# Monetary Policy and the Redistribution Channel Online Appendix 

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## A Proofs for sections I and II

## A. 1 The standard New Keynesian model

This section shows that, in the standard New Keynesian model with sticky Calvo prices, the impulse response to the path for prices $P_{t}$, real discount rates $q_{t}$, real wages $w_{t}$ and unearned income are those given by my main experiment in figure 1. I only outline the elements of the model relevant to my argument, the reader is referred to the textbook treatments of Woodford (2003) or Galí (2008) for details.

I consider the model in its 'cashless limit', with no aggregate uncertainty. The model features a representative agent with separable utility trading in one-period nominal bonds and holding a fixed stock of capital $k$, so equation (1) simplifies to

$$
\begin{gathered}
\sum \beta^{t}\left\{u\left(c_{t}\right)-v\left(n_{t}\right)\right\} \\
P_{t} c_{t}+\left({ }_{t} Q_{t+1}\right) B_{t+1}=P_{t} \pi_{t}+W_{t} n_{t}+B_{t}+P_{t} \rho_{t} k
\end{gathered}
$$

Here $\rho_{t}$ denotes the real rental rate of capital, so $\rho_{t} k$ are total real rents, and $\pi_{t}$ are real firm profits. Together, rents and profits make up the unearned income in this economy. Consumption $c_{t}$ is an aggregate of intermediate goods, with constant elasticity of substitution $\epsilon$. Hence the price index, aggregating the individual goods prices $p_{j t}$, is $P_{t}=\left(\int_{0}^{1} p_{j t}^{1-\epsilon} d j\right)^{\frac{1}{1-\epsilon}}$.

Each good $j$ is produced under monopolistic competition with constant returns to scale and unit productivity. The production function is

$$
y_{j t}=F\left(k_{j t}, l_{j t}\right)=k_{j t}^{\alpha} 1_{j t}^{1-\alpha}
$$

Firms can only adjust their price with probability $\theta$ each period, independent across firms and periods (the Calvo assumption). Nominal wages $W_{t}$ and nominal rents are flexible. Cost minimization by the firm therefore implies

$$
\begin{aligned}
\rho_{t} P_{t} & =\Lambda_{j t} F_{k}\left(k_{j t}, l_{j t}\right) \\
W_{t} & =\Lambda_{j t} F_{l}\left(k_{j t}, l_{j t}\right)
\end{aligned}
$$

for some scalar $\Lambda_{j t}$ representing the nominal marginal cost of production for firm $j$. Hence

$$
\frac{F_{k}\left(k_{j t}, l_{j t}\right)}{F_{l}\left(k_{j t}, l_{j t}\right)}=\frac{F_{k}\left(\frac{k_{j t}}{l_{j t}, 1}\right)}{F_{l}\left(\frac{k_{j t}}{l_{j t}}, 1\right)}=\frac{\rho_{t}}{w_{t}}
$$

[^0]so all firms have the same capital-labor ratio $\frac{k_{j t}}{l_{j t}}=\frac{k_{t}}{l_{t}}$, and hence all firms have the same nominal marginal cost of production $\Lambda_{t}$.

As is well-known, a first-order approximation to the equilibrium equations of this model is given by the system of three equations

$$
\begin{align*}
\log \left(\frac{c_{t}}{\bar{c}}\right) & =\log \left(\frac{c_{t+1}}{\bar{c}}\right)-\sigma\left(i_{t}-\log \left(\frac{P_{t+1}}{P_{t}}\right)-\varrho\right)  \tag{A.1}\\
\log \left(\frac{P_{t}}{P_{t-1}}\right) & =\beta \log \left(\frac{P_{t+1}}{P_{t}}\right)+\kappa \log \left(\frac{c_{t}}{\bar{c}}\right)  \tag{A.2}\\
i_{t} & =\varrho+\phi_{\pi} \log \left(\frac{P_{t}}{P_{t-1}}\right)+\epsilon_{t} \tag{A.3}
\end{align*}
$$

where $\bar{c}$ is the level of consumption that would prevail under flexible prices, which (normalizing $k=1$ ) solves

$$
\frac{v^{\prime}\left((\bar{c})^{\frac{1}{1-\alpha}}\right)}{u^{\prime}(\bar{c})}=\frac{\epsilon-1}{\epsilon} \frac{(1-\alpha)}{\bar{c}} \equiv \bar{w}
$$

$\varrho=\beta^{-1}-1$ is the steady-state net real interest rate, $\sigma=-\frac{u^{\prime}(\bar{c})}{c \mu^{\prime \prime}(\bar{c})}$ is the elasticity of substitution around $\bar{c}$, and $\kappa$ is the slope of the Phillips curve (a function of model parameters). Equation (A.3) is a Taylor rule describing the behavior of monetary policy. We assume that $\phi_{\pi}>1$, which guarantees equilibrium uniqueness. We consider the effects of a time- 0 monetary policy loosening, $\epsilon_{0}<0$ and $\epsilon_{t}=0$ for $t \geq 1$, assuming the system was at steady-state at $t=-1$, with constant price level $\bar{P}$.

It is easy to guess and verify that the equilibrium features $i_{t}=\rho, P_{t}=P_{t-1}$ and $c_{t}=\bar{c}$ for $t \geq 1$. Solving backwards, this implies that

$$
\begin{aligned}
i_{0} & =\rho+\frac{1}{1+\kappa \sigma \phi_{\pi}} \epsilon_{0} \\
\log \left(\frac{c_{0}}{\bar{c}}\right) & =-\frac{\sigma}{1+\kappa \sigma \phi_{\pi}} \epsilon_{0} \\
\log \left(\frac{P_{0}}{\bar{P}}\right) & =-\frac{\kappa \sigma}{1+\kappa \sigma \phi_{\pi}} \epsilon_{0}
\end{aligned}
$$

In other words, a monetary loosening raises $\mathcal{c}_{t}$ at $t=0$ only, and raises $P_{t}$ immediately and permanently. (Firms that get an opportunity to reset at $t=0$ all increase their price above $\bar{P}$, pulling up the price level to $P_{0}$. Thereafter, all firms that get a chance reset their price to $P_{0}$, so there is no inflation.) To a first-order approximation, the real wage satisfies

$$
w_{t}=\frac{v^{\prime}\left(c_{t}^{\frac{1}{1-\alpha}}\right)}{u^{\prime}\left(c_{t}\right)}
$$

so $w_{t}$ increases at $t=0$ only and then reverts to $\bar{w}$. Moreover, real rents are

$$
\rho_{t}=\frac{\alpha}{1-\alpha} w_{t} c_{t}^{\frac{1}{1-\alpha}}
$$

so they also increase at $t=0$ and then revert to $\bar{\rho}=\frac{\alpha}{1-\alpha} \bar{w}(\bar{c})^{\frac{1}{1-\alpha}} .{ }^{47}$ Date- 0 nominal and real state prices

[^1]are $Q_{0}=q_{0}=1$ and, for $t \geq 1$, given that $P_{t}=P_{0}$,
$$
q_{t}=Q_{t}=\prod_{s=0}^{t-1}\left(s_{s} Q_{t}\right)=\frac{1}{1+i_{0}} \beta^{t-1}
$$

Hence, the path of $q_{t}$ and $Q_{t}$ for $t \geq 1$ is shifted upwards by $\frac{d q_{t}}{q_{t}}=\frac{d Q_{t}}{Q_{t}}=-\frac{d R}{R}$ where the proportional real interest rate change is $\frac{d R}{R}=\frac{d \epsilon_{0}}{\left(1+\kappa \sigma \phi_{\pi}\right)} \frac{1}{1+\rho)}$. Finally, aggregate profits are, to first-order, given by

$$
\begin{equation*}
\pi_{t}=c_{t}-w_{t} n_{t}-\rho_{t} k=c_{t}\left(1-\frac{1}{1-\alpha} \frac{v^{\prime}\left(\left(c_{t}\right)^{\frac{1}{1-\alpha}}\right)}{u^{\prime}\left(c_{t}\right)} c_{t}\right) \tag{A.4}
\end{equation*}
$$

Hence they also deviate only at $t=0$ from their steady state value of $\bar{d}=\frac{\bar{c}}{\epsilon}$. The first term in (A.4) is volume, which rises with $c_{0}$. The second term is the markup, which falls with $c_{0}$. In typical calibrations, the markup effect dominates and profits fall in response to an expansionary monetary shock $\epsilon_{0}<0$.

Collecting results, the timing of changes for $w_{t}, P_{t}$ and $q_{t}$, as well as unearned income $\rho_{t} k+\pi_{t}$, is exactly that depicted in figure 1 , as claimed in the main text.

## A. 2 Proof of theorem 1

The proof is greatly simplified by first applying a simple renormalization of discount factors. Instead of the present value normalization $q_{0}=1$, I normalize $q_{1}=1$ and let $q_{0}$ vary. Then, setting

$$
\begin{equation*}
\frac{d q_{0}}{q_{0}}=\frac{d R}{R} \tag{A.5}
\end{equation*}
$$

yields the experiment in figure 1. Intuitively, a rise in the relative price of future goods relative to a current good is the same as a fall in the price of that current good relative to all future goods. This renormalization is innocuous since there is a degree of freedom in choosing discount factors.

Given the experiment, we can hold $q_{t}$ fixed for $t \geq 1$. Hence, only three parameters $y_{0}, w_{0}$ and $q_{0}$ vary, together with the sequence $\left\{P_{t}\right\}$.

With this renormalization, the proof has three steps: first, I apply Slutsky's theorem to break down $d c$ and $d n$ into income and substitution effects. Second, I work out explicit expressions for MPC and MPN. Finally, I calculate compensated derivatives, and use my expressions from the second step to simplify their expressions.

Step 1: Slutsky's theorem. Recall that the sequences $\left\{q_{t}\right\}$ and $\left\{w_{t}\right\}$ are fixed in the experiment, except for $q_{0}$ and $w_{0}$. Define the following expenditure function

$$
\begin{equation*}
e\left(q_{0}, w_{0}, U\right)=\min \left\{\sum_{t} q_{t}\left(c_{t}-w_{t} n_{t}\right) \quad \text { s.t. } \quad \sum_{t} \beta^{t}\left\{u\left(c_{t}\right)-v\left(n_{t}\right)\right\} \geq U\right\} \tag{A.6}
\end{equation*}
$$

and let $c_{0}^{h}, n_{0}^{h}$ be the resulting compensated (Hicksian) demands for time- 0 consumption and hours. Applying the envelope theorem, we obtain a version of Shephard's lemma:

$$
\begin{align*}
e_{q_{0}} & =c_{0}-w_{0} n_{0}  \tag{A.7}\\
e_{w_{0}} & =-q_{0} n_{0} \tag{A.8}
\end{align*}
$$

Define 'unearned' wealth as

$$
\widetilde{\omega} \equiv \sum_{t \geq 0} q_{t}\left(y_{t}+\left(-1 b_{t}\right)+\left(\frac{-1 B_{t}}{P_{t}}\right)\right)
$$

and note that, given the variation we consider,

$$
\begin{equation*}
d \widetilde{\omega}=\left(y_{0}+\left({ }_{-1} b_{0}\right)+\left(\frac{-1 B_{0}}{P_{0}}\right)\right) d q_{0}+q_{0} d y_{0}-\sum_{t \geq 0} q_{t}\left(\frac{-1 B_{t}}{P_{t}}\right) \frac{d P_{t}}{P_{t}} \tag{A.9}
\end{equation*}
$$

Using the Fisher equation $\frac{q_{t}}{P_{t}}=\frac{Q_{t}}{P_{0}}$, and the fact that $\frac{d P_{t}}{P_{t}}=\frac{d P}{P}$ is a constant, the last term rewrites

$$
\sum_{t \geq 0} q_{t}\left(\frac{-1 B_{t}}{P_{t}}\right) \frac{d P_{t}}{P_{t}}=\sum_{t \geq 0} Q_{t}\left(\frac{-1 B_{t}}{P_{0}}\right) \frac{d P}{P}=q_{0} N N P \frac{d P}{P}
$$

where we have defined the household's net nominal position as the present value of his nominal assets

$$
q_{0} N N P \equiv \sum_{t \geq 0} Q_{t}\left(\frac{-1 B_{t}}{P_{0}}\right)
$$

Moreover, defining

$$
U R E \equiv w_{0} n_{0}+y_{0}+\left({ }_{-1} b_{0}\right)+\left(\frac{-1 B_{0}}{P_{0}}\right)-c_{0}
$$

we can rewrite (A.9) as

$$
\begin{equation*}
d \widetilde{\omega}=\left(U R E+c_{0}-w_{0} n_{0}\right) d q_{0}+q_{0} d y_{0}-q_{0} N N P \frac{d P}{P} \tag{A.10}
\end{equation*}
$$

Next, define the indirect utility function that attains $\widetilde{\omega}$ as

$$
\begin{equation*}
V\left(q_{0}, w_{0}, \widetilde{\omega}\right)=\max \left\{\sum_{t} \beta^{t}\left\{u\left(c_{t}\right)-v\left(n_{t}\right)\right\} \quad \text { s.t. } \quad \sum_{t} q_{t}\left(c_{t}-w_{t} n_{t}\right)=\widetilde{\omega}\right\} \tag{A.11}
\end{equation*}
$$

Let $c_{0}, n_{0}$ denote the resulting Marshallian demands. Applying the envelope theorem, we find

$$
\begin{align*}
\frac{\partial V}{\partial q_{0}} & =-\frac{u^{\prime}\left(c_{0}\right)}{q_{0}}\left(c_{0}-w_{0} n_{0}\right)  \tag{A.12}\\
\frac{\partial V}{\partial w_{0}} & =\frac{u^{\prime}\left(c_{0}\right)}{q_{0}} q_{0} n_{0}  \tag{A.13}\\
\frac{\partial V}{\partial \widetilde{\omega}} & =\frac{u^{\prime}\left(c_{0}\right)}{q_{0}} \tag{A.14}
\end{align*}
$$

As in the proof of Slutsky's theorem, we next differentiate along the identities

$$
\begin{aligned}
c_{0}^{h}\left(q_{0}, w_{0}, U\right) & =c_{0}\left(q_{0}, w_{0}, e\left(q_{0}, w_{0}, U\right)\right) \\
n_{0}^{h}\left(q_{0}, w_{0}, U\right) & =n_{0}\left(q_{0}, w_{0}, e\left(q_{0}, w_{0}, U\right)\right)
\end{aligned}
$$

to find that Marshallian and Hicksian derivatives are related via

$$
\begin{array}{cl}
\frac{\partial c_{0}^{h}}{\partial q_{0}}=\frac{\partial c_{0}}{\partial q_{0}}+\frac{\partial c_{0}}{\partial \widetilde{\omega}} e_{q_{0}} & \frac{\partial c_{0}^{h}}{\partial w_{0}}=\frac{\partial c_{0}}{\partial w_{0}}+\frac{\partial c_{0}}{\partial \widetilde{\omega}} e_{w_{0}} \\
\frac{\partial n_{0}^{h}}{\partial q_{0}}=\frac{\partial n_{0}}{\partial q_{0}}+\frac{\partial n_{0}}{\partial \widetilde{\omega}} e_{q_{0}} & \frac{\partial n_{0}^{h}}{\partial w_{0}}=\frac{\partial n_{0}}{\partial w_{0}}+\frac{\partial n_{0}}{\partial \widetilde{\omega}} e_{w_{0}} \tag{A.16}
\end{array}
$$

Next, define

$$
\begin{align*}
M P C & \equiv q_{0} \frac{\partial c_{0}}{\partial \widetilde{\omega}}  \tag{A.17}\\
M P N & \equiv q_{0} \frac{\partial n_{0}}{\partial \widetilde{\omega}} \tag{A.18}
\end{align*}
$$

these express the dollar-for-dollar (or hour-for-dollar) marginal propensities to consume and work at date 0 : indeed,

$$
\frac{\partial c_{0}}{\partial y_{0}}=\frac{\partial c_{0}}{\partial \widetilde{\omega}} \frac{\partial \widetilde{\omega}}{\partial y_{0}}=\frac{M P C}{q_{0}} q_{0}=M P C
$$

and similarly $\frac{\partial n_{0}}{\partial y_{0}}=M P N$.
Totally differentiating the Marshallian consumption function and using (A.10), we find

$$
d c_{0}=\frac{\partial c_{0}}{\partial q_{0}} d q_{0}+\frac{\partial c_{0}}{\partial w_{0}} d w_{0}+\frac{\partial c_{0}}{\partial \widetilde{\omega}}\left(\left(U R E+c_{0}-w_{0} n_{0}\right) d q_{0}+q_{0} d y_{0}-q_{0} N N P \frac{d P}{P}\right)
$$

Using (A.15)-(A.16),

$$
\begin{aligned}
d c_{0}= & \left(\frac{\partial c_{0}^{h}}{\partial q_{0}}-\frac{\partial c_{0}}{\partial \widetilde{\omega}} e_{q_{0}}\right) d q_{0}+\left(\frac{\partial c_{0}^{h}}{\partial w_{0}}-\frac{\partial c_{0}}{\partial \widetilde{\omega}} e_{w_{0}}\right) d w_{0} \\
& \quad+\frac{\partial c_{0}}{\partial \widetilde{\omega}}\left(\left(U R E+c_{0}-w_{0} n_{0}\right) d q_{0}+q_{0} d y_{0}-q_{0} N N P \frac{d P}{P}\right) \\
= & \frac{\partial c_{0}}{\partial \widetilde{\omega}}\left(-e_{w_{0}} d w_{0}+q_{0} d y_{0}+\left(-e_{q_{0}}+U R E+c_{0}-w_{0} n_{0}\right) d q_{0}-N N P \frac{d P}{P}\right)+\frac{\partial c_{0}^{h}}{\partial q_{0}} d q_{0}+\frac{\partial c_{0}^{h}}{\partial w_{0}} d w_{0}
\end{aligned}
$$

and using (A.7), (A.8) and (A.17) to replace $e_{w_{0}}, e_{q_{0}}$ and $\frac{\partial c_{0}}{\partial \tilde{\omega}}$, we find

$$
\begin{aligned}
d c_{0} & =\frac{M P C}{q_{0}}\left(q_{0} n_{0} d w_{0}+q_{0} d y_{0}+U R E d q_{0}-q_{0} N N P \frac{d P}{P}\right)+\frac{\partial c_{0}^{h}}{\partial q_{0}} d q_{0}+\frac{\partial c_{0}^{h}}{\partial w_{0}} d w_{0} \\
& =M P C\left(n_{0} d w_{0}+d y_{0}+U R E \frac{d q_{0}}{q_{0}}-N N P \frac{d P}{P}\right)+c_{0}\left(\frac{q_{0}}{c_{0}} \frac{\partial c_{0}^{h}}{\partial q_{0}} \frac{d q_{0}}{q_{0}}+\frac{w_{0}}{c_{0}} \frac{\partial c_{0}^{h}}{\partial w_{0}} \frac{d w_{0}}{w_{0}}\right)
\end{aligned}
$$

