A Simple Approach for Measuring Higher-Order Risk Attitudes

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Outline

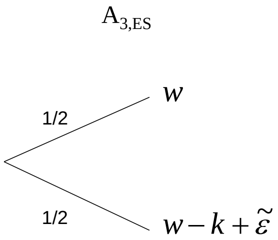
- Introduction
- Theory
- Experiments
- Conclusions

The importance of higher-order risk preference

- Risk attitudes of an economic agent have long been a fundamental issue in economics.
- Prudence
  - The direction: $u''' > 0$
  - The intensity of prudence: Absolute prudence ($-\frac{u'''}{u''}$) and relative prudence ($-\frac{xu'''}{u''}$)
- Decisions:
  - Menezes et al. (1980, AER): Downside risk aversion
  - Kimball (1990, Econometrica): Precautionary saving
  - Eso and White (2004, Econometrica): Auctions

A simple model to understand the direction of higher order risk preferences

- Eeckhoudt and Schinger (2006, AER): Risk apportionment
- Prudence:
  - $u''' \geq 0$ iff $Eu(B_{3,ES}) \geq Eu(A_{3,ES})$ for all $w, k > 0$ and $E(\tilde{\varepsilon}) = 0$



Research question

- Could we use the idea of risk apportionment to understand the intensity of higher order risk preference?

Purpose

- Refining the risk apportionment approach, we provide a non-parametric approach for comparative higher-order risk aversion.
- We further implement our approach in a controlled laboratory setting to find the upper and lower bounds of a subject's higher-order Arrow-Pratt risk aversion coefficients.

Contributions

- Our work goes beyond the seminal paper by Eeckhoudt and Schlesinger (2006, AER).
- Compared with the theoretical literature on comparative risk aversion (e.g., Chiu (2005, MS), Denuit and Eeckhoudt (2010, MS)), our approach
  - does not assume the utility functional form;
  - is simple, systematic and generalizable;
  - directly connects the Arrow-Pratt coefficients of higher order risk aversion with risk apportionment;
- Compared with the experimental literature (e.g., Ebert and Wiesen (2014, JRU), Noussair et al. (2014, RES)), our method can estimate the Arrow-Pratt coefficients coefficients at each order by using one task.

Theoretical Model

2nd order

- Direction: Let $E(\tilde{\varepsilon}) = 0$.

$$w + \tilde{\varepsilon} \;\; \text{v.s.} \;\; w$$

- Our lottery pair:

$$A_2 = w + \tilde{\delta} \;\; \text{v.s.} \;\; B_2 = w$$

where $E(\tilde{\delta})$ can be $>, =, < 0$.

- We find that

$$Ev(A_2) = Ev(B_2) \text{ always implies } Eu(A_2) \leq Eu(B_2) \; \forall A_2, B_2$$

$$\Leftrightarrow -\frac{u''(x)}{u'(x)} \geq -\frac{v''(x)}{v'(x)} \; \forall x$$

- Note that $-\frac{u''(x)}{u'(x)} \geq -\frac{v''(x)}{v'(x)} \; \forall x \Leftrightarrow -\frac{xu''(x)}{u'(x)} \geq -\frac{xv''(x)}{v'(x)} \; \forall x$

Intuition for the 2nd order lotteries

- Let $\tilde{\delta} = \tilde{\varepsilon} + l$.
- Our lotteries

$$w + \tilde{\varepsilon} + l \quad \text{v.s.} \quad w$$

- First order: Let $l > 0$. For all $u' > 0$,

$$w + l \quad \text{v.s.} \quad w$$

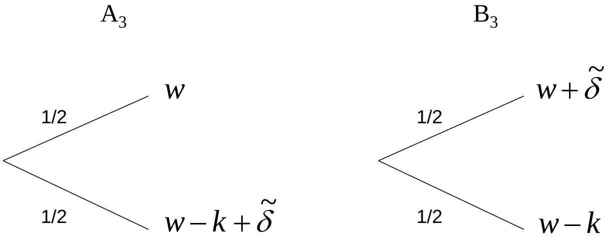
- Second order: For all $u'' < 0$,

$$w + \tilde{\varepsilon} \quad \text{v.s.} \quad w.$$

- If v ($v' > 0$) is indifferent between $w + \tilde{\varepsilon} + l$ and w, but u still prefer w, this means u is more risk averse than v.

