Household Savings and Monetary Policy under Individual and Aggregate Stochastic Volatility

Yuriy Gorodnichenko,[1] Lilia Maliar,[2] Serguei Maliar,[3] & Christopher Naubert[4]

[1]University of California at Berkeley and NBER; [2]CUNY Graduate Center and CEPR; [3]Santa Clara University;

[4]CUNY Graduate Center

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Motivation

- Modern macroeconomics:
  - Questions:
    - heterogeneity in income or productivity
    - assets with differing liquidity (machines, liquid bonds)
    - aggregate (and idiosyncratic) risk and uncertainty
    - redistribution
  - Tools:
    - global solutions
- These are usually done in isolation.
- **This paper:** We do all in one framework.

HANK Model

- Households
  - Two assets: bonds (liquid) and machines (illiquid)
  - Two (occasionally binding) borrowing constraints
  - Idiosyncratic shocks to productivity level (risk)
  - Aggregate shock to productivity variance (uncertainty)
  - Sticky wages
  - Households
- Firms
  - CRS with machines and labor
  - Aggregate shocks to TFP level (risk)
  - Aggregate shocks to TFP variance (uncertainty)
  - Firms
- Government
  - Fiscal policy (progressive income taxation as in Heathcote, Storesletten, and Violante 2017)
  - Monetary policy (Taylor rule with ZLB)
  - Government

Risk and Uncertainty

- Household productivity: $e^{\left(\eta_{\ell,t}(j)-\frac{\bar{\sigma}_\ell^2}{1-(\rho^\ell)^2}\right)}$

$$\text{Individual risk: } \eta_{\ell,t}(j) = \rho^\ell \eta_{\ell,t-1}(j) + \exp\left(\sigma_{\ell,t-1} - \frac{1}{2}\frac{\sigma_{\sigma_\ell}^2}{1-(\rho^{\sigma_\ell})^2}\right)\bar{\sigma}_\ell \varepsilon_{\ell,t}(j)$$

$$\text{Individual uncertainty: } \sigma_{\ell,t} = \rho^{\sigma_l}\sigma_{\ell,t-1} + \sigma_{\sigma_\ell}\varepsilon_{\sigma_\ell,t}$$

- Aggregate TFP: $e^{\left(\eta_{\theta,t}-\frac{\bar{\sigma}_\theta^2}{1-(\rho^\theta)^2}\right)}$

$$\text{TFP risk: } \eta_{\theta,t} = \rho^\theta \eta_{\theta,t-1} + \exp\left(\sigma_{\theta,t-1} - \frac{1}{2}\frac{\sigma_{\sigma_\theta}^2}{1-(\rho^{\sigma_\theta})^2}\right)\bar{\sigma}_\theta \varepsilon_{\theta,t}$$

$$\text{TFP uncertainty: } \sigma_{\theta,t} = \rho^{\sigma_\theta}\sigma_{\theta,t-1} + \sigma_{\sigma_\theta}\varepsilon_{\sigma_\theta,t}$$

where $\varepsilon_{\ell,t}$, $\varepsilon_{\sigma_\ell,t}$, $\varepsilon_{\theta,t}$, $\varepsilon_{\sigma_\theta,t} \sim \mathcal{N}(0,1)$

Related Literature

- Uncertainty shocks ▸ Uncertainty
- HANK ▸ HANK
- Numerical solutions to heterogeneous agent models ▸ Numerics

Model Generated Statistics

- Global solutions following L. Maliar, S. Maliar, and Winant 2021

	Wealth Gini	Consumption Gini	Net Income Gini
Data	0.78	0.36	0.43
Model	0.66	0.276	0.360
95% CI	(0.655,0.684)	(0.274,0.277)	(0.359,0.361)

