Simple Distribution Sensitive Welfare Measures

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Motivation

- Simple welfare measures (e.g. mean income, poverty headcount) are ubiquitous but not distribution sensitive

- Distribution sensitive welfare measures (e.g. Atkinson Index, Sen Index, Watts Index) are rarely used in practice, often because they have complex mathematical formulations and/or units that are difficult for non-specialists to understand

- This situation is unappealing given widely-held views that distributional considerations are important for policy analysis

Table 1. Frequency of Distribution Sensitive Poverty and Welfare Measures in Academic, Policy, and Public Discourse

	Academic Use	**Policy Use**	**Public Discourse**
	*EconLit**	*Multilateral Development Banks at nlp4dev.org**	*Google N-gram***
Poverty Measures			
Poverty Rate OR Poverty Headcount ***	40832	26730	40.49
Squared Poverty Gap	473	681	0.32
Watts Index	47	52	0.09
Welfare Measures			
Average income	61493	15492	36.11
Atkinson Index	744	122	0.49
Sen Index	258	70	0.12

Contributions

- We develop a novel axiomatic framework that formalizes simplicity properties of welfare measures that represent welfare orderings
  - Restrictions on units
  - Sign and orientation
  - Restrictions on number of parameters
  - Analytical properties such as subgroup decomposability

- We document sharp tradeoffs between ethical properties of welfare orderings and simplicity properties of welfare measures that represent them
  - e.g., there is no subgroup decomposable welfare measure with income units that represents a distribution sensitive welfare ordering

What we do, cont'd

- We use the axiomatic framework to provide a full "if-and-only-if" characterization of a simple new sub-group decomposable distribution sensitive welfare measure, the **welfare gap**:

$$W(y,z) = \frac{1}{N}\sum_{i=1}^{N}\frac{z}{y_i}$$

- The welfare gap is "*the average factor by which incomes must be multiplied to reach the reference income level* z"

- We provide an empirical illustration using the world distribution of income across individuals 1990-2019

Roadmap

1. Axiomatic framework for ethical and simplicity properties
2. Main theoretical results
3. Properties of the Welfare Gap
4. Empirical illustration

Framework

- $y = (y_1, \ldots, y_N) \in Y$ is an **income distribution** with mean $\bar{y}$

- $O \in \Omega$ is a **welfare ordering** that ranks income distributions and may satisfy *ethical properties* such as monotonicity and Pigou-Dalton sensitivity

- $M(y, z)$ is a **welfare measure** which maps income distributions into scalar values and may satisfy *simplicity properties*. It depends on a vector $z \in P$ of parameters with income units.
  - Specialize to scalar z for this talk (see paper for general case)
  - May also depend on unitless parameters like inequality aversion

- $M(y, z)$ is an **increasing (decreasing) representation** of O if $y \succcurlyeq_{O_z} y' \iff M_z(y) \geq (\leq) M_z(y')$ for all $y, y' \in Y$

Two simplicity properties

SP1 – $M(y, z)$ is subgroup decomposable with population weights

- Simplifies analysis of welfare measure
- Simplifies interpretation since welfare measure must be average of individual contributions $M(y, z) = \frac{1}{N}\sum_{i=1}^{N} f(y_i, z)$
- Rules out welfare measures that depend on relative aspects of income (example of tradeoff between ethical and simplicity properties)

SP2 – $M(y, z)$ has normalized income units

- Formally defining "has income units" is surprisingly hard!
- As shortcut, require $M(ky, kz) = kM(y, z)$ and $M(a\mathbb{1}_N, z) = a$
- Weaker normalization is possible (see paper)

An impossibility result

Theorem 1: *M is a **parametric welfare measure** that satisfies SP1 and SP2 if and only if for all $y \in Y$ and all $z \in P$,*

$$M(y,z) = \pm\left(\frac{1}{N}\sum_{i=1}^{N} y_i\right) = \pm\,\bar{y}$$

Remarks:

- The only parametric welfare measure that satisfies SP1 and SP2 is $\pm$ mean income
- Impossibility result: *There is no welfare measure that satisfies SP1 and SP2 and represents a Pigou-Dalton sensitive welfare ordering*
- Stark illustration of tradeoff between ethical and simplicity properties

Escaping the impossibility

SP3 – Weak Intuitive Units: *M has either income units or is the ratio of two incomes. M is the ratio of two incomes if there exists a parametric measure M^* with normalized income units such that either:*

$$M\left(y, z^{ref}\right) = \frac{M^*(y)}{M^*(z^{ref})} \text{ or}$$

$$M\left(y, z^{ref}\right) = \frac{M^*\left(z^{ref}\right)}{M^*(y)}$$

Remarks:

