# Competition and Strategic Incentives in the Market for Credit Ratings: Empirics of the Financial Crisis of 2007

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## ONLINE APPENDIX

## APPENDIX C Alternative specification with shadow ratings specifying subordinate securities

The baseline model in the main text assumes that bidders' "shadow rating" specifies the subordination amount for the AAA security, based on the salience of this figure as an institutional feature of CMBS. In this appendix, we consider an alternative assumption under which the shadow rating stipulates subordination amounts for all of the securities, both AAA and non-AAA.

For tractability, we make an assumption that allows us to continue regarding bids as one-dimensional (as in the baseline case). Namely, we assume that the loss distribution on the pool principal is drawn from a stochastically ordered family of distributions (assumed to be common knowledge). A "bid" can be thought of as the bidder's assertion about the specific distribution of losses, with the stochastic-ordering assumption allowing bids to be ranked along a single dimension.

As in the baseline case, in order for the model to be identified, we must normalize the scaling of the bids to something observable. For this purpose, we define a measure of how favorably an entire deal is rated overall—the weighted-average "rating-implied yield spread" (WARIS). Let $\zeta_r$ be the proportion of securities rated $r$. For a set of ratings $1 \dots R$:

$$WARIS \quad = \quad \textstyle\sum_r^R \zeta_r \cdot RIS_r,$$

The constant $RIS_r$ proxies for the level of risk associated with rating $r$. Following Efing and Hau (2015), we define $RIS_r$ to be the fixed effect associated with rating $r$ in a regression of observed bond yields at issuance on various deal- and security characteristics.[35] We take as given that a lower WARIS is more favorable to the issuer—conceptually, a lower WARIS implies a lower cost of funding due to lower yields that must be paid to investors.

Consider the problem of minimizing the WARIS over alternative choices of $\zeta_r$ given some set of constraints on the permissible loss rate for each rating. The stochastic-ordering assumption implies that a bid can equivalently be thought of as stipulating the lowest feasible value of WARIS given such constraints.[36]

The final deal structure is determined in the following manner. Recall that in the baseline case, the final AAA share is determined by the $K$'th lowest winning bid, for an auction with $K$ winners. Analogously, here the issuer minimizes the WARIS with respect to $\{proportion\ of\ securities\ rated\ r\}_{1,\dots R}$, subject to the constraints stipulated by each of the $K$ winning bidders.

[35]The regression is (yield spread at issuance$_{ir}$) $= \alpha' W_i + \beta' Z_{ir} + RIS_t + \varepsilon_{ir}$, where $W_i$ are observed deal characteristics, $Z_{ir}$ are observed characteristics of securities rated $r$, and the dependent variable is the yield premium on securities from deal $i$ that have rating $r$ at the time of issuance. Data on security yields are from Commercial Mortgage Alert.

[36]For example, according to the agencies' published investor guidelines, "AAA", "A1" and "B1" have idealized loss rates over four years of 0.001 percent, 0.104 percent, and 7.6175 percent, respectively. See "Probability of Default Ratings and Loss Given Default Assessments for Non-Financial Speculative-Grade Corporate Obligors in the United States and Canada," Moody's Investors Service (August 2006), Appendix 1 (available online).

We can specify bounds on the bidders' actual bids, based on the observed ratings of the winning bidders. Let $WARIS^*_{ij}$ denote the WARIS on deal $i$ computed using bidder $j$'s observed ratings.[37] For convenience, we transform $WARIS^*_{ij}$ by a known, monotone-decreasing, normalizing function $q(\cdot)$, in order to have a higher bid correspond to a more favorable outcome for the issuer (as in the baseline model).[38]

- Suppose, there is a single observed winner and, without loss of generality, let this be bidder 1. By the definition of a bid, $b_{i1} \geq q(WARIS^*_{i1})$. Moreover, by the optimality of the issuer's behavior, this inequality must hold strictly: $b_{i1} = q(WARIS^*_{i1})$. Otherwise, the issuer could structure the deal in a way that achieves a lower WARIS while still complying with bidder 1's requirements.
- Suppose there are two observed winners and, without loss of generality, let these be bidders 1 and 2. By the definition of a bid, $b_{i1} \geq q(WARIS^*_{i1})$ and $b_{i2} \geq q(WARIS^*_{i2})$. Moreover, the optimality of the issuer's behavior implies that one of these inequalities must hold strictly: otherwise the issuer could structure the deal in a way that achieves a lower WARIS while still complying with both bidders' requirements. Also, bidder 3 not being a winner implies $b_{i3} \leq \min\{b_{i1}, b_{i2}\}$.
- When all three agencies win, it must be the case that $b_{i1} \geq q(WARIS^*_{i1})$, $b_{i2} \geq q(WARIS^*_{i2})$, and $b_{i3} \geq q(WARIS^*_{i3})$. Almost surely, exactly one of these inequalities must also hold strictly.

