Contagion, Migration and Misallocation in a Pandemic

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2022 AEA Poster Session

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Introduction

Motivating Facts

- By the end of 2021, 280 million infected and 5.4 million deaths from this disease had been confirmed worldwide.
- The academic literature related to this disease has burgeoned after the outbreak, giving rise to different lines of research.
    - Topics on the restrictions on movements between regions, and the agents' decisions as a result of these restrictions have not been sufficiently analyzed.
    - The efficiency in the use and allocation of hospitalization resources across regions has also been under-studied.
- If there is no severe misallocation, the death rate for COVID-19 should be approximately the same across regions and close to the national average.
    - However, this is not the case when we look into the data of China.

Introduction

Motivating Facts

Table: Heterogeneous COVID-19 Death Rates

Countries (Provinces)	Date	Cases	Deaths	$\frac{\text{Deaths}}{\text{Cases}}$	Deaths per 100k People	Normalized SD of Death Rate	Hospital Beds per 1k People
Cross-Country Comparison							
United States	Aug. 26th	5,343,498	145,803	2.73%	45	0.69	2.9
India	Aug. 27th	3,234,474	59,449	1.84%	4.4	0.75	0.7
Brazil	Aug. 26th	3,717,156	117,665	3.17%	56	0.49	2.2
Germany	Aug. 21th	230,048	9,260	4.03%	11	0.23	8.3
South Korea	Aug. 26th	16,620	310	1.87%	0.60	1.11	11.5
Japan	Aug. 26th	63,973	1,229	1.92%	0.97	1.01	13.4
Mainland China	Aug. 2th	83,882	4,634	5.52%	0.33	1.23	4.2
Comparison within Mainland China							
Hubei	Aug. 2th	68,135	4,512	6.62%	7.6	-	6.7
(Wuhan of Hubei)	Aug. 2th	50,340	3,869	7.69%	35	-	9.2
Henan	Aug. 2th	1,276	22	1.72%	0.022	-	6.3
Heilongjiang	Aug. 2th	947	13	1.37%	0.034	-	6.6
Beijing	Aug. 2th	929	9	0.97%	0.042	-	9.1
Guangdong	Aug. 2th	1,672	8	0.48%	0.007	-	4.6
Shandong	Aug. 2th	799	7	0.88%	0.007	-	6.1
Shanghai	Aug. 2th	741	7	0.94%	0.029	-	9.6

Introduction
Our Works

- Our model emphasizes the endogenous migration decisions of different population groups during a pandemic, which has not been paid sufficient attention in related research.
  - An uninfected agent might want to move to a city with less infected people.
  - An infected patient would intend to migrate to a city with better medical treatment.
- We find closed-form solutions of our model, which can facilitate the understanding of pandemic economics and policy design.

Introduction

Related Literature

- The classical SIR model first proposed by Kermack et al. (1927).
  - Some other models have extended this framework in order to make it more meaningful (e.g., Chowell et al., 2003; Stehlé et al., 2011).
- The estimation of the economic impact due to COVID-19: Fernàndez-Villaverde and Jones (2020), Hall et al. (2020), and Guerrieri et al. (2020).
- Lockdown policy: Alvarez et al. (2021), Bobashev et al. (2011), Chinazzi et al. (2020).
- Our paper studies the misallocation of hospitalization resources during a pandemic (e.g., Hsieh and Klenow ,2009; Dower and Markevich, 2018; Hsieh et al., 2019; Tombe and Zhu, 2019).

Outline

The Model

Agents

- Agents only care about their health states and consumption.

$$u(c_t; h_t) = c_t + \phi h_t,$$

  - If the agent is healthy, $h_t = 1$.
  - If the agent becomes infected, $0 < h_t = u_I < 1$.
  - When an agent is recovered from the disease, the utility will return to the same level as those susceptible ones.
  - If an agent is dead, there will be a high disutility value, i.e., $h_t = u_D \ll 0$.