3rd order: Lottery pair

- Let A_3 and B_3 be



3rd order: Intensity

- Given $u'' < 0$ and $v'' < 0$,

$$Ev(A_3) = Ev(B_3) \text{ always implies } Eu(A_3) \leq Eu(B_3)\ \forall A_3, B_3$$

$$\Leftrightarrow -\frac{u'''(x)}{u''(x)} \geq -\frac{v'''(x)}{v''(x)}\ \forall x$$

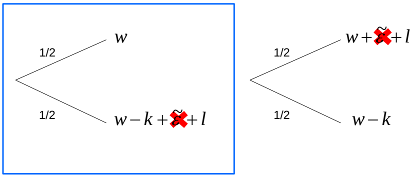
- Given $u'' > 0$ and $v'' > 0$,

$$Ev(A_3) = Ev(B_3) \text{ always implies } Eu(A_3) \geq Eu(B_3)\ \forall A_3, B_3$$

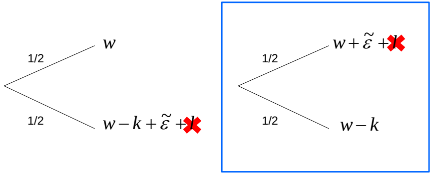
$$\Leftrightarrow -\frac{u'''(x)}{u''(x)} \geq -\frac{v'''(x)}{v''(x)}\ \forall x$$

Intuition for the 3rd order

- $\tilde{\delta} = \tilde{\varepsilon} + l$. Let $l > 0$.
- For all $u'' < 0$,



- For all $u''' > 0$,



Higher orders

- The above findings can be extended to higher orders systematically by modifying the lottery pairs in Eeckhoudt and Schlesinger (2006)
  - one zero mean lottery $\rightarrow$ a non-zero mean lottery
  - other zero mean lotteries $\rightarrow$ 50-50 zero mean lotteries

Experiments

The benchmark utility

- To measure the nth order absolute risk aversion:

$$-\frac{v^{(n)}(x)}{v^{(n-1)}(x)} = \theta_1$$

- To measure the relative degree of risk aversion:

$$-\frac{v^{(n)}(x)}{v^{(n-1)}(x)} = \frac{\theta_2}{x}$$

Design

- We test for the degrees of 2nd, 3rd, and 4th orders
- One task in each order
- 9 choices in each task

The task of order 2

- A series of risk apportionment choices:

	A_2	B_2	$-\frac{v''(x)}{v'(x)}$	$-\frac{xv''(x)}{v'(x)}$
1.	$18 + [-8; 0]$	18		
2.	$18 + [-7; 1]$	18	−0.69	−11.81
3.	$18 + [-6; 2]$	18	−0.31	−5.19
4.	$18 + [-5; 3]$	18	−0.14	−2.27
5.	$18 + [-4; 4]$	18	0.00	0.00
6.	$18 + [-3; 5]$	18	0.14	2.40
7.	$18 + [-2; 6]$	18	0.31	5.72
8.	$18 + [-1; 7]$	18	0.69	12.96
9.	$18 + [0; 8]$	18		

In lab

Phase 2

The nine tasks in Phase 2 involve the same set of nine 50-50 lotteries as Phase 1.

What has changed is that rather than starting with an endowment of 18, you are equally likely to start with an endowment of 13 or 23. This is why there is a large oval with the amounts 13 and 23 in it.

What you have to decide is if you would rather play the lottery associated with the task when you start with an endowment of 13 or when you start with an endowment of 23. This is why the options show the 50-50 lottery associated with the task inside the bigger 50-50 lottery about the endowment being either 13 or 23.

Notice that you cannot reject a lottery in Phase 2. Rather, you have to decide if you want to play the lottery if you get the "Smaller" initial endowment or if you get the "Larger" initial endowment.

An Example

Let's look at the 2nd task (the one with payouts of 1 and -7).

➔ If you choose "Smaller" there is a 50% chance that you will earn 13 + the 1 or -7 lottery & there is a 50% chance you will earn 23. This means there is

→ 25% chance you earn 13+1 = 14

→ 25% chance you earn 13-7 = 6

→ 50% chance you earn 23

➔ If you choose "Larger" there is a 50% chance that you will earn 13 & there is a 50% chance you will earn 23 + the 1 or -7 lottery. This means there is

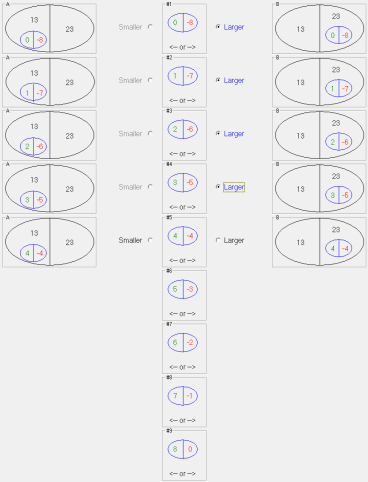
→ 50% chance you will earn 13

→ 25% chance you earn 23+1 = 24

→ 25% chance you earn 23-7 = 16

Questions?

