Vulnerable Growth
Online Appendix

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A.1 Downside and Upside Entropy

In this section, we illustrate the properties of downside and upside entropy using two examples: one where GDP growth evolves according to a first-order autoregressive (AR(1)) process with normal innovations and one where GDP growth evolves according to an AR(1) process with non-normal innovations. These examples illustrate that downside entropy depends on the moments of the distribution of GDP growth conditional on the realization of GDP growth being below the median, and upside entropy depends on the moments of the distribution of GDP growth conditional on the realization of GDP growth being above the median.

Example 1. Assume that GDP growth evolves as an AR(1) process with normal innovations, so that

$$y_{t+1} = \mu_g + \varphi_g y_t + \epsilon_{t+1}, \qquad \epsilon \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right),$$

where y_{t+1} is the one quarter annualized average growth rate of GDP and $\varphi_g < 1$ is the speed of mean-reversion of GDP growth to the unconditional growth rate $\left(1 - \varphi_g\right)^{-1} \mu_g$. In this case, the (one-step-ahead) conditional distribution of GDP growth is

$$f_t\left(y\right) = \left(2\pi\sigma_\epsilon^2\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2\sigma_\epsilon^2}\left(y - \mu_g - \varphi_g y_t\right)^2\right),$$

and the unconditional distribution is

$$g\left(y\right) = \sqrt{1 - \varphi_g^2}\left(2\pi\sigma_\epsilon^2\right)^{-\frac{1}{2}} \exp\left(-\frac{1 - \varphi_g^2}{2\sigma_\epsilon^2}\left(y - \frac{\mu_g}{1 - \varphi_g}\right)^2\right).$$

The downside and upside entropy in this case is given by

$$
\begin{aligned}
\mathcal{L}_t^D\left(f_t; g\right) &= -\int_{-\infty}^{\mu_t}\left(\log\sqrt{1-\varphi_g^2}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(y-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{1}{2\sigma_\epsilon^2}\left(y-\mu_t\right)^2\right)f_t(y)dy\\
&= -\left(\log\sqrt{1-\varphi_g^2}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(m_{-,t}-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{\left(m_{-,t}-\mu_t\right)^2}{2\sigma_\epsilon^2}\right)\int_{-\infty}^{\mu_t}f_t(y)dy\\
&\quad -\frac{1}{\sigma_\epsilon^2}\left(m_{-,t}-\mu_t-\left(1-\varphi_g^2\right)\left(m_{-,t}-\frac{\mu_g}{1-\varphi_g}\right)\right)\int_{-\infty}^{\mu_t}\left(y-m_{-,t}\right)f_t(y)dy\\
&\quad +\frac{\varphi_g^2}{2\sigma_\epsilon^2}\int_{-\infty}^{\mu_t}\left(y-m_{-,t}\right)^2 f_t\left(y\right)dy\\
&= -\frac{1}{2}\left(\log\sqrt{1-\varphi_g^2}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(m_{-,t}-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{1}{\pi}\right)+\frac{\varphi_g^2}{4}\left(1-\frac{2}{\pi}\right);
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_t^U\left(f_t; g\right) &= -\int_{\mu_t}^{\infty}\left(\log\sqrt{1-\varphi_g^2}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(y-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{1}{2\sigma_\epsilon^2}\left(y-\mu_t\right)^2\right)f_t(y)dy\\
&= -\left(\log\sqrt{1-\varphi_g^2}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(m_{+,t}-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{\left(m_{+,t}-\mu_t\right)^2}{2\sigma_\epsilon^2}\right)\int_{\mu_t}^{+\infty}f_t(y)dy\\
&\quad -\frac{1}{\sigma_\epsilon^2}\left(m_{+,t}-\mu_t-\left(1-\varphi_g^2\right)\left(m_{+,t}-\frac{\mu_g}{1-\varphi_g}\right)\right)\int_{\mu_t}^{+\infty}\left(y-m_{+,t}\right)f_t(y)dy\\
&\quad +\frac{\varphi_g^2}{2\sigma_\epsilon^2}\int_{\mu_t}^{+\infty}\left(y-m_{+,t}\right)^2 f_t\left(y\right)dy\\
&= -\frac{1}{2}\left(\log\sqrt{1-\varphi_g^2}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(m_{+,t}-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{1}{\pi}\right)+\frac{\varphi_g^2}{4}\left(1-\frac{2}{\pi}\right),
\end{aligned}
$$