- Every period, each agent receives w units of endowment.

$$c_t + f_t = w.$$

  - c_t is the consumption level.
  - f_t is the fees paid when pandemic comes (discussed as follows).

The Model

Agents (Cont.)

- Consider n cities, with populations N_1, N_2,..., N_n, where n is finite and no smaller than 2.
  - There exist natural migration rates $\bar{\eta}_{ij}$ which satisfy the following equations simultaneously.

$$N_i \sum_{j=1, j \neq i}^{n} \bar{\eta}_{ij} = \sum_{j=1, j \neq i}^{n} \bar{\eta}_{ji} N_j,$$

  - When a pandemic comes, agents pay to make their own migration rates deviate from the corresponding natural level.
  - The fee an agent who lives in City i has to pay to achieve these rates is set as

$$f_i = \sum_{j=1, j \neq i}^{n} k_{ij} (\eta_{ij} - \bar{\eta}_{ij})^2.$$

The Model

Aggregate Moving Equations

- When a pandemic comes, agents in each city are divided into four types: susceptible (S), infected (I), recovered (R) and dead (D).
- We define the actual number of these types of agents after migration at the current period as $U_1'(t)$, $U_2'(t)$,..., $U_n'(t)$, $U = S, I, R$, which are

$$U_i'(t) = \left(1 - \sum_{j=1, j\neq i}^{n} \eta_{U,ij}\right) U_i(t) + \sum_{j=1, j\neq i}^{n} \eta_{U,ji} U_j(t), i = 1, 2, ..., n. \tag{1}$$

The Model

Aggregate Moving Equations (Cont.)

- Then, the aggregate moving equations of agents in City i, $i = 1, 2, ..., n$, are:

$$
\begin{aligned}
S_i(t+1) =& S_i(t) - \frac{\beta}{N_i(t)} S_i'(t) I_i'(t) - S_i(t) \sum_{j=1, j\neq i}^{n} \eta_{S,ij} + \sum_{j=1, j\neq i}^{n} \eta_{S,ji} S_j(t), \\
I_i(t+1) =& I_i(t) + \frac{\beta}{N_i(t)} S_i'(t) I_i'(t) - [\gamma_i(t) + \lambda_i(t)] I_i'(t) - I_i(t) \sum_{j=1, j\neq i}^{n} \eta_{I,ij} + \\
& \sum_{j=1, j\neq i}^{n} \eta_{I,ji} I_j(t), \\
R_i(t+1) =& R_i(t) + \gamma_i(t) I_i'(t) - R_i(t) \sum_{j=1, j\neq i}^{n} \eta_{R,ij} + \sum_{j=1, j\neq i}^{n} \eta_{R,ji} R_j(t), \\
D_i(t+1) =& D_i(t) + \lambda_i(t) I_i'(t).
\end{aligned}
$$

The Model

Aggregate Moving Equations (Cont.)

- In every period, the probability of recovering from sickness in City i is

$$\gamma_i(t) = \bar{\gamma} - \kappa_1 \left(\frac{I_i'(t)}{H_i} \right),$$

- Similarly, we set the probability of dying from the disease in every period as

$$\lambda_i(t) = \bar{\lambda} + \kappa_2 \left(\frac{I_i'(t)}{H_i} \right),$$

- Considering migration, these rates can be written as follows.

$$p_{i,t} = \beta \frac{I_i'(t)}{N_i'(t)}, q_{i,t} = \gamma_i(t) = \bar{\gamma} - \kappa_1 \left(\frac{I_i'(t)}{H_i} \right), r_{i,t} = \lambda_i(t) = \bar{\lambda} + \kappa_2 \left(\frac{I_i'(t)}{H_i} \right),$$

where

$$N_i'(t) = S_i'(t) + I_i'(t) + R_i'(t).$$

The Model

Aggregate Moving Equations (Cont.)