- z^{ref} is a reference income distribution, e.g., $y = y^{US}$, $M^*(y^{US}) = \bar{y}^{US}$, $M\left(y, z^{ref}\right) = \bar{y}/\bar{y}^{US}$
- "ratio of two incomes" is not the same as "ratio units", e.g., headcount is ratio of number of people

Unique characterization of the Welfare Gap

Theorem 3: M is a **parametric welfare measure** *that satisfies SP1 and SP3 if and only if for all* $y \in Y$ *and all* $z > 0$ *:*

- ***Case 1:*** $M(y,z) = \pm\bar{y}$***; or***
- ***Case 2:*** $M(y,z) = \frac{\bar{y}}{z}$***; or***
- ***Case 3:*** $M(y,z) = \frac{1}{N}\sum_{i=1}^{N}\frac{z}{y_i}$

Remarks:

- Case 1 and Case 2 are transformations of mean income and therefore are simple but not distribution sensitive
- Case 3 characterizes the Welfare Gap as the unique welfare measure that satisfies SP1 and SP3 and represents a Pigou-Dalton sensitive welfare ordering

You can’t always get what you want!

- Simplicity restricts the degree of inequality aversion
  - Note that the welfare gap is $z/A_2(y)$ where $A_\varepsilon(y) = \left(\frac{1}{N}\sum_{i=1}^{N} y_i^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$ is the Atkinson index, so simplicity properties SP1 and SP3 restrict ethical properties (coefficient of inequality aversion must be $\varepsilon = 2$)
  - However, both micro evidence and macro calibration conventions are generally consistent with $\varepsilon \approx 2$
- The welfare gap is a *decreasing* welfare measure, which imposes some cognitive costs on users
  - But negating it to obtain an *increasing* welfare measure imposes different cognitive cost of negative units

Features of the Welfare Gap

- The welfare gap is:

$$W(y,z) = \frac{1}{N}\sum_{i=1}^{N}\frac{z}{y_i}$$

- Features:
  - New representation of an existing welfare ordering $A_2(y)$
  - Simple interpretation: "*the average factor by which incomes must be multiplied to reach the reference income level* z"
  - Satisfies Pigou-Dalton, transfer, and growth sensitivity
  - Is sub-group decomposable
  - Ordering of distributions is invariant to the choice of z

Related inequality measure

- Welfare gap decomposes into mean and inequality measure:

$$W(y,z) = \left(\frac{z}{\bar{y}}\right) I(y,\bar{y}), \qquad I(y,\bar{y}) \equiv \frac{1}{N}\sum_{i=1}^{N}\frac{\bar{y}}{y_i}$$

- Inequality measure has between-within decomposition:

$$I(y,\bar{y}) = \underbrace{\left(\sum_{g=1}^{G}\frac{N_g}{N}\frac{\bar{y}}{\bar{y}_g}\right)}_{\substack{between-group \\ inequality\ I_{btw}}} \underbrace{\left(\sum_{g=1}^{G} w_g\, I(y_g,\bar{y}_g)\right)}_{\substack{within-group \\ inequality\ I_{wth}}}, \qquad w_g = \frac{N_g/\bar{y}_g}{\sum_{g'=1}^{G} N_{g'}/\bar{y}_{g'}}$$

Related poverty measure

- We can interpret z as a poverty line to obtain a variant on the welfare gap that satisfies the Sen (1976) focus axiom:

$$P(y,z) = \underbrace{\left(\frac{N_z}{N}\right)}_{\substack{Poverty \\ Headcount}} \times \underbrace{\frac{1}{N_z}\sum_{i=1}^{N_z}\left(\frac{z}{y_i} - 1\right)}_{\substack{Average\ Growth \\ Needed\ to\ Reach\ z}}$$

- Average *growth rate* among the poor needed to reach the poverty line, multiplied by the poverty headcount

- Unlike $W(y,z)$, $P(y,z)$ is not invariant to choice of z

Empirical illustration

- World interpersonal income distribution as calculated in the World Bank's Poverty and Inequality Platform (PIP)
  - Based on over 2000 harmonized household surveys for 168 countries
  - Measured in 2017 $PPP per person per day
  - Caveat: as in World Bank's global poverty estimates, abstract from distinction between consumption and income

- Welfare gap is highly sensitive to very low incomes since it depends on z/y_i
  - This is a feature, not a bug!
  - Bottom-code very low incomes at $\$0.25$
  - Results are robust to reasonable alternative thresholds