We perform the first-step estimation by maximizing a likelihood function that is similar to expression (9), but conditioning on the observed values of $WARIS^*_{ij}$ and the above inequalities, instead of the share of AAA.

The structural estimation is similar to the baseline case. First, for each auction $i$ and each winning bidder $j$, we use bidder $j$'s optimality condition to solve for the belief that would rationalize bidder $i$'s bidding $b^*_{ij}$, which we denote by $\xi_{ij}(b^*_{ij}, x_i, w_i, z_{ij}; \theta, \beta)$. As in the baseline specification, we then compute moment conditions based on the expectation of the pivotal bidder's belief, $\hat{u}^*_i$, similar to Equation (10).

[37]Note that, in general, $WARIS^*_{ij} \neq WARIS^*_{ij'}$ for two bidders $j$ and $j'$. This is the case due to the existence of "split ratings"—where the observed rating differs across rating agencies—for securities other than the AAA one.

[38]We define $q(\cdot)$ it to be an affine transformation such that $q(WARIS^*_{ij})$ has the same mean and standard deviation across deals $i$ and bidders $j$ as the observed AAA share $b^*_i$.

## APPENDIX D Estimates for Additional Specifications

Tables D.1 and D.2 report the first-step estimates corresponding to the "Alternative Specification" reported in Table 3).
Tables D.3 and D.4 report the first-step estimates corresponding to the robustness check involving bids that specify the structure of both AAA and non-AAA securities, reported in Table 5.
Tables D.5 and D.6 report the post-estimation regressions of ex post deal outcomes on the ordinal distortion measure implied by the robustness checks in Sections VII.A and VII.B, respectively.
Tables D.7, D.8, and D.9 report the first-step and structural estimates for another specification in which we endogenize the number of winning bidders. To accomplish this, we modify the issuer's maximization (Equation 1) as follows[39]:

$$\max_{d,K}\left[\left(\min\left\{B(d): d\in\{0,1\}^J, \sum_{j=1}^{J} d_j = K\right\}\right)\left(1+\kappa_2 \mathbb{1}\left\{K>1\right\}+\kappa_3 \mathbb{1}\left\{K=3\right\}\right)\right]$$

The objective function from Equation 1 is in the first parenthesis. The term in the second parenthesis is new. This equation implies the issuer maximizes the pivotal bid times a function of how many ratings the issuer obtains. This modified specification captures the following tradeoff faced by the issuer. On the one hand, choosing fewer winners increases the AAA proportion, which equals the $K$'th highest bid when the issuer chooses $K$ winners. On the other hand, investors may place a premium on deals with more ratings—either because they value corroborating opinions or because they are sophisticated and recognize issuers' incentive to ratings shop. We do not explicitly model investor demand, but rather, specify that the issuer's payoff depends in an exogenous way on the number of bids, with $\kappa_2$ representing the premium for having two ratings versus only one, and $\kappa_3$ representing the premium for having three ratings versus two.

In principle, the issuer's premia on having at least two ratings ($\kappa_2$) or three ratings ($\kappa_3$) are identified by the relative frequency of auctions for which there are one, two, or three winners, and can be estimated jointly with the remaining first-step parameters. However, because the $z_{ij}$'s for losing bidders are known only in distribution, we have only weak identification of $\kappa_2$ and $\kappa_3$ separately from the degree of correlation in the bids (determined by the first-step parameters $\Omega$ and $\{\gamma\}_{j=1,2,3}$).[40] Intuitively, both greater correlation in the bids (which compresses the various order statistics of the bid profile) and a greater issuer premium on having more ratings would tend to result in more winners being selected. To

[39]This modification changes the relationship between an agency's bid and the probability of winning, which we account for in computing the expectation terms in the first-order condition (4).

[40]The extent of correlation in the bids is determined by the covariance matrix $\Omega$ and by the magnitude of the coefficients for the bidder-deal-specific covariates relative to the magnitude of those for deal-specific covariates among the elements of the parameter vector $\{\gamma\}_{j=1,2,3}$.

finesse this issue, we do not attempt to estimate $\kappa_2$ and $\kappa_3$. Rather, we fix their values at a level that is *higher* than seems reasonable (5 percent and 2.5 percent respectively)—which maximally alters the likelihood function for the remaining parameters, relative to the base specification in which the number of winners is exogenous. Therefore, if the number of winners were truly endogenous, the difference between these estimates and the base specification estimates would "bound" the impact of erroneously assuming an exogenous number of winners. In fact, we do not find any qualitative differences between the estimates that endogenize the number of winners and the base specification.