where $\mu_t = \mu_g + \varphi_g y_t$ is the conditional median of the distribution, $m_{+,t} \equiv \mathbb{E}_t\left[y_{t+1} \mid y_{t+1} \geq \mu_t\right] = \mu_t + \sigma_\epsilon\sqrt{\frac{2}{\pi}}$ is the mean of GDP growth conditional on being above the conditional median, and $m_{-,t} \equiv \mathbb{E}_t\left[y_{t+1} \mid y_{t+1} < \mu_t\right] = \mu_t - \sigma_\epsilon\sqrt{\frac{2}{\pi}}$ is the mean of GDP growth conditional on being below the conditional median.

Thus, when both the unconditional and conditional distributions are Gaussian, downside (upside) entropy is expressed in terms of the square difference between the mean conditional on the realization of GDP growth falling below (above) the mean and the unconditional mean and the variance conditional on the realization of GDP growth falling below (above) the mean. When the conditional mean μ_t coincides with the unconditional mean $\left(1-\varphi_g\right)^{-1}\mu_g$, so that the only difference between the conditional and the unconditional distribution is in the variance, downside entropy equals the upside entropy, and risks to the upside and downside are balanced. When the conditional mean is higher than the unconditional mean, downside entropy is higher than upside entropy, and downside risk exceeds upside risk. This property of upside and downside entropy is illustrated in Figure A.5c.

Example 2. Assume now that GDP growth evolves as an AR(1) process with a mixture of normal innovations, so that

$$y_{t+1} = \mu_g + \varphi_g y_t + \epsilon_{t+1}, \qquad \epsilon \sim p_+ \mathcal{N}_+ \left(0, \sigma^2_{\epsilon,+}\right) + p_- \mathcal{N}_- \left(0, \sigma^2_{\epsilon,-}\right),$$

where p_+ is the probability of a positive innovation, and $\mathcal{N}_+$ ($\mathcal{N}_-$) denotes normal distributions truncated from below (above) at 0. That is, we allow for negative shocks to GDP growth to have a different variance than positive shocks to GDP growth, so that $\sigma_{\epsilon,-} > \sigma_{\epsilon,+}$. In this case, the (one-step-ahead) conditional distribution of GDP growth is

$$\begin{aligned} f_t(y) = {} & p_+ \sqrt{\frac{2}{\pi\sigma^2_{\epsilon,+}}} \exp\left(-\frac{1}{2\sigma^2_{\epsilon,+}} \left(y - \mu_g - \varphi_g y_t\right)^2\right) \mathbf{1}_{y \geq \mu_g + \varphi_g y_t} \\ & + \left(1 - p_+\right) \sqrt{\frac{2}{\pi\sigma^2_{\epsilon,-}}} \exp\left(-\frac{1}{2\sigma^2_{\epsilon,-}} \left(y - \mu_g - \varphi_g y_t\right)^2\right) \mathbf{1}_{y < \mu_g + \varphi_g y_t}. \end{aligned}$$

Assume further that the unconditional distribution is

$$g(y) = \sqrt{1 - \varphi_g^2} \left(2\pi\sigma^2_\epsilon\right)^{-\frac{1}{2}} \exp\left(-\frac{1 - \varphi_g^2}{2\sigma^2_\epsilon} \left(y - \frac{\mu_g}{1 - \varphi_g}\right)^2\right).$$

For simplicity, we will focus on the case $p_+ = 1/2$, so that there is an equal probability of positive and negative innovations to GDP growth. In this case, the conditional median is the same as in the conditionally Gaussian case, so that $\mu_t = \mu_g + \varphi_g y_t$. The downside and upside entropy are then given by