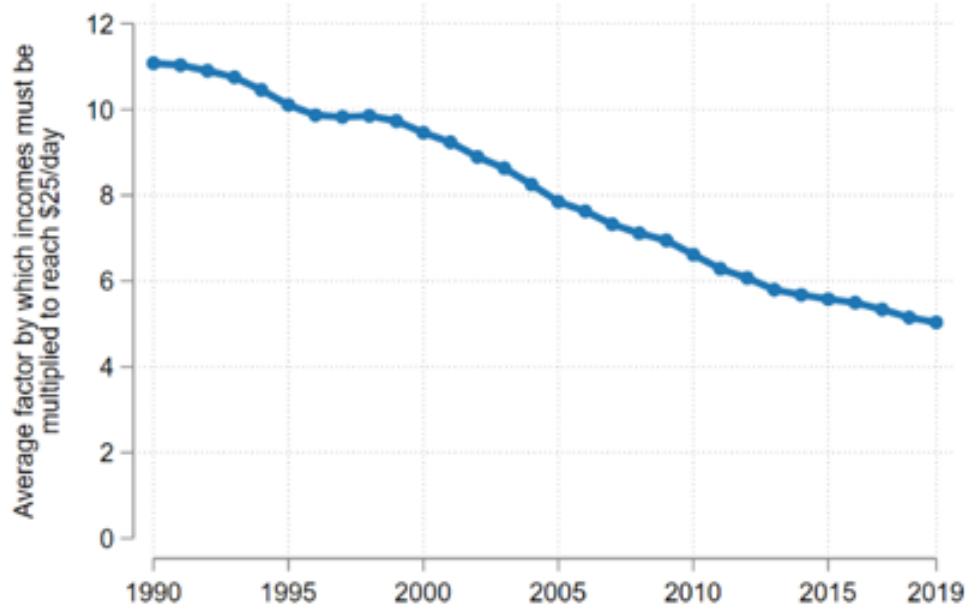
Empirical illustration, cont'd

- Set reference income level to $z = \$25$
  - Typical poverty line in high-income countries
  - Typical level of household income per capita for countries about to graduate from middle- to high-income status

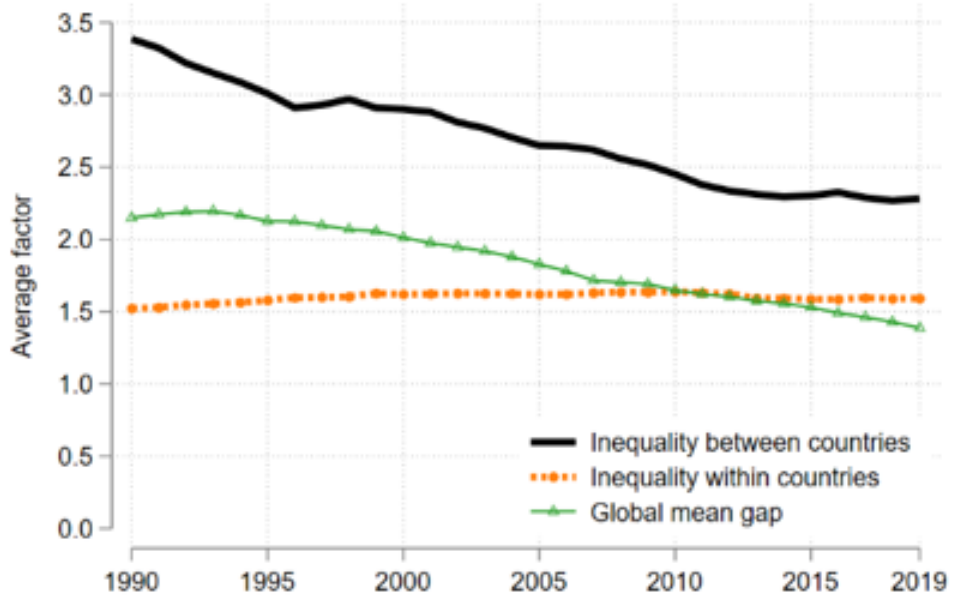
- Global welfare gap with this parameterization is the World Bank's "official" measure for tracking shared prosperity (labeling it the "global prosperity gap")

