Moral Hazard Under Contagion

Boli Xu

Northwestern University

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Motivation

- In some partnerships,
  - partners can exit at any time
  - partners who have exited still enjoy some free-riding benefits as long as remaining partners keep contributing to the partnership
  - free-riding makes it harder for remaining partners to run the partnership
- Trade-off:
  - free-riding is discouraged by the contagion of defections it may trigger
- Examples:
  - European Super League
  - public protests
  - group lending programs
  - ...

Example: European Super League

- A soccer competition proposed by 12 top European clubs (vs. UEFA Champions League)
- Contagion of defections:
  - fan opposition caused Manchester City to exit
  - 5 other English clubs exited within one day, and more followed suit
  - only 3 clubs remain until today
- Free-riding problem:
  - clubs that have exited continue to benefit from ESL's ongoing operation
    - in particular, from the checks it places on UEFA
  - UEFA made compromises that favor the top clubs
    - e.g., raised prize money allocated to knock-out stage of Champions League

In a Nutshell

Dynamic free-riding + Irreversible defections

- A stopping game where players run a joint project
  - project's flow output evolves stochastically
  - players can irreversibly exit at any time
  - players who have exited continue to enjoy some free-riding benefits, which depend on the number of remaining players
  - players' exits exert negative externalities on remaining players

Irreversibility

- Preview of some findings
  - curse of productivity: a better project may harm all the players
  - a partnership's ability to cooperate is non-monotonic in its size
    - vs. traditional wisdom that large size exacerbates free-riding (Olson, 1965)
  - ...

Application to ESL

Related Work

- **Moral hazard in teams**: McMillan (1979), Holmstrom (1982), etc.
- **Dynamic contribution games**: Fershtman & Nitzan (1991), Admati & Perry (1991), Marx & Matthews (2000), Compte & Jehiel (2004), Harstad (2012), Battaglini, Nunnari, & Palfrey (2014), Georgiadis (2015), Ramos & Sadzik (2019), Cetemen, Hwang, & Kaya (2020), etc.
- **Stochastic stopping games**: Rosenberg, Solan, & Vieille (2007), Moscarini & Squintani (2010), Murto & Valimaki (2011), Rosenberg, Salomon, & Vieille (2013), Guo & Roesler (2018), Margaria (2020), Kirpalani & Madsen (2021), Awaya & Krishna (2021), Cetemen, Urgun, & Yariv (2021), etc.
- **Voluntary partnerships**: Angeletos, Hellwig, & Pavan (2007), Fujiwara-Greve & Okuno-Fujiwara (2009), Chassang (2010), McAdams (2011), Fujiwara-Greve & Yasuda (2021), etc.
- **Real options games**: Dutta & Rustichini (1993), Grenadier (1996), Weeds (2002), Steg (2015), etc.
- **Farsightedness in cooperative games**: Harsanyi (1974), Chwe (1994), Ray & Vohra (2015), etc.

Outline

Outline

Payoff

- Time is continuous $t \in [0, \infty)$
- 2 players ($i = 1, 2$) run a joint project
  - $\Pi_i = \int_0^\infty e^{-rt} \pi_{it} dt$ where π_{it} is the flow payoff
- Flow payoff at time t

	Contribute	Defect
Contribute	$X_t - c\ ,\ X_t - c$	$X_t - \beta c\ ,\ \alpha X_t$
Defect	$\alpha X_t\ ,\ X_t - \beta c$	$0\ ,\ 0$

  - $X_t > 0$: the project's productivity/output, follows $\frac{dX_t}{X_t} = \mu dt + \sigma dZ_t$
  - $\beta > 1$: the *reliance parameter*
  - $\alpha \in (0, 1)$: the *free-riding parameter*

Interpretation of α

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Interpretation of α

Timeline

- In the baseline model
  - Defections are irreversible
  - Players' past actions are public

Irreversibility

- Timeline (*à la* Murto & Valimaki, 2013)
  - Stage 1: given that no one exited yet, i chooses exit region $\mathcal{X}^i \subseteq \mathcal{X}$
    - if both intend to exit at the same time: flip a coin so that only one of them exits successfully (each w.p. $\frac{1}{2}$)
    - one player exits at Stage 1 and becomes the *first mover*
  - Stage 2: the *second mover* chooses exit region $\mathcal{X}^s \subseteq \mathcal{X}$
    - possible for second mover to immediately exit: a de facto joint exit

Coin-flipping

- Main result: unique Pareto-undominated MPE at Stage 1
  - After reducing Stage 2, Stage 1 is a canonical stopping game
- Properties of the equilibrium:
  - Curse of productivity: A better project can harm both players
  - Blessing of reliance: Heavy reliance ensures cooperation