Table: Elements in the Transition Matrix

Health states in current period	Health states in the last period			
	S_i	I_i	R_i	D_i
S_i	$\left(1-\sum_{k\neq i}\tilde{\eta}_{S,ik}\right)(1-p_{i,t})$	0	0	0
I_i	$\left(1-\sum_{k\neq i}\tilde{\eta}_{S,ik}\right)p_{i,t}$	$\left(1-\sum_{k\neq i}\tilde{\eta}_{I,ik}\right)(1-q_{i,t}-r_{i,t})$	0	0
R_i	0	$\left(1-\sum_{k\neq i}\tilde{\eta}_{I,ik}\right)q_{i,t}$	$1-\sum_{k\neq i}\bar{\eta}_{ik}$	0
D_i	0	$\left(1-\sum_{k\neq i}\tilde{\eta}_{I,ik}\right)r_{i,t}$	0	1
S_j	$\tilde{\eta}_{S,ij}(1-p_{j,t})$	0	0	0
I_j	$\tilde{\eta}_{S,ij}p_{j,t}$	$\tilde{\eta}_{I,ij}(1-q_{j,t}-r_{j,t})$	0	0
R_j	0	$\tilde{\eta}_{I,ij}q_{j,t}$	$\bar{\eta}_{ij}$	0
D_j	0	$\tilde{\eta}_{I,ij}r_{j,t}$	0	0

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Migration Decisions

- The current expected utility of an agent in City i now if he/she was a susceptible one in the last period is

$$
\begin{aligned}
&u(c_t; h_t, h_{t-1} = \text{susceptible}, i)\\
=&w - f(\tilde{\eta}_{S,ij}; j \neq i) + \phi \left[\left(1 - \sum_{j\neq i} \tilde{\eta}_{S,ij}\right)(1 - p_{i,t}) + u_I \left(1 - \sum_{j\neq i} \tilde{\eta}_{S,ij}\right) p_{i,t} + \right.\\
&\left. \sum_{j\neq i} (\tilde{\eta}_{S,ij}(1 - p_{j,t}) + \tilde{\eta}_{S,ij} p_{j,t} u_I) \right].
\end{aligned}
$$

- The current expected utility for an agent that was infected in the last period can be derived as

$$
\begin{aligned}
&u(c_t; h_t, h_{t-1} = \text{infected}, i)\\
=&w - f(\tilde{\eta}_{I,ij}; j \neq i) + \phi \left[\left(1 - \sum_{j\neq i} \tilde{\eta}_{I,ij}\right)(1 - q_{i,t} - r_{i,t}) u_I + \left(1 - \sum_{j\neq i} \tilde{\eta}_{I,ij}\right) q_{i,t} \right.\\
&\left. + \left(1 - \sum_{j\neq i} \tilde{\eta}_{I,ij}\right) r_{i,t} u_D + \sum_{j\neq i} (\tilde{\eta}_{I,ij}(1 - q_{j,t} - r_{j,t}) u_I + \tilde{\eta}_{I,ij} q_{j,t} + \tilde{\eta}_{I,ij} r_{j,t} u_D) \right].
\end{aligned}
$$

Migration Decisions

- Denote the solutions of the migration rates as

$$\boldsymbol{\eta} = [\eta_{S,12}, \eta_{S,21}, ..., \eta_{S,(n-1)n}, \eta_{S,n(n-1)}, \eta_{I,12}, \eta_{I,21}, ..., \eta_{I,(n-1)n}, \eta_{I,n(n-1)}]',$$

  which is a $2n(n-1) \times 1$ vector.
- Given the total number of different types of agents I_i, D_i, $i = 1, 2, ..., n$, in the last period, these migration rates can be obtained from the system of linear equations

$$\boldsymbol{A}\eta = \boldsymbol{B},$$

  where $\boldsymbol{A}$ is a $2n(n-1) \times 2n(n-1)$ matrix, and $\boldsymbol{B}$ is a $2n(n-1) \times 1$ vector.
- Solutions in two cases.
  - Laissez-Faire Equilibrium: Each agent make their own decision given the belief of other agents' behavior.
  - Optimal Policy: A social planner decide all the migration rate simultaneously.

