

Exploring the Variance Risk Premium Across Assets

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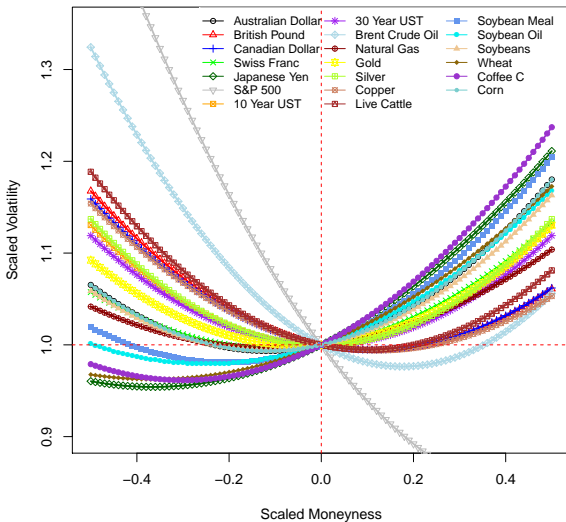
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What Do We Know about Options?

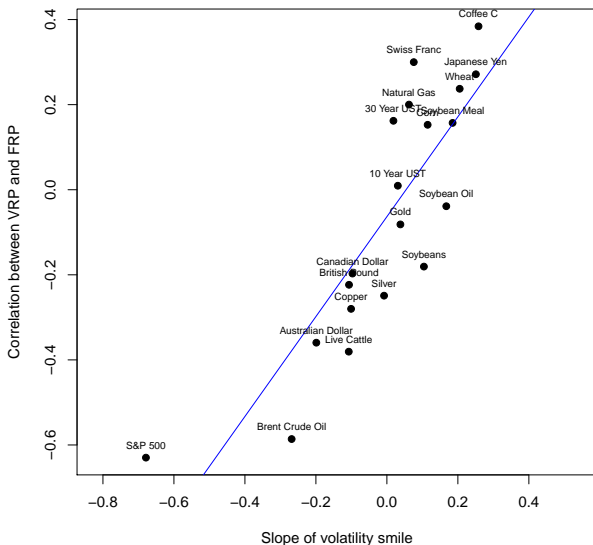
- Literature has focused on U.S. equities and equity indices:
 - Option smile.
 - Negative option smirk
 - Negative correlation of volatility and spot returns.
 - Implied volatility exceeds realized volatility.
 - Negative variance premium.
 - Dew-Becker and Giglio (2022) dispute this.
- What about prices of options on other assets?!
- What explains the variance premium returns across assets?
 - Risk ✗
 - Liquidity ✓ ✗
 - Conditional premiums ✓

Implied volatility smiles

$$\sigma(m)/\sigma^{ATM} = \bar{\alpha} + \bar{\beta}m + \bar{\gamma}m^2, \quad m = \log(K/F)/\sigma^{ATM}$$



Variance correlation and the slope of the volatility smile



Easy Measure of Variance Premium

- VIX^2 = price of a portfolio of out-of-the-money options.
 - Model-free weights $w_i \propto 2/\text{strike}_i^2$.
 - CBOE does an adjustment that hurts accuracy and sometimes gives negative "variance".
 - Simpson's adjustment improves accuracy.
 - Model-free delta = $2/F_0 - 2/F_t$
 - Buy and hold \$2 of stock, and daily sell \$2 forward.
- Measure the daily hedged return on this portfolio:
$$\text{return}(t + 1) \approx \frac{\text{Realised Variance}(t+1) - \text{Implied Variance}(t)}{\text{Implied Variance}(t)}$$
- Researchers have used Realized Variance - Implied Variance:
 - Christensen and Prabhala (1998),
 - Jiang and Tian (2005),
 - Bekaert and Hoerova (2014),
 - Johnson (2017)
 - Grishchenko, Song, Zhou (2022).