Backward Induction: Stage 2

- *Second mover*'s optimal stopping problem
  - flow payoff $= X_t - \beta c$
  - lump-sum exiting payoff $= 0$
- *Second mover*'s optimal decision: $\mathcal{X}^s = (0, x^*)$
  - $x^* = \frac{r-\mu}{r}\frac{\gamma}{\gamma-1}\beta c$, where $\gamma = \frac{\sigma^2 - 2\mu - \sqrt{(\sigma^2-2\mu)^2 + 8r\sigma^2}}{2\sigma^2}$
- *Second mover*'s value function

Derivation

$$S(X_t) = \begin{cases} 0 & \text{, when } X_t < x^* \\ \underbrace{\frac{X_t}{r-\mu} - \frac{\beta c}{r}}_{\text{value if never exit}} + \underbrace{k_1 X_t^{\gamma}}_{\text{option value}} & \text{, when } X_t \geq x^*. \end{cases}$$

Backward Induction: First Mover's Exit Payoff

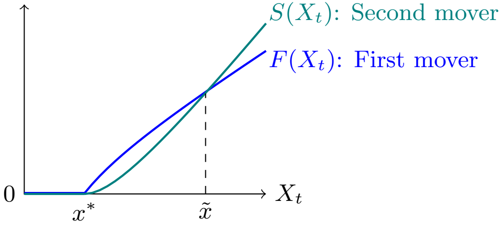
- After exit, first mover gets αX_t until second mover terminates the project.
- *First mover*'s value function at the moment of exit

Derivation

$$F(X_t) = \begin{cases} 0 & \text{, when } X_t < x^* \\ \underbrace{\frac{\alpha X_t}{r-\mu}}_{\text{value if never terminated}} - \underbrace{k_2 X_t^{\gamma}}_{\text{termination loss}} & \text{, when } X_t \geq x^*. \end{cases}$$

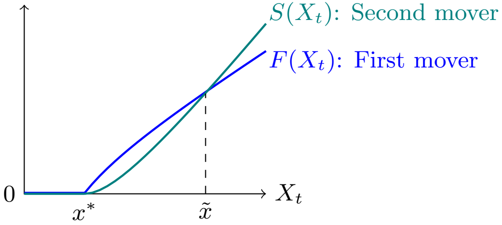
- Lemma 1: First-mover advantage in $(x^*, \tilde{x})$

Concavity Intersection



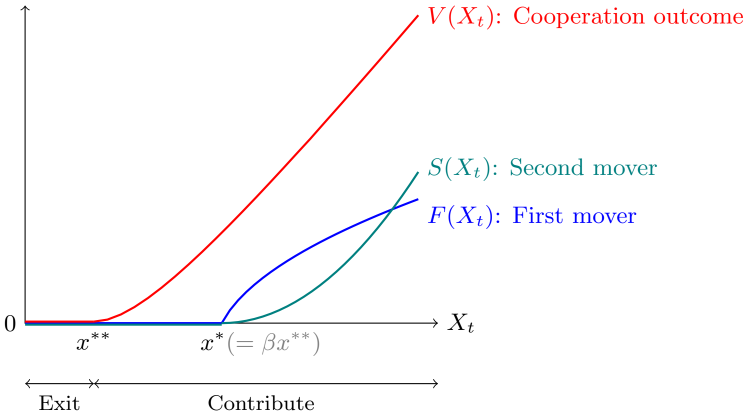
The Stopping Game

- Stage 1 is a canonical stopping game:
  - before any player exits: the flow payoff is $X_t - c$
  - the one who exits gets $F(X_t)$, the remaining player gets $S(X_t)$



Case 1: $\beta \geq \beta^*$

- Notice that $F(X_t)$ point-wise decreases in β.
- When $\beta \geq \beta^* := \left[\frac{1-(1-\alpha)^\gamma}{\alpha\gamma}\right]^{\frac{1}{1-\gamma}}$: cooperative equilibrium