$$\begin{aligned} \mathcal{L}^D_t(f_t; g) = {} & -\int_{-\infty}^{\mu_t} \left(\log\sqrt{1 - \varphi_g^2} + \log\frac{\sigma_{\epsilon,-}}{\sigma_\epsilon} - \frac{1 - \varphi_g^2}{2\sigma^2_\epsilon} \left(y - \frac{\mu_g}{1 - \varphi_g}\right)^2 + \frac{1}{2\sigma^2_{\epsilon,-}} \left(y - \mu_t\right)^2\right) f_t(y) dy \\ = {} & -\left(\log\sqrt{1 - \varphi_g^2} + \log\frac{\sigma_{\epsilon,-}}{\sigma_\epsilon} - \frac{1 - \varphi_g^2}{2\sigma^2_\epsilon} \left(m_{-,t} - \frac{\mu_g}{1 - \varphi_g}\right)^2 + \frac{\left(m_{-,t} - \mu_t\right)^2}{2\sigma^2_{\epsilon,-}}\right) \int_{-\infty}^{\mu_t} f_t(y) dy \\ & -\left(\frac{m_{-,t} - \mu_t}{\sigma^2_{\epsilon,-}} - \frac{\left(1 - \varphi_g^2\right)}{\sigma^2_\epsilon} \left(m_{-,t} - \frac{\mu_g}{1 - \varphi_g}\right)\right) \int_{-\infty}^{\mu_t} \left(y - m_{-,t}\right) f_t(y) dy \\ & -\left(\frac{1 - \varphi_g^2}{2\sigma^2_\epsilon} - \frac{1}{2\sigma^2_{\epsilon,-}}\right) \int_{-\infty}^{\mu_t} \left(y - m_{-,t}\right)^2 f_t(y)\, dy \\ = {} & -\frac{1}{2}\left(\log\sqrt{1 - \varphi_g^2} + \log\frac{\sigma_{\epsilon,-}}{\sigma_\epsilon} - \frac{1 - \varphi_g^2}{2\sigma^2_\epsilon} \left(m_{-,t} - \frac{\mu_g}{1 - \varphi_g}\right)^2 + \frac{1}{\pi}\right) \\ & + \frac{1}{4}\left(1 - \left(1 - \varphi_g^2\right) \frac{\sigma^2_{\epsilon,-}}{\sigma^2_\epsilon}\right) \left(1 - \frac{2}{\pi}\right); \end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_t^U(f_t;g) &= -\int_{\mu_t}^{\infty}\left(\log\sqrt{1-\varphi_g^2}+\log\frac{\sigma_{\epsilon,+}}{\sigma_\epsilon}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(y-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{1}{2\sigma_{\epsilon,+}^2}(y-\mu_t)^2\right)f_t(y)dy \\
&= -\left(\log\sqrt{1-\varphi_g^2}+\log\frac{\sigma_{\epsilon,+}}{\sigma_\epsilon}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(m_{+,t}-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{(m_{+,t}-\mu_t)^2}{2\sigma_{\epsilon,+}^2}\right)\int_{\mu_t}^{+\infty}f_t(y)dy \\
&\quad -\left(\frac{m_{+,t}-\mu_t}{\sigma_{\epsilon,+}^2}-\frac{\left(1-\varphi_g^2\right)}{\sigma_\epsilon^2}\left(m_{+,t}-\frac{\mu_g}{1-\varphi_g}\right)\right)\int_{\mu_t}^{+\infty}(y-m_{+,t})f_t(y)dy \\
&\quad -\left(\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}-\frac{1}{2\sigma_{\epsilon,+}^2}\right)\int_{\mu_t}^{+\infty}(y-m_{+,t})^2 f_t(y)\,dy \\
&= -\frac{1}{2}\left(\log\sqrt{1-\varphi_g^2}+\log\frac{\sigma_{\epsilon,+}}{\sigma_\epsilon}-\frac{1-\varphi_g^2}{2\sigma_\epsilon^2}\left(m_{+,t}-\frac{\mu_g}{1-\varphi_g}\right)^2+\frac{1}{\pi}\right) \\
&\quad +\frac{1}{4}\left(1-\left(1-\varphi_g^2\right)\frac{\sigma_{\epsilon,+}^2}{\sigma_\epsilon^2}\right)\left(1-\frac{2}{\pi}\right),
\end{aligned}
$$

where $m_{+,t}\equiv\mathbb{E}_t\left[y_{t+1}|\,y_{t+1}\geq\mu_t\right]=\mu_t+\sigma_{\epsilon,+}\sqrt{\frac{2}{\pi}}$ is the mean of GDP growth conditional on being above the conditional median, and $m_{-,t}\equiv\mathbb{E}_t\left[y_{t+1}|\,y_{t+1}<\mu_t\right]=\mu_t-\sigma_{\epsilon,-}\sqrt{\frac{2}{\pi}}$ is the mean of GDP growth conditional on being below the conditional median.

