Self-Enforcing Power Dynamics

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Abstract

When can a ruler and an opposition share power? Why do power sharing agreements vary in the extent to which they constrain ruler's power? Why are these agreements fragile and inherently entail the risk of conflict? To answer these questions, I develop a model of strategic power dynamics in a polity that generates a flow of spoils, which the ruler divides. The opposition may initiate conflicts to contend for the spoils and the ruler's power is defined as the probability to defeat the opposition if a conflict occurs. Over time, the opposition gets opportunities to fight the ruler, while the ruler gets opportunities to grab power up to a context-specific and exogenously given physical limit. The possibility that ruler's power may increase renders the opposition an *opportunistic incentive* to fight before the ruler becomes too powerful. Whether conflict erupts depends on the extent to which the ruler can *credibly commit* to restrain power grab. Power sharing is impossible when the physical limit of the ruler's power is either extremely high or extremely low. When this limit is intermediate, the ruler is able to maintain peace with the opposition via power sharing agreements that vary in the extent to constrain the ruler's power. Under each of these agreements, the ruler increases power up to an endogenously emerging "permissible" limit that is strictly bounded below the exogenously given physical limit and refrains from further power grab. As long as the physical limit of the ruler's power is not too low, however, mutual suspicions between the ruler and the opposition about each other's intention are self-fulfilling, which prevents them from sharing power and generates conflicts.

"Every piece of land under the heaven, the king must rule.

Every single person upon the earth, the king must dominate."

— *Book of Odes*

"It will not be denied, that power is of an encroaching nature, and that it ought to be effectually restrained from passing the limits assigned to it."

— *James Madison in Federalist No.48*

1. Introduction

The question of whether and when is the sharing of power possible is essential to the study of authoritarian politics. Defined by Svolik (2012) as "arrangements over the sharing of spoils from joint rule," power sharing has become the benchmark to understand how authoritarian regimes can be ruled with peace and prosperity. To maintain support and/or discourage opposition from initiating costly conflicts, rulers of authoritarian regimes promise to share spoils with relevant political actors. To make the sharing of spoils credible, rulers must also share power, ultimately, physical force, in such a way that they would be punished for failing to honor their promises. According to Myerson (2008) and Boix and Svolik (2013), forums such as courts and parliaments make the sharing of spoils credible by enabling oppositions to punish rulers. Przeworski, Rivero and Xi (2015) find more directly that when the distribution of physical force between a ruler and an opposition is sufficiently balanced, they can credibly share spoils inter-temporally through regularly held competitive elections under the shadow of conflict.[1] But what makes the sharing of power — a balanced distribution of physical force — credible?

The conventional wisdom, dating back to to James Madison, resorts to institutions or, more generally, third-party enforcers. Based on the insight that power has a transitory

[1]The idea that rulers must share power to make their promises of sharing spoils credible is widespread. Other examples include constitutions (North and Weingast, 1989; Ginsburg and Simpser, 2013), ruling parties (Gandhi and Przeworski, 2006, 2007; Gehlbach and Keefer, 2011), elections (Fearon, 2011), and cabinet assignments (Meng, 2020; Meng and Paine, 2022).

nature, Acemoglu and Robinson (2000, 2001, 2005) and, more recently, Castañeda Dower et al. (2018, 2020) and Little and Paine (2023), argue that when potential challengers temporarily gain advantages against rulers in the distribution of power, rulers are willing to yield power to institutions that allow them to commit on a plan of sharing spoils that is credible even after challengers have lost their advantages. These "window-of-opportunity" models explain well why rulers have incentives to yield power to institutions, yet they assume the strength of these institutions in preventing rulers from regaining power. When institutions are weak, as Powell (2021) observes, rulers lose their ability to credibly commit on sharing spoils and thus are unable to buy off potential challengers. The question that remains unaddressed by the conventional wisdom, therefore, is where does the strength of institutions come from?

According to Svolik (2012), the logic of authoritarian politics has two foundational premises: "First, dictatorships inherently lack an independent authority with the power to enforce agreements among key political actors. Second, violence is an ever-present and ultimate arbiter of conflicts in authoritarian politics." Therefore, the strength of institutions under authoritarian contexts must itself be a result of a more fundamental and *self-enforcing* arrangement of sharing power. In line with Svolik's two premises, Paine (2022) and Kenkel and Paine (2023) offer an argument of "optimality" without resorting to any institution or third-party enforcer. Namely, they derive a set of conditions under which rulers, out of their self-interests, choose intermediate levels of power even though they can be more powerful. A crucial assumption, however, is that rulers can only choose their levels of power at the beginning of their reigns and, afterwards, the distribution of power is fixed. This is equivalent to assuming that after rulers have chosen their levels of power, they credibly commit on not altering them. The puzzle, therefore, is what prevents rulers from altering their levels of power and thus makes the distribution of power stationary? After all, even if rulers initiate plans to share power, temporarily fixing the distribution, they may later encounter opportunities to aggrandize power and reverse these plans.

Power evolves constantly and endogenously in the strategic interaction between rulers and the political actors they share spoils with. Given that the distribution of power is *dynamic* and *endogenous*, how would it evolve as a result of the strategic calculations by rulers and relevant political actors? How would the dynamics of power affect arrangements of sharing spoils? Is power sharing ever possible? And most importantly, can authoritarian regimes be ruled peacefully given that the rulers and relevant political actors are playing the game of power dynamics sophisticatedly? If the answer is yes, under what conditions?

To address these questions, I develop a model to study the strategic power dynamics in a polity consists of a "ruler" and an "opposition." The polity generates flow of spoils that by default are at the ruler's disposal. The ruler may share spoils with the opposition, while the opposition may contend for more via conflict. Power is relative, defined as one's probability to defeat the other in a conflict. Over time, the opposition gets opportunities to fight the ruler at a frequency depending on the ruler's power — the more powerful the ruler is, the rarer the opposition has opportunities to fight.[2] Whenever the opposition is unable to fight, the ruler has a chance to get an opportunity of power grab and with such an opportunity, the ruler is able to choose how powerful to become up to an exogenously given *physical limit*, which is the physically possible largest probability to defeat the opposition in a conflict. The physical limit is context-specific and can be affected by various factors outside of the ruler's and the opposition's control, including but not limited to military technology, geography, infrastructure, and logistics.[3]

Despite its parsimonious setup, this model helps to answer the question of whether and when is the sharing of power between the ruler and the opposition credible, given that the ruler's power is both dynamic and endogenous. The main contribution of the model,

[2]This relation between power and opposition's opportunity to fight can be justified by positively correlated stochastic processes that determine whether the opposition is able to fight and whether the ruler wins the fight. It is assumed in Paine (2022) and Little and Paine (2023). In a recent paper, Paine (2023) endogenizes the opposition's decision to mobilize and become able to fight the ruler.

[3]See Fearon (1997) for a detailed discussion about how the evolution of offensive and defensive military technologies affect the decisiveness of conflicts. For geography, infrastructure, and logistics, see Zhao (2015) for a measure of the power of states in a certain period of time by the longest distance they march troops.

therefore, is to provide a foundation for authoritarian power sharing that is completely self-enforcing — without relying on assumptions about either the strength of institutions or the ruler's commitment on a fixed level of power. Moreover, because the ruler's power is dynamic and endogenous, power sharing in this model has two features that are generally absent in existing models that treat power as exogenous and/or static. First, when power sharing is possible, the ruler and the opposition may agree on multiple ways to share power that vary in the extent to constrain the ruler's power. Second, power sharing could be fragile, dangerous, and inherently entail the risk of conflict.

Regarding the multiple ways to share power, the empirical literature (Cheibub, Gandhi and Vreeland, 2010; Geddes, Wright and Frantz, 2018; Xi, 2019; Gandhi and Sumner, 2020; Little and Meng, 2023; Jiang, Xi and Xie, 2020) has documented rich evidence about variation in the concentration of power across different authoritarian regimes and across time within these regimes. Such a variation cannot be explained by the "window-of-opportunity" models, where power is assumed to be exogenous. Neither can it be explained by the argument of "optimality." There, the ruler either refuses to share power or chooses a uniquely defined optimal level of power sharing. Hence, by offering a rational explanation for the coexistence of multiple power sharing agreements that vary in the extent to constrain the ruler's power, my model contributes by reconciling this gap between the theoretical and empirical literature.

Regarding the fragility and danger of power sharing, Meng, Paine and Powell (2023) showcase in their recent summary of the literature a divergence between scholarship on authoritarian stability and research on civil conflict about the implication of power sharing. The former emphasizes the role of power sharing in stabilizing authoritarian rule (for instance, Myerson, 2008; Boix and Svolik, 2013; Przeworski, Rivero and Xi, 2015; Meng, 2019, 2020), while the latter stresses the danger of power sharing in producing conflicts (see Roessler, 2016; Roessler and Ohls, 2018).[4] The seemingly paradoxical feature of power

[4] According to Meng, Paine and Powell (2023): "On the one hand, extensive scholarship on authoritarian stability usually stresses how sharing power facilitates regime survival by making ruler's promises to distribute spoils to elites and social groups more credible. On the other hand, most research on conflict and civil-military relations view power sharing as dangerous for the ruler."

sharing to simultaneously promote stability and breed conflicts cannot be explained by either the "window-of-opportunity" models or the argument of "optimality." In these models, whenever power sharing is possible, the ruler and the opposition share power and enjoy peace. In contrast, when power sharing is possible in my model, its realization may require joint efforts by the ruler and the opposition to coordinate. Their mutual suspicions about each other's intention could lead to coordination failure and generate conflicts.[5]

All the results of the model are derived from two driving forces. On the one hand, because power is dynamic, the possibility that the ruler may become more powerful renders the opposition an *opportunistic incentive* to fight before the ruler becomes too powerful. On the other hand, because power is endogenous, whether conflict erupts depends on how well the ruler solves a *commitment problem* with regard to power grab, that is, on the extent to which the ruler can credibly commit to restrain power grab.

Power sharing is impossible when the physical limit of the ruler's power is either extremely high or extremely low. In these cases, the ruler has no commitment in restraining power grab at all and conflict could occur as a result of the opposition's opportunistic incentive.

When the physical limit of the ruler's power is intermediate, power sharing is possible, it varies in the extent to constrain the ruler's power, and it is fragile and dangerous as long as the physical limit of the ruler's power is not too low. On the one hand, the ruler is able to maintain peace with the opposition via multiple "power sharing agreements" under which the ruler credibly commits to restrain power below endogenously emerging "permissible" limits. Each of these permissible limits is strictly bounded above by the physical limit of the ruler's power and measures the extent to which the corresponding power sharing agreement constrains the ruler's power. In contrast to the physical limit which is exogenously given, these permissible limits are *self-enforcing*: the opposition credibly commits to punish the ruler for increasing power over these limits and given the punishment, the ruler prefers to

[5] Meng and Paine (2022) find empirically that rulers and oppositions in rebel regimes are more likely to establish power sharing agreements. Their finding is consistent with the logic of the model: the shared experience of rulers and oppositions in establishing rebel regimes help them to avoid mutual suspicions and coordinate on sharing power.

restrain power within these limits. On the other hand, as long as the physical limit of the ruler's power is not too low, though power sharing agreements exist in theory, the ruler and the opposition could fail to coordinate on any one of them. When this happens, the mutual suspicions between the ruler and the opposition about each other's intention are self-fulfilling prophecies that prevent them from sharing power and make conflict inevitable.