Finally, dropping time subscripts for ease of notation, using (A.5), and defining compensated elasticities
by

$$
\begin{aligned}
\epsilon_{c, q}^{h} & \equiv \frac{q_{0}}{c_{0}} \frac{\partial c_{0}^{h}}{\partial q_{0}} \\
\epsilon_{c, w}^{h} & \equiv \frac{w_{0}}{c_{0}} \frac{\partial c_{0}^{h}}{\partial w_{0}}
\end{aligned}
$$

we obtain

$$
\begin{equation*}
d c=M P C\left(n d w+d y+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)+c\left(\epsilon_{c, q}^{h} \frac{d R}{R}+\epsilon_{c, w}^{h} \frac{d w}{w}\right) \tag{A.19}
\end{equation*}
$$

In a completely analogous way, we also find

$$
\begin{equation*}
d n=M P N\left(n d w+d y+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)+n\left(\epsilon_{n, q}^{h} \frac{d R}{R}+\epsilon_{n, w}^{h} \frac{d w}{w}\right) \tag{A.20}
\end{equation*}
$$

The rest of the proof calculates the compensated elasticities and relates them to MPC and MPN, which will yield our expressions for consumption and labor supply. To get my expression for welfare, totally differentiate the indirect utility function and use (A.12)-(A.14) and (A.10) to obtain

$$
\begin{aligned}
d U & =\frac{\partial V}{\partial q_{0}} d q_{0}+\frac{\partial V}{\partial w_{0}} d w_{0}+\frac{\partial V}{\partial \widetilde{\omega}} d \widetilde{\omega} \\
& =\frac{u^{\prime}\left(c_{0}\right)}{q_{0}} \cdot\left(U R E d q_{0}+q_{0} n_{0} d w_{0}+q_{0} d y_{0}-q_{0} N N P \frac{d P}{P}\right)
\end{aligned}
$$

This yields my expression in (5),

$$
d U=u^{\prime}(c) \cdot\left(d y+n d w+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)
$$

Step 2: Marginal propensities. I now derive explicit expressions for marginal propensities to consume, that is, the Marshallian derivatives of the consumption and labor supply functions that are solutions to (A.11). Inverting the first-order conditions

$$
\begin{align*}
u^{\prime}\left(c_{t}\right) & =\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) u^{\prime}\left(c_{0}\right)  \tag{A.21}\\
v^{\prime}\left(n_{t}\right) & =\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) v^{\prime}\left(n_{0}\right) \tag{A.22}
\end{align*}
$$

and inserting the resulting values for $c_{t}$ and $n_{t}$ into the budget constraint (redefining $W=\frac{\widetilde{\omega}}{q_{0}}$ as presentvalue wealth for simplicity)

$$
\sum_{t \geq 0} \frac{q_{t}}{q_{0}}\left(c_{t}-w_{t} n_{t}\right)=W
$$

we obtain

$$
\begin{equation*}
c_{0}+\sum_{t \geq 1} \frac{q_{t}}{q_{0}}\left(u^{\prime}\right)^{-1}\left[\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) u^{\prime}\left(c_{0}\right)\right]-w_{0}\left(n_{0}+\sum_{t \geq 1} \frac{q_{t}}{q_{0}} \frac{w_{t}}{w_{0}}\left(v^{\prime}\right)^{-1}\left[\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) v^{\prime}\left(n_{0}\right)\right]\right)=W \tag{A.23}
\end{equation*}
$$

Recall that MPC $=\frac{\partial c_{0}}{\partial W}$ and $M P N=\frac{\partial n_{0}}{\partial W}$. Differentiating (A.23) with respect to $W$, we obtain

$$
\begin{equation*}
\operatorname{MPC}\left(1+\sum_{t \geq 1} \frac{q_{t}}{q_{0}} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) \frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime \prime}\left(c_{t}\right)}\right)-w_{0} M P N\left(1+\sum_{t \geq 1} \frac{q_{t}}{q_{0}} \frac{w_{t}}{w_{0}} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime \prime}\left(n_{t}\right)}\right)=1 \tag{A.24}
\end{equation*}
$$

moreover, the intratemporal first order condition

$$
\begin{equation*}
v^{\prime}\left(n_{0}\right)=w_{0} u^{\prime}\left(c_{0}\right) \tag{A.25}
\end{equation*}
$$

implies

$$
\begin{aligned}
v^{\prime \prime}\left(n_{0}\right) M P N & =w_{0} u^{\prime \prime}\left(c_{0}\right) M P C \\
\frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime}\left(n_{0}\right)} M P N & =\frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime}\left(c_{0}\right)} \text { MPC }
\end{aligned}
$$

so, using the definition of the local elasticities of substitution,

$$
\begin{align*}
-\sigma\left(c_{t}\right) c_{t} u^{\prime \prime}\left(c_{t}\right) & =u^{\prime}\left(c_{t}\right)  \tag{A.26}\\
\psi\left(n_{t}\right) n_{t} v^{\prime \prime}\left(n_{t}\right) & =v^{\prime}\left(n_{t}\right) \tag{A.27}
\end{align*}
$$

we see that MPC and MPN are related through

$$
M P N=-\frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \frac{n_{0}}{c_{0}} M P C
$$

Inserting into (A.24), this gives

$$
\begin{equation*}
M P C=\left(1+\sum_{t \geq 1} \frac{q_{t}}{q_{0}} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) \frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime \prime}\left(c_{t}\right)}+\frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \frac{w_{0} n_{0}}{c_{0}}\left(1+\sum_{t \geq 1} \frac{q_{t}}{q_{0}} \frac{w_{t}}{w_{0}} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime \prime}\left(n_{t}\right)}\right)\right)^{-1} \tag{A.28}
\end{equation*}
$$

as well as

$$
\begin{align*}
M P S= & 1-M P C+w_{0} M P N \\
= & M P C\left(\sum_{t \geq 1} \frac{q_{t}}{q_{0}} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) \frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime \prime}\left(c_{t}\right)}\right. \\
& \left.\quad+\frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \frac{w_{0} n_{0}}{c_{0}} \sum_{t \geq 1} \frac{q_{t}}{q_{0}} \frac{w_{t}}{w_{0}} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime \prime}\left(n_{t}\right)}\right) \tag{A.29}
\end{align*}
$$

Expressions (A.28) and (A.29) can also be rewritten using the fact that (A.21)-(A.22) together with (A.26)(A.27) yield

$$
\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) \frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime \prime}\left(c_{t}\right)}=\frac{\sigma\left(c_{t}\right) c_{t}}{\sigma\left(c_{0}\right) c_{0}} \quad \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime \prime}\left(n_{t}\right)}=\frac{\psi\left(n_{t}\right) n_{t}}{\psi\left(n_{0}\right) n_{0}}
$$

So, we also have

$$
M P C=\left(1+\sum_{t \geq 1} \frac{q_{t}}{q_{0}} \frac{\sigma\left(c_{t}\right) c_{t}}{\sigma\left(c_{0}\right) c_{0}}+\frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \frac{w_{0} n_{0}}{c_{0}}\left(1+\sum_{t \geq 1}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{\psi\left(n_{t}\right) n_{t}}{\psi\left(n_{0}\right) n_{0}}\right)\right)^{-1}
$$

Step 3: Hicksian elasticities. The solution to the expenditure minimization problem in (A.6) also involves the first-order conditions (A.21)-(A.22) , from which we obtain

$$
u\left(c_{t}\right)=u\left(\left(u^{\prime}\right)^{-1}\left[\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) u^{\prime}\left(c_{0}\right)\right]\right) \quad v\left(n_{t}\right)=v\left(\left(v^{\prime}\right)^{-1}\left[\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) v^{\prime}\left(n_{0}\right)\right]\right)
$$

attaining utility $U$ requires that the initial values $c_{0}, n_{0}$ satisfy

$$
\begin{align*}
& u\left(c_{0}\right)+\sum_{t \geq 1} \beta^{t} u\left(\left(u^{\prime}\right)^{-1}\left[\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) u^{\prime}\left(c_{0}\right)\right]\right)-v\left(n_{0}\right) \\
& \quad-\sum_{t \geq 1} \beta^{t} v\left(\left(v^{\prime}\right)^{-1}\left[\beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) v^{\prime}\left(n_{0}\right)\right]\right)=U \tag{A.30}
\end{align*}
$$

Differentiating with respect to $q_{0}$ along the indifference curve (A.30) results in

$$
\begin{array}{r}
\frac{\partial c_{0}}{\partial q_{0}}\left(u^{\prime}\left(c_{0}\right)+\sum_{t \geq 1} \beta^{t} u^{\prime}\left(c_{t}\right) \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) \frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime \prime}\left(c_{t}\right)}\right) \\
-\frac{\partial n_{0}}{\partial q_{0}}\left(v^{\prime}\left(n_{0}\right)+\sum_{t \geq 1} \beta^{t} v^{\prime}\left(n_{t}\right) \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime \prime}\left(n_{t}\right)}\right) \\
-\sum_{t \geq 1} \beta^{t} \frac{u^{\prime}\left(c_{t}\right)}{u^{\prime \prime}\left(c_{t}\right)}\left(\beta^{-t} \frac{q_{t}}{q_{0}^{2}} u^{\prime}\left(c_{0}\right)\right)-\sum_{t \geq 1} \beta^{t} \frac{v^{\prime}\left(n_{t}\right)}{v^{\prime \prime}\left(n_{t}\right)}\left(\beta^{-t} \frac{q_{t}}{q_{0}^{2}}\left(\frac{w_{t}}{w_{0}}\right) v^{\prime}\left(n_{0}\right)\right)=0
\end{array}
$$

dividing by $u^{\prime}\left(c_{0}\right)$ and using (A.21), (A.25), (A.26) and (A.27) we find

$$
\begin{array}{r}
\frac{\partial c_{0}}{\partial q_{0}}\left(1+\sum_{t} \frac{q_{t}}{q_{0}} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) \frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime \prime}\left(c_{t}\right)}\right)-\frac{\partial n_{0}}{\partial q_{0}} w_{0}\left(1+\sum_{t \geq 1} \frac{q_{t}}{q_{0}} \frac{w_{t}}{w_{0}} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime \prime}\left(n_{t}\right)}\right) \\
\quad=\frac{1}{u^{\prime}\left(c_{0}\right)}\left(\sum_{t \geq 1} \beta^{t} \frac{u^{\prime}\left(c_{t}\right)}{u^{\prime \prime}\left(c_{t}\right)}\left(\beta^{-t} \frac{q_{t}}{q_{0}^{2}} u^{\prime}\left(c_{0}\right)\right)+\sum_{t \geq 1} \beta^{t} \frac{v^{\prime}\left(n_{t}\right)}{v^{\prime \prime}\left(n_{t}\right)}\left(\beta^{-t} \frac{q_{t}}{q_{0}^{2}}\left(\frac{w_{t}}{w_{0}}\right) v^{\prime}\left(n_{0}\right)\right)\right)
\end{array}
$$

moreover, differentiating (A.25) we also find

$$
\frac{\partial n_{0}}{\partial q_{0}}=-\frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \frac{n_{0}}{c_{0}} \frac{\partial c_{0}}{\partial q_{0}}
$$

Gathering results, we recognize, on the left-hand-side, the MPC expression in (A.28). We then use firstorder conditions on the right hand side to obtain

$$
\begin{aligned}
\frac{\partial c_{0}}{\partial q_{0}} M P C^{-1} & =\frac{1}{u^{\prime}\left(c_{0}\right)}\left\{\sum_{t \geq 1} \beta^{t} \frac{u^{\prime}\left(c_{t}\right)}{u^{\prime \prime}\left(c_{t}\right)}\left(\beta^{-t} \frac{q_{t}}{q_{0}^{2}} u^{\prime}\left(c_{0}\right)\right)-\sum_{t \geq 1} \beta^{t} \frac{v^{\prime}\left(n_{t}\right)}{v^{\prime \prime}\left(n_{t}\right)}\left(\beta^{-t} \frac{q_{t}}{q_{0}^{2}}\left(\frac{w_{t}}{w_{0}}\right) v^{\prime}\left(n_{0}\right)\right)\right\} \\
& =\frac{1}{q_{0}}\left(\sum_{t \geq 1} \frac{u^{\prime}\left(c_{t}\right)}{u^{\prime \prime}\left(c_{t}\right)} \frac{q_{t}}{q_{0}}-w_{0} \sum_{t \geq 1} \frac{v^{\prime}\left(n_{t}\right)}{v^{\prime \prime}\left(n_{t}\right)} \frac{q_{t}}{q_{0}}\left(\frac{w_{t}}{w_{0}}\right)\right)
\end{aligned}
$$

Manipulating the right-hand side, we recognize the expression for (A.29) as

$$
\begin{aligned}
\frac{\partial c_{0}}{\partial q_{0}} M P C^{-1}= & -\frac{1}{q_{0}} \sigma\left(c_{0}\right) c_{0}\left\{\sum_{t \geq 1} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right) \frac{u^{\prime \prime}\left(c_{0}\right)}{u^{\prime \prime}\left(c_{t}\right)} \frac{q_{t}}{q_{0}}\right. \\
& \left.+\frac{w_{0} n_{0}}{c_{0}} \frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \sum_{t \geq 1} \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime \prime}\left(n_{t}\right)} \frac{q_{t}}{q_{0}}\left(\frac{w_{t}}{w_{0}}\right)\right\} \\
= & -\frac{1}{q_{0}} \sigma\left(c_{0}\right) c_{0} \frac{M P S}{M P C}
\end{aligned}
$$

and therefore, we finally simply have

$$
\left.\frac{\partial c_{0}}{\partial q_{0}}\right|_{U}=-\frac{c_{0}}{q_{0}} \sigma\left(c_{0}\right) M P S
$$

which corresponds to a Hicksian elasticity of

$$
\begin{equation*}
\epsilon_{c_{0}, q_{0}}^{h}=-\sigma\left(c_{0}\right) M P S \tag{A.31}
\end{equation*}
$$

A similar procedure can be used to differentiate with respect to $w_{0}$ : from (A.25) we obtain

$$
\frac{\partial n_{0}}{\partial w_{0}}=-\frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \frac{n_{0}}{c_{0}} \frac{\partial c_{0}}{\partial w_{0}}+\psi\left(n_{0}\right) \frac{n_{0}}{w_{0}}
$$

and differentiating along (A.30) we therefore obtain

$$
\begin{aligned}
& \frac{\partial c_{0}}{\partial w_{0}} u^{\prime}\left(c_{0}\right) M P C^{-1}-\psi\left(n_{0}\right) \frac{n_{0}}{w_{0}}\left(v^{\prime}\left(n_{0}\right)+\sum_{t \geq 1} \beta^{t} v^{\prime}\left(n_{t}\right) \beta^{-t}\left(\frac{q_{t}}{q_{0}}\right)\left(\frac{w_{t}}{w_{0}}\right) \frac{v^{\prime \prime}\left(n_{0}\right)}{v^{\prime \prime}\left(n_{t}\right)}\right) \\
& \quad=\sum_{t \geq 1} \beta^{t} \frac{v^{\prime}\left(n_{t}\right)}{v^{\prime \prime}\left(n_{t}\right)} \beta^{-t} \frac{q_{t}}{q_{0}}\left(\frac{w_{t}}{w_{0}^{2}}\right) v^{\prime}\left(n_{0}\right)
\end{aligned}
$$

We conclude by noticing that $v^{\prime}\left(n_{0}\right)=\psi\left(n_{0}\right) n_{0} v^{\prime \prime}\left(n_{0}\right)$, so

$$
\frac{\partial c_{0}}{\partial w_{0}} \int_{U}=\operatorname{MPC} \psi\left(n_{0}\right) n_{0}
$$

and

$$
\begin{equation*}
\epsilon_{c_{0}, w_{0}}^{h}=\operatorname{MPC}\left(\psi\left(n_{0}\right) \frac{w_{0} n_{0}}{c_{0}}\right) \tag{A.32}
\end{equation*}
$$

Finally, elasticities for $n_{0}$ result from a final differentiation of (A.25):

$$
\begin{align*}
\epsilon_{n_{0}, q_{0}}^{h} & =-\frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \epsilon_{c_{0}, q_{0}}^{h}  \tag{A.33}\\
\epsilon_{n_{0}, w_{0}}^{h} & =\psi\left(n_{0}\right)\left(1-\frac{1}{\sigma\left(c_{0}\right)} \epsilon_{c_{0}, w_{0}}^{h}\right) \\
& =\psi\left(n_{0}\right)\left(1-\frac{\psi\left(n_{0}\right)}{\sigma\left(c_{0}\right)} \frac{w_{0} n_{0}}{c_{0}} M P C\right) \\
& =\psi\left(n_{0}\right)\left(1+w_{0} M P N\right) \tag{A.34}
\end{align*}
$$

Step 4: Putting all expressions together. For consumption, equations (A.31)-(A.32) can be inserted into (A.19) to yield

$$
d c=M P C\left(n d w+d y+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)+c\left(-\sigma M P S \frac{d R}{R}+\psi M P C \frac{w n}{c} \frac{d w}{w}\right)
$$

The first term is the wealth effect, and the last two terms the substitution effects with respect to interest rates and wages. We then simplify the expression to

$$
\begin{equation*}
d c=M P C\left(d y+n(1+\psi) d w+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)-\sigma c M P S \frac{d R}{R} \tag{A.35}
\end{equation*}
$$

which is our equation (3).
Similarly, equations (A.33)-(A.34) can be inserted into (A.20) to yield

$$
d n=M P N\left(n d w+d y+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)+n\left(\psi M P S \frac{d R}{R}+\psi(1+w M P N) \frac{d w}{w}\right)
$$

and we again naturally separate the latter piece to obtain

$$
\begin{equation*}
d n=M P N\left(d y+n(1+\psi) d w+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)+\psi n M P S \frac{d R}{R}+\psi n \frac{d w}{w} \tag{A.36}
\end{equation*}
$$

which is equation (4).

