Endogenous Monetary Non-Neutrality

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This Paper

Contribution towards filling the gap between:

1. RBC theory $\begin{cases} (+) & \text{Strongly microfounded} \\ (-) & \text{Neutrality} \end{cases}$

2. New Keynesian framework $\begin{cases} (+) & \text{Short-run non-neutrality} \\ (-) & \text{Critical role of ad hoc restrictions} \end{cases}$

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Simple RBC framework with two main ingredients:

1. **Industry Structure featuring Horizontal Differentiation.**
2. **Bounded strategic reasoning.**

Outline

Model: Overview. Industry Structure

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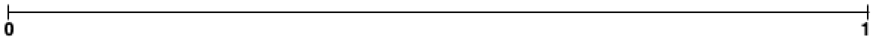

Figure 1: An industry j.

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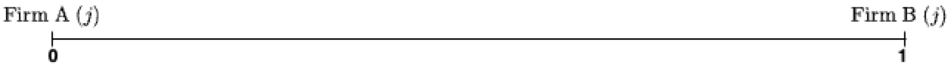

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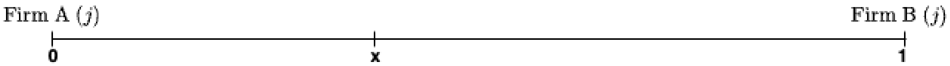

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Interpretation:

1. Linear city as product space. Heterogeneous tastes, utility cost.

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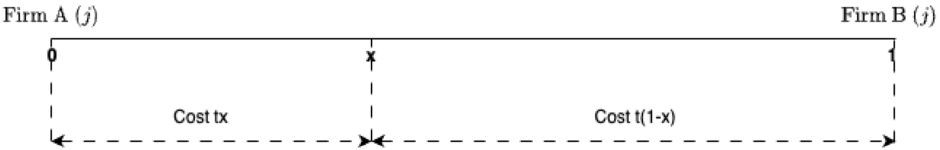

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t governs the degree of horizontal differentiation:

1. When $t = 0$, back to the Bertrand paradox.
2. As t increases: more costly for consumers to move, more market power.

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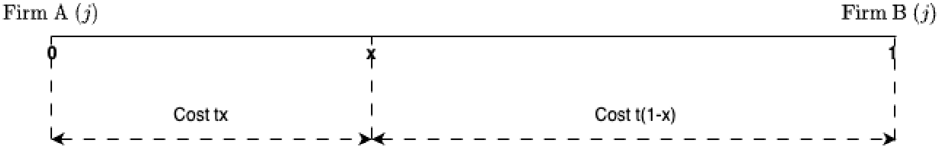

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Households

- Measure-one continuum of households, $i \in [0, 1]$
- Identical, except for their location in the product space of each industry $j \in [0, 1]$
- For every pair industry j - household i, location randomly assigned $x(j, i) \overset{\text{iid}}{\sim} \mathbb{U}[0, 1]$

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Figure 2: Location of household i in the product space of industry j.

Details

Firms

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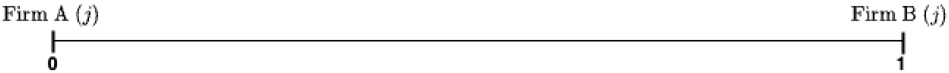

Figure 3: Distribution of the customer base in industry j.

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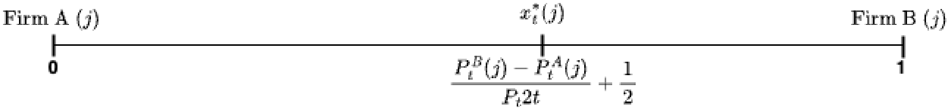

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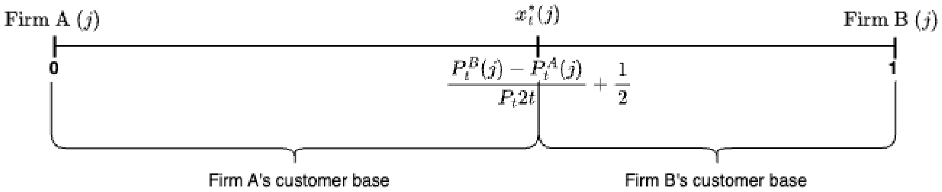

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- Each period, both firms choose their price simulteneously after observing the realization of the shocks.
- While this happens across industries, inflation is realized.
- Best response function: price of the industry competitor, overall inflation rate, marginal cost.

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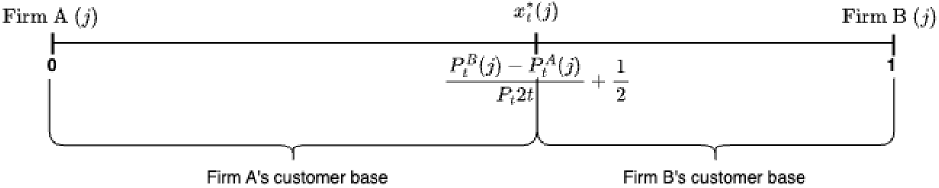

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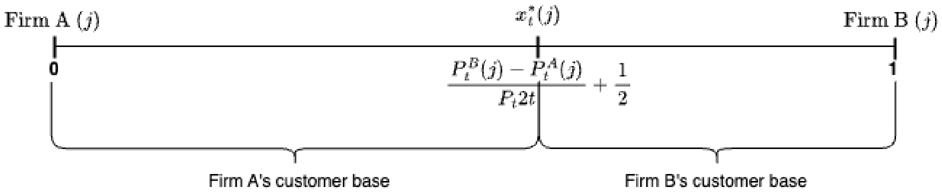

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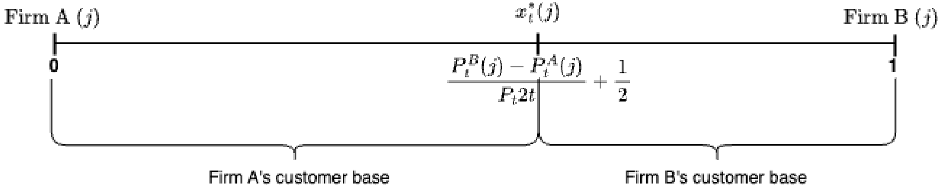

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- Instead, firms form consistent beliefs about how the competitor's price will react to the shocks.
- In particular, level-k thinking.

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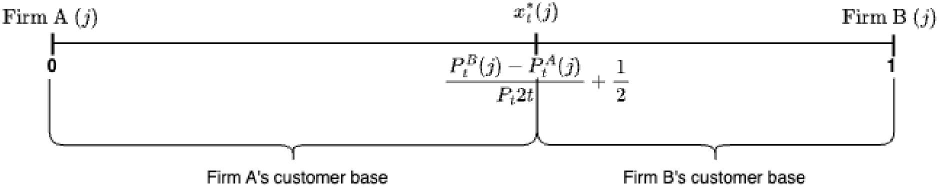

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Profits of Firm A (j) (CRS technology in labor):

$$\Pi_t^A(j) = P_t^A(j) Y_t^A(j) - W_t N_t^A(j) = \left[P_t^A(j) - \left(\frac{W_t}{A_t} \right) \right] Y_t^A(j)$$

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- Raising—ceteris paribus—the price of firm A:
  1. (+) Higher markup per unit.
  2. (−) Firm A's customers will now be purchasing less units.
  3. (−) Completely lose some customers, move away to industry competitor.
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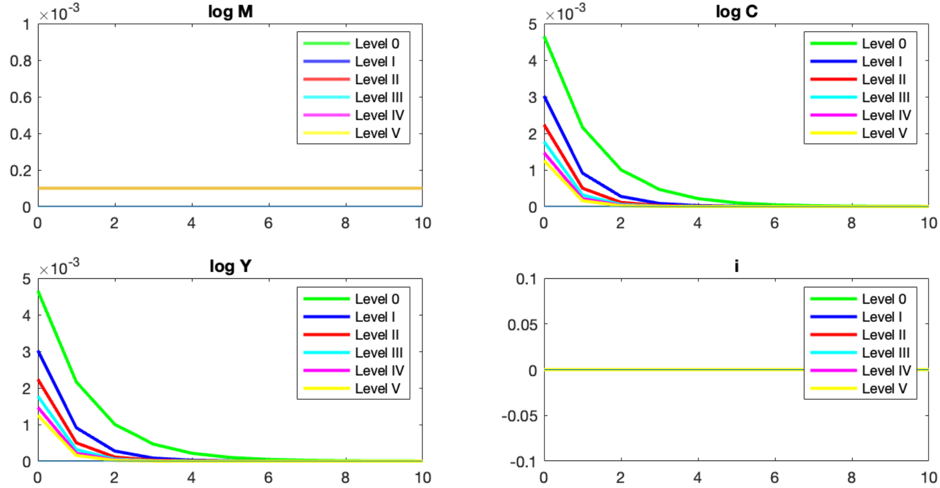
$$\pi_t^A(j) = BR^A\left(\pi_t^B(j), \pi_t, \frac{w_t}{A_t}\right)$$

Details

Outline

Quantitative Analysis



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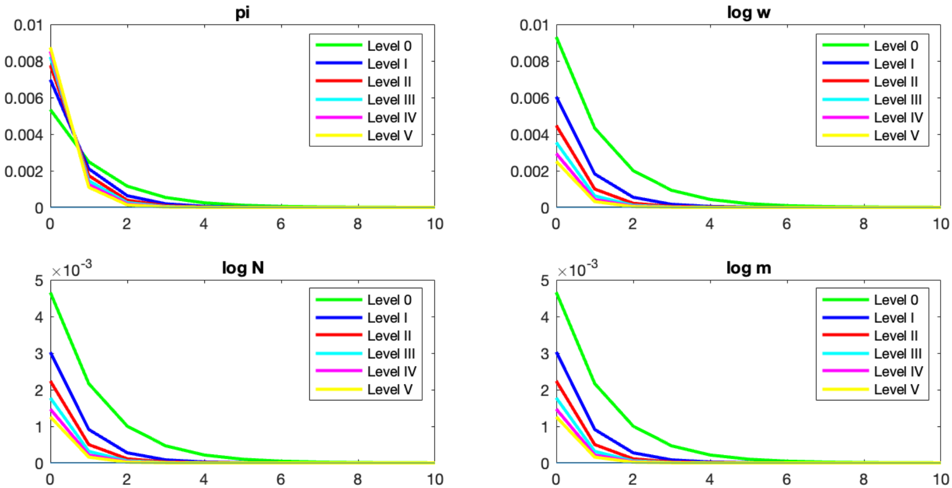

Figure 4: Impulse Responses.