Communication helps the ruler and the opposition to avoid mutual suspicions. However, communication also makes it drastically more difficult for the ruler to solve the commitment problem with regard to power grab and, therefore, could lead to conflict, as a result of failing to address the opposition's opportunistic incentive.

The physical limit of the ruler's power and the availability of the opportunity of power grab, as parameters of the model, affect how much the ruler's power can be constrained via power sharing agreements. The comparative statics are counter-intuitive: increasing both could make it easier to constrain the ruler's power by adding credibility to the opposition's commitment to punish the ruler for power grab. In adjusting these parameters, one faces a tradeoff between constraining power and ensuring peace. To maximally constrain the ruler's power, one must bear the risk of coordination failure between the ruler and the opposition that would ultimately lead to conflict. To ensure peace, making it independent of how well the ruler and the opposition coordinate with each other, one must have a degree of forbearance in not constraining the ruler's power as much as possible.

Other than the literature of authoritarian power sharing, the model is related to models of conflict bargaining (Fearon, 2004; Powell, 2004, 2006, 2012; Krainin, 2017). These models emphasize how exogenous shifts in the distribution of power affect the prospect for peace. In my model, the opposition's opportunistic incentive to fight due to the growth potential of the ruler's power follows the same logic. The difference is that here the dynamics in the ruler's power is endogenous. The model is also related to the broader literature on power dynamics (Powell, 2013; Acemoglu, Egorov and Sonin, 2012, 2013, 2015; Howell, Shepsle and Wolton, 2023; Luo and Przeworski, 2023). However, the focuses of these models are not how the sharing of spoils is made credible by self-enforcing power sharing. Powell (2013) focuses on the path civil conflicts end. Acemoglu, Egorov and Sonin (2012, 2013, 2015) focus on characterizing long-term composition of ruling coalitions, while Howell, Shepsle and Wolton (2023) focus on how the executive gradually aggrandizes its policy authority against the judiciary within democracies. Luo and Przeworski (2023) have a similar setup about power dynamics, but they focus on dynamics in the incumbent's electoral advantage under democracies rather than the ruler's power, that is, physical force, under authoritarian regimes.

2. Model

2.1. Setup

The players are a *ruler* (she) and an *opposition* (it) of a polity interacting in discrete times. In each period, the polity generates a flow of spoils with size normalized to 1. The two players may contend for the spoils via conflict. After a conflict, the loser is eliminated, getting 0 in the current and all subsequent periods; while the winner pays a lump-sum cost of $k > 0$ but enjoys all the spoils, getting 1 in the current and all subsequent periods. The ruler's probability to defeat the opposition in a conflict at t, denoted as p_t, defines her *power* at t. At $t = 0$, the ruler has the initial power of $p_0 \in (0, \pi]$, where $\pi \in (0, 1)$ is the *physical limit* of the ruler's power — her physically possible largest probability to defeat the opposition in a conflict.

Let $\mathcal{C}_t \in \{\emptyset, \mathcal{R}, \mathcal{O}\}$ keep track of the onset of conflict and its result: $\mathcal{C}_t = \emptyset$ means that conflict has not yet occurred before t, $\mathcal{C}_t = \mathcal{R}$ means that the ruler has defeated the opposition in a conflict before t, and $\mathcal{C}_t = \mathcal{O}$ means that the opposition has defeated the ruler before t. Then, $\mathcal{C}_0 = \emptyset$ and $\mathcal{C}_{t+1} = \mathcal{C}_t$ for any $t > 0$ such that $\mathcal{C}_t \in \{\mathcal{R}, \mathcal{O}\}$. The stage game at any t such that $\mathcal{C}_t \in \{\mathcal{R}, \mathcal{O}\}$ is trivial: the winner of the conflict that occurred before t consumes all the spoils. At any $t \geq 0$ such that $\mathcal{C}_t = \emptyset$, the ruler controls the spoils and decides how to divide them between herself and the opposition: let $s_t \in [0,1]$ denote the share of spoils the ruler allocates to the opposition. The stage game at t has two possible branches after the ruler's choice of s_t depending on whether the opposition *poses a threat* against the ruler, represented by $\theta_t \in \{0,1\}$.

First, if $\theta_t = 1$, the opposition has the option to start a conflict and fight the ruler. Let $a_t \in \{0,1\}$ denote the opposition's choice. Conflict breaks out if $a_t = 1$. If $a_t = 0$, the game proceeds to the next period peacefully with the ruler's power remaining unchanged, $p_{t+1} = p_t$. If $\theta_t = 0$, the opposition is unable to fight the ruler, in which case the game always proceeds to the next period peacefully.

At any $t \geq 0$ such that $\mathcal{C}_t = \emptyset$, the probability that $\theta_t = 0$, so that the ruler is free from the opposition's threat, is $\mu(p_t)$, where $\mu : [0,1] \to [0,1]$ is a twice differentiable, strictly increasing, and strictly concave function such that $\mu(0) = 0$ and $\mu(1) = 1$. The monotonicity of μ implies that the ruler's power improves her safety: the more powerful the ruler is, the less likely she has to worry about the opposition's threat. The concavity of μ implies that the marginal effect of the ruler's power on improving her safety is diminishing: increasing the ruler's power has a larger effect in reducing the opposition's probability to pose a threat when the ruler is less powerful than when she is more power.

Second, if $\theta_t = 0$, the opposition poses no threat and the ruler may get an *opportunity of power grab*, represented by $b_t \in \{0,1\}$. If $b_t = 1$, the ruler is able to increase her power at the next period to any $p_{t+1} = q_t \in [p_t, \pi]$.[6] If $b_t = 0$, the ruler's power remains unchanged, $p_{t+1} = p_t$. At any $t \geq 0$ such that $\mathcal{C}_t = \emptyset$ and $\theta_t = 0$, the probability that $b_t = 1$, so that an opportunity of power grab is available for the ruler is $\beta \in (0,1)$.

To summarize, the stage game at any t such that $\mathcal{C}_t = \emptyset$ begins with both players observing the ruler's power p_t, whether the opposition poses any threat θ_t, and whether the ruler has any opportunity of power grab b_t. The ruler then chooses how to allocate the spoils, $s_t \in [0,1]$. If $\theta_t = 1$, the opposition chooses whether to fight the ruler, $a_t \in \{0,1\}$, after

[6] Allowing the ruler to reduce power, that is, to choose any $q_t \in [\epsilon, \pi]$ for a sufficiently small $\epsilon > 0$ does not qualitatively change the results.

observing her allocation choice s_t. If $\theta_t = 0$ and $b_t = 1$, the ruler chooses her power at the next period, $q_t \in [p_t, \pi]$. The law of motion is given by

$$\Pr(\mathcal{C}_{t+1} = \emptyset | \mathcal{C}_t = \emptyset) = 1 - \theta_t a_t$$
$$\Pr(\mathcal{C}_{t+1} = \mathcal{R} | \mathcal{C}_t = \emptyset) = \theta_t a_t p_t$$
$$\Pr(\mathcal{C}_{t+1} = \mathcal{O} | \mathcal{C}_t = \emptyset) = \theta_t a_t (1 - p_t)$$

and conditional on $\mathcal{C}_{t+1} = \emptyset$,

$$p_{t+1} = (1 - (1 - \theta_t) b_t) \, p_t + (1 - \theta_t) b_t q_t.$$

The two players care only about the spoils and the cost of conflict and they discount future payoffs by a common factor $\delta \in (0, 1)$. At any $t \geq 0$, the ruler gets the flow payoff of

$$u_t = \mathbf{1}\{\mathcal{C}_t = \emptyset\} \left((1 - \theta_t a_t)(1 - s_t) + \theta_t a_t p_t (1 - k) \right) + \mathbf{1}\{\mathcal{C}_t = \mathcal{R}\}$$

and she chooses s_t and q_t to maximize the average discounted payoff

$$(1 - \delta) E \left(\sum_{t'=t}^{\infty} \delta^{t'-t} u_{t'} \mid \mathcal{C}_t = \emptyset, p_t, \theta_t, b_t \right);$$

while the opposition has the flow payoff of

$$v_t = \mathbf{1}\{\mathcal{C}_t = \emptyset\} \left((1 - \theta_t a_t) s_t + \theta_t a_t (1 - p_t)(1 - k) \right) + \mathbf{1}\{\mathcal{C}_t = \mathcal{O}\}$$

and it chooses a_t to maximize the average discounted payoff

$$(1 - \delta) E \left(\sum_{t'=t}^{\infty} \delta^{t'-t} v_{t'} \mid \mathcal{C}_t = \emptyset, p_t, \theta_t, b_t \right).$$

To break indifference, assume that the opposition pays an infinitesimal cost $\zeta > 0$ to fight the ruler; and that the ruler pays a cost $\eta h(q_t - p_t) \geq 0$ for increasing her power to q_t, where $\eta > 0$ is infinitesimal and $h : [0,1] \to \mathbb{R}_+$ is strictly increasing and $h(0) = 0$.

The solution concept is *pure strategy Markov perfect equilibrium* (equilibrium) with $\mathcal{C}_t, p_t, \theta_t, b_t$ being the state variables. For any $p \leq \pi$, the *continuation game at* p refers to the continuation game from any $t \geq 0$ such that $\mathcal{C}_t = \emptyset$ and $p_t = p$. In the continuation game at p, the ruler's strategy $\sigma = (\sigma_s, \sigma_q)$ consists of two parts: σ_s is referred to as the ruler's *spoils sharing strategy* that determines her allocation choice $s_t = \sigma_s(p_t, \theta_t, b_t) \in [0,1]$ conditional on p_t, θ_t, b_t; while σ_q is referred to as the ruler's *power grab strategy* that determines the level of power she chooses $q_t = \sigma_q(p_t) \in [p_t, \pi]$ conditional on p_t when $\theta_t = 0$ and $b_t = 1$. The opposition's strategy α determines its decision on whether to fight the ruler $a_t = \alpha(p_t, s_t) \in \{0,1\}$ conditional on p_t and the ruler's allocation choice s_t when $\theta_t = 1$. A strategy profile (σ, α) is an *equilibrium* of the continuation game at p if σ maximizes the ruler's payoff given α and α maximizes the opposition's payoff given σ.

