Innovation and Production in the Global Economy
Online Appendix

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The Online Appendix includes a number of results. First, we include the proofs and supporting results of propositions in the main paper. Second, we discuss a number of results that appear, or are discussed, in the main paper. Finally, we incorporate a number of robustness checks. We make use and cite various definitions and equations from the main paper, which, for brevity, we do not reintroduce here but cite accordingly.

1 Proof of Proposition 2

It immediately follows from (13) and (14), that as $\kappa \to \infty$ workers become perfect substitutes. Hence, an interior equilibrium in which both innovation and production occur in country i requires $w_i^e = w_i^p$. As such, we refer to the single wage w_i for country i.

Part (i) First, as a preliminary result, we establish that if $\bar{L}_1 > \bar{L}_2$, then $\omega \equiv w_1/w_2 > 1$. The absence of trade costs implies that $\lambda_{in}^E \equiv \lambda_i^E$ for any i, n (For future reference, note that this implies that $\lambda_1^E + \lambda_2^E = \lambda_{11}^E + \lambda_{21}^E = 1$) and that $\Psi_{in} \equiv \Psi_i$ for any i, n. The zero-profit condition in (16) implies that

$$L_1^e = \eta \lambda_1^E (\bar{L}_1 + \bar{L}_2/\omega), \tag{O.1}$$

$$L_2^e/\omega = \eta \lambda_2^E (\bar{L}_1 + \bar{L}_2/\omega). \tag{O.2}$$

Using these equations together with the definition of λ_i^E, which implies that

$$\lambda_i^E = \frac{M_i \Psi_{in}}{\sum_k M_k \Psi_{kn}} = \frac{M_i \Psi_i}{\sum_k M_k \Psi_k}, \tag{O.3}$$

and $M_i f^e = L_i^e$, we have $\omega = \Psi_1/\Psi_2$.

Using the definition of Ψ_{in} and the assumption of $A_1 = A_2$, we can obtain after some derivations

$$\frac{\bar{L}_1}{\bar{L}_2} = \omega^{\frac{\theta}{1-\rho}} \frac{\omega^{1/(1-\rho)} - \gamma^{-\theta/(1-\rho)}}{1 - \gamma^{-\theta/(1-\rho)} \omega^{1/(1-\rho)}}. \tag{O.4}$$

The right hand side of this equation is increasing in ω which implies that ω is increasing in $\bar{L}_1/\bar{L}_2$. Since $\bar{L}_1/\bar{L}_2 = 1$ implies that $\omega = 1$, then $\bar{L}_1/\bar{L}_2 > 1$ implies that $\omega > 1$, which proves the preliminary result.

Second, using the previous result we can prove that if $\bar{L}_1 > \bar{L}_2$ then $r_1 > r_2$. The proof is by contradiction. Suppose that $r_1 < r_2$. From the labor market clearing condition in (15) and from (17) and $\lambda_{in}^T = \lambda_{ii}^T \equiv \lambda_i^T$, we have

$$\begin{aligned} w_i L_i^e &= w_i \bar{L}_i \left(1 - \frac{\theta - \sigma + 1}{\sigma\theta}\right) - \frac{1}{\tilde{\tilde{\sigma}}} \lambda_i^T \sum_k w_k \bar{L}_k \implies \\ r_i &= \eta + 1 - 1/\sigma - \frac{1}{\tilde{\tilde{\sigma}}} \frac{\lambda_i^T \sum_k w_k \bar{L}_k}{w_i \bar{L}_{i.}} \end{aligned}$$

Assuming that $r_1 < r_2$ then labor market clearing in the two countries requires

$$\frac{\lambda_1^T}{w_1 \bar{L}_1} > \frac{\lambda_2^T}{w_2 \bar{L}_2}. \tag{O.5}$$

Using the definition for λ_l^T, the result $\lambda_{in}^E = \lambda_i^E$, and $(O.1)$ and $(O.2)$, after some derivations expression (O.5) implies that

$$\bar{L}_2 r_2 \omega^{\frac{\rho}{1-\rho}} \left(\gamma^{-\theta/(1-\rho)} - \omega^{\theta/(1-\rho)+1}\right) > \bar{L}_1 r_1 \left(\omega^{\theta/(1-\rho)+1} \gamma^{-\theta/(1-\rho)} - 1\right),$$

which will finally allow us to prove the result by contradiction. Note that when $\bar{L}_1 > \bar{L}_2$ we have $\omega > 1$, so that the term in parentheses on the left-hand-side of this inequality is negative. If $\omega^{\theta/(1-\rho)+1} \gamma^{-\theta/(1-\rho)} \geq 1$, then the inequality is violated and the desired contradiction is shown. Alternately, if $\omega^{\theta/(1-\rho)+1} \gamma^{-\theta/(1-\rho)} < 1$ we can substitute out $\bar{L}_2/\bar{L}_1$ from the inequality using $(O.4)$ to arrive at an expression that given the assumption that $\theta > 1$ contradicts the initial assertion that $r_1 < r_2$. Thus, since this assertion leads to a contradiction in all cases, we conclude that $r_1 > r_2$, which completes the proof of part i).

Part (ii) Denote the size of the labor forces as L and note that with $\kappa \to \infty$ $w_i^e = w_i^p = w_i$. Let $\omega \equiv w_1/w_2$. From the free entry conditions for countries 1 and 2 and the assumption of no trade costs, it follows that $\omega = \Psi_1/\Psi_2$. Using the definitions, this implies that

$$\omega^{\frac{1}{1-\rho}} = \frac{1 + \omega^{\frac{\theta}{1-\rho}} \left(\gamma_{12}\right)^{-\frac{\theta}{1-\rho}}}{\left(\gamma_{21}\right)^{-\frac{\theta}{1-\rho}} + \omega^{\frac{\theta}{1-\rho}}}. \tag{O.6}$$

We first show that $\gamma_{12} < \gamma_{21}$ implies that $\omega > 1$. Note that for $\gamma_{12} = \gamma_{21}$ the only solution to (O.6) is $\omega = 1$. Totally differentiating (O.6), yields

$$\frac{d\omega}{d\gamma_{21}} = \frac{\theta\omega}{\gamma_{21}} \frac{\omega^{\frac{1}{1-\rho}} \left(\gamma_{21}\right)^{-\frac{\theta}{1-\rho}}}{\omega^{\frac{1}{1-\rho}} \left(\gamma_{21}\right)^{-\frac{\theta}{1-\rho}} + (\theta+1)\omega^{\frac{\theta+1}{1-\rho}} - \theta\omega^{\frac{\theta}{1-\rho}} \left(\gamma_{12}\right)^{-\frac{\theta}{1-\rho}}}.$$

Equation (O.6) then confirms that $d\omega/d\gamma_{21} > 0$. Hence for $\gamma_{12} < \gamma_{21}$, $\omega > 1$.

Now, we turn to the labor market clearing, equation (15). With identical countries and free trade, the labor market clearing condition for country 1 can be written.

$$w_1 L_1^p = \left(1 - \frac{1+\theta}{\theta}\frac{\sigma-1}{\sigma}\right) X_1 + \frac{\sigma-1}{\sigma} \left(\lambda_1^E \psi_{111} + \lambda_2^E \psi_{211}\right) \left(X_1 + X_2\right).$$

After some manipulation, this reduces to

$$r_1 = \eta + \frac{\sigma-1}{\sigma} \left(1 - \left(\lambda_1^E \psi_{111} + \lambda_2^E \psi_{211}\right) \left(1 + \omega^{-1}\right)\right). \tag{O.7}$$

Free entry in each country implies that

$$\frac{r_1}{\eta} = \lambda_1^E \left(1 + \omega^{-1}\right), \text{ and } \frac{r_2}{\eta\omega} = \lambda_2^E \left(1 + \omega^{-1}\right)$$

so that (O.7) can be written

$$r_1 = \frac{\eta(1+\theta) - \theta\frac{r_2}{\omega}\psi_{211}}{1 + \theta\psi_{111}}.$$

Noting that $r_2 = \eta(1+\omega) - \omega r_1$ we can further consolidate terms, arriving at

$$r_1 = \eta\frac{1 + \theta - (1+\omega)\frac{\theta}{\omega}\psi_{211}}{1 + \theta\psi_{111} + \theta\psi_{211}}.$$

Using the definitions of ψ_{211} and ψ_{111}, we arrive at

$$r_1 = \eta \frac{1+\theta - \frac{(1+\omega)\frac{\theta}{\omega}}{1+\omega^{\frac{\theta}{1-\rho}}(\gamma_{21})^{\frac{\theta}{1-\rho}}}}{1+\theta\frac{1}{1+\omega^{\frac{\theta}{1-\rho}}(\gamma_{12})^{-\frac{\theta}{1-\rho}}}+\theta\frac{1}{1+\omega^{\frac{\theta}{1-\rho}}(\gamma_{21})^{\frac{\theta}{1-\rho}}}}$$

Now suppose that $r_1 < \eta$, then we must the ratio on the right-hand side of this expression be less than one. After simplification the required inequality can be reduced to

$$1-\omega^{-1} < \omega^{-1}\left(\frac{\gamma_{12}}{\omega}\right)^{\frac{\theta}{1-\rho}} - \omega^{\frac{\theta}{1-\rho}}(\gamma_{21})^{\frac{\theta}{1-\rho}}.$$

The left-hand side of this inequality must be positive because $\gamma_{12} < \gamma_{21}$ implies that $\omega > 1$. Moreover, $\omega > 1$ and $\gamma_{12} < \gamma_{21}$ imply that the right-hand side of this inequality must be negative. Hence, this is a contradiction and we conclude that $r_1 > \eta$. By the equilibrium conditions, we have $r_2 = \eta + \omega(\eta - r_1)$ which implies that $r_2 < \eta$. **QED**.

2 Real Wage in Terms of Flows

We start with the definition of λ^T_{ln} in (10). Using also the definitions of ψ_{iln} and ξ_{iln}, setting $l = n$ and solving for w^p_n, we have

$$w^p_n = \left[\frac{\lambda^T_{nn}}{\sum_k \left(T_{kn}\gamma^{-\theta}_{kn}/\Psi_{kn}\right)^{1/(1-\rho)}\lambda^E_{kn}}\right]^{-(1-\rho)/\theta}.$$

Using the result for the Dixit-Stiglitz price index in $(A.6)$, and noting that the definition of λ^E_{in} implies that $\sum_k M_k\Psi_{kn} = M_n\Psi_{nn}/\lambda^E_{nn}$, we can write

$$P_n = \zeta^{-1}\left[\left(\frac{w^p_n F_n}{X_n}\right)^{1-\theta/(\sigma-1)}\frac{M_n\Psi_{nn}}{\lambda^E_{nn}}\right]^{-1/\theta},$$

where ζ is a constant that is defined above. Combining the two previous expressions and using $T_{in} = T_i^e T_n^p$, we get

$$\frac{w_n^p}{P_n} = \zeta \left(T_n^e T_n^p M_n\right)^{1/\theta} \left(\lambda_{nn}^T\right)^{\frac{\rho-1}{\theta}} \left(\lambda_{nn}^E\right)^{-\frac{1}{\theta}} \left[\sum_k \left(T_k^e \gamma_{kn}^{-\theta} \frac{\Psi_{nn}}{\Psi_{kn}}\right)^{\frac{1}{1-\rho}} \lambda_{kn}^E\right]^{\frac{1-\rho}{\theta}} \left(\frac{w_n^p F_n}{X_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}}. \tag{O.8}$$

Using (7), the definition of ψ_{iln}, and simplifying, we get

$$\sum_k \left(T_k^e \gamma_{kn}^{-\theta} \frac{\Psi_{nn}}{\Psi_{kn}}\right)^{\frac{1}{1-\rho}} \lambda_{kn}^E = \frac{\left(T_n^e\right)^{\frac{1}{1-\rho}} \lambda_{nn}^E}{X_{nnn} / \sum_i X_{inn}}. \tag{O.9}$$

Plugging this expression into $(O.8)$, and using the definitions of λ_{nn}^T and λ_{nn}^E yields

$$\frac{w_n^p}{P_n} = \zeta \left(T_n^e T_n^p M_n\right)^{1/\theta} \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1-\rho}{\theta}} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\frac{\rho}{\theta}} \left(\frac{w_n^p F_n}{X_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}}. \tag{O.10}$$

Finally, to write w_n^p / X_n in terms of flows, note that

$$\frac{w_n^p \bar{L}_n}{X_n} = \frac{\bar{L}_n}{L_n^p} \frac{w_n^p L_n^p}{w_n^p L_n^p + w_n^e L_n^e} = \frac{1 - r_n}{\left[1 + \left(\frac{w_n^e}{w_n^p}\right)^{\kappa}\right]^{1/\kappa - 1}}. \tag{O.11}$$

Combined with expression (22), $w_n^e / w_n^p = \left(r_n / (1 - r_n)\right)^{1/\kappa}$ yields

$$\frac{w_n^p \bar{L}_n}{X_n} = (1 - r_n)^{1/\kappa}. \tag{O.12}$$

Plugging into (O.10) then yields

$$\frac{w_n^p}{P_n} = \zeta \left(\frac{F_n}{\bar{L}_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} \left(T_n^e T_n^p M_n\right)^{1/\theta} \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1-\rho}{\theta}} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\frac{\rho}{\theta}} (1 - r_n)^{\frac{1}{\kappa} \frac{\sigma-1-\theta}{\theta(\sigma-1)}}. \tag{O.13}$$

We obtain $\left(X_n / \bar{L}_n\right) / P_n = \left(X_n / w_n^p \bar{L}_n\right) \times \left(w_n^p / P_n\right)$ by combining the last two equations to get

$$\frac{X_n / \bar{L}_n}{P_n} = \zeta \left(\frac{F_n}{\bar{L}_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} \left(T_n^e T_n^p M_n\right)^{1/\theta} \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1-\rho}{\theta}} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\frac{\rho}{\theta}} (1 - r_n)^{\frac{1}{\kappa}\left(\frac{\sigma-1-\theta}{\theta(\sigma-1)} - 1\right)}.$$

This allows us to write gains from openness as

$$GO_n = \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1-\rho}{\theta}} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\frac{\rho}{\theta}} \left(\frac{1-r_n}{1-\eta}\right)^{\frac{1}{\kappa}\left(\frac{\sigma-1-\theta}{\theta(\sigma-1)}-1\right)} \left(\frac{M_n}{M_n^A}\right)^{1/\theta},$$

where M_n^A is mass of products introduced in an autarky equilibrium. Writing M_n in terms of r_n as in (23) implies that

$$\frac{M_n}{M_n^A} = \left(\frac{r_n}{\eta}\right)^{1-1/\kappa},$$

leaving us with

$$GO_n = \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1-\rho}{\theta}} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\frac{\rho}{\theta}} \left(\frac{1-r_n}{1-\eta}\right)^{\frac{1}{\kappa}\left(\frac{\sigma-1-\theta}{\theta(\sigma-1)}-1\right)} \left(\frac{r_n}{\eta}\right)^{\frac{\kappa-1}{\kappa\theta}},$$

as in the text.

3 Proof of Proposition 3

We start with part (i) of the proposition and begin by showing that the mass of varieties is proportional to the population of the economy. For any given relative wage, the limit as $\kappa \to 1$ of the measure of innovation workers is

$$\begin{aligned}
\lim_{\kappa\to 1} L_i^e &= \lim_{\kappa\to 1} \bar{L}_i \left(1+\left(\frac{w_i^e}{w_i^p}\right)^{-\kappa}\right)^{\frac{1}{\kappa}-1} \\
&= \bar{L}_i \frac{\lim_{\kappa\to 1}\left(1+\left(\frac{w_i^e}{w_i^p}\right)^{-\kappa}\right)^{\frac{1}{\kappa}}}{\lim_{\kappa\to 1}\left(1+\left(\frac{w_i^e}{w_i^p}\right)^{-\kappa}\right)} = \bar{L}_i
\end{aligned}$$

Similiarly, $L_i^p = \bar{L}_i$. It immediately follows that $M_i = \bar{L}_i / f^e$. Let $l_k \equiv \bar{L}_k / \sum_j \bar{L}_j$, $t_i \equiv T_i^e$, and $t \equiv \sum_j \frac{\bar{L}_j T_j^e}{\sum_k \bar{L}_k} = \sum_j l_j t_j$. The assumption that $A_i = A$, for all i, together with the definition $A_i \equiv \left(T_i^p\right)^{1/(1-\rho)} / L_i^p$, implies that $T_{ii} = T_i^e T_i^p = T^e \left(L_i^p\right)^{1-\rho}$. Let W_i (X_i) be the real wage (expenditure) in country i under frictionless trade and no MP, and let W_i^* (X_i^*) be the real wage (expenditure) in country i under frictionless trade and MP. We first characterize the expressions for welfare under restricted entry in the following Lemma, which is proved in Section 6.

Lemma O.1 *Consider a world with no worker mobility, $\kappa \to 1$, where $A_i = A$, and $\frac{t_i}{t} < \frac{(\theta+1)(\theta\sigma-\sigma+1)}{(\theta-\sigma+1)}$ $\forall i$, and assume $\rho \to 1$ for all i. The ratio of the real wage under frictionless trade and MP to the real wage under free trade and no MP, $\mathcal{W}_i \equiv W_i^*/W_i$, is given by the expression:*

$$(\mathcal{W}_i)^\theta = \frac{[(1-\eta)\,t + \eta t_i]^v \, t^{1-v}}{t_i^{\theta/(1+\theta)} \sum_k t_k^{1/(1+\theta)} l_k}\,, \tag{O.14}$$

where $v = \theta/\,(\sigma-1) - 1$.