Calibration Alternative calibrations Taylor Rule

Quantitative Analysis

Why are not prices strongly and inmediately rising to the extend of undoing the effect of the shock as in the RBC model?

- Consider the case of Firm A (j) of industry j.
- A positive demand shock will raise the marginal cost.
- However, the marginal cost-price pass-through will depend on how the firm predicts that the industry competitor will react:
  - If firm rises its price unilaterally, part of its customer base will move away to its industry competitor.
  - Therefore, reluctant to rise its price *too much* over the price of the industry competitor.
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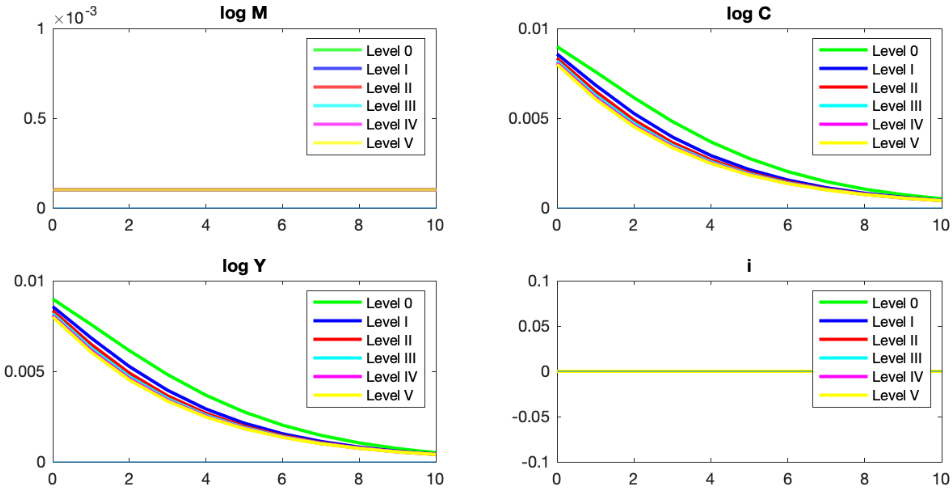
Quantitative Analysis: Wage Inertia



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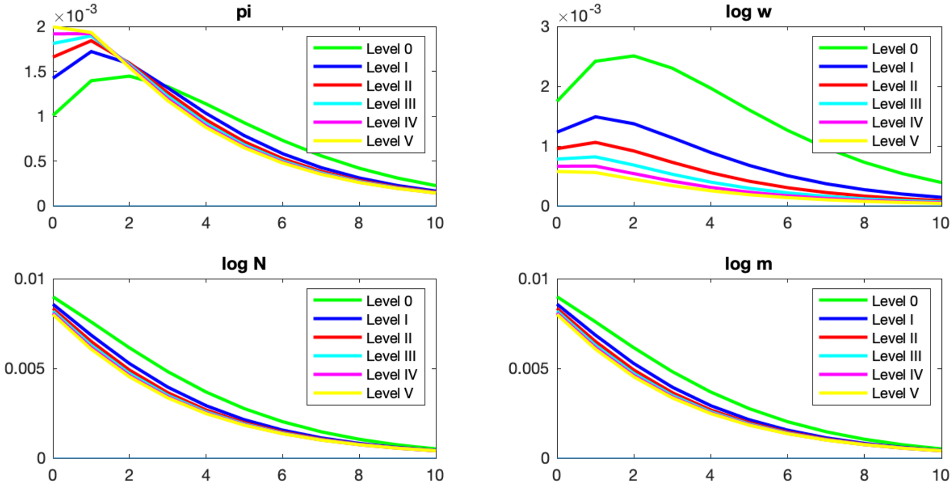

Figure 5: Impulse Responses under Sticky Wages.

Outline

Empirical Foundations

- **Main Mechanism:** incomplete cost-to-price pass-through [Nakamura and Zerom, 2010; Gorodnichenko and Talavera, 2017]

- **Two key assumptions:**

  1. Strategic complementarities [Amiti et al., 2019; Koga et al., 2020]
  2. Bounded strategic reasoning
     - $\rightarrow$ Evolutionary game theory [Mailath, 1998; Samuelson, 2002; Shapiro et al., 2014]
     - $\rightarrow$ Experimental Economics [Nagel, 1995; Bosch-Domenech et al., 2002]

Implications

1. **Monetary/Demand shocks have stronger real effects in markets with higher levels of competition**
    - Empirical support: Duvan et al. (2024), Hong and Li (2017).
    - Policy: if market power is trending upward over time (Autor et al., 2020; De Loecker et al., 2020), effectiveness of monetary policy may be structurally declining.

2. **The real effects of the Monetary/Demand shocks depend on the inflationary environment through the firms' evolving cognitive sophistication**

    → Model: real effects diminish with the level-k of thinking.

    → Experimental economics: repeated exposure to games increase strategic sophistication [Duffy and Nagel, 1997; Cooper et al., 2024]

    - Empirical support: Alvarez et al. (2019), Gagnon (2009), Nakamura et al. (2018).
    - Policy: effectiveness of monetary policy may be limited in economies with a recent history of high-inflation.

Outline

Concluding Remarks

- DSGE model: Horizontally differentiated markets + bounded strategic reasoning.
- Short-run non-neutrality of monetary and demand shocks wrt real variables.
- In contrast to previous literature and leading DSGE: do not impose any sort of nominal rigidities or ad hoc restrictions to agents.
- Instead, price inertia derives as an equilibrium outcome.
- This paper provides microfoundations to the phenomenon of price inertia and short-run non-neutrality.
- Framework reasonably robust for policy evaluation.

This paper abstracts from

Thank You!

Outline

A.1. Optimal Allocation of Consumption Expenditures

For any given household i, optimal behavior requires $C_t(i)$ to be maximized for any given level of expenditures

$$X_t(i) \equiv \int_0^1 \left[P_t^A(j) C_t^A(j,i) + P_t^B(j) C_t^B(j,i) \right] dj + P_t t \int_0^1 \left[x(j,i) C_t^A(j,i) + [1 - x(j,i)] C_t^B(j,i) \right] dj.$$

This first requires the household to optimally choose the firm that minimizes the effective cost per unit (price plus transportation cost) in each industry or, in other words, to purchase from firm A in those industries where $x(j,i) \leq x_t^*(j)$, and from B otherwise.

A.1. Optimal Allocation of Consumption Expenditures

We can define the effective price of a unit of industry good j as

$$\tilde{P}_t(j,i) \equiv [P_t^A(j) + P_t t x(j,i)]\mathbb{1}\{x(j,i) \leq x_t^*(j)\} + [P_t^B(j) + P_t t(1 - x(j,i))]\mathbb{1}\{x(j,i) > x_t^*(j)\},$$

which is the effective cost of a unit of industry good j for household i when behaving optimally.

The solution to the previous problem is then the same as the solution to the problem of maximizing $C_t(i)$ subject to

$$\int_0^1 \tilde{P}_t(j,i)C_t(j,i)dj = X_t(i).$$

A.1. Optimal Allocation of Consumption Expenditures

This can be formalized with the Lagrangian

$$\mathcal{L} = \left(\int_0^1 C_t(j,i)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} + \lambda_t \left[X_t(i) - \int_0^1 \tilde{P}_t(j,i) C_t(j,i) dj \right].$$

The implied first order condition is

$$C_t(i)^{\frac{1}{\epsilon}} C_t(j,i)^{-\frac{1}{\epsilon}} = \lambda_t \tilde{P}_t(j,i)$$

so, for any two industries $j, j' \in [0,1]$, we have that

$$C_t(j,i) = C_t(j',i) \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t(j',i)} \right)^{-\epsilon}.$$

A.1. Optimal Allocation of Consumption Expenditures

This expression can be plugged in the constraint to get

$$C_t(j,i) = \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t(i)}\right)^{-\epsilon} \frac{X_t(i)}{\tilde{P}_t(i)},$$

where $\tilde{P}_t(i) \equiv \left(\int_0^1 \tilde{P}_t(j,i)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$, which can be substituted into the definition of $C_t(i)$ to get that

$$\tilde{P}_t(i) C_t(i) = X_t(i).$$

A.1. Optimal Allocation of Consumption Expenditures

Combining the two previous equations we finally get

$$C_t(j,i) = \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t(i)}\right)^{-\epsilon} C_t(i),$$

which, for a given household i, defines a set of demand equations for all $j \in [0,1]$.

Back to Households

A.2. Homogeneity of the Effective Aggregate Price Index

This appendix shows that $\tilde{P}_t(i) = \tilde{P}_t(i') = \tilde{P}_t$ for any $i, i' \in [0, 1]$.

For a given household i notice that, by definition,

$$\tilde{P}_t(i)^{1-\epsilon} = \int_0^1 \tilde{P}_t(j,i)^{1-\epsilon} dj =$$

$$\int_0^1 \left[[P_t^A(j) + P_t t x(j,i)] \mathbb{1}\{x(j,i) \leq x_t^*(j)\} + [P_t^B(j) + P_t t(1 - x(j,i))] \mathbb{1}\{x(j,i) > x_t^*(j)\} \right]^{1-\epsilon} dj.$$

A.2. Homogeneity of the Effective Aggregate Price Index

- Let me define $\theta \equiv \int_0^1 \mathbb{1}\{x(j,i) \leq x_t^*(j)\}dj$ as the proportion of industries where household i optimally purchases from firm A.
- Notice that, given that each $x(j,i)$ location is iid drawn from a continuous uniform $[0,1]$ distribution, the distribution of locations—for that particular household—across industries $[x(j,i)]_{j\in[0,1]}$ is also continuously uniform along the interval $[0,1]$.

As no particular order matters, we can relabel the j's to break up the integral in the following way:

$$\tilde{P}_t(i)^{1-\epsilon} = \int_0^\theta \left[[P_t^A(j) + P_t t x(j,i)] \right]^{1-\epsilon} dj + \int_\theta^1 \left[[P_t^B(j) + P_t t(1 - x(j,i))] \right]^{1-\epsilon} dj.$$

A.2. Homogeneity of the Effective Aggregate Price Index

- First, the weak law of large numbers immediately gives us that

$$\int_0^\theta P_t^A(j)dj = \theta\mathbb{E}[P_t^A(j)|x \leq x_t^*(j)]$$

  or, in other words, that the integral (sum) of the individual prices of all the industries where household i's location falls to the left of the threshold is proportional to the weighted sum over the entire unit interval, where the weights are the conditional probabilities given the thresholds $x_t^*(j)$ for all $j \in [0,1]$, and where the proportion is equal to the subset of industries where $x(j,i) \leq x_t^*(j)$[1].