An equilibrium of the game is an equilibrium of the continuation game at p_0. A *peaceful equilibrium* is one in which conflict never occurs on its equilibrium path. On the contrary, a *conflictual equilibrium* is one in which conflict always occurs, that is, with probability 1, on its equilibrium path. To ease notation, the subscript "t" is omitted in what follows whenever it causes no confusion.

2.2. *Formal definition of equilibrium*

For any strategy of the opposition α, let U_α be the ruler's value function induced by α, which is the unique solution to the Bellman equation

$$
\begin{aligned}
(1)\qquad & U_\alpha(\theta, b|p) \\
&= \max_{s\in[0,1], q\in[p,\pi]} (1-\theta\alpha(p,s))\Big((1-\delta)(1-s) + \delta U_\alpha\left((1-(1-\theta)b)\,p + (1-\theta)bq\right)\Big) \\
&\qquad + \theta\alpha(p,s)p\,(1-(1-\delta)k)\,,
\end{aligned}
$$

where

$$U_\alpha(p) := (1-\mu(p))\, U_\alpha(1,0|p) + \mu(p)(1-\beta)U_\alpha(0,0|p) + \mu(p)\beta U_\alpha(0,1|p)$$

is the expectation conditional on p. Similarly, for any strategy of the ruler σ, let V_σ be the opposition's value function induced by σ, which is the unique solution to

$$\begin{aligned}(2)\qquad & V_\sigma(\theta,b,s|p)\\ &= \max_{a\in\{0,1\}}(1-\theta a)\Big((1-\delta)s + \delta V_\sigma\left(\left(1-(1-\theta)b\right)p + (1-\theta)b\sigma_q(p)\right)\Big)\\ &\qquad + \theta a(1-p)\left(1-(1-\delta)k\right),\end{aligned}$$

where

$$\begin{aligned}V_\sigma(p) &:= (1-\mu(p))\, V_\sigma\left(1,0,\sigma_s(p,1,0)|p\right) + \mu(p)(1-\beta)V_\sigma\left(0,0,\sigma_s(p,0,0)|p\right)\\ &\quad + \mu(p)\beta V_\sigma\left(0,1,\sigma_s(p,0,1)|p\right)\end{aligned}$$

is the expectation conditional on p given $s=\sigma_s(p,\theta,b)$.

A strategy profile (σ,α) is an equilibrium of the continuation game at $p\in(0,\pi]$ if: given α, the ruler's strategy σ solves the maximization problem in (1) for all $p'\in[p,\pi]$, $\theta\in\{0,1\}$, and $b\in\{0,1\}$; and given σ, the opposition's strategy α solves the maximization problem in (2) for all $p'\in[p,\pi]$, $\theta\in\{0,1\}$, $b\in\{0,1\}$, and $s\in[0,1]$.

2.3. Analysis

Consider an equilibrium (σ,α) of the continuation game at p. Observe that the sum of the ruler's value and that of the opposition admits an upper and a lower bound:

$$(3)\qquad 1-(1-\delta)k \le U_\alpha(p) + V_\sigma(p) \le 1.$$

The upper bound 1 is attained if the equilibrium is peaceful, in which case the ruler and the opposition simply divide up the spoils at every period; while the lower bound $1-(1-\delta)k$ is attained if conflict occurs immediately, in which case one of the two players wins, pays the cost of conflict, and consumes all the spoils every period afterwards.

The opposition has the option to fight the ruler only when $\theta = 1$. In this case, by fighting the ruler, the opposition prevails and eliminates the ruler with probability $1-p$, so that it expects to get

$$(1-p)\left(1-(1-\delta)k\right).$$

While given the ruler's allocation s, the opposition expects to get

$$(1-\delta)s + \delta V_\sigma(p)$$

by giving up fighting. Therefore, the opposition prefers not to fight the ruler, so that $\alpha(p,s) = 0$, if and only if

$$s \geq S_\sigma(p) := \frac{(1-p)\left(1-(1-\delta)k\right) - \delta V_\sigma(p)}{1-\delta}. \tag{4}$$

When $\theta = 0$, the ruler chooses $s \in [0,1]$ to maximize

$$(1-\delta)(1-s) + \delta U_\alpha\left((1-b)p + bq\right).$$

In this case, because the opposition is unable to fight, the ruler's choice of s is inconsequential for whether conflict occurs. Hence, $\sigma_s(p,0,b) = 0$ must hold for each $b \in \{0,1\}$. The ruler need not share any spoils absent the opposition's threat. In addition, if $b = 1$, the ruler's choice of power grab, $q = \sigma_q(p)$, must maximize her continuation value $U_\alpha(q)$ in the future periods.

Lemma 1. *If (σ, α) is an equilibrium of the continuation game at $p \leq \pi$, then:*

1. *for all $p' \in [p, \pi]$ and $s \in [0, 1]$,*

$$\alpha(p', s) = \mathbf{1}\left\{s < S_\sigma(p')\right\},$$

   where $S_\sigma(p') > 0$;

2. *for all $p' \in [p, \pi]$ such that $S_\sigma(p') \leq 1$,*

$$\sigma_s(p', 1, 0) = S_\sigma(p');$$

3. *for all $p' \in [p, \pi]$ and $b \in \{0, 1\}$,*

$$\sigma_s(p', 0, b) = 0;$$

4. *there exists a unique $Q_\sigma(p) \in [p, \pi]$ such that for all $p' \in [p, Q_\sigma(p)]$,*

$$\sigma_q(p') = Q_\sigma(p).$$

Lemma 1 summarizes basic equilibrium properties that facilitate further analysis. First, whenever the opposition poses a threat, it fights the ruler if and only if she fails to share enough spoils, $s < S_\sigma(p)$, and the amount needed to buy off the opposition is always nontrivial, as $S_\sigma(p) > 0$.

Second, facing the opposition's threat, the ruler always prefers to buy it off rather than fighting. In particular, if $S_\sigma(p) \leq 1$, so that the ruler can afford to buy off the opposition, she shares exactly $S_\sigma(p)$ with the opposition to avoid conflict. The ruler has no incentive to share more than $S_\sigma(p)$. By sharing exactly $S_\sigma(p)$ to avoid conflict, the ruler expects to get

$$\begin{aligned}(1-\delta)\left(1-S_\sigma(p)\right)+\delta U_\alpha(p) &\geq (1-\delta)\left(1-S_\sigma(p)\right)+\delta\left(1-(1-\delta)k-V_\sigma(p)\right)\\ &= p\left(1-(1-\delta)k\right)+(1-\delta)^2 k > p\left(1-(1-\delta)k\right),\end{aligned}$$

where the first inequality follows (3) and the second follows the definition of $S_\sigma(p)$ in (4). Hence, the ruler always prefers to spend $S_\sigma(p)$ to buy peace, rather than sharing less and immediately fighting the opposition.

Third, the ruler withholds all the spoils absent the opposition's threat. The ruler controls all the spoils at any period that conflict has not yet occurred. The only reason to share spoils is to avoid conflict, which is possible only when the opposition poses a threat. Consequently, when the opposition does pose a threat, the ruler cannot credibly commit on future payments — if the opposition gives up fighting, the ruler would not share any spoils until it comes up with a threat again. Therefore, to buy off the opposition, the ruler must share at least $S_\sigma(p)$ immediately at the current period. Because the ruler can at most transfer all the spoils she currently controls, if $S_\sigma(p) > 1$, she is unable to buy off the opposition and must fight.[7]

Fourth, in any equilibrium of the continuation game at p, the ruler alters her power at most once on the equilibrium path to a unique *target of power grab* $Q_\sigma(p)$. As long as her power is below the target, the ruler would increase her power to $Q_\sigma(p)$ as soon as an opportunity of power grab is available and refrain from further increasing power. Suppose the ruler would increase power from p to some $q \geq p$ the first time she gets an opportunity of power grab and then from q to some $q' > q$ the second time such an opportunity is available. Because $q' > q$ and because such an adjustment costs $\eta h(q'-q) > 0$, the ruler must strictly prefer q' to q, so that $U_\alpha(q') > U_\alpha(q)$. But because $q' > p$ and $U_\alpha(q') > U_\alpha(q)$, the ruler would not have chosen to increase her power from p to q the first time she gets an

[7]Similarly to all models that have a "window-of-opportunity" structure (Acemoglu and Robinson, 2000, 2001, 2005; Castañeda Dower et al., 2018, 2020; Powell, 2021; Paine, 2022, 2023), posing a threat provides the opposition a "window-of-opportunity" to fight the ruler as well as getting paid; while the ruler faces a commitment problem in promising future payments.

opportunity of power grab given that she is able to directly increase her power to q'.

3. Strategic power dynamics, peace, and conflict

3.1. Impact of power dynamics

According to Lemma 1, if (σ, α) is an equilibrium of the continuation game at p, then the opposition's value conditional on p is

$$(5) \qquad V_\sigma(p) = (1-\mu(p)) \overbrace{(1-p)\left(1-(1-\delta)k\right)}^{\theta=1} + \mu(p)(1-\beta) \overbrace{\delta V_\sigma(p)}^{\theta=0,b=0} + \mu(p)\beta \overbrace{\delta V_\sigma\left(Q_\sigma(p)\right)}^{\theta=0,b=1}.$$

Specifically, three cases are possible at any period that conflict has not yet occurred. First, $\theta = 1$, so that the opposition poses a threat, which happens with probability $1-\mu(p)$. In this case, the ruler either allocates less than $S_\sigma(p)$ to the opposition and the two players fight, or she shares exactly $S_\sigma(p)$, leaving the opposition indifferent on whether to fight. Either way, the opposition expects to get $(1-p)\left(1-(1-\delta)k\right)$. Second, $\theta = 0$ and $b = 0$, so that the ruler is free from the opposition's threat but is unable to increase her power, which happens with probability $\mu(p)(1-\beta)$. In this case, the ruler shares no spoils and her power remains unchanged, so that the opposition expects to get $\delta V_\sigma(p)$. Third, $\theta = 0$ and $b = 1$, so that the ruler is free from the opposition's threat and gets an opportunity of power grab, which happens with probability $\mu(p)\beta$. In this case, the ruler shares no spoils and increases her power to $Q_\sigma(p)$, so that the opposition expects to get $\delta V_\sigma\left(Q_\sigma(p)\right)$.