VSR vs hedged VIX^2 returns and ATM straddles

- Median correlation btw VSR and hedged VIX^2 returns **99%**

| | mean r^{VIX} | SR r^{VIX} | mean r^{VSR} | SR r^{VSR} | mean r^{str} | SR r^{str} | $\rho(r^{VSR}, r^{VIX})$ | $\rho(r^{VSR}, r^{str})$ |
|-------------------|----------------|--------------|----------------|--------------|----------------|--------------|--------------------------|--------------------------|
| Australian Dollar | -2.08 | -0.14 | -6.37 | -0.41 | 0.05 | 0.00 | 0.99 | 0.41 |
| British Pound | -9.40 | -0.46 | -11.82 | -0.58 | -0.51 | -0.03 | 0.96 | 0.31 |
| Canadian Dollar | -8.81 | -0.52 | -13.46 | -0.82 | -3.91 | -0.21 | 0.99 | 0.35 |
| Swiss Franc | 14.79 | 0.19 | 14.20 | 0.17 | 6.44 | 0.27 | 1.00 | 0.39 |
| Japanese Yen | -16.56 | -0.98 | -19.55 | -1.20 | -3.93 | -0.17 | 0.97 | 0.42 |
| S&P 500 | -11.93 | -0.23 | -13.08 | -0.24 | -0.75 | -0.04 | 1.00 | 0.51 |
| 10 Year UST | -25.79 | -2.97 | -31.94 | -3.68 | -6.14 | -0.29 | 0.97 | 0.53 |
| 30 Year UST | -14.44 | -1.41 | -18.77 | -1.76 | 5.18 | 0.21 | 0.99 | 0.60 |
| Brent Crude Oil | -12.17 | -0.46 | -17.18 | -0.61 | -8.98 | -0.59 | 0.99 | 0.30 |
| Natural Gas | -14.13 | -0.80 | -15.31 | -0.86 | -13.45 | -0.68 | 1.00 | 0.44 |
| Gold | -11.71 | -0.42 | -12.70 | -0.47 | 7.45 | 0.30 | 1.00 | 0.41 |
| Silver | -5.50 | -0.11 | -3.69 | -0.07 | 6.43 | 0.32 | 0.89 | 0.49 |
| Copper | -17.64 | -1.41 | -18.73 | -1.35 | -13.30 | -0.65 | 0.92 | 0.45 |
| Live Cattle | -21.95 | -1.52 | -25.17 | -1.77 | -5.93 | -0.27 | 1.00 | 0.60 |
| Soybean Meal | -15.66 | -1.40 | -23.22 | -2.07 | -0.65 | -0.03 | 0.99 | 0.16 |
| Soybean Oil | -16.69 | -1.40 | -19.15 | -1.63 | -10.00 | -0.42 | 0.95 | 0.17 |
| Soybeans | -22.91 | -1.95 | -24.51 | -2.14 | 4.82 | 0.20 | 0.99 | 0.35 |
| Wheat | -13.79 | -1.30 | -15.16 | -1.41 | -5.83 | -0.32 | 0.99 | 0.18 |
| Coffee C | -18.49 | -1.57 | -23.26 | -2.00 | -7.23 | -0.34 | 0.98 | 0.54 |
| Corn | -23.26 | -1.71 | -22.79 | -1.67 | -8.51 | -0.37 | 0.76 | 0.50 |

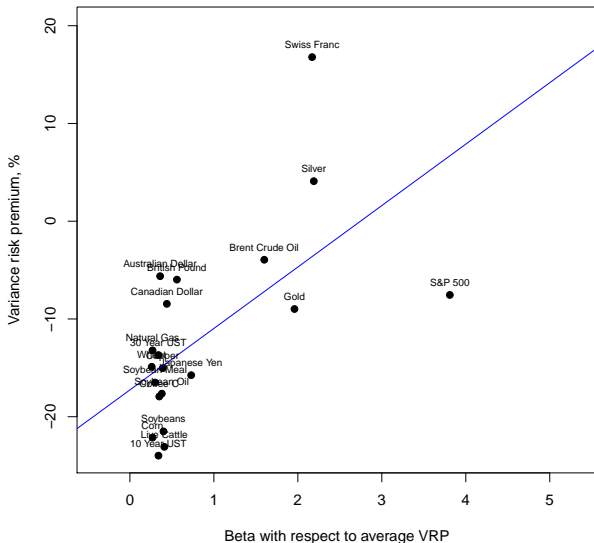
VIX summary statistics

| | Mean | Median | Std (%) | Min | Max | Skew | Kurt | Mean RVol |
|-----------------|------|--------|---------|------|-------|------|------|-----------|
| British Pound | 10.3 | 9.4 | 4.0 | 4.9 | 29.7 | 2.3 | 9.9 | 8.1 |
| Swiss Franc | 10.2 | 9.8 | 3.6 | 3.9 | 24.2 | 1.2 | 5.1 | 9.4 |
| Japanese Yen | 10.9 | 10.2 | 3.8 | 4.9 | 30.1 | 1.6 | 7.8 | 8.1 |
| S&P 500 | 22.2 | 17.6 | 12.8 | 9.7 | 78.0 | 1.8 | 6.2 | 17.3 |
| 10 Year UST | 6.3 | 5.8 | 2.0 | 3.4 | 14.7 | 1.3 | 5.1 | 4.2 |
| Brent Crude Oil | 38.5 | 31.0 | 24.8 | 17.3 | 155.0 | 3.0 | 13.1 | 30.1 |
| Natural Gas | 50.6 | 45.7 | 17.8 | 21.9 | 111.7 | 1.0 | 3.7 | 37.7 |
| Gold | 19.0 | 18.0 | 6.5 | 9.5 | 51.2 | 1.6 | 7.3 | 14.5 |
| Soybeans | 25.2 | 24.1 | 8.1 | 12.1 | 56.1 | 1.4 | 5.0 | 18.6 |
| Wheat | 33.5 | 30.4 | 10.8 | 19.1 | 71.9 | 1.2 | 4.2 | 25.1 |