Thus, in this case as well, downside (upside) entropy is expressed in terms of the square difference between the mean conditional on the realization of GDP growth falling below (above) the median and the unconditional mean and the variance conditional on the realization of GDP growth falling below (above) the median. Unlike the fully Gaussian case described below, however, the conditional variance used is different for the upside and downside entropy. Hence, when the conditional median μ_t coincides with the unconditional median $(1-\varphi_g)^{-1}\mu_g$, downside entropy is lower than the upside entropy. Instead, to make the risks to the upside and downside balance, the conditional median of GDP growth has to be below the unconditional median, as can be seen in the numerical example in Figure A.5d.

A.2 GDP Vulnerability and Other Financial Indicators

The results in the main body of the paper relied on the NFCI, a composite financial conditions indicator that relies on information of 105 measures from money markets, debt and equity markets and the traditional and shadow banking systems. In order to shed light on the importance of the contribution of individual series, we now investigate three financial indicators that are of particular interest: equity market volatility, the credit spread, and the term spread.

Equity market volatility has been used as indicator for the price of risk. Rey (2015) shows that global capital flows, global credit growth, and global asset prices comove tightly with the VIX. Longstaff, Pan, Pedersen, and Singleton (2011) estimate that the price of sovereign risk is strongly correlated with the VIX. Furthermore, Adrian, Crump, and Vogt (2015) show that a nonlinear transformation of the VIX forecasts stock and bond returns, suggesting that the pricing of risk depends on the VIX. In general equilibrium, pricing of risk is associated with GDP growth, and risk to GDP growth. Hence we expect the VIX to be a significant forecasting variable for GDP vulnerability.

A recent literature has linked downside risk to GDP, particularly during financial crises, to credit conditions as measured by credit spreads. Gilchrist and Zakrajšek (2012) construct the excess bond premium, a residual credit spread orthogonal to firm specific information on defaults, and show that that premium has considerable predictive power for future real activity. Using U.S. data from 1929 to 2013, López-Salido, Stein, and Zakrajšek (2017) demonstrate that elevated credit-market sentiment is associated with a decline in economic activity two and three years in the future, driven by mean reversion in credit spreads. Using a long time series across a panel of countries, Krishnamurthy and Muir (2016) document that the transition into a crisis occurs when credit spreads increase markedly, indicating that crises involve a dramatic shift in expectations and are a surprise.

Finally, an extensive literature has shown the forecasting power of the term spread for recession (see Estrella and Hardouvelis, 1991, and the subsequent literature). The term spread is shown to predict recessions 12-18 months in advance, both in sample and out of sample, and is generally a more powerful predictor of recessions than other variables. The term spread generally works best as individual predictor (see Estrella and Mishkin, 1998). Harvey (1988) shows that a consumption Euler equation naturally gives rise to forecasting of the term spread for real activity.

Figure A.6 shows the quantile coefficients for equity market option-implied volatility, the BAA-AAA credit spread, and the 10-year/3-month term spread.[15] Comparing the loadings on NFCI to the loading on these three individual components, we see that the conditional quantile function is most sensitive to the NFCI, followed by option-implied volatility, term spread and the credit spread. The term spread has the curious property of having a non-monotonic relationship with respect to the upper quantiles for the four quarter ahead prediction. At intermediate quantiles, the conditional quantile function has almost constant loadings on volatility. At very low quantiles, however, the quantile function has a significant negative relationship with volatility: high option-implied volatility is associated with large downside risks to GDP growth at both one and four quarter horizons. The credit spread carries surprisingly little information, as indicated by a very flat quantile coefficient curve, which is close to zero. In sum, these findings suggest that the NFCI financial conditions index is a robust proxy for how financial conditions affect the predicted distribution for GDP growth.

Figure A.7 reports the coefficients obtained by replacing the NFCI with its risk, credit and leverage subindexes. Results broadly confirm the stronger relationship of the predictive distribution of real GDP growth with financial conditions at the lower quantiles.