Table: Data from Krueger, Mitman, and Perri 2016

Generalized Impulse Response

- Koop, Pesaran, and Potter 1996
- TFP uncertainty

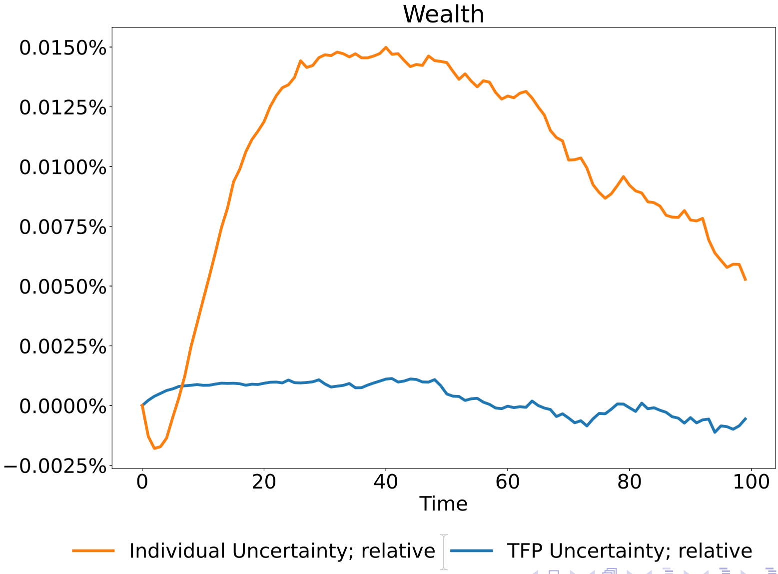
$$\sigma_{\theta,t} = \rho^{\sigma_\theta} \sigma_{\theta,t-1} + \sigma_{\sigma_\theta} \left(\varepsilon_{\sigma_\theta,t} + 1 \right)$$

- Individual uncertainty

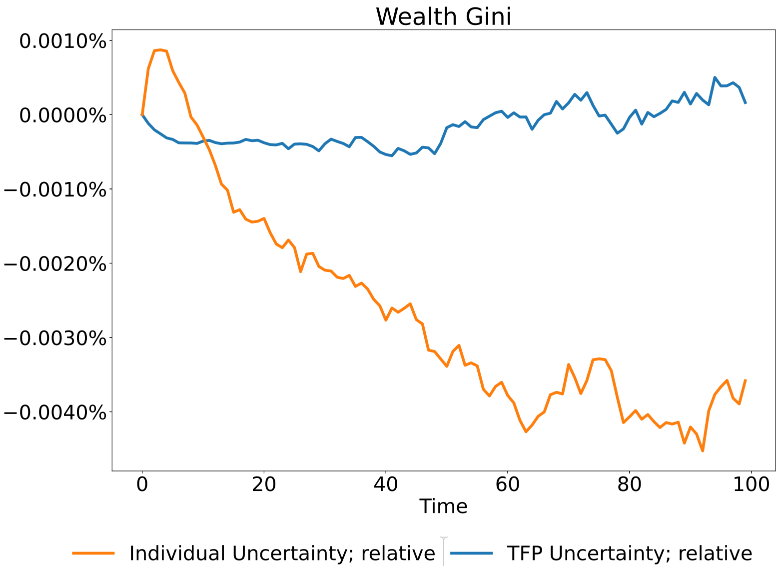
$$\sigma_{\ell,t} = \rho^{\sigma_l} \sigma_{\ell,t-1} + \sigma_{\sigma_\ell} \left(\varepsilon_{\sigma_\ell,t} + 1 \right)$$

- 1 standard deviation innovation
- 100 initial conditions
- 100 draws of innovations for each initial condition
- Time period: 1 quarter
- 200 agents