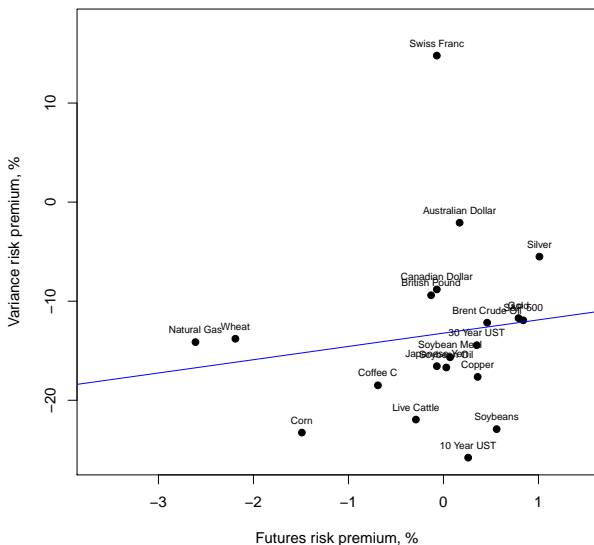
Summary statistics of hedged VIX^2 returns

| | Mean (%) | Std (%) | Sharpe | Skew | Kurt | Volume ratio | Put-call (%) | $\rho(r_T^E, r_T^{VIX})$ |
|-------------------|----------|---------|----------|------|-------|--------------|--------------|--------------------------|
| British Pound | -9.4 | 75.2 | -0.46 | 8.44 | 94.31 | 10.7 | 0.08 | -0.22 |
| Swiss Franc | 14.8 | 274.3 | 0.19 | 11.2 | 130.5 | 12.7 | 0.08 | 0.30 |
| Japanese Yen | -16.6 | 61.4 | -0.98*** | 2.7 | 13.1 | 5.7 | 0.08 | 0.27 |
| S&P 500 | -11.9 | 192.0 | -0.23 | 8.6 | 90.7 | 14.5 | 0.05 | -0.63 |
| S&P 500 pre-Covid | -22.9 | 100.0 | -0.86*** | 4.56 | 26.01 | 14.6 | 0.05 | -0.65 |
| 10 Year UST | -25.8 | 31.5 | -2.97*** | 1.4 | 6.2 | 12.7 | 0.09 | 0.01 |
| Brent Crude Oil | -12.2 | 96.2 | -0.46 | 5.4 | 39.7 | 16.3 | 0.05 | -0.59 |
| Natural Gas | -14.1 | 62.2 | -0.80*** | 6.4 | 61.3 | 10.7 | 0.09 | 0.20 |
| Gold | -11.7 | 101.0 | -0.42 | 8.4 | 88.5 | 15.9 | 0.08 | -0.08 |
| Soybeans | -21.4 | 42.4 | -1.95*** | 1.5 | 6.0 | 15.8 | 0.08 | -0.19 |
| Wheat | -13.4 | 39.6 | -1.30*** | 1.1 | 4.6 | 15.7 | 0.08 | 0.23 |

Variance risk premium and β of systematic variance



Variance risk premium and futures risk premium



Futures risk and systematic risk do not explain the VRP

- Risk in the underlying futures returns does not explain returns on the hedged VIX^2 portfolios
 - $r_t^{VIX} = \alpha + \beta r_t^F + \epsilon_t$
 - α is significant for 14/20 assets
- Systematic risk (cross-sectional average VIX^2 returns) also does not explain returns on the hedged VIX^2 portfolios
 - $r_t^{VIX} = \alpha_{EW} + \beta_{EW} r_t^{VIX_{EW}} + \epsilon_t$
 - α_{EW} is significant for 16/20 assets

Liquidity and fat tails are mildly related to the VRP

- Run cross-sectional Fama-Macbeth regressions of hedged VIX^2 returns on measures of liquidity, hedging risk and fat tails
- Liquidity
 - log-ratio of futures trading volume to that of options
 - put-call-parity deviation: absolute difference between call price and the price of the replicating portfolio (put plus spot price minus the present value of the strike price)
- Fat tails: skewness and kurtosis of futures returns
- Regression results:
 - log-ratio is only significant in the univariate specification but not in the multivariate one
 - put-call parity deviations are not significant at conventional levels
 - skewness is insignificant, whereas kurtosis is strongly significant

Conditional Returns and Predictive Regressions

- Run Bollerslev-Tauchen-Zhou (2009) predictive regressions

$$r_{t \rightarrow T}^F = \alpha + \beta(VIX_t^2 - RV_{t-1,t}) + \epsilon_t$$

- Most of the estimates (16/20 assets) are insignificant across maturities from one month (T=1) to one year (T=12)
- Some β -s are negative, contrary to the positive and significant coefficients found in BTZ (2009)