The opposition's value $V_\sigma(p)$ depends on the ruler's strategy σ only through her target of power $Q_\sigma(p)$. In particular, if $Q_\sigma(p) = p$, equation (5) solves

$$V_\sigma(p) = \overline{V}(p) := \frac{1-\mu(p)}{1-\mu(p)\delta}(1-p)\left(1-(1-\delta)k\right),$$

which is the opposition's value given that the ruler's power is fixed at p. In general, for each

$p \in (0,1)$, define

$$\lambda_\beta(p) := \frac{\mu(p)\beta\delta}{1-\mu(p)(1-\beta)\delta} \in (0,1).$$

Then, equation (5) solves

$$V_\sigma(p) = (1-\lambda_\beta(p))\,\overline{V}(p) + \lambda_\beta(p)\overline{V}\left(Q_\sigma(p)\right)$$

as a weighted average between the opposition's payoff when the ruler's power is fixed at p, that is, $\overline{V}(p)$, and its payoff when the ruler's power is fixed at $Q_\sigma(p)$, that is, $\overline{V}\left(Q_\sigma(p)\right)$. Because

$$\overline{V}'(p) = -\left(\frac{\mu'(p)(1-\delta)}{(1-\mu(p)\delta)^2}(1-p) + \frac{1-\mu(p)}{1-\mu(p)\delta}\right)(1-(1-\delta)k) < 0,$$

$\overline{V}(p)$ is strictly decreasing in p. Hence, $V_\sigma(p) \leq \overline{V}(p)$ and the inequality holds strictly if $Q_\sigma(p) > p$. Conditional on the ruler's power p, the opposition is worse off when the ruler seeks to increase her power to $Q_\sigma(p) > p$ than when her power is fixed at p.

The weight $\lambda_\beta(p)$ measures how fast the ruler's power grows at p. Because

$$\begin{aligned} \lambda'_\beta(p) &= \frac{\mu'(p)\beta\delta}{(1-\mu(p)(1-\beta)\delta)^2} > 0 \\ \frac{\partial}{\partial\beta}\lambda_\beta(p) &= \frac{\mu(p)\delta\left(1-\mu(p)\delta\right)}{(1-\mu(p)(1-\beta)\delta)^2} > 0, \end{aligned}$$

$\lambda_\beta(p)$ is strictly increasing in both p and β. Remember that the ruler is able to increase power when she is free from the opposition's threat and has an opportunity of power grab. The larger p is, the more likely that the ruler is free from the opposition's threat and given $\theta = 0$, the larger β is, the more likely an opportunity of power grab is available.

According to Lemma 1, whether conflict is possible to occur depends on whether the ruler can afford to buy off the opposition, that is, on whether $S_\sigma(p)$ is larger or smaller than

1. For each $p \in (0,1)$, let

$$\underline{S}(p) := \frac{(1-p)\left(1-(1-\delta)k\right)-\delta\overline{V}(p)}{1-\delta}$$

denote the amount of spoils needed to buy off the opposition when the ruler's power is fixed at p. Then,

$$S_\sigma(p) = \overbrace{\underline{S}(p)}^{\text{base}} + \overbrace{\frac{\delta}{1-\delta}\lambda_\beta(p)\left(\overline{V}(p)-\overline{V}\left(Q_\sigma(p)\right)\right)}^{\text{premium}}.$$

Because $\overline{V}(p)$ is strictly decreasing in p, $S_\sigma(p) \geq \underline{S}(p)$ and the inequality holds strictly if $Q_\sigma(p) > p$. Conditional on having the power of p, the ruler has to share a larger amount of spoils to buy off the opposition when she seeks to increase power to $Q_\sigma(p) > p$ than the base amount she must pay when her power is fixed at p. The gap is the *premium* to compensate the opposition's loss, $\overline{V}(p) - \overline{V}\left(Q_\sigma(p)\right)$, due to the growth of the ruler's power. Intuitively, the premium is a result of the opposition's *opportunistic incentive* to fight. Given that the ruler is actively seeking to increase power, the opposition has an incentive to fight as soon as he is able to, so as to curb the growth of the ruler's power.[8] The faster the ruler's power grows as reflected by a larger $\lambda_\beta(p)$, the stronger is the opposition's opportunistic incentive, and therefore the larger is the premium.

Due to the opposition's opportunistic incentive and the resulted premium, the dynamics in the ruler's power always makes peace between the two players harder to maintain. The following results are very helpful in characterizing the conditions for peace.

Lemma 2. *$\underline{S}(p)$ is strictly quasiconcave in p and $\underline{S}(p) + \frac{\delta}{1-\delta}\lambda_\beta(p)\left(\overline{V}(p)-\overline{V}(q)\right)$ is strictly quasiconcave in $p \leq q$, strictly increasing in $q \geq p$, and strictly increasing in β if $q > p$.*

Lemma 2 implies that conditional on the ruler's power, it is most expensive to buy off the

[8] This incentive is also present in models of conflict bargaining, see Fearon (2004); Powell (2004, 2006, 2012); Krainin (2017).

opposition when the ruler's power is intermediate.[9] An extremely high level of power allows the ruler to pay a low price for peace by leaving the opposition an extremely low expected payoff to fight. An extremely low level of power allows the ruler to pay a low price because it leaves the opposition an extremely high payoff by giving up fighting and waiting for the next time to pose a threat and get paid.[10] An intermediate level of power has neither advantage.

Moreover, buying off the opposition is more expensive when the ruler has a more ambitious target of power grab and/or when she can more frequently get opportunities of power grab. The former imposes on the opposition a greater loss, that is, a larger $\overline{V}(p) - \overline{V}(q)$, while the latter implies a faster transition, that is, a larger $\lambda_\beta(p)$. Both reinforce the opposition's opportunistic incentive to fight and therefore result in a larger premium the ruler has to pay to buy off the opposition. Hence, the dynamics in the ruler's power does not only *qualitatively* make peace harder to maintain; *quantitatively*, it is even more difficult if the transition has a greater magnitude and/or happens at a faster pace.

3.2. Peace under power consolidation

To have the richest set of results, the following assumption is imposed throughout.

Assumption 1. *The ruler and the opposition care sufficiently about the future, $\delta > \overline{\delta}$, where $\overline{\delta} \in (0,1)$.*

Assumption 1 implies the following result about when peace can be maintained between the two players given that the ruler's power is fixed at p.

Lemma 3. [11] *There are two thresholds $\underline{p}, \overline{p} \in (0,1)$ such that $\underline{p} < \overline{p}$ and $\underline{S}(p) > 1$ holds if and only if $p \in (\underline{p}, \overline{p})$.*

[9]This result depends on the concavity of μ. Paine (2022) assumes a functional form $\mu(p) = 1 - (1-p)^\gamma$ for $\gamma \in (0,1)$ and proves the quasiconcavity of $\underline{S}(p)$.

[10]This logic is generic in "window-of-opportunity" models, see (Acemoglu and Robinson, 2000, 2001, 2005; Castañeda Dower et al., 2018, 2020; Powell, 2021; Paine, 2022, 2023).

[11]For Lemma 3, it is sufficient to assume $\delta > \underline{\delta}$, where $\underline{\delta} := \min_{p\in(0,1)} \frac{(1-p)k+p}{(1-p)k+\mu(p)} < \overline{\delta}$.

Recall that $\underline{S}(p)$ is the amount of spoils needed to buy off the opposition when the ruler's power is fixed at p. Lemma 3 therefore implies that when the ruler's power is fixed, peace can be maintained if and only if the ruler is either extremely powerful, $p \geq \overline{p}$, or extremely powerless, $p \leq \underline{p}$. If the ruler has an intermediate level of power $p \in (\underline{p}, \overline{p})$, she cannot afford to buy off the opposition, so that conflict occurs as soon as the opposition is able to fight.

After the ruler *consolidates* power, that is, increases her power to its physical limit π, her power becomes fixed at π. Hence, Lemma 3 also shows the consequence of the ruler's power consolidation: it brings peace if $\pi \notin (\underline{p}, \overline{p})$ and leads to conflict if $\pi \in (\underline{p}, \overline{p})$.

Proposition 1. *If $\pi \notin (\underline{p}, \overline{p})$, then for any $p \leq \pi$, the continuation game at p has a unique equilibrium (σ, α) in which $Q_\sigma(p) = \pi$ and this equilibrium is peaceful if and only if*

$$p \notin (\underline{p}, \overline{p}) \text{ and } \pi \leq \hat{q}_\beta(p),$$

where for each $p \notin (\underline{p}, \overline{p})$,

$$\hat{q}_\beta(p) := \max\left\{q \in [p, 1] : \ \underline{S}(p) + \frac{\delta}{1-\delta}\lambda_\beta(p)\left(\overline{V}(p) - \overline{V}(q)\right) \leq 1\right\} \in [p, 1],$$

$\hat{q}_\beta(p) = 1$ if p is sufficiently closed to either 0 or 1, and otherwise, $\hat{q}_\beta(p) < 1$ is strictly quasiconvex in p.

According to Proposition 1, the ruler is always seeking power consolidation provided that it brings peace. This happens when the physical limit of the ruler's power is either extremely high or extremely low, $\pi \notin (\underline{p}, \overline{p})$. In this case, the ruler increases her power to π as soon as an opportunity of power grab is available. Recall that when $\theta = 0$ and $b = 1$, so that the ruler is free from the opposition's threat and gets an opportunity of power grab, she chooses $q \in [p, \pi]$ to maximize her continuation value $U_\alpha(q)$ in the future periods. Because power consolidation brings peace, the ruler's continuation value by choosing $q = \pi$ is exactly the size of the spoils net the opposition's continuation value,

$$U_\alpha(\pi) = 1 - \overline{V}(\pi),$$

which strictly dominates her continuation value by choosing any $q < \pi$, as

$$U_\alpha(q) \leq 1 - V_\sigma(q) = 1 - (1 - \lambda_\beta(q))\,\overline{V}(q) - \lambda_\beta(q)\overline{V}\left(Q_\sigma(q)\right) < 1 - \overline{V}(\pi),$$

where the first inequality follows (3) and the last inequality is due to $q < \pi$ and $\overline{V}$ being strictly decreasing.

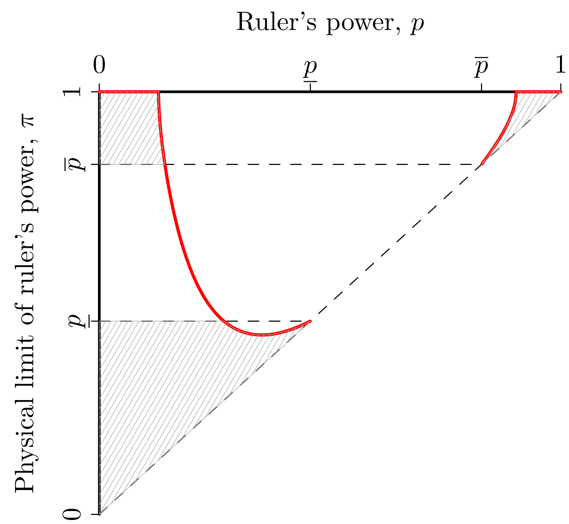
When the physical limit of the ruler's power is extreme, peace between the two players is guaranteed *after* the ruler consolidates power. Formally, if $\pi \notin (\underline{p}, \overline{p})$, the unique equilibrium of the continuation game at π must be peaceful. This condition, however, is insufficient to guarantee peace *before* the ruler consolidates power. According to Proposition 1, this additionally requires the ruler's power before consolidation to be either extremely high or extremely low, $p \notin (\underline{p}, \overline{p})$, and its physical limit to be sufficiently low, $\pi \leq \hat{q}_\beta(p)$.