- Given that the thresholds $x_t^*(j)$'s are common across households, $\mathbb{E}[P_t^A(j)|x \leq x_t^*(j)]$ is also common across households. Moreover, given iidness and the continuum of industries, so is the proportion $\theta \in [0,1]$.

[1] Actually, the different laws of large numbers do not state equality but different notions of convergence. However, since we have a continuum of industries, we are already in the limit.

A.2. Homogeneity of the Effective Aggregate Price Index

The following result—which is a version of the law of large numbers that does not require an identical distribution but just uncorrelation and finite variance—will be very useful for the rest of this section.

Result 1(Chebyshev's L_2 law of large numbers). *Let X_1, X_2, ... be random variables such that $\mathbb{E}|X_i|^2 < \infty$ for every i. If*

$$\frac{1}{n^2} Var(X_1 + ... + X_n) \xrightarrow[n \to \infty]{} 0,$$

then denoting $S_n = X_1 + ... + X_n$,

$$\frac{S_n}{n} - \mathbb{E}\frac{S_n}{n} \xrightarrow[n \to \infty]{L_2} 0.$$

In particular, this holds when the X_i are uncorrelated with bounded variance, that is $Var(X_i) \leq M$ for every i for some M.

A.2. Homogeneity of the Effective Aggregate Price Index

Proof. We have

$$\mathbb{E}\left|\frac{S_n}{n} - \mathbb{E}\frac{S_n}{n}\right|^2 = \frac{1}{n^2}\mathbb{E}|S_n - \mathbb{E}S_n|^2 = \frac{1}{n^2}Var(X_1 + \ldots + X_n) \xrightarrow[n\to\infty]{} 0.$$

Since

$$Var(X_1 + \ldots + X_n) = \sum_{i=1}^{n} Var(X_i) + 2\sum_{1\leq i<j\leq n} Cov(X_i, X_j),$$

when the X_i are uncorrelated with bounded variance, we have

$$\frac{1}{n^2}Var(X_1 + \ldots + X_n) \leq \frac{Mn}{n^2} = \frac{M}{n}$$

which goes to 0 as $n \to \infty$. $\square$

A.2. Homogeneity of the Effective Aggregate Price Index

Now, for our given household i, consider the set of $x(j,i)$'s across $j \in [0,1]$ such that $x(j,i) \leq x_t^*(j)$.

- We know that, conditional on such a set, $x(j,i) \sim \mathbb{U}[0, x_t(j)]$.
- Therefore, and again conditional on such a set, $Var(x(j,i)) = \frac{1}{12}(x_t^*(j))^2 < \frac{1}{12}$.
- If we randomly selected n variable x(j,i)'s out of that set and call them $X_1, ..., X_n$, we would have that

$$Var(X_1 + ... + X_n) = \sum_{z=1}^{n} Var(X_z) + 2 \sum_{1 \leq z < z' \leq n} Cov(X_z, X_{z'}).$$

But, as these variables are independent, we would have that

$$Var(X_1 + ... + X_n) = \sum_{z=1}^{n} Var(X_z) = \sum_{z=1}^{n} \frac{1}{12}(x_t^*(j))^2 = \frac{1}{12} \sum_{z=1}^{n} (x_t^*(j))^2 < \frac{n}{12}.$$

A.2. Homogeneity of the Effective Aggregate Price Index

Therefore,

$$\frac{1}{n^2} Var(X_1 + ... + X_n) < \frac{1}{n^2}\frac{n}{12} = \frac{1}{12n} \xrightarrow[n\to\infty]{} 0.$$

Since convergence in L_2 implies convergence in probability, the same result would apply in this case, so that

$$\int_0^\theta x(j,i)dj = \theta\mathbb{E}[x(j,i)|x(j,i) \leq x_t^*(j)],$$

which is again a function of the set of $x_t^*(j)$ for all $j \in [0,1]$ and hence common across households.

A.2. Homogeneity of the Effective Aggregate Price Index

The rest of the proof of this section follows from the continuous mapping theorem, which states that, given a sequence of random variables $\{X_n\}_{n=1}^{\infty}$ and X taking values in a metric space $(\mathcal{X}, d)$,

$$X_n \xrightarrow{p} X \implies g(X_n) \xrightarrow{p} g(X),$$

for all bounded continuous functions g.

By the continuous mapping theorem, $\int_0^{\theta} P_t tx(j,i)dj = \theta \mathbb{E}[P_t tx(j,i) | x(j,i) \leq x_t^*(j)]$ (where $g(x) = cx$ for a constant c).

A.2. Homogeneity of the Effective Aggregate Price Index

Applying again the continuous mapping theorem yields

$$\int_0^\theta P_t^A(j) + P_t t x(j,i) dj = \theta \mathbb{E}[P_t^A(j) + P_t t x(j,i) | x(j,i) \leq x_t^*(j)]$$

where $g(x,y) = x + y$ for two variables x and y. And finally,

$$\int_0^\theta [P_t^A(j) + P_t t x(j,i)]^{1-\epsilon} dj = \theta \mathbb{E}\left[[P_t^A(j) + P_t t x(j,i)]^{1-\epsilon} | x(j,i) \leq x_t^*(j) \right],$$

which is again common across households. An equivalent argument follows for the B part of the integral, $\int_\theta^1 [P_t^B(j) + P_t t(1 - x(j,i))]^{1-\epsilon} dj$.

Back to Households

A.3. Derivation of Equation (14)

Given prices, we can write the profits function of firm A from industry j as

$$
\begin{aligned}
\Pi_t^A(j) = \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] Y_t^A(j) &= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] \int_0^{\frac{P_t^B(j)-P_t^A(j)}{P_t 2t}+\frac{1}{2}} C_t(j,x)dx \\
&= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] \int_0^{\frac{P_t^B(j)-P_t^A(j)}{P_t 2t}+\frac{1}{2}} \left(\frac{\tilde{P}_t(j,x)}{\tilde{P}_t(x)}\right)^{-\epsilon} C_t(x)dx \\
&= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] \int_0^{\frac{P_t^B(j)-P_t^A(j)}{P_t 2t}+\frac{1}{2}} \left(\frac{\tilde{P}_t(j,x)}{\tilde{P}_t}\right)^{-\epsilon} C_t dx \\
&= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] \tilde{P}_t^{\epsilon} C_t \int_0^{\frac{P_t^B(j)-P_t^A(j)}{P_t 2t}+\frac{1}{2}} [\tilde{P}_t(j,x)]^{-\epsilon} dx
\end{aligned}
$$

where the third line follows from the fact that the effective aggregate price index is homogeneous across households.

A.3. Derivation of Equation (14)

$$= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right]\tilde{P}_t^{\epsilon} C_t \int_0^{\frac{P_t^B(j)-P_t^A(j)}{P_t 2t}+\frac{1}{2}} [\tilde{P}_t(j,x)]^{-\epsilon} dx$$

$$= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right]\tilde{P}_t^{\epsilon} C_t \int_0^{\frac{P_t^B(j)-P_t^A(j)}{P_t 2t}+\frac{1}{2}} [P_t^A(j) + P_t t x]^{-\epsilon} dx$$

$$= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right]\tilde{P}_t^{\epsilon} C_t \left[\frac{[P_t^A(j) + P_t t x]^{-\epsilon+1}}{(-\epsilon+1)P_t t}\Bigg|_0^{\frac{P_t^B(j)-P_t^A(j)}{P_t 2t}+\frac{1}{2}}\right]$$

$$= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right]\tilde{P}_t^{\epsilon} C_t \frac{1}{(-\epsilon+1)P_t t}$$

$$\left\{\left[P_t^A(j) + P_t t\left(\frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}\right)\right]^{-\epsilon+1} - [P_t^A(j)]^{-\epsilon+1}\right\}.$$

Back to Firms

A.4. $\tilde{\pi}_t$ equation

From the definition and homogeneity of the effective aggregate price index, we have that

$$\tilde{P}_t^{1-\epsilon} = \tilde{P}_t(i)^{1-\epsilon} = \int_0^1 \tilde{P}_t(j,i)^{1-\epsilon} dj,$$

for a given $i \in [0,1]$.

- Given that we are restricting to equilibria where all firms apply the same level-k thinking to the formation of their beliefs,
- and given that—for any given industry $j \in [0,1]$—both firms A and B face a symmetric problem

$P_t^A(j) = P_t^B(j)$ in equilibrium, so $x^*(j) = \frac{1}{2}$.

A.4. $\tilde{\pi}_t$ equation

$P_t^A(j) = P_t^B(j)$ in equilibrium, so $x^*(j) = \frac{1}{2}$.

- From random assignment and the continuum of industries, this leads to the fact that household i will optimally choose to buy from firm A in half of the industries, and from B in the other half.
- Moreover, as locations are drawn from a continuous uniform $[0, 1]$ distribution, the distribution of locations—for that particular household i—across industries $\left[x(j, i)\right]_{j\in[0,1]}$ is also continuously uniform along the interval $[0, 1]$.
- Since the particular order of the $j \in [0, 1]$ index does not matter, one can relabel the j's to be equal to the exact location of household i in such particular industry, and then we can break up the integral of the expression above in the following way

$$\tilde{P}_t^{1-\epsilon} = \tilde{P}_t(i)^{1-\epsilon} = \int_0^{\frac{1}{2}} \left[P_t^A(j) + P_t tj\right]^{1-\epsilon} dj + \int_{\frac{1}{2}}^1 \left[P_t^B(j) + P_t t(1-j)\right]^{1-\epsilon} dj.$$

A.4. $\tilde{\pi}_t$ equation

$$\tilde{P}_t^{1-\epsilon} = \tilde{P}_t(i)^{1-\epsilon} = \int_0^{\frac{1}{2}} \left[P_t^A(j) + P_t tj\right]^{1-\epsilon} dj + \int_{\frac{1}{2}}^1 \left[P_t^B(j) + P_t t(1-j)\right]^{1-\epsilon} dj.$$

Dividing by $P_{t-1}^{1-\epsilon}$ and noticing that—since all industries are identical and beliefs are homogeneous—the optimal prices for firms A and B will be the same across industries in equilibrium, we get that

$$(1+\tilde{\pi}_t)^{1-\epsilon} = \int_0^{\frac{1}{2}} \left[(1+\pi_t^A) + (1+\pi_t)tj\right]^{1-\epsilon} dj + \int_{\frac{1}{2}}^1 \left[(1+\pi_t^B) + (1+\pi_t)t(1-j)\right]^{1-\epsilon} dj,$$

where $\pi_t^A = \pi_t^A(j)$ and $\pi_t^B = \pi_t^B(j)$ for all $j \in [0,1]$.