Table D.1—First-step estimates (alternative specification): distribution of presale-report variables

**Means ($\mu_z$) relative to sample means**

| | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|
| | DSCR | LTV | DSCR | LTV | DSCR | LTV |
| Estimate | 1.350 | 0.928 | 1.083 | 0.885 | 1.288 | 0.861 |
| Std Error | 0.043 | 0.007 | 0.025 | 0.010 | 0.027 | 0.008 |

**Covariances ($\Omega_z$)**

| | | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|---|
| | | DSCR | LTV | DSCR | LTV | DSCR | LTV |
| S&P | DSCR | 0.145 | | | | | |
| | LTV | -0.008 | 0.011 | | | | |
| Moody's | DSCR | 0.013 | -0.003 | 0.091 | | | |
| | LTV | 0.005 | 0.011 | -0.008 | 0.018 | | |
| Fitch | DSCR | 0.069 | -0.006 | 0.042 | -0.002 | 0.062 | |
| | LTV | 0.004 | 0.011 | -0.007 | 0.016 | -0.004 | 0.017 |

**Standard errors of covariance**

| | | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|---|
| | | DSCR | LTV | DSCR | LTV | DSCR | LTV |
| S&P | DSCR | 0.007 | | | | | |
| | LTV | 0.004 | 0.001 | | | | |
| Moody's | DSCR | 0.006 | 0.006 | 0.013 | | | |
| | LTV | 0.005 | 0.002 | 0.004 | 0.002 | | |
| Fitch | DSCR | 0.005 | 0.003 | 0.005 | 0.003 | 0.003 | |
| | LTV | 0.005 | 0.002 | 0.004 | 0.002 | 0.003 | 0.002 |

Tables D.1 and D.2 report maximum likelihood estimates for the first-step parameters of the "Alternative Specification" (whose structural parameter estimates are in Table 3). All of the first-step parameters are jointly estimated but are reported in two separate tables due to space considerations. Table D.1 shows the estimated joint distribution of the weighted-average reunderwritten DSCR and LTV for each agency (at the deal-level), assuming joint normality.

Table D.2—First-step estimates (alternative specification): bid functions

**Sieve parameters ($\{\gamma_j\}_{j=1,2,3}$)**

| **Bidder-deal-specific covariates** | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|
| | Estimate | Std Error | Estimate | Std Error | Estimate | Std Error |
| Bidder fixed effect | 1.034 | 0.099 | 0.950 | 0.123 | 1.077 | 0.098 |
| Reunderwritten DSCR | 0.222 | 0.078 | 0.499 | 0.091 | 0.007 | 0.045 |
| Reunderwritten LTV | -0.207 | 0.123 | 0.247 | 0.126 | -0.267 | 0.102 |
| Bidder produced no pre-sale report | 0.165 | 0.066 | 0.229 | 0.089 | 0.189 | 0.083 |
| Own share of last 10 deals by different bank | 0.032 | 0.046 | 0.076 | 0.068 | 0.028 | 0.040 |
| Avg. competitors' share of last 10 deals by different bank | 0.030 | 0.077 | 0.101 | 0.102 | -0.064 | 0.080 |

| **Deal covariates** | Estimate | Std Error |
|---|---|---|
| Balloon payment (wtd avg) | -0.303 | 0.079 |
| Cross-collateralization (wtd avg) | -0.032 | 0.037 |
| Deal size (total principal) | 0.008 | 0.012 |
| Originator HHI | -0.088 | 0.032 |
| Property type HHI | -0.383 | 0.076 |
| Region HHI | -0.134 | 0.055 |
| 2001 vintage | 0.216 | 0.036 |
| 2002 vintage | 0.284 | 0.032 |
| 2003 vintage | 0.383 | 0.036 |
| 2004 vintage | 0.484 | 0.038 |
| 2005 vintage | 0.580 | 0.037 |
| 2006 vintage | 0.623 | 0.036 |
| 2007 vintage | 0.601 | 0.045 |
| 2010-2012 vintages | 0.254 | 0.047 |

**Covariance of residual ($\Omega$), point estimates**

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.020 | | |
| Moody's | 0.019 | 0.020 | |
| Fitch | 0.021 | 0.021 | 0.024 |

**Standard errors of covariance of residual**

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.011 | | |
| Moody's | 0.008 | 0.012 | |
| Fitch | 0.013 | 0.012 | 0.026 |

*Note:* Tables D.1 and D.2 report maximum likelihood estimates for the first-step parameters of the "Alternative Specification" (whose structural parameter estimates are in Table 3). All of the first-step parameters are jointly estimated but are reported in two separate tables due to space considerations. Table D.2 shows the estimated equilibrium bidding behavior. The sieve parameters capture the effect of covariates on bidding behavior for individual agencies. The covariance parameters capture the joint distribution of the component of agencies' bids that is not explained by covariates.

