Interim Strategy-Proof Mechanisms: Designing Simple Mechanisms in Complex Environments

Tangren Feng[1] Qinggong Wu [2]

[1]Bocconi

[2]HKUST

AEA, January 6, 2023

Intro

- Simple mechanism: each agent has one strategy that is optimal regardless of others' strategies

Intro

- Simple mechanism: each agent has one strategy that is optimal regardless of others' strategies
    - 2nd price auction, VCG, majority voting...

Intro

- Simple mechanism: each agent has one strategy that is optimal regardless of others' strategies
  - 2nd price auction, VCG, majority voting...
- Complex environment: one agent's preferences may depend on others' private info; informational externalities

Intro

- Simple mechanism: each agent has one strategy that is optimal regardless of others' strategies
  - 2nd price auction, VCG, majority voting...
- Complex environment: one agent's preferences may depend on others' private info; informational externalities
  - job market, school choice, drilling right auction ...

Interim strategy-proof (ISP)/ dominant strategy mechanisms with interdependent values.

ISP Example 0: single unit auction

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's valuation: $v_i(\theta) = \theta_i$
- Buyer i's payoff: $q_i\theta_i - \tau_i$
  - q_i: the probability i gets the good
  - τ_i: the transfer i pays

ISP Example 0: single unit auction

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's valuation: $v_i(\theta) = \theta_i$
- Buyer i's payoff: $q_i\theta_i - \tau_i$
  - q_i: the probability i gets the good
  - τ_i: the transfer i pays
- Bidding in 2nd-price auctions is simple: just bid the true value

ISP Example 1: single unit auction with informational externalities

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$

ISP Example 1: single unit auction with informational externalities

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0, 1]$
- Buyer i's ex post valuation: $v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$

ISP Example 1: single unit auction with informational externalities

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's ex post valuation: $v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$
  - $\beta > 0$: informational externality

ISP Example 1: single unit auction with informational externalities

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's ex post valuation: $v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$
  - $\beta > 0$: informational externality
  - Buyer i's interim valuation: $E[v_i|\theta_i] = \theta_i$
- Buyer i's payoff: $q_i v_i(\theta) - \tau_i$

ISP Example 1: single unit auction with informational externalities

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's ex post valuation: $v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$
  - $\beta > 0$: informational externality
  - Buyer i's interim valuation: $E[v_i|\theta_i] = \theta_i$
- Buyer i's payoff: $q_i v_i(\theta) - \tau_i$

Bidding in 2nd-price auctions is NOT simple anymore!

ISP Example 1: single unit auction with informational externalities

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's ex post valuation: $v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$
  - $\beta > 0$: informational externality
  - Buyer i's interim valuation: $E[v_i|\theta_i] = \theta_i$
- Buyer i's payoff: $q_i v_i(\theta) - \tau_i$

Bidding in 2nd-price auctions is NOT simple anymore!

- Suppose $\theta_1 = \frac{1}{2}$. Consider the following b_2

$$b_2(\theta_2) = \begin{cases} 1 & \text{if } \theta_2 \geq \frac{1}{2} \\ \frac{1}{2} - \epsilon & \text{if } \theta_2 < \frac{1}{2} \end{cases}.$$

ISP Example 1: single unit auction with informational externalities

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's ex post valuation: $v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$
  - $\beta > 0$: informational externality
  - Buyer i's interim valuation: $E[v_i|\theta_i] = \theta_i$
- Buyer i's payoff: $q_i v_i(\theta) - \tau_i$

Bidding in 2nd-price auctions is NOT simple anymore!

- Suppose $\theta_1 = \frac{1}{2}$. Consider the following b_2

$$b_2(\theta_2) = \begin{cases} 1 & \text{if } \theta_2 \geq \frac{1}{2} \\ \frac{1}{2} - \epsilon & \text{if } \theta_2 < \frac{1}{2} \end{cases}.$$

- The expected payoff of bidding $\frac{1}{2}$ is negative.

Question

Question: Does there exist any ISP mechanisms?

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

$$\tau_i^*(b_1, b_2) = \frac{b_i^2}{4}.$$

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

$$\tau_i^*(b_1, b_2) = \frac{b_i^2}{4}.$$

- Suppose i has type θ_i. How about overbid by Δ?

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

$$\tau_i^*(b_1, b_2) = \frac{b_i^2}{4}.$$

- Suppose i has type θ_i. How about overbid by Δ?
  - Benefit: Extra winning $\mathbb{P} : \dfrac{\Delta}{2}$

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

$$\tau_i^*(b_1, b_2) = \frac{b_i^2}{4}.$$

- Suppose i has type θ_i. How about overbid by Δ?
  - Benefit: $\frac{\Delta}{2}\theta_i$

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

$$\tau_i^*(b_1, b_2) = \frac{b_i^2}{4}.$$

- Suppose i has type θ_i. How about overbid by Δ?
  - Benefit: $\frac{\Delta}{2}\theta_i$
  - Cost: $\frac{(\theta_i + \Delta)^2}{4} - \frac{\theta_i^2}{4}$

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

$$\tau_i^*(b_1, b_2) = \frac{b_i^2}{4}.$$

- Suppose i has type θ_i. How about overbid by Δ?
  - Benefit: $$\frac{\Delta}{2}\theta_i$$
  - Cost: $$\frac{\Delta^2}{4} + \frac{\Delta}{2}\theta_i$$

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

$$\tau_i^*(b_1, b_2) = \frac{b_i^2}{4}.$$

- Suppose i has type θ_i. How about overbid by Δ?
  - Benefit: $\frac{\Delta}{2}\theta_i$
  - Cost: $\frac{\Delta^2}{4} + \frac{\Delta}{2}\theta_i$
- Overbidding is dominated!