Given the ruler's target of power grab $Q_\sigma(p) = \pi$, before getting any opportunity to actually increase power, she must share at least

$$S_\sigma(p) = \underline{S}(p) + \frac{\delta}{1-\delta}\lambda_\beta(p)\left(\overline{V}(p) - \overline{V}(\pi)\right)$$

spoils to avoid conflict when the opposition poses a threat. This minimal amount of spoils to buy off the opposition consists of two parts: the base payment $\underline{S}(p)$ that is required when the ruler's power is fixed at p and the premium $\frac{\delta}{1-\delta}\lambda_\beta(p)\left(\overline{V}(p) - \overline{V}(\pi)\right)$ for its growth from p to π. First, if $p \in (\underline{p}, \overline{p})$, then $\underline{S}(p) > 1$, so that the ruler cannot even afford the base payment. Second, if $p \notin (\underline{p}, \overline{p})$ but $\pi > \hat{q}_\beta(p)$, then although the ruler is able to buy off the opposition when her power is fixed at p, she cannot additionally afford the premium for increasing her power from p to π. In this case, if the ruler commits on restraining her power grab below $\hat{q}_\beta(p)$, she would be able to buy off the opposition and avoid conflict. This commitment, however, is not credible: provided that power consolidation brings peace, when an opportunity of power grab is available, the ruler gets a strictly better payoff by increasing her power to π than any other choices.

Figure 1: Conditions for peace when $\pi \notin (\underline{p}, \overline{p})$



The red curve represents $\hat{q}_\beta(p)$. When $\pi \notin (\underline{p}, \overline{p})$, the unique equilibrium of the continuation game at p is peaceful if and only if (p, π) lies in the shaded regions.

Figure 1 illustrates the conditions for peace when the physical limit of the ruler's power is either extremely high or extremely low: (p, π) must be in the shaded regions. The shaded region at the lower-left corner represents the case when the ruler is powerless and is physically impossible to become too powerful; while the upper-right corner corresponds to the case when the ruler already is very powerful and can become slightly more powerful. In both cases, peace is maintained because the ruler's power has only limited growth potential. The shaded region in the upper-left corner is most interesting: it represents the case when the ruler is extremely powerless but can become extremely powerful after consolidating power. In this case, peace can be maintained before the ruler consolidates power because of her powerlessness. Recall that $\lambda_\beta(p)$ measures how fast the ruler's power grows at p and it is strictly increasing in p. With an extremely low p, $\lambda_\beta(p)$ is extremely small, which means an extremely long time for the ruler to increase her power to π. This is because the ruler's power consolidation is constantly interrupted by the opposition's threat. As a result, the opposition only has a very weak opportunistic incentive to fight the ruler before she consolidates power, as reflected by the very small premium $\frac{\delta}{1-\delta}\lambda_\beta(p)\left(\overline{V}(p) - \overline{V}(\pi)\right)$.

3.3. Peace via power sharing

When the physical limit of the ruler's power is intermediate, $\pi \in (\underline{p}, \overline{p})$, her power consolidation leads to conflict. Hence, constraining the ruler's power is a premise for peace. In fact, an equilibrium (σ, α) of the continuation game at p is peaceful in this case only if $p \leq Q_\sigma(p) \leq \underline{p}$: the ruler must have a sufficiently low level of power and a sufficiently restrained target of power grab. In what follows, I first investigate when for a given $p^* \leq \underline{p}$, the ruler can have p^* as her target of power grab provided that her current level of power is below p^*. Then, I analyze when there exists a $p^* \leq \underline{p}$ for which the ruler can credibly commit to restrain her power below p^* and such a commitment guarantees the peace between the ruler and the opposition.

Consider $p^* \leq \underline{p}$ and $p \leq p^*$. Suppose the continuation game at p has an equilibrium (σ, α) in which $Q_\sigma(p) = p^*$. Consider the case when the ruler gets an opportunity of power grab. By following the equilibrium power grab strategy to choose $q = Q_\sigma(p) = p^*$, the ruler expects to get

$$U_\alpha(p^*) = 1 - V_\sigma(p^*) = 1 - \overline{V}(p^*)$$

in the future periods. This is because $p^* \leq \underline{p}$, so that after increasing power to p^*, the ruler is able to buy off the opposition and avoid conflict.

By deviating to $q = \pi$, the ruler expects to get

$$U_\alpha(\pi) = (1-\mu(\pi))\overbrace{p\,(1-(1-\delta)k)}^{\theta=1} + \mu(\pi)\overbrace{(1-\delta+\delta U_\alpha(\pi))}^{\theta=0} \tag{6}$$

in the future periods. This is because $\pi \in (\underline{p}, \overline{p})$, so that after increasing power to π, the ruler is unable to buy off the opposition and must fight as soon as it poses a threat. Hence, the ruler consumes all the spoils, getting $1-\delta+\delta U_\alpha(\pi)$ every period until the opposition poses a threat, in which case conflict occurs and the ruler expects to get $p\,(1-(1-\delta)k)$ by fighting the opposition. Equation (6) solves

$$U_\alpha(\pi) = 1 - \overline{W}(\pi),$$

where

$$\overline{W}(\pi) := \overline{V}(\pi) + \overbrace{\frac{1-\mu(\pi)}{1-\mu(\pi)\delta}(1-\delta)k}^{\text{deadweight loss}}$$

and the ruler's loss additional to the opposition's payoff $\overline{V}(\pi)$ is the deadweight loss caused by the upcoming conflict. Because p^* is the ruler's target of power grab in equilibrium, she must prefer to choose $q = p^*$ rather than deviating to $q = \pi$. Hence, necessarily,

$$\overline{V}(p^*) \leq \overline{W}(\pi). \tag{7}$$

Moreover, to prevent the ruler from deviating to any q, the opposition must punish her for doing so. The only way for the opposition to punish the ruler is to fight her whenever it poses a threat. As a result, the opposition can credibly commit to punish the ruler for increasing power to some q only if

$$\underline{S}(q) + \frac{\delta}{1-\delta}\lambda_\beta(q)\left(\overline{V}(q) - \overline{V}(\pi)\right) > 1. \tag{8}$$

Otherwise, according to Lemma 2,

$$S_{\sigma'}(q) = \underline{S}(q) + \frac{\delta}{1-\delta}\lambda_\beta(q)\left(\overline{V}(q) - \overline{V}\left(Q_{\sigma'}(q)\right)\right) \le \underline{S}(q) + \frac{\delta}{1-\delta}\lambda_\beta(q)\left(\overline{V}(q) - \overline{V}(\pi)\right) \le 1$$

holds in any equilibrium (σ', α') of the continuation game at q, which implies that the opposition would not fight the ruler after she increases power to q, as she can always share enough spoils to buy off the opposition. Because p^* is the ruler's target of power grab in equilibrium, the opposition must be able to credibly commit to punish the ruler for increasing her power to any $q > p^*$. Hence, necessarily, inequality (8) must hold for all $q \in (p^*, \pi]$, which, because the left hand side is strictly quasiconcave in q and because $\underline{S}(\pi) > 1$ as a result of $\pi \in (\underline{p}, \overline{p})$, is equivalent to

$$\underline{S}(p^*) + \frac{\delta}{1-\delta}\lambda_\beta(p^*)\left(\overline{V}(p^*) - \overline{V}(\pi)\right) \ge 1. \tag{9}$$

Proposition 2. [12] *If $\pi \in (\underline{p}, \overline{p})$, then:*

1. *if $p > \underline{p}$ or $\pi > \overline{\pi}$, the continuation game at p has a unique equilibrium (σ, α) in which $Q_\sigma(p) = \pi$ and this equilibrium is conflictual, where*

$$\overline{\pi} := \overline{W}^{-1}\left(\overline{V}(\underline{p})\right) \in (\underline{p}, \overline{p});$$

2. *if $\pi \le \overline{\pi}$, then for every*

$$p^* \in \left[\underline{q}_\beta(\pi), \underline{p}\right],$$

the continuation game at any $p \le p^*$ admits an equilibrium (σ, α) in which $Q_\sigma(p) = p^*$,

[12] The threshold $\overline{\delta}$ is defined as the smallest $\delta \ge \underline{\delta}$ that ensures $\overline{\pi} < \overline{p}$.

where for each $\pi \in (\underline{p}, \overline{\pi}]$,

$$\underline{q}_\beta(\pi) := \max\left\{q^R(\pi), q^O_\beta(\pi)\right\} \in (0, \underline{p}]$$
$$q^R(\pi) := \overline{V}^{-1}\left(\overline{W}(\pi)\right) \in (0, \underline{p}]$$
$$q^O_\beta(\pi) := \min\left\{p \in [0, \underline{p}] : \underline{S}(p) + \frac{\delta}{1-\delta}\lambda_\beta(p)\left(\overline{V}(p) - \overline{V}(\pi)\right) \geq 1\right\} \in (0, \underline{p}).$$

The first part of Proposition 2 shows the conditions under which constraining the ruler's power is impossible provided that $\pi \in (\underline{p}, \overline{p})$, so that her power consolidation brings conflict. This happens when the ruler is already too powerful to buy off the opposition, $p > \underline{p}$, or when the physical limit of her power is too large, $\pi > \overline{\pi}$. Given the definition of $\overline{\pi}$, if $\pi > \overline{\pi}$, condition (7) fails for all $p^* \leq \underline{p}$. As a result, the ruler prefers to consolidate power and then wait to fight the opposition, rather than restraining her power at any level below $\underline{p}$ in exchange for peace. In this case, even if the ruler currently is powerless enough and thus can afford to buy off the opposition, she ultimately would consolidate power and become unable to maintain peace afterwards. Therefore, conflict erupts sooner or later.

The second part of Proposition 2 indicates that when $\pi \in (\underline{p}, \overline{p})$, as long as the physical limit of the ruler's power is not too large, that is, $\pi \leq \overline{\pi}$, there are multiple equilibria in which the ruler's power is constrained and to different degrees.[13] Specifically, each p^* bounded between $\underline{p}$ and $\underline{q}_\beta(\pi)$ can be the ruler's target of power grab in an equilibrium of the continuation game at any $p \leq p^*$. Such an equilibrium can be interpreted and therefore is referred to as a "*power sharing agreement*" between the ruler and the opposition that imposes a "*permissible limit*" p^* on the ruler's power. Under a power sharing agreement, the ruler's power grab within the permissible limit is considered "*legitimate*," while her power grab beyond the limit is regarded as a "*transgression*" and would result in the opposition's punishment. In contrast to the physical limit of the ruler's power which is exogenously given, the permissible limit in any power sharing agreement emerges endogenously and is

[13]The condition for multiple equilibria is generic: only in the knife-edge case of $\pi = \overline{\pi}$, there exists a unique equilibrium (σ, α) in which the ruler's power is constrained and in this equilibrium, $Q_\sigma(p) = \underline{p}$.

self-enforcing. The opposition credibly commits to punish the ruler for increasing her power beyond the permissible limit, while given the punishment, the ruler prefers to restrain her power within the limit.