Conditional Returns and Predictive Regressions

- Run forecasting equation for the variance premium that parallels Bollerslev-Tauchen-Zhou (2009)

$$r_{t \rightarrow t+1}^{\text{hedged VIX}} \times VIX_t^2 = \alpha + \beta(VIX_t^2 - RV_{t-1,t}) + \epsilon_t$$

- The results are more significant than for futures returns
 - Except for currencies, most assets have $\beta > 0$, meaning that larger realized implied variance (in excess of historical variance) is associated with larger future variance returns
- The results show that the returns on hedged VIX^2 portfolios are predictable and reject the variance expectations hypothesis
- We find stronger evidence for conditional variability in the variance risk premium than for predictability of the equity premium

Conclusions

- All 20 assets have option smiles
 - S&P 500 has biggest skew and most negative correlation between returns and variance
 - Model-free returns are 99% correlated with realized variance
- Negative variance premium (except Swiss Franc)
- Variance premium:
 - Not related to risk
 - Related to kurtosis and weakly related to liquidity
 - Predictable (conditional risk premium)

Appendix

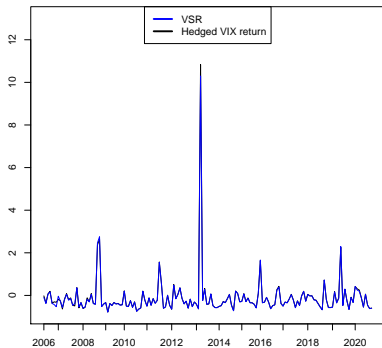
Futures returns

| | Mean | Std | Sharpe | Skew | Kurt | Obs. | Start date | End date | Exchange |
|-------------------|-------|-------|----------|-------|-------|------|------------|----------|----------|
| Australian Dollar | 0.17 | 3.30 | 0.18 | 0.07 | 4.12 | 168 | 2007-02 | 2020-10 | CME |
| British Pound | -0.13 | 2.59 | -0.18 | -0.60 | 3.96 | 166 | 2006-01 | 2020-10 | CME |
| Canadian Dollar | -0.07 | 2.30 | -0.11 | 0.02 | 3.47 | 152 | 2006-01 | 2020-09 | CME |
| Swiss Franc | -0.07 | 2.94 | -0.11 | -0.03 | 5.22 | 140 | 2007-01 | 2020-08 | CME |
| Japanese Yen | -0.07 | 2.57 | -0.11 | -0.24 | 3.64 | 167 | 2006-01 | 2020-09 | CME |
| S&P 500 | 0.84 | 5.33 | 0.58** | -1.16 | 12.36 | 167 | 2006-01 | 2020-06 | CME |
| 10 Year UST | 0.26 | 1.37 | 0.69** | 0.21 | 4.11 | 120 | 2006-02 | 2020-04 | CBOT |
| 30 Year UST | 0.35 | 2.74 | 0.47 | 0.07 | 3.83 | 127 | 2006-02 | 2020-03 | CBOT |
| Brent Crude Oil | 0.46 | 10.97 | 0.14 | 0.61 | 14.49 | 94 | 2011-08 | 2020-09 | NYMEX |
| Natural Gas | -2.61 | 11.41 | -0.83*** | 0.26 | 4.34 | 162 | 2006-01 | 2020-10 | NYMEX |
| Gold | 0.79 | 4.52 | 0.61** | 0.15 | 2.64 | 160 | 2006-01 | 2020-09 | COMEX |
| Silver | 1.01 | 8.68 | 0.43 | 0.30 | 3.22 | 152 | 2006-08 | 2020-09 | COMEX |
| Copper | 0.36 | 6.48 | 0.18 | 0.30 | 3.95 | 155 | 2006-01 | 2020-07 | COMEX |
| Live Cattle | -0.29 | 4.74 | -0.22 | -0.30 | 4.55 | 127 | 2006-01 | 2020-08 | CME |
| Soybean Meal | 0.07 | 7.38 | 0.04 | 0.37 | 3.26 | 145 | 2006-02 | 2020-08 | CBOT |
| Soybean Oil | 0.03 | 5.56 | 0.04 | 0.51 | 3.60 | 143 | 2007-01 | 2020-08 | CBOT |
| Soybeans | 0.56 | 6.59 | 0.29 | -0.50 | 4.30 | 144 | 2006-02 | 2020-08 | CBOT |
| Wheat | -2.19 | 8.18 | -0.97*** | 0.05 | 3.37 | 131 | 2006-02 | 2020-07 | CBOT |
| Coffee C | -0.69 | 8.24 | -0.29 | 0.42 | 3.00 | 110 | 2007-01 | 2020-09 | ICE |
| Corn | -1.49 | 8.02 | -0.69** | 0.25 | 5.98 | 126 | 2006-02 | 2020-07 | CBOT |