Table D.3—First-step estimates for robustness check with bids specifying structure of both AAA and non-AAA securities: distribution of presale-report variables

**Means ($\boldsymbol{\mu_z}$) relative to sample means**

| | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|
| | DSCR | LTV | DSCR | LTV | DSCR | LTV |
| Estimate | 1.441 | 0.919 | 1.207 | 0.885 | 1.248 | 0.860 |
| Std Error | 0.038 | 0.007 | 0.026 | 0.009 | 0.026 | 0.009 |

**Covariances ($\boldsymbol{\Omega_z}$)**

| | | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|---|
| | | DSCR | LTV | DSCR | LTV | DSCR | LTV |
| S&P | DSCR | 0.127 | | | | | |
| | LTV | -0.005 | 0.011 | | | | |
| Moody's | DSCR | 0.051 | -0.007 | 0.076 | | | |
| | LTV | 0.006 | 0.011 | -0.008 | 0.018 | | |
| Fitch | DSCR | 0.067 | -0.005 | 0.045 | -0.003 | 0.062 | |
| | LTV | 0.005 | 0.011 | -0.008 | 0.016 | -0.004 | 0.017 |

**Standard errors of covariance**

| | | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|---|
| | | DSCR | LTV | DSCR | LTV | DSCR | LTV |
| S&P | DSCR | 0.005 | | | | | |
| | LTV | 0.003 | 0.001 | | | | |
| Moody's | DSCR | 0.004 | 0.002 | 0.008 | | | |
| | LTV | 0.004 | 0.002 | 0.003 | 0.002 | | |
| Fitch | DSCR | 0.005 | 0.003 | 0.005 | 0.003 | 0.003 | |
| | LTV | 0.004 | 0.002 | 0.003 | 0.002 | 0.003 | 0.002 |

*Note:* Tables D.3 and D.4 report maximum likelihood estimates for the first-step parameters of the robustness check with bids specifying structure of both AAA and non-AAA securities: distribution of presale-report variables (whose structural parameter estimates are in Table 5). All of the first-step parameters are jointly estimated but are reported in two separate tables due to space considerations. Table D.3 shows the estimated joint distribution of the weighted-average reunderwritten DSCR and LTV for each agency (at the deal-level), assuming joint normality.

TABLE D.4—FIRST-STEP ESTIMATES FOR ROBUSTNESS CHECK WITH BIDS SPECIFYING STRUCTURE OF BOTH AAA AND NON-AAA SECURITIES: BID FUNCTIONS

**Sieve parameters ($\{\gamma_j\}_{j=1,2,3}$)**

| **Bidder-deal-specific covariates** | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|
| | Estimate | Std Error | Estimate | Std Error | Estimate | Std Error |
| Bidder fixed effect | 1.349 | 0.083 | 1.377 | 0.077 | 1.376 | 0.086 |
| Reunderwritten DSCR | 0.002 | 0.033 | 0.006 | 0.039 | 0.009 | 0.047 |
| Reunderwritten LTV | -0.045 | 0.069 | -0.016 | 0.065 | -0.034 | 0.063 |
| Bidder produced no pre-sale report | 0.038 | 0.019 | 0.030 | 0.009 | 0.023 | 0.019 |
| Own share of last 10 deals | 0.005 | 0.045 | -0.035 | 0.043 | -0.026 | 0.042 |
| Competitors' share of last 10 deals | -0.079 | 0.081 | -0.091 | 0.080 | -0.073 | 0.083 |
| Own share of last 3 deals by same bank | 0.002 | 0.024 | 0.018 | 0.025 | 0.016 | 0.022 |
| Avg. competitors' share of last 10 deals by different bank | 0.041 | 0.046 | 0.033 | 0.044 | 0.014 | 0.042 |

| **Deal covariates** | Estimate | Std Error |
|---|---|---|
| Balloon payment (wtd avg) | 0.006 | 0.060 |
| Cross-collateralization (wtd avg) | -0.035 | 0.020 |
| Deal size (total principal) | -0.015 | 0.007 |
| Originator HHI | -0.068 | 0.024 |
| Property type HHI | 0.027 | 0.051 |
| Region HHI | -0.032 | 0.047 |
| 2001 vintage | 0.140 | 0.032 |
| 2002 vintage | 0.189 | 0.039 |
| 2003 vintage | 0.282 | 0.034 |
| 2004 vintage | 0.388 | 0.027 |
| 2005 vintage | 0.381 | 0.028 |
| 2006 vintage | 0.376 | 0.027 |
| 2007 vintage | 0.455 | 0.036 |
| 2010-2012 vintages | 0.320 | 0.050 |