The permissible limit inversely measures the extent the ruler's power is constrained under the corresponding power sharing agreement. A power sharing agreement is *more constraining* than the other if the former has a lower permissible limit than the latter. The most constraining power sharing agreement has the permissible limit of $\underline{q}_\beta(\pi)$, which is defined as the smallest p^* that simultaneously satisfies conditions (7) and (9). Condition (9) is equivalent to $p^* \geq q^O_\beta(\pi)$, which guarantees that the opposition can credibly commit to punish the ruler for increasing power beyond p^*. Condition (7) is equivalent to $p^* \geq q^R(\pi)$, which guarantees that the opposition's commitment to punish power grab beyond p^* deters the ruler from doing so. These two conditions together enable the ruler to credibly commit to restrain her power grab within p^*.[14]

As mentioned above, when $\pi \in (\underline{p}, \overline{p})$, so that the ruler's power consolidation leads to conflict, constraining the ruler's power is a premise for peace. In this case, a peaceful equilibrium, if exists, must be a power sharing agreement. However, given that $\pi \leq \underline{\pi}$, a power sharing agreement is not necessarily peaceful. Each power sharing agreement guarantees peace *after* the ruler increases her power to its permissible limit, as $p^* \leq \underline{p}$. But *before* the ruler increases her power to p^*, she is able to maintain peace with the opposition only if $p^* \leq \hat{q}_\beta(p)$, where $\hat{q}_\beta(p)$ is defined in Proposition 1. Otherwise, the growth of the ruler's power from p to p^* renders the opposition a strong enough opportunistic incentive, so that it cannot be bought off and would fight the ruler when posing a threat before the ruler gets any opportunity to increase her power to p^*.[15] Therefore, given that $\pi \in (\underline{p}, \overline{p})$,

[14] Meng, Paine and Powell (2023) argue conceptually that three conditions are necessary for power sharing: challenger credibility, challenger willingness, and ruler willingness. My model formalizes their conceptual framework. Here, condition (7) corresponds to "ruler willingness," while condition (9) corresponds to "challenger credibility."

[15] Here, conditions $p^* \leq \underline{p}$ and $p^* \leq \hat{q}_\beta(p)$ together ensure that the ruler is able to buy off the opposition both before and after she increases power to p^*. Because the ruler's power is dynamic in my model, Meng, Paine and Powell (2023)'s "challenger willingness" can be decomposed into two parts: condition $p^* \leq \underline{p}$ corresponds to "challenger willingness" after the ruler increases power, while $p^* \leq \hat{q}_\beta(p)$ corresponds to that

peace can be maintained between the ruler and the opposition if and only if there exists a power sharing agreement with a permissible limit p^* that is sufficiently high, $p^* \geq \underline{q}_\beta(\pi)$, so that the ruler can credibly commit to restrain power grab within the limit, and sufficiently low, $p^* \leq \hat{q}_\beta(p)$, so that before the ruler legitimately increases her power to p^*, she can afford to buy off the opposition and avoid conflict.

Proposition 3. *If $\pi \in (\underline{p}, \overline{p})$, then for any $p \leq \pi$, the continuation game at p admits a peaceful equilibrium if and only if*

$$p \leq \underline{p} \text{ and } \pi \leq \hat{\pi}_\beta(p),$$

where for each $p \in (0, \underline{p}]$,

$$\hat{\pi}_\beta(p) := \overline{W}^{-1}\left(\overline{V}\left(\min\left\{\underline{p}, \hat{q}_\beta(p)\right\}\right)\right) \in (\underline{p}, \overline{\pi}]$$

and $\hat{\pi}_\beta(p) = \overline{\pi}$ if p is sufficiently small or $p = \underline{p}$.

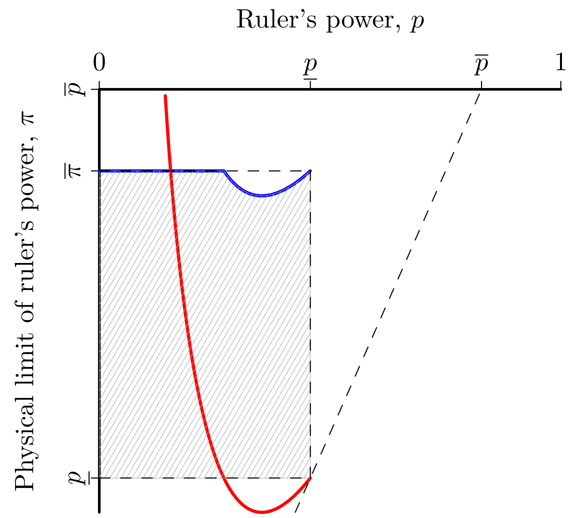
Proposition 3 shows the conditions for peace when the ruler's power consolidation leads to conflict. These conditions are illustrated by the shaded region in Figure 2. When $p \leq \underline{p}$ and $\pi \in (\hat{\pi}_\beta(p), \overline{\pi}]$, corresponding to the sector-shape region bounded between $\hat{\pi}_\beta(p)$ and $\overline{\pi}$, power sharing agreements exist, but none would allow the ruler to buy off the opposition before she legitimately increases her power.

3.4. Mutual suspicions and conflict

Given that the physical limit of the ruler's power is intermediate, $\pi \in (\underline{p}, \overline{p})$, Proposition 2 implies that if the ruler's current level of power is sufficiently low, $p \leq \underline{p}$, and if the physical limit is not too large, $\pi \leq \overline{\pi}$, then the two players may coordinate on a power sharing agreement that allows them to avoid conflict at least after the ruler increases power. One

before the ruler increases power.

Figure 2: Conditions for peace when $\pi \in (\underline{p}, \overline{p})$



The red curve represents $\hat{q}_\beta(p)$, while the blue curve shows $\hat{\pi}_\beta(p)$. When $\pi \in (\underline{p}, \overline{p})$, peaceful equilibrium exists if and only if (p, π) lies in the shaded region.

question remains to fully characterize the set of all equilibria when $\pi \in (\underline{p}, \overline{p})$: when $p \leq \underline{p}$ and $\pi \leq \overline{\pi}$, are all equilibria power sharing agreements, so that the two players never fail to coordinate on one?

Proposition 4. *Let*

$$\underline{\pi}_\beta := \min\left\{\pi \in [\underline{p}, \overline{\pi}] : \ 1 - \mu\left(q_\beta^O(\pi)\right) \geq \overline{W}(\pi)\right\} \in [\underline{p}, \overline{\pi}).$$

If $p \leq \underline{p}$ and $\pi \in (\underline{p}, \overline{\pi}]$, then:

1. *if $\pi > \tilde{\pi}_\beta(p)$, the continuation game at p admits an equilibrium (σ, α) with $Q_\sigma(p) = \pi$, where for each $p \in (0, \underline{p}]$,*

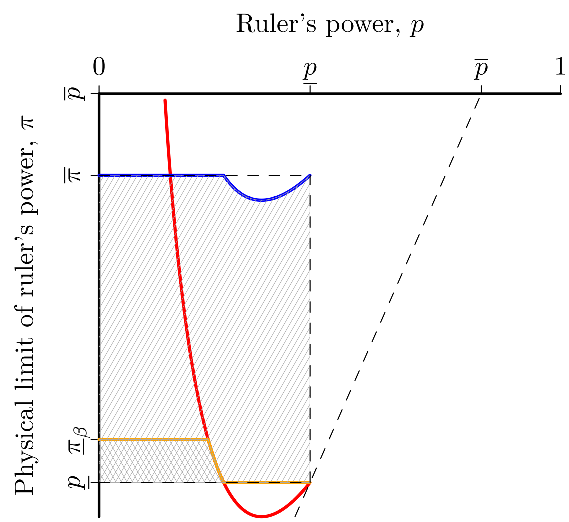
$$\tilde{\pi}_\beta(p) := \max\left\{\underline{p}, \min\left\{\underline{\pi}_\beta, \hat{q}_\beta(p)\right\}\right\} \in \left[\underline{p}, \hat{\pi}_\beta(p)\right),$$

is decreasing in p, and $\tilde{\pi}_\beta(p) = \underline{\pi}_\beta$ if p is sufficiently small;

2. if $\pi \leq \tilde{\pi}_\beta(p)$, every equilibrium of the continuation game at p is peaceful.

According to Proposition 4, even if $p \leq \underline{p}$ and $\pi \leq \overline{\pi}$, as long as the physical limit of the ruler's power is not too low, $\pi > \tilde{\pi}_\beta(p)$, a conflictual equilibrium coexists with other equilibria in the form of power sharing agreements. In this case, although power sharing agreements exist in theory, the ruler and the opposition could fail to coordinate on any one of them and as a result of coordination failure, they would ultimately fight against each other. Only when the physical limit of the ruler's power is very low, $\pi \leq \tilde{\pi}_\beta(p)$, every equilibrium is a power sharing agreement and must be peaceful. Only when this is the case, the two players need not worry about coordination failure: no matter how they coordinate, they share power and enjoy peace.

Figure 3: Equilibria when $\pi \in (\underline{p}, \overline{p})$



The red curve represents $\hat{q}_\beta(p)$, the blue curve represents $\hat{\pi}_\beta(p)$, and the yellow curve represents $\tilde{\pi}_\beta(p)$. Power sharing agreement exists if (p, π) lies within the square region to the left of $p = \underline{p}$ and below $\pi = \overline{\pi}$; conflictual equilibrium exists if p is to the right of $\underline{p}$ or π is above the yellow curve; peaceful equilibrium exists in the shaded region; only peaceful equilibrium exists in the cross-shaded region at the lower-left corner.

To understand the intuition behind Proposition 4, suppose $p \leq \underline{p}$ and consider first the case when $\pi > \hat{q}_\beta(p)$, so that $\pi > \tilde{\pi}_\beta(p)$. In this case, given that the ruler is seeking to consolidate power, having $Q_\sigma(p) = \pi$ as her target of power grab, she must pay at least

$$\underline{S}(p) + \frac{\delta}{1-\delta}\lambda_\beta(p)\left(\overline{V}(p) - \overline{V}(\pi)\right) > 1$$

to buy off the opposition, which is out of her budget because of $\pi > \hat{q}_\beta(p)$ and the definition of $\hat{q}_\beta(p)$. Hence, if the opposition *suspects* that the ruler has the ambition to consolidate power, the ruler would be unable to buy off it and must fight as soon as the opposition poses a threat. In turn, if the ruler anticipates that she must fight the opposition as soon as it poses a threat, her most sensible action before the opposition is able to fight is to increase her power as much and as soon as possible, so as to best her odds of defeating the opposition in the upcoming conflict. Therefore, when $\pi > \hat{q}_\beta(p)$, the opposition's suspicion about the ruler's ambition to consolidate power is a *self-fulfilling* prophecy that results into the conflictual equilibrium.

