The Rise and Fall of Investment: Rethinking Q theory in Equilibrium

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Overview

Question: Why has the investment-Q correlation been unstable over time?

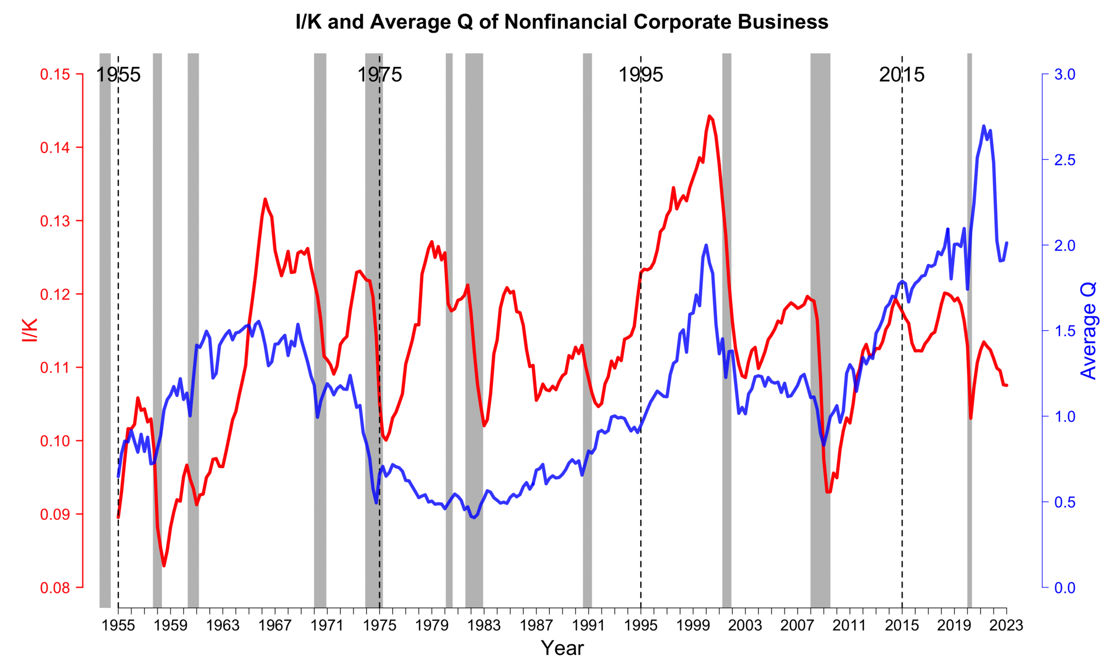


Figure 1. The Aggregate Investment and Tobin's Q

This paper shows theoretically and numerically that *investment supply shocks*, shocks to the marginal cost of investment, are crucial for determining the joint dynamics of investment and Q.

- Demand: positive shocks to the marginal benefit of investment drive up both investment and Q
- Supply: positive shocks to the marginal cost of investment could drive up Q and reduce investment
- The comovement of investment and Q depends on the source / nature of shocks
- *Q is to investment in neoclassical models as price is to quantity in demand-supply systems*

Two-period Model

- Investment and Q are jointly determined by demand and supply
- Demand side: shocks to expected profitability and discount rate
- $MQ_t = [\nu \mathbb{E}_t A_{t+1}{}^{1-\nu} K_{t+1}^{\nu-1} + (1-\delta)]/R$