Wealth

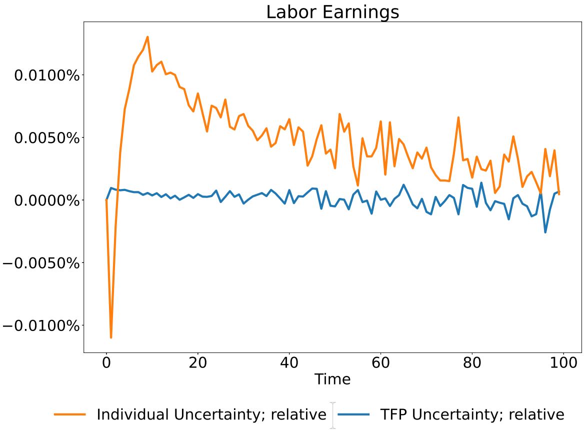

Wealth
0.0150%
0.0125%
0.0100%
0.0075%
0.0050%
0.0025%
0.0000%
−0.0025%
0
20
40
60
80
100
Time
Individual Uncertainty; relative
TFP Uncertainty; relative

Wealth Gini



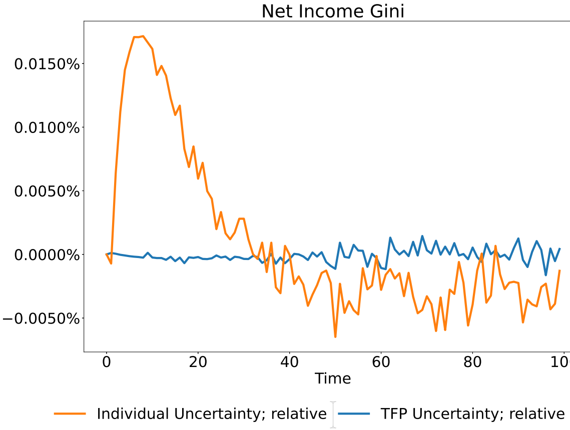
Wealth Gini
0.0010%
0.0000%
−0.0010%
−0.0020%
−0.0030%
−0.0040%
0
20
40
60
80
100
Time
Individual Uncertainty; relative
TFP Uncertainty; relative

Average Labor Earnings



$$\text{Average Labor Earnings} = w_t H_t \int e^{\left(\eta_{\ell,t}(j) - \frac{\overline{\sigma}_\ell^2}{1-(\rho^\ell)^2}\right)} dj$$

Net Income Gini



$$\text{Net Income} = \left(\frac{R_{t-1}}{\pi_t} - 1\right) b_{t-1}(j) + \left[r_t^k - d\right] k_{t-1}(j) + \tau_t(j) + \tau_1 \left[w_t H_t \exp\left(\eta_{\ell,t}(j) - \frac{1}{2}\frac{\overline{\sigma}_\ell^2}{1-\left(\rho^\ell\right)^2}\right)\right]^{1-\tau_2}$$

Conclusion

- Response to individual uncertainty shocks is much larger than the response to TFP uncertainty shocks
- Individual uncertainty shocks lead to a persistent increase in wealth following an initial reduction.
- Individual uncertainty shocks increase income inequality.
- **Future versions**: Correlation between individual and aggregate uncertainty.

Thank you!

Appendix

Model and Calibration

Household Problem

$$\max_{c_t, i_t, b_t, k_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t(j)^{1-\gamma} - 1}{1-\gamma}$$

$$\text{s.t. } c_t(j) + i_t(j) + b_t(j) + \Psi(i_t(j), k_{t-1}(j)) =$$

$$\frac{R_{t-1}}{\pi_t} b_{t-1}(j) + \tau_1 \left[w_t H_t \exp\left(\eta_{\ell,t}(j) - \frac{1}{2} \frac{\overline{\sigma}_\ell^2}{1-(\rho^\ell)^2} \right) \right]^{1-\tau_2} + \tau_t(j)$$

$$k_t(j) = \left[1 - d + r_t^k\right] k_{t-1}(j) + i_t(j)$$

$$k_t(j) \geq 0 \qquad b_t(j) \geq \overline{b}$$

- $\Psi(\cdot,\cdot)$: adjustment cost on machines and shares
- $\tau_t(j)$: transfers