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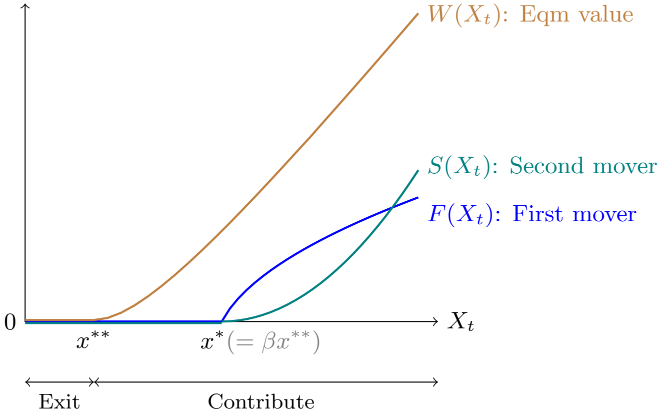
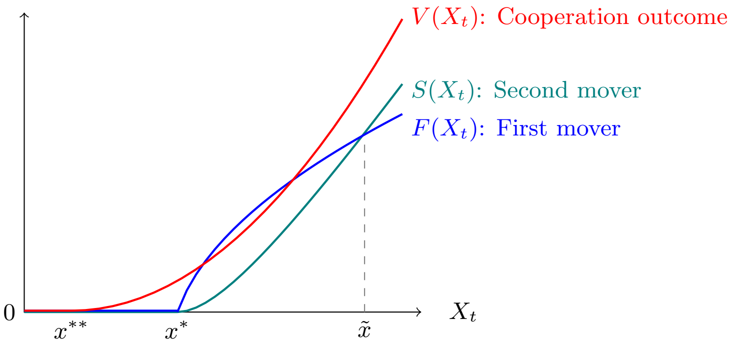


Case 2: $\beta < \beta^*$

- Pre-emption: players exit in entire interval w/ first-mover advantage
-



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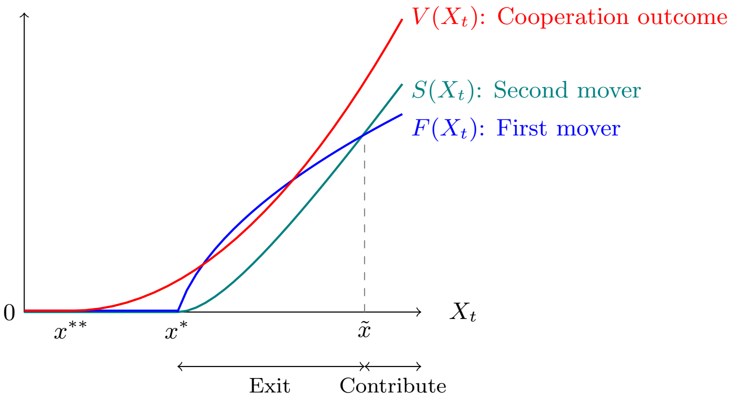
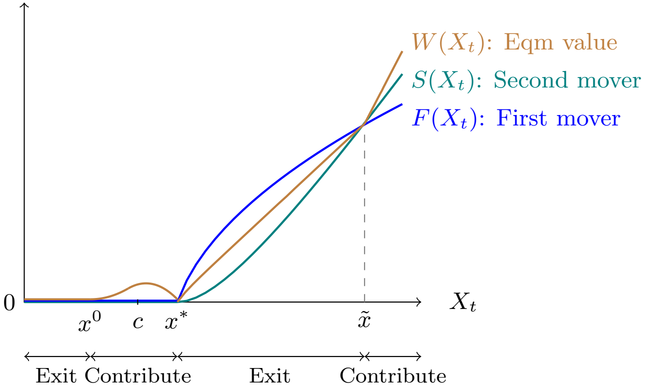


Case 2: $\beta < \beta^*$

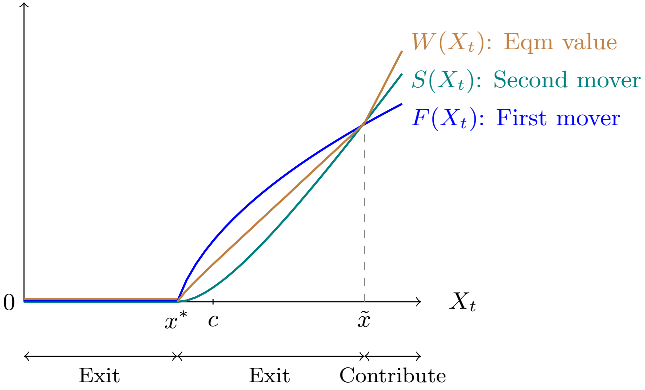
- Pre-emption: players exit in entire interval w/ first-mover advantage
- Case 2(a): when $x^* > c(\Leftrightarrow \beta > \beta^{**} = \frac{r}{r-\mu}\frac{\gamma-1}{\gamma})$



- **Curse of Productivity**: A large X_t generates more revenue, but also stimulates free-riding.