[15]We use the VXO instead of the VIX as it has a slightly longer time series, and we backfill the data to 1973 using realized equity market volatility (see Bloom, 2009).

Appendix References

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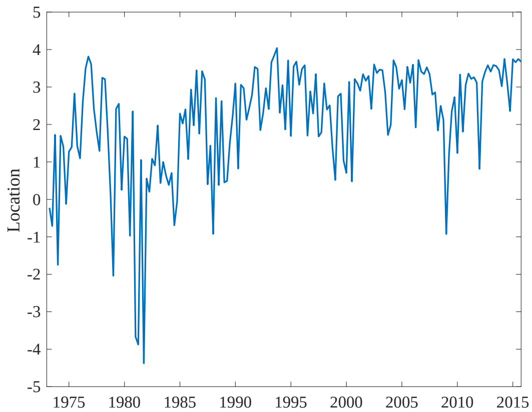
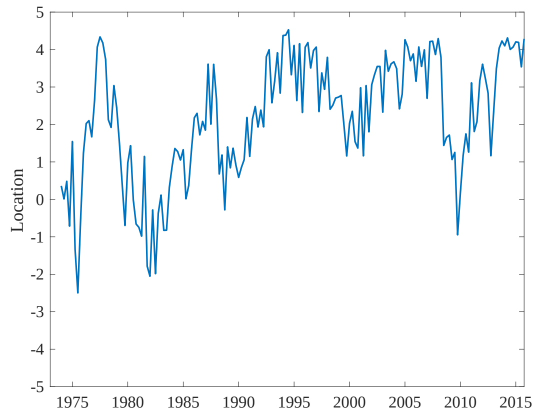
Rey, H. (2015): "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence," Working Paper N. 21162, National Bureau of Economic Research.

Figure A.1. Predictive Distribution of GDP Growth: Location and Scale Parameters over Time. The figure shows the time series evolution of location μ_t and scale σ_t.

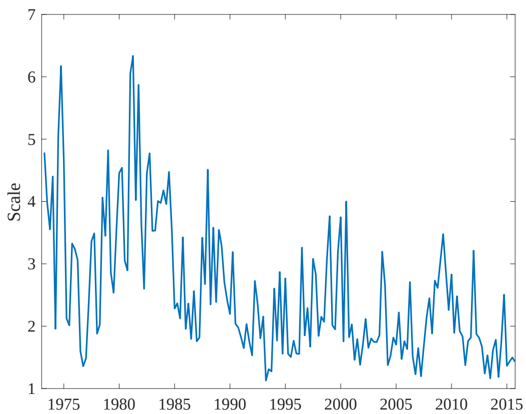
(a) **Location** μ_t: One quarter ahead



(b) **Location** μ_t: One year ahead



(c) **Scale** σ_t: One quarter ahead



(d) **Scale** σ_t: One year ahead

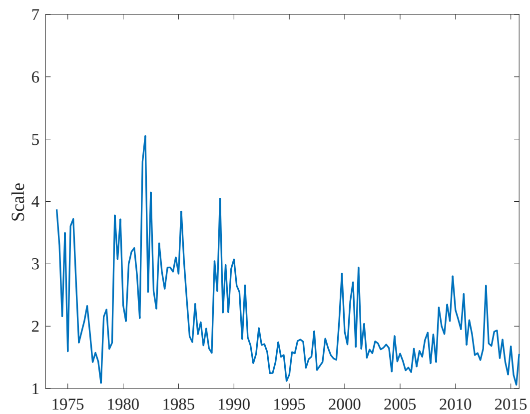
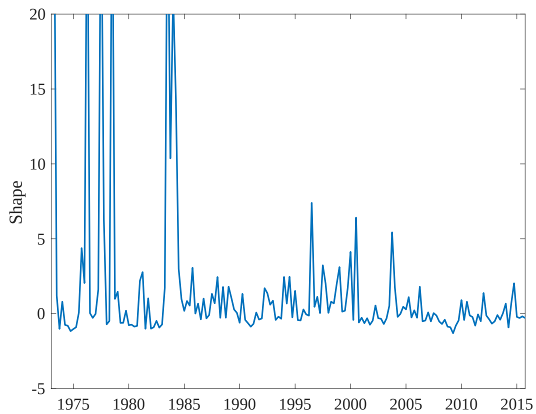
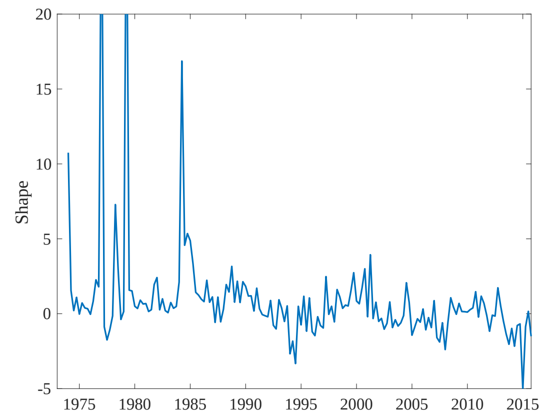