Labor Union

$$\max_{W_t(m)} E_0 \sum_{t=0}^{\infty} \beta^t \left[\int \left(\frac{c_t(j)^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t(j)^{1+\vartheta}}{1+\vartheta} \right) dj - \frac{\mu_w}{1-\mu_w} \frac{1}{2\kappa_w} \left[\log \left(\frac{W_t(m)}{W_{t-1}(m)} \frac{1}{\pi^*} \right) \right]^2 \right]$$

$$\text{s.t. } H_t(m) = \left(\frac{W_t(m)}{W_t} \right)^{\mu_w - 1} H_t$$

$$h_t(j) = \int \left(\frac{W_t(m)}{W_t} \right)^{\mu_w - 1} H_t dm$$

Labor Union (Continued)

$$\log\left(\frac{\pi_t}{\pi^*}\right) = \kappa_w \left(\psi H_t^{1+\vartheta} - \mu_w (1-\tau_2) Z_t \widetilde{\Lambda}_t\right) + \beta E_t \log\left(\frac{\pi_{t+1}}{\pi^*}\right)$$

$$Z_t \equiv \tau_0 (w_t L_t)^{(1-\tau_2)} \int_0^1 \left(\exp\left(\eta_{\ell,t}(j) - \frac{1}{2}\frac{\overline{\sigma}_\ell^2}{1-(\rho^\ell)^2}\right)\right)^{(1-\tau_2)} dj$$

$$\widetilde{\Lambda}_t \equiv \int_0^1 \frac{\left(\exp\left(\eta_{\ell,t}(j) - \frac{1}{2}\frac{\overline{\sigma}_\ell^2}{1-(\rho^\ell)^2}\right)\right)^{(1-\tau_2)} c_t(j)^{-\gamma}}{\int_0^1 \left(\exp\left(\eta_{\ell,t}(j) - \frac{1}{2}\frac{\overline{\sigma}_\ell^2}{1-(\rho^\ell)^2}\right)\right)^{(1-\tau_2)} dj} dj.$$

▸ Model

Firms

- Production function

$$Y_t = \overline{A} \exp\left(\eta_{\theta,t} - \frac{\overline{\sigma}_\theta^2}{1-(\rho^\theta)^2}\right) K_{t-1}^\alpha H_t^{1-\alpha}$$

▸ Model

Central Bank

- Taylor rule subject to ZLB

$$R_t \equiv \max\left\{1.0, R_*\left(\frac{R_{t-1}}{R_*}\right)^{\mu}\left[\left(\frac{\pi_t}{\pi_*}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_*}\right)^{\phi_y}\right]^{1-\mu}\exp\left(\eta_{R,t}-\frac{1}{2}\frac{\sigma_R^2}{1-(\rho^R)^2}\right)\right\}$$

- Monetary policy shock

$$\eta_{R,t}=\rho^R\eta_{R,t-1}+\sigma_R\varepsilon_{R,t},\qquad \varepsilon_{R,t}\sim\mathcal{N}(0,1)$$

▸ Model

Market Clearing

- Market clearing

$$\int_0^1 b_t(j)\, dj = 0$$

$$C_t + K_t - (1-d)\, K_{t-1} + \int_0^1 \mathrm{AC}(i)\, di = Y_t$$

- $\int_0^1 \mathrm{AC}(i)\, di = \int_0^1 \Psi\left(i_t(j), k_{t-1}(j)\right) dj$: aggregate cost of adjustment
- $C_t = \int_0^1 c_t(j)\, dj$
- $K_{t-1} = \int_0^1 k_{t-1}(j)\, dj$