**Covariance of residual ($\Omega$), point estimates**

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.021 | | |
| Moody's | 0.020 | 0.021 | |
| Fitch | 0.020 | 0.020 | 0.021 |

**Standard errors of covariance of residual**

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.019 | | |
| Moody's | 0.009 | 0.003 | |
| Fitch | 0.011 | 0.006 | 0.013 |

*Note:* Tables D.3 and D.4 report maximum likelihood estimates for the first-step parameters of the robustness check with bids specifying structure of both AAA and non-AAA securities: distribution of presale-report variables (whose structural parameter estimates are in Table 5). All of the first-step parameters are jointly estimated but are reported in two separate tables due to space considerations. Table D.4 shows the estimated joint distribution of the weighted-average reunderwritten DSCR and LTV for each agency (at the deal-level), assuming joint normality.

Table D.5—Tobit regressions for ex post deal outcomes (principal losses and interest shortfall) on distortion ($\lambda_i$) and control variables, for robustness check involving alternative bidder preferences (i.e., the uniform-"price" specification)

| | **Full sample of deals** | | **Deals rated by Moody's and S&P** | |
|---|---|---|---|---|
| | Estimate | Std. Error | Estimate | Std. Error |
| Distortion | 0.031 | 0.013 | 0.051 | 0.021 |
| Splines for AAA share | Included | | Included | |
| Deal rated by 3 agencies | 0.0003 | 0.004 | | |
| Deal rated by 1 agency | -0.022 | 0.004 | | |
| Constant | -0.010 | 0.014 | -0.017 | 0.022 |
| Square root of error variance | 0.026 | 0.003 | 0.020 | 0.001 |
| Observations | 579 | | 203 | |

| | **Deals rated by S&P and Fitch** | | **Deals rated by Moody's and Fitch** | |
|---|---|---|---|---|
| Distortion | 0.004 | 0.019 | 0.068 | 0.027 |
| Splines for AAA share | Included | | Included | |
| Constant | 0.014 | 0.033 | -0.024 | 0.042 |
| Square root of error variance | 0.031 | 0.006 | 0.026 | 0.004 |
| Observations | 147 | | 131 | |

*Note:* Dependent variable is sum of principal loss and interest payment shortfalls on the deal's loan pool, as of the censoring date (September 2012), expressed as a share of the original pool principal. Letting $\psi$ denote linear coefficients and $\varepsilon$ a normal error, the assumed model is:

$$\begin{aligned} \text{dependent variable} &= \psi'(\text{covariates}) + \varepsilon \text{ if } \psi'(\text{covariates}) + \varepsilon > 0, \\ &= 0 \text{ otherwise.} \end{aligned}$$

* Standard errors are bootstrapped and take into account estimation error in both the first step and in structural estimation.

Table D.6—Tobit regressions for ex post deal outcomes (principal losses and interest shortfall) on distortion ($\lambda_i$) and control variables, for robustness check involving bids that specify the structure of non-AAA securities

| | **Full sample of deals** | | **Deals rated by Moody's and S&P** | |
|---|---|---|---|---|
| | Estimate | Std. Error | Estimate | Std. Error |
| Distortion | 0.001 | 0.002 | -0.013 | 0.004 |
| Splines for AAA share | Included | | Included | |
| Deal rated by 3 agencies | -0.0055 | 0.002 | | |
| Deal rated by 1 agency | -0.025 | 0.003 | | |
| Constant | 0.014 | 0.006 | 0.033 | 0.013 |
| Square root of error variance | 0.026 | 0.003 | 0.020 | 0.002 |
| Observations | 579 | | 203 | |
| | **Deals rated by S&P and Fitch** | | **Deals rated by Moody's and Fitch** | |
| Distortion | 0.013 | 0.007 | 0.023 | 0.009 |
| Splines for AAA share | Included | | Included | |
| Constant | 0.018 | 0.034 | 0.013 | 0.011 |
| Square root of error variance | 0.031 | 0.007 | 0.026 | 0.003 |
| Observations | 147 | | 131 | |

*Note:* Dependent variable is sum of principal loss and interest payment shortfalls on the deal's loan pool, as of the censoring date (September 2012), expressed as a share of the original pool principal. Letting $\psi$ denote linear coefficients and $\varepsilon$ a normal error, the assumed model is:

$$\begin{aligned}\text{dependent variable} &= \psi'(\text{covariates}) + \varepsilon \text{ if } \psi'(\text{covariates}) + \varepsilon > 0,\\ &= 0 \text{ otherwise.}\end{aligned}$$

* Standard errors are bootstrapped and take into account estimation error in both the first step and in structural estimation.