## A. 3 Extension of Theorem 1 to general preferences and persistent changes

Theorem 1 in the main text is a special case of a general decomposition that holds for arbitrary nonsatiable preferences $U$ over $\left\{c_{t}\right\}$ and $\left\{n_{t}\right\}$ and for any change in the price level $\left\{P_{0}, P_{1} \ldots\right\}$, the real term structure $\left\{q_{0}=1, q_{1}, q_{2} \ldots\right\}$, the agent's unearned income sequence $\left\{y_{0}, y_{1} \ldots\right\}$ and the stream of real wages $\left\{w_{0}, w_{1} \ldots\right\}$, with the nominal term structure adjusting instantaneously to make the Fisher equation hold at the post-shock sequences of interest rates and prices. The utility maximization problem is then

$$
\begin{array}{cl}
\max & U\left(\left\{c_{t}, n_{t}\right\}\right) \\
\text { s.t. } & P_{t} c_{t}=P_{t} y_{t}+W_{t} n_{t}+\left({ }_{t-1} B_{t}\right)+\sum_{s \geq 1}\left({ }_{t} Q_{t+s}\right)\left({ }_{t-1} B_{t+s}-{ }_{t} B_{t+s}\right) \\
& +P_{t}\left({ }_{t-1} b_{t}\right)+\sum_{s \geq 1}\left({ }_{t} q_{t+s}\right) P_{t+s}\left({ }_{t-1} b_{t+s}-{ }_{t} b_{t+s}\right)
\end{array}
$$

and the first order date-0 responses of consumption, labor supply and welfare to the considered change are, in this case, given by

$$
\begin{aligned}
d c_{0} & =M P C d \Omega+c_{0}\left(\sum_{t \geq 0} \epsilon_{c_{0}, q_{t}}^{h} \frac{d q_{t}}{q_{t}}+\sum_{t \geq 0} \epsilon_{c_{0}, w_{t}}^{h} \frac{d w_{t}}{w_{t}}\right) \\
d n_{0} & =M P N d \Omega+n_{0}\left(\sum_{t \geq 0} \epsilon_{n_{0}, q_{t}}^{h} \frac{d q_{t}}{q_{t}}+\sum_{t \geq 0} \epsilon_{n_{0}, w_{t}}^{h} \frac{d w_{t}}{w_{t}}\right) \\
d U & =U_{c_{0}} d \Omega
\end{aligned}
$$

where $\epsilon_{x_{0}, y_{t}}^{h}=\frac{\partial x_{0}^{h}}{\partial y_{t}} \frac{y_{t}}{x_{0}}$ for $x \in\{c, n\}$ and $y \in\{q, w\}$ are Hicksian elasticities and $d \Omega=d W-\sum_{t \geq 0} c_{t} d q_{t}$, the net-of-consumption wealth change, is given by

$$
\begin{align*}
d \Omega= & \underbrace{\sum_{t \geq 0}\left(q_{t} y_{t}\right) \frac{d y_{t}}{y_{t}}}_{\text {Real unearned income change }}+\underbrace{\sum_{t \geq 0}\left(q_{t} w_{t} n_{t}\right) \frac{d w_{t}}{w_{t}}}_{\text {Real earned income change }} \\
& +\underbrace{\sum_{t \geq 0} q_{t}\left(y_{t}+w_{t} n_{t}+\left(\frac{-1 B_{t}}{P_{t}}\right)+\left(-1 b_{t}\right)-c_{t}\right) \frac{d q_{t}}{q_{t}}}_{\text {Revaluation of net savings flows }}-\underbrace{\sum_{t \geq 0} Q_{t}\left(\frac{-1 B_{t}}{P_{0}}\right) \frac{d P_{t}}{P_{t}}}_{\text {Revaluation of net nominal position }} \tag{A.37}
\end{align*}
$$

The proof is a generalization of that in section A.2. I omit it here in the interest of space.

Values of all elasticities with separable preferences in a steady-state with no growth. Following once more the steps of section A.2, it is possible to derive the value of Hicksian elasticities for a change at any horizon. Here I just report the values of these elasticities in the case of an infinite horizon model where $\frac{q_{s}}{q_{0}}=\beta^{s}$ and $w_{s}=w^{*}, \forall s$. These prices correspond to those prevailing in a steady-state with no growth of any such model, and the resulting elasticities are relevant, for example, to determine the impulse responses in many RBC and DSGE models. The first order conditions imply that consumption and labor supply are constant. Let us call the solutions $c^{*}$ and $n^{*}$, respectively. Writing $\vartheta \equiv \frac{w^{*} n^{*}}{c^{*}}$ for the share of earned income in consumption and $\kappa \equiv \frac{\frac{\psi}{\sigma} \vartheta}{1+\frac{\psi}{\sigma} \vartheta} \in(0,1)$, obtain values of elasticities summarized in table A.1.

Table A.1: Steady-state moments, separable preferences

| $\epsilon^{h}$ | $q_{0}$ | $q_{s} s \geq 1$ | $w_{0}$ | $w_{s}, s \geq 1$ | Marg. propensity |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $c_{0}$ | $-\sigma \beta$ | $\sigma(1-\beta) \beta^{s}$ | $\sigma \kappa(1-\beta)$ | $\sigma \kappa(1-\beta) \beta^{s}$ | $M P C$ | $(1-\kappa)(1-\beta)$ |
| $n_{0}$ | $\psi \beta$ | $-\psi(1-\beta) \beta^{s}$ | $\psi(1-\kappa(1-\beta))$ | $-\psi \kappa(1-\beta) \beta^{s}$ | $M P N$ | $-\frac{1}{w^{*}} \kappa(1-\beta)$ |
|  |  |  |  |  | $M P S$ | $(1-\beta)$ |

## A. 4 Proof of corollary 1

Rewrite equations (A.35) and (A.36) as

$$
\begin{aligned}
d c & =\operatorname{MPC}\left(d Y+\psi d w-w d n+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)-\sigma c M P S \frac{d R}{R} \\
w d n-\psi n d w & =w M P N\left(d Y+\psi d w-w d n+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)+\psi w n M P S \frac{d R}{R}
\end{aligned}
$$

Hence

$$
w d n-\psi n d w=\frac{1}{1+w M P N}\left\{w M P N\left(d Y+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)+\psi w n M P S \frac{d R}{R}\right\}
$$

which, inserted into the expression for $d c$ yields
$d c=M P C\left(1-\frac{w M P N}{1+w M P N}\right)\left(d Y+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)-\sigma c M P S\left(1+M P C \frac{\psi w n}{\sigma c} \frac{1}{1+w M P N}\right) \frac{d R}{R}$
But $M P C \frac{\psi n}{\sigma c}=-M P N$ so this is

$$
d c=\left(\frac{M P C}{1+w M P N}\right)\left(d Y+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)-\sigma c \frac{M P S}{1+w M P N} \frac{d R}{R}
$$

and noting that

$$
1+w M P N=M P C+M P S
$$

we can finally rewrite this in terms of $M \hat{P P C}=\frac{M P C}{M P C+M P S}$ as

$$
d c=M \hat{P} C\left(d Y+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)-\sigma c(1-M \hat{P} C) \frac{d R}{R}
$$

as claimed.

## A. 5 Adding durable goods

This section shows the consequences of adding durable goods to the model.
I consider a standard durable goods problem. For simplicity, I ignore labor supply and nominal assets, neither of which interacts with the conclusions below. A consumer maximizes a separable intertemporal utility function

$$
\begin{array}{cl}
\max & \sum^{t} \beta^{t}\left\{u\left(C_{t}\right)+w\left(D_{t}\right)\right\} \\
\text { s.t. } & C_{t}+p_{t} I_{t}=Y_{t}+\left({ }_{t-1} b_{t}\right)+\sum_{s \geq 1}\left({ }_{t} q_{t+s}\right)\left({ }_{t-1} b_{t+s}-{ }_{t} b_{t+s}\right) \\
& D_{t}=I_{t}+D_{t-1}(1-\delta) \\
& D_{-1},\left\{{ }_{-1} b_{t}\right\} \quad \text { given }
\end{array}
$$

where $C_{t}$ is now nondurable consumption, $D_{t}$ is the consumer's stock of durables, and $p_{t}$ is the relative price of durable goods in period $t$.

I am interested in the response of the demand for nondurable goods $C_{t}$ and durables goods $I_{t}$, as well as that of total expenditures

$$
\begin{equation*}
X_{t} \equiv C_{t}+p_{t} I_{t} \tag{A.38}
\end{equation*}
$$

to a change in the time- 0 nondurable real interest rate $R_{0}$ and (potentially) a simultaneous change in the price of durables $p_{0}$. As I argue below, the notion of aggregate demand makes most sense when the relative price of durables does not change with $R_{0}$, but I start by covering the general case in which $p_{0}$ can change.

The intertemporal budget constraint reads

$$
\sum_{t \geq 0} q_{t}\left(C_{t}+p_{t} I_{t}\right)=\sum_{t \geq 0} q_{t} Y_{t}+\sum_{t \geq 0} q_{t}\left(-1 b_{t}\right)
$$

Defining $R_{t} \equiv \frac{q_{t}}{q_{t+1}}$, the first-order conditions of this problem are, for all $t \geq 0$

$$
\begin{align*}
u^{\prime}\left(C_{t}\right) & =\beta R_{t} u^{\prime}\left(C_{t+1}\right)  \tag{A.39}\\
w^{\prime}\left(D_{t}\right) & =u^{\prime}\left(C_{t}\right)\left[p_{t}-\frac{(1-\delta) p_{t+1}}{R_{t}}\right] \tag{A.40}
\end{align*}
$$

Equation (A.39) is the standard Euler equation for nondurable consumption. Equation (A.40) shows that the consumer equates the marginal rate of substitution between the stock of durables and consumption to the user cost of durables, $p_{t}-\frac{(1-\delta) p_{t+1}}{R_{t}}$. A fall in the nondurable real interest rate at date $0, R_{0}$, increases the desired level of nondurable consumption and of the stock of nondurables (an intertemporal substitution effect). Holding $p_{1}$ constant, it also reduces the user cost of durables, increasing the desired stock of durables relative to nondurable consumption. A fall in $p_{0}$ has the same effect of reducing the durable user cost, but it does not affect intertemporal substitution in consumption.

Suppose that the path for interest rates $\left\{R_{t}\right\}$, relative prices $\left\{p_{t}\right\}$ and income $\left\{Y_{t}\right\}$ delivers the solution $\left\{C_{t}, D_{t}\right\}$. Consider the solution under the alternative paths $\left\{\overline{R_{0}}, R_{1}, R_{2} \ldots\right\},\left\{\overline{p_{0}}, p_{1}, p_{2} \ldots\right\}$, and $\left\{\overline{Y_{0}}, \Upsilon_{1}, \Upsilon_{2} \ldots\right\}$. Let $d R=\overline{R_{0}}-R_{0}, d p=\overline{p_{0}}-p_{0}$ and $d Y=\overline{Y_{0}}-Y_{0}$. I am interested in the response of the paths of nondurable and durable expenditures to these changes. To obtain this, I find the paths for consumption $\left\{C_{t}\right\}$ and durables $\left\{D_{t}\right\}$, and then find the implied path for durable expenditures $\left\{p_{t} I_{t}\right\}$.

Marshallian demand. In order to determine the Marshallian demands, I could follow the same proof as that of section A.2, but here I follow an alternative and somewhat more intuitive procedure. The procedure is in two steps. First, I determine a variation that respects all the first-order conditions (A.39)-(A.40) at the new prices. This gives $d C^{*}$ and $d D^{*}$, which result in a budgetary cost $d \Omega^{*}$ at the old prices. Second, I determine the change in net wealth $d \Omega$ that results from the change in prices. The Marshaling demands are then

$$
\begin{align*}
d C & =d C^{*}+M P C\left(d \Omega-d \Omega^{*}\right)  \tag{A.41}\\
d D & =d D^{*}+M P D\left(d \Omega-d \Omega^{*}\right) \tag{A.42}
\end{align*}
$$

where $M P D=\frac{\partial D}{\partial Y}$ is the increase in the stock of date-0 durables that results from a date- 0 increase in income. Note that MPC and MPD are related: differentiating (A.40), we find

$$
w^{\prime \prime}\left(D_{0}\right) M P D=u^{\prime \prime}\left(C_{0}\right) M P C\left[p_{0}-\frac{(1-\delta) p_{1}}{R_{0}}\right]
$$

$$
M P D=\frac{\sigma_{D}}{\sigma_{C}} \frac{D_{0}}{C_{0}} M P C
$$

where $\sigma_{C} \equiv-\frac{u^{\prime}\left(C_{0}\right)}{u^{\prime \prime}\left(C_{0}\right) C_{0}}$ and $\sigma_{D} \equiv-\frac{w^{\prime}\left(D_{0}\right)}{w^{\prime \prime}\left(D_{0}\right) D_{0}}$ are the elasticities of intertemporal substitution in consumption and in the stock of durables. Since $D_{0}=I_{0}+D_{-1}(1-\delta)$ and the initial stock $D_{-1}$ is fixed, the total constant- $p$ marginal propensity to spend at date 0 is

$$
\begin{aligned}
M P X \equiv \frac{\partial(C+p I)}{\partial Y}=\frac{\partial C}{\partial Y}+p \frac{\partial D}{\partial Y} & =M P C+p M P D \\
& =M P C\left(1+\frac{\sigma_{D}}{\sigma_{C}} \frac{p D}{C}\right)
\end{aligned}
$$

Step 1: variation respecting FOCs. The simplest variation that respects all FOCs holds the paths $\left\{C_{t}\right\}$ and $\left\{D_{t}\right\}$ fixed for all $t \geq 1$ and adjusts $C_{0}$ and $D_{0}$ by $d C$ (respectively $d D$ ) such that (A.39) and (A.40) are satisfied at $t=0$. Differentiating these equations, I obtain

$$
\begin{aligned}
-\frac{1}{\sigma_{C}} \frac{d C}{C} & =\frac{d R}{R} \\
-\frac{1}{\sigma_{D}} \frac{d D}{D} & =-\frac{1}{\sigma_{C}} \frac{d C}{C}+\frac{p_{1} \frac{1-\delta}{R}}{p_{0}-p_{1} \frac{1-\delta}{R}} \frac{d R}{R}+\frac{p_{0}}{p_{0}-p_{1} \frac{1-\delta}{R}} \frac{d p}{p}
\end{aligned}
$$

Hence we find

$$
\begin{equation*}
d C^{*}=-\sigma_{C} C \frac{d R}{R} \tag{A.43}
\end{equation*}
$$

and

$$
\begin{equation*}
d D^{*}=-\sigma_{D} D\left[\frac{p_{0}}{p_{0}-p_{1} \frac{1-\delta}{R}}\right]\left(\frac{d R}{R}+\frac{d p}{p}\right) \tag{A.44}
\end{equation*}
$$

These responses are very intuitive: one way to respond to a fall in real interest rates is to raise nondurable consumption and the stock of durables. The relevant elasticity for durables is higher than $\sigma_{D}$ because of the additional substitution effect coming from the change in the user cost. A lower current relative price of durables has a symmetric effect on the demand for durables as that of a lower real interest rate (in other words, it is the real interest rate in terms of durables that matters for durables demand).

We are now ready to determine the net cost of this variation. Since

$$
\begin{aligned}
& D_{0}=(1-\delta) D_{-1}+I_{0} \\
& D_{1}=(1-\delta) D_{0}+I_{1}
\end{aligned}
$$

the sequence of investment that achieves this variation consists naturally in an increase of $d D^{*}$ followed by a subsequent decrease:

$$
\begin{aligned}
d I_{0}^{*} & =d D^{*} \\
d I_{1}^{*} & =-(1-\delta) d D^{*}
\end{aligned}
$$

Hence the total budgetary cost of this 'star' variation at the old prices $p$ and $R$ has the simple form

$$
\begin{aligned}
d \Omega^{*} & =d C^{*}+p_{0} d I_{0}^{*}+p_{1} \frac{d I_{1}^{*}}{R} \\
& =d C^{*}+\left(p_{0}-p_{1} \frac{1-\delta}{R}\right) d D^{*} \\
& =-\left(\sigma_{C} C+p_{0} \sigma_{D} D\right) \frac{d R}{R}-\sigma_{D} p_{0} D \frac{d p}{p}
\end{aligned}
$$

Step 2: change in net wealth. Let $\Omega$ be defined as

$$
\Omega \equiv \sum_{t \geq 0} q_{t}\left\{Y_{t}+\left({ }_{-1} b_{t}\right)-C_{t}-p_{t} I_{t}\right\}
$$

At the initial prices, the intertemporal budget constraint implies $\Omega=0$. The exogenous variation $d R, d p$ and $d Y$ yields

$$
\begin{align*}
d \Omega & =d Y-I d p+\sum_{t \geq 0} d q_{t}\left\{Y_{t}+\left({ }_{-1} b_{t}\right)-C_{t}-p_{t} I_{t}\right\} \\
& =d Y-I d p-\sum_{t \geq 1} q_{t}\left\{Y_{t}+\left({ }_{-1} b_{t}\right)-C_{t}-p_{t} I_{t}\right\} \frac{d R}{R} \\
& =d Y-p I_{0} \frac{d p}{p}+(\underbrace{Y_{0}+\left({ }_{-1} b_{0}\right)-C_{0}-p_{0} I_{0}}_{U R E}) \frac{d R}{R} \tag{A.45}
\end{align*}
$$

The intuition is as follows. Suppose that the nondurable real interest rate falls at date 0 . As before, this benefits consumers that have a negative $U R E$, that is, maturing liabilities $C_{0}+p_{0} I_{0}$ in excess maturing assets $Y_{0}+\left({ }_{-1} b_{0}\right)$. Note that, for this effect, total expenditures including expenditures on durables are counted as part of $U R E$. In that sense, $U R E$ measures the true balance-sheet exposure to a change in the real interest rate. In particular, ceteris paribus, when investment is higher today the consumer benefits more from a fall in real interest rates.