Calibration

Parameter	*Description*	*Target/Source*
	Household	
$\gamma = 2.0$	Risk aversion	standard
$\beta = .975$	Discount factor	standard
$d = 0.0135$	Depreciation rate	standard
$\Gamma_2 = 1.1686$	Illiquid asset adjustment cost	
$\Gamma_3 = 2.0$	Illiquid asset adjustment cost	
$\xi = 0.0$	Illiquid asset adjustment cost	
$\varepsilon = 0.25$	Illiquid asset adjustment cost	
$\overline{b} = -0.1$	Liquid asset borrowing constraint	75% of people have liquid assets Kaplan, Violante, and Weidner 2014
$\tau_1 = 0.8$	Tax function parameter	Heathcote, Storesletten, and Violante 2017
$\tau_2 = 0.181$	Tax function parameter	Heathcote, Storesletten, and Violante 2017
	Labor Union	
$\vartheta = 1.0$	Labor supply elasticity	standard
$\psi = 0.8796$	Disutility of labor shift	$H = 1$ in model without agg risk
$\mu_w = 1.1$	Elasticity of substitution among goods	profits share of 10%
$\kappa_w = 0.15$	Slope of wage Phillips curve	Auclert et al. 2021
	Firm	
$\alpha = 0.325$	Capital share	standard
$\overline{A} = 0.4735$	Constant in production function	$Y = 1$ in model without agg risk

Calibration (cont.)

Parameter	*Description*	*Target/Source*
	Monetary Policy	
$\mu = 0.0$	Nominal rate persistence	
$R_* = 1.0175$	Long run nominal rate	
$Y_* = 1$	Long run output	
$\pi_* = 1.005$	Inflation target	
$\phi_\pi = 1.5$	MP response to inflation	
$\phi_y = \frac{.25}{4}$	MP response to output	
	Exogenous Variables	
$\rho^\ell = 0.966$	Persistence of idiosyncratic shocks	Auclert et al. 2021
$\overline{\sigma}_\ell = 0.2379$	Standard deviation of idios.-level shocks (in the absence of uncertainty shocks)	Auclert et al. 2021
$\rho^{\sigma_\ell} = 0.84$	Persistence of idios.-volatility shocks	Based on Bayer et al. (2019)
$\sigma_{\sigma_\ell} = 0.02$	Standard deviation of idios.-volatility shocks	Based on Bayer et al. (2019)
$\rho^\theta = 0.9$	Persistence of TFP-level shocks	standard
$\overline{\sigma}_\theta = 0.016$	Standard deviation of TFP-level shocks (in the absence of uncertainty shocks)	standard
$\rho^{\sigma_\theta} = 0.73$	Persistence of TFP-volatility shocks	Based on Fernandez-Villaverde et al. (2015)
$\sigma_{\sigma_\theta} = 0.04$	Standard deviation of TFP-volatility shocks	Based on Fernandez-Villaverde et al. (2015)
$\rho^R = 0.5$	Persistence of monetary-policy shocks	standard
$\sigma_R = 0.01$	Standard deviation of monetary-policy shocks	standard

Solution Algorithm

Deep Learning Analysis of Maliar, Maliar, Winant (2019)

1. **HANK model**: $\begin{cases} E_\epsilon\left[f_1\left(X\left(s\right),\epsilon\right)\right]=0 \\ \quad\quad ... \\ E_\epsilon\left[f_n\left(X\left(s\right),\epsilon\right)\right]=0 \end{cases}$

   s = state, $X\left(s\right)$ = decision function, ϵ = innovations.
2. Parameterize $X\left(s\right)\simeq\varphi\left(s;\theta\right)$ with a **deep neural network**.
3. Construct **objective function** for DL training

$$\min_{\theta}\left(E_\epsilon\left[f_1\left(\varphi\left(s;\theta\right),\epsilon\right)\right]\right)^2+...+\left(E_\epsilon\left[f_n\left(\varphi\left(s;\theta\right),\epsilon\right)\right]\right)^2\rightarrow 0$$