Table D.7—First-step estimates (specification endogenizing number of winning bidders): distribution of presale-report variables

**Sieve parameters ($\{\gamma_j\}_{j=1,2,3}$)**

| **Bidder-specific covariates** | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|
| | Estimate | Std Error | Estimate | Std Error | Estimate | Std Error |
| Bidder fixed effect | 1.147 | 0.098 | 1.221 | 0.095 | 1.200 | 0.101 |
| Reunderwritten DSCR | -0.004 | 0.033 | 0.027 | 0.038 | 0.044 | 0.043 |
| Reunderwritten LTV | -0.275 | 0.083 | -0.137 | 0.074 | -0.186 | 0.072 |
| Bidder produced no pre-sale report | 0.069 | 0.016 | 0.066 | 0.010 | 0.048 | 0.028 |
| Own share of last 10 deals | 0.050 | 0.048 | -0.021 | 0.046 | 0.010 | 0.048 |
| Competitors' share of last 10 deals | -0.017 | 0.089 | -0.099 | 0.091 | -0.060 | 0.098 |
| Own share of last 3 deals by same bank | -0.051 | 0.027 | -0.027 | 0.029 | -0.027 | 0.026 |
| Avg. competitors' share of last 10 deals by different bank | -0.050 | 0.053 | -0.047 | 0.054 | -0.079 | 0.054 |

| **Deal covariates** | Estimate | Std Error |
|---|---|---|
| Balloon payment (wtd avg) | -0.330 | 0.068 |
| Cross-collateralization (wtd avg) | -0.072 | 0.039 |
| Deal size (total principal) | 0.008 | 0.011 |
| Originator HHI | -0.092 | 0.030 |
| Property type HHI | -0.480 | 0.063 |
| Region HHI | -0.015 | 0.044 |
| 2001 vintage | 0.212 | 0.030 |
| 2002 vintage | 0.291 | 0.027 |
| 2003 vintage | 0.405 | 0.032 |
| 2004 vintage | 0.487 | 0.035 |
| 2005 vintage | 0.578 | 0.033 |
| 2006 vintage | 0.613 | 0.030 |
| 2007 vintage | 0.592 | 0.039 |
| 2010-2012 vintages | 0.274 | 0.045 |

**Covariance of residual ($\Omega$), point estimates**

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.021 | | |
| Moody's | 0.020 | 0.022 | |
| Fitch | 0.020 | 0.020 | 0.022 |

**Standard errors of covariance of residual**

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.021 | | |
| Moody's | 0.011 | 0.004 | |
| Fitch | 0.013 | 0.009 | 0.017 |

*Note:* Tables D.7 and D.8 report maximum likelihood estimates for the first-step parameters of the specification endogenizing the number of winning bidders (discussed earlier in this Appendix), fixing $\kappa_2 = 0.05$ and $\kappa_3 = 0.025$. (Structural estimates are reported in Table D.9). All of the first-step parameters are jointly estimated but are reported in two separate tables due to space considerations. Table D.7 shows the estimated joint distribution of the weighted-average reunderwritten DSCR and LTV for each agency (at the deal-level), assuming joint normality.

Table D.8—First-step estimates (specification endogenizing number of winning bidders): bid functions

**Sieve parameters ($\{\gamma\}_{j=1,2,3}$)**

**Bidder-specific covariates**

| | S&P | | Moody's | | Fitch | |
|---|---|---|---|---|---|---|
| | Estimate | Std error | Estimate | Std error | Estimate | Std error |
| Bidder fixed effect | 0.8410 | 0.1157 | 0.9212 | 0.1136 | 0.8985 | 0.1139 |
| Reunderwritten DSCR | -0.0336 | 0.0237 | 0.0167 | 0.0275 | 0.2781 | 0.0359 |
| Reunderwritten LTV | -0.8494 | 0.0605 | -0.6242 | 0.0499 | -0.4438 | 0.0496 |
| Bidder produced no pre-sale report | -0.0346 | 0.0252 | 0.0065 | 0.0141 | -0.0340 | 0.0241 |
| Own share of last 10 deals | 0.1728 | 0.0411 | 0.0489 | 0.0405 | 0.0717 | 0.0426 |
| Avg competitors' share of last 10 deals* | 0.1426 | 0.0714 | 0.0899 | 0.0740 | 0.1014 | 0.0807 |
| Own share of last 3 deals by same bank | -0.0330 | 0.0293 | -0.0083 | 0.0301 | -0.0167 | 0.0291 |
| Avg. competitors' share of last 3 deals by same bank* | -0.0351 | 0.0574 | -0.0064 | 0.0560 | -0.0233 | 0.0580 |