With the help of this Lemma, we can now proceed to prove the two parts of the proposition. Notice that around $t_i \simeq t$, the restriction specified in the Lemma is always satisfied, so that we can make use of the Lemma for proving Proposition 3.

Part (i) We first show that real wages increase iff $\sigma < \bar{\theta} \equiv \frac{(1+\theta)^2}{1+\theta+\theta^2}$, by using Lemma O.1. Taking logs in (O.20), differentiating with respect to the size of one country t_i, and evaluating it at $t_i = t$, for all i, we get that the sign of this derivative is determined by

$$v\,[(1-\eta)\,l_i + \eta] + (1-v)\,l_i - \frac{\theta}{1+\theta} - \frac{1}{1+\theta} l_i\,,$$

or equivalently, by the sign of $v\eta - \theta/(1+\theta)$. Having $v\eta > \theta/(1+\theta)$ is equivalent to $\sigma < \bar{\theta} \equiv \frac{(1+\theta)^2}{1+\theta+\theta^2}$, which proves part i).

Part (ii) Now consider real expenditures. Total real expenditure in country i is $X_i = \left(w_i^p + w_i^e\right)\bar{L}_i/P_i = \left(1 + w_i^e/w_i^p\right)\bar{L}_i W_i$.

In the no-MP equilibrium, we must have $w_i^e/w_i^p = 1-\eta$, whereas in the MP equilibrium , labor market clearing for innovation labor, $\sum_k w_k^p \overline{L}_k = (1-\eta)\sum_k X_k$, and production wage equalization for $\rho \to 1$, yield

$$\frac{w_i^e}{w_i^p} = \frac{\eta}{1-\eta}\frac{t_i}{t}.$$

Consider the ratio $\mathcal{X}_i \equiv \left(X_i^*/P_i^*\right)/\,(X_i/P_i)$. The total expenditure gains from MP are

$$\mathcal{X}_i = \left(1 - \eta + \eta\frac{t_i}{t}\right)\frac{W_i^*}{W_i},$$

and hence, using (O.20),

$$\mathcal{X}_i = \left(\frac{[(1-\eta)\,t + \eta t_i]^{v+\theta}\, t^{1-v-\theta}}{t_i^{\theta/(1+\theta)} \sum_k t_k^{1/(1+\theta)} l_k}\right)^{1/\theta}.$$

This expression is similar to what we had above for real wages, only that instead of v we now have $v+\theta$. Thus, the condition for real income to increase with MP is that $(v+\theta)\,\eta >$

$\theta/(1+\theta)$. Notice that this condition is equivalent to $\theta > \sigma - 1$, which we always require for the various integrals to have a finite mean. Thus, real expenditure must increase with MP.

In a similar manner we can show that the innovation wage increases under frictionless trade and MP. With frictionless trade and no MP we have $W_i^e = W_i\eta/(1-\eta)$, while with frictionless trade and MP we have $W_i^{e*} = W_i^* \frac{t_i}{t}\eta/(1-\eta)$. This implies that

$$\frac{W_i^{e*}}{W_i^e} = \frac{t_i}{t}\frac{W_i^*}{W_i},$$

and hence,

$$\frac{W_i^{e*}}{W_i^e} = \left(\frac{\left[(1-\eta)\,t + \eta t_i\right]^v t^{1-v-\theta}}{t_i^{-\frac{\theta^2}{1+\theta}} \sum_k t_k^{1/(1+\theta)} l_k}\right)^{\frac{1}{\theta}}.$$

Taking logs, differentiating, and evaluating around $t_i \simeq t$, we obtain

$$\frac{1}{\sigma\theta}\left(1 - \frac{\sigma-1}{\theta}\right) + \frac{1}{t}\frac{1}{1+\theta}\left(\theta^2 - l_i\right),$$

which is always positive because $\theta > \max\{1, \sigma-1\}$, implying that real profits are higher with frictionless MP than with no MP.

Now consider part (ii) of the proposition. We begin by solving for the equilibrium real income of innovation labor in country j. In the absence of MP, there can be no specialization in trade or MP so we relative wages are fixed at

$$w_j^e = \frac{\eta}{1-\eta} w_j^p.$$

The price index continues to be given by

$$P_j = \zeta^{-1}\left(\left(\frac{w_j^p F_j}{X_j}\right)^{1-\frac{\theta}{\sigma-1}} \sum_k \overline{L}_k \Psi_{kj}\right)^{-\frac{1}{\theta}},$$

but in our simplified case, we have

$$\sum_k \overline{L}_k \Psi_{kj} = \overline{L}_j T^e T_j^p (w_j^p)^{-\theta}.$$

These three equations completely pin down the real income of a unit of innovation labor in the no MP equilibrium.

In the equilibrium with MP, the fact that there are not trade barriers and there are free flow of ideas requires that factor price equalization prevails for innovation labor: $w_i^e = w_j^e = w^e$ for all i. For all $i \neq j$ production labor can only be used for marketing fixed costs so that

$$w_i^p \overline{L}_i = \left(1 - \eta(1+\theta)\right) X_i$$

As in the MP equilibrium, production labor from country j must unilaterally serve global demand. Hence, the labor market clearing condition becomes

$$w_j^p \overline{L}_j = \left(1 - \eta(1+\theta)\right) X_j + \frac{\sigma - 1}{\sigma} \sum_n X_n,$$

which, after substituting for w_i^e and w_i^p, simplifies to

$$w_1^p = \left(\frac{1-\eta}{\eta} + \frac{\theta(N-1)}{\eta(1+\theta)} \right) w_1^e,$$

where N is the number of countries in the world economy. The price index can now be written

$$P_1 = \zeta^{-1} \left(\left(\frac{w_j^p F_j}{\left(w_j^p + w_j^e\right) \overline{L}_j} \right)^{1-\frac{\theta}{\sigma-1}} \left(\sum_i \overline{L}_i T_i^e \right) T_j^p (w_j^p)^{-\theta} \right)^{-\frac{1}{\theta}}.$$

Combining the wages and price indexes for the two equilibrium yields

$$\frac{\widehat{w}_j^e}{\widehat{P}_j} = \left(\left(\frac{(1-\eta)\left(1 + \frac{\theta(N-1)}{1+\theta}\right)}{1 - \eta + \frac{\theta(N-1)}{1+\theta}} \right)^{\frac{\theta}{\sigma-1}-1} N \right)^{\frac{1}{\theta}} \frac{1-\eta}{1 - \eta + \frac{\theta(N-1)}{1+\theta}}.$$

Taking the logarithm of this expression and differentiating with respect to N we obtain after some simplification:

$$\frac{d \ln\left(\widehat{w}_j^e / \widehat{P}\right)}{dN} = \frac{1}{\theta}\frac{1}{N} - \left[\left(\frac{\theta}{\sigma-1} - 1 \right) \frac{\eta(1+\theta)}{1+\theta N} + \theta \right] \frac{1}{1 - \eta\left(1+\theta\right) + \theta N}.$$

We complete the proof by contradiction. Suppose that the real wage of country j innovation workers were to increase with an increase in the number of countries that the country engages in MP. Then, we would have

$$\frac{1}{\theta}\frac{1}{N} \geq \left[\left(\frac{\theta}{\sigma-1}-1\right)\frac{\eta(1+\theta)}{1+\theta N}+\theta\right]\frac{1}{1-\eta\left(1+\theta\right)+\theta N},$$

but rearranging this expression yields

$$1-\eta\left(1+\theta\right) \geq \left(\frac{\theta}{\sigma}-\eta\theta\right)\frac{1+\theta}{\frac{1}{N}+\theta}+\theta N\left(\theta-1\right)$$

Note that the left hand side of this inequality is strictly increasing in N so that if this condition fails for $N=1$, then if must fall for all N. Evaluating this expression at $N=1$, yields

$$1 \geq \theta^2.$$

Note that the requirement that $\theta > \max(1, \sigma-1)$ implies that the term on the right-hand side of this inequality must be greater than one. This contradicts the assertion that income must rise. Hence, the real income of innovation workers in country j must fall.

Finally, using the equations above, aggregate real expenditure change is given by

$$\frac{\widehat{X}_j}{\widehat{P}_j} = \left(1+\frac{\theta(N-1)}{1+\theta}\right)\frac{\widehat{w}_j^e}{\widehat{P}_j}.$$

Totally differentiating this expression with respect to N it can be shown by contradiction that an increase in N must be associated with an increase in real expenditure. **QED**

4 Gains from MP: Frictionless Trade and Homogenous Workers

We now establish the claim in Section 2.5.3 that a move from frictionless trade but no MP to frictionless trade and frictionless MP increases the common real wage paid to workers employed in the innovation and production sector under perfect worker mobility, or homogeneous workers, $\kappa \to \infty$.

To prove the result we first compute the real wage under two scenarios: (i) frictionless trade and frictionless MP; and (ii) frictionless trade but no MP. Then we compare the two cases. Note that when $\kappa \to \infty$, wages in the innovation and production sector are equalized, $w_i = w_i^e$.

(i) Frictionless trade and frictionless MP. From $(A.9)$ and the normalization $w_N = 1$, we get

$$w_n = T_n^e / T_N^e \,. \tag{O.15}$$

Using (A.11), which holds in the case of frictionless trade and MP, together with (A.7), (17) and (O.15), and replacing into the price index in (A.6), we obtain the real wage in country n under frictionless trade and MP,

$$\frac{w_n}{P_n} = \zeta \eta^{1/\theta} (T_n^e / f^e) \left[\left(\frac{F_n}{\bar{L}_n} \right)^{\frac{\theta-\sigma+1}{1-\sigma}} \left\{ \sum\nolimits_k \left[T_k^p \left(T_k^e / f^e \right)^{-\theta} \right]^{1/(1-\rho)} \right\}^{1-\rho} \left(\sum\nolimits_k \bar{L}_k (T_k^e / f^e) \right) \right]^{1/\theta} . \tag{O.16}$$

(ii) Frictionless trade but no MP. Given that there is no MP, trade is balanced so that $X_n = Y_n$ and $L_n^e = \eta \bar{L}_n$ for all n. Therefore the current account balance in (17) together with the fact that all income is accrued to labor, $X_n = w_n \bar{L}_n$, and $L_n^e = \eta \bar{L}_n$ imply that $w_n \bar{L}_n = \sum_k \lambda_{nk}^E X_k$. But since there is frictionless trade but no MP, then by replacing for the definition of λ_{in}^E, the current account balance can be written as

$$w_n \bar{L}_n = \frac{M_n T_n^e T_n^p w_n^{-\theta}}{\sum_k M_k T_k^e T_k^p w_k^{-\theta}} \sum_k X_k.$$

Normalizing $w_N = 1$, and using $M_n = r_n \bar{L}_n / f^e$—for which $r_n = \eta$ as there is no MP—the above expression implies that wages can be expressed as

$$w_n = \left(\frac{T_n^e T_n^p}{T_N^e T_N^p} \right)^{\frac{1}{1+\theta}} . \tag{O.17}$$

Also, using $(O.17)$, $M_n = r_n \bar{L}_n / f^e$, $r_n = \eta$ and $\Psi_{in} = T_i^e T_i^p w_i^{-\theta}$, we have that

$$\sum_k M_k \Psi_{kn} = \eta \left(T_N^e T_N^p / f^e \right)^{\frac{\theta}{1+\theta}} \sum_k L_k \left(T_k^e T_k^p / f^e \right)^{\frac{1}{1+\theta}} .$$

Finally, we get the real wage by substituting the above relationship and $X_n = w_n \bar{L}_n$ into the price index in (A.6), and using (O.17),

$$\frac{w_n}{P_n} = \zeta \eta^{1/\theta} \left[\left(\frac{F_n}{\bar{L}_n} \right)^{\frac{\theta-\sigma+1}{1-\sigma}} \sum_k \bar{L}_k \left(T_k^e T_k^p / f^e \right)^{\frac{1}{1+\theta}} \right]^{1/\theta} \left(T_n^e T_n^p / f^e \right)^{\frac{1}{1+\theta}} . \tag{O.18}$$

Comparison. To prove our result we simply need to show that (O.16) is larger than (O.18), or equivalently,

$$\left\{\sum_k \left[T_k^p\left(\frac{T_k^e}{f^e}\right)^{-\theta}\right]^{1/(1-\rho)}\right\}^{1-\rho} \geq \left[T_n^p\left(\frac{T_n^e}{f^e}\right)^{-\theta}\right]^{\frac{\theta}{1+\theta}} \sum_j \frac{\bar{L}_j T_j^e}{\sum_k \bar{L}_k T_k^e}\left[T_j^p\left(\frac{T_j^e}{f^e}\right)^{-\theta}\right]^{\frac{1}{1+\theta}}. \quad \text{(O.19)}$$

Note that the right-hand side of this expression is less than or equal to $\max_k T_k^p\left(T_k^e/f^e\right)^{-\theta}$. We can then write the inequality as,

$$\sum_k \left[T_k^p\left(\frac{T_k^e}{f^e}\right)^{-\theta}\right]^{1/(1-\rho)} \geq \left[\max_k T_k^p\left(\frac{T_k^e}{f^e}\right)^{-\theta}\right]^{1/(1-\rho)},$$

which is always true. **QED.**

5 Gains From Openness: Homogeneous Labor

As reported in the paper, the gains from openness are given by

$$GO_n = \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1-\rho}{\theta}}\left(\frac{\sum_l X_{nln}}{X_n}\right)^{-\frac{\rho}{\theta}}\left(\frac{1-\eta}{1-r_n}\right)^{\frac{1}{\kappa}\left(\frac{\sigma}{\sigma-1}-\frac{1}{\theta}\right)}\left(\frac{r_n}{\eta}\right)^{\frac{\kappa-1}{\kappa\theta}}.$$

Letting $\kappa \to \infty$ reorganizing the expression and using $r_n X_n = w_n L_n^e$, the gains from openness can be written

$$GO_n = \left((X_{nnn})^{\rho-1}\left(\sum_l X_{nln}\right)^{-\rho}\frac{w_n L_n^e}{\eta}\right)^{\frac{1}{\theta}}.$$

Free entry requires $w_n L_n^p = \eta \sum_{l,j} X_{nlj}$ so we may rewrite the gains from openness as

$$GO_n = \left(\left(\frac{\sum_{l,j} X_{nlj}}{X_{nnn}}\right)^{1-\rho}\left(\frac{\sum_{l,j} X_{nlj}}{\sum_l X_{nln}}\right)^{\rho}\right)^{\frac{1}{\theta}}.$$

Because $\sum_{l,j} X_{nlj} > X_{nnn}$ and $\sum_{l,j} X_{nlj} > \sum_l X_{nln}$, $GO_n > 1$. When labor within a country is homogeneous, a country cannot lose from openness.

6 Proof Lemma O.1

We first define $l_k = \frac{L_k}{\sum_j L_j}$, $t_i \equiv T_i^e$, and $t \equiv \frac{\sum_j L_j T_j^e}{\sum_k L_k} = \sum_j l_j t_j$.

Lemma O.1 Consider a world with no worker mobility, $\kappa \to 1$, where $A_i = A$, and

$\frac{t_i}{t} < \frac{(\theta+1)(\theta\sigma-\sigma+1)}{(\theta-\sigma+1)}$ $\forall i$, and assume $\rho \to 1$ for all i. The ratio of the real wage under frictionless trade and MP to the real wage under free trade and no MP, $\mathcal{W}_i \equiv W_i^*/W_i$, is given by the expression:

$$(\mathcal{W}_i)^\theta = \frac{[(1-\eta)\, t + \eta t_i]^{v}\, t^{1-v}}{t_i^{\theta/(1+\theta)} \sum_k t_k^{1/(1+\theta)} l_k}\,, \tag{O.20}$$

where $v = \theta/(\sigma-1) - 1$.

Proof: Notice that $\kappa \to 1$, $L_i^p = L_i^e = \overline{L}_i$, and $M_i = \overline{L}_i/f^e$. Because we focus on frictionless trade, we have

$$\Psi_{in} = T_i^e \left\{ \sum_k \left[T_k^p (\gamma_{ik} w_k^p)^{-\theta} \right]^{\frac{1}{1-\rho}} \right\}^{1-\rho} \equiv \Psi_i, \tag{O.21}$$

$$\psi_{iln} = \left[T_i^e T_l^p \left(\gamma_{il} w_l^p \right)^{-\theta} / \Psi_i \right]^{\frac{1}{1-\rho}} \equiv \psi_{il}, \tag{O.22}$$

and

$$\lambda_{in}^E = \frac{M_i \Psi_i}{\sum M_j \Psi_j} = \frac{\overline{L}_i \Psi_i}{\sum \overline{L}_j \Psi_j} \equiv \lambda_i^E.$$

Using the definition of λ_{ln}^T and imposing free trade we have

$$w_n^p = \left[\frac{\lambda_{nn}^T}{\sum_i \left(T_i^e T_n^p \gamma_{in}^{-\theta} / \Psi_{in} \right)^{1/(1-\rho)} \lambda_{in}^E} \right]^{-(1-\rho)/\theta}.$$

Using equation (O.21) and $\lambda_{in}^E = \lambda_i^E$, we can write this expression as

$$w_n^p = \left[\frac{\lambda_n^T}{\left(T_n^p\right)^{1/(1-\rho)} \sum_i \left(T_i^e \gamma_{in}^{-\theta} / \Psi_i \right)^{1/(1-\rho)} \lambda_i^E} \right]^{-(1-\rho)/\theta}. \tag{O.23}$$

We will use this expression and consider separately the two cases: zero MP costs and infinite MP costs.