A.4. $\tilde{\pi}_t$ equation

The two components of this equation can be written as

$$
\int_0^{\frac{1}{2}} \left[(1+\pi_t^A)+(1+\pi_t)tj\right]^{1-\epsilon} dj = \left.\frac{\left[(1+\pi_t^A)+(1+\pi_t)tj\right]^{2-\epsilon}}{(2-\epsilon)(1+\pi_t)t}\right|_0^{\frac{1}{2}}
$$

$$
= \frac{\left[(1+\pi_t^A)+(1+\pi_t)t\frac{1}{2}\right]^{2-\epsilon}}{(2-\epsilon)(1+\pi_t)t} - \frac{(1+\pi_t^A)^{2-\epsilon}}{(2-\epsilon)(1+\pi_t)t},
$$

and

$$
\int_{\frac{1}{2}}^{1} \left[(1+\pi_t^B)+(1+\pi_t)t(1-j)\right]^{1-\epsilon} dj = \left.\frac{\left[(1+\pi_t^B)+(1+\pi_t)t(1-j)\right]^{2-\epsilon}}{(2-\epsilon)(-(1+\pi_t)t)}\right|_{\frac{1}{2}}^{1}
$$

$$
= \frac{\left[(1+\pi_t^B)+(1+\pi_t)t\frac{1}{2}\right]^{2-\epsilon}}{(2-\epsilon)(1+\pi_t)t} - \frac{(1+\pi_t^B)^{2-\epsilon}}{(2-\epsilon)(1+\pi_t)t},
$$

A.4. $\tilde{\pi}_t$ equation

so the $\tilde{\pi}_t$ equation reduces to

$$(1+\tilde{\pi}_t)^{1-\epsilon} = \frac{\left[(1+\pi_t^A)+(1+\pi_t)t\frac{1}{2}\right]^{2-\epsilon} + \left[(1+\pi_t^B)+(1+\pi_t)t\frac{1}{2}\right]^{2-\epsilon}}{(2-\epsilon)(1+\pi_t)t} - \frac{(1+\pi_t^A)^{2-\epsilon}+(1+\pi_t^B)^{2-\epsilon}}{(2-\epsilon)(1+\pi_t)t}.$$

Back to Aggregation

A.4. π_t equation

Recall the definition of the aggregate price index

$$P_t Y_t \equiv \int_0^1 P_t^A(j) Y_t^A(j) + P_t^B(j) Y_t^B(j) dj,$$

and consider a particular industry $j \in [0, 1]$.

- Since both firms choose the same price in equilibrium, the market will be equally split and $x_t^*(j) = \frac{1}{2}$.
- Since households are uniformly distributed across the product space of industry j, and since the particular order of the $i \in [0, 1]$ index does not matter, one can relabel the i's to be equal to the exact location of each household in the $[0, 1]$ interval of industry j.

A.4. π_t equation

Then, we can compute $Y_t^A(j)$ of industry j as

$$
\begin{aligned}
Y_t^A(j) &= \int_0^{\frac{1}{2}} \frac{1+\tilde{\pi}_t}{1+\pi_t} C_t(j,i)di = \int_0^{\frac{1}{2}} \frac{1+\tilde{\pi}_t}{1+\pi_t} \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t(i)}\right)^{-\epsilon} C_t(i)di \\
&= \int_0^{\frac{1}{2}} \frac{1+\tilde{\pi}_t}{1+\pi_t} \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t}\right)^{-\epsilon} C_t di = \int_0^{\frac{1}{2}} \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t}\right)^{-\epsilon} Y_t di \\
&= \int_0^{\frac{1}{2}} \tilde{P}_t(j,i)^{-\epsilon} \tilde{P}_t^{\epsilon} Y_t di = \tilde{P}_t^{\epsilon} Y_t \int_0^{\frac{1}{2}} \tilde{P}_t(j,i)^{-\epsilon} di = \tilde{P}_t^{\epsilon} Y_t \int_0^{\frac{1}{2}} \left[P_t^A + P_t ti\right]^{-\epsilon} di \\
&= \tilde{P}_t^{\epsilon} Y_t \left[\frac{[P_t^A + P_t ti]^{-\epsilon+1}}{(-\epsilon+1)P_t t} \Bigg|_0^{\frac{1}{2}} \right] = \tilde{P}_t^{\epsilon} Y_t \frac{[P_t^A + P_t t\frac{1}{2}]^{-\epsilon+1} - [P_t^A]^{-\epsilon+1}}{(-\epsilon+1)P_t t},
\end{aligned}
$$

A.4. π_t equation

and we can compute $Y_t^B(j)$ of industry j as

$$
\begin{aligned}
Y_t^B(j) &= \int_{\frac{1}{2}}^{1} \frac{1+\tilde{\pi}_t}{1+\pi_t} C_t(j,i)di = \int_{\frac{1}{2}}^{1} \frac{1+\tilde{\pi}_t}{1+\pi_t} \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t(i)} \right)^{-\epsilon} C_t(i)di \\
&= \int_{\frac{1}{2}}^{1} \frac{1+\tilde{\pi}_t}{1+\pi_t} \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t} \right)^{-\epsilon} C_t di = \int_{\frac{1}{2}}^{1} \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t} \right)^{-\epsilon} Y_t di \\
&= \int_{\frac{1}{2}}^{1} \tilde{P}_t(j,i)^{-\epsilon} \tilde{P}_t^{\epsilon} Y_t di = \tilde{P}_t^{\epsilon} Y_t \int_{\frac{1}{2}}^{1} \tilde{P}_t(j,i)^{-\epsilon} di = \tilde{P}_t^{\epsilon} Y_t \int_{\frac{1}{2}}^{1} \left[P_t^B + P_t t(1-i) \right]^{-\epsilon} di \\
&= \tilde{P}_t^{\epsilon} Y_t \left[\frac{[P_t^B + P_t t(1-i)]^{-\epsilon+1}}{(-\epsilon+1)(-P_t t)} \Bigg|_{\frac{1}{2}}^{1} \right] = \tilde{P}_t^{\epsilon} Y_t \frac{[P_t^B + P_t t \frac{1}{2}]^{-\epsilon+1} - [P_t^B]^{-\epsilon+1}}{(-\epsilon+1) P_t t}.
\end{aligned}
$$

A.4. π_t equation

Plugging both expressions in the definition of the aggregate price index and noticing that, in equilibrium, both prices and quantities are homogeneous across industries, we get

$$P_t Y_t = P_t^A \tilde{P}_t^{\epsilon} Y_t \frac{[P_t^A + P_t t \frac{1}{2}]^{-\epsilon+1} - [P_t^A]^{-\epsilon+1}}{(-\epsilon+1) P_t t} + P_t^B \tilde{P}_t^{\epsilon} Y_t \frac{[P_t^B + P_t t \frac{1}{2}]^{-\epsilon+1} - [P_t^B]^{-\epsilon+1}}{(-\epsilon+1) P_t t},$$

where $P_t^A = P_t^A(j)$ and $P_t^B = P_t^B(j)$ for all $j \in [0,1]$. This equation can be written in terms of inflation rates as

$$\begin{aligned}(1+\pi_t) = {} & (1+\pi_t^A)\left(\frac{(1+\tilde{\pi}_t)}{(1+\pi_t)}\right)^{\epsilon} \frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t}\right) + \frac{t}{2}\right]^{-\epsilon+1} - \left(\frac{1+\pi_t^A}{1+\pi_t}\right)^{-\epsilon+1}}{(-\epsilon+1)t} \\ & + (1+\pi_t^B)\left(\frac{(1+\tilde{\pi}_t)}{(1+\pi_t)}\right)^{\epsilon} \frac{\left[\left(\frac{1+\pi_t^B}{1+\pi_t}\right) + \frac{t}{2}\right]^{-\epsilon+1} - \left(\frac{1+\pi_t^B}{1+\pi_t}\right)^{-\epsilon+1}}{(-\epsilon+1)t}.\end{aligned}$$

Back to Aggregation

A.4. Aggregate Production Function

Recall that the production function of firm A from industry j is $Y_t^A(j) = A_t N_t^A(j)$. From labor market clearing we have that

$$\begin{aligned} A_t N_t = A_t \int_0^1 N_t(j)dj = \int_0^1 A_t N_t(j)dj = \int_0^1 A_t[N_t^A(j) + N_t^B(j)]dj \\ = \int_0^1 A_t N_t^A(j)dj + \int_0^1 A_t N_t^B(j)dj = \int_0^1 Y_t^A(j)dj + \int_0^1 Y_t^B(j)dj. \end{aligned}$$

A.4. Aggregate Production Function

Using the results from the previous section we get that

$$A_t N_t = \tilde{P}_t^{\epsilon} Y_t \left\{ \frac{[P_t^A + P_t t \frac{1}{2}]^{-\epsilon+1} - [P_t^A]^{-\epsilon+1}}{(-\epsilon+1) P_t t} + \frac{[P_t^B + P_t t \frac{1}{2}]^{-\epsilon+1} - [P_t^B]^{-\epsilon+1}}{(-\epsilon+1) P_t t} \right\},$$

which can be written as

$$A_t N_t = Y_t \left(\frac{1+\tilde{\pi}_t}{1+\pi_t} \right)^{\epsilon} \left\{ \frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t} \right) + \frac{t}{2} \right]^{-\epsilon+1} + \left[\left(\frac{1+\pi_t^B}{1+\pi_t} \right) + \frac{t}{2} \right]^{-\epsilon+1}}{(-\epsilon+1) t} \right.$$
$$\left. - \frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t} \right) \right]^{-\epsilon+1} + \left[\left(\frac{1+\pi_t^B}{1+\pi_t} \right) \right]^{-\epsilon+1}}{(-\epsilon+1) t} \right\}.$$

Understanding the Aggregate Production Function | Back to Aggregation

A.4. $\pi_t = \pi_t^A = \pi_t^B$

Claim: $\pi_t = \pi_t^A = \pi_t^B$

- I have argued that, in equilibrium, $P_t^A(j) = P_t^B(j)$ for any industry $j \in [0,1]$.
- Moreover, the fact that all industries are alike leads to $\pi_t^A(j) = \pi_t^A$ and $\pi_t^B(j) = \pi_t^B$ for any $j \in [0,1]$.
- Therefore, $\pi_t^A = \pi_t^B$.