VSR and hedged VIX^2 returns

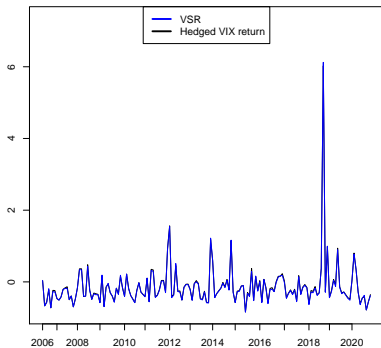
S&P 500

Hedged VIX return (Simpson's method) and VSR. Corr= 1



Natural gas

Hedged VIX return (Simpson's method) and VSR. Corr= 1



Futures correlations

| | Australian Dollar | British Pound | Canadian Dollar | Swiss Franc | Japanese Yen | S&P 500 | 10 Year UST | 30 Year UST | Brent Oil | Natural Gas | Gold | Silver | Copper | Live Cattle | Soybean Meal | Soybean Oil | Soybeans | Wheat | Coffee C | Corn | |
|-----------------|-------------------|---------------|-----------------|-------------|--------------|---------|-------------|-------------|-----------|-------------|-------|--------|--------|-------------|--------------|-------------|----------|-------|----------|------|--|
| British Pound | 0.45 | | | | | | | | | | | | | | | | | | | | |
| Canadian Dollar | 0.72 | 0.44 | | | | | | | | | | | | | | | | | | | |
| Swiss Franc | 0.39 | 0.53 | 0.35 | | | | | | | | | | | | | | | | | | |
| Japanese Yen | 0.05 | -0.02 | -0.07 | 0.30 | | | | | | | | | | | | | | | | | |
| S&P 500 | 0.07 | -0.06 | 0.09 | -0.13 | -0.20 | | | | | | | | | | | | | | | | |
| 10 Year UST | -0.01 | -0.15 | -0.25 | -0.12 | 0.16 | -0.47 | | | | | | | | | | | | | | | |
| 30 Year UST | -0.10 | -0.21 | -0.30 | -0.32 | 0.10 | -0.26 | 0.94 | | | | | | | | | | | | | | |
| Brent Crude Oil | 0.23 | 0.24 | 0.27 | -0.03 | -0.14 | 0.27 | -0.17 | -0.23 | | | | | | | | | | | | | |
| Natural Gas | -0.10 | -0.01 | -0.10 | -0.01 | -0.11 | 0.15 | -0.10 | -0.04 | -0.17 | | | | | | | | | | | | |
| Gold | 0.03 | -0.06 | 0.04 | -0.03 | 0.20 | -0.12 | 0.26 | 0.18 | 0.16 | -0.08 | | | | | | | | | | | |
| Silver | 0.24 | 0.11 | 0.13 | -0.01 | 0.11 | 0.07 | 0.10 | 0.05 | 0.20 | 0.10 | 0.50 | | | | | | | | | | |
| Copper | 0.20 | 0.19 | 0.29 | 0.29 | -0.10 | 0.31 | -0.34 | -0.42 | 0.31 | 0.07 | 0.09 | 0.24 | | | | | | | | | |
| Live Cattle | 0.17 | 0.18 | 0.08 | -0.09 | -0.22 | -0.01 | 0.02 | 0.03 | 0.26 | 0.04 | -0.01 | 0.03 | -0.07 | | | | | | | | |
| Soybean Meal | 0.20 | 0.27 | 0.20 | 0.32 | 0.00 | 0.10 | -0.05 | -0.15 | 0.08 | 0.19 | -0.07 | 0.04 | 0.24 | -0.04 | | | | | | | |
| Soybean Oil | 0.27 | 0.22 | 0.27 | 0.25 | 0.09 | 0.11 | -0.09 | -0.10 | 0.21 | 0.05 | 0.06 | 0.12 | 0.25 | 0.05 | 0.39 | | | | | | |
| Soybeans | 0.22 | 0.22 | 0.19 | 0.21 | -0.07 | 0.28 | -0.12 | -0.14 | 0.09 | 0.24 | -0.12 | 0.08 | 0.14 | -0.02 | 0.74 | 0.6 | | | | | |
| Wheat | -0.12 | 0.00 | -0.18 | -0.10 | 0.04 | 0.03 | 0.09 | 0.08 | -0.10 | 0.08 | -0.06 | 0.08 | 0.05 | 0.10 | 0.39 | 0.14 | 0.43 | | | | |
| Coffee C | 0.17 | -0.01 | 0.22 | -0.14 | -0.12 | 0.08 | 0.03 | 0.02 | 0.01 | 0.25 | 0.26 | 0.16 | 0.05 | 0.10 | 0.07 | 0.22 | 0.26 | 0.22 | | | |
| Corn | -0.01 | 0.08 | -0.10 | 0.01 | 0.04 | 0.15 | 0.09 | 0.10 | 0.01 | 0.33 | -0.05 | 0.12 | -0.04 | 0.11 | 0.52 | 0.34 | 0.68 | 0.66 | 0.23 | | |
| Eq. Weighted | 0.37 | 0.33 | 0.31 | 0.23 | 0.02 | 0.36 | -0.01 | -0.08 | 0.51 | 0.48 | 0.18 | 0.49 | 0.39 | 0.24 | 0.66 | 0.57 | 0.74 | 0.52 | 0.43 | 0.73 | |