Migration Decisions

Comparison of the Laissez-Faire Equilibrium and the Optimal Policy

Table 2: Elements in the Matrix $\boldsymbol{A}$ and Vector $\boldsymbol{B}$ in Laissez-Faire Equilibrium and in Optimal Policy

Elements in Matrix $\boldsymbol{A}$				
Column	Row $(U, ij), U = S, I$			
	Laissez-Faire Equilibrium, $\boldsymbol{A}_L$		Optimal Policy, $\boldsymbol{A}_O$	
	(S, ij)	(I, ij)	(S, ij)	(I, ij)
(S, ij)	1	0	1	$\frac{C_S S_i}{k_{ij}}\left(\frac{1}{N_i - D_i} + \frac{1}{N_j - D_j}\right)$
(S, ji)	0	0	0	$-\frac{C_S S_j}{k_{ij}}\left(\frac{1}{N_i - D_i} + \frac{1}{N_j - D_j}\right)$
(S, ik)	0	0	0	$\frac{1}{k_{ij}} C_S \frac{S_i}{N_i - D_i}$
(S, ki)	0	0	0	0
(S, jk)	0	0	0	$-\frac{1}{k_{ij}} C_S \frac{S_j}{N_j - D_j}$
(S, kj)	0	0	0	0

Migration Decisions

Comparison of the Laissez-Faire Equilibrium and the Optimal Policy (Cont.)

(I,ij)	$\frac{C_S I_i}{k_{ij}}\left(\frac{1}{N_i-D_i}+\frac{1}{N_j-D_j}\right)$	$\frac{C_I I_i}{k_{ij}}\left(\frac{1}{H_i}+\frac{1}{H_j}\right)+1$	$\frac{C_S I_i}{k_{ij}}\left(\frac{1}{N_i-D_i}+\frac{1}{N_j-D_j}\right)$	$\frac{C_I I_i}{k_{ij}}\left(\frac{1}{H_i}+\frac{1}{H_j}\right)+1$
(I,ji)	$-\frac{C_S I_j}{k_{ij}}\left(\frac{1}{N_i-D_i}+\frac{1}{N_j-D_j}\right)$	$-\frac{C_I I_j}{k_{ij}}\left(\frac{1}{H_i}+\frac{1}{H_j}\right)$	$-\frac{C_S I_j}{k_{ij}}\left(\frac{1}{N_i-D_i}+\frac{1}{N_j-D_j}\right)$	$-\frac{2C_I I_j}{k_{ij}}\left(\frac{1}{H_i}+\frac{1}{H_j}\right)$
(I,ik)	$\frac{1}{k_{ij}}C_S\frac{I_i}{N_i-D_i}$	$\frac{1}{k_{ij}}C_I\frac{I_i}{H_i}$	$\frac{1}{k_{ij}}C_S\frac{I_i}{N_i-D_i}$	$\frac{2}{k_{ij}}C_I\frac{I_i}{H_i}$
(I,ki)	$-\frac{1}{k_{ij}}C_S\frac{I_k}{N_i-D_i}$	$-\frac{1}{k_{ij}}C_I\frac{I_k}{H_i}$	$-\frac{1}{k_{ij}}C_S\frac{I_k}{N_i-D_i}$	$-\frac{2}{k_{ij}}C_I\frac{I_k}{H_i}$
(I,jk)	$-\frac{1}{k_{ij}}C_S\frac{I_j}{N_j-D_j}$	$-\frac{1}{k_{ij}}C_I\frac{I_j}{H_j}$	$-\frac{1}{k_{ij}}C_S\frac{I_j}{N_j-D_j}$	$-\frac{2}{k_{ij}}C_I\frac{I_j}{H_j}$
(I,kj)	$\frac{1}{k_{ij}}C_S\frac{I_k}{N_j-D_j}$	$\frac{1}{k_{ij}}C_I\frac{I_k}{H_j}$	$\frac{1}{k_{ij}}C_S\frac{I_k}{N_j-D_j}$	$\frac{1}{k_{ij}}C_I\frac{I_k}{H_j}$
(S,kl)	0	0	0	0
(I,kl)	0	0	0	0