Suppose however that, in parallel, the relative price of durables rises. In the general equilibrium model of Barsky, House and Kimball (2007), for example, this happens in response to an accommodative monetary policy shock when durable goods prices are more flexible than nondurable goods prices. In that case, equation (A.45) shows that there is an additional capital loss on wealth due to the rise in the durable relative price. While conceptually distinct, these two effects could be consolidated into a single one, if we restrict ourselves to variations that feature a constant elasticity of the durable-good price to the nondurable real interest rate

$$
\begin{equation*}
\epsilon_{p R} \equiv-\frac{\partial p}{p} \frac{R}{\partial R} \tag{A.46}
\end{equation*}
$$

The benchmark case where $p$ is constant corresponds to $\epsilon_{p R}=0$, the case where the durable real interest rate is constant to $\epsilon_{p R}=1$. Then,

$$
\begin{equation*}
d \Omega=d Y+(\underbrace{Y_{0}+\left({ }_{-1} b_{0}\right)-C_{0}-p_{0} I_{0}\left(1-\epsilon_{p R}\right)}_{U R E^{\epsilon}}) \frac{d R}{R} \tag{A.47}
\end{equation*}
$$

In other words, once we net out the capital revaluation effect, an alternative measure of $U R E$ becomes $U R E^{\epsilon}$, which subtracts a fraction $\left(1-\epsilon_{p R}\right)$ of durable expenditures.

Step 3: demand for durables and nondurables. Combining (A.41)-(A.42) with (A.43), (A.44) and (A.45), I obtain the Marshallian demands (recall that $d I=d D$ at time 0 )

$$
\begin{aligned}
d C= & M P C\left(d Y+U R E \frac{d R}{R}+\left(\sigma_{C} C+p \sigma_{D} D\right) \frac{d R}{R}+\left(p \sigma_{D} D-p I_{0}\right) \frac{d p}{p}\right)-\sigma_{C} C \frac{d R}{R} \\
d D= & M P D\left(d Y+U R E \frac{d R}{R}+\left(\sigma_{C} C+p \sigma_{D} D\right) \frac{d R}{R}+\left(p \sigma_{D} D-p I_{0}\right) \frac{d p}{p}\right) \\
& \quad-\sigma_{D} D\left[\frac{p_{0}}{p_{0}-p_{1} \frac{1-\delta}{R}}\right]\left(\frac{d R}{R}+\frac{d p}{p}\right)
\end{aligned}
$$

This separates out the separate effects from changing $R$ and $p$. Given the elasticity $\epsilon_{p R}$ in (A.46), we can also rewrite this as

$$
\begin{align*}
d C=M P C & \left(d Y+U R E^{\epsilon} \frac{d R}{R}\right)-\sigma_{C} C(1-M P C) \frac{d R}{R} \\
& +\sigma_{D} \cdot p D \cdot M P C \cdot\left(1-\epsilon_{p R}\right) \cdot \frac{d R}{R}  \tag{A.48}\\
d D= & M P D\left(d Y+U R E^{\epsilon} \frac{d R}{R}\right)+\sigma_{C} \cdot M P D \cdot C \cdot \frac{d R}{R} \\
& \quad-\sigma_{D} \cdot p D \cdot\left(1-\epsilon_{p R}\right) \cdot(1-M P D) \cdot\left[\frac{1}{p_{0}-p_{1} \frac{1-\delta}{R}}\right] \frac{d R}{R} \tag{A.49}
\end{align*}
$$

Where $U R E^{\epsilon}$ is defined in (A.47).

Special case with constant durable real interest rate $\left(\epsilon_{p R}=1\right)$. When $\epsilon_{p R}=1$, equations (A.48)-(A.49) simplify to

$$
\begin{aligned}
d C & =M P C\left(d Y+U R E^{1} \frac{d R}{R}\right)-\sigma_{C} C(1-M P C) \frac{d R}{R} \\
d D & =M P D\left(d Y+U R E^{1} \frac{d R}{R}\right)+\sigma_{C} \cdot M P D \cdot C \cdot \frac{d R}{R}
\end{aligned}
$$

which are simple extensions of expressions in the main text, with $U R E^{1}$ (which does not subtract durable expenditures) replacing $U R E$. Note that to the extent that $U R E^{1} \geq 0$, the expression for $d D$ implies a contraction in durable goods from an increase in real interest rates, as in Barsky, House and Kimball (2007). This is counterfactual, suggesting that $\epsilon_{p R}=1$ may be too high an elasticity in practice.

Special case with constant relative price ( $\epsilon_{p R}=0$ ). While the cases where $\epsilon_{p R} \neq 0$ are interesting in principle, they prevent a straightforward definition of aggregate demand $X=C+p I$ : if the relative price of two goods can change, then the relative demands for these two goods (as well as their relative supplies) will matter for general equilibrium. Therefore, the case where $\epsilon_{p R}=0$ is the most relevant for my purposes. Assume then that $p_{0}=p_{1}=p$. In this case, we can combine (A.48) and (A.49) to obtain an expression for the change in aggregate demand $d X=d C+p d D$ as a function of the marginal
propensity to spend $M P X=M P C+p M P D$ and other variables

$$
d X=M P X\left(d Y+U R E \frac{d R}{R}+\sigma_{C} C+\sigma_{D} p D\right)-\left(\sigma_{C} C+\frac{\sigma_{D} p D}{1-\frac{1-\delta}{R}}\right) \frac{d R}{R}
$$

This can further be simplified to yield an expression with the same form as the expression in the main text,

$$
\begin{equation*}
d X=M P X\left(d Y+U R E \frac{d R}{R}\right)-\sigma_{X}(1-M P X) X \frac{d R}{R} \tag{A.50}
\end{equation*}
$$

where $\sigma_{X}$ is defined as

$$
\begin{equation*}
\sigma_{X} \equiv \frac{C}{X} \cdot \sigma_{C}+\left(1-\frac{C}{X}\right) \cdot \sigma_{D} \cdot \frac{p D}{p I} \cdot \frac{\frac{1}{1-\frac{1-\delta}{R}}-M P X}{1-M P X} \tag{A.51}
\end{equation*}
$$

In other words, $\sigma_{X}$ is a weighted average of $\sigma_{\mathcal{C}}$ and the relevant elasticity of substitution in durable expenditures: the product of $\sigma_{D}$ by the stock-flow ratio $\frac{p D}{p I}$, multiplied by a term that increases in the elasticity of the user cost to the real interest rate.

Quantitatively, the second term is likely to be much larger than the first. If initially durable expenditures cover replacement costs $I=D \delta$, then the stock-flow ratio is $\frac{1}{\delta}$. Hence, with $\delta=5 \%$ and $R=1.05$ at annual rates, the second term in (A.51) is at least as large as $\frac{1}{20} \times \frac{1}{10} \times \sigma_{D}=200 \sigma_{D}$. This makes aggregate demand very sensitive to given changes in the real interest rate because of the large substitution effect that results from the presence of long-lived durables, a point made by Barsky et al. (2007).

## A. 6 Proof of theorem 2

After dividing through by $P_{t}$, defining the real bond position as $\lambda_{t} \equiv \frac{\Lambda_{t}}{P_{t-1}}$ and writing $\Pi_{t} \equiv \frac{P_{t}}{P_{t-1}}$ for the inflation rate between $t-1$ and $t$, the budget constraint (9) becomes

$$
c_{t}+Q_{t}\left(\lambda_{t+1}-\delta \frac{\lambda_{t}}{\Pi_{t}}\right)+\left(\theta_{t+1}-\theta_{t}\right) \cdot \mathbf{s}_{t}=y_{t}+w_{t} n_{t}+\frac{\lambda_{t}}{\Pi_{t}}+\theta_{t} \cdot \mathbf{d}_{t}
$$

In this notation, the consumer's date- $t$ net nominal position is

$$
N N P_{t}=\left(1+Q_{t} \delta\right) \frac{\lambda_{t}}{\Pi_{t}}
$$

while his unhedged interest rate exposure is:

$$
U R E_{t}=y_{t}+w_{t} n_{t}+\frac{\lambda_{t}}{\Pi_{t}}+\theta_{t} \cdot \mathbf{d}_{t}-c_{t}=Q_{t}\left(\lambda_{t+1}-\delta \frac{\lambda_{t}}{\Pi_{t}}\right)+\left(\theta_{t+1}-\theta_{t}\right) \cdot \mathbf{S}_{t}
$$

His optimization problem can be represented using the recursive formulation

$$
\begin{array}{ll} 
& \max _{c, n, \lambda^{\prime}, \theta^{\prime}} u(c)-v(n)+\underbrace{\beta \mathbb{E}\left[V\left(\lambda^{\prime}, \theta^{\prime} ; y^{\prime}, w^{\prime}, Q^{\prime}, \Pi^{\prime}, \mathbf{d}^{\prime}, \mathbf{S}^{\prime}\right)\right]}_{\equiv W\left(\lambda^{\prime}, \theta^{\prime}\right)} \\
\text { s.t. } & c+Q\left(\lambda^{\prime}-\delta \frac{\lambda}{\bar{\Pi}}\right)+\left(\theta^{\prime}-\theta\right) \mathbf{S}=y+w n+\frac{\lambda}{\Pi}+\theta \mathbf{d}  \tag{A.52}\\
& Q \lambda^{\prime}+\theta^{\prime} \mathbf{S} \geq \frac{\bar{D}}{R}
\end{array}
$$

The function $V$ corresponds to the value from optimizing given a starting real level of bonds $\lambda^{\prime}$ and shares $\theta^{\prime}$, and includes the possibility of hitting future borrowing constraints.

I consider the predicted effects on $c$ and $n$ resulting from a simultaneous unexpected change in unearned income $d y$, the real wage $d w$, the price level $\frac{d P}{P}=\frac{d \Pi}{\Pi}$ and the real interest rate $d R$, which result in a change in asset prices $\frac{d Q}{Q}=\frac{d S_{j}}{S_{j}}=-\frac{d R}{R}$ for $j=1 \ldots N$. By leaving the future unaffected, this purely transitory change does not alter the value from future optimization starting at $\left(\lambda^{\prime}, \theta^{\prime}\right)$ - that is, the function $W$ is unchanged. I claim that, provided the consumption and labor supply functions are differentiable, their first order differentials are

$$
\begin{align*}
& d c=M P C\left(d y+n(1+\psi) d w+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)-\sigma c M P S \frac{d R}{R}  \tag{A.53}\\
& d n=M P N\left(d y+n(1+\psi) d w+U R E \frac{d R}{R}-N N P \frac{d P}{P}\right)+\psi n M P S \frac{d R}{R}+\psi n \frac{d w}{w} \tag{A.54}
\end{align*}
$$

where $\sigma \equiv-\frac{u^{\prime}(c)}{c u \prime^{\prime \prime}(c)}$ and $\psi=\frac{v^{\prime}(n)}{n v^{\prime \prime}(n)}$ are the local elasticities of intertemporal substitution and labor supply, respectively, MPC $=\frac{\partial c}{\partial y}, M P N=\frac{\partial n}{\partial y}$ and MPS $=1-M P C+w M P N$.

In order to prove (A.53) and (A.54), there are two cases to consider. In the first case, the consumer is at a binding borrowing limit or lives hand-to-mouth. The problem is then a static choice between $c$ and $n$. In the second case, the consumer is at an interior optimum. The result then follows from application of the implicit function theorem to the set of $N+2$ first-order conditions which, together with the budget constraint, characterize the solution to the problem in (A.52). Here, to simplify the notation and the proof, I first prove the statement in the case where all variables are changing but $N=0$, and then consider the case with stocks $(N>0)$ but without bonds and assuming only $R$ is changing.

## Case 1. Binding borrowing limit and hand-to-mouth agents.

Proof. The consumption of an agent at the borrowing limit is given by

$$
\begin{equation*}
c=w n+Z \tag{A.55}
\end{equation*}
$$

where

$$
Z=z+(1+Q \delta) \frac{\lambda}{\Pi}+\theta \cdot(\mathbf{d}+\mathbf{S})+\frac{\bar{D}}{R}
$$

Similarly, the consumption of an agent that lives hand to mouth is

$$
c=w n+z
$$

Given that $d \mathbf{S}=-\frac{\mathrm{S}}{R} d R, d Q=-\frac{Q}{R} d R$ and $d\left(\frac{1}{\Pi}\right)=-\frac{1}{\Pi^{2}} d \Pi=-\frac{1}{\Pi} \frac{d P}{P}$, we have, if the agent is at the
borrowing limit

$$
\begin{equation*}
d Z=d z-\underbrace{(1+Q \delta) \frac{\lambda}{\Pi}}_{\text {NNP }} \frac{d P}{P}+\underbrace{\left(Q \delta \frac{\lambda}{\Pi}+\theta \cdot \mathbf{S}+\frac{\bar{D}}{R}\right)}_{- \text {URE }}\left(-\frac{d R}{R}\right) \tag{A.56}
\end{equation*}
$$

and, if the agent lives hand to mouth,

$$
d Z=d z
$$

but since that agent also has

$$
N N P=U R E=0
$$

equation (A.56) still applies. In both cases, the consumer is making a static choice between $c$ and $n$ given the budget constraint (A.55), and hence has MPS $=0$. We can then apply the results of section A. 2 to find

$$
\begin{aligned}
d c & =\operatorname{MPC}(d Z+w(1+\psi)) \\
d n & =\operatorname{MPN}(d Z+w(1+\psi))+\psi n d w
\end{aligned}
$$

which yields the desired result.
Case 2a). $N=0$, all variables changing I first prove the following lemma.
Lemma A.1. Let $c(z, w, q, b)$ and $n(z, w, q, b)$ be the solution to the following separable consumer choice problem under concave preferences over current consumption $u(c)$ and assets $V(a)$, and convex preferences over hours worked v (n):

$$
\begin{array}{cl}
\max & u(c)-v(n)+V(a) \\
\text { s.t. } & c+q(a-b)=w n+z
\end{array}
$$

Assume $c()$ and $n()$ are differentiable. Then the first order differentials are

$$
\begin{aligned}
d c & =M P C(d z+n(1+\psi) d w-(a-b) d q+q d b)-\sigma c M P S \frac{d q}{q} \\
d n & =M P N(d z+n(1+\psi) d w-(a-b) d q+q d b)+\psi n M P S \frac{d q}{q}+\psi n \frac{d w}{w}
\end{aligned}
$$

where $M P C=\frac{\partial c}{\partial z}, M P N=\frac{\partial n}{\partial z}$ and $M P S=1-M P C+w M P N=1-M P C\left(1+\frac{w n}{c} \frac{\psi}{\sigma}\right)$.
Proof. The following first-order conditions are necessary and sufficient for optimality:

$$
\begin{equation*}
u^{\prime}(c)=\frac{1}{w} v^{\prime}(n)=\frac{1}{q} V^{\prime}(a) \tag{A.57}
\end{equation*}
$$

I first obtain the expression for MPC by considering an increase in income $d z$ alone. Consider how that increase is divided between current consumption, leisure and assets. (A.57) implies

$$
\begin{equation*}
u^{\prime \prime}(c) d c=\frac{1}{w} v^{\prime \prime}(n) d n=\frac{1}{q} V^{\prime \prime}(a) d a \tag{A.58}
\end{equation*}
$$

where the changes $d c, d n$ and $d a$ are related to $d z$ through the budget constraint

$$
\begin{equation*}
d c+q d a=w d n+d z \tag{A.59}
\end{equation*}
$$

Define $M P C=\frac{\partial c}{\partial z}, M P N=\frac{\partial n}{\partial z}$ and $M P S=q \frac{\partial a}{\partial z}$. Then (A.58) implies

$$
\begin{aligned}
& \frac{M P N}{M P C}=w \frac{u^{\prime \prime}(c)}{v^{\prime \prime}(n)}=\frac{u^{\prime \prime}(c)}{u^{\prime}(c)} \frac{v^{\prime}(n)}{v^{\prime \prime}(n)}=-\frac{n}{c} \frac{\psi}{\sigma} \\
& \frac{M P S}{M P C}=\frac{q^{2} u^{\prime \prime}(c)}{V^{\prime \prime}(a)}=\frac{q}{c} \frac{V^{\prime}(a)}{\sigma V^{\prime \prime}(a)}
\end{aligned}
$$

where $\sigma \equiv-\frac{u^{\prime}(c)}{c u^{\prime \prime}(c)}$ and $\psi \equiv \frac{v^{\prime}(n)}{n v^{\prime \prime}(n)}$. Hence the total marginal propensity to spend is

$$
\begin{equation*}
1-M P S=\frac{\partial c}{\partial z}-w \frac{\partial n}{\partial z}=M P C\left(1+\frac{w n}{c} \frac{\psi(n)}{\sigma(c)}\right)=1-\frac{q^{2} u^{\prime \prime}(c)}{V^{\prime \prime}(a)} M P C \tag{A.60}
\end{equation*}
$$

and the marginal propensity to consume is

$$
M P C=\frac{1}{1+q^{2} \frac{u^{\prime \prime}(c)}{V^{\prime \prime}(a)}-w^{2} \frac{u^{\prime \prime}(c)}{v^{\prime \prime}(n)}}=\frac{V^{\prime \prime}(a) v^{\prime \prime}(n)}{V^{\prime \prime}(a) v^{\prime \prime}(n)+q^{2} u^{\prime \prime}(c) v^{\prime \prime}(n)-w^{2} u^{\prime \prime}(c) V^{\prime \prime}(a)}
$$