This section shows that $\pi_t = \pi_t^A = \pi_t^B$.

To show this, we can depart from the $\tilde{\pi}_t$ equation, which reduces to

$$(1+\tilde{\pi}_t)^{1-\epsilon} = 2\frac{\left[(1+\pi_t^A) + (1+\pi_t)t\frac{1}{2}\right]^{2-\epsilon} - (1+\pi_t^A)^{2-\epsilon}}{(2-\epsilon)(1+\pi_t)t}$$

A.4. $\pi_t = \pi_t^A = \pi_t^B$

After applying the fact that $\pi_t^A = \pi_t^B$. This equation can be written as

$$\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right)^{-\epsilon+1} = 2\frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t}\right)+\frac{t}{2}\right]^{2-\epsilon} - \left(\frac{1+\pi_t^A}{1+\pi_t}\right)^{2-\epsilon}}{(2-\epsilon)t},$$

which is true for all $\epsilon > 2$. Let's make the change of variables $\epsilon = y + 1$, so

$$\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right)^{-y} = 2\frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t}\right)+\frac{t}{2}\right]^{1-y} - \left(\frac{1+\pi_t^A}{1+\pi_t}\right)^{1-y}}{(1-y)t},$$

which is true for all $y > 1$.

A.4. $\pi_t = \pi_t^A = \pi_t^B$

But this implies that

$$\frac{1}{2} = \left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right)^y \frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t}\right) + \frac{t}{2}\right]^{1-y} - \left(\frac{1+\pi_t^A}{1+\pi_t}\right)^{1-y}}{(1-y)t}$$

for all $y > 1$. Now, recall the π_t equation

$$(1+\pi_t) = (1+\pi_t^A)\left(\frac{(1+\tilde{\pi}_t)}{(1+\pi_t)}\right)^{\epsilon} \frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t}\right) + \frac{t}{2}\right]^{-\epsilon+1} - \left(\frac{1+\pi_t^A}{1+\pi_t}\right)^{-\epsilon+1}}{(-\epsilon+1)t}$$
$$+(1+\pi_t^B)\left(\frac{(1+\tilde{\pi}_t)}{(1+\pi_t)}\right)^{\epsilon} \frac{\left[\left(\frac{1+\pi_t^B}{1+\pi_t}\right) + \frac{t}{2}\right]^{-\epsilon+1} - \left(\frac{1+\pi_t^B}{1+\pi_t}\right)^{-\epsilon+1}}{(-\epsilon+1)t}.$$

A.4. $\pi_t = \pi_t^A = \pi_t^B$

Combining the two previous results we get that

$$(1+\pi_t) = (1+\pi_t^A)\frac{1}{2} + (1+\pi_t^B)\frac{1}{2},$$

so $\pi_t = \pi_t^A = \pi_t^B$. Notice that this is true regardless of the specific beliefs of the firms, as long as these beliefs are homogeneous across firms so that they end up choosing the same price in equilibrium. $\square$

Back to Understanding Production Function

A.4. Understanding the Resource Constraint

The previous section shows that

$$\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right)^{\epsilon} \frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t}\right)+\frac{t}{2}\right]^{1-\epsilon}-\left(\frac{1+\pi_t^A}{1+\pi_t}\right)^{1-\epsilon}}{(1-\epsilon)t}=\frac{1}{2}.$$

Applying the fact that $\pi_t = \pi_t^A = \pi_t^B$, the equation reduces to

$$\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right)=\left[2\frac{\left[1+\frac{t}{2}\right]^{-\epsilon+1}-1}{(-\epsilon+1)t}\right]^{-\frac{1}{\epsilon}},$$

which is the term that multiplies consumption in the resource constraint and is a function of t.

This section explores the properties of this term. In particular, and given that $t > 0$, I analyze what happens as t goes to zero, and I compute the partial derivative with respect to t.

A.4. Understanding the Resource Constraint: when $t \to 0$

First, let me define

$$g(t) \equiv 2\frac{[1+\frac{t}{2}]^{-\epsilon+1}-1}{(-\epsilon+1)t}$$

and let us calculate the limit of $g(t)$ as t goes to zero with the application of L'Hopital's rule as

$$\lim_{t\to 0} g(t) = \lim_{t\to 0} 2\frac{[1+\frac{t}{2}]^{-\epsilon+1}-1}{(-\epsilon+1)t} = \lim_{t\to 0} 2\frac{(-\epsilon+1)[1+\frac{t}{2}]^{-\epsilon}\frac{1}{2}}{(-\epsilon+1)} = 1.$$

Therefore,

$$\lim_{t\to 0}\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right) = \lim_{t\to 0}[g(t)]^{-\frac{1}{\epsilon}} = 1.$$

A.4. Understanding the Resource Constraint: first derivative

Second, let us analyze the partial derivative of $\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right)$ with respect to t

$$\frac{\partial\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right)}{\partial t} = \left(-\frac{1}{\epsilon}\right)\left[2\frac{\left[1+\frac{t}{2}\right]^{-\epsilon+1}-1}{(-\epsilon+1)t}\right]^{-\frac{1}{\epsilon}-1} 2$$

$$* \frac{(-\epsilon+1)^2[1+\frac{t}{2}]^{-\epsilon}\frac{t}{2} - (-\epsilon+1)[(1+\frac{t}{2})^{-\epsilon+1}-1]}{(-\epsilon+1)^2 t^2}.$$

It is easy to see that this term will be positive if, and only if,

$$(-\epsilon+1)\left[\left(1+\frac{t}{2}\right)^{-\epsilon+1}-1\right] > (-\epsilon+1)^2\left[1+\frac{t}{2}\right]^{-\epsilon}\frac{t}{2}$$

$$\iff \left(1+\frac{t}{2}\right)^{-\epsilon}\left[1+\frac{t}{2}-(-\epsilon+1)\frac{t}{2}\right] < 1.$$

A.4. Understanding the Resource Constraint: first derivative

Let me define $f(t) \equiv \left(1+\frac{t}{2}\right)^{-\epsilon}\left[1+\frac{t}{2}-(-\epsilon+1)\frac{t}{2}\right]$. Notice that $f(0)=1$, and

$$
\begin{aligned}
\frac{\partial f(t)}{\partial t} &= -\epsilon\left(1+\frac{t}{2}\right)^{-\epsilon-1}\left(\frac{1}{2}\right)\left[1+\frac{t}{2}-(-\epsilon+1)\frac{t}{2}\right]+\left(1+\frac{t}{2}\right)^{-\epsilon}\left[\frac{1}{2}-\frac{1}{2}(-\epsilon+1)\right] \\
&= \left(1+\frac{t}{2}\right)^{-\epsilon}\left(\frac{\epsilon}{2}\right)\frac{(-\epsilon+1)t}{2+t} < 0,
\end{aligned}
$$

as $\epsilon > 2$. Therefore, $f(t) < 1$ for all $t > 0$, so $\frac{\partial\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right)}{\partial t} > 0$.

A.4. Understanding the Resource Constraint: second derivative

One can also show that the second derivative is negative, although that involves a significant amount of algebra. Given that the expression has only one parameter, ϵ, figure 6 shows a graphical representation for the particular case where this parameter is, say, $\epsilon = 6$.

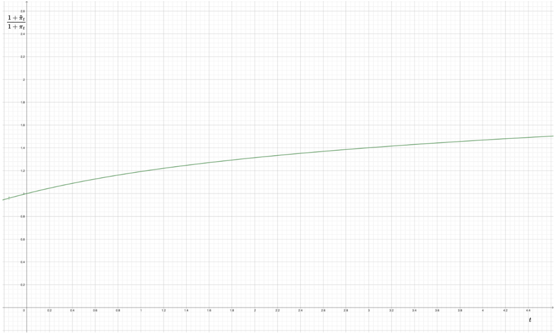

Figure 6: $\frac{1+\tilde{\pi}_t}{1+\pi_t}$ as a function of t.

Back to Aggregation

A.4. Understanding the Aggregate Production Function

The aggregate production function, as calculated above, is

$$A_t N_t = Y_t \left(\frac{1 + \tilde{\pi}_t}{1 + \pi_t} \right)^{\epsilon} \left\{ \frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t} \right) + \frac{t}{2} \right]^{-\epsilon+1} + \left[\left(\frac{1+\pi_t^B}{1+\pi_t} \right) + \frac{t}{2} \right]^{-\epsilon+1}}{(-\epsilon + 1)t} - \frac{\left[\left(\frac{1+\pi_t^A}{1+\pi_t} \right) \right]^{-\epsilon+1} + \left[\left(\frac{1+\pi_t^B}{1+\pi_t} \right) \right]^{-\epsilon+1}}{(-\epsilon + 1)t} \right\}.$$

A.4. Understanding the Aggregate Production Function

Applying the fact that, in equilibrium, $\pi_t = \pi_t^A = \pi_t^B$ we get

$$A_t N_t = Y_t \left(\frac{1+\tilde{\pi}_t}{1+\pi_t} \right)^{\epsilon} 2 \frac{\left(1+\frac{t}{2}\right)^{-\epsilon+1} - 1}{(-\epsilon+1)t},$$

but in the previous section we found that

$$\left(\frac{1+\tilde{\pi}_t}{1+\pi_t} \right) = \left[2 \frac{\left[1+\frac{t}{2}\right]^{-\epsilon+1} - 1}{(-\epsilon+1)t} \right]^{-\frac{1}{\epsilon}},$$

so the aggregate production function reduces to

$$A_t N_t = Y_t.$$

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Households

The Lagrangian for household i would then be

$$\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\sigma}}{1-\sigma} - \Psi\frac{N_t^{1+\eta}}{1+\eta} + \theta\ln\left(\frac{M_t}{P_t}\right) + \right.$$
$$\left.\lambda_t\left(M_{t-1} + W_t N_t + \Pi_t + (1+i_{t-1}^n)B_t - P_t T_t - \tilde{P}_t C_t - B_{t+1} - M_t\right)\right],$$