The global welfare gap

Panel A: Global welfare gap

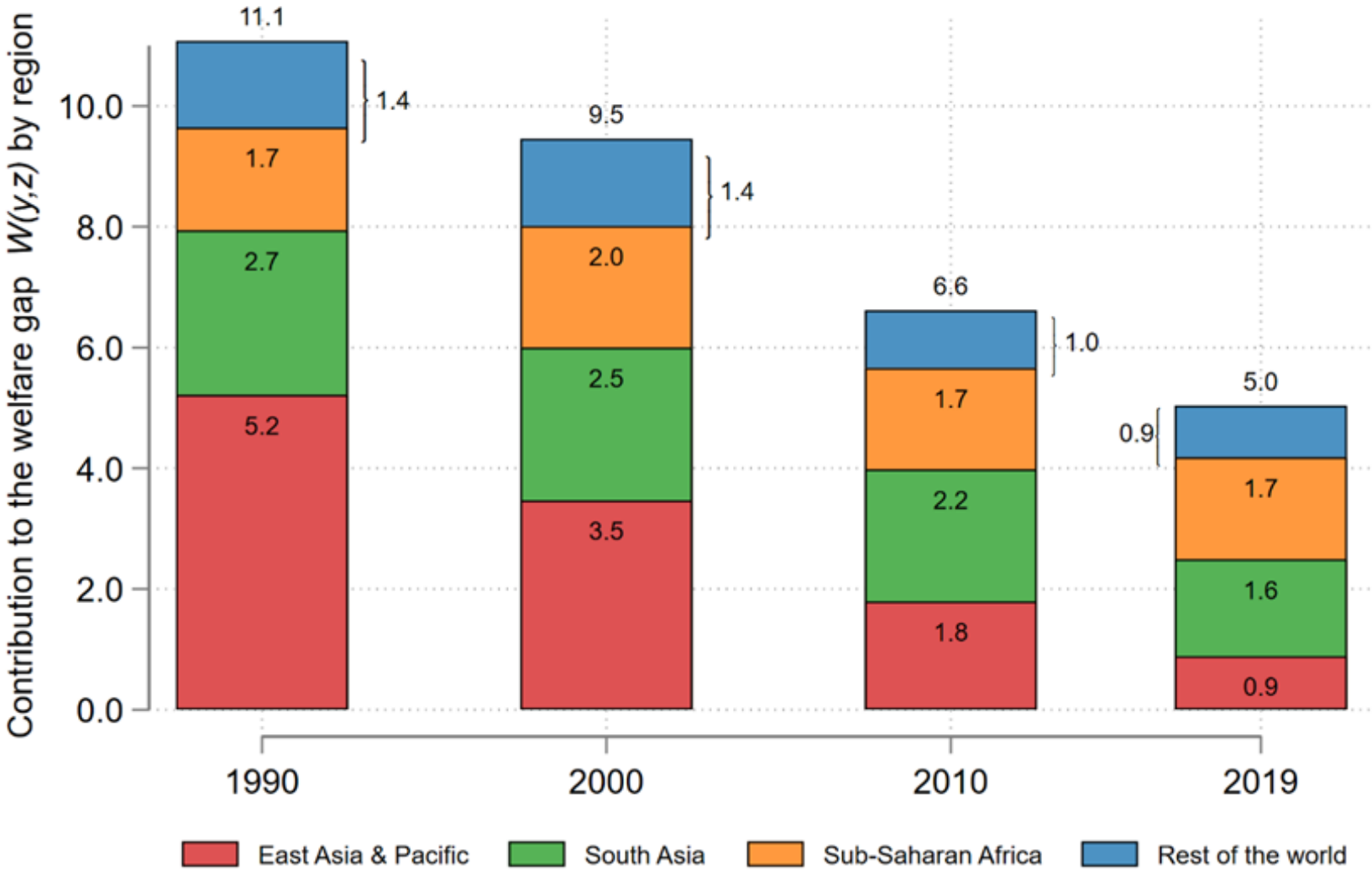


Panel B: Components of the global welfare gap



$$\underbrace{W(y_t, z)}_{Welfare\ gap} = \underbrace{\frac{z}{\bar{y}_t}}_{global\ mean\ gap} \times \underbrace{I_{btw}(y_t, \bar{y}_t)}_{\substack{between-country \\ inequality}} \times \underbrace{I_{wth}(y_t, \bar{y}_t)}_{\substack{within-country \\ inequality}}$$

Illustrating value of subgroup decomposability



Summing Up

- Existing simple welfare measures (mean, headcount) are ubiquitous but not distribution sensitive
- Distribution-sensitive measures are rarely used, likely because they lack simple units and interpretation
- We resolve this dilemma by formally defining simplicity properties and using them to uniquely characterize a simple new distribution sensitive subgroup decomposable welfare measure, the **welfare gap**.