An ISP Mechanism

Consider the following mechanism:

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2},$$

$$\tau_i^*(b_1, b_2) = \frac{b_i^2}{4}.$$

- Suppose i has type θ_i. How about overbid by Δ?
  - Benefit: $$\frac{\Delta}{2}\theta_i$$
  - Cost: $$\frac{\Delta^2}{4} + \frac{\Delta}{2}\theta_i$$
- Overbidding is dominated! So is underbidding.

More Questions

Question 1: Any other ISP mechanisms?

Question 2: Why is (q^*, τ^*) ISP while 2nd price auction isn't?

More Questions

Question 1: Any other ISP mechanisms?

More Questions

Question 1: Any other ISP mechanisms?

Answer 1: Yes, many more...

$$\hat{q}_i(b_1, b_2) = \frac{1}{2} + \frac{b_i^2 - b_j^2}{2},$$

$$\hat{\tau}_i(b_1, b_2) = \frac{b_i^3}{6} + c_i.$$

More Questions

Question 1: Any other ISP mechanisms?

Answer 1: Yes, many more...

$$\hat{q}_i(b_1, b_2) = \frac{1}{2} + \frac{b_i^2 - b_j^2}{2},$$

$$\hat{\tau}_i(b_1, b_2) = \frac{b_i^3}{6} + c_i.$$

In general,

$$q_i(b_1, b_2) = f_i(b_i) - g_i(b_j),$$

$$\tau_i(b_1, b_2) = \int_0^{b_i} f_i(x) dx + c_i,$$

where f_i and g_i are increasing functions.

More Questions

Question 2: Why is (q^*, τ^*) ISP but 2nd price auction isn't?

Answer 2: Let's compare them...

A Comparison

Mechanism: 2nd price auction

$$q_i^{2nd}(b_i, b_j) = \begin{cases} 1 & \text{if } b_i > b_j \\ 0 & \text{if } b_i \leq b_j \end{cases}.$$

$$\frac{\partial q_i^{2nd}}{\partial b_i} = \begin{cases} 0 & \text{if } b_i \neq b_j \\ \infty & \text{if } b_i = b_j \end{cases}.$$

Depends on b_j.

Extreme strategic externality.

Private value environment:
$v_i(\theta) = \theta_i$
No informational externality.

Mechanism: (q^*, t^*)

$$q_i^*(b_1, b_2) = \frac{1}{2} + \frac{b_i - b_j}{2}$$

$$\frac{\partial q_i^*}{\partial b_i} = \frac{1}{2}$$

Independent of b_j.

No strategic externality.

Interdependent value environment:
$v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$
Some informational externality.

Beyond the Example?

The toy example.

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's ex post valuation: $v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$

(q^*, τ^*) ISP.

Beyond the Example?

The toy example.

- 1 seller with single unit of good; 2 buyers
- Buyer i's type: $\theta_i \overset{\text{iid}}{\sim} U[0,1]$
- Buyer i's ex post valuation: $v_i(\theta) = \theta_i + \beta(\theta_j - \frac{1}{2})$

(q^*, τ^*) ISP.

The general model.

- 1 seller with single unit of good; N buyers
- Any type distribution
- Buyer i's ex post valuation: $v_i(\theta)$

Goal: All ISP mechanisms.

Characterizing ISP Auctions

Theorem
Under some regularity conditions,

$$M + PE + AS \Leftrightarrow ISP$$

where

M=Monotonicity

PE=Payoff Equivalence

AS=Additive Separability

Characterizing ISP Auctions

Theorem

Under some regularity conditions,

$$M + PE + AS \Leftrightarrow ISP$$

where

M=Monotonicity, $q_i(t_i, t_{-i})$ *is increasing in* t_i*;*

PE=Payoff Equivalence

AS=Additive Separability

Characterizing ISP Auctions

Theorem

Under some regularity conditions,

$$M + PE + AS \Leftrightarrow ISP$$

where

M=Monotonicity, $q_i(t_i, t_{-i})$ *is increasing in* t_i*;*

PE=Payoff Equivalence, $\tau_i(t_i, t_{-i}) = \int_{\underline{t}_i}^{t_i} q_i(x, t_{-i})dx + c_i$*;*

AS=Additive Separability

Characterizing ISP Auctions

Theorem

Under some regularity conditions,

$$M + PE + AS \Leftrightarrow ISP$$

where

M=Monotonicity, $q_i(t_i, t_{-i})$ *is increasing in* t_i*;*

PE=Payoff Equivalence, $\tau_i(t_i, t_{-i}) = \int_{\underline{t}_i}^{t_i} q_i(x, t_{-i})dx + c_i$*;*

AS=Additive Separability, $q_i(t_i, t_{-i}) = f_i(t_i) - g_i(t_{-i})$*.*

Beyond Auctions

What are the optimal ISP auctions that...

- maximizes revenue
- maximizes efficiency
- ....

What are ISP mechanisms in

- bilateral trade, public goods provision
- collective decision without money
- some other restricted domains

General theory on ISP mechanisms?

Why ISP?

ISP is desirable:

- better prediction
- outcome doesn't depend much on agents' cognitive abilities
- fair
- prevents waste from espionage
- helps agents to avoid strategic mistakes
- generates better information about true preferences