Households

which yields the intra-temporal (labor/leisure) optimality condition

$$\Psi N_t^{\eta} = C_t^{-\sigma} \frac{W_t}{\tilde{P}_t},$$

the inter-temporal (consumption/saving) optimality condition

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left[C_{t+1}^{-\sigma} (1 + i_t^n) \frac{\tilde{P}_t}{\tilde{P}_{t+1}} \right],$$

and the optimality condition for money (money demand)

$$\theta \left(\frac{M_t}{\tilde{P}_t} \right)^{-1} = C_t^{-\sigma} \frac{i_t^n}{1 + i_t^n}.$$

Back to Households

Firms: FOC

We can then take the first order condition to get the best response function as a function of prices

$$\left\{\left[P_t^A(j) + P_t t\left(\frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}\right)\right]^{-\epsilon+1} - [P_t^A(j)]^{-\epsilon+1}\right\}$$
$$= (\epsilon - 1)\left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right]$$
$$\left\{\left[P_t^A(j) + P_t t\left(\frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}\right)\right]^{-\epsilon}\frac{1}{2} - [P_t^A(j)]^{-\epsilon}\right\},$$

Firms: FOC

which can be written in terms of inflation rates as

$$\left\{\left[(1+\pi_t^A(j)) + (1+\pi_t)t\left(\frac{\pi_t^B(j)-\pi_t^A(j)}{(1+\pi_t)2t}+\frac{1}{2}\right)\right]^{-\epsilon+1} - [1+\pi_t^A(j)]^{-\epsilon+1}\right\}$$

$$= (\epsilon-1)\left[(1+\pi_t^A(j)) - \left(\frac{w_t}{A_t}(1+\pi_t)\right)\right]$$

$$\left\{\left[(1+\pi_t^A(j)) + (1+\pi_t)t\left(\frac{\pi_t^B(j)-\pi_t^A(j)}{(1+\pi_t)2t}+\frac{1}{2}\right)\right]^{-\epsilon}\frac{1}{2} - [1+\pi_t^A(j)]^{-\epsilon}\right\},$$

where the real wage is defined as $w_t \equiv \frac{W_t}{P_t}$, and which implicitly defines the best response function of firm A, $BR^A\left(\pi_t^B(j), \pi_t, \frac{w_t}{A_t}\right)$, as a function of the price inflation of the industry competitor $\pi_t^B(j)$, the overall inflation π_t, and the real marginal cost $\frac{w_t}{A_t}$.

Back to Firms

Firms: Firm B

Following the same argument as before, one can show that firm B's best response to its beliefs about the price inflation of the industry competitor and the overall inflation is implicitly defined from the expression

$$
\begin{aligned}
&\left\{[1+\pi_t^B(j)]^{-\epsilon+1} - \left[(1+\pi_t^B(j)) + (1+\mathcal{B}_{\pi_t})t\left(\frac{1}{2} - \frac{\pi_t^B(j) - \mathcal{B}_{\pi_t^A(j)}}{(1+\mathcal{B}_{\pi_t})2t}\right)\right]^{-\epsilon+1}\right\} \\
&\qquad = (\epsilon-1)\left[(1+\pi_t^B(j)) - \left(\frac{w_t}{A_t}(1+\mathcal{B}_{\pi_t})\right)\right] \\
&\left\{[1+\pi_t^B(j)]^{-\epsilon} - \left[(1+\pi_t^B(j)) + (1+\mathcal{B}_{\pi_t})t\left(\frac{1}{2} - \frac{\pi_t^B(j) - \mathcal{B}_{\pi_t^A(j)}}{(1+\mathcal{B}_{\pi_t})2t}\right)\right]^{-\epsilon}\frac{1}{2}\right\}.
\end{aligned}
$$

Back to Firms

Resource Constraint

Integrating the budget constraint across all households and applying the fiscal rule, the fact that bonds are in zero net supply, and the results from optimal allocation of consumption expenditures, we get

$$\int_0^1 \tilde{P}_t(i) C_t(i) di = W_t N_t + \Pi_t,$$

which, given that the effective price index is homogeneous across households, reduces to

$$\tilde{P}_t C_t = W_t N_t + \Pi_t.$$

Resource Constraint

Total profits are the aggregation of the profits of all firms across industries

$$
\begin{aligned}
\Pi_t &= \int_0^1 \pi_t^A(j) + \pi_t^B(j) dj \\
&= \int_0^1 [P_t^A(j) Y_t^A(j) - W_t N_t^A(j)] + [P_t^B(j) Y_t^B(j) - W_t N_t^B(j)] dj \\
&= \int_0^1 P_t^A(j) Y_t^A(j) + P_t^B(j) Y_t^B(j) dj - W_t \int_0^1 N_t(j) dj \\
&= \int_0^1 P_t^A(j) Y_t^A(j) + P_t^B(j) Y_t^B(j) dj - W_t N_t,
\end{aligned}
$$

where $N_t(j) \equiv N_t^A(j) + N_t^B(j)$ for any $j \in [0, 1]$, and where labor market clearing implies $N_t = \int_0^1 N_t(j) dj$.

Resource Constraint

Combining both equations we get

$$\tilde{P}_t C_t = \int_0^1 P_t^A(j) Y_t^A(j) + P_t^B(j) Y_t^B(j) dj, \tag{1}$$

which, from the definition of the aggregate price index, reduces to

$$\tilde{P}_t C_t = P_t Y_t,$$

or

$$\frac{1+\tilde{\pi}_t}{1+\pi_t} C_t = Y_t, \tag{2}$$

which is the resource constraint of this economy.

Back to Aggregation
Understanding Resource Constraint

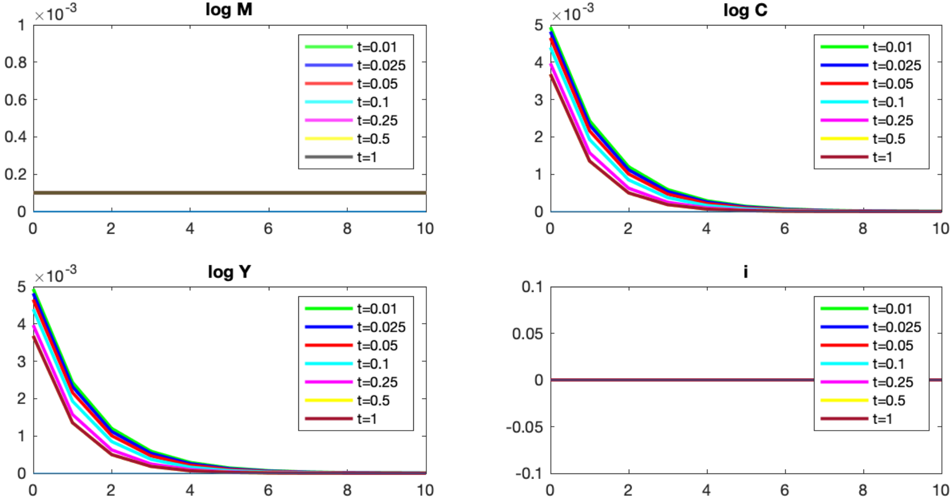
This paper abstracts from:

1. An alternative way to introduce wage inertia would be to model the labor market as in Christiano et al. (2016).
2. No capital and investment, but easy to introduce them.
3. No learning.
4. Take the degree of market competition as given.
5. No micro-level infrequency of price adjustment.

Back to Conclusions

Outline

B.1. Alternative Calibrations: different t's



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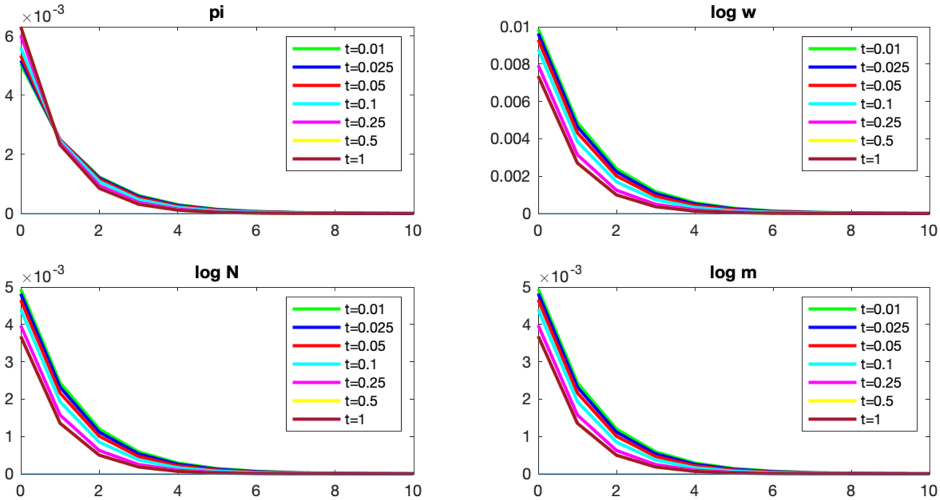

Figure 7: Impulse Responses for different t calibrations and level-0 thinking.

Back to Quantitative Analysis

B.2. Sticky Wages

- Sticky wages in a standard way, as in Erceg et al. (2000)
- Households supply differenciated labor input that is then "packed" into a bundled labor input that is sold to firms.

B.2. Sticky Wages

The profit maximization problem of a *labor packer* that aggregates labor with a CES function—with elasticity of substitution $\epsilon_w > 1$—produces the following demand for labor

$$N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_w} N_t,$$

where

$$N_t = \left(\int_0^1 N_t(i)^{\frac{\epsilon_w - 1}{\epsilon_w}} di\right)^{\frac{\epsilon_w}{\epsilon_w - 1}},$$

and

$$W_t = \left(\int_0^1 W_t(i)^{1-\epsilon_w} di\right)^{\frac{1}{1-\epsilon_w}}.$$

B.2. Sticky Wages

- The problem of the households is slightly different because they are now supplying differentiated labor and setting their own wage.
- The optimal allocation of consumption expenditures result and the homogeneity of the effective aggregate price index result still hold without changes.
- Moreover, the fact that preferences are separable in consumption and leisure and markets are complete, allows us to apply the Erceg et al. (2000) result so that households will only differ in the wage they charge and their labor supply.
- The rest of derivations are standard and, hence, omited.