Case 2: $\beta < \beta^*$

- Pre-emption: players exit in entire interval w/ first-mover advantage
- Case 2(b): when $x^* \leq c (\Leftrightarrow \beta \leq \beta^{**} = \frac{r}{r-\mu}\frac{\gamma-1}{\gamma})$



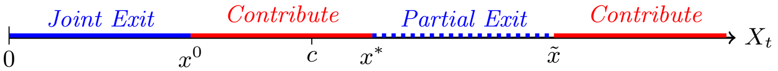
Main Result

Theorem (1)

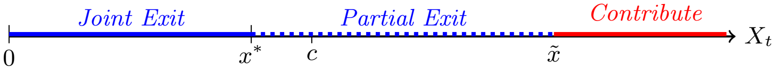
Pareto-undominated MPE is (almost) unique. Suppose $\beta^{**} < \beta^*$. If $\beta^{**} \geq \beta^*$

- *(1) When $\beta \geq \beta^*$: cooperative equilibrium*

- *(2a) when $\beta^{**} < \beta < \beta^*$: pre-emptive equilibrium (non-monotonic)*



- *(2b) when $1 < \beta \leq \beta^{**}$: pre-emptive equilibrium (monotonic)*



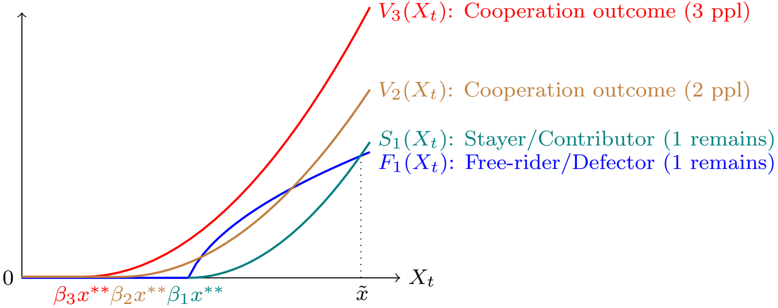
- **Blessing of Reliance**: cooperative equilibrium exists when players rely heavily on each other (large β)

Outline

Effect of Group Size

- Setup
  - Generalize to $N \geq 2$ players
  - Denote n_t as the number of players still contributing at time t
  - Flow payoff if *Contribute* $= X_t - \beta_{n_t} c$
    - assumption: $\beta_1 \geq \beta_2 \geq ... \geq \beta_{N-1} \geq \beta_N$
  - Flow payoff if *Defect* $= \alpha_{n_t} X_t$
    - for ease of exposition: $\alpha_0 = 0$ and $\alpha_{n_t} = \alpha$ if $n_t \geq 1$
- Key intuition: Domino effect
- Main finding: a group's ability to cooperate is non-monotonic in its size

Example



- Suppose $\frac{\beta_1}{\beta_2} < \beta^*$ but $\frac{\beta_1}{\beta_3} \geq \beta^*$
- $N = 2$: $V_2(X_t)$ vs. $F_1(X_t) \Rightarrow$ cooperation outcome is NOT sustainable
- $N = 3$: $V_3(X_t)$ vs. $F_1(X_t) \Rightarrow$ cooperation outcome is sustainable

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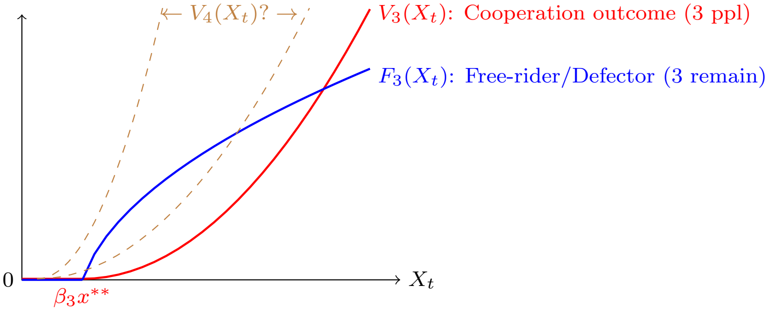

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- $N = 3$: $V_3(X_t)$ vs. $F_1(X_t) \Rightarrow$ cooperation outcome is sustainable
- $N = 4$: $V_4(X_t)$ vs. $F_3(X_t) \Rightarrow$ depend on whether $\frac{\beta_3}{\beta_4} \geq \beta^*$
- ...