Figure A.2. Predictive Distribution of GDP Growth: Shape and Degrees of Freedom over Time. The figure shows the time series evolution of location shape α_t and degrees of freedom ν_t.

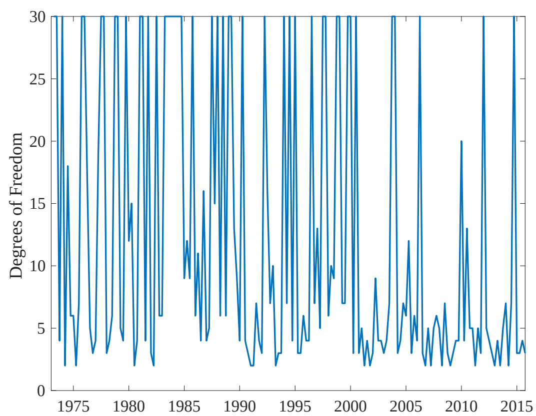
(a) Shape α_t One quarter ahead



(b) Shape α_t One year ahead



(c) Degrees of Freedom ν_t One quarter ahead



(d) Degrees of Freedom ν_t One year ahead

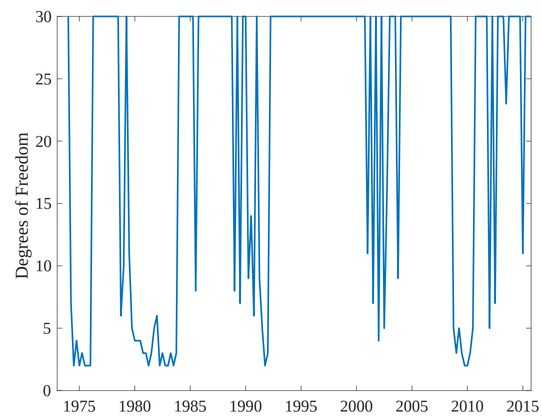
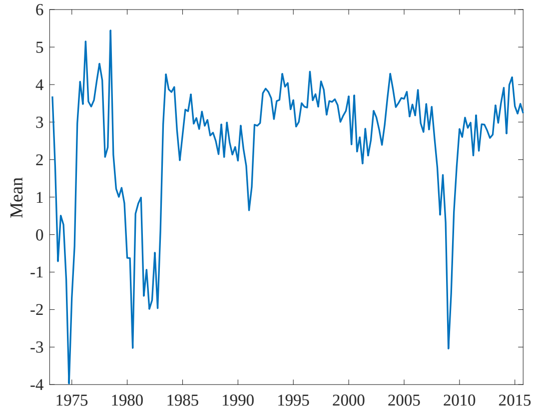
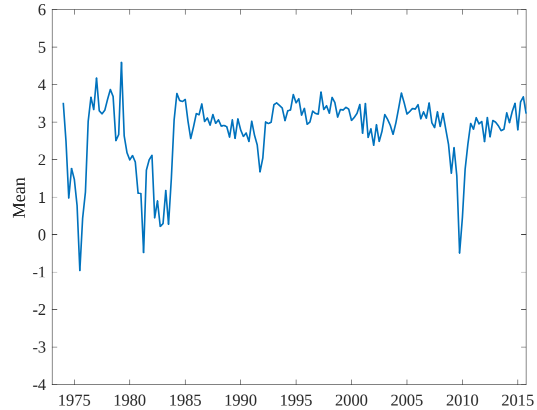


Figure A.3. Predictive Distribution of GDP Growth: Moments over Time. The figure shows the time series evolution of the conditional mean and variance.