**Deal covariates**

| | Estimate | Std error |
|---|---|---|
| Balloon payment (wtd avg) | -0.3140 | 0.0887 |
| Cross-collateralization (wtd avg) | -0.0927 | 0.0445 |
| Deal size (total principal) | 0.0301 | 0.0158 |
| Originator HHI | -0.0748 | 0.0384 |
| Property type HHI | -0.4701 | 0.0769 |
| Region HHI | -0.1205 | 0.0505 |
| 2001 vintage | 0.2347 | 0.0375 |
| 2002 vintage | 0.3191 | 0.0350 |
| 2003 vintage | 0.4037 | 0.0419 |
| 2004 vintage | 0.4987 | 0.0436 |
| 2005 vintage | 0.6018 | 0.0411 |
| 2006 vintage | 0.6566 | 0.0393 |
| 2007 vintage | 0.6696 | 0.0452 |
| 2010 vintage | 0.3587 | 0.0550 |

**Covariance of residual ($\Omega$), point estimates**

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.0267 | | |
| Moody's | 0.0256 | 0.0271 | |
| Fitch | 0.0251 | 0.0253 | 0.0262 |

**Standard errors of covariance of residual**

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.0132 | | |
| Moody's | 0.0073 | 0.0099 | |
| Fitch | 0.0076 | 0.0064 | 0.0089 |

*Note:* Tables D.7 and D.8 report maximum likelihood estimates for the first-step parameters of the specification endogenizing the number of winning bidders (discussed earlier in this Appendix), fixing $\kappa_2 = 0.05$ and $\kappa_3 = 0.025$. (Structural estimates are reported in Table D.9). All of the first-step parameters are jointly estimated but are reported in two separate tables due to space considerations. Table D.8 shows the estimated equilibrium bidding behavior. The sieve parameters capture the effect of covariates on bidding behavior for individual agencies. The covariance parameters capture the joint distribution of the component of agencies' bids that is not explained by covariates.

Table D.9—Structural estimates (specification endogenizing number of winning bidders)

| **Bidder-specific covariates, commonly observed ($\beta_1$)** | Estimate | Std. Err. |
|---|---|---|
| Own share of last 10 deals | 0.317 | 0.480 |
| Avg. competitors' share of last 10 deals | 1.028 | 0.939 |
| Own share of last 3 deals by same bank | 0.201 | 0.429 |
| Avg. competitors' share of last 3 deals by same bank | -0.310 | 0.493 |
| **Deal covariates ($\beta_2$)** | | |
| Constant | 0.596 | 1.074 |
| Balloon payment (wtd avg) | -0.699 | 0.446 |
| Cross-collateralization (wtd avg) | -0.052 | 0.063 |
| Deal size (total principal) | 0.013 | 0.010 |
| Originator HHI | -0.138 | 0.051 |
| Property type HHI | -0.591 | 0.341 |
| Region HHI | 0.081 | 0.143 |
| 2001 vintage | 0.253 | 0.055 |
| 2002 vintage | 0.381 | 0.069 |
| 2003 vintage | 0.461 | 0.054 |
| 2004 vintage | 0.591 | 0.053 |
| 2005 vintage | 0.602 | 0.060 |
| 2006 vintage | 0.664 | 0.052 |
| 2007 vintage | 0.572 | 0.092 |
| 2010-2012 vintages | 0.540 | 0.180 |
| **Bidder-specific covariates, private information ($\beta_3$)** | | |
| Reunderwritten DSCR | 0.034 | 0.160 |
| Reunderwritten LTV | -0.070 | 0.288 |
| Bidder produced no pre-sale report | 0.003 | 0.172 |

Covariance of residual $u_{ij}$

| | S&P | Moody's | Fitch |
|---|---|---|---|
| S&P | 0.122 | | |
| Moody's | 0.094 | 0.139 | |
| Fitch | 0.092 | 0.110 | 0.129 |

Number of observations: 578

*Note:* Table shows structural estimates for specification endogenizing the number of winning bidders (discussed earlier in this Appendix), fixing $\kappa_2 = 0.05$ and $\kappa_3 = 0.025$. The distribution of the residual $u_{ij}$ is computed by simulating the distribution of beliefs for the full set of bidders and netting out the effects of the covariates. First-step estimates corresponding to the specification reported in this table are in Tables D.7 and D.8.