Consider now the overall effect on $c, n$ and $a$ of a change in $q, w, z$ and $b$. Applying the implicit function theorem to the system of equations

$$
\left\{\begin{array}{l}
v^{\prime}(n)-w u^{\prime}(c)=0 \\
V^{\prime}(a)-q u^{\prime}(c)=0 \\
c+q(a-b)-w n-z=0
\end{array}\right.
$$

results in the following expression for partial derivatives:

$$
\begin{align*}
& {\left[\begin{array}{llll}
\frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial w} & \frac{\partial c}{\partial b} \\
\frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b} \\
\frac{\partial a}{\partial q} & \frac{\partial a}{\partial z} & \frac{\partial a}{\partial w} & \frac{\partial a}{\partial b}
\end{array}\right]} \\
& =-\underbrace{\left[\begin{array}{ccc}
-w u^{\prime \prime}(c) & v^{\prime \prime}(n) & 0 \\
-q u^{\prime \prime}(c) & 0 & V^{\prime \prime}(a) \\
1 & -w & q
\end{array}\right]^{-1}\left[\begin{array}{cccc}
0 & 0 & -u^{\prime}(c) & 0 \\
-u^{\prime}(c) & 0 & 0 & 0 \\
(a-b) & -1 & -n & -q
\end{array}\right]}_{\equiv A} \tag{A.61}
\end{align*}
$$

now

$$
\operatorname{det}(A)=v^{\prime \prime}(n) V^{\prime \prime}(a)-w^{2} u^{\prime \prime}(c) V^{\prime \prime}(a)+q^{2} u^{\prime \prime}(c) v^{\prime \prime}(n)=\frac{V^{\prime \prime}(a) v^{\prime \prime}(n)}{M P C}
$$

and so

$$
A^{-1}=\frac{M P C}{V^{\prime \prime}(a) v^{\prime \prime}(n)}\left[\begin{array}{ccc}
w V^{\prime \prime}(a) & -v^{\prime \prime}(n) q & v^{\prime \prime}(n) V^{\prime \prime}(a) \\
q^{2} u^{\prime \prime}(c)+V^{\prime \prime}(a) & -w q u^{\prime \prime}(c) & w u^{\prime \prime}(c) V^{\prime \prime}(a) \\
q w u^{\prime \prime}(c) & w^{2} u^{\prime \prime}(c)-v^{\prime \prime}(n) & q u^{\prime \prime}(c) v^{\prime \prime}(n)
\end{array}\right]
$$

therefore, the first row of (A.61)

$$
\left[\begin{array}{cccc}
\frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial w} & \frac{\partial c}{\partial b}
\end{array}\right]=M P C\left[\begin{array}{ccc}
-\frac{w}{v^{\prime \prime}(n)} & \frac{q}{V^{\prime \prime}(a)} & -1
\end{array}\right]\left[\begin{array}{cccc}
0 & 0 & -u^{\prime}(c) & 0  \tag{A.62}\\
-u^{\prime}(c) & 0 & 0 & 0 \\
(a-b) & -1 & -n & -q
\end{array}\right]
$$

Using (A.60) we find

$$
-q \frac{u^{\prime}(c)}{V^{\prime \prime}(a)} M P C=\frac{\sigma c}{q} q^{2} \frac{u^{\prime \prime}(c)}{V^{\prime \prime}(a)} M P C=\frac{\sigma c}{q} M P S
$$

so that the first column of the matrix equation (A.62) reads

$$
\frac{\partial c}{\partial q}=\frac{\sigma c}{q} M P S-(a-b) M P C
$$

The second and fourth column of (A.62) yield directly

$$
\begin{aligned}
& \frac{\partial c}{\partial z}=M P C \\
& \frac{\partial c}{\partial b}=q M P C
\end{aligned}
$$

Finally, using (A.57) we have

$$
w \frac{u^{\prime}(c)}{v^{\prime \prime}(n)}=\frac{v^{\prime}(n)}{v^{\prime \prime}(n)}=\psi n
$$

so that the third column of (A.62) reads

$$
\begin{aligned}
\frac{\partial c}{\partial w} & =M P C \psi n+M P C n \\
& =M P C(1+\psi) n
\end{aligned}
$$

The first-order total differential $d c$ is then

$$
\begin{align*}
d c & =\frac{\partial c}{\partial z} d z+\frac{\partial c}{\partial b} d b+\frac{\partial c}{\partial q} d q+\frac{\partial c}{\partial w} d w \\
& =M P C(d z+q d b-(a-b) d q+(1+\psi) n d w)+\sigma c M P S \frac{d q}{q} \tag{A.63}
\end{align*}
$$

as claimed. Similarly, after using $M P N=M P C w \frac{u^{\prime \prime}(c)}{\overline{v^{\prime \prime}(n)}}$, the second row of (A.61) is

$$
\left[\begin{array}{cccc}
\frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b}
\end{array}\right]=M P N\left[\begin{array}{ccc}
-\frac{q^{2}+V^{\prime \prime}(a) / u^{\prime \prime}(c)}{w V^{\prime \prime}(a)} & \frac{q}{V^{\prime \prime}(a)} & -1
\end{array}\right]\left[\begin{array}{cccc}
0 & 0 & -u^{\prime}(c) & 0  \tag{A.64}\\
-u^{\prime}(c) & 0 & 0 & 0 \\
(a-b) & -1 & -n & -q
\end{array}\right]
$$

Using (A.60) we find

$$
-q \frac{u^{\prime}(c)}{V^{\prime \prime}(a)} M P N=\frac{\sigma c}{q} q^{2} \frac{u^{\prime \prime}(c)}{V^{\prime \prime}(a)} M P C\left(\frac{-n \psi}{\sigma c}\right)=-\frac{n \psi}{q} M P S
$$

Again the first column yields

$$
\frac{\partial c}{\partial q}=-\frac{n \psi}{q} M P S-(a-b) M P N
$$

The second and fourth column of (A.62) yield directly

$$
\begin{aligned}
& \frac{\partial c}{\partial z}=M P N \\
& \frac{\partial c}{\partial b}=q M P N
\end{aligned}
$$

Finally, since

$$
\begin{aligned}
\left(q^{2}+\frac{V^{\prime \prime}(a)}{u^{\prime \prime}(c)}\right) \frac{u^{\prime}(c)}{V^{\prime \prime}(a)} & =-\sigma c\left(q^{2} \frac{u^{\prime \prime}(c)}{V^{\prime \prime}(a)}+1\right) \\
& =\psi n \frac{M P C}{M P N}\left(\frac{M P S}{M P C}+1\right)
\end{aligned}
$$

the third column yields

$$
\frac{\partial n}{\partial w}=\frac{1}{w} \psi n(M P S+M P C)+M P N n=\frac{1}{w} \psi n(1+w M P N)+M P N n=\psi n \frac{1}{w}+M P N(n+\psi n)
$$

The first-order total differential $d n$ is then

$$
\begin{align*}
d n & =\frac{\partial n}{\partial z} d z+\frac{\partial n}{\partial b} d b+\frac{\partial n}{\partial q} d q+\frac{\partial n}{\partial w} d w \\
& =M P N(d z+q d b-(a-b) d q+(1+\psi) n d w)-\psi n M P S \frac{d q}{q}+\psi n \frac{d w}{w} \tag{A.65}
\end{align*}
$$

Proof of theorem 2 in case 2a). If the policy functions are differentiable and the consumer is at an interior optimum, then the conditions of lemma A. 1 are satisfied: the borrowing constraint is not binding so can be ignored, and the value function is concave per standard dynamic programming arguments. The notation of theorem 2 can be cast using that of the lemma by using the mapping

$$
q \equiv Q \quad z \equiv y+\frac{\lambda}{\Pi} \quad a \equiv \lambda^{\prime} \quad b \equiv \delta \frac{\lambda}{\Pi}
$$

with $\frac{d P}{P}=\frac{d \Pi}{\Pi}$ and $\frac{d Q}{Q}=-\frac{d R}{R}$. Hence $d z=d y-\frac{\lambda}{\Pi} \frac{d P}{P}, d b=-\delta \frac{\lambda}{\Pi} \frac{d P}{P}$ and $\frac{d q}{q}=-\frac{d R}{R}$; so

$$
d z+q d b-(a-b) d q=d y-\underbrace{(1+Q \delta) \frac{\lambda}{\Pi}}_{\text {NNP }} \frac{d P}{P}+\underbrace{\left(\lambda^{\prime}-\delta \frac{\lambda}{\Pi}\right) Q}_{\text {URE }} \frac{d R}{R}
$$

Inserting this equation into (A.63) and (A.65) yields the desired result.
Case 2b) $N>0$, no bonds, only $R$ changing. Since we are not considering changes in wages, it is sufficient to restrict the analysis to a choice between consumption and assets. The following lemma
then proves the result for $d c$. The result for $d n$ follows as a straightforward extension.
Lemma A.2. Let $c(\theta, Y, R)$ be the solution to the following consumer choice problem under concave preferences over current consumption $u(c)$ and assets $W\left(\theta^{\prime}\right)$

$$
\begin{array}{cl}
\max _{c, \theta^{\prime}} & u(c)+W\left(\theta^{\prime}\right) \\
\text { s.t. } & c+\left(\theta^{\prime}-\theta\right) \mathbf{S}=Y+\theta \mathbf{d}
\end{array}
$$

where $\frac{d \mathbf{S}}{d R}=-\frac{\mathbf{S}}{R}$. Then, to first order

$$
d c=M P C\left(d Y+U R E \frac{d R}{R}\right)-\sigma(c) c(1-M P C) \frac{d R}{R}
$$

where $\sigma(c) \equiv-\frac{u^{\prime}(c)}{c u^{\prime \prime}(c)}$ is the local elasticity of intertemporal substitution, MPC $=\frac{\partial c}{\partial Y}$, and $U R E=Y+\theta \mathbf{d}-c$ Proof. The following first-order conditions characterize the solution

$$
\begin{equation*}
S^{i} u^{\prime}\left(Y+\theta \mathbf{d}-\left(\theta^{\prime}-\theta\right) \mathbf{S}\right)=W_{\theta^{i}}\left(\theta^{\prime}\right) \quad \forall i=1 \ldots N \tag{A.66}
\end{equation*}
$$

Consider first an increase in income $d Y$ alone. Differentiating along (A.66) we find

$$
\begin{equation*}
S^{i} u^{\prime \prime}(c)\left(1-\sum_{j} S^{j} \frac{d \theta^{\prime} j}{d Y}\right)=\sum_{j} W_{\theta^{i} \theta}\left(\theta^{\prime}\right) \frac{d \theta^{\prime} j}{d Y} \quad \forall i \tag{A.67}
\end{equation*}
$$

Define $\eta^{j} \equiv S^{j} \frac{d \theta^{\prime} j}{d Y}$. Then (A.67) rewrites

$$
\sum_{j}\left(\frac{1}{S^{i} S^{j}} W_{\theta^{i} \theta^{j}}\left(\theta^{\prime}\right)+u^{\prime \prime}(c)\right) \eta^{j}=u^{\prime \prime}(c) \quad \forall i
$$

Defining the matrix $M$ with elements

$$
m_{i j} \equiv \frac{1}{S^{i} S^{j}} W_{\theta^{i} \theta^{j}}\left(\theta^{\prime}\right)+u^{\prime \prime}(c)
$$

this system can also be written in matrix form as

$$
M \eta=u^{\prime \prime}(c) \mathbf{1}
$$

or

$$
\eta=u^{\prime \prime}(c) M^{-1} \mathbf{1}
$$

The budget constraint then implies that

$$
\begin{equation*}
M P C=\frac{d c}{d Y}=1-\sum_{j} \eta^{j}=1-u^{\prime \prime}(c) m \tag{A.68}
\end{equation*}
$$

where $m$ is defined as

$$
\begin{equation*}
m \equiv \mathbf{1} M^{-1} \mathbf{1} \tag{A.69}
\end{equation*}
$$

Next, consider an increase in the real interest rate $d R$. Differentiating along (A.66) we now have

$$
\frac{d S^{i}}{d R} u^{\prime}(c)+S^{i} u^{\prime \prime}(c)\left(-\sum_{j} S^{j} \frac{d \theta^{\prime} j}{d R}-\sum_{j} \frac{d S^{j}}{d R}\left(\theta^{\prime j}-\theta^{j}\right)\right)=\sum_{j} W_{\theta^{i} \theta^{j}}\left(\theta^{\prime}\right) \frac{d \theta^{\prime} j}{d R} \quad \forall i
$$

Using $\frac{d S^{i}}{S^{i}}=-\frac{d R}{R}$ this rewrites

$$
\begin{equation*}
-\frac{S^{i}}{R} u^{\prime}(c)+S^{i} u^{\prime \prime}(c)\left(-\sum_{j} S^{j} \frac{d \theta^{\prime} j}{d R}+\sum_{j} \frac{S^{j}}{R}\left(\theta^{\prime j}-\theta^{j}\right)\right)=\sum_{j} W_{\theta^{i} \theta^{j}}\left(\theta^{\prime}\right) \frac{d \theta^{\prime} j}{d R} \quad \forall i \tag{A.70}
\end{equation*}
$$

Defining now $\gamma^{j} \equiv S^{j} \frac{d \theta i^{\prime}}{d R}$,(A.70) shows that $\gamma^{j}$ solves

$$
\sum_{j} m_{i j} \gamma^{j}=-\frac{1}{R} u^{\prime}(c)+u^{\prime \prime}(c) \sum_{j} \frac{S^{j}}{R}\left(\theta^{\prime j}-\theta^{j}\right) \quad \forall i
$$

which rewrites in matrix form

$$
M \gamma=\left(-\frac{1}{R} u^{\prime}(c)+u^{\prime \prime}(c) \sum_{j} \frac{S^{j}}{R}\left(\theta^{\prime j}-\theta^{j}\right)\right) \mathbf{1}
$$

or

$$
\begin{equation*}
\gamma=\left(-\frac{1}{R} u^{\prime}(c)+u^{\prime \prime}(c) \sum_{j} \frac{S^{j}}{R}\left(\theta^{\prime} j-\theta^{j}\right)\right) M^{-1} \mathbf{1} \tag{A.71}
\end{equation*}
$$

Differentiating with respect to $R$ along the budget constraint $c=Y+\theta \mathbf{d}-\left(\theta^{\prime}-\theta\right) \mathbf{S}$, we next see that

$$
\frac{d c}{d R}=-\sum_{j} S^{j} \frac{\theta^{\prime} j}{d R}+\sum_{j} \frac{S^{j}}{R}\left(\theta^{j}-\theta^{\prime} j\right)=-\sum_{j} \gamma^{j}+\sum_{j} \frac{S^{j}}{R}\left(\theta^{j}-\theta^{\prime j}\right)
$$

inserting (A.71) and using the definition of $m$,

$$
\begin{equation*}
\frac{d c}{d R}=-\left(-\frac{1}{R} u^{\prime}(c)+u^{\prime \prime}(c) \sum_{j} \frac{S^{j}}{R}\left(\theta^{\prime} j-\theta^{j}\right)\right) m+\sum_{j} \frac{S^{j}}{R}\left(\theta^{j}-\theta^{\prime j}\right) \tag{A.72}
\end{equation*}
$$

rearranging terms and using $u^{\prime}(c) \equiv-c \sigma(c) u^{\prime \prime}(c)$ we find

$$
\frac{d c}{d R}=-\sigma(c) \frac{c}{R} u^{\prime \prime}(c) m+\sum_{j} \frac{S^{j}}{R}\left(\theta^{\prime j}-\theta^{j}\right)\left(1-u^{\prime \prime}(c) m\right)
$$

But using the expression for MPC in (A.68), this is simply

$$
\frac{d c}{d R}=-\sigma(c) \frac{c}{R}(1-M P C)+\sum_{j} \frac{S^{j}}{R}\left(\theta^{j^{\prime}}-\theta^{j}\right) M P C
$$

and using the budget constraint $\sum_{j} S^{j}\left(\theta^{j^{\prime}}-\theta^{j}\right)=\left(\theta^{\prime}-\theta\right) \cdot \mathbf{S}_{t}=U R E$ we obtain

$$
\begin{equation*}
\frac{d c}{d R}=-\sigma(c) \frac{c}{R}(1-M P C)+\frac{1}{R} U R E \cdot M P C \tag{A.73}
\end{equation*}
$$

Finally, considering a simultaneous change in income and the real interest rate, combining (A.68) and (A.73) we obtain the first order differential

$$
d c=M P C\left(d Y+U R E \frac{d R}{R}\right)-\sigma(c) c(1-M P C) \frac{d R}{R}
$$

as was to be shown.