B.2. Sticky Wages

The problem of a household that chooses its own wage to maximize utility subject to labor demand, and subject to the Calvo assumption, results in the wage setting equation

$$w_t^{\#,\epsilon_w \eta+1} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{H_{1,t}}{H_{2,t}},$$

where $w_t^{\#}$ is the reset wage, and where

$$H_{1,t} = \Psi w_t^{\epsilon_w(1+\eta)} N_t^{1+\eta} + (\beta\phi_w)\mathbb{E}_t(1+\pi_{t+1})^{\epsilon_w(1+\eta)} H_{1,t+1},$$

and

$$H_{2,t} = C_t^{-\sigma} \frac{1+\pi_t}{1+\tilde{\pi}_t} w_t^{\epsilon_w} N_t + (\beta\phi_w)\mathbb{E}_t(1+\pi_{t+1})^{\epsilon_w - 1} H_{2,t+1}.$$

B.2. Sticky Wages

Lastly, in equilibrium,

$$w_t^{1-\epsilon_w} = (1-\phi_w) w_t^{\#,1-\epsilon_w} + \phi_w w_{t-1}^{1-\epsilon_w} (1+\pi_t)^{\epsilon_w - 1}.$$

The resource constraint does not change and the rest of equilibrium results also hold.

Outline

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1. There is ample evidence that demand shocks affect output, and a
2. "widely held belief (certaintly among central bankers) in the power of [monetary] policy to influence output and employment developments, at least in the short run" (Galí, 2015).

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- Large scale macro models based on behavioral equations and ad hoc assumptions

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- Theoretical predictions vs evidence.
- Normative implications vs policy practice.

1. Menu cost models (e.g. Caplin and Spulber, 1987; Golosov and Lucas, 2007).
2. Rational inattention models (e.g. Mankiw and Reis, 2002; Reis, 2006).
3. New Keynesian Framework (e.g. Smets and Wouters, 2003 and 2007; Christiano et al., 2005).

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Introduction

New Keynesian Framework

- Rests on the DSGE structure characteristic of RBC models.
- Major workhorse for policy evaluation.
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  - Depends on the pivotal role of the respective exogenous device that imposes nominal rigidities.
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This Paper: Industry Structure

Measure-one continuum of industries, $j \in [0, 1]$

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Figure 8: An industry j.

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- Consumers uniformly distributed.
- Each periods, both firms choose their price simultaneously after observing realization shocks.
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- Results: large and persistent short-run non-neutrality to monetary and demand shocks.

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  - Therefore, reluctant to rise its price *too much* over the price of the industry competitor.
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- However, marginal cost-price pass-through shaped by firm competition:
  - If firm rises its price unilaterally, part of its customer base will move away to its industry competitor.
  - Therefore, reluctant to rise its price *too much* over the price of the industry competitor.
  - Firm only willing to rise its price significantly when the competitor is doing something similar, so not as negatively perceived by its customer base.

- Incorporating wage inertia $\uparrow$ size and duration, $+$ more robust to high levels of thinking.

Back to intro

Equilibrium and Aggregation

- Process for A_t :

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_{a,t},$$

  where $\rho_a \in (0,1)$, and $\epsilon_{a,t}$ is a productivity shock.

- For simplicity, restrict to equilibria where all firms same level-k thinking.

- Define the aggregate price index P_t as

$$P_t Y_t \equiv \int_0^1 P_t^A(j) Y_t^A(j) + P_t^B(j) Y_t^B(j) dj,$$

  where aggregate output is

$$Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Equilibrium and Aggregation

Resource constraint of this economy:

$$\frac{1+\tilde{\pi}_t}{1+\pi_t} C_t = Y_t,$$

where, in equilibrium,

$$\left(\frac{1+\tilde{\pi}_t}{1+\pi_t}\right) = \left[2\frac{\left[1+\frac{t}{2}\right]^{-\epsilon+1} - 1}{(-\epsilon+1)t}\right]^{-\frac{1}{\epsilon}}.$$

The aggregate production function is:

$$A_t N_t = Y_t.$$

Resource Constraint | Understanding Resource Constraint | $\tilde{\pi}$ equation | π equation | Aggregate Production Function | Back to government

Quantitative Analysis

Calibration

$\sigma = \Psi = \eta = \theta = 1$	Parameters utility function
$\beta = 0.98$	Subjective discount factor
$\epsilon = 6$	Elasticity of substitution between differentiated industry goods
$\pi_{ss} = 0$	Zero inflation steady state
$t = 0.05$	Transportation cost per unit of distance

Shock

- Recall $\Delta \ln M_t = (1 - \rho_m)\pi_{ss} + \rho_m \Delta \ln M_{t-1} + \epsilon_{m,t}$
- Set $\rho_m = 0$
- Set $\sigma_m = 0.01$
- Carry out six independent exercises: economies that differ in the level of thinking.

Back to Quantitative Analysis

Quantitative Analysis: Taylor Rule

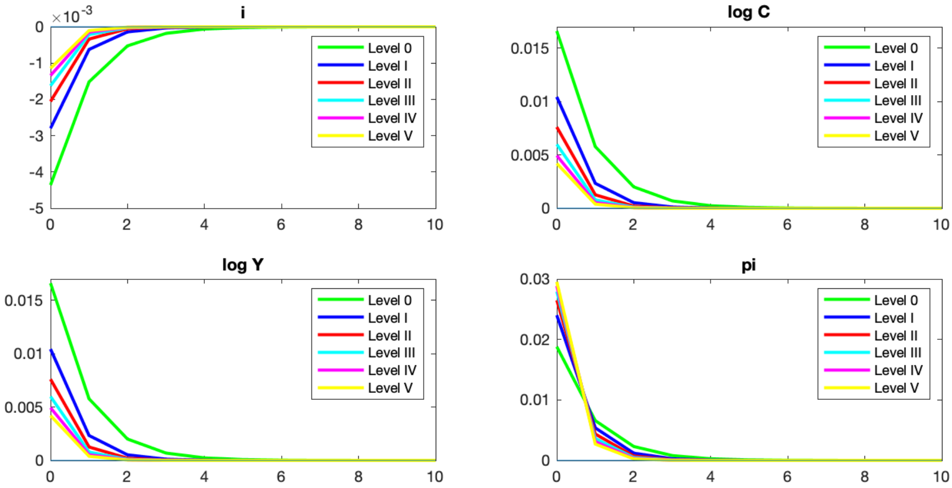
Substitute the money supply process for the following Taylor rule:

$$i_t = (1-\rho_i) i_{ss} + \rho_i i_{t-1} + (1-\rho_i)\phi_\pi(\pi_t - \pi_{ss}) + \epsilon_i,$$

where $i_{ss} = \frac{1+\pi_{ss}}{\beta} - 1$ is the steady state nominal interest rate, and ϵ_i is a monetary policy shock.

- Set $\rho_i = 0.8$, $\phi_\pi = 1.5$. $\sigma_i = 0.01$.
- Expansionary monetary policy shock. As before, six independent exercises.

Quantitative Analysis: Taylor Rule



Quantitative Analysis: Taylor Rule

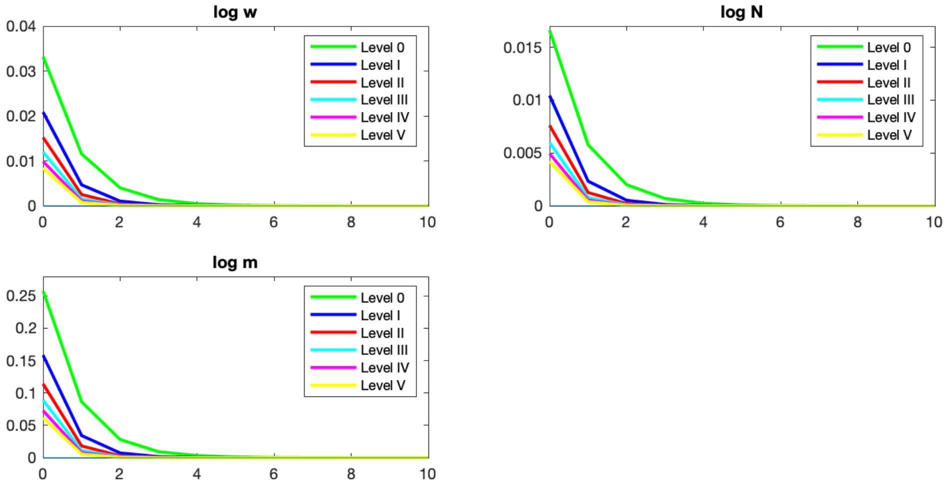


Figure 9: Impulse Responses under a Taylor rule.

Back to Quantitative Analysis

Households

Optimal consumer behavior also yields the set of demand equations

$$C_t(j,i) = \left(\frac{\tilde{P}_t(j,i)}{\tilde{P}_t(i)}\right)^{-\epsilon} C_t(i) \qquad \text{for all } j \in [0,1]$$

where

$$\tilde{P}_t(j,i) \equiv [P_t^A(j) + P_t t x(j,i)]\mathbb{1}\{x(j,i) \leq x_t^*(j)\} + [P_t^B(j) + P_t t(1 - x(j,i))]\mathbb{1}\{x(j,i) > x_t^*(j)\}$$

is the effective price (or cost) of a unit of industry good j optimally purchased by household i,

and where

$$\tilde{P}_t(i) \equiv \left(\int_0^1 \tilde{P}_t(j,i)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$

is the *effective* aggregate price index. Proof

Households

Moreover,

$$\tilde{P}_t(i)C_t(i) = \int_0^1 \left[P_t^A(j)C_t^A(j,i) + P_t^B(j)C_t^B(j,i)\right]dj$$
$$+P_t t \int_0^1 \left[x(j,i)C_t^A(j,i) + [1 - x(j,i)]C_t^B(j,i)\right]dj,$$

which allows us to write the budget constraint of the household as

$$\tilde{P}_t(i)C_t(i) + B_{t+1}(i) + M_t(i) {=} M_{t-1}(i) + W_t N_t(i) + \Pi_t + (1 + i_{t-1}^n)B_t(i) - P_t T_t.$$

Lastly, as a result of the continuum of industries and random assignment assumptions, $\tilde{P}_t(i) = \tilde{P}_t(i') = \tilde{P}_t$ for any $i, i' \in [0,1]$. Proof Back to households, main

Outline

Introduction

1. There is ample evidence that demand shocks affect output, and a
2. "widely held belief (certaintly among central bankers) in the power of [monetary] policy to influence output and employment developments, at least in the short run" (Galí, 2015).