Hedged VIX^2 returns correlations

| | Australian Dollar | British Pound | Canadian Dollar | Swiss Franc | Japanese Yen | S&P 500 | 10 Year UST | 30 Year UST | Brent Oil | Natural Gas | Gold | Silver | Copper | Live Cattle | Soybean Meal | Soybean Oil | Soybeans | Wheat | Coffee C | Corn | |
|-----------------|-------------------|---------------|-----------------|-------------|--------------|---------|-------------|-------------|-----------|-------------|------|--------|--------|-------------|--------------|-------------|----------|-------|----------|------|--|
| British Pound | 0.64 | | | | | | | | | | | | | | | | | | | | |
| Canadian Dollar | 0.77 | 0.68 | | | | | | | | | | | | | | | | | | | |
| Swiss Franc | 0.13 | 0.44 | 0.27 | | | | | | | | | | | | | | | | | | |
| Japanese Yen | 0.32 | 0.17 | 0.29 | 0.03 | | | | | | | | | | | | | | | | | |
| S&P 500 | 0.15 | 0.10 | 0.10 | 0.03 | 0.47 | | | | | | | | | | | | | | | | |
| 10 Year UST | 0.08 | -0.03 | 0.07 | 0.00 | 0.23 | 0.11 | | | | | | | | | | | | | | | |
| 30 Year UST | 0.08 | -0.06 | 0.06 | -0.04 | 0.28 | 0.33 | 0.81 | | | | | | | | | | | | | | |
| Brent Crude Oil | 0.06 | 0.14 | 0.01 | 0.23 | 0.52 | 0.75 | 0.19 | 0.18 | | | | | | | | | | | | | |
| Natural Gas | 0.01 | 0.00 | 0.00 | -0.05 | 0.10 | 0.08 | -0.07 | -0.09 | 0.25 | | | | | | | | | | | | |
| Gold | -0.01 | 0.06 | 0.02 | 0.00 | 0.20 | 0.26 | 0.02 | 0.17 | 0.11 | 0.03 | | | | | | | | | | | |
| Silver | 0.00 | 0.06 | -0.05 | 0.04 | 0.34 | 0.66 | 0.08 | 0.12 | 0.66 | 0.11 | 0.53 | | | | | | | | | | |
| Copper | 0.11 | -0.06 | 0.06 | -0.06 | 0.26 | 0.33 | 0.06 | 0.18 | 0.27 | 0.04 | 0.27 | 0.37 | | | | | | | | | |
| Live Cattle | 0.35 | 0.41 | 0.35 | 0.02 | 0.27 | 0.22 | -0.15 | -0.07 | 0.47 | 0.16 | 0.05 | 0.24 | 0.10 | | | | | | | | |
| Soybean Meal | -0.11 | -0.06 | -0.04 | 0.00 | 0.05 | -0.01 | -0.04 | -0.05 | -0.05 | 0.02 | 0.01 | 0.09 | 0.13 | 0.04 | | | | | | | |
| Soybean Oil | 0.01 | 0.00 | -0.07 | 0.08 | 0.09 | 0.13 | 0.16 | 0.23 | 0.11 | -0.04 | 0.24 | 0.11 | 0.10 | -0.02 | 0.33 | | | | | | |
| Soybeans | 0.06 | -0.04 | 0.06 | -0.02 | 0.13 | 0.19 | 0.01 | 0.06 | 0.07 | -0.02 | 0.04 | 0.12 | 0.21 | 0.13 | 0.57 | 0.25 | | | | | |
| Wheat | -0.06 | -0.02 | 0.00 | 0.06 | 0.08 | 0.08 | 0.05 | 0.08 | 0.10 | -0.09 | 0.13 | 0.16 | 0.19 | -0.02 | 0.19 | 0.10 | 0.33 | | | | |
| Coffee C | 0.18 | 0.14 | 0.16 | -0.01 | 0.14 | 0.03 | 0.15 | 0.15 | 0.26 | 0.19 | 0.08 | 0.13 | -0.09 | 0.27 | 0.11 | 0.14 | 0.21 | 0.20 | | | |
| Corn | -0.07 | -0.09 | -0.10 | -0.01 | -0.01 | -0.22 | 0.25 | 0.12 | 0.11 | -0.12 | 0.15 | 0.22 | 0.23 | -0.09 | 0.28 | 0.04 | 0.33 | 0.47 | 0.04 | | |
| Eq. Weighted | 0.32 | 0.32 | 0.34 | 0.37 | 0.35 | 0.81 | 0.25 | 0.31 | 0.84 | 0.17 | 0.45 | 0.75 | 0.39 | 0.39 | 0.2 | 0.31 | 0.32 | 0.28 | 0.28 | 0.21 | |