Group Sizes Sustaining Cooperation

Theorem (2)

- *Denote* $n^{(0)} = 1$ *and* $n^{(k)} = \min\{n : \frac{\beta_{n^{(k-1)}}}{\beta_n} \geq \beta^*\}$. *The set of cooperation-sustaining group size is* $\{n^{(1)}, n^{(2)}, ...\}$

- Numerical example:
  - suppose $\beta_n = \frac{N}{n}$, and $\beta^* = 2.2$
  - C-sustaining: $N = 3$ (i.e., $\lceil \beta^* \rceil$), 7 (i.e., $\lceil 3 * \beta^* \rceil$), 16 (i.e., $\lceil 7 * \beta^* \rceil$) ,...
  - not C-sustaining: $N = 2, 4, 5, 6, 8, ..., 15, 17, ...$

- Takeaway message:
  - A group size sustains cooperation not because it is sufficiently large/small, but because it properly deters players from free-riding

Renegotiation Exit waves

Outline

Last-Exit Commitment by Leaders

- Motivation:
  - some partnerships have leaders, e.g., Real Madrid in ESL
  - leaders implicitly commit not to exit before others
- Setup:
  - designated first mover = the follower
  - designated second mover = the leader
- Timeline: a Stackelberg setting
  - Stage 1: follower chooses exit region $\mathcal{X}^f \subseteq \mathcal{X}$
  - Stage 2: afterward, leader chooses exit region $\mathcal{X}^l \subseteq \mathcal{X}$

Main Result Preview

Proposition (1)

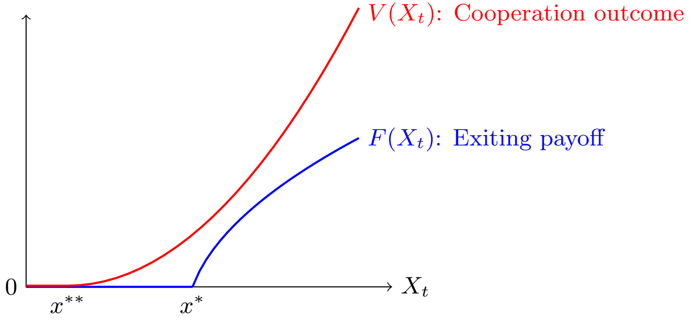
- *When β is large, follower adopts a cooperation strategy*

- *When β is small, follower's exit decision is non-monotonic*

- Main finding: Last-exit commitment can be a Pareto improvement
  - naturally, follower is better off compared with baseline
  - surprisingly, leader can be better off as well
    - intuition: cost (abandon option to exit first) $<$ benefit (avoid pre-emption)

Backward Induction: Stage 1 (Large β)

- Follower is facing an optimal stopping problem
  - flow payoff $= X_t - c$
  - lump-sum exiting payoff $= F(X_t)$
- When β is large ($\beta \geq \beta^*$)



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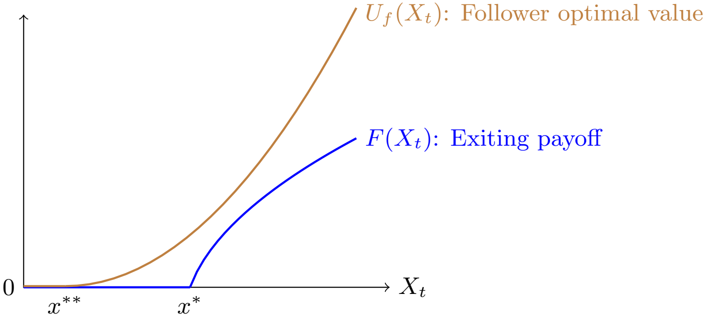
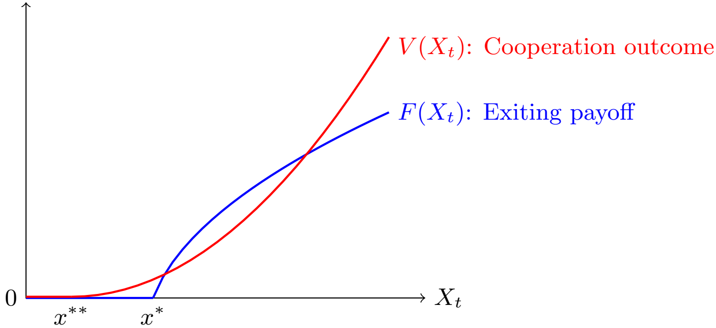


Backward Induction: Stage 1 (Small β)

- When β is small ($\beta < \beta^*$):