More intro

Households

A particular household i seeks to maximize the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t(i)^{1-\sigma}}{1-\sigma} - \Psi \frac{N_t(i)^{1+\eta}}{1+\eta} + \theta \ln \left(\frac{M_t(i)}{P_t} \right) \right]$$

where

$$C_t(i) = \left(\int_0^1 C_t(j,i)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

and $C_t(j,i) = C_t^A(j,i) + C_t^B(j,i)$.

Households

The period budget constraint is

$$\int_0^1 \left[P_t^A(j)C_t^A(j,i) + P_t^B(j)C_t^B(j,i)\right]dj +$$

$$P_t t \int_0^1 \left[x(j,i)C_t^A(j,i) + [1 - x(j,i)]C_t^B(j,i)\right]dj +$$

$$B_{t+1}(i) + M_t(i) = M_{t-1}(i) + W_t N_t(i) + \Pi_t + (1 + i_{t-1}^n)B_t(i) - P_t T_t$$

Households

The period budget constraint is

$$\underbrace{\int_0^1 \left[P_t^A(j)C_t^A(j,i) + P_t^B(j)C_t^B(j,i)\right] dj}_{\text{total cost of the industry goods}} +$$

$$P_t t \int_0^1 \left[x(j,i)C_t^A(j,i) + [1 - x(j,i)]C_t^B(j,i)\right] dj +$$

$$B_{t+1}(i) + M_t(i) = M_{t-1}(i) + W_t N_t(i) + \Pi_t + (1 + i_{t-1}^n)B_t(i) - P_t T_t$$

Households

The period budget constraint is

$$\underbrace{\int_0^1 \left[P_t^A(j)C_t^A(j,i) + P_t^B(j)C_t^B(j,i)\right]dj}_{\text{total cost of the industry goods}} +$$

$$\underbrace{P_t t \int_0^1 \left[x(j,i)C_t^A(j,i) + [1 - x(j,i)]C_t^B(j,i)\right]dj}_{\text{total transportation costs}} +$$

$$B_{t+1}(i) + M_t(i) = M_{t-1}(i) + W_t N_t(i) + \Pi_t + (1 + i_{t-1}^n)B_t(i) - P_t T_t$$

Households

Given a pair industry j - household i, with location $x(j,i) \in [0,1]$:

- Effective cost per unit when purchasing from firm A: $P_t^A(j) + P_t t x(j,i)$
- Effective cost per unit when purchasing from firm B: $P_t^B(j) + P_t t(1 - x(j,i))$
- Optimal consumer behavior defines the threshold
- Households located to the left, $x(j,i) \leq x_t^*(j) \Rightarrow$ Firm A
- Households located to the right, $x(j,i) > x_t^*(j) \Rightarrow$ Firm B

Households

Given a pair industry j - household i, with location $x(j,i) \in [0,1]$:

- Effective cost per unit when purchasing from firm A: $P_t^A(j) + P_t t x(j,i)$
- Effective cost per unit when purchasing from firm B: $P_t^B(j) + P_t t(1 - x(j,i))$
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Households

Given a pair industry j - household i, with location $x(j,i) \in [0,1]$:

- Effective cost per unit when purchasing from firm A: $P_t^A(j) + P_t t x(j,i)$
- Effective cost per unit when purchasing from firm B: $P_t^B(j) + P_t t(1 - x(j,i))$
- Optimal consumer behavior defines the threshold

$$x_t^*(j) = \frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}$$

- Households located to the left, $x(j,i) \leq x_t^*(j) \Rightarrow$ Firm A
- Households located to the right, $x(j,i) > x_t^*(j) \Rightarrow$ Firm B

Households

Given a pair industry j - household i, with location $x(j,i) \in [0,1]$:

- Effective cost per unit when purchasing from firm A: $P_t^A(j) + P_t t x(j,i)$
- Effective cost per unit when purchasing from firm B: $P_t^B(j) + P_t t(1 - x(j,i))$
- Optimal consumer behavior defines the threshold

$$x_t^*(j) = \frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}$$

- Households located to the left, $x(j,i) \leq x_t^*(j) \Rightarrow$ Firm A
- Households located to the right, $x(j,i) > x_t^*(j) \Rightarrow$ Firm B

Households

Under optimal consumer behavior, the problem of the household reduces to

$$\max_{C_t(i),N_t(i),B_{t+1}(i),M_t(i)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t(i)^{1-\sigma}}{1-\sigma} - \Psi \frac{N_t(i)^{1+\eta}}{1+\eta} + \theta \ln\left(\frac{M_t(i)}{P_t}\right) \right]$$

$$s.t.\ \tilde{P}_t C_t(i) + B_{t+1}(i) + M_t(i) = M_{t-1}(i) + W_t N_t(i) + \Pi_t + (1+i^n_{t-1}) B_t(i) - P_t T_t.$$

- However, all households face same problem with same set of prices.
- Drop the i index, back to the representative household framework.
- Beauty and simplicity of this setting:
  1. Different households will distribute consumption spending differently across and within industries.
  2. But same overall consumption/saving and labor/leisure choices. Optimality Conditions

Details Back

Firms

Constant returns to scale technology in labor with a common productivity shock A_t:

$$Y_t^A(j) = A_t N_t^A(j)$$

and

$$Y_t^B(j) = A_t N_t^B(j).$$

- Labor is homogeneous across industries.
- Marginal cost: $\frac{W_t}{A_t}$
- Firm A will optimally choose $P_t^A(j)$ and $N_t^A(j)$ s.t. producing enough to meet demand.

Firms

Given prices, we can write the profit function of Firm A (j) as

$$\Pi_t^A(j) = \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] Y_t^A(j) = \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] \int_0^{\frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}} C_t(j,x)dx$$

$$= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] \tilde{P}_t^{\epsilon} C_t \frac{1}{(-\epsilon+1)P_t t}$$

$$\left\{\left[P_t^A(j) + P_t t\left(\frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}\right)\right]^{-\epsilon+1} - [P_t^A(j)]^{-\epsilon+1}\right\},$$

Profits Function

Firms

Given prices, we can write the profit function of Firm A (j) as

$$\Pi_t^A(j) = \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] Y_t^A(j) = \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] \int_0^{\frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}} C_t(j,x)dx$$

$$= \left[P_t^A(j) - \left(\frac{W_t}{A_t}\right)\right] \tilde{P}_t^\epsilon C_t \frac{1}{(-\epsilon+1)P_t t}$$

$$\left\{\left[P_t^A(j) + P_t t\left(\frac{P_t^B(j) - P_t^A(j)}{P_t 2t} + \frac{1}{2}\right)\right]^{-\epsilon+1} - [P_t^A(j)]^{-\epsilon+1}\right\},$$

which implicitly defines a best response function that can be written as

$$\pi_t^A(j) = BR^A\left(\pi_t^B(j), \pi_t, \frac{w_t}{A_t}\right)$$

Profits Function FOC

Firms

Firm A (j) forms beliefs about:

- Price inflation of the industry competitor, $\mathcal{B}_{\pi_t^B(j)}$.
- Overall inflation, $\mathcal{B}_{\pi_t}$.
- Level-k thinking.

Firms

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Firms

Firm A (j) forms beliefs about:

- Price inflation of the industry competitor, $\mathcal{B}_{\pi_t^B(j)}$.
- Overall inflation, $\mathcal{B}_{\pi_t}$.
- Level-k thinking.

Level-0 thinking:

$$\mathcal{B}^0_{\pi_t^B(j)} \equiv \pi_{ss}^B(j),$$

and

$$\mathcal{B}^0_{\pi_t} \equiv \pi_{ss}.$$

Level-k thinking:

$$\mathcal{B}^k_{\pi_t^B(j)} \equiv \mathcal{B}^k_{\pi_t} \equiv BR^B\left(\mathcal{B}^{k-1}_{\pi_t^A(j)}, \mathcal{B}^{k-1}_{\pi_t}, \frac{w_t}{A_t}\right).$$

Firms

Firm A's best response to its beliefs about the price inflation of the competitor and the overall inflation—$\mathcal{B}_{\pi_t^B(j)}$ and $\mathcal{B}_{\pi_t}$—would then be $BR^A\left(\mathcal{B}_{\pi_t^B(j)}, \mathcal{B}_{\pi_t}, \frac{w_t}{A_t}\right)$, implicitly defined from

$$\left\{\left[(1+\pi_t^A(j)) + (1+\mathcal{B}_{\pi_t})t\left(\frac{\mathcal{B}_{\pi_t^B(j)} - \pi_t^A(j)}{(1+\mathcal{B}_{\pi_t})2t} + \frac{1}{2}\right)\right]^{-\epsilon+1} - [1+\pi_t^A(j)]^{-\epsilon+1}\right\}$$
$$= (\epsilon-1)\left[(1+\pi_t^A(j)) - \left(\frac{w_t}{A_t}(1+\mathcal{B}_{\pi_t})\right)\right]$$
$$\left\{\left[(1+\pi_t^A(j)) + (1+\mathcal{B}_{\pi_t})t\left(\frac{\mathcal{B}_{\pi_t^B(j)} - \pi_t^A(j)}{(1+\mathcal{B}_{\pi_t})2t} + \frac{1}{2}\right)\right]^{-\epsilon}\frac{1}{2} - [1+\pi_t^A(j)]^{-\epsilon}\right\}.$$

Firm B Back to Firms

Government

The government in this economy consists on:

1. A monetary policy rule that determines M_t:

$$\Delta \ln M_t = (1 - \rho_m)\pi_{ss} + \rho_m \Delta \ln M_{t-1} + \epsilon_{m,t},$$

where $\Delta \ln M_t \equiv \ln M_t - \ln M_{t-1}$, $\rho_m \in (0, 1)$, and $\epsilon_{m,t}$ is a money supply shock.

2. A fiscal rule that determines T_t:

$$T_t = -\frac{M_t - M_{t-1}}{P_t}.$$

Equilibrium and Aggregation

Quantitative Analysis: Sticky Wages

Main mechanism of this model $\Rightarrow$ incomplete marginal cost-price pass-through

- Hence, natural to wonder how the model behaves under more realistic wage dynamics:
  - ~~Christiano et al. (2016) derive wage inertia from wage negotiation~~.
  - Instead—with the aim of maximizing simplicity and clarity—standard Calvo sticky wages as in Erceg et al (2000).

- Results:
  1. Interaction of wage inertia with the incomplete wage-price pass-through mechanism $\Longrightarrow$ much more sluggish behavior of prices.
  2. Reponses of all real variables roughly double both in size and duration.
  3. Much more robust to high levels of thinking.