## A. 7 Proof of theorem 3

Given the assumption of fixed balance sheets and purely transitory shocks, Theorem 2 shows that

$$
d c_{i}=M \hat{P} C_{i}\left(d Y_{i}-d t_{i}+U R E_{i} \frac{d R}{R}-N N P_{i} \frac{d P}{P}\right)-\sigma_{i} c_{i}\left(1-M \hat{P} C_{i}\right) \frac{d R}{R}
$$

where, where $d Y_{i}=n_{i} e_{i} d w+w e_{i} d n_{i}+d\left(d_{i}\right)$ is the change in gross income at the individual level and $d t_{i}$ the change in taxes. We can further decompose the change in gross income as

$$
d Y_{i}=\frac{Y_{i}}{Y} d Y+d Y_{i}-\frac{Y_{i}}{Y} d Y
$$

and note that, since $\mathbb{E}_{I}\left[Y_{i}\right]=Y$,

$$
\begin{equation*}
\mathbb{E}_{I}\left[d Y_{i}-\frac{Y_{i}}{Y} d Y\right]=d Y-\frac{\mathbb{E}_{I}\left[Y_{i}\right]}{Y} d Y=0 \tag{A.74}
\end{equation*}
$$

Hence,

$$
d c_{i}=M \hat{P} C_{i}\left(\frac{Y_{i}}{Y} d Y+d Y_{i}-\frac{Y_{i}}{Y} d Y-d t_{i}+U R E_{i} \frac{d R}{R}-N N P_{i} \frac{d P}{P}\right)-\sigma_{i} c_{i}\left(1-M \hat{P} C_{i}\right) \frac{d R}{R}
$$

and taking a cross-sectional average

$$
\begin{align*}
d C= & \mathbb{E}_{I}\left[\frac{Y_{i}}{Y} M \hat{P} C_{i}\right] d Y+\mathbb{E}_{I}\left[M \hat{P} C_{i}\left(d Y_{i}-\frac{Y_{i}}{Y} d Y\right)\right]-\mathbb{E}_{I}\left[M \hat{P} C_{i}\left(d t_{i}\right)\right]-\mathbb{E}_{I}\left[M \hat{P} C_{i} N N P_{i}\right] \frac{d P}{P} \\
& +\left(\mathbb{E}_{I}\left[M \hat{P} C_{i} U R E_{i}\right]-\mathbb{E}_{I}\left[\sigma_{i}\left(1-M \hat{P} C_{i}\right) c_{i}\right]\right) \frac{d R}{R} \tag{A.75}
\end{align*}
$$

Now, the government budget (13) with the fiscal rule $G_{t}=\bar{G}$ and target $\frac{B_{t}}{P_{t}}=\bar{b}$ reads

$$
\mathbb{E}_{I}\left[t_{i t}\right]=\bar{G}+\frac{B_{t}}{P_{t}}-\frac{\bar{b}}{R_{t}}
$$

Using the fact that at the margin, taxes are adjusted lump-sum, and the fact that $N N P_{g}=-\bar{b}$ as well as $U R E_{g}=-\frac{\bar{b}}{R}$, this implies

$$
d t_{i}=d t=N N P_{g} \frac{d P}{P}-U R E_{g} \frac{d R}{R}
$$

In other words, taxes fall with unexpected increases in prices which reduce the government debt burden, and they fall with reductions in real interest rates which reduces the government's debt servicing costs. But the market clearing conditions (17) and (18) imply that these gains and losses have counterparts at the household level:

$$
\begin{equation*}
d t_{i}=d t=-\mathbb{E}_{I}\left[N N P_{i}\right] \frac{d P}{P}+\mathbb{E}_{I}\left[U R E_{i}\right] \frac{d R}{R} \tag{A.76}
\end{equation*}
$$

Hence, (A.75) rewrites

$$
\begin{aligned}
d C= & \mathbb{E}_{I}\left[\frac{Y_{i}}{Y} M \hat{P} C_{i}\right] d Y+\mathbb{E}_{I}\left[M \hat{P} C_{i}\left(d Y_{i}-\frac{Y_{i}}{Y} d Y\right)\right]-\mathbb{E}_{I}\left[M \hat{P} C_{i}\right](d t)-\mathbb{E}_{I}\left[M \hat{P} C_{i} N N P_{i}\right] \frac{d P}{P} \\
& +\left(\mathbb{E}_{I}\left[M \hat{P} C_{i} U R E_{i}\right]-\mathbb{E}_{I}\left[\sigma_{i}\left(1-M \hat{P} C_{i}\right) c_{i}\right]\right) \frac{d R}{R}
\end{aligned}
$$

so

$$
\begin{aligned}
d C= & \mathbb{E}_{I}\left[\frac{Y_{i}}{Y} M \hat{P} C_{i}\right] d Y+\mathbb{E}_{I}\left[M \hat{P} C_{i}\left(d Y_{i}-\frac{Y_{i}}{Y} d Y\right)\right]+\mathbb{E}_{I}\left[M \hat{P} C_{i}\right] \mathbb{E}_{I}\left[N N P_{i}\right] \frac{d P}{P}-\mathbb{E}_{I}\left[M \hat{P} C_{i} N N P_{i}\right] \frac{d P}{P} \\
& +\left(\mathbb{E}_{I}\left[M \hat{P} C_{i} U R E_{i}\right]-\mathbb{E}_{I}\left[M \hat{P} C_{i}\right] \mathbb{E}_{I}\left[U R E_{i}\right]-\mathbb{E}_{I}\left[\sigma_{i}\left(1-M \hat{P} C_{i}\right) c_{i}\right]\right) \frac{d R}{R}
\end{aligned}
$$

and finally, using (A.74)

$$
\begin{aligned}
d C= & \mathbb{E}_{I}\left[\frac{Y_{i}}{Y} M \hat{P} C_{i}\right] d Y+\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, d Y_{i}-Y_{i} \frac{d Y}{Y}\right)-\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, N N P_{i}\right) \frac{d P}{P} \\
& +\left(\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, U R E_{i}\right)-\mathbb{E}_{I}\left[\sigma_{i}\left(1-M \hat{P} C_{i}\right) c_{i}\right]\right) \frac{d R}{R}
\end{aligned}
$$

as claimed.

Case with heterogeneous taxes. If the taxes were not lump-sum, equation (A.76) would be replaced by

$$
\mathbb{E}_{I}\left[d t_{i}\right]=-\mathbb{E}_{I}\left[N N P_{i}\right] \frac{d P}{P}+\mathbb{E}_{I}\left[U R E_{i}\right] \frac{d R}{R}
$$

we would therefore use the fact that

$$
\mathbb{E}_{I}\left[M \hat{P} C_{i}\left(d t_{i}\right)\right]=\mathbb{E}_{I}\left[M \hat{P} C_{i}\right] \mathbb{E}_{I}\left[d t_{i}\right]+\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, d t_{i}\right)
$$

to finally obtain

$$
\begin{aligned}
d C= & \mathbb{E}_{I}\left[\frac{Y_{i}}{Y} M \hat{P} C_{i}\right] d Y+\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, d Y_{i}-Y_{i} \frac{d Y}{Y}\right)-\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, N N P_{i}\right) \frac{d P}{P} \\
& +\left(\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, U R E_{i}\right)-\mathbb{E}_{I}\left[\sigma_{i}\left(1-M \hat{P} C_{i}\right) c_{i}\right]\right) \frac{d R}{R}-\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, d t_{i}\right)
\end{aligned}
$$

The additional heterogeneous-taxation term is very natural. Suppose for example that, at the margin, gains from the government budget $\left(\mathbb{E}_{I}\left[d t_{i}\right]<0\right)$ lead to disproportionate reductions of taxes on high-

MPC agents. Then $\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, d t_{i}\right)<0$, so aggregate consumption increases by more than the benchmark from Theorem 1. The opposite happens when tax reductions fall disproportionately on low-MPC agents.

## A. 8 Proof of corollary 2

From the definition of $\gamma_{i}$ in (24), we have

$$
d\left(\frac{Y_{i}}{Y}\right)=\gamma_{i}\left(\frac{Y_{i}}{Y}-1\right) \frac{d Y}{Y}
$$

Moreover,

$$
\begin{equation*}
d Y_{i}-Y_{i} \frac{d Y}{Y}=Y d\left(\frac{Y_{i}}{Y}\right)=\gamma_{i}\left(\frac{Y_{i}}{Y}-1\right) d Y \tag{A.77}
\end{equation*}
$$

Next, rewrite equation (19) in elasticity terms by dividing by per-capita consumption $C=\mathbb{E}_{I}\left[c_{i}\right]$ and using (A.77). We find

$$
\begin{aligned}
\frac{d C}{C}= & \underbrace{\mathbb{E}_{I}\left[\frac{Y_{i}}{\mathbb{E}_{I}\left[c_{i}\right]} M \hat{P} C_{i}\right] \frac{d Y}{Y}}_{\text {Aggregate income channel }}+\underbrace{\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, \gamma_{i} \frac{Y_{i}}{\mathbb{E}_{I}\left[c_{i}\right]}\right)}_{\text {Earnings heterogeneity channel }} \frac{d Y}{Y}-\underbrace{\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, \frac{N N P_{i}}{\mathbb{E}_{I}\left[c_{i}\right]}\right) \frac{d P}{P}}_{\text {Fisher channel }} \\
& +(\underbrace{\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, \frac{U R E_{i}}{\mathbb{E}_{I}\left[c_{i}\right]}\right)}_{\text {Interest rate exposure channel }}-\underbrace{\mathbb{E}_{I}\left[\sigma_{i}\left(1-M \hat{P} C_{i}\right) \frac{c_{i}}{\mathbb{E}_{I}\left[c_{i}\right]}\right]}_{\text {Substitution channel }}) \frac{d R}{R}
\end{aligned}
$$

Imposing $\gamma_{i}=\gamma$ and $\sigma_{i}=\sigma$ for all $i$, this equation writes

$$
\begin{aligned}
&\frac{d C}{C}=\underbrace{\mathbb{E}_{I}\left[\frac{Y_{i}}{\mathbb{E}_{I}\left[c_{i}\right]} M \hat{P} C_{i}\right]}_{\mathcal{M}} \frac{d \gamma}{Y}+\gamma \times \underbrace{\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, \frac{Y_{i}}{\mathbb{E}_{I}\left[c_{i}\right]}\right)}_{\mathcal{E}_{Y}} \frac{d Y}{Y}-\underbrace{\operatorname{Cov}_{I}\left(M \hat{P} C_{i}\right.}_{\mathcal{E}_{P}}, \frac{N N P_{i}}{\mathbb{E}_{I}\left[c_{i}\right]}) \frac{d P}{P} \\
&+(\underbrace{\operatorname{Cov}_{I}\left(M \hat{P} C_{i}, \frac{U R E_{i}}{\mathbb{E}_{I}\left[c_{i}\right]}\right)}_{\mathcal{E}_{R}}-\sigma \times \underbrace{\mathbb{E}_{I}\left[\left(1-M \hat{P} C_{i}\right) \frac{c_{i}}{\mathbb{E}_{I}\left[c_{i}\right]}\right]}_{S}) \frac{d R}{R}
\end{aligned}
$$

which is equation (25).

## B From quarterly to annual MPCs

In this appendix I derive a simple theoretical relationship between quarterly MPC $\left(M P C^{\ell}\right)$ and annual MPC (MPC ${ }^{A}$ ), namely

$$
\begin{equation*}
M P C^{A}=1-\left(1-M P C^{Q}\right)^{4} \tag{B.1}
\end{equation*}
$$

This relationship holds exactly in some models of consumption, and tends to be a good approximation in many others.

Time is discrete, $t=0,1,2, \ldots, \infty$ and represents quarters. A consumer faces a constant real interest rate $r$, and chooses consumption $c_{t}$ in each period $t$. His budget constraint along any realized path of income $y_{t}$ is

$$
\begin{equation*}
\sum_{t \geq 0}\left(\frac{1}{1+r}\right)^{t} c_{t}=\sum_{t \geq 0}\left(\frac{1}{1+r}\right)^{t} y_{t}+\omega \tag{B.2}
\end{equation*}
$$

Denote by $m_{t}=\frac{\partial \mathbb{E}\left[c_{t}\right]}{\partial \omega}$ the average response of consumption response at date $t$ following a transfer at date 0 . By definition, $M P C^{Q} \equiv m_{0}$, while the annual MPC cumulates spending for the first four quarters, $M P C^{A} \equiv \sum_{t=0}^{3} m_{t}$.

Consider now a simple model where the response of consumption is exponential

$$
\begin{equation*}
m_{t}=m_{0} \lambda^{t} \text { for } \lambda \in(0,1), \quad t>0 \tag{B.3}
\end{equation*}
$$

This rule is exact, for example, in any model in which agents have CRRA utility and consume only out of wealth $\omega$ (so $y_{t}=0$ ), so that consumption is proportional to wealth. (B.2) implies that the discounted sum of the quarterly responses is 1 :

$$
\sum_{t \geq 0}\left(\frac{1}{1+r}\right)^{t} m_{t}=m_{0} \frac{1}{1-\frac{\lambda}{1+r}}=1
$$

Hence, $m_{0}$ and $\lambda$ are related via $\lambda=(1+r)\left(1-m_{0}\right)$. The annual MPC is then

$$
M P C^{A}=\sum_{t=0}^{3} m_{t}=m_{0} \sum_{t=0}^{3} \lambda^{t}=m_{0} \frac{1-\lambda^{4}}{1-\lambda}=M P C^{Q} \frac{1-\left[(1+r)\left(1-M P C^{Q}\right)\right]^{4}}{1-\left[(1+r)\left(1-M P C^{Q}\right)\right]}
$$

In the special case where $r=0$, this delivers equation (B.1). Given the simplicity and robustness ${ }^{48}$ of this formula, I apply it to convert quarterly into annual MPCs in the CE, where annual MPCs are not available. ${ }^{49}$

[^2]
## C Data appendix

This section starts out by providing more details about the data and the MPC identification strategies for the SHIW (section C.1), the PSID (section C.2), and the CE (section C.3).

Section C. 4 then performs a sensitivity analysis along several dimensions. Section C.4.1 considers the consequence of using total consumption expenditure to estimate MPC in the PSID and CE. Section C.4.2 considers the effect of varying the fraction of durable expenditures included in URE, corresponding to different assumptions about the elasticity of the relative durable price to the real interest rate $\epsilon$. Section C.4.3 considers robustness to the number of bins used to stratify the population in the PSID and in the CE. Finally, section C.4.4 calculates redistribution elasticities for URE at a quarterly level in the CE.

Section C. 5 cuts the data in various ways to examine the empirical drivers of the correlations I document in the data. Section C.5.1 looks at the influence of age, and section C.5.2 examines the role of income. Section C.5.3 generalizes my covariance decomposition procedure from section D to multiple covariates, and reports the decomposition when all of Jappelli and Pistaferri (2014)'s covariates are simultaneously used in this decomposition.

Section C. 6 concludes this appendix by contrasting the financial asset and liability information available in the PSID and the CE, and comparing it to the same information reported in the Survey of Consumer Finances (SCF).

## C. 1 SHIW

My first dataset comes from the 2010 wave of the Italian Survey of Household Income and Wealth, which is publicly available from the Bank of Italy's website. This is the data source employed by Jappelli and Pistaferri (2014), and it is very useful for my purposes because it contains a direct household-level measure of MPC, reported as part of a survey question. ${ }^{50}$ An additional benefit of this dataset is that it presents detailed information on financial assets and liabilities, allowing a fairly precise measurement of URE and NNP for each household.

## C.1.1 Exposure measures

The survey is annual, so I do not need to make adjustments to the raw data. ${ }^{51}$ Table C. 1 presents summary statistics in euros.

URE: $Y-T-C+A-L$. To construct my measure of unhedged interest rate exposure, I use net annual disposable income (which includes taxes, transfers, interest income and realized capital gains) as my measure of income net of taxes $Y-T$. My consumption measure $C$ includes expenditures on both durables and non durables goods as well as interest and principal payments (the SHIW records up to three mortgages for each household). I also count house purchases and extraordinary maintenance towards $C$.

[^3]Table C.1: Summary statistics, SHIW

|  | N | mean | p5 | p25 | p50 | p75 | p95 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Net Income | 7,951 | 36,187 | 9,629 | 20,013 | 30,838 | 45,515 | 81,320 |
| Consumption | 7,951 | 30,442 | 10,800 | 17,200 | 24,200 | 34,500 | 65,603 |
| Maturing assets | 7,951 | 28,280 | 0 | 2,000 | 10,467 | 30,242 | 100,000 |
| Maturing liabilities | 7,951 | 9,440 | 0 | 0 | 0 | 305 | 49,000 |
| URE | 7,951 | 24,586 | $-43,958$ | 1,903 | 15,622 | 38,984 | 115,403 |
| Nominal assets | 7,951 | 22,499 | 0 | 1,274 | 6,796 | 22,000 | 77,272 |
| Nominal liabilities | 7,951 | 15,133 | 0 | 0 | 0 | 4,285 | 99,000 |
| Net nominal position | 7,951 | 7,366 | $-81,712$ | -1 | 3,830 | 17,113 | 71,216 |
| Gross income | 7,951 | 38,691 | 7,907 | 19,102 | 31,059 | 48,377 | 92,193 |
| MPC | 7,951 | 0.47 | 0.00 | 0.20 | 0.50 | 0.80 | 1.00 |

For remaining assets maturing in the year ( $A$ ), I consider as "deposits" the amounts held in checking accounts, savings accounts, certificates of deposits, and repurchase agreements. I consider as "bonds" government and corporate bonds, for which I make separate maturity assumptions, as reported in table 2. Given an assumed maturity of $N_{j}$ years for a given asset or liability $j$, I scale the observed amounts by $\frac{1}{N_{j}}$ to obtain an annual measure of maturing flows.

For liabilities maturing in the year ( $L$ ), I scale the principal balance outstanding on adjustable rate mortgages and on credit cards by $\frac{1}{N_{j}}$, given my assumptions for $N_{j}$.

NNP and income. To construct my measure of net nominal position, I include in nominal assets the full amount held in checking accounts, savings accounts, certificates of deposits and repurchase agreements. I also include the full amounts held in bonds from Italian banks and firms, with the exception of inflation-indexed BTP bonds. I assume that two-thirds of foreign bonds are denominated in euros, and count that amount in nominal assets. I then include all the shares of money market mutual funds and bonds mutual funds, in keeping with Doepke and Schneider (2006). For shares held at 'mixed' mutual funds, I assume that half of those are indirectly invested in bonds. Finally, I count all credit originating from commercials or private party loans.

For nominal liabilities, my measure includes all debt due to banks, other financial institutions, and other households, as well as commercial loans.

My results are not influenced in any meaningful way by altering the share of 'mixed' mutual funds invested in bonds, the share of foreign bonds that are euro-denominated, or by excluding commercials and private party loans from both nominal assets and liabilities.

For my income exposure measure $Y$, I use a measure of gross income from the Household Finance and Consumption Survey for Italy, which I merge into my main dataset.