4. **All-in-one expectation** operator is a critical novelty:

$$\left(E_\epsilon\left[f_j\left(\varphi\left(s;\theta\right),\epsilon\right)\right]\right)^2=E_{\left(\epsilon_1,\epsilon_2\right)}\left[f_j\left(\varphi\left(s;\theta\right),\epsilon_1\right)\cdot f_j\left(\varphi\left(s;\theta\right),\epsilon_2\right)\right]$$

   with ϵ_1,ϵ_2 = two independent draws.
4. **Stochastic gradient descent** for training (random grids)
5. Google **TensorFlow** platform

Solution Algorithm

- Use algorithm of Maliar, Maliar and Winant (2021)
- 13 agg. variables $\left\{ \begin{array}{c} C_t, H_t, K_t, I_t, Y_t, \pi_t, w_t, r_t^k, \\ R_{t-1}, \eta_{R,t}, \eta_{\theta,t}, \sigma_{\theta,t}, \sigma_{\ell,t} \end{array} \right\}$
- 8 individual variables $\left\{ \begin{array}{c} c_t(j), k_t(j), i_t(j), b_t(j), \\ q_t(j), \eta_{\ell,t}(j), \upsilon_t(j), \varphi_t(j) \end{array} \right\}$,

  $\upsilon_t(j), \varphi_t(j) =$ Lagrange multipliers; $q_t(j) =$ value of an additional unit of illiquid assets
- 5 aggregate state variables:

$$\underbrace{\{R_{t-1}\}}_{endogenous} \qquad \underbrace{\{\eta_{R,t}, \eta_{\theta,t}, \sigma_{\theta,t}, \sigma_{\ell,t}\}}_{exogenous} \tag{1}$$

- 3 individual state variables:

$$\underbrace{\{k_{t-1}(j), b_{t-1}(j)\}}_{endogenous} \qquad \underbrace{\{\eta_{\ell,t}(j)\}}_{exogenous} \tag{2}$$

- $3J + 5$ dimensional state space, where $J =$ number of agents

Neural Networks

- 2 neural networks (NN) with 4 hidden layers each and 128 neurons in each layer.
- Leaky relu as activation function. ADAM optimization algorithm. Batch size of 10.
- **Outputs of NNs**:
  - 1st NN ($\mathcal{N}^{agg}$): aggregate variables $\{H_t, \pi_t\}$
  - 2d NN $\left(\mathcal{N}^{indiv}\right)$: individual variables $\left\{\xi_t^k(j), \xi_t^c(j), \upsilon_t(j), \varphi_t(j)\right\}$
    - $\xi_t^a(j)$=share of illiquid assets out of income net of consumption and the borrowing limit
    - $\xi_t^k(j)$=share of capital in illiquid assets
    - $\upsilon_t(j)$, $\varphi_t(j)$ = multipliers
- Need to approximate just six $3J+5$–dimensional decision function to characterize the labor choices of all J agents.

Recovering Aggregate Variables

- Use weights of NNs to compute **aggregate** variables

$$
\begin{gathered}
\mathcal{N}^{agg}(\Sigma) \rightarrow (H_t, \pi_t) \\
k(j) \rightarrow K_t \\
(H_t, K_t, \eta_{\theta,t}) \rightarrow \left(w_t, r_t^k, Y_t\right) \\
(\pi_t, Y_t, R_{t-1}, \eta_{R,t}) \rightarrow R_t
\end{gathered}
$$

Recovering Individual Variables

- NN for individuals

$$\mathcal{N}^{indiv}(\Sigma) \to \left(\xi_t^k(j), \xi_t^c(j), \upsilon_t(j), \varphi_t(j)\right) \quad (3)$$

- resources

$$M_t(j) \equiv \frac{R_{t-1}}{\pi_t} b_{t-1}(j) + \left[1 - d + r_t^k\right] k_{t-1}(j) + \tau_t(j) + \tau_1 \left[w_t H_t \exp\left(\eta_{\ell,t}(j) - \frac{1}{2}\frac{\overline{\sigma}_\ell^2}{1-(\rho^\ell)^2}\right)\right]^{1-\tau_2} \quad (4)$$