Elements in Vector $\boldsymbol{B}$

	Laissez-Faire Equilibrium, $\boldsymbol{B}_L$	Optimal Policy, $\boldsymbol{B}_O$
$B_{(S,ij)}$	$\frac{1}{k_{ij}}C_S\left(\frac{I_i}{N_i-D_i}-\frac{I_j}{N_j-D_j}\right)+\bar{\eta}_{ij}$	$\frac{1}{k_{ij}}C_S\left(\frac{I_i}{N_i-D_i}-\frac{I_j}{N_j-D_j}\right)+\bar{\eta}_{ij}$
$B_{(I,ij)}$	$\frac{1}{k_{ij}}C_I\left(\frac{I_i}{H_i}-\frac{I_j}{H_j}\right)+\bar{\eta}_{ij}$	$\frac{1}{k_{ij}}C_S\left(\frac{S_i}{N_i-D_i}-\frac{S_j}{N_j-D_j}\right)+\frac{2}{k_{ij}}C_I\left(\frac{I_i}{H_i}-\frac{I_j}{H_j}\right)+\bar{\eta}_{ij}$

Migration Decisions

Comparison of the Laissez-Faire Equilibrium and the Optimal Policy (Cont.)

Laissez-Faire Equilibrium

In the laissez-faire equilibrium, the elements of matrix $\boldsymbol{A}_L$ and vector $\boldsymbol{B}_L$ are shown in Table 2. Specifically, matrix $\boldsymbol{A}_L$ can be divided into the following four blocks:

$$\boldsymbol{A}_L = \begin{bmatrix} \boldsymbol{A}_{L,SS} & \boldsymbol{A}_{L,SI} \\ \boldsymbol{A}_{L,IS} & \boldsymbol{A}_{L,II} \end{bmatrix}.$$

These four block matrices are all $n(n-1) \times n(n-1)$ matrices, and they have the following properties:

1. $\boldsymbol{A}_{L,SS}$ is an $n(n-1) \times n(n-1)$ identity matrix.
2. $\boldsymbol{A}_{L,IS}$ is an $n(n-1) \times n(n-1)$ null matrix.
3. $\boldsymbol{A}_{L,SI}$ and $\boldsymbol{A}_{L,II}$ are non-singular matrices.

Since $\boldsymbol{A}_L$ is non-singular and $\boldsymbol{B}_L$ is non-zero, we can uniquely determine the migration decisions of the agents.

Migration Decisions

Comparison of the Laissez-Faire Equilibrium and the Optimal Policy (Cont.)

Optimal Policy

In the optimal policy, the elements of matrix $\boldsymbol{A}_O$ and vector $\boldsymbol{B}_O$ are shown in Table 2. Specifically, matrix $\boldsymbol{A}_O$ can be divided into the following four blocks:

$$\boldsymbol{A}_O = \begin{bmatrix} \boldsymbol{A}_{O,SS} & \boldsymbol{A}_{O,SI} \\ \boldsymbol{A}_{O,IS} & \boldsymbol{A}_{O,II} \end{bmatrix}.$$

These four block matrices are all $n(n-1) \times n(n-1)$ matrices, and they have the following properties:

1. $\boldsymbol{A}_{O,SS}$ is an $n(n-1) \times n(n-1)$ identity matrix.
2. $\boldsymbol{A}_{O,SI}$, $\boldsymbol{A}_{O,IS}$ and $\boldsymbol{A}_{O,II}$ are all non-singular matrices.
3. $\boldsymbol{A}_{O,SI} = \boldsymbol{A}_{L,SI}$ but $\boldsymbol{A}_{O,II} \neq \boldsymbol{A}_{L,II}$.