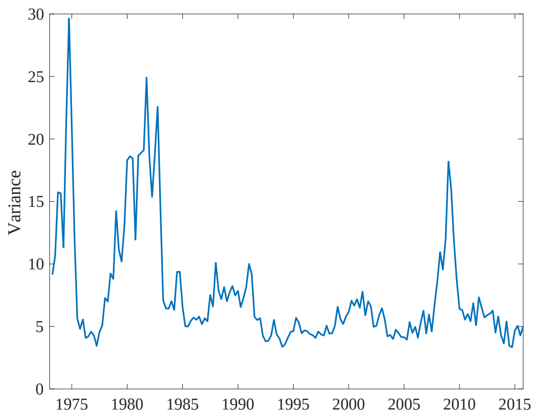
(a) **Mean**: One quarter ahead



(b) **Mean**: One year ahead



(c) **Variance**: One quarter ahead



(d) **Variance**: One year ahead

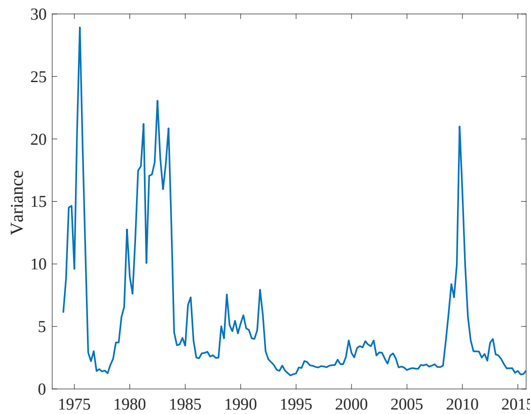
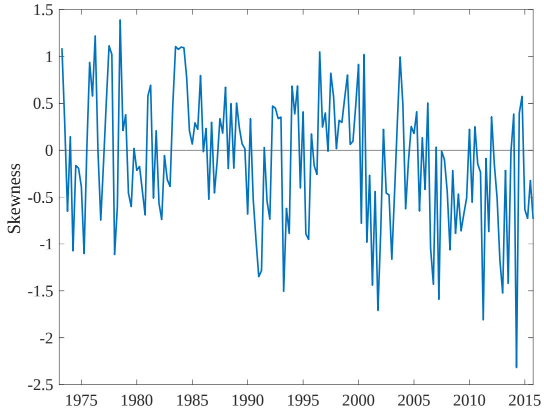
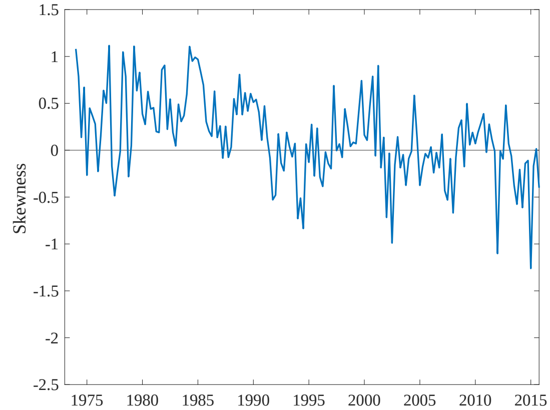


Figure A.4. Predictive Distribution of GDP Growth: Moments over Time. The figure shows the time series evolution of the conditional skewness, and kurtosis.