- consumption

$$c_t(j) = \xi_t^c\left(M_t(j) - \overline{b}\right) \quad (5)$$

Recovering Individual Variables (Continued)

- machines

$$k_t(j) = \max\left(\xi_t^k(j) \cdot \left[M_t(j) - \overline{b} - c_t(j)\right], 0.0\right) \quad (6)$$

- adjustment cost

$$(k_t(j), k_{t-1}(j)) \rightarrow i_t(j) \rightarrow (\Psi_t(j), q_t(j)) \quad (7)$$

- bonds

$$b(j) = \max\left(\left[M_t(j) - \overline{b} - c_t(j) - k_t(j) - \Psi_t(j)\right], \overline{b}\right) \quad (8)$$

Related Literature

Relation to Literature about Uncertainty

1. *Aggregate uncertainty in RA models.*
   - *TFP:* Basu and Bundick (2017), Born and Pfeifer (2014), Fernandez-Villaverde et al. (2015).
   - *Other sources of uncertainty:* Born and Pfeifer (2014) (monetary and fiscal policy), Kelly, Pastor and Veronesi (2016) and Pastor and Veronesi (2012, 2013) (political factors), Basu and Bundick (2017) (preference shocks), Fernandez-Villaverde et al. (2015) (fiscal instruments), Nodari (2014) (financial regulation policy), Stokey (2015) (future tax rates).
2. *Idiosyncratic uncertainty on the production side.*
   - Arellano, Bai and Kehoe (2019), Bloom, Floetotto, Jaimovich, Saporta-Ekstein and Terry (2018), Gilchrist, Sim and Zakrajšek (2014), Bahmann and Bayer (2013, 2014).
   - Assume representative household –uncertainty does not affect households of different income and wealth levels.
3. *Stochastic volatility in HA models.*
   - Bayer et al. (2019) and Schabb (2020).

Relation to HANK Literature

- *Aggregate MIT risk shocks + No uncertainty shocks*
  - Kaplan, Moll and Violante (2018), Alves, Kaplan, Moll and Violante (2020)
- *Aggregate MIT uncertainty shocks*
  - Bayer, Luetticke, Pham-Dao and Tjaden (2019) and Schabb (2020)

This paper: the first HANK model with both

- aggregate uncertainty shocks
- aggregate risk shocks

Related Literature

Relation to HA Computational Lliterature

Novel numerical methods for solving HANK models with distribution

- Based on **Reiter (2009)**:
  - *Idea:* local (perturbation) solutions at the aggregate level + Global solutions at the individual level
  - *Papers:* Ahn et al. (2018), Boppart et al. (2018), Bayer and Luetticke (2019), and Auclert et al. (2020). $\Rightarrow$ No TFP dynamics over time.
- Based on **Fernandez-Villaverde, Hurtado and Nuno** (2020):
  - *Idea:* use neural networks to approximate aggregate law of motion (ALM)
  - *Paper:* Fernandez-Villaverde, Marbet, Nuno and Rachedi (2021)
  - ALM is approximated with a general function of distributional moments $\Rightarrow$ Krusell and Smith (1998) type of algorithm

▸ Related Literature

Uncertainty Shocks and Global Solutions

Global versus local solution methods

Solving models with uncertainty shocks

- *Fernandez-Villaverde:*
  – Perturbation solutions must be at least of order three
  $\Longrightarrow$ Volatility of shocks nontrivially enters decision rules
- *Groot (2020)*:
  – Even third-order perturbation methods may not be sufficient.

$$\Downarrow$$

Need global solutions to capture effects of volatility on decision rules

Approaches to Uncertainty Shocks in the Literature

- MIT aggregate shocks
- Low-order perturbation
- Reduced state space approximations

We address these problems with AI and deep learning (DL)

- Aggregate shocks in the solution procedure
- Global nonlinear solutions
- True state space