Since $\boldsymbol{A}_O$ is non-singular and $\boldsymbol{B}_O$ is non-zero, we can uniquely determine the migration decisions of the agents.

Outline

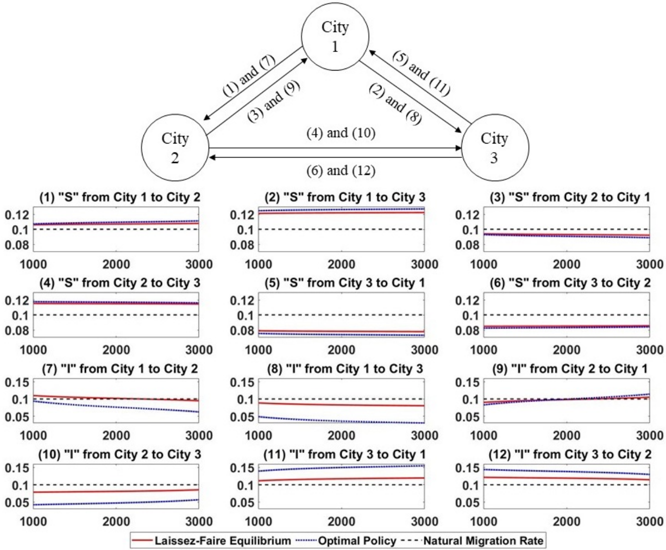
Numerical Results

Methodology and Calibration

- We study a three-city model as an example.
- These cities have the same population N, and the same natural migration rates $\bar{\eta}$ as well as their corresponding fee rates k between each other.
- The three cities are different.
  - City 1 has the largest number of infected agents, but has a medium level of hospital resources without satisfying the needs of its infected agents.
  - City 2 has a medium number of infected agents, but has the most abundant hospital resources.
  - Infected agents in City 3 are nearly zero, and has few hospital resources.
- Other parameters (estimated from data): $\beta = 0.4$, $\bar{\gamma} = 0.04$, $\bar{\lambda} = 0.0008$, $\kappa_1 = 0.01$ and $\kappa_2 = 0.0005$, $\bar{\eta} = 0.1$, and we extend them into two-week time span.

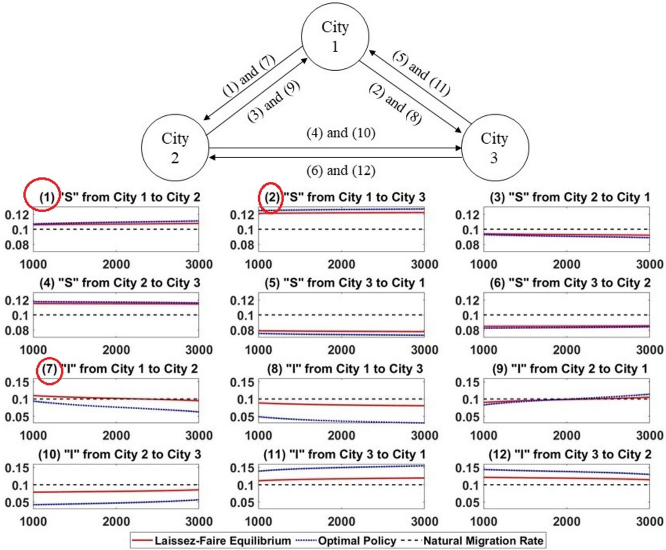
Numerical Results

Allocations of Hospitalization Resources



Numerical Results

Allocations of Hospitalization Resources (Cont.)