Frictionless trade but no MP. With frictionless trade and infinite MP costs we have $\lambda_n^T = \lambda_n^E$, and $\Psi_n = T_i^e T_n^p (w_n^p)^{-\theta}$ so that expression (O.23) yields

$$w_n^p = \left[\frac{\lambda_n^T}{\left(T_n^e T_n^p / \Psi_n \right)^{1/(1-\rho)} \lambda_n^E} \right]^{-(1-\rho)/\theta},$$

whereas equation (24) becomes

$$\lambda_l^E = \lambda_l^T = \frac{\overline{L}_l T_l^e T_l^p (w_l^p)^{-\theta}}{\sum_j \overline{L}_j T_j^e T_j^p (w_j^p)^{-\theta}}, \tag{O.24}$$

which gives us

$$\left(\frac{w_i^p}{w_l^p}\right)^{-\theta} = \frac{\lambda_i^T}{\overline{L}_i T_l^p T_i^p} / \frac{\lambda_l^T}{\overline{L}_l T_l^e T_l^p}. \tag{O.25}$$

No MP implies that $X_l = Y_l$, and equation (15) implies that

$$w_l^p L_l^p = (1-\eta)\,\lambda_l^T \sum_n X_n,$$

and thus

$$\frac{w_l^p}{w_i^p} = \frac{\lambda_l^T}{\overline{L}_l} / \frac{\lambda_i^T}{\overline{L}_i}.$$

Combined with $(O.25)$ this equation yields

$$\frac{w_l^p}{w_i^p} = \left(\frac{\overline{L}_i}{\overline{L}_l} \frac{M_l T_l^p T_l^e}{M_i T_i^p T_i^e}\right)^{1/(1+\theta)}. \tag{O.26}$$

Further, combining $(O.24)$, $(O.25)$, and$(O.26)$ yields the following closed form solution for wages:

$$w_i^p = \left(\frac{T_i^e T_i^p}{T_n^e T_n^p}\right)^{\frac{1}{1+\theta}}. \tag{O.27}$$

Using expression $(O.13)$, with $\kappa \to 1$, and noting that with no MP we have $X_{nln} = 0$ except for $l = n$, $\lambda_{nn}^T = X_{nnn}/X_n$, $X_n = Y_n$, and $r_n = \eta$, the expression for welfare is

$$W_n = \zeta \left(\frac{F_n}{\overline{L}_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} \left(T_n^e T_n^p \overline{L}_n / f^e\right)^{1/\theta} \left(\frac{X_{nnn}}{X_n}\right)^{-\frac{1}{\theta}} (1-\eta)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}}.$$

Using expression $(O.24)$, we can write

$$W_n = \zeta \left(\frac{F_n}{\overline{L}_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} (1-\eta)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} \left(\frac{f^e (w_n^p)^{-\theta}}{\sum_j \overline{L}_j T_j^e T_j^p (w_j^p)^{-\theta}}\right)^{-\frac{1}{\theta}}.$$

We can now substitute $(O.27)$ to finally obtain

$$W_n = \zeta \left(\frac{F_n}{\overline{L}_n} \right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} (1-\eta)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} \left(\frac{\sum_j \overline{L}^e_j \left(T^e_j T^p_j \right)^{\frac{1}{1+\theta}}}{f^e \left(T^e_n T^p_n \right)^{-\frac{\theta}{1+\theta}}} \right)^{\frac{1}{\theta}} . \tag{O.28}$$

Frictionless trade and MP. Using (8) and (10) together with the definition of ψ_{iln} and imposing zero MP costs we get

$$\lambda^T_l = \sum_k \left(\frac{T^p_l (w^p_l)^{-\theta}}{\Psi_k} \right)^{\frac{1}{1-\rho}} \frac{M_k \Psi_k}{\sum_j M_j \Psi_j}.$$

But now we have $\Psi_i = \Psi \equiv T^e_i \left[\sum_j \left(T^p_j (w^p_j)^{-\theta} \right)^{\frac{1}{1-\rho}} \right]^{1-\rho}$, hence

$$\lambda^E_i = \frac{\overline{L}_i T^e_i}{\sum_j \overline{L}_j T^e_j} , \tag{O.29}$$

and

$$\lambda^T_l = \frac{\left(T^p_l (w^p_l)^{-\theta} \right)^{\frac{1}{1-\rho}}}{\sum_k \left(T^p_k (w^p_k)^{-\theta} \right)^{\frac{1}{1-\rho}}}. \tag{O.30}$$

Therefore, relative trade shares are

$$\frac{\lambda^T_l}{\lambda^T_i} = \frac{\left(T^p_l \right)^{1/(1-\rho)} (w^p_l)^{-\theta/(1-\rho)}}{\left(T^p_i \right)^{1/(1-\rho)} (w^p_i)^{-\theta/(1-\rho)}}. \tag{O.31}$$

Using (7) and noting that $\lambda^E_{nn} \equiv \frac{\sum_l X_{nln}}{X_n}$, from the definition of λ^E_{in}, and recalling that $\lambda^E_{nn} = \lambda^E_n$, then $(O.13)$ can be rewritten as

$$W^*_n = \zeta \left(\frac{F_n}{\overline{L}_n} \right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} \left(\frac{T^e_n T^p_n \overline{L}_n}{f^e} \right)^{1/\theta} (\psi_{nnn})^{-\frac{1-\rho}{\theta}} \left(\lambda^E_n \right)^{-\frac{1}{\theta}} (1-r_n)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}}$$

Since $\psi_{iln} = \frac{\left(T_l^p\left(w_l^p\right)^{-\theta}\right)^{\frac{1}{1-\rho}}}{\sum_k\left(T_k^p(w_k^p)^{-\theta}\right)^{\frac{1}{1-\rho}}} \equiv \psi_n$, $\lambda_n^E = \frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e}$ and $T_l^p = \tilde{A}^{1-\rho}\left(L_l^p\right)^{1-\rho}$ we have

$$W_n^* = \zeta\left(\frac{F_n}{\overline{L}_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}}\left(\psi_n\right)^{-\frac{1-\rho}{\theta}}\left(\frac{\sum_j \overline{L}_j T_j^e}{f^e/T_n^p}\right)^{\frac{1}{\theta}}\left(1-r_n\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}}. \tag{O.32}$$

We want to find the expression for W_n^* when $\rho \to 1$. We first conjecture that under this limit wages equalize and we *a)* derive an expression for the last parenthetical term of the welfare expression; *b)* show that ψ_n tends to a constant, which is finite and bounded away from zero; and then *c)* show that the wage equalization conjecture is true. Combining these three results, the limit of the expression (O.32) as $\rho \to 1$ is

$$W_n^* = \zeta\left(\frac{F_n}{\overline{L}_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}}\left(\frac{\sum_j \overline{L}_j T_j^e}{f^e/T_n^p}\right)^{\frac{1}{\theta}}\left(\frac{(1-\eta)t}{(1-\eta)t+\eta t_n}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}}. \tag{O.33}$$

a) First show that wages equalize as $\rho \to 1$ and show that

$$1-r_n = \frac{(1-\eta)t}{(1-\eta)t+\eta t_n}.$$

From the free entry condition, we have

$$w_n^e L_n^e = \eta \sum_k \lambda_{ik}^E X_k$$

Using $\lambda_{nk}^E = \frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e}$ and multiplying by X_n we have $r_n = \eta \frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e}\frac{\sum_k X_k}{X_n}$ so

$$1-r_n = 1-\eta\frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e}\frac{\sum_k X_k}{X_n}. \tag{O.34}$$

From the current account balance condition (17) combined with the labor market clearing condition (15), we have

$$w_n^p L_n^p + \eta\sum_k \lambda_{in}^E X_k = X_n,$$

and given that $\lambda_{in}^E = \lambda_i^E = \frac{\overline{L}_i T_i^e}{\sum_j \overline{L}_j T_j^e}$, we obtain

$$\frac{w_n^p L_n^p}{\sum_k X_k} + \eta\frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e} = \frac{X_n}{\sum_k X_k}. \tag{O.35}$$

Substituting this expression into (O.34) and using $\sum_k w_k^p L_k^p = (1-\eta)\sum_k X_k$, we obtain

$$1 - r_n = 1 - \eta \frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e} \frac{1}{(1-\eta)\frac{w_n^p L_n^p}{\sum_k w_k^p L_k^p} + \eta \frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e}}.$$

Finally, imposing wage equalization, $L_n^p = \overline{L}_n$ and then reorganizing the resulting expression using the definitions of t, t_n and l_n yields

$$1 - r_n = \frac{(1-\eta)\, t}{(1-\eta)\, t + \eta t_n}.$$

completing the derivation.

b) Now we want to show that under the condition in the proposition in this limit equilibrium all countries have, $\psi_l > 0$. To show that we compute the limit

$$\begin{aligned}
\lim_{\rho \to 1} \psi_l &= \lim_{\rho \to 1} \frac{\left(T_l^p (w_l^p)^{-\theta}\right)^{\frac{1}{1-\rho}}}{\sum_k \left(T_k^p (w_k^p)^{-\theta}\right)^{\frac{1}{1-\rho}}} \\
&= \lim_{\rho \to 1} \frac{1}{\sum_k \left(\frac{T_k^p (w_k^p)^{-\theta}}{T_l^p (w_l^p)^{-\theta}}\right)^{\frac{1}{1-\rho}}} \\
&= \lim \lambda_l^T.
\end{aligned}$$

Thus, we simply need to construct the trade shares in the case of wage equalization with $\rho \to 1$. The equilibrium conditions in a frictionless equilibrium are the current account balance,

$$X_i = w_i^p L_i^p + \eta \frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e} \sum_n X_n,$$

and labor market clearing,

$$\frac{\theta - \sigma + 1}{\sigma\theta} X_i + (1 - 1/\sigma)\, \lambda_i^T \sum_n X_n = w_i^p L_i^p\ ,$$

with λ_i^T given by (O.30), with $T_i^p = \tilde{A}^{1-\rho}\left(L_i^p\right)^{1-\rho}$. Adding up across the current account balance conditions implies that

$$\sum_k X_k = \frac{1}{1-\eta} \sum_k w_k^p L_k^p.$$

Combining the current account balance with labor market clearing and using this last result together with the expression for λ_i^T implies that

$$\frac{\theta-\sigma+1}{\sigma\theta}\left(w_i^p L_i^p + \frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e}\frac{\eta}{1-\eta}\sum_k w_k^p L_k^p\right) + (1-1/\sigma)\frac{L_i^p (w_i^p)^{-\theta/(1-\rho)}}{\sum_k L_k^p (w_k^p)^{-\theta/(1-\rho)}}\frac{1}{1-\eta}\sum_k w_k^p L_k^p = w_i^p L_i^p$$

$$\implies \frac{\eta}{1-\eta}\frac{\theta-\sigma+1}{\sigma\theta}\frac{\overline{L}_n T_n^e}{\sum_j \overline{L}_j T_j^e} + \frac{1}{\widetilde{\sigma}}\frac{1}{1-\eta}\frac{l_n (w_n^p)^{-\theta/(1-\rho)}}{\sum_k l_k (w_k^p)^{-\theta/(1-\rho)}} = \left(1-\frac{\theta-\sigma+1}{\sigma\theta}\right)\frac{w_n^p L_n^p}{\sum_k w_k^p L_k^p}. \tag{O.36}$$

This system, together with a normalization for wages, gives us a system of N non-linear equations in N unknowns. Since wages are equalized in the limit as $\rho \to 1$, we can let $w_i^p = 1$, and we have from (O.36) that

$$\lambda_n^T = \lim_{\rho\to 1}\frac{l_n (w_n^p)^{-\theta/(1-\rho)}}{\sum_k l_k (w_k^p)^{-\theta/(1-\rho)}} = \widetilde{\sigma}\,(1-\eta)\left(1-\frac{\theta-\sigma+1}{\sigma\theta}\right) l_n - \widetilde{\sigma}\frac{\theta-\sigma+1}{\sigma\theta}\eta\frac{l_n t_n}{t}\ ,$$

and in order for that to be positive we need to assume that

$$\frac{(\sigma\theta-\sigma+1)}{(\theta-\sigma+1)}(1+\theta) > \frac{t_n}{t}. \tag{O.37}$$

This is the condition required in the proposition for interior solution. Notice that, around symmetry, $t_n = t$, since the left-hand side of this equation is always strictly greater than 1.

c) In the last step, we want to show that in the limit as $\rho \to 1$ equilibrium wages are equalized. Equation (O.36) can be rewritten as

$$a_i/l_i + \left\{\frac{w_i^p}{\left[\sum_k l_k (w_k^p)^{-\theta/(1-\rho)}\right]^{-(1-\rho)/\theta}}\right\}^{-\theta/(1-\rho)} = b w_i^p, \tag{O.38}$$

where

$$a_i \equiv \widetilde{\sigma}\frac{\theta-\sigma+1}{\sigma\theta}\eta\frac{t_{ii}}{t} \text{ and } b \equiv \widetilde{\sigma}\,(1-\eta)\left(1-\frac{\theta-\sigma+1}{\sigma\theta}\right).$$

Assumption $(O.37)$ is then

$$0 < b - t_i/l_i.$$

Since, given the normalization of one wage to one, $\max w_v^p \geq 1$, so that letting $j = \arg\max_v w_v^p$, we then have $b \max w_v^p - a_j/l_j > 0$, and $(O.38)$ implies that

$$\frac{\max w_v^p}{\left[\sum_k l_k (w_k^p)^{-\theta/(1-\rho)}\right]^{-(1-\rho)/\theta}} = \left(b \max w_v^p - a_j/l_j\right)^{-(1-\rho)/\theta}\ . \tag{O.39}$$

Note that

$$\lim_{\rho\to 1}\left\{\left[\sum_k l_k (w_k^p)^{-\theta/(1-\rho)}\right]^{-(1-\rho)/\theta}\right\} = \min w_k^p,$$

and thus the left-hand side of (O.39) when we take the limit is,

$$\lim_{\rho\to 1}\left\{\frac{\max w_v^p}{\left[\sum_k l_k (w_k^p)^{-\theta/(1-\rho)}\right]^{-(1-\rho)/\theta}}\right\} = \lim_{\rho\to 1}\frac{\max w_v^p}{\min w_v^p}.$$

In addition, taking the limit on the right-hand side of (O.39)

$$\lim_{\rho\to 1}\left\{\left(b\max w_v^p - a_j/l_j\right)^{-(1-\rho)/\theta}\right\} = 1\ ,$$

since $b\max w_v^p - a_j/l_j$ is bounded away from zero and must be $b\max w_v^p - a_j/l_j \le 1$ since the left-hand-side of (O.39) is always greater or equal to 1. Hence, taking limits of $(O.39)$ we have

$$\lim_{\rho\to 1}\left(\frac{\max w_v^p}{\min w_v^p}\right) = 1.$$

which means that wages equalize.

We have completed the derivations of the two analytical equations for no MP and frictionless MP. Combining equations $(O.28)$ and $(O.33)$ we obtain

$$\begin{aligned}
\mathcal{W}_n &= \left(\frac{\left(T_n^e\right)^{-\frac{\theta}{1+\theta}}\left(T_n^p\right)^{\frac{1}{1+\theta}}\sum_j \overline{L}_j T_j^e}{\sum_j \overline{L}_j^e\left(T_j^e\right)^{-\frac{1}{1+\theta}}\left(T_j^p\right)^{\frac{1}{1+\theta}}}\right)^{\frac{1}{\theta}}\left(\frac{t}{(1-\eta)t+\eta t_i}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} \Longrightarrow \\
\mathcal{W}_n &= \left(\frac{\left(L_n^{1-\rho}\right)^{\frac{1}{1+\theta}} t}{t_n^{\frac{\theta}{1+\theta}}\sum_j l_j t_j^{-\frac{1}{1+\theta}}\left(L_j^{1-\rho}\right)^{\frac{1}{1+\theta}}}\right)^{\frac{1}{\theta}}\left(\frac{t}{(1-\eta)t+\eta t_i}\right)^{\frac{\sigma-1-\theta}{\theta(\sigma-1)}} \Longrightarrow \\
\mathcal{W}_n^\theta &= \frac{\left((1-\eta)t+\eta t_i\right)^v t^{1-v}}{t_n^{\frac{\theta}{1+\theta}}\sum_j l_j t_j^{\frac{1}{1+\theta}}}
\end{aligned}$$

which is expression (O.20), with $v \equiv \theta/\left(\sigma-1\right)-1$ where notice that under symmetry $\mathcal{W}_n = 1$. This last derivation completes the proof of the Lemma. **Q.E.D.**

7 Gains from Openness: Unilateral MP Liberalization

Proposition O.1 *Consider a two-country world with perfect worker mobility (i.e., $\kappa \to \infty$) and with frictionless trade. Assume that $A_1 = A_2$, $T_1^e = T_2^e$, and $L_1 = L_2$ and that countries are not fully specialized in innovation or production. Let $\gamma^* \equiv (2\theta - 1)^{(1-\rho)/\theta}$ and assume that $\gamma_{21} < \gamma^*$, $\gamma_{12} \leq \gamma_{21}$. Then country 1 gains when γ_{21} increases.*

Proof: We will derive an expression for the real wage in country 1, w_1/P_1, and then use that expression to prove part (ii). Notice that given that $\omega \equiv w_1^p/w_2^p$ we can normalize the wage of country 2, $w_2^p = 1$, so that $\omega = w_1^p$. Also, frictionless trade implies that the price index is the same across the two countries, $P_1 = P_2 \equiv P$.