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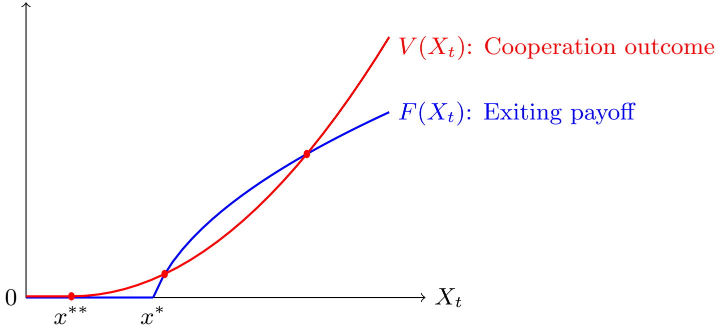

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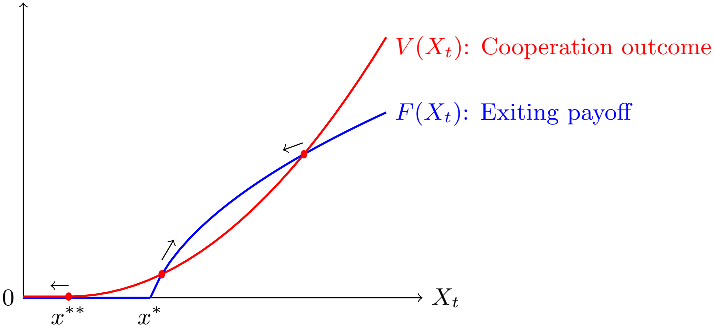

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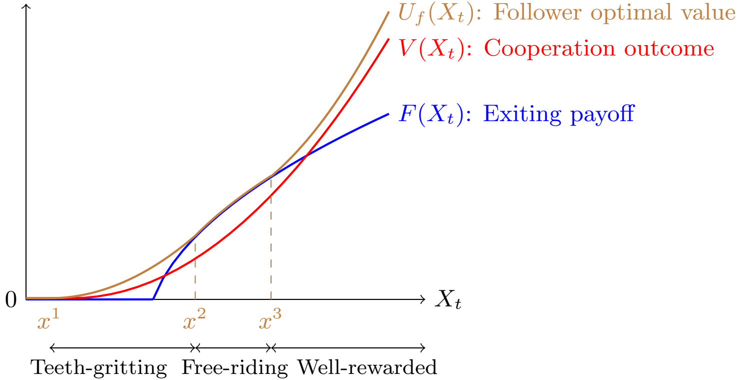


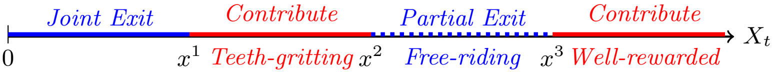
- Pin down thresholds: for $j = 1, 2, 3$
  - value matching: $U_f(X^j) = F(X^j)$
  - smooth pasting: $U_f'(X^j) = F'(X^j)$

Main Results

Proposition (1)

- *When $\beta \geq \beta^*$: cooperation outcome is implemented*

- *When $\beta < \beta^*$: free-riding occurs*



Proposition (2)

If $\beta < \beta^*$, last-exit commitment is a Pareto improvement when $X_t \geq \tilde{x}$.

Outline

Reversibility

- Motivation: some partnerships admit reversible defections Example
- Now, players can freely switch between *Contribute* or *Defect*

	Contribute	Defect
Contribute	$X_t - c\ ,\ X_t - c$	$X_t - \beta c\ ,\ \alpha X_t$
Defect	$\alpha X_t\ ,\ X_t - \beta c$	$0\ ,\ 0$

Proposition (3)

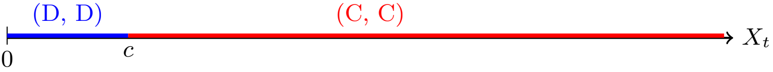
Under reversibility, FB outcome is implementable via a grim trigger strategy.

- Takeaway:
  - Classic repeated games: free-riding problem can be eliminated in a dynamic setting (McMillan, 1979)
  - Our baseline: irreversibility of defections explains observed free-riding in dynamic partnerships

When $\beta > \frac{1}{1-\alpha}$

	Contribute	Defect
Contribute	$X_t - c\ ,\ X_t - c$	$X_t - \beta c\ ,\ \alpha X_t$
Defect	$\alpha X_t\ ,\ X_t - \beta c$	$0\ ,\ 0$

- FB outcome:



- Stage-game NE:

- FB outcome is implementable with the following grim trigger strategy
  - upon deviation, switch to *Nash revision profile*: both defect iff $X_t < \frac{1}{1-\alpha}c$
  - reasons:
    - no one-period deviation benefit in CT
    - FB outcome Pareto improves stage-game NE for $\forall X_t$