Second, consider the case when $\pi \leq \hat{q}_\beta(p)$ but $\pi > \underline{\pi}_\beta$, so that $\pi > \tilde{\pi}_\beta(p)$. Note that $\pi \leq \hat{q}_\beta(p)$ is equivalent to $p \leq q^O_\beta(\pi)$ due to the definition of $\hat{q}_\beta(p)$ and $q^O_\beta(\pi)$. In this case,

$$S_{\sigma'}(q) = \underline{S}(q) + \frac{\delta}{1-\delta}\lambda_\beta(q)\left(\overline{V}(q) - \overline{V}\left(Q_{\sigma'}(q)\right)\right) \leq \underline{S}(q) + \frac{\delta}{1-\delta}\lambda_\beta(q)\left(\overline{V}(q) - \overline{V}(\pi)\right) \leq 1$$

holds in each equilibrium (σ', α') of the continuation game at any $q \leq q^O_\beta(\pi)$. Hence, because the ruler is extremely powerless, she can always buy off the opposition as long as her power is kept below $q^O_\beta(\pi)$. Now suppose the ruler *suspects* that the opposition has a suspicion about her ambition to consolidate power. As a result, after the ruler increases her power to any q, the opposition needs to be paid at least

$$\underline{S}(q) + \frac{\delta}{1-\delta}\lambda_\beta(q)\left(\overline{V}(q) - \overline{V}(\pi)\right)$$

to give up fighting, which is affordable if and only if $q \leq q^O_\beta(\pi)$. As long as the physical limit of the ruler's power is not too low, $\pi > \underline{\pi}_\beta$, she prefers to consolidate power and then wait to fight the opposition rather than keeping her power below $q^O_\beta(\pi)$ and paying the opposition every time it comes up with a threat. Therefore, when $\pi \leq \hat{q}_\beta(p)$ but $\pi > \underline{\pi}_\beta$, the ruler's suspicion about the opposition's suspicion concerning her ambition to consolidate power is a *self-fulfilling* prophecy that motivates the ruler's power consolidation, which ultimately leads to a conflict.

At last, suppose both $\pi \leq \hat{q}_\beta(p)$ and $\pi \leq \underline{\pi}_\beta$ hold, so that $\pi \leq \tilde{\pi}_\beta(p)$. In this case, because $\pi \leq \hat{q}_\beta(p)$, the ruler is able to avoid conflict by keeping her power below $q^O_\beta(\pi)$. Because $\pi \leq \underline{\pi}_\beta$, even if the opposition suspects that the ruler is seeking to consolidate power, the ruler prefers to restrain her power below $q^O_\beta(\pi)$ to maintain peace rather than consolidating power and then fighting the opposition in a conflict. As a result, neither the opposition's suspicion about the ruler's ambition nor the ruler's suspicion about the opposition's suspicion fulfills itself into a conflictual equilibrium.

3.5. Summary of equilibria

Proposition 1 to 4 together characterize the set of all possible equilibria of the continuation game given the ruler's power $p \in (0, \pi]$ and its physical limit $\pi \in (0, 1)$. Applying them, the following corollary summarizes what happens in the polity between the ruler and the opposition when the ruler has a sufficiently low initial level of power p_0 at the beginning of her reign.

Corollary 1. *Given that the ruler has a sufficiently low initial power p_0:*

1. *if $\pi \geq \overline{p}$, there is a unique equilibrium (σ, α) which is peaceful and $Q_\sigma(p_0) = \pi$;*
2. *if $\overline{\pi} < \pi < \overline{p}$, there is a unique equilibrium (σ, α) which is conflictual and $Q_\sigma(p_0) = \pi$;*
3. *if $\underline{\pi}_\beta < \pi \leq \overline{\pi}$, there is a conflictual equilibrium (σ, α) with $Q_\sigma(p_0) = \pi$ and for each $p^* \in \left[\underline{q}_\beta(\pi), \underline{p}\right]$, there is a peaceful equilibria (σ', α') with $Q_{\sigma'}(p_0) = p^*$;*

4. *if* $\underline{p} < \pi \leq \underline{\pi}_\beta$*, then for each* $p^* \in \left[\underline{q}_\beta(\pi), \underline{p}\right]$*, there is a peaceful equilibrium* (σ, α) *with* $Q_\sigma(p_0) = p^*$*, and these are all the equilibria;*

5. *if* $\pi \leq \underline{p}$*, there is a unique equilibrium* (σ, α) *which is peaceful and* $Q_\sigma(p_0) = \pi$*.*

When the physical limit of the ruler's power is extremely high, $\pi \geq \overline{p}$, she would maintain peace with the opposition and ultimately consolidate power, though it would take a long time given her powerlessness to begin with. When the physical limit is quite high but not too high, $\overline{\pi} < \pi < \overline{p}$, the ruler seeks power consolidation as well, but conflict ultimately would erupt after she consolidates power. When the physical limit is trivially low, $\pi \leq \underline{p}$, the ruler would maintain peace with the opposition and consolidate power, though the growth potential of her power is extremely limited.

The most interesting cases are the middle two with multiple equilibria. First, when the physical limit of the ruler's power is intermediate, so that $\underline{\pi}_\beta < \pi \leq \overline{\pi}$, what happens depends on how the two players coordinate with each other. If they coordinate on a power sharing agreement, peace can be maintained under which the ruler would legitimately increase her power to the permissible limit the agreement imposes and refrains from further power grab. However, if they fail to coordinate on any power sharing agreement, the ruler would consolidate power, after which a conflict would ultimately break out. Second, when the physical limit of the ruler's power is low but not trivial, $\underline{p} < \pi \leq \underline{\pi}_\beta$, the ruler and the opposition always share power and enjoy peace. Peace and power sharing are robust in this case as they are independent of how well the two players coordinate.

4. Discussion

4.1. Welfare and communication

Two power sharing agreements are not Pareto comparable provided that they both guarantee peace. Under a particular power sharing agreement, the opposition gets the payoff

$$(1 - \lambda_\beta(p))\, \overline{V}(p) + \lambda_\beta(p)\overline{V}(p^*),$$

depending on the ruler's power p and the permissible limit p^* that the agreement imposes; and if the power sharing agreement is peaceful, the ruler gets exactly the total size of the spoils net the opposition's payoff,

$$1 - (1 - \lambda_\beta(p))\, \overline{V}(p) - \lambda_\beta(p)\overline{V}(p^*).$$

Hence, between two power sharing agreements that guarantee peace, the opposition always prefers the more constraining one, while the ruler always prefers the less constraining one.

Proposition 5. *If $p \leq \underline{p}$ and $\pi \in (\tilde{\pi}_\beta(p), \overline{\pi}]$, then in the continuation game at p, the conflictual equilibrium is Pareto dominated by each equilibrium (σ, α) with $Q_\sigma(p) \leq \underline{p}$.*

According to Proposition 5, a power sharing agreement is always Pareto superior to the conflictual equilibrium if they both exist. Hence, although the ruler and the opposition may disagree with regard to which power sharing agreement to put into effect, they both benefit from coordinating on at least one of them.

As shown in the discussion below Proposition 4, when $p \leq \underline{p}$ and $\pi \in (\tilde{\pi}_\beta(p), \overline{\pi}]$, the ruler and the opposition may fail to coordinate on any power sharing agreement due to their mutual suspicions about each other's intention. The natural question is whether the two players can clear these suspicions and avoid getting into the conflictual equilibrium by *communicating* their good wills. This question can be addressed by the *renegotiation-proof* refinement (Farrell and Maskin, 1989). Specifically, an equilibrium is renegotiation-proof if it is *Pareto optimal* in all the continuation games on and off the equilibrium path, so that after any deviation, the two players cannot agree on switching to any other Pareto superior equilibrium.

Corollary 2. *If $p \leq \underline{p}$ and $\pi \in (\underline{p}, \overline{\pi}]$, then the continuation game at p has a unique renegotiation-proof equilibrium (σ, α) in which $Q_\sigma(p) = \underline{p}$ and this equilibrium is peaceful if and only if $\hat{q}_\beta(p) \geq \underline{p}$ or, equivalently, $\hat{\pi}_\beta(p) = \overline{\pi}$.*

According to Corollary 2, the effect of communication is twofold. On the one hand, it does help the ruler and the opposition to avoid conflict by clearing mutual suspicions. Specifically, when $p \leq \underline{p}$ and $\pi \in (\tilde{\pi}_\beta(p), \overline{\pi}]$, the conflictual equilibrium fails to be renegotiation-proof. Because any power sharing agreement is Pareto superior to the conflictual equilibrium, the two players need not worry about failing to coordinate on one given that they can always communicate and renegotiate on which equilibrium to coordinate into.

On the other hand, communication makes constraining the ruler's power more difficult and, therefore, may lead to conflict. Recall that by Proposition 2, when $p \leq \underline{p}$ and $\pi \in (\underline{p}, \overline{\pi}]$, each $p^* \in \left[\underline{q}_\beta(\pi), \underline{p}\right]$ can be imposed as the permissible limit of a power sharing agreement provided that $p \leq p^*$. Among these power sharing agreements, however, only the least constraining one with $p^* = \underline{p}$ is renegotiation-proof. Under this power sharing agreement, if $\hat{q}_\beta(p) < \underline{p}$ or, equivalently, $\hat{\pi}_\beta(p) < \overline{\pi}$ (the sector-shaped region in Figure 2 or 3 bounded between $\hat{\pi}_\beta(p)$ and $\overline{\pi}$), the ruler cannot afford to buy off the opposition and conflict occurs as a result, when the opposition poses a threat before the ruler increases power. In particular, if $p \leq \underline{p}$ and $\underline{p} < \pi \leq \hat{\pi}_\beta(p) < \overline{\pi}$, the two players are able to maintain peace under a more constraining power sharing agreement with $p^* \leq \hat{q}_\beta(p) < \underline{p}$ if they cannot communicate. Such a power sharing agreement is enforced by the opposition's commitment to punish the ruler for increasing power to any level above p^*. If the two players can always communicate to decide which equilibrium to coordinate into, however, the ruler can benefit by breaching the power sharing agreement, choosing $q = \underline{p} > p^*$, and then communicating her "good will" to the opposition so as to avoid its punishment. Because in the continuation game at $\underline{p}$, the new and less constraining power sharing agreement with the permissible limit of $\underline{p}$ Pareto dominates the conflictual equilibrium, both players would agree to coordinate into the former rather than the latter.