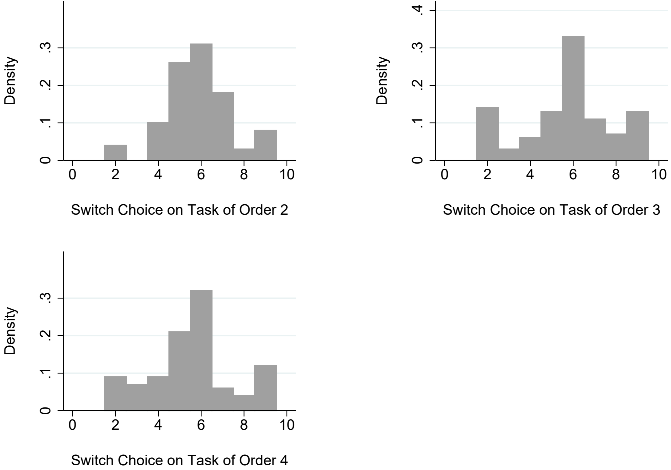
When you are ready, press Continue to make your nine choices for the second phase. But if you have a question at this point, please raise your hand before pressing the button.



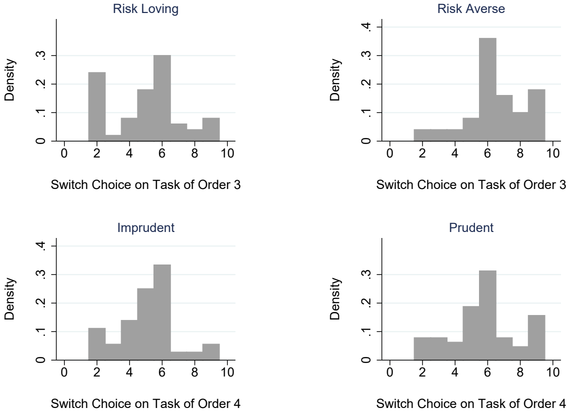
Basic Information

- University of Alabama
- 100 Subjects
- The average salient earnings were $18.74
- Risk aversion: 60%
- Prudence: 67%
- Temperance: 54%

Histogram of Switching in Task of Order n



Histogram of Switching in Task of Order n Given Sign of (n-1)th Order Risk Attitude



Correlation of Risk Intensities across Orders

	All Subjects		Risk Averters		Risk Lovers	
	Order 3	Order 4	Order 3	Order 4	Order 3	Order 4
Order 2	0.21**	0.42***	0.30**	0.56***	0.13	0.30**
Order 3		0.26***		0.32**		0.23

Results of Individual Level Calibration

Utility Function	Model	Parameters	Accuracy
exponential	$-\frac{1}{\gamma}e^{-rx}$	$\gamma = 0.15$	80%
power	$\frac{1}{1-\gamma}x^{1-r}$	$\gamma = 1.53$	86%
HARA	$\frac{\gamma}{1-\gamma}\left(\frac{x}{\gamma}+\theta\right)^{1-r}$	$\gamma = 2.20$	83%
		$\theta = 152.76$	
polynomial	$ax^4 + bx^3 + cx^2 + x$	$a = -0.08$	87%
		$b = 1.42$	
		$c = 266.40$	
exponential power	$-\frac{1}{\gamma}e^{\frac{1}{1-\theta}x^{1-\theta}}$	$\gamma = 0.26$	91%
		$\theta = 0.30$	

Conclusions

Conclusions

- This paper introduces a simple and systematic procedure for identifying the intensity of risk attitudes using the notion of risk apportionment.
  - Our process is systematic in that it involves a series of binary comparisons where each comparisons differs from the others in the same incremental manner.
  - Our approach can be used to identify both relative and absolute risk aversion of any arbitrary degree without relying upon assumptions regarding the respondent's underlying preference structure.
- We also demonstrate the implementation of our approach in a laboratory setting.
  - Our data also demonstrate that individuals often exhibit both risk averse and risk seeking behavior, both prudent and imprudent behavior, and both temperate and intemperate behavior.