Outline

Summary

- We analyze:
  - contagion of defections in a partnership
  - its implications on free-riding problem in teams
- Main results:
  - Curse of productivity & Blessing of reliance
  - A group's ability to cooperate is not monotonic in its size.
- Other results:
  - Last-exit commitment by leaders: potentially a Pareto improvement
  - Reversibility gives first-best outcome

Main Results Applied to ESL

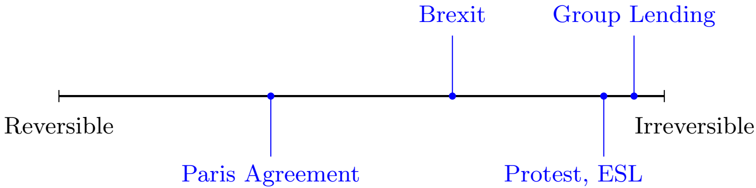
- Curse of productivity:
  - initial withdrawal of English clubs occurred soon after a Madrid commercial court's ruling that prohibited UEFA from sanctioning ESL's founding clubs
- Non-monotonicity in size:
  - a size of 3 sustains cooperation
  - a size much larger than 12 is also expected to push UEFA to compromise
  - a size of 12 did not work

Back

Thank You!

Irreversibility

- Returning to some partnerships is either impossible or very costly
- ESL: returning to ESL is snobbish and damages reputation among fans
- Public protest: demonstrative power lost upon withdrawal
- Group lending: upon default, future borrowing opportunities lost
- Brexit, Paris Agreement: reversible at a cost



Back to motivation Back to timeline Back to reversibility

Interpretations of $\alpha < 1$

	Contribute	Defect
Contribute	$X_t - c\ ,\ X_t - c$	$X_t - \beta c\ ,\ \alpha X_t$
Defect	$\alpha X_t\ ,\ X_t - \beta c$	$0\ ,\ 0$

- ESL: less "membership benefits"
- Public protest: loss of social influence
- Group lending: social sanctions
- ...

Back

Coin-Flipping Assumption

- ESL:
  - AC Milan & Juventus expressed intention to exit at the same time
  - AC Milan successfully exited
  - Juventus became one of the three remaining
- Standard in stochastic stopping games: Dutta & Rustichini (1993), Grenadier (1996), Weeds (2002), etc.
- Commons justifications:
  - exit decisions need to go through an authority who can approve only one application at a time
  - a random delay between a player's exit decision and the actual exercise of that decision

Back

Second Mover's Optimal Decision

- HJB equation

$$0 = max\{-S(X), -rS(X) + X - \beta c + S'(X)\mu X + \frac{\sigma^2}{2}S''(X)x^2\}$$

- General solution for homogeneous part (plus TVC condition)

$$S(X) = \underbrace{\frac{X}{r-\mu} - \frac{\beta c}{r}}_{\text{value if never exit}} + \underbrace{k_1 X^\gamma}_{\text{option value}}, \text{when } X \geq x^*.$$

- Pin down the solution
  - value matching: $S(x^*) = 0$
  - smooth pasting: $S'(x^*) = 0$
- Property:
  - $x^* < c$: option value of waiting

Back

First Mover's Value Function at Stage 2

- First mover does not make a decision at Stage 2
- Feynman-Kac equation

$$0 = -rF(X) + \alpha X + F'(X)\mu X + \frac{\sigma^2}{2} F''(X)x^2,$$

- General solution (plus TVC condition)

$$F(X) = \underbrace{\frac{\alpha X}{r-\mu}}_{\text{value if never terminated}} - \underbrace{k_2 X^\gamma}_{\text{termination loss}}, \text{when } X \geq x^*.$$

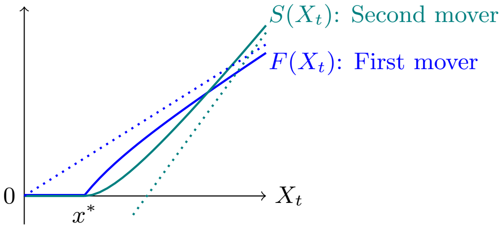
- Pin down the solution with exogenous exit $F(x^*) = 0$

Back

Concavity of $F(X_t)$

$$S(X_t) = \underbrace{\frac{X_t}{r-\mu} - \frac{\beta c}{r}}_{\text{value if never exit}} + \underbrace{k_1 X_t^{\gamma}}_{\text{option value}} \quad \text{, when } X_t \geq x^*$$

$$F(X_t) = \underbrace{\frac{\alpha X_t}{r-\mu}}_{\text{value if never terminated}} - \underbrace{k_2 X_t^{\gamma}}_{\text{termination loss}} \quad \text{, when } X_t \geq x^*$$