Regressions of hedged VIX^2 returns

| | $r_t^{VIX} = \alpha + \beta r_t^F + \epsilon_t$ | | | | | $r_t^{VIX} = \alpha_{EW} + \beta_{EW} r_t^{VIX_{EW}} + \epsilon_t$ | | | | |
|-------------------|---|-------------|---------|------------|----------|--|------------------|--------------|-----------------|------------------|
| | α | α se | β | β se | IR | α_{EW} | α_{EW} se | β_{EW} | β_{EW} se | IR _{EW} |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| Australian Dollar | -0.01 | 0.04 | -6.25 | 1.26 | -0.07 | 0.03 | 0.04 | 0.41 | 0.09 | 0.20 |
| British Pound | -0.10 | 0.06 | -6.49 | 2.21 | -0.49* | -0.02 | 0.06 | 0.56 | 0.13 | -0.10 |
| Canadian Dollar | -0.09 | 0.05 | -5.25 | 2.13 | -0.54* | -0.04 | 0.05 | 0.45 | 0.10 | -0.25 |
| Swiss Franc | 0.17 | 0.22 | 28.00 | 7.58 | 0.23 | 0.37 | 0.22 | 2.11 | 0.45 | 0.52* |
| Japanese Yen | -0.16 | 0.05 | 6.49 | 1.79 | -0.97*** | -0.07 | 0.04 | 0.77 | 0.09 | -0.49* |
| S&P 500 | 0.07 | 0.12 | -22.67 | 2.18 | 0.17 | 0.31 | 0.09 | 3.47 | 0.19 | 1.00*** |
| 10 Year UST | -0.26 | 0.03 | 0.22 | 2.11 | -2.97*** | -0.22 | 0.03 | 0.27 | 0.10 | -2.59*** |
| 30 Year UST | -0.15 | 0.03 | 2.21 | 1.20 | -1.46*** | -0.10 | 0.03 | 0.36 | 0.10 | -1.01*** |
| Brent Crude Oil | -0.10 | 0.08 | -5.14 | 0.74 | -0.46 | 0.10 | 0.06 | 1.54 | 0.10 | 0.68* |
| Natural Gas | -0.11 | 0.05 | 1.09 | 0.42 | -0.65** | -0.11 | 0.05 | 0.23 | 0.11 | -0.65** |
| Gold | -0.10 | 0.08 | -1.82 | 1.77 | -0.36 | 0.10 | 0.08 | 1.53 | 0.24 | 0.40 |
| Silver | -0.01 | 0.11 | -4.13 | 1.31 | -0.03 | 0.20 | 0.08 | 2.30 | 0.17 | 0.75*** |
| Copper | -0.17 | 0.04 | -1.97 | 0.54 | -1.40*** | -0.13 | 0.03 | 0.39 | 0.07 | -1.12*** |
| Live Cattle | -0.23 | 0.04 | -4.22 | 0.92 | -1.71*** | -0.18 | 0.04 | 0.42 | 0.09 | -1.34*** |
| Soybean Meal | -0.16 | 0.03 | 0.87 | 0.46 | -1.42*** | -0.12 | 0.04 | 0.27 | 0.11 | -1.08*** |
| Soybean Oil | -0.17 | 0.04 | -0.30 | 0.66 | -1.41*** | -0.09 | 0.04 | 0.47 | 0.12 | -0.79*** |
| Soybeans | -0.21 | 0.03 | -1.22 | 0.53 | -1.82*** | -0.15 | 0.04 | 0.45 | 0.11 | -1.34*** |
| Wheat | -0.11 | 0.03 | 1.11 | 0.41 | -1.03*** | -0.08 | 0.04 | 0.36 | 0.11 | -0.76** |
| Coffee C | -0.17 | 0.04 | 1.99 | 0.46 | -1.55*** | -0.14 | 0.04 | 0.37 | 0.12 | -1.23*** |
| Corn | -0.21 | 0.04 | 1.17 | 0.55 | -1.53*** | -0.18 | 0.05 | 0.33 | 0.14 | -1.31*** |