We first compute the price index. Irrespective of MP costs, under frictionless trade profits in country i are $\eta L \left(w_1^p + w_2^p\right) \lambda_i^E$, where $L \equiv L_1 = L_2$. But frictionless trade also implies that

$$\lambda_i^E = \frac{M_i \Psi_i}{M_1 \Psi_1 + M_2 \Psi_2},$$

hence free entry requires

$$w_i^p = \eta \frac{L}{f^e} \left(w_1^p + w_2^p\right) \frac{\Psi_i}{M_1 \Psi_1 + M_2 \Psi_2}.$$

Adding up wages we get

$$w_1^p + w_2^p = \eta \frac{L}{f^e} \left(w_1^p + w_2^p\right) \frac{\Psi_1 + \Psi_2}{M_1 \Psi_1 + M_2 \Psi_2},$$

and hence

$$M_1 \Psi_1 + M_2 \Psi_2 = \eta \frac{L}{f^e} \left(\Psi_1 + \Psi_2\right). \tag{O.40}$$

Using (O.40) and the definition of the price index, equation (A.6), we have that

$$P = \zeta^{-1} \left[\left(\frac{F}{L}\right)^{(\theta - \sigma + 1)/(1-\sigma)} \eta \frac{L}{f^e} \right]^{-1/\theta} \left(\Psi_1 + \Psi_2\right)^{-1/\theta}. \tag{O.41}$$

By the definition of Ψ_i we have

$$\Psi_1 + \Psi_2 = T \left[\omega^{-\frac{\theta}{1-\rho}} + \left(\gamma_{12}\right)^{-\frac{\theta}{1-\rho}}\right]^{1-\rho} + T \left[\left(\omega \gamma_{21}\right)^{-\frac{\theta}{1-\rho}} + 1\right]^{1-\rho}. \tag{O.42}$$

Using the definition of $\omega = \Psi_1/\Psi_2$ we can show that $\partial \ln \omega / \partial \ln \gamma_1 > 0$ (see part i). Also, manipulating (O.42), we get

$$\Psi_1+\Psi_2 = T\left[\omega^{-\frac{\theta}{1-\rho}}+(\gamma_{12})^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}+T\left[(\omega\gamma_{21})^{-\frac{\theta}{1-\rho}}+1\right]^{1-\rho} =$$

$$= T\left[\omega^{-\frac{\theta}{1-\rho}}+(\gamma_{12})^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}+T\left[\omega^{-\frac{\theta}{1-\rho}}+(\gamma_{12})^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}/\omega \tag{O.43}$$

$$= T\left[\omega^{-\frac{\theta}{1-\rho}}+(\gamma_{12})^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\left(1+\frac{1}{\omega}\right) \tag{O.44}$$

We can now write the real wage of country 1, using (O.41), as

$$\begin{aligned}\frac{w_1}{P} &= \frac{\omega}{\zeta^{-1}\left[\left(\frac{F}{L}\right)^{(\theta-\sigma+1)/(1-\sigma)}\eta\frac{L}{f^e}\right]^{-1/\theta}\left\{T\left[\omega^{-\frac{\theta}{1-\rho}}+(\gamma_{12})^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\left(1+\frac{1}{\omega}\right)\right\}^{-1/\theta}} \\ &= \frac{\left\{\left[\omega^{-\frac{\theta}{1-\rho}}\omega^{\frac{\theta}{1-\rho}}+\omega^{\frac{\theta}{1-\rho}}(\gamma_{12})^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\left(1+\frac{1}{\omega}\right)\right\}^{1/\theta}}{\zeta^{-1}\left[\left(\frac{F}{L}\right)^{(\theta-\sigma+1)/(1-\sigma)}\eta\frac{L}{f^e}\right]^{-1/\theta}T^{-1/\theta}} \\ &= \frac{\left\{\left[1+\omega^{\frac{\theta}{1-\rho}}(\gamma_{12})^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\left(1+\frac{1}{\omega}\right)\right\}^{1/\theta}}{\zeta^{-1}\left[\left(\frac{F}{L}\right)^{(\theta-\sigma+1)/(1-\sigma)}\eta\frac{L}{f^e}\right]^{-1/\theta}T^{-1/\theta}}\end{aligned}$$

Thus, changes in the welfare in country 1 are given by

$$\frac{\partial\left(w_1^p/P\right)}{\partial\gamma_1}=\frac{\partial\left\{\left[1+\omega^{\frac{\theta}{1-\rho}}(\gamma_{12})^{-\frac{\theta}{1-\rho}}\right]^{1-\rho}\left(1+\frac{1}{\omega}\right)\right\}}{\partial\omega}\frac{\partial\omega}{\partial\gamma_1}\,.$$

Since, from part (i) we know that $\partial\omega/\partial\gamma_1>0$, it suffices to show that the first term is positive. But the sign of this first term is determined by the sign of

$$\theta\frac{\omega^{\frac{\theta}{1-\rho}}(\gamma_{12})^{-\frac{\theta}{1-\rho}}}{1+\omega^{\frac{\theta}{1-\rho}}(\gamma_{12})^{-\frac{\theta}{1-\rho}}}-\frac{1}{\omega+1}\,. \tag{O.45}$$

Notice that (O.45) is increasing in ω, thus if we show that is positive for $\gamma_{12}=\gamma_{21} \implies \omega=1$ it will be satisfied for any $\gamma_{21}>\gamma_{12} \implies \omega>1$. We will show that if γ_2 satisfies a certain condition, this expression is positive and thus $\frac{\partial\left(w_1^p/P\right)}{\partial\gamma_{21}}>0$. Notice that for (O.45) to be positive we need

$$
\begin{aligned}
\theta\omega^{\frac{\theta}{1-\rho}}\left(\gamma_{12}\right)^{-\frac{\theta}{1-\rho}}(\omega+1) &> 1+\omega^{\frac{\theta}{1-\rho}}\left(\gamma_{12}\right)^{-\frac{\theta}{1-\rho}} \implies \\
\omega^{\frac{\theta}{1-\rho}}\left[\theta(\omega+1)-1\right] &> \left(\gamma_{12}\right)^{\frac{\theta}{1-\rho}}
\end{aligned}
$$

For $\gamma_{12}=\gamma_{21} \implies \omega=1$ and as long as $\gamma_{12}<\gamma^*$,

$$
\gamma^* \equiv (2\theta-1)^{(1-\rho)/\theta} .
$$

the inequality is satisfied, proving the second part of the proposition. **QED**.

8 Anti Home Market Effects in a Two-Sector Trade Model

Consider two countries the Home and the Foreign, denoted by H and F, respectively. Labor is the only factor of production and $L_H < L_F$. There are two goods and consumers in the two countries have symmetric Cobb-Douglas preferences over the goods, with α of total expenditure devoted to the ek-good, and a share $1-\alpha$ of expenditure devoted to the k-good.

Each country can potentially produce both goods. The ek good, is produced under perfect competition and is composed of a continuum of varieties $\omega_{ek} \in [0,1]$ that are aggregated in a CES fashion with an elasticity of substitution $\sigma_{ek} > 1$. These varieties are produced in country $i = H, F$ with a linear technology and productivity $z_i(\omega)$. There is an iceberg cost of shipping the ek-good $\tau_{in} \geq 1$, for $i \neq n$ and $\tau_{ii} = 1$. Productivity is drawn from a Fréchet distribution of the form $F_i = e^{T_i z^{-\theta}}$, with $T_i > 0$ and $\theta > 1$. We assume that $T_H/L_H = T_F/L_F = 1$. The technology in this sector is identical to the technology postulated in the perfect competition setup of Eaton & Kortum (2002).

The k-good is produced using a continuum of differentiated varieties, aggregated CES with elasticity of substitution $\sigma_k > 1$. Each variety in country $i = H, F$ is produced with a linear production function using labor and with identical productivity across countries. In order to produce a firm has to incur a fixed cost of entry f^e. There is an iceberg cost of shipping the k-good, $\gamma_{in} \geq 1$, for $i \neq n$, and $\gamma_{ii} = 1$. Firms are homogeneous and compete monopolistically and there is free entry into this sector. The (endogenous) number of firms is denoted by M_i, $i = H, F$. The technology in this sector is identical to the technology postulated in the monopolistic competition setup of Krugman (1980).

The equilibrium is defined as firm entry, M_i, and wages, w_i, for the two countries $i = H, F$ such that labor markets clear and free entry drives profits to zero in both countries.

We prove the following Lemma:

Lemma O.2 *Assume that $\gamma_{HF} = \gamma_{FH} = 1$, $\tau_{HF} = \tau_{FH} > 1$, and $L_H > L_F$. Then the smaller country specializes in the Krugman sector.*

Proof: First, notice the free entry condition implies that the equilibrium entry in the Krugman sector is given by

$$M_i = \frac{r_i}{\sigma_k f^e} L_i, \; i = H, F. \tag{O.46}$$

Thus, the market shares for each of the countries in the two sectors can be written as

$$\lambda_{in}^{ek} = \frac{T_i \left(w_i \tau_{in}\right)^{-\theta}}{\sum_k T_k \left(w_k \tau_{kn}\right)^{-\theta}} = \frac{L_i \left(w_i \tau_{in}\right)^{-\theta}}{\sum_k L_k \left(w_k \tau_{kn}\right)^{-\theta}},$$

$$\lambda_{in}^{k} = \frac{M_i \left(w_i \gamma_{in}\right)^{1-\sigma}}{\sum_k M_k \left(w_k \gamma_{kn}\right)^{1-\sigma}} = \frac{r_i L_i \left(w_i \gamma_{in}\right)^{1-\sigma}}{\sum_k r_k L_k \left(w_k \gamma_{kn}\right)^{1-\sigma}} .$$

Using those expressions we now have to solve for equilibrium wages, w_i, and entry r_i, using the three of the four labor market clearing conditions for the two sectors and one wage normalization, say $w_H = 1$. Therefore, the equilibrium conditions for r_H , r_L and w_H and w_F are $w_H = 1$ and

$$w_F \left(1 - r_F\right) L_F = \alpha \sum_n \lambda_{Fn}^{ek} w_n L_n, \tag{O.47}$$

$$w_i r_i L_i = \left(1 - \alpha\right) \sum_n \lambda_{in}^{k} w_n L_n \text{ for } i = H, F. \tag{O.48}$$

We can now prove the lemma. We start by proving that wages equalize in both countries when $\gamma_{in} = 1$. Using the equilibrium condition for the k-sector,

$$\left(w_i\right)^{\sigma} = \left(1 - \alpha\right) \frac{\sum_n w_n L_n}{\sum_k r_k L_k \left(w_k\right)^{1-\sigma}} \text{ for } i = H, F \,,$$

which implies that $w_H = w_F$. Imposing the normalization and using the equation above we obtain a condition

$$\sum_k r_k L_k = \left(1 - \alpha\right) \sum_n L_n \,, \tag{O.49}$$

which combined with (O.47) can be used, in turn, to solve for r_i, $i = H, F$, and thus

$$r_F = 1 - \alpha \sum_n \frac{\left(\tau_{Fn}\right)^{-\theta} L_n}{\sum_k L_k \left(\tau_{kn}\right)^{-\theta}} . \tag{O.50}$$

We substitute (O.50) into (O.49) to obtain

$$r_H L_H + \left(1 - \alpha \sum_n \frac{(\tau_{Fn})^{-\theta} L_n}{\sum_k L_k (\tau_{kn})^{-\theta}}\right) L_F = (1-\alpha) \sum_n L_n \implies \tag{O.51}$$

$$r_H = (1-\alpha) + \alpha \frac{L_F}{L_H} \left(-1 + \frac{1}{\sum_k \frac{L_k}{L_F} (\tau_{kF})^{-\theta}} + \frac{(\tau_{FH})^{-\theta}}{\sum_k \frac{L_k}{L_H} (\tau_{kH})^{-\theta}}\right) \tag{O.52}$$

$$r_H = (1-\alpha) + \alpha \frac{L_F}{L_H} \left(\frac{-\frac{L_H}{L_F} (\tau_{HF})^{-\theta}}{1 + \frac{L_H}{L_F} (\tau_{HF})^{-\theta}} + \frac{(\tau_{FH})^{-\theta}}{1 + \frac{L_F}{L_H} (\tau_{FH})^{-\theta}}\right) . \tag{O.53}$$

We show that $L_H > L_F$ implies that $r_H < 1 - \alpha$, under symmetry of trade costs. Notice that this term is negative iff

$$\begin{aligned}
(\tau_{FH})^{-\theta} \left(1 + \frac{L_H}{L_F} (\tau_{HF})^{-\theta}\right) &< \frac{L_H}{L_F} (\tau_{HF})^{-\theta} \left(1 + \frac{L_F}{L_H} (\tau_{FH})^{-\theta}\right) \implies \\
(\tau_{FH})^{-\theta} + \frac{L_H}{L_F} (\tau_{HF})^{-\theta} (\tau_{FH})^{-\theta} &< \frac{L_H}{L_F} (\tau_{HF})^{-\theta} + (\tau_{HF})^{-\theta} (\tau_{FH})^{-\theta} \implies \\
\frac{L_H}{L_F} \frac{\left((\tau_{HF})^{-\theta} (\tau_{FH})^{-\theta} - (\tau_{HF})^{-\theta}\right)}{\left((\tau_{HF})^{-\theta} (\tau_{FH})^{-\theta} - (\tau_{FH})^{-\theta}\right)} &> 1 ,
\end{aligned}$$

and under symmetry of trade costs, the last parenthetical term is negative iff $L_H > L_F$, proving the result. It is straightforward to prove that $r_F > 1 - \alpha$ using equation (O.49). **QED**

9 Plant-Level Fixed Location Costs: The case with $\rho = 0$

In this Section we discuss the incorporation of plant-level fixed production cost under the special case with $\rho = 0$. While we maintain the rest of the assumptions of the model, we assume that for firms from i to open an affiliate in l there is a fixed cost φ_l in units of production labor of country l. For convenience, we write this cost as $\varphi_l = v_l F_l$.

We show that fixed costs of opening a plant can be incorporated into the special case of our model with $\rho = 0$ in such a way that the resulting extension is isomorphic to the existing model without plant-level fixed costs.

As noted in the discussion of the multivariate Pareto distribution, for this parameterization an entrant from a country i will draw a single country where it can produce with the probability of this country being l is given by $T_l^p / \sum_k T_k^p$ and the productivity level given by a Pareto distribution with shape parameter θ. Additionally, we consider only equilibria in which the following restriction is satisfied:

$$(1+v_l)\,\frac{w_l^p F_l/X_l}{w_n^p F_n/X_n} < \left(\frac{\tau_{ln} P_l}{P_n}\right)^{\sigma-1} \quad \text{for all } i,l \text{ and } n \neq l. \tag{O.54}$$

Note that under symmetry, this condition becomes $1+v < \tau^{\sigma-1}$. Hence, this condition naturally extends a similar condition in Helpman, Melitz & Yeaple (2004) to the case of asymmetric countries.

Given these assumptions, we first show that the equilibrium is such that all affiliates in l sell in l and that there is a positive measure of those affiliates that do not sell in n for all $n \neq l$ – that is, there is selection of foreign affiliates into all export markets. We begin by defining the variable profits for a firm from i with productivity z_l producing in l and selling to n, which are given by

$$\widetilde{\pi}_{iln}(z_l) = \frac{1}{\sigma}\left(\frac{\widetilde{\sigma}\xi_{iln}}{z_l}\right)^{1-\sigma} P_n^{\sigma-1} X_n.$$

The productivity cutoff for a domestic firm to export to country n, denoted here by z_{ln}^T, are defined implicitly by $\widetilde{\pi}_{lln}(z_{ln}^T) = w_n^p F_n$. From the definition of variable profits, the cutoff is given by

$$z_{ln}^T = \left(\frac{\sigma w_n^p F_n}{X_n}\right)^{1/(\sigma-1)} \frac{\widetilde{\sigma}\xi_{lln}}{P_n}.$$

Because the variable profits of any foreign firm from $i \neq l$ satisfy $\widetilde{\pi}_{iln}(z_l) = \gamma_{il}^{1-\sigma}\widetilde{\pi}_{lln}(z_l)$, the cutoff productivity for all firms from i that have a plant in country l to sell to country n is $z_{iln}^T \equiv \gamma_{il} z_{ln}^T$.