- $S(X_t)$: "option value" increases super-linearly as X_t decreases towards x^*
- $F(X_t)$: "termination loss" exhibits the same feature

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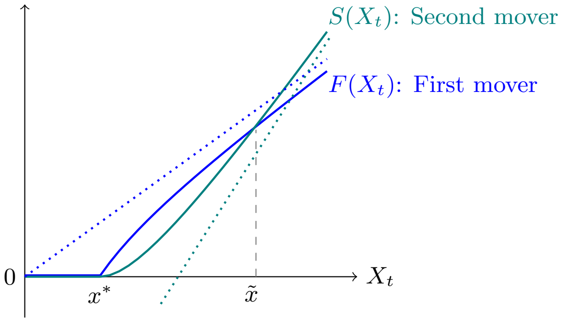
Intersection of $F(X_t)$ and $S(X_t)$



- Asymptotic lines of $F(X_t)$ and $S(X_t)$ have slopes $\frac{\alpha}{r-\mu}$ and $\frac{1}{r-\mu}$ respectively
- $F(X_t)$ and $S(X_t)$ have a unique intersection $\tilde{x} \in (x^*, \infty)$

Cooperation Outcome

- An outcome where players decide when to jointly exit
  - ex-post Pareto-optimal outcome
- Derivation: single-agent optimal stopping problem
  - flow payoff $= X_t - c$
  - lump-sum exiting payoff $= 0$
- Solution: exit iff $X_t \leq x^{**} = \frac{r-\mu}{r} \frac{\gamma}{\gamma-1} c$

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If $\beta^{**} \geq \beta^*$

Theorem (1')

- *When $\beta \geq \beta^*$, cooperative equilibrium*

- *when $\beta < \beta^*$: pre-emptive equilibrium*

- The medium scenario vanishes.

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Absence of Renegotiation

- Theorem 2 presumes that n-player cooperative equilibrium (if existing) will be played by n remaining players.
- This presumption is backed up by allowing renegotiation among players
  - the equilibrium is unique Pareto-undominated (Safronov & Strulovici 2018)
- If we disallow renegotiation:
  - we can use Pareto-dominated equilibrium to punish one who free rides
  - a group size sustains cooperation iff $N \geq n^{(1)}$
- Takeaway message:
  - renegotiation can backfire
  - without renegotiation, a large group is better

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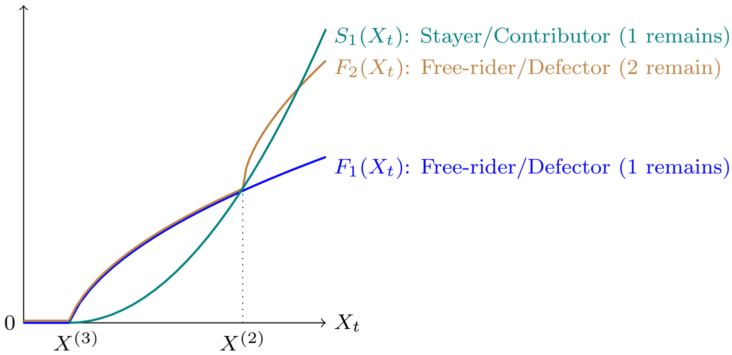
Exit Waves

- Example: initially $N = 3$, and $\alpha_2 > \alpha_1$



Exit Waves

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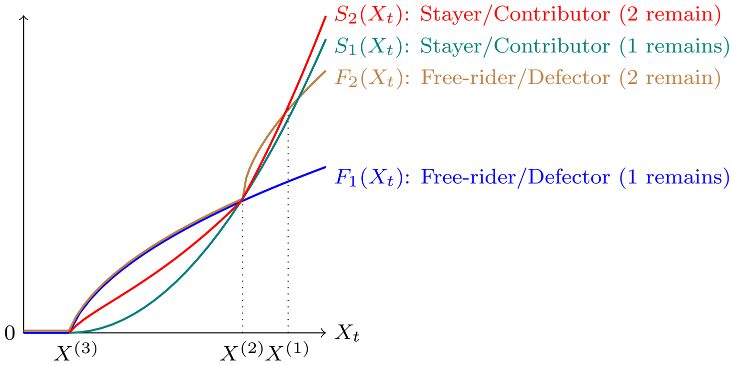

- Exit waves:
  - one player exits when $X^{(1)}$ is reached
  - a second player exits when $X^{(2)}$ is reached
  - a third player exits when $X^{(3)}$ is reached
- An algorithm is available to determine the exit waves

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