Sheng(2018), de Paula, Richards-Shubik and Tamer(2018)
- Dickstein and Morales(2018)
- Chesher and Rosen(2020)
- ...

Summary of Main Results

- Negative Result:
  outer sets can be misleading / discordant if based on nonsharp id. restrictions.
- Positive Result:
  outer sets may not be discordant under some conditions.
- Related Result:
  ideas on how to summarize discordant results from different assumptions.

Discordant Outer Sets: an Example

Example: Intersection Bounds

$$\mathbb{E}[\underline{Y}|Z] \leqslant \theta \leqslant \mathbb{E}[\overline{Y}|Z] \quad a.s.$$

Example: heterogeneous treatment model

(Y_0, Y_1)	potential outcome
D	binary treatment
$Y = Y_D$	observed outcome
Z	instrument
$\underline{Y}_d$	$\underline{y}\mathbb{1}(D \neq d) + Y\mathbb{1}(D = d))$
$\overline{Y}_d$	$\overline{y}\mathbb{1}(D \neq d) + Y\mathbb{1}(D = d))$
θ_d	$\mathbb{E}[Y_d]$
$\mathbb{E}[Y_d|Z] = \mathbb{E}[Y_d]$	mean independence

Example: Intersection Bounds

$$\mathbb{E}[\underline{Y}|Z] \leqslant \theta \leqslant \mathbb{E}[\overline{Y}|Z] \quad a.s.$$

Then,

$$\theta \in [\underline{\gamma}, \overline{\gamma}] := \left[\sup_z \mathbb{E}[\underline{Y}|Z = z], \inf_z \mathbb{E}[\overline{Y}|Z = z]\right]$$

it implies for any $h(\cdot) \geqslant 0$,

$$\mathbb{E}\left[h(Z)(\underline{Y} - \theta)\right] \leqslant 0, \quad \mathbb{E}\left[h(Z)(\theta - \overline{Y})\right] \leqslant 0$$

its id. set $\Theta_I(h)$.

Interpreting $\Theta_I(h)$

Recall $\underline{\gamma} = \sup_z \mathbb{E}[\underline{Y}|Z=z] \quad \overline{\gamma} = \inf_z \mathbb{E}[\overline{Y}|Z=z]$.

- when $\underline{\gamma} \leqslant \overline{\gamma}$, $\quad [\underline{\gamma}, \overline{\gamma}] \subseteq \Theta_I(h)$
- when $\underline{\gamma} > \overline{\gamma}$, $\quad [\underline{\gamma}, \overline{\gamma}] = \varnothing$, $\quad$ what is $\Theta_I(h)$?

Discordant Outer Sets

Theorem

Suppose $\mathbb{E}[\underline{Y}|Z] \leqslant \mathbb{E}[\overline{Y}|Z]$ a.s.

When full model refuted, i.e. when $\underline{\gamma} > \overline{\gamma}$ and $[\underline{\gamma}, \overline{\gamma}] = \varnothing$,

$$\forall \theta \in (\overline{\gamma}, \underline{\gamma}),\ \exists h \geqslant 0,\ s.t.\ \Theta_I(h) = \{\theta\}$$

Takeaway: an outer set can be very tight but misleading

Discordant Outer Sets

Here, when the model is refuted, outer sets are discordant:

there exist h_1, h_2 such that $\Theta_I(h_1) \neq \emptyset$, $\Theta_I(h_2) \neq \emptyset$,

$$\Theta_I(h_1) \cap \Theta_I(h_2) = \emptyset$$

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In general,

nonsharp id. restrictions $\Rightarrow$ discordant outer sets

examples including

- Artstein inequality on random set and Choquet capacity
- conditional moment inequality models

Compatible Outer Sets: an Example

Example: Adaptive Monotone IV

Potential Outcome Model:

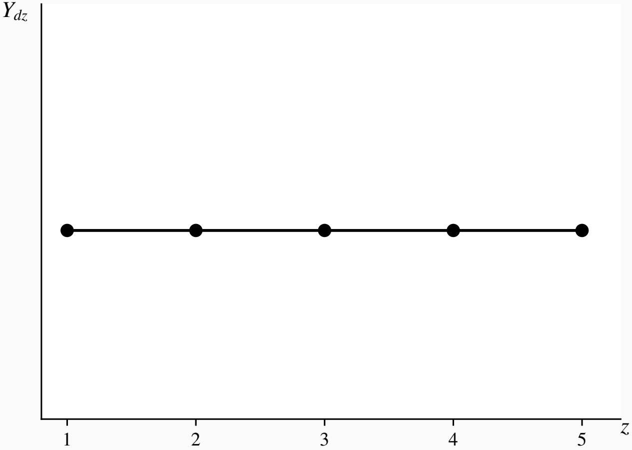
D	binary treatment: college education or not
Z	instrument: max. years of education of parents
Y_{dz}	potential outcome: potential wage
$Y = Y_{DZ}$	observed outcome: observed wage
θ	ATE

Assume mean independence: $\forall(d, z)$, $\mathbb{E}[Y_{dz}|Z] = \mathbb{E}[Y_{dz}]$.

Assumptions

exclusion restriction in Manski (1990) a_{exclude}:

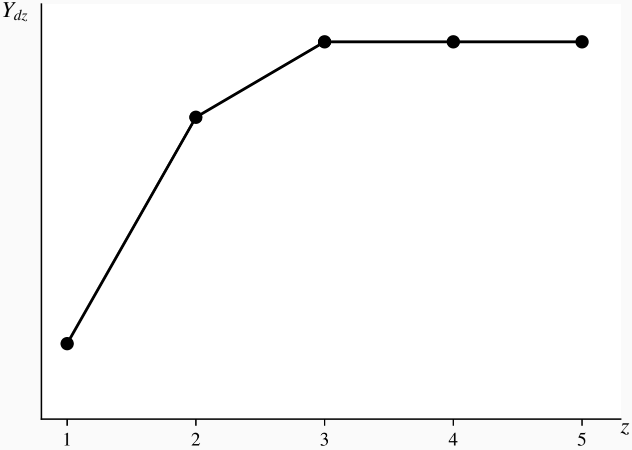
instruments z has no impact on potential outcome Y_{dz}



Assumptions

Adaptive Monotone IV assumption a_k:

Y_{dz} weakly increases with z then remain flat after $z \geqslant k$



Outer Sets and Minimum Relaxation

Note that

- assumptions are nested,

$$a_{\text{exclude}} \Leftrightarrow a_1 \Rightarrow a_2 \Rightarrow \cdots \Rightarrow a_K$$

- id. set $\Theta_I(a_k)$ is an outer set of the id. set $\Theta_I(a_{\text{exclude}})$

However,

- outer sets are compatible even if model $a_{exclude}$ is refuted

$$\text{either } \Theta_I(a_k) \subseteq \Theta_I(a_{k'}) \text{ or } \Theta_I(a_{k'}) \subseteq \Theta_I(a_k)$$

- Importantly, data reveals the unique minimum relaxation needed to restore data-consistency.

In our paper,

- we derived the iff condition for the uniqueness of minimum data-consistent relaxation

- we also discussed what conclusion could be drawn if minimum data-consistent relaxations are not unique.