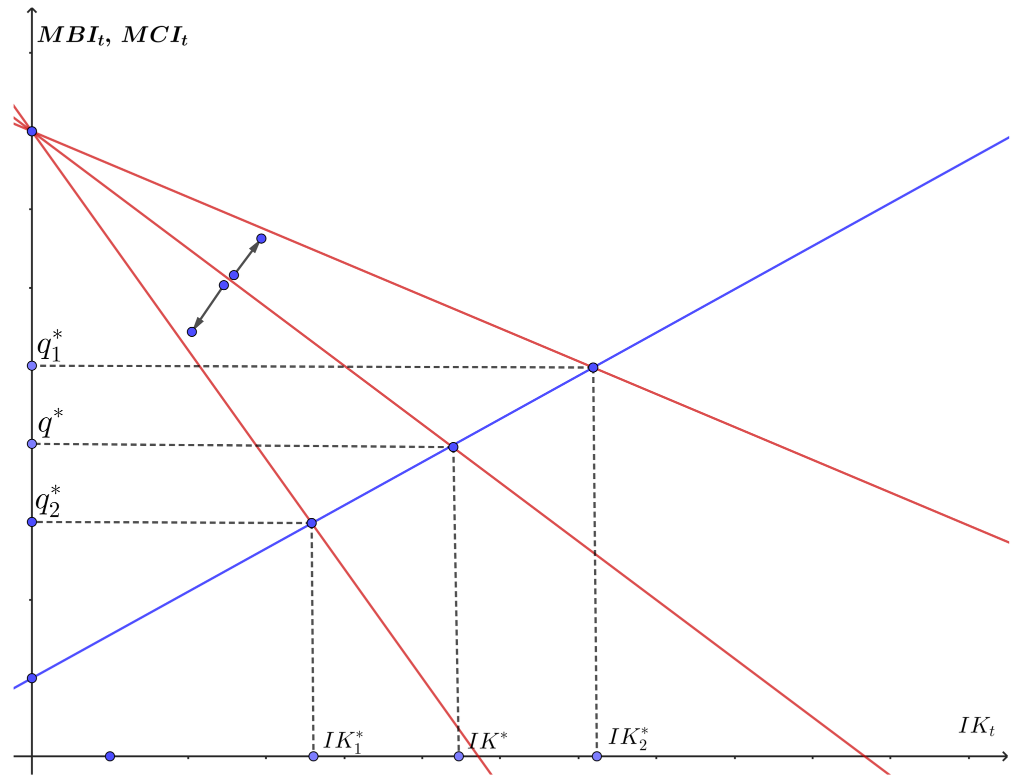


Figure 2. Marginal benefit curve shifting

- Supply side: shocks to investment adjustment costs
- $MCI_t = 1 + C_t \, IK_t^\eta$

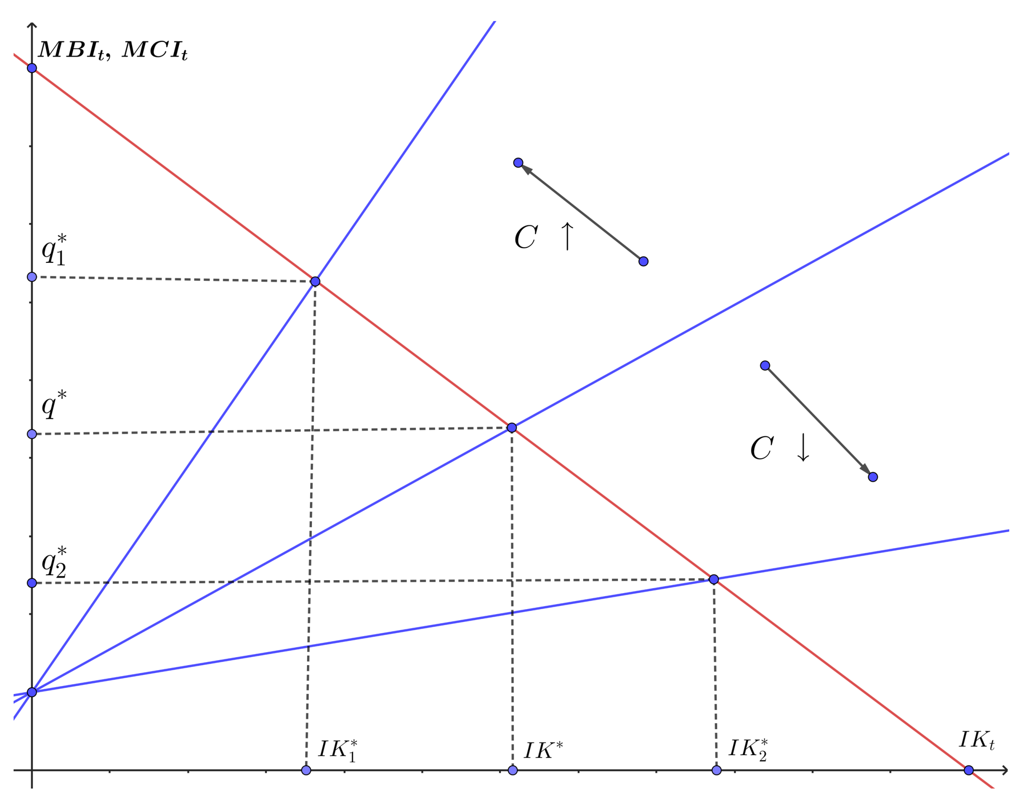


Figure 3. Marginal cost curve shifting

Dynamic Model

- Optimality condition

$$Q_t = 1 + C_t IK_t^\eta$$

$$Q_t = \frac{1}{R}\mathbb{E}_t\Big[\nu A_{t+1}{}^{1-\nu} K_{t+1}^{\nu-1} + \frac{\eta C_{t+1}}{\eta+1} IK_{t+1}^{\eta+1} + (1-\delta)\big[1 + C_{t+1} IK_{t+1}^\eta\big]\Big]$$