## C. 2 Panel Study of Income Dynamics

The procedure to identify MPC out of transitory income shocks that I employ for the PSID closely follows Blundell, Pistaferri and Preston (2008) (BPP), Kaplan, Violante and Weidner (2014), and Berger et al. (2018). Since the PSID only starts recording detailed consumption information in 1999, my sample period starts with the 1999 wave, and ends in 2013. I use the core sample of the PSID (made up of the

Table C.2: Summary statistics, PSID

|  | N | mean | p5 | p25 | p50 | p75 | p95 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Net Income | 41,820 | 57,943 | 11,338 | 26,882 | 45,356 | 72,554 | 136,969 |
| Consumption | 41,820 | 40,906 | 8,104 | 16,047 | 24,380 | 36,889 | 114,037 |
| Maturing assets | 41,820 | 53,180 | 0 | 1,035 | 8,454 | 35,712 | 220,000 |
| Maturing liabilities | 41,820 | 20,776 | 0 | 74 | 8,290 | 20,410 | 67,690 |
| URE | 41,820 | 49,440 | $-89,630$ | 10 | 18,009 | 57,797 | 256,536 |
| Nominal assets | 41,820 | 48,289 | 0 | 700 | 6,209 | 34,861 | 230,248 |
| Nominal liabilities | 41,820 | 70,728 | 0 | 0 | 17,000 | 106,839 | 282,000 |
| Net Nominal Position | 41,820 | $-22,438$ | $-248,383$ | $-80,228$ | $-4,421$ | 7,084 | 183,004 |
| Gross income | 41,820 | 69,131 | 899 | 23,169 | 50,212 | 89,994 | 184,603 |

SCR, SEO and Immigrant samples) and drop households with intermittent headship, those appearing only once, and those with missing information on the head's race, education or the state of residence. I then drop households whose income or consumption increases by more than $500 \%$ or falls by more than $80 \%$ over two consecutive surveys, as well as households whose consumption is below $\$ 100$ in any period. I treat top-coded income or consumption data as missing data.

While the literature usually restricts the sample to working-age households, in my benchmark scenario I keep all families whose head is between 20 and 90 years old, in order to have a more accurate picture of the cross-sectional distribution of UREs and NNPs by age. ${ }^{52}$ This sample selection leaves me with 41,820 observations from 7,287 different households.

## C.2.1 Exposure measures

The PSID is annual, so I do not need to perform a frequency adjustment. I deflate all nominal variables to 2009 dollars using the CPI. Table C. 2 reports summary statistics.

URE: $Y-T-C+A-L . \quad$ For URE, I use an annual measure of net disposable income for $Y-T$ (which includes interest and capital gains), and an annual consumption measure $C$ that includes only the consumption categories continuously available in the survey since 1999 (my first sample year). Those consists of expenditures on food, rent, property taxes, home insurance, utilities, telecommunications, transportations, education, childcare and healthcare. I also add a measure of housing expenditures, as well as interest and principal payments.

For assets maturing in the year $(A)$, the PSID contains a variable that groups together checking accounts, saving accounts, money market mutual funds, certificates of deposit, government savings bonds and T-bills. I treat this category as "deposits", to which I apply the maturity assumptions of table 2. The PSID contains another variable that includes bonds, trusts, estates, cash value of life insurance and collection. I assume that half of this amount is "bonds" and that the rest is equity-like, with an infinite maturity.

[^4]For the remainder of liabilities $(L)$, the PSID reports up to two mortgages for each household, and reports whether they are ARMs or FRMs. The PSID also contains a variable that includes credit cards debt, student loans, medical bills, legal debt and loan from relatives. From 2011 onwards, a breakdown of categories is available, and credit cards account for an average of $40 \%$ of the total. I assume that this fraction has been constant over time to form my "credit cards" variable.

NNP and income. To construct a household's net nominal position, I count as nominal assets all the amount held in checking accounts, saving accounts, money market mutual funds, certificates of deposit, government savings bonds and T-bills, as well as half of "bonds, trusts, estates, cash value of life insurance and collection", which I assume to be all nominal. I include the whole amount in IRAs invested in bonds, and half the amount in IRAs invested in a mix of stocks and bonds.

For nominal liabilities, I count the principal balance outstanding on each mortgage and the whole amount due in the form of credit cards debt, student loans, medical bills, legal debt and loan from relatives.

For my income exposure measure, I use the PSID measure of gross income before taxes and government transfers.

## C.2.2 Identification of MPC

As mentioned in main text, the literature exploits the panel dimension of the data in PSID in order to estimate the MPC out of transitory income shocks. I follow BPP and construct my consumption measure for MPC using all non durable consumption categories. ${ }^{53}$ For my income measure, I use labor income plus government transfers, as in Kaplan, Violante and Weidner (2014). Following BPP and Kaplan, Violante and Weidner (2014), I first regress the log of consumption and the $\log$ of income on observables characteristics of the households, including dummy variables for year of birth, family size, number of children, and income coming from other members of the family, as well as dummies for interactions between year with education, race, employment status and region. I then use the residuals of these regressions (call them $y_{i t}$ and $c_{i t}$ ) to estimate the MPC out of transitory income shocks. Specifically, for each exposure measure, in each year, I stratify the population in $J$ bins. I then estimate $\psi_{j}=\frac{\operatorname{Cov}_{j}\left(\Delta c_{t}, \Delta y_{t+1}\right)}{\operatorname{Cov}_{j}\left(\Delta y_{t}, \Delta y_{t+1}\right)}$ as the pass-through coefficient of $\log$ income on log consumption, pooling all years together. ${ }^{54}$ I finally recover a measure of the marginal propensity to consume $M P C_{j}$ by multiplying $\psi_{j}$ by the ratio of average consumption to average income in each bin $j$.

Next, for each exposure measure, I calculate the average value of exposure in each bin, $E X P_{j}$, normalized by average consumption in the sample. I finally compute my estimators as ${ }^{55}$

[^5]\[

$$
\begin{aligned}
\widehat{\mathcal{E}_{E X P}^{N R}} & =\frac{1}{J} \sum_{j=1}^{J} M P C_{j} E X P_{j} \\
\widehat{\mathcal{E}_{E X P}} & =\widehat{\mathcal{E}_{E X P}^{N R}}-\left(\frac{1}{J} \sum_{j=1}^{J} M P C_{j}\right)\left(\frac{1}{J} \sum_{j=1}^{J} E X P_{j}\right) \\
\widehat{S} & =1-\left(\frac{1}{J} \sum_{j=1}^{J} M P C_{j}\right)
\end{aligned}
$$
\]

In order to take into account sampling uncertainty, I compute the distribution of these estimators using a Monte-Carlo procedure, resampling the panel at the household level with replacement. Section C.4.3 considers robustness to using $J=3$ to 8 bins to stratify the sample.

## C. 3 Consumer Expenditure Survey, 2001-2002 (JPS sample)

My data for the Consumer Expenditure Survey comes from the Johnson, Parker and Souleles (2006) (JPS) dataset, which I merge with the main survey data and detailed expenditure files to obtain additional information on households's consumption expenditures, financial assets and liabilities. The dataset covers households with interviews between February 2001 and March 2002. Relative to the full CE sample, JPS drop the bottom $1 \%$ of nondurable expenditure in levels, households living in student housing, those with age less than 21 or greater than 85 , those with age changing by more than a unit or by a negative amount between quarters, and those whose family size changes by more than three members between quarters. Since the 2001 CE survey has several observations with missing values for income-which is a crucial component of URE and a measure of exposure in its own right-I do not consider observations with incomplete income information when analyzing the interest rate exposure or the earnings heterogeneity channel. My sample is therefore made of 9,983 observations from 4,833 different households when computing statistics relevant to these two channels, and contains 12,227 observations from 5,900 households when analyzing the Fisher channel.

## C.3.1 Exposure measures

As discussed in the main text, I measure all variables at an annual rate, summing across quarterly survey observations when necessary and adjusting MPC measures using the formula from appendix B. Table C. 3 presents summary statistics in dollars.

URE: $Y-T-C+A-L$. In order to construct my annual measure of URE, I use annual net disposable income as my measure of income $Y-T$. For $C$, I sum durables and non durables goods as well as house purchases, obtained from the CE's supplemental expenditure files.

I count checking accounts and savings accounts as "deposits", and I assume that half of the "securities" variables is bonds ("securities" contains the amount held in stocks, mutual funds, private sector bonds, government bonds or Treasury notes).

I proceed as usual for liabilities. The CE also contains information on adjustable-rate home equity loans, which I add to my ARM liability measure.

NNP and income. To construct my NNP measure, I include in nominal assets all the amount in savings and checking accounts, half of the "securities" variable, and all the amount held in US savings

Table C.3: Summary statistics, CE

|  | N | mean | p5 | p25 | p50 | p75 | p95 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Net income | 9,983 | 46,482 | 5,780 | 18,200 | 35,980 | 62,572 | 118,824 |
| Consumption | 9,983 | 40,702 | 9,361 | 19,272 | 32,245 | 51,820 | 96,228 |
| Maturing assets | 9,983 | 21,721 | 0 | 0 | 400 | 9,000 | 100,000 |
| Maturing liabilities | 9,983 | 20,976 | 0 | 0 | 1,200 | 8,478 | 137,609 |
| URE | 9,983 | 7,464 | $-115,773$ | $-11,408$ | 2,588 | 22,355 | 120,033 |
| Nominal assets | 12,227 | 19,006 | 0 | 0 | 9 | 5,000 | 100,000 |
| Nominal liabilities | 12,227 | 49,671 | 0 | 0 | 12,786 | 73,951 | 200,794 |
| Net Nominal Position | 12,227 | $-27,859$ | $-174,317$ | $-58,440$ | $-6,800$ | 0 | 58,078 |
| Gross income | 9,983 | 50,082 | 6,923 | 19,257 | 38,000 | 67,000 | 130,000 |

Units: 2001 USD. All flow variables are annualized. All statistics are computed using survey weights.
bonds and in private party loans owed. Using the supplemental expenditure files, my measure of nominal liabilities is fairly detailed. I take the sum of principal balances outstanding on mortgages, home equity loans, home equity line of credit, loans on vehicles, personal debt and credit card debt. For my income exposure measure, I use the CE's annual measure of gross income before taxes.

## C.3.2 MPC identification strategy

JPS identified the propensity to consume out of the 2001 tax rebate by exploiting random variation in the timing of its receipt across households. I closely follow their procedure for analyzing responses to the rebate among different exposure groups. Specifically, for each of my redistribution channels, I rank households in equally-sized bins according to their measure of exposure as at the time of the first interview. I then regress changes in the level of consumption expenditures ( $\Delta C_{i t}$ in JPS's notation) on the amount of the tax rebate ( Rebate $_{i t}$ ). I follow their instrumental-variable specification, instrumenting Rebate $_{i t}$ with a dummy indicator for whether the debate was received. I include month effects and control for age and changes in family composition, and I allow both the intercept and the rebate coefficients to differ across households bins.

My benchmark estimate uses food consumption expenditures as dependent variable. This allows for substantially more precise estimates, as it does in JPS. Section C.4.1 below reports all results using total consumption expenditures as dependent variable instead.

The procedure to compute estimators is the same as the one I use for the PSID-confidence intervals are constructed using a Monte-Carlo procedure, resampling the panel at the household level with replacement. Section C.4.3 reports redistribution elasticities by stratifying the sample in 3 to 8 bins.

## C. 4 Sensitivity analysis

In this section I perform several robustness checks. As a general matter, my results are remarkably stable across all scenarios.

## C.4.1 Using total expenditure to estimate MPC

Table C. 4 replicates the right two columns of table 4 when all available consumption expenditures are used to estimate MPC in the PSID and in the CE, instead of my benchmark scenario (which uses non-
durable consumption in the PSID and food consumption in the CE). The results are intuitive: the confidence intervals get larger, so are the average MPCs, and all my point elasticity estimates for redistribution elasticities turn more negative. In particular, the point estimate $\widehat{\mathcal{E}_{P}}$ turns negative, just as it is in the other two surveys. Interestingly, this is true despite the fact that the point estimate for the average income-weighted MPC is actually a little lower in the PSID that it is when using nondurable consumption alone.

Table C.4: Using total expenditures to estimate MPC in the PSID and CE

| Survey | PSID |  | CE |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Estimate | $95 \%$ C.I. | Estimate | $95 \%$ C.I. |
| $\widehat{\mathcal{E}_{R}}$ | -0.16 | $[-0.23,-0.10]$ | -0.59 | $[-1.34,0.17]$ |
| $\widehat{\mathcal{E}_{R R}^{N R}}$ | -0.01 | $[-0.07,0.04]$ | -0.45 | $[-1.21,0.30]$ |
| $\widehat{S}$ | 0.87 | $[0.84,0.91]$ | 0.65 | $[0.15,1.16]$ |
| $\widehat{\mathcal{E}_{P}}$ | -0.03 | $[-0.10,0.03]$ | -0.15 | $[-0.97,0.67]$ |
| $\widehat{\mathcal{E}_{P}^{N R}}$ | -0.07 | $[-0.14,0.00]$ | -0.83 | $[-1.78,0.12]$ |
| $\widehat{\mathcal{E}_{Y}}$ | -0.08 | $[-0.12,-0.04]$ | -0.25 | $[-0.72,0.22]$ |
| $\widehat{\mathcal{M}}$ | 0.05 | $[-0.01,0.11]$ | 0.51 | $[-0.34,1.36]$ |

This figure recomputes the right two columns of table 4, but uses total expenditures to estimate MPC.

## C.4.2 Excluding durable consumption from the URE calculation

Section B shows that, if relative durable goods prices have an elasticity $\epsilon$ with respect to the real interest rate, then a theoretically-consistent measure of URE counts a fraction $1-\epsilon$ of nondurable expenditures. Figure C. 1 plots my estimated $\widehat{\mathcal{E}_{R}}$ against $\epsilon$ in all three datasets. The left-most part of the graph corresponds to $\epsilon=0$, which is my benchmark scenario. In all the surveys, excluding durable goods make the estimated value of $\mathcal{E}_{R}$ more negative. This effect is most pronounced in the CE.


This figure plots the estimated covariance between MPCs and UREs $\left(\widehat{\mathcal{E}_{R}}\right)$ under various assumptions about the fraction of durable purchases, including house purchases, excluded from the computation of URE. $\epsilon=0$ is the benchmark from table 4 in which all durable purchases are included in the URE consumption measure. $\epsilon=1$ counts no durable purchase instead.

Figure C.1: Estimating $\widehat{\mathcal{E}_{R}}$ assuming alternative values of $\epsilon$.

## C.4.3 Number of bins in the PSID and CE

Recall that my estimates of MPCs in the PSID and the CE are obtained by stratifying the population in three equally-sized groups. Table C. 5 reports the full redistribution elasticities of all three channels by progressively increasing the number of bins from 3 to 8 bin in both samples. As is evident from the table, the number of bins used to stratify the sample does not have a meaningful impact on my main estimates, though magnitudes in the PSID tend to be a little larger with more bins.

Table C.5: Redistribution elasticities using 3 to 8 bins in the PSID and the CE

|  |  | Number of bins |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 3 (Benchmark) | 4 | 5 | 6 | 7 | 8 |
| PSID | $\widehat{\mathcal{E}_{R}}$ | $\begin{gathered} -0.12 \\ {[-0.16,-0.08]} \end{gathered}$ | $\begin{gathered} -0.13 \\ {[-0.17,-0.09]} \end{gathered}$ | $\begin{gathered} -0.12 \\ {[-0.16,-0.07]} \end{gathered}$ | $\begin{gathered} -0.10 \\ {[-0.15,-0.06]} \end{gathered}$ | $\begin{gathered} -0.11 \\ {[-0.16,-0.06]} \end{gathered}$ | $\begin{gathered} -0.11 \\ {[-0.16,-0.06]} \end{gathered}$ |
|  | $\widehat{\mathcal{E}_{P}}$ | $\begin{gathered} 0.02 \\ {[-0.02,0.07]} \end{gathered}$ | $\begin{gathered} 0.03 \\ {[-0.01,0.08]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[-0.03,0.06]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[-0.03,0.06]} \end{gathered}$ | $\begin{gathered} 0.01 \\ {[-0.04,0.06]} \end{gathered}$ | $\begin{gathered} 0.02 \\ {[-0.03,0.07]} \end{gathered}$ |
|  | $\widehat{\mathcal{E}_{Y}}$ | $\begin{gathered} -0.06 \\ {[-0.09,-0.04]} \end{gathered}$ | $\begin{gathered} -0.07 \\ {[-0.10,-0.04]} \end{gathered}$ | $\begin{gathered} -0.07 \\ {[-0.10,-0.04]} \end{gathered}$ | $\begin{gathered} -0.07 \\ {[-0.10,-0.04]} \end{gathered}$ | $\begin{gathered} -0.07 \\ {[-0.10,-0.04]} \end{gathered}$ | $\begin{gathered} -0.07 \\ {[-0.10,-0.04]} \end{gathered}$ |
| CE | $\widehat{\mathcal{E}_{R}}$ | $\begin{gathered} -0.23 \\ {[-0.60,0.15]} \end{gathered}$ | $\begin{gathered} -0.26 \\ {[-0.66,0.15]} \end{gathered}$ | $\begin{gathered} -0.27 \\ {[-0.70,0.16]} \end{gathered}$ | $\begin{gathered} -0.23 \\ {[-0.66,0.21]} \end{gathered}$ | $\begin{gathered} -0.44 \\ {[-0.88,-0.01]} \end{gathered}$ | $\begin{gathered} -0.21 \\ {[-0.65,0.23]} \end{gathered}$ |
|  | $\widehat{\mathcal{E}_{P}}$ | $\begin{gathered} -0.09 \\ {[-0.51,0.33]} \end{gathered}$ | $\begin{gathered} -0.14 \\ {[-0.56,0.28]} \end{gathered}$ | $\begin{gathered} -0.18 \\ {[-0.61,0.25]} \end{gathered}$ | $\begin{gathered} -0.14 \\ {[-0.58,0.30]} \end{gathered}$ | $\begin{gathered} -0.22 \\ {[-0.67,0.23]} \end{gathered}$ | $\begin{gathered} -0.41 \\ {[-0.82,-0.00]} \end{gathered}$ |
|  | $\widehat{\mathcal{E}_{Y}}$ | $\begin{gathered} -0.13 \\ {[-0.36,0.10]} \end{gathered}$ | $\begin{gathered} -0.13 \\ {[-0.34,0.08]} \end{gathered}$ | $\begin{gathered} -0.17 \\ {[-0.36,0.02]} \end{gathered}$ | $\begin{gathered} -0.14 \\ {[-0.33,0.05]} \end{gathered}$ | $\begin{gathered} -0.15 \\ {[-0.31,0.01]} \end{gathered}$ | $\begin{gathered} -0.17 \\ {[-0.35,0.02]} \end{gathered}$ |

## C.4.4 Quarterly measurement in the CE

To ensure comparability across surveys, in the main text I measure MPCs and UREs at an annual level. In the $C E$, this requires me to use the formula from appendix $B$ to map quarterly into annual MPCs. An alternative is to measure both UREs and MPCs at a quarterly level in that dataset. This requires adjusting UREs accordingly: I divide annual income by 4 to obtain $Y-T$, use quarterly consumption for $C$, and scale all maturities for assets and liabilities according to my benchmark assumptions from table 2. Table C. 6 reports the outcome of this exercise. Calculating elasticities in this way cuts $\widehat{\mathcal{E}_{R}}$ is cut in half but the point estimate remains negative.