## APPENDIX E Solving for Counterfactual Equilibrium of Bidding Game

*Existence of Pure Strategy Bayesian Nash Equilibrium*

In this Appendix, we argue that a pure-strategy Bayesian Nash Equilibrium (PSNE) exists for the bidding game described in the model. If the possible set of actions were discrete (e.g., if bidders could bid only in increments of 0.01), the existence of a PSNE would be guaranteed so long as the game satisfies the Single-Crossing Condition (SCC) and certain other regularity conditions (see Definition 3 and Theorem 1 in Athey, 2001). The SCC can easily be shown to hold in our setup, and stipulates that, for each player $j = 1, \ldots, J$, whenever every opponent $j' \neq j$ uses a strategy that is nondecreasing in its type, player $j$'s objective function satisfies the *single crossing property of incremental returns* in $(b_{ij}, t_{ij})$. Because the objective function $\pi_{ij}(t_{ij}, b_i)$ is differentiable, it suffices to observe that $\frac{\partial \pi_{ij}(t_{ij}, b_i)}{\partial t_{ij} \partial b_{ij}} > 0$.

In the case of continuous actions, existence of a PSNE could be shown constructively by taking the limit of the finite-action equilibrium for successively finer action sets *if* the limit of this series were guaranteed to be an equilibrium of the continuous game. A complication arises in bidding games, such as in our setup, because the outcome (namely, the set of winners) is discontinuous in the actions. However, this problem goes away if, in the limit as the action becomes successively finer, "mass points" do not arise and the payoffs are continuous. The conditions for this to hold are discussed in Theorem 6 of Athey (2001), and are either standard or hold trivially in the current setting by virtue of the assumption of private values.

*Solution method*

The solution method falls under the general approach of mathematical programming with equilibrium constraints (MPEC). Let $\varphi_{ij}(b_{ij}) = \tilde{b}_{ij}$ denote the inverse bid function for bidder $j$ in auction $i$ (we assume a bidder's bid function is monotone and thus invertible). Following Hubbard and Paarsch (2009) and Bajari (2001), we approximate $j$'s inverse bid function $\varphi_{ij}(b_{ij})$ by $\psi_{ij}' f(b_{ij})$, where $f(\cdot)$ is a family of basis functions (which we choose to be Chebyshev polynomials). Solving for the PSNE entails finding coefficients $\{\psi_{ij}\}$ for bidders $j = 1, \ldots J$ that best fit a set of equilibrium conditions, evaluated on a set of grid points over the domain of possible bids. For all grid points $b$ and $b'$, the imposed equilibrium conditions are as follows:

- Monotonicity: $b' > b \Leftrightarrow \psi_{ij}' f(b') > \psi_{ij}' f(b)$.
- Optimality: setting $b_{ij} = b$ and $\tilde{b}_{ij} = \psi_{ij}' f(b)$ satisfies bidder $j$'s first-order condition (4).

- Individual rationality: $V - (\Phi^{-1}(b) - w_{ij}\beta_1 - \psi'_{ij}f(b))^2 > 0$, where $\Phi^{-1}(\cdot)$ and $\beta'_3 w_{ij}$ are as defined in expression (3).

Because the support of bidders' type distributions is the real number line and thus infinite, in order to make the solution method feasible, we truncate the bidders' type spaces from above at an auction-specific truncation point $\bar{b}_i$ (chosen to be sufficiently high such that it is in the tails of the type distributions), and "fix" the upper bound of the domain of bids by imposing the boundary condition $\psi'_{ij}f(\bar{b}_i) = \bar{b}_i$. The lower bound of the domain of bids is determined in equilibrium, and, in principle could be estimated by imposing an additional boundary condition (see Hubbard and Paarsch, 2009). However, we found that doing so resulted in numerically unstable results. Instead, we attempt to estimate the inverse bid function only for bids greater than or equal to 0.40, a lower cutoff that we chose based on the observation that 0.40 is less than the minimum observed pivotal bid, 0.508.

Simulation is central to the computation. We simulate the joint distribution of bids conditional on $x$ and $w$, treating $z$ and $\varepsilon$ in expression 8 as random variables distributed according to the first-stage estimates. We then compute the implied joint distribution of types based on the bidders' first-order conditions. At a given bid value, bidder $j$'s first-order condition depends on the distribution of the competitors' bids. For each bidder $j$, we specify a discrete grid of $T$ bid values $\{a(t)_{ij}, t = 1 \ldots T\}$ spanning a range that covers all but the extreme tails of the marginal distribution of $j$'s bid distribution.

We use MPEC (Judd and Su, 2012) to minimize:

$$\sum_{j=1,2,3} \sum_{t=1,\ldots,T} [\omega(t,1)\varsigma_1 + \omega(t,2)\varsigma_2 + \omega(3)\varsigma_3]$$

where $\varsigma_i$ refers to the squared norm of the violation of monotonicity, optimality and individual rationality for $i = 1, 2, 3$. The terms $\omega(t,1)$, $\omega(t,2)$ and $\omega(3)$ are weights that we chose heuristically. In practice, we do this computation for only one auction, a representative auction as described in the text.