- Elasticities of Q/investment to demand and supply shocks

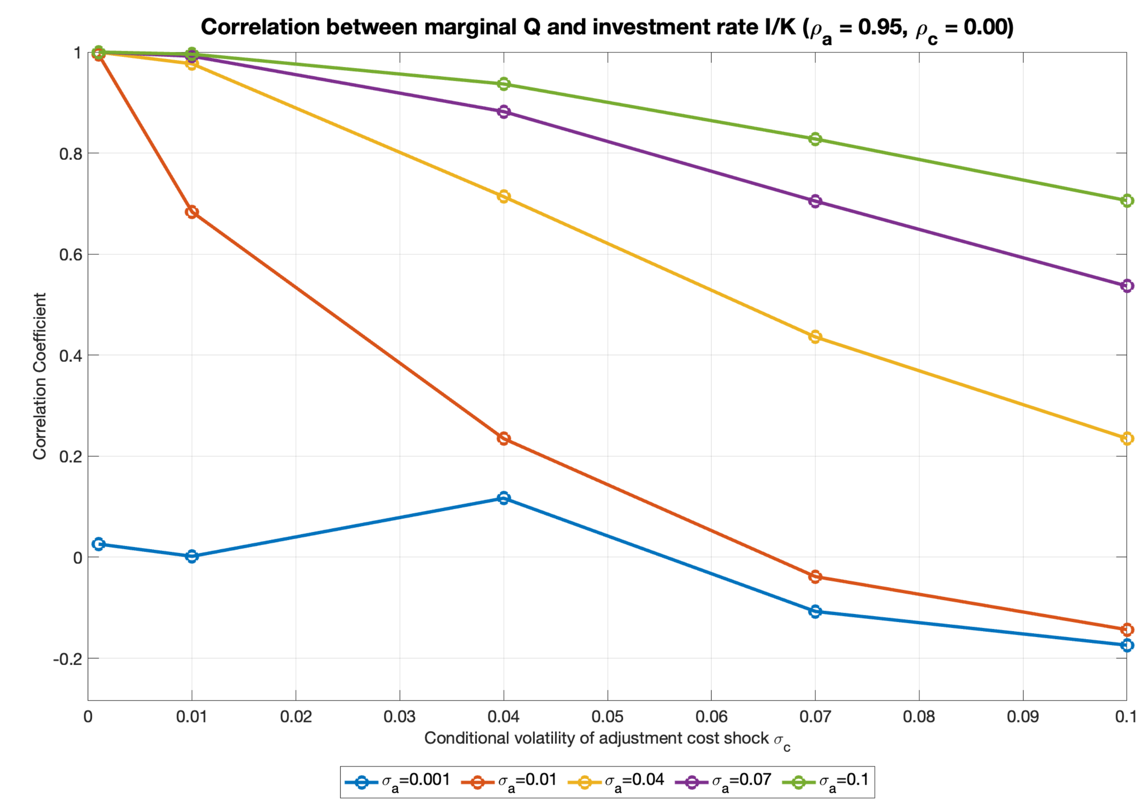
$$\beta_t^Q \equiv \frac{\partial(Q_t^*-1)/(Q_t^*-1)}{\partial \mathbb{E}_t A_{t+1}/\mathbb{E}_t A_{t+1}} = \eta \frac{\partial IK_t^*/IK_t^*}{\partial \mathbb{E}_t A_{t+1}/\mathbb{E}_t A_{t+1}} \equiv \eta \beta_t$$

$$\phi_t^Q \equiv \frac{\partial(Q_t^*-1)/(Q_t^*-1)}{\partial C_t/C_t} = 1 + \eta \frac{\partial IK_t^*/IK_t^*}{\partial C_t/C_t} \equiv 1 + \eta \phi_t$$