Interestingly, suppose $p \leq \underline{p}$ and $\pi \in \left(\underline{p}, \tilde{\pi}_\beta(p)\right]$, so that every equilibrium is peaceful. Then, the two players disagree on whether they should or should not be able to communicate with each other. Without worrying about conflict, the ruler always prefers communication to be allowed, which enables her to push through the least constraining power sharing agreement. In contrast, the opposition hopes communication to be banned, so that the two players may coordinate on a more constraining power sharing agreement, preferably the most constraining one that imposes the permissible limit of $p^* = \underline{q}_\beta(\pi)$.

4.2. Comparative statics and tradeoff between constraining power and ensuring peace

According to Proposition 2, constraining the ruler's power is possible only when she is not too powerful, $p \leq \underline{p}$, and the physical limit of her power is intermediately low, $\pi \in (\underline{p}, \overline{\pi}]$. In this case, the ruler and the opposition can coordinate on a set of possible power sharing agreements that impose various permissible limits on the ruler's power, each of which is strictly bounded above by its physical limit π. The question is how constrained can a power sharing agreement be and how does it depend on the physical limit of the ruler's power, π, and her probability to get an opportunity of power grab absent the opposition's threat, β.

Proposition 6. *For all $\pi \in (\underline{p}, \overline{\pi}]$, $q^R(\pi)$ is strictly increasing in π, $q^O_\beta(\pi)$ is strictly decreasing in π and β, and $q^R(\pi) \geq q^O_\beta(\pi)$ if and only if $\pi \geq \pi^*_\beta$, where*

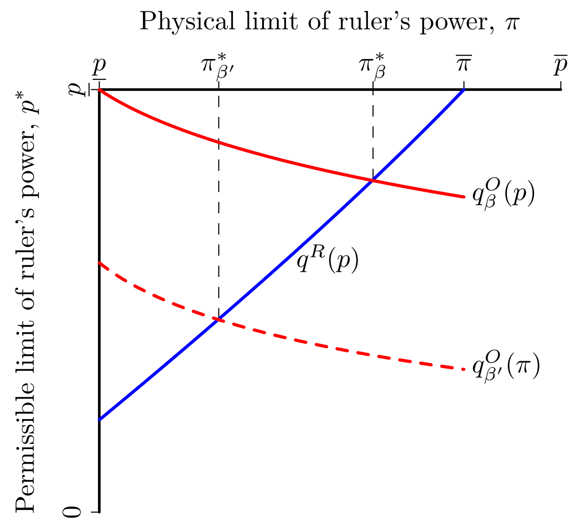
$$\pi^*_\beta := \min\left\{\pi \in [\underline{p}, \overline{\pi}] : \ \overline{V}\left(q^O_\beta(\pi)\right) \geq \overline{W}(\pi)\right\} \in [\underline{p}, \overline{\pi}).$$

Moreover, for two uniquely defined thresholds $\underline{\beta}, \beta^* \in (0, 1]$ such that $\underline{\beta} \leq \beta^*$, $\underline{\pi}_\beta = \underline{p}$ if $\beta \geq \underline{\beta}$ and $\underline{\pi}_\beta > \underline{p}$ and is strictly decreasing in β if $\beta < \underline{\beta}$; while $\pi^*_\beta = \underline{\pi}_\beta = \underline{p}$ if $\beta \geq \beta^*$ and

$\pi^*_\beta > \overline{\pi}_\beta \geq \underline{p}$ *and is strictly decreasing in* β *if* $\beta < \beta^*$.

As shown in Proposition 2, given $\pi \in (\underline{p}, \overline{\pi}]$ and a sufficiently small p, any p^* bounded between $\underline{q}_\beta(\pi)$ and $\underline{p}$ can be imposed as the permissible limit in some power sharing agreement. Proposition 6 shows how the lowest possible one $\underline{q}_\beta(\pi) = \max\left\{q^R(\pi), q^O_\beta(\pi)\right\}$, corresponding to the most constraining power sharing agreement, varies as a function of π and β. The results are illustrated in Figure 4.

Figure 4: Power sharing agreements when $\pi \in (\underline{p}, \overline{\pi}]$



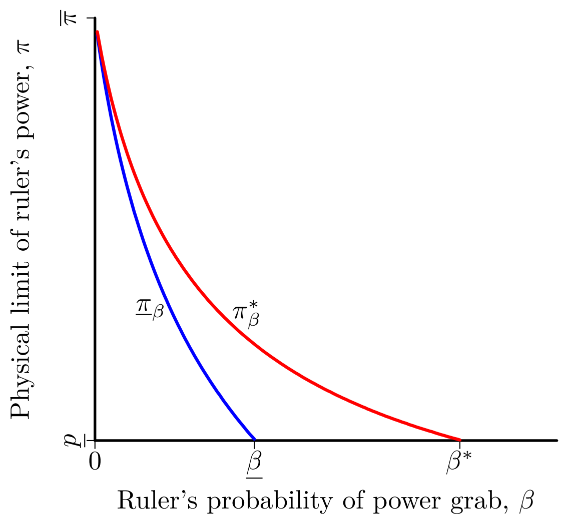
The solid red curve represents $q^O_\beta(\pi)$, the dashed red curve represents $q^O_{\beta'}(\pi)$ for $\beta' > \beta$, and the blue curve represents $q^R(\pi)$.

As long as $\beta < \beta^*$, there holds $\pi^*_\beta > \underline{p}$, so that $\underline{q}_\beta(\pi)$ is nonmonotone in π. As π increases from $\underline{p}$ to $\overline{\pi}$, $\underline{q}_\beta(\pi) = \underline{q}^O_\beta(\pi)$ decreases until reaching to its minimum at $\pi = \pi^*_\beta$, after which $\underline{q}_\beta(\pi) = q^R(\pi)$ increases. To understand the intuition behind this nonmonotonicty, recall that $\underline{q}_\beta(\pi)$ is defined as the lowest possible target of power grab for the ruler that simultaneously satisfies conditions (7) and (9). Condition (7) requires the ruler to be willing to restrain her power when she has an opportunity of power grab. This condition is more stringent when π is larger, as power consolidation becomes more attractive for the ruler. Condition (9) requires the opposition to be willing to punish the ruler for excessive power grab. This condition is more stringent when π is smaller, as the ruler's power consolidation becomes more tolerable for the opposition. Therefore, when $\pi < \pi^*_\beta$, condition (9) is binding, so that $\underline{q}_\beta(\pi) = q^O_\beta(\pi)$ and with a slightly larger π, $\underline{q}_\beta(\pi)$ decreases as the condition become more loose. When $\pi > \pi^*_\beta$, condition (7) is binding, so that $\underline{q}_\beta(\pi) = q^R(\pi)$ and with a slightly larger π, $\underline{q}_\beta(\pi)$ increases as the condition becomes more stringent. Similarly, given π, $\underline{q}_\beta(\pi) = q^O_\beta(\pi)$ is strictly decreasing in β if β is small enough, so that $\pi^*_\beta > \pi$; and $\underline{q}_\beta(\pi) = q^R(\pi)$ is constant in β if β is large enough, so that $\pi^*_\beta < \pi$.

The substantively interesting implication of Proposition 6 is that when $\pi < \pi^*_\beta$, because the ruler has a sufficiently low physical limit of power and/or a sufficiently low probability to get any opportunity of power grab, increasing either or both could help constraining the ruler's power. Because $\pi < \pi^*_\beta$, there must exist (π', β') such that $\pi' > \pi$ and $\beta' > \beta$ for which $\underline{q}_{\beta'}(\pi') < \underline{q}_\beta(\pi)$. Under (π', β'), power sharing agreements with the permissible limits in $\left[\underline{q}_{\beta'}(\pi'), \underline{q}_\beta(\pi)\right)$ exist, while these limits are too constraining to be imposed by any power sharing agreement under (π, β). This counter-intuitive tactic to constrain the ruler's power works for two reasons. First, because $\pi < \pi^*_\beta$, a limit that is slightly lower than $\underline{q}_\beta(\pi)$ cannot be imposed in any power sharing agreement under (π, β) because it would make the opposition unable to credibly commit to punish the ruler for increasing power slightly over it. Second, increasing (π, β) to (π', β') enables the ruler to grab power with a greater magnitude and at a faster pace, both of which reinforce the opposition's opportunistic incentive and therefore adds credibility to its commitment of punishing the ruler.

The two parameters π and β are crucial in characterizing the dynamics of the ruler's power. As the physical limit of the ruler's power, π measures the magnitude of its growth; while as the ruler's probability of being able to increase power absent the opposition's threat, β determines the pace at which her power grows. From the perspective of a *designer* (they, for example, founding persons of a new regime or reformers of an existing one), it worths analyzing how these parameters can be adjusted for certain normative goals. Two goals are particularly interesting: to constrain the ruler's power and to ensure peace. Assume that the ruler has a sufficiently low initial power and that the designer has *enough but limited* capacity to adjust these parameters: they can choose any (π, β) such that $\pi \geq \pi^{\min}$ and $\beta \geq \beta^{\min}$, where $\pi^{\min} - \underline{p}$ and $\beta^{\min}$ are positive and sufficiently close to 0. Then, Proposition 4 and 6 together imply a tradeoff between constraining the ruler's power and ensuring peace between the ruler and the opposition.

Figure 5: Constraining power v.s. ensuring peace



On the one hand, suppose the designer wants to constrain the ruler's power as much as possible. Then, according to Proposition 6, they should choose (π, β) such that $\pi = \pi^*_\beta$, which minimizes the permissible limit $\underline{q}_\beta(\pi)$ that the most constraining power sharing agreement imposes. In Figure 5, the designer must choose (π, β) that lies on the red curve. But this implies that $\pi = \pi^*_\beta > \underline{\pi}_\beta$, so that the conflictual equilibrium exists and, as a result, whether the two players can maintain peace via power sharing depends on how well they coordinate with each other.

On the other hand, suppose the designer wants to ensure peace between the ruler and the opposition no matter how well they coordinate with each other. Then, according to

Proposition 4, they should choose (π, β) such that $\pi \leq \underline{\pi}_\beta$, so that every possible equilibrium the two players may coordinate into is a power sharing agreement that guarantees peace. In Figure 5, the designer must choose (π, β) that is below the blue curve. But this implies that $\pi \leq \underline{\pi}_\beta < \pi^*_\beta$. As a result, there always exists another choice (π', β') such that $\pi' > \pi$ and $\beta' > \beta$ under which $\underline{q}_{\beta'}(\pi') < \underline{q}_\beta(\pi)$, so that more constraining power sharing agreements are available.

Therefore, given that the designer is capable enough to choose π as close as possible to $\underline{p}$ and β as close as possible to 0, they can *either* maximally constrain the ruler's power *or* perfectly ensure peace between the two players. They cannot do both. If the primary goal is to constrain power, the designer must bear the risk of coordination failure that ultimately leads to conflict. If the primary goal is to ensure peace, the designer needs a degree of forbearance: they must not be tempted to constrain the ruler's power as much as they can.

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