Numerical Results

Welfare Analysis

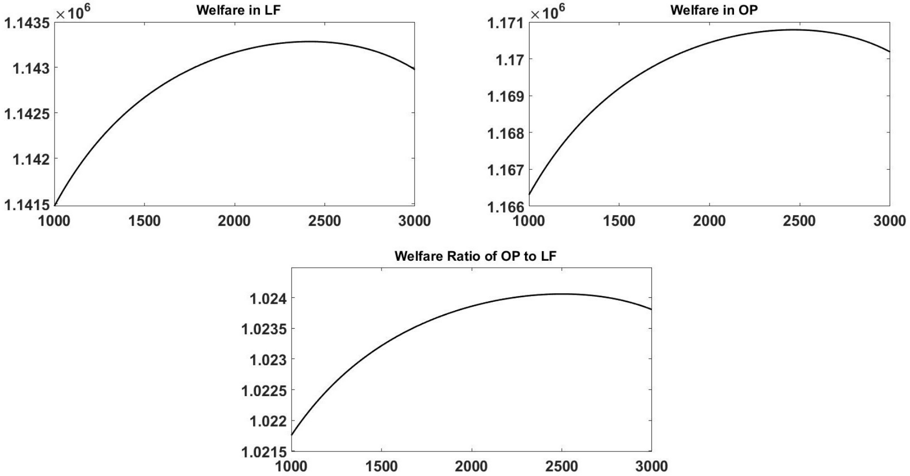


Figure: Relationship between Total Welfare and the Hospitalization Resources Allocated in City 1, Two-Week Time Span

Numerical Results

Misallocation in a Pandemic

Table: Simulated Results in Different Cases, Two-Week Time Span

Cases	Initial Number of Different Types of Agents and Hospital Resources	States	$\frac{\text{Deaths}}{\text{Cases}}$	Deaths per 10k People	Cases per 10k People	Normalized Standard Deviation of Death Rate	
						With Contagion	No Contagion
I	1 epidemic focus +	Initial	0.53%	10	1,048	1.3693	1.3693
($n = 5$,	1 large city +	LF	0.71%	41.17	4,735	0.2202	0.1517
$\eta_{max} = 0.8$)	3 small cities	OP	2.21%	36.47	513	1.7829	0.1377
II	1 epidemic focus +	Initial	0.54%	13	1332	0.8607	0.8607
($n = 10$,	5 large city +	LF	0.68%	51.22	6,192	0.3383	0.3237
$\eta_{max} = 0.8$)	4 small cities	OP	0.60%	38.43	4,575	0.9202	0.3196
III	1 epidemic focus +	Initial	0.52%	7	706	1.5672	1.5672
($n = 20$,	5 large cities +	LF	0.66%	30.22	3,971	0.1380	0.1353
$\eta_{max} = 0.8$)	14 small cities	OP	0.48%	20.67	3,104	0.5424	0.1217
IV	1 epidemic focus +	Initial	0.53%	7	817	1.3367	1.3367
($n = 30$,	10 large cities +	LF	0.68%	36.09	4,546	0.2103	0.2213
$\eta_{max} = 0.8$)	19 small cities	OP	0.49%	26.22	4,041	0.2283	0.1820
V	1 epidemic focus +	Initial	0.51%	4.60	384	1.9021	1.9021
($n = 50$,	10 large cities +	LF	0.59%	20.95	3,049	0.0463	0.0493
$\eta_{max} = 0.8$)	39 small cities	OP	0.45%	17.52	2,901	0.1731	0.1145

Outline

Conclusion

- We develop an endogenous migration model during pandemics based on a multi-city framework with hospitalization resource constraints, integrated with a traditional SIR epidemic model.
  - Several explicit solutions on migration decisions are provided.
  - The relationship between allocation of hospitalization resources and migration decisions.
  - Simulated results are consistent with what we find from the data.
- The framework we develop can be used to understand the behavior of people when facing an unknown epidemic disease like COVID-19, and provide a tool for governments to efficiently allocate hospitalization resources and different types of agents during these uncertain times.

Thanks for your attention!