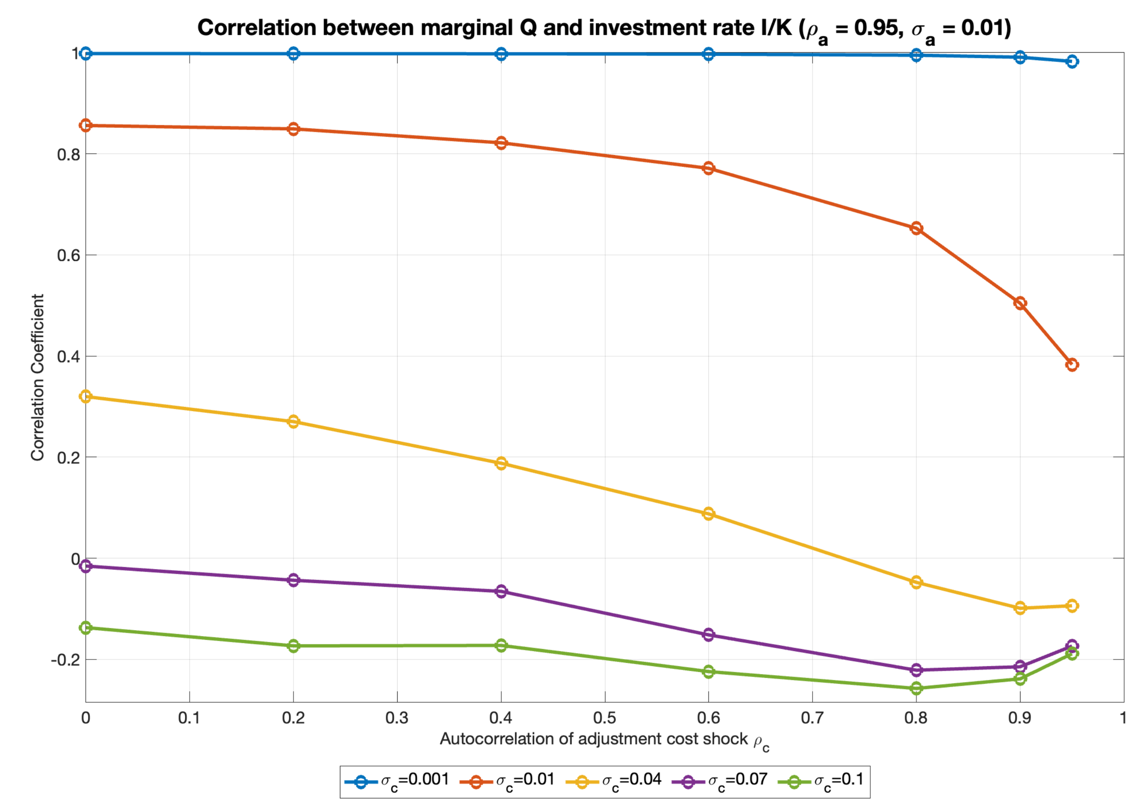
- **Proposition 1**: When $\mathbb{E}_t A_{t+1}$ changes, Q_t^* and IK_t^* always comove positively if $\beta_t \neq 0$.
- **Proposition 2**: When C_t changes, Q_t^* and IK_t^* comove positively if $\phi_t \in (-\infty, -\frac{1}{\eta}) \cup (0, +\infty)$, and comove negatively if $\phi_t \in (-\frac{1}{\eta}, 0)$. IK_t^* is invariant to C_t if $\phi_t = 0$, and Q_t^* is invariant to C_t if $\phi_t = -\frac{1}{\eta}$.
- When the technology exhibits CRS:
- **Lemma 1**: For $\rho_a \in [0, 1]$, $\beta \in [0, \frac{-\omega_1}{(\lambda_1 - \lambda_2)}]$ is non-negative and increasing in ρ_a
- **Lemma 2**: For $\rho_c \in [0, 1]$, $\phi \in [\frac{-(\theta_1-\theta_2)}{(\lambda_1-\lambda_2)}, -\frac{1}{\eta}]$ is negative and decreasing in ρ_c
- Prop 2 and Lemma 2 show that, under CRS, Q_t^* and IK_t^* can't comove negatively due to large ϕ

Numerical Results

- $Corr(MQ, IK)$ decreases with adjustment cost shock volatility



- $Corr(MQ, IK)$ increases with the adjustment cost shock persistence



Conclusion

- Marginal Q is not sufficient for investment with supply shocks