Now, define $\pi_{il}(z_l)$ to be the profits net of marketing costs (but gross of fixed investment costs) for a firm from i with productivity z_l producing in l,

$$\pi_{il}(z_l) = \sum_n \max\left\{\widetilde{\pi}_{iln}(z_l) - w_n^p F_n, 0\right\}.$$

The MP productivity cutoffs for firms from i to open an affiliate in country l, z_{il}^{MP}, are defined implicitly by

$$\pi_{il}(z_{il}^{MP}) = w_l^p \varphi_l.$$

We can now prove the following lemma that establishes that there exists affiliates that do not export:

Lemma O.3 *If condition (O.54) is satisfied, then $z_{il}^{MP} < z_{iln}^T$ for all i,l and $n \neq l$.*

Proof. Consider a firm from i in l with productivity

$$z_{il}^{*}=\gamma_{il}\left[\sigma w_{l}^{p}\left(F_{l}+\varphi_{l}\right)/X_{l}\right]^{1/(\sigma-1)}\frac{\widetilde{\sigma}w_{l}}{P_{l}}.$$

Because $\widetilde{\pi}_{ill}(z_{il}^{*})-w_{l}^{p}F_{l}=w_{l}^{p}\varphi_{l}$ by construction, this firm would make zero profits, and so would break even, if it sells only in market l. Now consider the additional profits that could be earned by selling in market $n\neq l$:

$$\begin{aligned}
\widetilde{\pi}_{iln}(z_{il}^{*})-w_{n}^{p}F_{ln} &= \frac{1}{\sigma}\left(\widetilde{\sigma}\xi_{iln}/z_{il}^{*}\right)^{1-\sigma}P_{n}^{\sigma-1}X_{n}-w_{n}^{p}F_{n}\\
&= w_{n}^{p}F_{n}\left(\left(\frac{\widetilde{\sigma}\gamma_{il}w_{l}^{p}\tau_{ln}}{z_{il}^{*}P_{n}}\right)^{1-\sigma}\frac{X_{n}}{\sigma w_{n}^{p}F_{n}}-1\right)\\
&= w_{n}^{p}F_{n}\left(\frac{\tau_{ln}P_{l}}{P_{n}}\right)^{1-\sigma}\left(\frac{w_{l}^{p}\left(F_{l}+\varphi_{l}\right)/X_{l}}{w_{n}^{p}F_{n}/X_{n}}-\left(\frac{\tau_{ln}P_{l}}{P_{n}}\right)^{\sigma-1}\right)<0,
\end{aligned}$$

where the inequality at the end follows from the assumption in the lemma. This implies that

$$\sum_{n}\max\left\{\widetilde{\pi}_{iln}(z_{il}^{*})-w_{n}^{p}F_{n},0\right\}=\widetilde{\pi}_{ill}(z_{il}^{*})-w_{l}^{p}F_{l}=w_{l}^{p}\varphi_{l}$$

and hence

$$z_{il}^{MP}=z_{il}^{*}=\gamma_{il}\left[\sigma w_{l}^{p}\left(F_{l}+\varphi_{l}\right)/X_{l}\right]^{1/(\sigma-1)}\frac{\widetilde{\sigma}w_{l}^{p}}{P_{l}}.$$

Finally, $z_{il}^{MP}<z_{iln}^{T}$ then follows directly from the definition of z_{iln}^{T} and the assumption in the lemma. ■

The next step is to solve for price indices as a function of entry levels and wages. Integrating using the Pareto distribution of entrants and substituting for the cutoffs for exporting, the price indices must satisfy

$$\begin{aligned}
P_{n}^{1-\sigma} &= \sum_{i}\sum_{l\neq n}M_{i}T_{il}\left(\widetilde{\sigma}\gamma_{il}w_{l}^{p}\tau_{ln}\right)^{1-\sigma}\left(\frac{\theta}{\sigma-\theta-1}\right)\left(\left(\frac{\sigma w_{n}^{p}F_{n}}{X_{n}}\right)^{1/(\sigma-1)}\frac{\widetilde{\sigma}\gamma_{in}w_{n}^{p}\tau_{ln}}{P_{n}}\right)^{\sigma-\theta-1}\\
&\quad+\sum_{i}M_{i}T_{in}\left(\widetilde{\sigma}\gamma_{in}w_{n}^{p}\right)^{1-\sigma}\left(\frac{\theta}{\sigma-\theta-1}\right)\left(\left[\frac{\sigma w_{n}^{p}F_{n}}{X_{n}}\left(1+v_{n}\right)\right]^{1/(\sigma-1)}\frac{\widetilde{\sigma}\gamma_{in}w_{n}^{p}}{P_{n}}\right)^{\sigma-\theta-1}
\end{aligned}$$

Note that we have substituted $\varphi_{l}=v_{l}F_{l}$ to obtain this expression. Letting $\mu_{ll}=1$ and $\mu_{ln}=0$

if $l \neq n$, we can solve for P_n, which can be written compactly as

$$P_n^{-\theta} = \left(\frac{\theta}{\theta-(\sigma-1)}\right)\left(\frac{\sigma w_n^p F_n}{X_n}\right)^{1-\theta/(\sigma-1)} \sum_{i,l} M_i T_{il}\left(\widetilde{\sigma}\gamma_{il} w_l^p \tau_{ln}\right)^{-\theta}\left(1+\mu_{ln} v_n\right)^{1-\theta/(\sigma-1)}. \tag{O.55}$$

Now, integrating over the sales of the individual exporters that originate from i and sell to n from country l, we obtain

$$X_{iln} = M_i T_{il}\left(\widetilde{\sigma}\xi_{iln}\right)^{1-\sigma} P_n^{\sigma-1} X_n \left(\frac{\theta}{\theta-(\sigma-1)}\right)\left(\gamma_{il} z_{ln}^T\right)^{\sigma-\theta-1} \quad \text{for } n \neq l$$

and

$$X_{ill} = M_i T_{il}\left(\widetilde{\sigma}\xi_{ill}\right)^{1-\sigma} P_l^{\sigma-1} X_l \left(\frac{\theta}{\theta-(\sigma-1)}\right)\left(\gamma_{il} z_{ll}^{MP}\right)^{\sigma-\theta-1}.$$

Substituting for the price index using (O.55), we have

$$X_{iln} = \frac{M_i T_{il}\left(\xi_{iln}\right)^{-\theta}\left(1+\mu_{ln} v_n\right)^{1-\theta/(\sigma-1)}}{\sum_{j,k} M_k T_{jk}\left(\gamma_{jkl} w_k^p \tau_{kn}\right)^{-\theta}\left(1+\mu_{kn} v_n\right)^{1-\theta/(\sigma-1)}} X_n$$

for all n and l.

Finally, the equilibrium (with $\Delta = 0$) is a set of firms, production worker wages, and innovation worker wages for each country, $\boldsymbol{M}, \boldsymbol{w^p}, \boldsymbol{w^e}$ such that the market for innovation labor clears,

$$\eta \sum_{n,l} X_{iln} = w_i^e L_i^e, \tag{O.56}$$

and the market for production labor clears,

$$w_l^p L_l^p = \frac{1}{\widetilde{\sigma}} \sum_{i,n} X_{iln} + \left(1-\eta-\frac{1}{\widetilde{\sigma}}\right) X_l. \tag{O.57}$$

Note that these two equations are the same as in the case without fixed costs up to the calculation of X_{iln}, which we now address.[1]

To link the equilibrium without fixed costs to the one with fixed costs, let τ_{ln} and T_l^p be the parameters for trade costs and productivity in the model without fixed costs and denote with tildes the ones in the world with fixed costs. Then, in the world without fixed costs, we

[1]One may have thought that we needed to subtract $M_{il} w_l \varphi_l$ from $\eta \sum_{n,l} X_{iln}$, but recall that ηX_{ill} are the profits net of marketing costs for firms from i producing in l and selling domestically in l. When the cutoff is determined by $w_l^p \left(F_l + \varphi_l\right)$ rather than $w_l^p F_l$, then this means that ηX_{ill} is already net of $w_l^p \left(F_l + \varphi_l\right)$, and hence it should not be subtracted again.

have

$$X_{iln} = \frac{M_i T_i^e T_l^p \left(\gamma_{il} w_l^p \tau_{ln}\right)^{-\theta}}{\sum_{j,k} M_k T_j^e T_k^p \left(\gamma_{jkl} w_k^p \tau_{kn}\right)^{-\theta}} X_n,$$

and in the world with fixed costs, we have

$$X_{iln} = \frac{M_i T_i^e \tilde{T}_l^p \left(\gamma_{il} w_l^p \tilde{\tau}_{ln}\right)^{-\theta} \left(1 + \mu_{ln} v_n\right)^{1-\theta/(\sigma-1)}}{\sum_{j,k} M_k T_j^e \tilde{T}_k^p \left(\gamma_{jkl} w_k^p \tilde{\tau}_{kn}\right)^{-\theta} \left(1 + \mu_{kn} v_n\right)^{1-\theta/(\sigma-1)}} X_n.$$

Thus, if we set

$$\tilde{T}_l^p \equiv T_l^p$$

and

$$\tilde{\tau}_{ln} \equiv \frac{\tau_{ln}}{\left(1 + \mu_{ln} v_n\right)^{1/(\sigma-1)-1/\theta}}$$

then the two equilibria yield the same $(\mathbf{M}, \mathbf{w}^p, \mathbf{w}^e)$. Note that

$$\tilde{\tau}_{ll} \equiv \tau_{ll} = 1 \text{ for all } l$$

and

$$\tilde{\tau}_{ln} \equiv \tau_{ln} \left(1 + v_n\right)^{1/(\sigma-1)-1/\theta} > 1 \text{ for } l \neq n,$$

where the inequality is strict for all $v_n > 0$ since $\theta > \sigma - 1$ by assumption. Note that prices are also the same in the two equilibria. Prices in the equilibrium without fixed costs are given by

$$P_n^{-\theta} = \left(\frac{\theta}{\theta - (\sigma - 1)}\right) \left(\sigma \frac{w_n^p F_n}{X_n}\right)^{1-\theta/(\sigma-1)} \sum_{i,l} M_i T_i^e T_l^p \left(\widetilde{\sigma} \gamma_{il} w_l^p \tau_{ln}\right)^{-\theta},$$

while in the equilibrium with fixed costs we would have

$$P_n^{-\theta} = \left(\frac{\theta}{\theta - (\sigma - 1)}\right) \left(\sigma \frac{w_n^p F_n}{X_n}\right)^{1-\theta/(\sigma-1)} \sum_{i,l} M_i T_i^e \tilde{T}_l^p \left(\widetilde{\sigma} \gamma_{il} w_l^p \tilde{\tau}_{ln}\right)^{-\theta} \left(1 + \mu_{ln} v\right)^{1-\theta/(\sigma-1)}.$$

Because $(\mathbf{M}, \mathbf{w}^p, \mathbf{w}^e)$ are the same and given the definition of the variables with tildes, then prices must also be the same. Finally, the gravity relationships of the two models are also the same as long as "own country" pairs are excluded.

Finally, because our counterfactuals entail moving $\tau's$ or $\gamma's$ by a certain percent change, the counterfactual implications of a percent change in the $\tau's$ (with no transformation of $\gamma's$) are the same if we lived in a world with fixed investment costs as long as the key inequality

(O.54) holds.

10 Isomorphism

In this Section we present a formal isomorphism of our model where the firm has a multivariate Pareto productivity to one where each firm's productivity in a location is the product of a core productivity and a location-specific efficiency shock as in Tintelnot (2017). The results are independent from the assumptions on labor mobility across sectors.

10.1 Environment

The basic environment of the model with location-specific productivity shocks is the same as in the main paper. The difference is that a firm's productivity is the product of two random variables: a "core productivity" parameter ϕ and a vector of location specific productivity adjustment parameters, $\mathbf{z} = (z_1, z_2, ..., z_N)$. A firm with productivity variables ϕ and $\mathbf{z}$ producing in country l has labor productivity $\phi \times z_l$.

These assumptions imply that a firm with productivity ϕ and $\mathbf{z}$ producing in location l to serve market n has a unit cost given by $w_l^p \tau_{ln} / (\phi z_l)$. Such a firm chooses to serve country n from the cheapest production location l, charging a price

$$\begin{aligned} p_{in} &= \min \{p_{iln}\} \\ &\frac{\sigma}{\sigma - 1} \min_l \left\{ \frac{\gamma_{il} w_l^p \tau_{ln}}{\phi z_l} \right\}. \end{aligned}$$

We assume that ϕ is drawn from a Pareto distribution,

$$\phi \sim F_i(\phi) = 1 - \left(\frac{b_i}{\phi}\right)^{\kappa},$$

where $\kappa + 1 - \sigma > 0$, while z_l for firms from i are drawn i.i.d from a Fréchet distribution with parameters θ and T_{il},

$$z_l \sim e^{-T_{il} z^{-\theta}}.$$

We again assume that firms incur a destination-specific fixed cost $w_n^p F_{in}$ in order to have the possibility of serving market n. In addition, we assume that these fixed costs are paid before the vector of location-specific efficiency shocks $\mathbf{z}$ is observed, but knowing the firm's core productivity ϕ. We assume that firms have to decide ex-ante how many markets they might end up serving; once, the vector $\mathbf{z}$ is observed, the set of possible markets to serve is given.

Hence, firms choose to pay the entry cost to the destination market n if expected profits are larger than entry costs. Firms make this calculation market-by-market.

10.2 Firm's Problem

The entry decision into each market n gives us a threshold productivity level ϕ_{in}^* for which firms from country i with $\phi \geq \phi_{in}^*$ pay the fixed cost $w_n^p F_{in}$ and serve n and firms with $\phi < \phi_{in}^*$ do not. To derive ϕ_{in}^*, imagine first that a firm from i with productivity vector $(\phi, \mathbf{z})$ is forced to supply a country n from l. The revenues of this firm would be $(P_n / p_{iln})^{\sigma-1} X_n$. Let $v_{iln} \equiv \frac{\sigma-1}{\sigma} \phi p_{iln}$. Expected profits (gross of the fixed marketing cost) for such a firm are

$$P_n^{\sigma-1} (\tilde{\sigma})^{1-\sigma} \phi^{\sigma-1} (X_n / \sigma) E(v_{iln}^{1-\sigma}),$$

where

$$\tilde{\sigma} = \frac{\sigma}{\sigma - 1}.$$

Letting $G(z; T) \equiv 1 - e^{-Tz^\theta}$, then we know that v_{iln} is distributed according to $G(v_{iln}; (\gamma_{il} w_l \tau_{ln})^{-\theta} T_{il})$. Next, notice that for every $c \geq 1$ we have

$$[E(v_{iln}^c)]^{1/c} = \left[\int_0^\infty v^c dG(v; (\gamma_{il} w_l \tau_{ln})^{-\theta} T_{il}) \right]^{1/c} = \Gamma \left(\frac{\theta + c}{\theta} \right)^{1/c} \left[(\gamma_{il} w_l \tau_{ln})^{-\theta} T_{il} \right]^{-1/\theta}, \tag{O.58}$$

where we denote $\gamma_c = \Gamma \left(\frac{\theta+c}{\theta} \right)^{1/c}$.

This result implies that expected profits in this case are

$$(\tilde{\sigma})^{1-\sigma} P_n^{\sigma-1} \phi^{\sigma-1} (X_n / \sigma) E(v_{iln}^{1-\sigma}) = P_n^{\sigma-1} \phi^{\sigma-1} (X_n / \sigma) \tilde{\gamma} \left[\left(\gamma_{il} w_l^p \tau_{ln} \right)^{-\theta} T_{il} \right]^{(\sigma-1)/\theta},$$

where

$$\tilde{\gamma} = (\tilde{\sigma})^{1-\sigma} \Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right).$$

In fact firms in i that are not forced to supply a country n from l, they will choose the production location that minimizes delivery cost, hence

$$E \left[\max_l P_n^{\sigma-1} (\tilde{\sigma})^{1-\sigma} \phi^{\sigma-1} (X_n / \sigma) (v_{iln}^{1-\sigma}) \right] = P_n^{\sigma-1} (\tilde{\sigma})^{1-\sigma} \phi^{\sigma-1} (X_n / \sigma) E(\min_l v_{iln})^{1-\sigma}.$$

However, $v_{in} \equiv \min_l v_{iln}$ is distributed according to $G(v_{in}; \Psi_{in})$, where

$$\Psi_{in} \equiv \sum_j \left(\gamma_{ij} w_j^p \tau_{jn}\right)^{-\theta} T_{ij}. \tag{O.59}$$

We finally have that the expected profits we are interested in are

$$(\tilde{\sigma})^{1-\sigma} P_n^{\sigma-1} \phi^{\sigma-1} (X_n/\sigma) B_{in},$$

where

$$B_{in} \equiv E(\min_l v_{iln})^{1-\sigma} = \gamma_{1-\sigma} \Psi_{in}^{(\sigma-1)/\theta}. \tag{O.60}$$

This definition implies that $P_n^{\sigma-1} \left(\tilde{\sigma}\phi_{in}^*\right)^{\sigma-1} (X_n/\sigma) B_{in} = w_n^p F_{in}$, and hence

$$\phi_{in}^* = \left[\sigma \frac{w_n^p F_{in}}{X_n} \frac{1}{B_{in}}\right]^{\frac{1}{\sigma-1}} \frac{\tilde{\sigma}}{P_n}. \tag{O.61}$$

Given that the expected marginal cost of producing and shipping the good from country i to n, using production location l is

$$\begin{aligned}
c_n^* &= E\left(\min_l \left\{\tilde{\sigma}\frac{\gamma_{il} w_l^p \tau_{ln}}{z_l \phi_{in}^*}\right\}\right) \\
&= E\left(\min_l \left\{\frac{\gamma_{il} w_l^p \tau_{ln}}{z_l}\right\}\right) \tilde{\sigma} \left[\sigma \frac{w_n^p F_{in}}{X_n} \frac{1}{B_{in}}\right]^{\frac{1}{1-\sigma}} P_n \\
&= \gamma_1 \Psi_{in}^{-1/\theta} \tilde{\sigma} \left[\sigma \frac{w_n^p F_{in}}{X_n} \frac{1}{\gamma_{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \Psi_{in}^{1/\theta} P_n \\
&= \gamma_1 \left(\gamma_{1-\sigma}\right)^{\frac{1}{\sigma-1}} \tilde{\sigma} \left[\sigma \frac{w_n^p F_{in}}{X_n}\right]^{\frac{1}{1-\sigma}} P_n.
\end{aligned}$$

where we have made use of definition (O.62). Notice that this expected cutoff cost, is the same as the realized cutoff cost, up to a constant, in the main paper.