Fama-MacBeth cross-sectional regressions

| <i>Dependent variable: hedged VIX² return</i> | | | | | | | |
|--|---------|----------|----------|----------|--------|----------|--------|
| Volume ratio | -0.04** | | | | | -0.02 | |
| | (0.02) | | | | | (0.02) | |
| Put-call deviations | 4.68 | | | | | 21.18 | |
| | (25.78) | | | | | (32.52) | |
| Skewness | | | -0.02 | | | -0.003 | |
| | | | (0.03) | | | (0.03) | |
| Kurtosis | | | 0.07*** | | | 0.07*** | |
| | | | (0.02) | | | (0.02) | |
| VIX level | | | | | | -0.01*** | |
| | | | | | | (0.002) | |
| VIX std | | | | | | 0.01 | |
| | | | | | | (0.02) | |
| Intercept | -0.10 | -0.15*** | -0.15*** | -0.37*** | 0.01 | -0.14*** | -0.13* |
| | (0.08) | (0.03) | (0.02) | (0.06) | (0.03) | (0.02) | (0.08) |
| Observations | 2,804 | 2,804 | 2,804 | 2,804 | 2,804 | 2,804 | 2,804 |

Bollerslev-Tauchen-Zhou predictive regressions

| | Panel A: $r_{t \rightarrow T}^F = \alpha + \beta(VIX_t^2 - RV_{t-1,t}) + \epsilon_t$ | | | | Panel B: $r_{t \rightarrow t+1}^{\text{hedged VIX}} \times VIX_t^2 = \alpha + \beta(VIX_t^2 - RV_{t-1,t}) + \epsilon_t$ |
|-------------------|--|------------------|------------------|------------------|---|
| | T=t+1m | T=t+3m | T=t+6m | T=t+12m | |
| | (1) | (2) | (3) | (4) | (5) |
| Australian Dollar | -5.44 (-0.92) | -3.41 (-1.03) | -4.98 (-1.95) | -2.07 (-1.03) | 0.49 (2.71) |
| British Pound | -7.30 (-3.39) | -1.11 (-0.88) | -1.32 (-1.42) | -0.13 (-0.26) | 0.11 (1.58) |
| Canadian Dollar | -6.53 (-1.73) | -0.32 (-0.13) | 2.64 (0.92) | 2.92 (2.48) | 0.16 (1.50) |
| Swiss Franc | 1.74 (3.18) | 0.11 (0.67) | 0.46 (3.86) | 0.30 (3.55) | 0.07 (2.35) |
| Japanese Yen | 1.62 (0.97) | -0.28 (-0.13) | -1.60 (-1.02) | -0.37 (-0.37) | -0.30 (-1.77) |
| S&P 500 | 0.12 (0.35) | 0.48 (2.57) | -0.07 (-0.32) | -0.08 (-0.68) | -0.63 (-2.51) |
| 10 Year UST | 26.79 (3.91) | 5.25 (1.23) | 2.51 (0.64) | 5.68 (1.94) | -0.56 (-13.41) |
| 30 Year UST | -4.89 (-0.79) | -7.40 (-2.24) | -7.14 (-1.89) | -1.18 (-0.48) | -0.33 (-2.34) |
| Brent Crude Oil | 1.03 (1.32) | 1.02 (6.29) | 1.29 (1.98) | 0.16 (0.42) | -1.02 (-10.53) |
| Natural Gas | 0.07 (0.67) | 0.19 (3.99) | 0.08 (4.36) | 0.05 (2.54) | -0.96 (-22.07) |
| Gold | 0.72 (0.27) | 0.79 (0.67) | 0.22 (0.25) | 0.18 (0.27) | -0.41 (-1.99) |
| Silver | 0.69 (0.69) | 1.07 (1.68) | -0.10 (-0.19) | -0.34 (-1.01) | 0.04 (0.13) |
| Copper | 2.12 (1.75) | 1.63 (1.74) | 1.37 (2.22) | 1.07 (3.11) | -0.17 (-0.53) |
| Live Cattle | -0.33 (-0.12) | -0.39 (-0.2) | 1.02 (0.81) | -0.17 (-0.08) | -1.04 (-2.80) |
| Soybean Meal | 0.00 (0.00) | 0.95 (0.93) | 0.22 (0.34) | 0.05 (0.12) | -0.15 (-1.09) |
| Soybean Oil | 1.26 (1.38) | -0.16 (-0.44) | 0.21 (0.77) | 0.37 (1.16) | -0.51 (-4.72) |
| Soybeans | 3.12 (2.45) | 2.89 (5.91) | 0.40 (0.56) | 0.14 (0.21) | -0.45 (-5.79) |
| Wheat | 1.06 (3.30) | -1.80 (-1.69) | -1.24 (-1.26) | -0.96 (-1.44) | -0.39 (-2.85) |
| Coffee C | 0.94 (0.40) | 0.15 (0.18) | 0.32 (0.65) | -0.11 (-0.26) | -0.13 (-0.74) |
| Corn | 1.99 (0.92) | 0.74 (0.96) | -0.55 (-0.70) | -0.42 (-0.58) | -0.37 (-1.94) |