Table C.6: Estimates with quarterly measurement in the CE

| Survey | Benchmark |  | Quarterly |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Estimate | $95 \%$ C.I. | Estimate | $95 \%$ C.I. |
| $\widehat{\mathcal{E}_{R}}$ | -0.23 | $[-0.60,0.15]$ | -0.09 | $[-0.29,0.11]$ |
| $\widehat{\mathcal{E}_{R} R}$ | -0.09 | $[-0.48,0.31]$ | -0.06 | $[-0.26,0.14]$ |
| $\widehat{S}$ | 0.64 | $[0.36,0.92]$ | 0.90 | $[0.77,1.03]$ |
| $\widehat{\mathcal{E}_{P}}$ | -0.09 | $[-0.51,0.33]$ | -0.11 | $[-0.82,0.60]$ |
| $\widehat{\mathcal{E}_{P}^{N R}}$ | -0.45 | $[-0.94,0.04]$ | -0.54 | $[-1.31,0.23]$ |
| $\widehat{\mathcal{E}_{Y}}$ | -0.13 | $[-0.36,0.10]$ | -0.05 | $[-0.15,0.06]$ |
| $\widehat{\mathcal{M}}$ | 0.46 | $[-0.06,0.98]$ | 0.14 | $[-0.12,0.39]$ |

## C. 5 Correlates of MPCs and exposures

This section complements section D by providing other perspectives on the empirical drivers of my main objects of interest.

## C.5.1 The role of age

This section examines the distribution of exposures and MPC by age in each survey. I divide the population in eight equally-sized age bins. This allows me to assess life-cycle dynamics. It also helps to visualize clearly the relative strengths and weaknesses of each survey.

Exposure measures. Figure C. 2 reports the average value of URE, NNP and income in each age bin, normalized by average consumption in the survey. Average URE (the blue line in the first row of graphs) is increasing in age across all three surveys, with a pattern of decline after retirement in the SHIW. This pattern is mostly due to a decumulation of financial assets in that survey (as represented by the green line). In terms of magnitudes, average URE is always positive in the SHIW and in the PSID, while in the CE average URE is negative for most working-age households. However, this is clearly driven by the different data flaws in each survey: the SHIW and the PSID greatly underreport consumption relative to income-notice the difference between the black and the red line. This tends to overestimate URE. By contrast, as documented above, the CE severely underreports assets, underestimating URE.

Regarding net nominal positions (the blue line in the second row of graphs), the life-cycle pattern in the SHIW is also increasing in age. By contrast, the PSID and the CE display an interesting U shape, with a minimum around age 40 . In particular, in the SHIW, nominal liabilities are declining almost monotonically with age, while nominal assets are sharply increasing until age 60 and then decline rapidly. By contrast, in the PSID and in the CE, nominal liabilities are increasing in age for young households, and then start to decline steadily after age 40-while nominal assets are almost monotonically increasing in age. In terms of magnitudes, average NNP is negative for most of working age population in the SHIW, while it is very negative in the CE and PSID for all households cohorts except the oldest ones. This highlights, once again, the issue that these surveys cover liabilities better than they cover assets.

MPC. Figure C. 2 also reports marginal propensities to consume by age bins in all three surveys. There is an overall declining pattern in age, except for a spike for the oldest cohort in the CE. Interestingly, all three surveys also suggest a rise in MPC around middle age. This pattern is not sensitive to the


This figure plots all three exposure measures and estimated MPCs by age in all three surveys. Households are grouped by 8 equally-sized age bins. The $x$ axis reports the average age in each group, the $y$ axis reports mean exposure as well as estimated MPC in each bin.

Figure C.2: Exposure measures by age bins in all three datasets
number of bins employed to stratify the population. Combining this graph with figure C.2, it appears that age is indeed a driver of the negative correlation between MPC and my exposure measures-as already apparent in table 6.

## C.5.2 The role of income

Figure C. 3 examine the distribution of URE and NNP in all three surveys, when the population is grouped into eight income bins. Unsurprisingly, average URE is increasing in income, especially in the SHIW and the PSID. In these surveys, average URE increases more than one for one with income at the top of the distribution, owing an increase in maturing assets. Interestingly, maturing liabilities (the orange line) also increase in income across all three surveys.

For net nominal position, patterns are different in Italy and in the United States. In the SHIW, net nominal position is initially flat, and then increases with income, owing to an increase in assets at the top of the income distribution. By contrast, in the PSID and in the CE, net nominal position initially declines in income, and then flattens out. This is because nominal liabilities initially increase strongly with income, while nominal assets only increase mildly.

## C.5.3 A general covariance decomposition

In section $D, I$ presented a covariance decomposition that projected observables on a single covariate. This approach can of course be generalized to include any number of covariates. The procedure is in two steps: first, run an OLS regression

$$
\begin{aligned}
M P C_{i} & =\left(\beta^{m}\right)^{\prime} \mathbf{Z}_{\mathbf{i}}+\epsilon_{i}^{m} \\
U R E_{i} & =\left(\beta^{u}\right)^{\prime} \mathbf{Z}_{\mathbf{i}}+\epsilon_{i}^{u}
\end{aligned}
$$

where $\mathbf{Z}_{\mathbf{i}}=\left(1, Z_{i 1}, \cdots, Z_{i J}\right)^{\prime}$ is now a vector of covariates. Then, recover fitted values

$$
\begin{aligned}
\widehat{M P C_{i}} & =\left(\widehat{\beta^{m}}\right)^{\prime} \mathbf{Z}_{\mathbf{i}} \\
\widehat{U R E_{i}} & =\left(\widehat{\beta^{u}}\right)^{\prime} \mathbf{Z}_{\mathbf{i}}
\end{aligned}
$$

and residuals $\widehat{\epsilon_{i}^{m}}, \widehat{\epsilon_{i}^{u}}$. The law of total covariance can now be expressed as

$$
\begin{equation*}
\operatorname{Cov}\left(M P C_{i}, U R E_{i}\right)=\operatorname{Cov}\left(\widehat{M P C_{i}}, \widehat{U R E_{i}}\right)+\operatorname{Cov}\left(\widehat{\epsilon_{i}^{m}}, \widehat{\epsilon_{i}^{u}}\right) \tag{C.1}
\end{equation*}
$$

The first term gives the component of explained covariance, and the second the component of unexplained covariance. The explained part of the covariance can be further decomposed as

$$
\begin{equation*}
\operatorname{Cov}\left(\widehat{M P C_{i}}, \widehat{U R E_{i}}\right)=\operatorname{Cov}\left(\sum_{j=1}^{J} \widehat{\beta_{j}^{m}} Z_{i j}, \sum_{k=1}^{J} \widehat{\beta_{k}^{m}} Z_{i k}\right)=\sum_{j=1}^{J} \sum_{k=1}^{J} \widehat{\beta_{j}^{m}} \widehat{\beta_{k}^{m}} \operatorname{Cov}\left(Z_{i j}, Z_{i k}\right) \tag{C.2}
\end{equation*}
$$

Of course, the 'share of explained covariance' attributed to one particular covariate through this procedure depends on which other covariates are included in $\mathbf{Z}_{\mathbf{i}}$.


This figure plots all three exposure measures and estimated MPCs by income in all three surveys. Households are grouped by 8 equally-sized bins of gross income. The $x$ axis reports the average age in each group, the $y$ axis reports mean exposure as well as estimated MPC in each bin.

Figure C.3: URE and NNP components by income bins in all three datasets

Implementation. Tables C.7-C. 9 report the full matrix described by equation (C.2) for each of my three main covariances $\mathcal{E}_{R}, \mathcal{E}_{P}$, and $\mathcal{E}_{Y}$ in the SHIW, when all covariates from table 6 are included simultaneously. In the PSID and the CE, this exercise is less interesting since MPCs are only available at the group level, but it is possible to do by using the average value of explanatory variables in each bin. These results can easily be generated using the code provided online.

Table C.7: Fraction of $\mathcal{E}_{R}$ explained by each pair of SHIW covariates

|  | Age bins | Male | Married | Years of ed. | Family size | Res. South | City size | Unemployed |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age bins | 9.83 | 0.19 | 0.00 | -2.24 | -0.07 | -0.02 | 0.01 | 0.08 |
| Male | 0.90 | 1.69 | -0.01 | 0.24 | 0.02 | 0.09 | 0.01 | 0.03 |
| Married | -0.29 | 0.28 | -0.02 | 0.27 | 0.05 | -0.04 | 0.01 | 0.01 |
| Years of ed. | -3.30 | 0.07 | -0.00 | 7.24 | 0.03 | 0.52 | -0.07 | 0.00 |
| Family size | 2.88 | -0.14 | 0.02 | -0.78 | -0.22 | 0.36 | -0.01 | 0.05 |
| Res. South | -0.37 | 0.32 | 0.00 | 5.79 | -0.15 | 11.24 | -0.00 | 0.25 |
| City size | 0.64 | 0.07 | -0.00 | -2.26 | 0.01 | -0.01 | 0.96 | 0.01 |
| Unemployed | 1.77 | 0.13 | -0.00 | 0.02 | -0.03 | 0.33 | 0.00 | 0.69 |

Table C.8: Fraction of $\mathcal{E}_{P}$ explained by each pair of SHIW covariates

|  | Age bins | Male | Married | Years of ed. | Family size | Res. South | City size | Unemployed |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age bins | 13.29 | 0.22 | -0.06 | -2.56 | 1.52 | -0.01 | -0.02 | -0.15 |
| Male | 1.22 | 1.98 | 0.28 | 0.28 | -0.35 | 0.04 | -0.01 | -0.05 |
| Married | -0.40 | 0.33 | 0.49 | 0.30 | -1.06 | -0.02 | -0.02 | -0.01 |
| Years of ed. | -4.47 | 0.09 | 0.08 | 8.27 | -0.60 | 0.22 | 0.12 | -0.00 |
| Family size | 3.89 | -0.16 | -0.43 | -0.89 | 4.48 | 0.15 | 0.01 | -0.10 |
| Res. South | -0.50 | 0.37 | -0.13 | 6.62 | 3.12 | 4.84 | 0.00 | -0.46 |
| City size | 0.87 | 0.08 | 0.10 | -2.58 | -0.14 | -0.00 | -1.73 | -0.01 |
| Unemployed | 2.39 | 0.15 | 0.03 | 0.03 | 0.60 | 0.14 | -0.00 | -1.26 |

Table C.9: Fraction of $\mathcal{E}_{Y}$ explained by each pair of SHIW covariates

|  | Age bins | Male | Married | Years of ed. | Family size | Res. South | City size | Unemployed |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age bins | 6.51 | 0.19 | -0.14 | -4.82 | -2.46 | -0.07 | -0.02 | 0.54 |
| Male | 0.60 | 1.71 | 0.64 | 0.52 | 0.56 | 0.28 | -0.01 | 0.18 |
| Married | -0.19 | 0.28 | 1.11 | 0.57 | 1.72 | -0.11 | -0.01 | 0.04 |
| Years of ed. | -2.19 | 0.08 | 0.19 | 15.56 | 0.98 | 1.57 | 0.09 | 0.01 |
| Family size | 1.91 | -0.14 | -0.97 | -1.67 | -7.28 | 1.09 | 0.01 | 0.35 |
| Res. South | -0.24 | 0.32 | -0.30 | 12.46 | -5.06 | 34.04 | 0.00 | 1.65 |
| City size | 0.42 | 0.07 | 0.22 | -4.86 | 0.23 | -0.02 | -1.33 | 0.04 |
| Unemployed | 1.17 | 0.13 | 0.06 | 0.05 | -0.97 | 1.00 | -0.00 | 4.52 |

## C. 6 Evaluating the quality of the financial information in U.S. surveys

In order to shed light on the quality of financial data in the PSID and the CE, tables C. 10 and C. 11 compare the median value of each class of assets and liabilities for households holding these instruments with the comparable number from the Survey of Consumer Finance. All three surveys are analyzed in 2001, the year in which they all overlap. As discussed above, the CE and the PSID group assets and liabilities into coarse categories, making a precise comparison difficult. However, table C. 10 illustrates that liabilities in both the CE and the PSID appear to be aligned with numbers from the SCF as far as medians are concerned. This is especially true in the CE. Regarding financial assets, PSID and SCF data are fairly comparable. By contrast, the CE appears to considerably underreport assets, confirming previous findings in the literature.

Table C.10: Median values for financial liabilities - CE v. PSID v. SCF

| Liabilities |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| SCF | CE | PSID | CE/SCF | PSID/SCF |  |
| Mortgages on primary residence | 72 | 72.3 | 73 | 1.00 | 1.01 |
| HELOC on primary residence | 15 | 18.9 | - | 1.26 | - |
| Other residential debt | 40 | 37.9 | 18 | 0.95 | 0.45 |
| Credit cards | 1.9 | 2 |  | 1.05 |  |
| Vehicle loans | 9.2 | 10.4 | 6 | 1.13 | 0.6 |
| Education loans, personal loans, other | 5 | 1.2 |  | 0.24 |  |
| Any debt | 38.7 | 40.1 | 49 | 1.04 | 1.26 |

Units: Thousands of 2001 USD.
Households holding those liabilities in 2001. Medians computed using survey weights.

Table C.11: Median values for financial assets - CE v. PSID v. SCF

| Financial Assets | SCF | CE | PSID | CE/SCF | PSID/SCF |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Transaction accounts | 3.9 | 1 |  | 0.26 |  |
| Certificates of deposit | 15 | 3 | 5 | 0.2 | 1 |
| Savings bonds | 1 | 0.8 |  | 0.8 |  |
| Retirement accounts | 29.4 | - | 31 | - | 1.12 |
| Stocks | 20 |  | 25 | 30 |  |
| Bonds, mutual funds, life insurance, other | 20 | 25 | 11 | 0.64 | 1.5 |
| Any financial asset | 28.3 | 4.5 | 12 | 0.16 | 0.65 |

Units: Thousands of 2001 USD.
Households holding those assets in 2001. Medians computed using survey weights.

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[^0]:    ${ }^{\dagger}$ Last update: September 2019. I thank Thomas Hintermaier for carefully reading through the proof and pointing out typos in the derivation of Theorem 1. (These do not affect the final result as stated in the paper.)

[^1]:    ${ }^{47}$ Since price dispersion rises as a result of the monetary policy shock, the nonlinear solution features a real wage that is different from steady state even beyond $t \geq 1$, but the difference is second order in $\epsilon_{0}$.

[^2]:    ${ }^{48}$ Equation (B.1) holds approximately in partial equilibrium Bewley models. For example, fix $r=0$, consider a standard lognormal income process, and simulate the model implied mapping between $M P C^{Q}$ and $M P C^{A}$ for different values of the discount factor $\beta$ and the elasticity of intertemporal substitution $\sigma$. The model-implied conversion gets very close to (B.1), especially for low values of $\sigma$. In general, the simplified model overstates the true $M P C^{A}$ a little since quarterly MPCs decay faster than exponentially, but in all of my simulations this never accounted for no more than 10 annual MPC points.
    ${ }^{49}$ While Johnson et al. (2006) cannot estimate annual MPCs given their identification strategy and the nature of the panel component of the CE, they are able to estimate 6-month MPCs. The formula MPC ${ }^{6 M}=1-\left(1-M P C^{Q}\right)^{2}$ provides a good approximation to to their findings. For example, for strictly nondurable goods they find MPCQ = 0.248 and $M P C^{6 M}=0.34$ while my formula delivers 0.43 . For nondurable goods they find $M P C^{Q}=0.386$ and $M P C^{6 M}=0.69$ while my formula delivers 0.62 .

[^3]:    50 "Imagine you unexpectedly receive a reimbursement equal to the amount your household earns in a month. How much of it would you save and how much would you spend? Please give the percentage you would save and the percentage you would spend."
    ${ }^{51}$ Note that the time frame for $M P C$ is not specified in the question, as issue that is left unresolved in Jappelli and Pistaferri (2014). A follow-up question in the 2012 SHIW separates durable and nondurable consumption, and specifies the time frame as a full year. The equivalent "MPC" out of both durable and nondurable consumption has close to the same distribution as that of MPC in the 2010 SHIW (respective means are 47 in 2010 and 45 in 2010) which suggests that households tended to assume that the question referred to the full year.

[^4]:    ${ }^{52}$ Figure C. 2 shows that young and old households tend to have the largest net nominal positions, and with opposite signs (see also Doepke and Schneider 2006) Since households' income processes tend to change upon entering retirement, however, including older households could lead to noisier estimates of MPCs. However, I verified that my elasticity estimates are essentially unchanged when I restrict the PSID sample to households heads between the ages of 25 and 55 .

[^5]:    ${ }^{53}$ This is also consistent with Kaplan et al. (2014) and Berger et al. (2018). In section C.4.1, I report instead an MPC calculated using all consumption expenditures available in the PSID.
    ${ }^{54}$ See Blundell et al. (2008) and Kaplan et al. (2014) for the structural assumptions under which this procedure correctly recovers the MPC out of transitory income shocks. The estimate can be recovered with an instrumental variable regression of $\triangle c_{t}$ on $\triangle y_{t}$, using $\triangle y_{t+1}$ as an instrument.
    ${ }^{55}$ Note that I simply take $\widehat{S}$ to be the sample counterpart to $1-\mathbb{E}_{I}[M P C]$. The procedure cannot simultaneously recover an estimate of the covariance between MPC and consumption. In the SHIW data, the difference between average MPC and consumption-weighted MPC is small, so this is unlikely to significantly affect the value of $S$.