We can also calculate the share of firms from i serving in n that choose to do so from location l, φ_{iln}. Since v_{iln} is distributed $G(v_{in}; (\gamma_{il} w_l^p \tau_{ln})^{-\theta} T_{il})$ and $l = \arg\min_l v_{iln}$, then standard results with the Fréchet distribution imply that φ_{iln} (which is also the share of sales by firms in i in n that is produced in location l) imply that

$$\psi_{iln} = \frac{\left(\gamma_{il} w_l^p \tau_{ln}\right)^{-\theta} T_{il}}{\Psi_{in}}. \tag{O.62}$$

10.3 Price index

To calculate the price index P_n, note that since $p_{iln} = \tilde{\sigma} v_{iln}/\phi$ and the measure of firms from country i with ϕ is $M_i dF_i(\phi)$ then

$$P_n^{1-\sigma} = (\tilde{\sigma})^{1-\sigma} \sum M_i \int_{\phi_{in}^*}^{\infty} E\left(\min_l v_{iln}\right)^{1-\sigma} \phi^{\sigma-1} dF_i(\phi)$$

and hence

$$P_n^{1-\sigma} = \bar{c}_1 \sum M_i \Psi_{in}^{(\sigma-1)/\theta} b_i^{\kappa} \left(\phi_{in}^*\right)^{-(\kappa+1-\sigma)}.$$

where

$$\bar{c}_1 \equiv \tilde{\gamma} \frac{\kappa}{\kappa+1-\sigma}.$$

Plugging in from (O.61) we get (after some simplifications)

$$P_n = \bar{c}_2 \left(\frac{w_n^p}{X_n}\right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)} \left[\sum_i b_i^{\kappa} M_i F_{in}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta}\right]^{-1/\kappa}, \tag{O.63}$$

where

$$c_2 \equiv \bar{c}_1^{-1/\kappa} (\sigma/\tilde{\gamma})^{(\kappa+1-\sigma)/\kappa(\sigma-1)}.$$

10.4 Market Shares

We will construct and make use of three market shares that are relevant for our model and potentially empirically relevant. To introduce these shares, let X_{iln} denote the sales to market n by firms originating in country i that are produced in country l. Notice that total spending and total output are given by $X_n = \sum_{i,l} X_{iln}$ and $Y_l = \sum_{i,n} X_{iln}$. The share λ_{in}^E, λ_{ln}^T, λ_{il}^M, can be derived using the formulas (8), (10), (11).

We can use standard arguments about price indices in this kind of environment to get

$$\lambda_{in}^E = \frac{M_i b_i^{\kappa} F_{in}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta}}{\sum_j M_j b_j^{\kappa} F_{jn}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{jn}^{\kappa/\theta}}. \tag{O.64}$$

In relation to the main paper, this expression is 'distorted' through the term F_{in} which, of course, cancels out if $F_{in} = F_n$. Notice that the term Ψ_{in} is slightly different from that paper, but very closely related.

To get trade shares, recall that firms from i that sell in n produce a share ψ_{iln} in country n (equation 6). Hence the import share by n from l is the weighted average of these production shares across i weighed by the λ^E_{in},

$$\lambda^T_{ln} = \sum_i \psi_{iln}\lambda^E_{in}. \tag{O.65}$$

To get MP shares, note that the value of goods produced in l by firms from i to be delivered to n is $\psi_{iln}\lambda^E_{in}X_n$, hence the total value of goods produced in l by firms from i is $\sum_n \psi_{iln}\lambda^E_{in}X_n$, and so

$$\lambda^M_{il} = \frac{\sum_n \psi_{iln}\lambda^E_{in}X_n}{\sum_{j,n} \psi_{jln}\lambda^E_{jn}X_n}. \tag{O.66}$$

Thus, these expression are all as in the main paper.

10.5 Profits

Let us now compute total profits made by firms from i from their production in country l, Π_{il}. We know that total variable profits made by firms from i in country l are $\sum_n \psi_{iln}\lambda^E_{in}X_n/\sigma$. What are the fixed costs paid by these firms? The measure of firms in country i that serve country n through location l is $\psi_{iln}M_i b_i^\kappa \left(\phi^*_{in}\right)^{-\kappa}$, hence

$$\Pi_{il} = \sum_n \left(\psi_{iln}\lambda^E_{in}X_n/\sigma - \psi_{iln}M_i b_i^\kappa \left(\phi^*_{in}\right)^{-\kappa} w^p_n F_{in}\right). \tag{O.67}$$

To proceed, note that from $(O.61)$ and $(O.60)$ we have

$$b_i^\kappa \left(\phi^*_{in}\right)^{-\kappa} = \frac{b_i^\kappa \left(\tilde{\gamma}\right)^{\kappa/(\sigma-1)} \Psi_{in}^{\kappa/\theta}}{\left(\left[\sigma \frac{w^p_n F_{in}}{X_n}\right]^{\frac{1}{\sigma-1}} \frac{1}{P_n}\right)^{\kappa}},$$

whereas from $(O.63)$ and (8) we have

$$P_n^{-\kappa} = \bar{c}_2^{-\kappa} \left(\frac{w^p_n}{X_n}\right)^{-(\kappa+1-\sigma)/(\sigma-1)} M_i b_i^\kappa F_{in}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta}/\lambda^E_{in},$$

hence

$$\begin{aligned}
b_i^\kappa \left(\phi_{in}^*\right)^{-\kappa} &= \frac{b_{i2}^\kappa \bar{c}_2^{-\kappa} \left(\tilde{\gamma}\right)^{\kappa/(\sigma-1)} \left(\tilde{\sigma}\Psi_{in}\right)^{\kappa/\theta}}{\left[\sigma \frac{w_n^p F_{in}}{X_n}\right]^{\frac{\kappa}{\sigma-1}} \left(\frac{w_n^p}{X_n}\right)^{-(\kappa+1-\sigma)/(\sigma-1)} M_i b_i^\kappa F_{in}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta} / \lambda_{in}^E} \\
&= \bar{c}_2^\kappa \left(\tilde{\gamma}/\sigma\right)^{\kappa/(\sigma-1)} \frac{\lambda_{in}^E X_n}{M_i w_n^p F_{in}}
\end{aligned}$$

Using expression (O.67) and the above definitions we have that

$$\begin{aligned}
\Pi_{il} &= \sum_n \psi_{iln} \lambda_{in}^E X_n/\sigma - \sum_n \psi_{iln}^\kappa \bar{c}_2^\kappa \left(\tilde{\gamma}/\sigma\right)^{\kappa/(\sigma-1)} \frac{\lambda_{in}^E X_n}{w_n^p F_{in}} w_n^p F_{in} \\
&= \left(1/\sigma - \bar{c}_2^\kappa \left(\tilde{\gamma}/\sigma\right)^{\kappa/(\sigma-1)}\right) \sum_n \psi_{iln} \lambda_{in}^E X_n
\end{aligned}$$

so

$$\Pi_{il} = \eta \lambda_{il}^M Y_l, \tag{O.68}$$

where

$$\eta \equiv \frac{\sigma-1}{\sigma\kappa},$$

with the interesting result the realized share of profits is constant but depends on κ.

10.6 Equilibrium

Wages, spending, and number of firms, w_i^p, X_i and M_i solve the exact same system of equation in Section 2.3 in the paper, provided that λ_{in}^E and ψ_{iln} are the same. Notice that the occupational choice model does not interfere with the rest of the derivations in the model as the equilibrium equations are the same given w_i^p, X_i and M_i, which in turn are fully determined by the equations above. Given that we proceed to characterize welfare in the model.

10.7 Welfare

To look at welfare, we start with

$$\lambda_{nn}^T = \sum_i \frac{\left(w_n^p\right)^{-v} T_{in}}{\sum_k \left(w_k^p \tau_{kn}\right)^{-\theta} T_{ik}} \lambda_{in}^E.$$

Using $\Psi_{in} \equiv \sum_l \left(w_l^p \tau_{ln}\right)^{-\theta} T_{il}$ this implies that

$$w_n^p = \left(\frac{\lambda_{nn}^T}{\sum_i \lambda_{in}^E T_{in}/\Psi_{in}}\right)^{-1/\theta}.$$

We also have

$$\lambda_{in}^E = \frac{\gamma b_i^\kappa M_i \Psi_{in}^{(\sigma-1)/\theta} \left(\phi_{in}^*\right)^{-(\kappa+1-\sigma)}}{P_n^{1-\sigma}}$$

Plugging in for $\phi_{in}^* = \left(\sigma \frac{w_n^p F_{in}}{X_n B_{in}}\right)^{1/(\sigma-1)} \frac{\tilde{\sigma}}{P_n}$ and $B_{in} = \Gamma \Psi_{in}^{(\sigma-1)/\theta}$ we then get

$$P_n^{1-\sigma} = \frac{\bar{c}_1^{-\kappa} b_i^\kappa M_i \Psi_{in}^{\kappa/\theta} \left(\frac{w_n^p F_{in}}{X_n}\right)^{-(\kappa+1-\sigma)/(\sigma-1)} P_n^{\kappa+1-\sigma}}{\lambda_{in}^E}$$

hence

$$P_n = \bar{c}_1 b_n^{-1} M_n^{-1/\kappa} \Psi_{nn}^{-1/\theta} \left(\frac{w_n^p F_{nn}}{X_n}\right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)} \left(\lambda_{nn}^E\right)^{1/\kappa} \tag{O.69}$$

and real wage is

$$\frac{w_n^p}{P_n} = \frac{\left(\lambda_{nn}^T\right)^{-1/\theta} \left(\lambda_{nn}^E\right)^{-1/\kappa} \left(\sum_i \lambda_{in}^E T_{in} \Psi_{nn}/\Psi_{in}\right)^{1/\theta} b_n M_n^{1/\kappa}}{\bar{c}_1 \left(\frac{w_n^p F_{nn}}{X_n}\right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)}}$$

Let $\varepsilon_{iln} \equiv X_{iln}/\sum_j X_{jln}$, then we can show that

$$\sum_i T_{in} \frac{\Psi_{nn}}{\Psi_{in}} \lambda_{in}^E = \frac{T_{nn} \lambda_{nn}^E}{\varepsilon_{nnn}},$$

so finally we have

$$\frac{X_n}{P_n} = \frac{\left[L_n/(1-\eta)\right]^{\frac{\kappa+1-\sigma}{\sigma-1}\frac{1}{\kappa}+1} \left(\frac{X_n}{Y_n}\right)^{\frac{\kappa+1-\sigma}{\sigma-1}\frac{1}{\kappa}+1} \left(\lambda_{nn}^T\right)^{-1/\theta} \left(\varepsilon_{nnn}\right)^{-1/\theta} \left(\lambda_{nn}^E\right)^{1/\theta-1/\kappa} \left(T_{nn}\right)^{1/\theta}}{\bar{c}_1 \left[F_{nn}^{\frac{-(\kappa+1-\sigma)}{\sigma-1}}\right]^{-1/\kappa}} b_n M_n^{1/\kappa}, \tag{O.70}$$

so the gains from openness are

$$
\begin{aligned}
GO &= \left(\frac{X_n}{Y_n}\right)^{\frac{\kappa+1-\sigma}{\sigma-1}\frac{1}{\kappa}+1} \left(\lambda_{nn}^T\right)^{-1/\theta} \left(\varepsilon_{nnn}\right)^{-1/\theta} \left(\lambda_{nn}^E\right)^{1/\theta-1/\kappa} \\
&= \left(\frac{X_n}{Y_n}\right)^{\frac{\kappa+1-\sigma}{\sigma-1}\frac{1}{\kappa}+1} \left(\frac{X_{nnn}}{X_n}\right)^{-1/\theta} \left(\frac{\sum_l X_{nln}}{X_n}\right)^{1/\theta-1/\kappa}
\end{aligned}
$$

where we used the definitions of λ_{nn}^T, ε_{nnn}, and λ_{nn}^E. This expression is the same as the main paper as long as $1/\theta = (1-\rho)/\tilde{\theta}$ and $1/\kappa - 1/\theta = \rho/\tilde{\theta}$ so that

$$1/\kappa = 1/\tilde{\theta}, 1/\theta = (1-\rho)/\tilde{\theta}\,.$$

Notice that the case of $\rho = 0$ in the main paper corresponds to the case $\kappa = \theta$ here.

10.8 Formally Connecting the Models

We finally formally connect this model to the model in the main text. The variables with tildes correspond to the variables of the model in the main text.

i) Parameters γ, τ, T and L are the same,

ii) We set $F_{in} = F_n = \tilde{F}_n$

iii) Set $1/\kappa = 1/\tilde{\theta}, 1/\theta = (1-\tilde{\rho})/\tilde{\theta} \implies \tilde{\theta} = \kappa, \frac{\tilde{\theta}}{1-\tilde{\rho}} = \theta$

iv) Set wages, $w_i = \tilde{w}_i$, spending, $X_i = \tilde{X}_i$, and entry, $M_i = \tilde{M}_i$, the same.

v) Given those, we can define $\Psi_{in} = \tilde{\Psi}_{in}^{1/(1-\tilde{\rho})}$, $\psi_{iln} = \tilde{\psi}_{iln}$ and $\lambda_{in}^E = \tilde{\lambda}_{in}^E$ since

$$\Psi_{in} \equiv \sum_j \left(\gamma_{ij} w_j \tau_{jn}\right)^{-\theta} T_{ij} = \sum_j \left(\tilde{\gamma}_{ij}\tilde{w}_j\tilde{\tau}_{jn}\right)^{-\frac{\tilde{\theta}}{1-\rho}} T_{ij} = \tilde{\Psi}_{in}^{1/(1-\tilde{\rho})},$$

and

$$
\begin{aligned}
\lambda_{in}^E &= \frac{M_i b_i^{\kappa} F_{in}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{in}^{\kappa/\theta}}{\sum_j M_j b_j^{\kappa} F_{jn}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{jn}^{\kappa/\theta}} \\
&= \frac{\tilde{M}_i \tilde{b}_i^{\kappa} \tilde{\Psi}_{in}}{\sum_j \tilde{M}_j \tilde{b}_j^{\kappa} \tilde{\Psi}_{in}} = \tilde{\lambda}_{in}^E
\end{aligned}
$$

and also

$$\begin{aligned}\psi_{iln} &= \frac{\left(\gamma_{il} w_l \tau_{ln}\right)^{-\theta} T_{il}}{\Psi_{in}} \\ &= \frac{\left(\tilde{\gamma}_{il} \tilde{w}_l \tilde{\tau}_{ln}\right)^{-\tilde{\theta}/(1-\tilde{\rho})} \tilde{T}_{il}}{\tilde{\Psi}_{in}^{1/(1-\tilde{\rho})}} = \tilde{\psi}_{iln}\end{aligned}$$

completing the isomorphism of all the variables in the two models. Notice that these derivations also imply that $\lambda^T_{ln} = \tilde{\lambda}^T_{ln}$ and $\lambda^M_{il} = \tilde{\lambda}^M_{il}$.

vi) We finally show that the price index is the same up to a constant. Using expression (O.69) we can write the price index as

$$P_n = \bar{c}_1 \left(\frac{w^p_n}{X_n}\right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)} \left[\frac{M_n b^\kappa_n F_{nn}^{-(\kappa+1-\sigma)/(\sigma-1)} \Psi_{nn}^{\kappa/\theta}}{\lambda^E_{nn}}\right]^{-1/\kappa}$$

and using the definitions above

$$\begin{aligned}P_n &= \bar{c}_1 \left(\frac{\tilde{w}^p_n}{\tilde{X}_n}\right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)} \left[\frac{\tilde{M}_n \tilde{b}^\kappa_n \tilde{F}_n^{-(\kappa+1-\sigma)/(\sigma-1)} \left(\tilde{\Psi}_{in}^{1/(1-\tilde{\rho})}\right)^{(1-\tilde{\rho})}}{\tilde{\lambda}^E_{nn}}\right]^{-1/\tilde{\theta}} \\ &= \bar{c}_1 \left(\frac{\tilde{w}^p_n}{\tilde{X}_n}\right)^{(\kappa+1-\sigma)/\kappa(\sigma-1)} \left[\frac{\tilde{M}_n \tilde{b}^\kappa_n \tilde{F}_n^{-(\kappa+1-\sigma)/(\sigma-1)} \tilde{\Psi}_{in}}{\tilde{\lambda}^E_{nn}}\right]^{-1/\tilde{\theta}} = \tilde{P}_{nn}\end{aligned}$$

given that $\lambda^E_{nn} = \tilde{\lambda}^E_{nn}$, so that this would imply that $P_n = \tilde{P}_n$.

vii) The final step is to show that the models solve the same equilibrium conditions. Given the above definitions and the discussion in Subsection 10.6 this holds completing the derivation of the isomorphism between the two setups.

11 Process Innovation

Process innovation involves a conscious effort on the part of firms to lower their marginal cost of production, but like all innovation involves uncertainty. Suppose that when firms enter, in our model, each entrant in country i can augment its T^e_i by a proportion a_i. The cost of this possibility is a^α_i/α in terms of home labor.

From the firm's perspective, the relevant concern is how process innovation will affect its expected costs of serving foreign markets. Adapting (A.2), we have

$$\begin{aligned}\Pr\left(C_{i1n} \geq c_{i1n}, ..., C_{iln} = c_{iln}, ..., C_{iNn} \geq c_{iNn}\right) &= \theta \left(\sum_{k=1}^{N} \left[\left(T_i^e a_i\right) T_k^p \left(\frac{\xi_{ikn}}{c_{ikn}} \right)^{-\theta} \right]^{\frac{1}{1-\rho}} \right)^{-\rho} \times \\ &\quad \left(\left(T_i^e a_i\right) T_l^p \xi_{iln}^{-\theta} \right)^{\frac{1}{1-\rho}} c_{iln}^{\theta/(1-\rho)-1}.\end{aligned}$$

Assuming as before that marketing fixed costs are such that firms do not operate on the support of the distribution, we still have

$$\begin{aligned}\Pr\left(\arg\min_k C_{ikn} = l \cap \min_k C_{ikn} = c\right) &= \Pr\left(C_{i1n} \geq c, ..., C_{iln} = c, ..., C_{iNn} \geq c\right) \\ &= \theta (\Psi_{in} a_i)^{-\frac{\rho}{1-\rho}} \left(\left(T_i^e a_i\right) T_l^p \xi_{iln}^{-\theta} \right)^{\frac{1}{1-\rho}} c^{\theta-1} \\ &= a_i \psi_{iln} \Psi_{in} \theta c^{\theta-1},\end{aligned}$$

where Ψ_{in} and ψ_{iln} continue to be given by the expressions in the text. Given this linearity, (A.3) becomes

$$\Pr\left(\arg\min_k C_{ikn} = l \cap \min_k C_{ikn} \leq c_n^*\right) = a_i \psi_{iln} \Psi_{in} \left(c_n^*\right)^{\theta}$$

and hence,

$$\Pr\left(\min_k C_{ikn} \leq c_n^*\right) = \sum_k a_i \psi_{ikn} \Psi_{in} \left(c_n^*\right)^{\theta} = a_i \Psi_{in} \left(c_n^*\right)^{\theta}.$$

The expected sales of the firm from i through l to n that has invested in process innovation of a_i are

$$\begin{aligned}\mathbf{E}\left(x_{iln}\right) &= a_i \psi_{iln} \Psi_{in} \widetilde{\sigma}^{1-\sigma} X_n P_n^{\sigma-1} \int_0^{c_n^*} \theta c^{\theta-\sigma} dc, \\ &= a_i \frac{\widetilde{\sigma}^{1-\sigma}\theta}{\theta-\sigma+1} \psi_{iln} \Psi_{in} X_n P_n^{\sigma-1} \left(c_n^*\right)^{\theta-\sigma+1},\end{aligned}$$

and the expected marketing costs are

$$\begin{aligned}\Pr\left(\min_k C_{ikn} \leq c_n^*\right) w_n^p F_n &= w_n^p F_n \sum_k a_i \psi_{ikn} \Psi_{in} \left(c_n^*\right)^{\theta}, \\ &= a_i w_n^p F_n \Psi_{in} \left(c_n^*\right)^{\theta}.\end{aligned}$$

Thus, the total expected profits (net of innovation costs) of serving n for a firm from i choosing process innovation level n are

$$\begin{aligned}
\mathbf{E}\pi_{in}(a_i) &= a_i \Psi_{in} \left(c_n^*\right)^{\theta} \left(\frac{\theta}{\theta-\sigma+1} \frac{\widetilde{\sigma}^{1-\sigma}}{\sigma} X_n P_n^{\sigma-1} \left(c_n^*\right)^{1-\sigma} - w_n^p F_n \right) \\
&= a_i \frac{\sigma-1}{\theta-\sigma+1} \Psi_{in} \left(\frac{X_n P_n^{\sigma-1}}{w_n^p F_n} \frac{\sigma}{\widetilde{\sigma}^{1-\sigma}} \right)^{\frac{\theta}{\sigma-1}} w_n^p F_n
\end{aligned}$$

Summing over all n and subtracting off the process innovation cost and the entry cost, the total expected profits of innovation level a_i

$$\begin{aligned}
\mathbf{E}\Pi_i(a_i) &= \sum_n E\pi_{in}(a_i) - w_i^e \left(\frac{(a_i)^{\alpha}}{\alpha} + f_i^e \right) \\
&= a_i \sum_n \frac{\sigma-1}{\theta-\sigma+1} \Psi_{in} \left(\frac{X_n P_n^{\sigma-1}}{w_n^p F_n} \frac{\sigma}{\widetilde{\sigma}^{1-\sigma}} \right)^{\frac{\theta}{\sigma-1}} w_n^p F_n - w_i^e \left(\frac{(a_i)^{\alpha}}{\alpha} + f_i^e \right).
\end{aligned}$$

The first-order condition for the choice of the optimal level of process innovation is

$$\sum_n \frac{\sigma-1}{\theta-\sigma+1} \Psi_{in} \left(\frac{X_n P_n^{\sigma-1}}{w_n^p F_n} \frac{\sigma}{\widetilde{\sigma}^{1-\sigma}} \right)^{\frac{\theta}{\sigma-1}} w_n^p F_n = w_i^e (a_i)^{\alpha-1}.$$

Substituting the first order condition into the zero expected profit condition yields

$$(a_i)^{\alpha} = \frac{\alpha}{\alpha-1} f_i^e.$$

To complete the analysis, we can define the country innovation parameter as

$$\widetilde{T}_i^e = T_i^e \left(\frac{\alpha}{\alpha-1} f_i^e \right)^{1/\alpha}, \tag{O.71}$$

and the country entry fixed cost parameter as

$$\begin{aligned}
\widetilde{f}_i^e &= \frac{(a_i)^{\alpha}}{\alpha} + f_i^e, \\
&= \frac{1}{\alpha} \left(\frac{\alpha}{\alpha-1} f_i^e \right)^{\alpha} + f_i^e.
\end{aligned} \tag{O.72}$$

This expression shows that total innovation costs are the sum of the process innovation costs (first term) and product innovation costs (second term). All the derivations in the paper are therefore consistent with a model in which firms can choose the level of process innovation at the time of entry with the parameters T_i^e and f_i^e adjusted to reflect the bundling of process and product innovation.

12 Gravity: Alternative Estimations

Table 1: Restricted and unrestricted gravity, instrumental variables (2SLS).

	OLS		IV	
	Restricted	Unrestricted	Restricted	Unrestricted
	(1)	(2)	(3)	(4)
Trade costs	-11.8	-6.9	-14.4	-9.7
	(1.02)	(3.10)	(4.98)	(5.22)
Observations	45	45	45	45
R-sq.	0.21	0.10	0.20	0.08

Notes: The dependent variable for the restricted gravity equation is $\log(X_{ili} \times X_{iil}) - \log(X_{ill} \times X_{iii})$ with $i = USA$, while for the unrestricted gravity equation is $\log(X_{li} \times X_{il}) - \log(X_{ll} \times X_{ii})$. The variable "trade costs" is the sum of freight costs (in logs), calculated from cif/fob U.S. imports and then assumed to be symmetric for U.S. exports, and inward and outward tariffs combined as $\log(1 + t_{il}/100) \times (1 + t_{li}/100)$ with t_{il} being the tariff rate for goods from i to l. Trade costs are instrumented in columns 3 and 4 using distance dummies, self, border, and language dummies, as well as an index of quality of infrastructure (from the *World Competitiveness Report*.). Robust standard errors in parenthesis.

13 Additional Tables

Table 2: Aggregate Variables. Data and Model.

Country Name	X_i^{data}	X_i^{model}	$\bar{L}_i$	Y_i^{data}	Y_i^{model}	r_i^{data}	r_i^{model}	Δ_i / X_i^{data}
Australia	0.042	0.024	0.053	0.04	0.03	0.137	0.128	0.151
Austria	0.024	0.020	0.028	0.02	0.02	0.140	0.140	0.101
Benelux	0.078	0.056	0.077	0.09	0.07	0.212	0.213	-0.189
Brazil	0.093	0.102	0.184	0.10	0.10	0.140	0.145	0.058
Canada	0.079	0.073	0.096	0.09	0.08	0.130	0.130	-0.012
China	0.303	0.292	1.501	0.33	0.32	0.139	0.140	0.003
Cyprus	0.001	0.001	0.002	0.00	0.00	0.158	0.157	0.392
Denmark	0.014	0.014	0.024	0.02	0.01	0.192	0.193	-0.057
Spain	0.089	0.066	0.084	0.09	0.07	0.133	0.135	0.086
Finland	0.017	0.010	0.019	0.02	0.01	0.203	0.201	-0.188
France	0.152	0.139	0.179	0.16	0.15	0.174	0.175	-0.028
United Kingdom	0.154	0.149	0.183	0.16	0.15	0.160	0.160	0.044
Germany	0.263	0.267	0.382	0.30	0.30	0.176	0.178	-0.073
Greece	0.016	0.010	0.013	0.01	0.01	0.151	0.156	0.271
Hungary	0.010	0.017	0.035	0.01	0.02	0.045	0.046	0.196
Ireland	0.011	0.005	0.011	0.02	0.01	0.092	0.090	-0.275
Italy	0.172	0.103	0.121	0.19	0.11	0.150	0.150	-0.018
Japan	0.598	0.511	0.523	0.66	0.56	0.182	0.180	-0.053
Korea	0.105	0.117	0.200	0.12	0.13	0.160	0.163	-0.031
Mexico	0.081	0.077	0.266	0.08	0.08	0.127	0.130	0.103
Poland	0.024	0.023	0.126	0.02	0.02	0.115	0.116	0.128
Portugal	0.018	0.020	0.033	0.02	0.02	0.084	0.087	0.182
Romania	0.007	0.006	0.065	0.01	0.01	0.149	0.158	0.090
Sweden	0.030	0.020	0.031	0.03	0.02	0.169	0.166	-0.104
Turkey	0.042	0.053	0.109	0.04	0.05	0.150	0.159	0.052
United States	1.000	1.000	1.000	1.00	1.00	0.179	0.180	0.034
Average	0.13	0.12	0.21	0.14	0.13	0.15	0.15	0.03

Note: Expenditure X_i^{data} is from the WIOD, an average over 1996-2001, for manufacturing; output Y_i^{data} is calculated using the expenditure data and bilateral trade shares in manufacturing, from the WIOD, an average over 1996-2001, as $Y_i^{data} = \sum_n (\lambda_{in}^T)^{data} X_n^{data}$; labor $\bar{L}_i$ is equipped labor from Klenow & Rodríguez-Clare (2005), an average for the nineties, adjusted by the share of manufacturing employment from UNIDO; the trade and MP deficits are calculated as $\Delta_i = X_i^{data} - Y_i^{data}(1/\widetilde{\sigma}) + X_i^{data}(1+\theta-\sigma)/(\sigma\theta) + \eta \sum_l (\lambda_{il}^M)^{data} Y_l^{data}$, where the bilateral MP shares are from Ramondo et al. (2015), an average over 1996-2001; and the innovation share is calculated as $r_i^{data} = 1 - (Y_i^{data}(1/\widetilde{\sigma}) + X_i^{data}(1+\theta - \sigma)/(\sigma\theta))/(X_i^{data} - \Delta_i)$. The variables X^{model}, Y^{model}, and r^{model} are as implied by the calibrated model with trade and MP imbalances. Variables X, Y, and $\bar{L}$ are relative to the United States.

Table 3: Gains from Openness, Trade, and MP. Baseline calibration.

	GO, overall	GO, direct	GO, indirect	GT	GMP
	(1)	(2)	(3)	(4)	(5)
Australia	1.207	1.422	0.848	0.924	1.117
Austria	1.346	1.383	0.973	1.033	1.110
Benelux	1.602	1.529	1.048	1.097	1.278
Brazil	1.038	1.069	0.970	0.987	1.003
Canada	1.490	1.572	0.949	1.056	1.068
China	1.034	1.074	0.963	0.978	1.000
Cyprus	1.372	1.372	1.000	1.370	1.004
Denmark	1.324	1.280	1.034	1.104	1.071
Spain	1.112	1.160	0.959	0.996	1.019
Finland	1.291	1.245	1.037	1.074	1.093
France	1.190	1.177	1.011	1.040	1.073
United Kingdom	1.267	1.281	0.989	1.029	1.130
Germany	1.182	1.171	1.009	1.025	1.102
Greece	1.130	1.140	0.992	1.071	1.000
Hungary	1.440	1.730	0.833	0.948	1.163
Ireland	1.895	2.182	0.868	0.990	1.277
Italy	1.111	1.135	0.978	1.014	1.015
Japan	1.051	1.035	1.015	1.026	1.027
Korea	1.049	1.056	0.993	1.011	1.012
Mexico	1.167	1.224	0.953	1.018	1.010
Poland	1.133	1.209	0.937	0.988	1.014
Portugal	1.265	1.405	0.900	0.968	1.067
Romania	1.139	1.150	0.990	1.076	0.998
Sweden	1.395	1.403	0.994	1.052	1.125
Turkey	1.058	1.068	0.990	1.026	0.995
United States	1.098	1.076	1.020	1.032	1.053
Average	1.246	1.290	0.971	1.036	1.070

Note: The gains from openness refer to changes in real expenditure between autarky and the calibrated equilibrium. The direct and indirect effects refer to the first and second terms, respectively, on the right-hand side of (27). The gains from trade (MP) refer to changes in real expenditure between an equilibrium with only MP (trade) and the calibrated equilibrium with both trade and MP. Changes are with respect to the baseline calibrated equilibrium without trade and MP imbalances, $\Delta = 0$.

Table 4: MP Liberalization. Baseline calibration.

% change in:	innovation share r	real expenditure X/P	real production wage w^p/P	real innovation wage w^e/P
Australia	-11.49	3.40	3.83	-2.72
Austria	-0.52	3.00	3.05	2.73
Benelux	10.63	5.61	4.17	11.08
Brazil	-4.14	0.26	0.61	-1.84
Canada	-3.06	3.03	3.27	1.44
China	-5.33	0.26	0.70	-2.45
Cyprus	0.00	0.06	0.06	0.06
Denmark	2.33	2.30	2.01	3.49
Spain	-5.94	0.93	1.41	-2.11
Finland	2.88	2.81	2.45	4.28
France	1.02	2.24	2.13	2.76
United Kingdom	2.16	3.36	3.14	4.46
Germany	1.29	2.71	2.57	3.37
Greece	-1.85	0.11	0.28	-0.82
Hungary	-14.80	3.76	4.24	-4.23
Ireland	7.52	4.94	4.60	8.81
Italy	-3.70	0.83	1.16	-1.05
Japan	0.99	0.72	0.61	1.22
Korea	-1.02	0.49	0.59	-0.03
Mexico	-8.26	0.81	1.45	-3.44
Poland	-9.40	0.95	1.60	-3.91
Portugal	-9.72	2.64	3.18	-2.47
Romania	-2.65	0.02	0.27	-1.32
Sweden	2.51	3.63	3.38	4.93
Turkey	-2.14	-0.01	0.20	-1.08
United States	0.75	1.32	1.23	1.70
Average	-2.00	1.93	2.01	0.88

Note: MP liberalization refers to a five-percent decrease in all MP costs with respect to the baseline calibrated values. The variable w^e is the wage per efficiency unit in the innovation sector, while w^p is the wage per efficiency unit in the production sector. Percentage changes are with respect to the baseline calibrated equilibrium without trade and MP imbalances, $\Delta = 0$.

Table 5: The Rise of the East. Baseline calibration.

	China in autarky				MP liberalization into China				Frictionless MP into CHN from US			
% change in:	r	w^p/P	w^e/P	X/P	r	w^p/P	w^e/P	X/P	r	w^p/P	w^e/P	X/P
Australia	-0.57	-0.47	-0.77	-0.49	0.67	0.10	0.45	0.12	-4.40	0.64	-1.75	0.48
Austria	-1.02	-0.80	-1.40	-0.89	-0.30	0.01	-0.17	-0.02	3.22	0.67	2.56	0.95
Benelux	-1.04	-0.51	-1.16	-0.64	0.95	0.10	0.70	0.22	-0.81	0.20	-0.31	0.09
Brazil	0.07	-0.18	-0.14	-0.17	-0.11	0.01	-0.06	0.00	-0.89	0.10	-0.43	0.02
Canada	-2.26	-0.48	-1.78	-0.65	1.16	0.14	0.80	0.22	-10.63	0.87	-5.38	0.09
China	19.55	-4.84	5.74	-3.29	-11.95	1.73	-5.45	0.75	-92.25	43.88	-62.65	34.20
Cyprus	0.00	-1.57	-1.57	-1.57	0.00	-0.08	-0.08	-0.08	0.00	0.75	0.75	0.75
Denmark	-1.18	-1.24	-1.96	-1.38	0.22	-0.03	0.11	0.00	0.69	1.31	1.75	1.40
Spain	0.19	-0.13	-0.02	-0.12	-0.10	0.02	-0.03	0.02	-0.03	0.22	0.20	0.22
Finland	-2.98	-0.56	-2.40	-0.91	1.68	0.17	1.22	0.38	-3.73	-0.05	-2.37	-0.50
France	-0.60	-0.67	-1.03	-0.73	0.32	0.02	0.21	0.06	-0.77	0.44	-0.03	0.36
United Kingdom	-0.58	-0.50	-0.84	-0.56	0.48	0.06	0.35	0.11	-1.32	0.33	-0.46	0.21
Germany	-1.57	-0.28	-1.23	-0.44	1.00	0.13	0.73	0.23	-4.66	-0.02	-2.85	-0.51
Greece	-0.03	-0.35	-0.36	-0.35	-0.07	-0.02	-0.07	-0.03	0.09	0.10	0.16	0.11
Hungary	0.06	-0.22	-0.19	-0.22	-0.31	0.08	-0.09	0.07	2.03	0.17	1.24	0.23
Ireland	-0.79	-0.33	-0.76	-0.37	0.58	0.12	0.43	0.14	-5.56	0.92	-2.16	0.68
Italy	-0.37	-0.57	-0.79	-0.60	-0.04	0.00	-0.03	-0.01	0.56	0.54	0.87	0.59
Japan	-1.81	-0.57	-1.67	-0.76	1.21	0.13	0.86	0.26	-0.86	0.09	-0.43	0.00
Korea	-1.18	-0.27	-0.98	-0.39	0.68	0.09	0.50	0.16	-1.60	-0.06	-1.02	-0.21
Mexico	0.34	-0.14	0.06	-0.11	-0.25	0.03	-0.11	0.01	-3.36	0.59	-1.36	0.33
Poland	0.24	-0.09	0.05	-0.07	-0.10	0.03	-0.03	0.02	0.72	0.08	0.49	0.13
Portugal	-0.51	-0.30	-0.58	-0.33	-0.18	0.03	-0.08	0.02	0.51	0.10	0.38	0.13
Romania	-0.03	-0.29	-0.31	-0.29	-0.05	-0.02	-0.05	-0.03	-0.07	0.04	0.00	0.03
Sweden	-1.13	-0.45	-1.12	-0.55	0.86	0.10	0.61	0.18	-2.34	0.24	-1.16	0.02
Turkey	0.02	-0.22	-0.21	-0.22	-0.06	0.00	-0.04	-0.01	0.04	0.18	0.20	0.18
United States	-2.53	-0.48	-2.03	-0.76	1.34	0.16	0.98	0.31	21.87	2.27	15.75	4.85
Average	0.01	-0.64	-0.67	-0.65	-0.09	0.12	0.06	0.12	-3.98	2.10	-2.23	1.72

Note: China in autarky refers to the counterfactual scenario in which trade and MP costs from/to China are set to infinity; MP liberalization into China refers to the counterfactual scenario in which MP costs into China are decreased by ten percent; frictionless MP into China from USA refers to the counterfactual scenario in which MP costs from the United States into China are set to one. The variables are: real expenditure, X/P; innovation share, r; real wage in production, w^p/P; and real wage in innovation, w^e/P. Percentage changes are with respect to the baseline calibrated equilibrium without trade and MP imbalances, $\Delta = 0$.

Table 6: The Fall of the West. Baseline calibration.

	Rise in MP barriers from US				"Brexit I"				"Brexit II"			
% change in:	r	w^p/P	w^e/P	X/P	r	w^p/P	w^e/P	X/P	r	w^p/P	w^e/P	X/P
Australia	26.55	-5.44	7.40	-4.53	0.09	0.01	0.05	0.01	0.14	0.01	0.08	0.01
Austria	5.72	-0.04	3.29	0.45	-0.15	-0.04	-0.12	-0.05	0.26	-0.07	0.08	-0.05
Benelux	9.31	-2.32	3.36	-1.14	0.02	-0.55	-0.53	-0.54	-0.33	-0.54	-0.75	-0.58
Brazil	6.10	-0.74	2.78	-0.22	0.02	0.01	0.03	0.01	0.01	0.02	0.03	0.02
Canada	35.30	-5.00	13.51	-2.41	0.01	0.00	0.01	0.00	0.10	0.02	0.08	0.03
China	6.20	-0.67	2.88	-0.17	0.00	0.00	0.01	0.00	0.01	0.01	0.01	0.01
Cyprus	0.00	-0.31	-0.31	-0.31	0.00	-1.42	-1.42	-1.42	0.00	-1.37	-1.37	-1.37
Denmark	5.73	0.00	3.54	0.69	0.01	-0.05	-0.04	-0.05	0.04	-0.16	-0.13	-0.15
Spain	3.73	-0.16	1.99	0.14	-0.45	-0.18	-0.44	-0.22	-0.14	-0.18	-0.26	-0.19
Finland	6.60	-0.20	3.88	0.61	-0.22	-0.17	-0.30	-0.20	-0.26	-0.31	-0.47	-0.34
France	8.78	-0.68	4.57	0.26	0.06	-0.01	0.03	0.00	0.05	-0.14	-0.11	-0.13
United Kingdom	15.68	-3.06	5.84	-1.60	-0.63	-0.47	-0.84	-0.53	-2.35	-1.35	-2.73	-1.57
Germany	9.25	-0.82	4.69	0.16	0.13	0.00	0.08	0.01	-0.07	-0.23	-0.28	-0.24
Greece	1.78	-0.15	0.91	0.02	0.03	-0.01	0.00	-0.01	0.11	-0.05	0.02	-0.04
Hungary	6.86	-0.69	2.88	-0.48	-1.18	-0.12	-0.74	-0.15	-0.48	-0.21	-0.46	-0.22
Ireland	35.56	-7.08	9.84	-5.66	0.03	-0.02	0.00	-0.02	-1.44	-0.47	-1.25	-0.53
Italy	4.78	-0.23	2.56	0.19	0.04	0.01	0.03	0.01	0.19	0.02	0.13	0.04
Japan	3.95	-0.27	2.12	0.16	0.02	0.00	0.02	0.01	0.07	0.01	0.05	0.02
Korea	4.11	-0.41	2.02	-0.01	0.01	0.01	0.02	0.01	0.02	0.02	0.03	0.02
Mexico	19.08	-2.23	8.28	-0.77	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poland	0.83	0.22	0.69	0.28	-0.34	-0.09	-0.28	-0.11	0.08	-0.12	-0.08	-0.12
Portugal	2.26	-0.01	1.23	0.11	-0.45	-0.10	-0.35	-0.12	0.92	-0.50	0.00	-0.45
Romania	0.16	-0.02	0.07	0.00	-0.01	-0.02	-0.02	-0.02	0.08	-0.04	0.01	-0.03
Sweden	9.08	-1.33	3.97	-0.45	-0.51	-0.34	-0.64	-0.38	-0.73	-0.48	-0.92	-0.55
Turkey	0.77	-0.04	0.41	0.03	0.01	0.02	0.03	0.02	0.03	0.04	0.06	0.04
United States	-16.67	-1.29	-11.52	-3.07	0.06	0.01	0.04	0.01	0.28	0.02	0.19	0.05
Average	8.13	-1.27	3.11	-0.68	-0.13	-0.13	-0.21	-0.14	-0.13	-0.23	-0.31	-0.24

Note: Rise in MP barriers from U.S. refers to an increase in γ_{il} of 20 percent, for i = US, $i \neq l$. "Brexit I" refers to an increase in τ_{il} and τ_{li} of five percent, for $i = GBR$, $l \in EU$, and $i \neq l$, "Brexit II" refers to an increase in τ_{il}, τ_{li}, γ_{il}, and γ_{li} of five percent each, for $i = GBR$, $l \in EU$, and $i \neq l$. The variables are: real expenditure, X/P; innovation share, r; real wage in production, w^p/P; and real wage in innovation, w^e/P. Percentage changes are with respect to the baseline calibrated equilibrium without trade and MP imbalances, $\Delta = 0$.

Table 7: Gains from Openness, Trade, and MP. Calibration with $\rho = 0$.

	r	GO, overall	GO, direct	GO, indirect	GT	GMP
	(1)	(2)	(3)	(4)	(5)	(6)
Australia	0.093	1.080	1.176	0.918	0.969	1.044
Austria	0.128	1.156	1.193	0.970	1.055	1.068
Benelux	0.184	1.378	1.320	1.044	1.135	1.256
Brazil	0.127	1.026	1.060	0.968	0.987	1.008
Canada	0.116	1.264	1.327	0.953	1.117	1.087
China	0.125	1.023	1.060	0.966	0.985	1.007
Cyprus	0.150	1.188	1.188	1.000	1.188	1.007
Denmark	0.165	1.170	1.147	1.020	1.129	1.059
Spain	0.121	1.068	1.112	0.960	1.009	1.021
Finland	0.175	1.190	1.153	1.032	1.113	1.104
France	0.155	1.117	1.110	1.007	1.056	1.067
United Kingdom	0.143	1.161	1.172	0.991	1.046	1.104
Germany	0.156	1.128	1.119	1.008	1.044	1.091
Greece	0.141	1.077	1.090	0.988	1.057	1.009
Hungary	0.042	1.156	1.407	0.822	0.949	1.087
Ireland	0.076	1.314	1.476	0.890	1.062	1.144
Italy	0.135	1.074	1.097	0.979	1.031	1.023
Japan	0.161	1.040	1.026	1.014	1.027	1.029
Korea	0.144	1.030	1.038	0.992	1.012	1.010
Mexico	0.115	1.101	1.157	0.952	1.038	1.016
Poland	0.105	1.070	1.142	0.937	1.004	1.016
Portugal	0.077	1.114	1.247	0.893	0.977	1.045
Romania	0.136	1.084	1.106	0.980	1.074	1.005
Sweden	0.148	1.215	1.218	0.997	1.098	1.107
Turkey	0.136	1.043	1.062	0.982	1.021	1.005
United States	0.164	1.086	1.067	1.018	1.041	1.063
Average	0.131	1.129	1.164	0.972	1.047	1.057

Note: The gains from openness refer to changes in real expenditure between autarky and the calibrated equilibrium. The direct and indirect effects refer to the first and second terms, respectively, on the right-hand side of (27). The gains from trade (MP) refer to changes in real expenditure between an equilibrium with only MP (trade) and the calibrated equilibrium with both trade and MP. Changes are with respect to the baseline calibrated equilibrium without trade and MP imbalances, $\Delta = 0$.

Table 8: MP Liberalization. Calibration with $\rho = 0$.

% change in:	innovation share r	real expenditure X/P	real production wage w^p/P	real innovation wage w^e/P
	(1)	(2)	(3)	(4)
Australia	-11.54	1.45	2.05	-4.58
Austria	-3.74	1.66	1.94	-0.26
Benelux	10.13	5.27	4.07	10.47
Brazil	-4.93	0.28	0.64	-2.23
Canada	-3.25	2.32	2.54	0.65
China	-5.33	0.26	0.64	-2.45
Cyprus	0.00	0.17	0.17	0.17
Denmark	1.42	1.28	1.14	2.00
Spain	-6.08	0.60	1.02	-2.50
Finland	2.63	2.17	1.89	3.50
France	0.26	1.52	1.50	1.65
United Kingdom	0.65	2.51	2.45	2.84
Germany	0.68	2.04	1.98	2.39
Greece	-1.94	0.24	0.40	-0.74
Hungary	-26.19	2.90	3.48	-11.60
Ireland	-4.96	3.57	3.78	0.96
Italy	-3.38	0.59	0.85	-1.13
Japan	1.18	0.65	0.53	1.24
Korea	-1.41	0.26	0.37	-0.45
Mexico	-7.46	0.53	1.02	-3.29
Poland	-10.29	0.63	1.24	-4.69
Portugal	-15.68	1.61	2.27	-6.70
Romania	-2.95	0.15	0.38	-1.34
Sweden	0.65	2.48	2.42	2.81
Turkey	-2.85	0.14	0.36	-1.30
United States	1.23	1.36	1.24	1.98
Average	-3.58	1.41	1.55	-0.48

Note: MP liberalization refers to a five-percent decrease in all MP costs with respect to the baseline calibrated values. The variable w^e is the wage per efficiency unit in the innovation sector, while w^p is the wage per efficiency unit in the production sector. Percentage changes are with respect to the baseline calibrated equilibrium without trade and MP imbalances, $\Delta = 0$.

Table 9: Gains from Openness, Trade, and MP. Calibration with $\kappa = 5$.

	r	GO, overall	GO, direct	GO, indirect	GT	GMP
	(1)	(2)	(3)	(4)	(5)	(6)
Australia	0.055	1.213	1.520	0.798	0.940	1.123
Austria	0.150	1.345	1.375	0.978	1.042	1.109
Benelux	0.197	1.597	1.538	1.039	1.081	1.273
Brazil	0.145	1.037	1.069	0.970	0.989	1.002
Canada	0.128	1.487	1.576	0.945	1.071	1.066
China	0.139	1.033	1.074	0.962	0.980	0.999
Cyprus	0.167	1.373	1.373	1.000	1.372	1.004
Denmark	0.194	1.323	1.278	1.035	1.099	1.070
Spain	0.137	1.110	1.159	0.958	1.000	1.018
Finland	0.193	1.288	1.246	1.033	1.065	1.090
France	0.175	1.190	1.176	1.012	1.040	1.073
United Kingdom	0.158	1.267	1.282	0.988	1.032	1.130
Germany	0.172	1.181	1.172	1.007	1.023	1.101
Greece	0.161	1.130	1.139	0.992	1.073	1.000
Hungary	0.067	1.384	1.668	0.830	0.982	1.122
Ireland	0.065	1.873	2.273	0.824	1.019	1.264
Italy	0.151	1.110	1.135	0.978	1.017	1.014
Japan	0.178	1.051	1.036	1.015	1.025	1.026
Korea	0.162	1.049	1.056	0.994	1.012	1.012
Mexico	0.133	1.165	1.223	0.952	1.026	1.008
Poland	0.122	1.128	1.206	0.935	0.996	1.010
Portugal	0.102	1.250	1.388	0.901	0.986	1.055
Romania	0.160	1.139	1.150	0.990	1.078	0.998
Sweden	0.160	1.394	1.407	0.991	1.054	71.124
Turkey	0.160	1.058	1.068	0.991	1.027	0.995
United States	0.183	1.098	1.075	1.021	1.030	1.053
Average	0.15	1.24	1.29	0.97	1.04	1.07

Note: The gains from openness refer to changes in real expenditure between autarky and the calibrated equilibrium. The direct and indirect effects refer to the first and second terms, respectively, on the right-hand side of (27). The gains from trade (MP) refer to changes in real expenditure between an equilibrium with only MP (trade) and the calibrated equilibrium with both trade and MP. Changes are with respect to the baseline calibrated equilibrium without trade and MP imbalances, $\Delta = 0$.

Table 10: MP Liberalization. Calibrations with $\kappa = 5$.

% change in:	innovation share r	real expenditure X/P	real production wage w^p/P	real innovation wage w^e/P
	(1)	(2)	(3)	(4)
Australia	-16.59	3.91	4.11	0.20
Austria	-1.12	2.98	3.02	2.75
Benelux	19.29	5.75	4.73	9.55
Brazil	-4.51	0.21	0.36	-0.71
Canada	-6.11	2.99	3.17	1.70
China	-5.91	0.20	0.39	-1.01
Cyprus	0.00	-0.04	-0.04	-0.04
Denmark	2.80	2.30	2.17	2.87
Spain	-7.13	0.85	1.07	-0.63
Finland	3.10	2.83	2.67	3.46
France	0.80	2.25	2.22	2.42
United Kingdom	2.24	3.36	3.27	3.82
Germany	0.90	2.78	2.74	2.96
Greece	-1.96	0.10	0.18	-0.29
Hungary	-20.66	3.62	3.92	-1.07
Ireland	13.28	5.05	4.85	7.70
Italy	-4.79	0.78	0.95	-0.20
Japan	0.92	0.71	0.67	0.90
Korea	-1.14	0.48	0.52	0.25
Mexico	-9.65	0.73	1.03	-1.29
Poland	-11.36	0.87	1.19	-1.53
Portugal	-12.30	2.53	2.81	-0.13
Romania	-2.65	0.01	0.11	-0.53
Sweden	2.04	3.65	3.57	4.07
Turkey	-2.22	-0.03	0.06	-0.47
United States	0.90	1.34	1.30	1.52
Average	-2.38	1.93	1.96	1.39

Note: MP liberalization refers to a five-percent decrease in all MP costs with respect to the baseline calibrated values. The variable w^e is the wage per efficiency unit in the innovation sector, while w^p is the wage per efficiency unit in the production sector. Percentage changes are with respect to the baseline calibrated equilibrium without trade and MP imbalances, $\Delta = 0$.

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