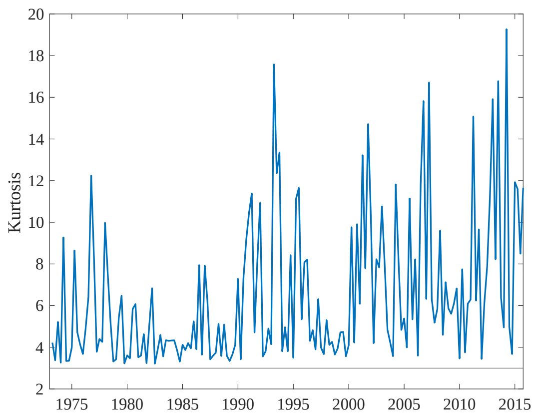
(a) **Skewness**: One quarter ahead



(b) **Skewness**: One year ahead



(c) **Kurtosis**: One quarter ahead



(d) **Kurtosis**: One year ahead

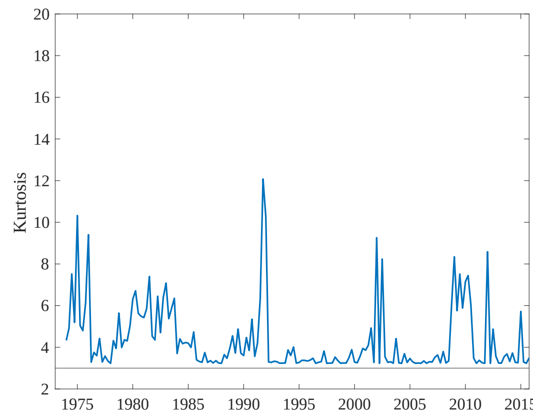
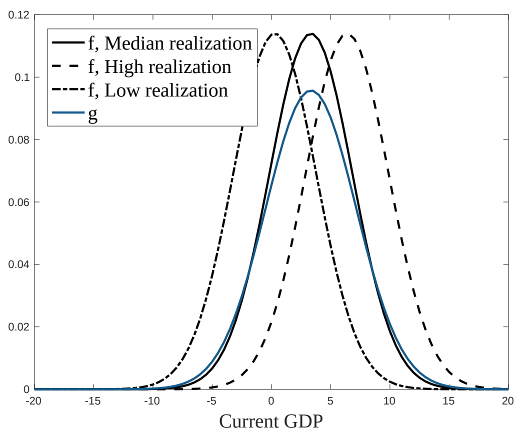
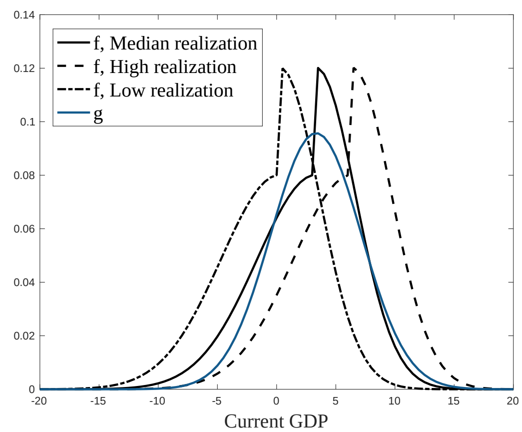


Figure A.5. Upside and Downside Entropy. The left column shows the conditional and unconditional distribution of GDP growth for the autoregressive GDP growth process with normal innovations in Example 1, and the corresponding downside and upside entropy as a function of the current realization of GDP growth. The right column shows the conditional and unconditional distribution of GDP growth for the autoregressive GDP growth process with truncated normal innovations in Example 2, and the corresponding downside and upside entropy as a function of the current realization of GDP growth. Parameters: $\mu_g = 0.2$, $\sigma_\epsilon = 3.5$, $\phi_g = 0.4$, $\sigma_{\epsilon,+} = 3.3$, $\sigma_{\epsilon,-} = 4.9$

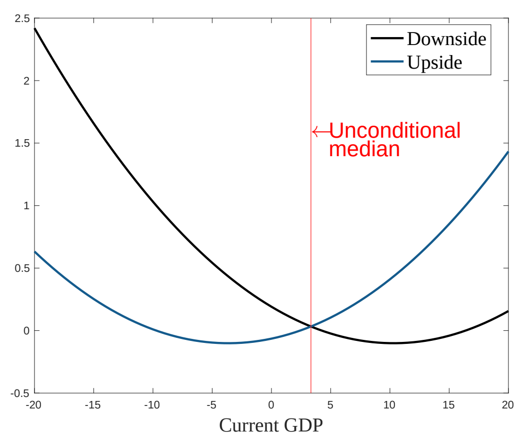
(a) **Distribution**: Gaussian innovations



(b) **Distribution**: Truncated innovations



(c) **Entropy**: Gaussian innovations



(d) **Entropy**: Truncated innovations

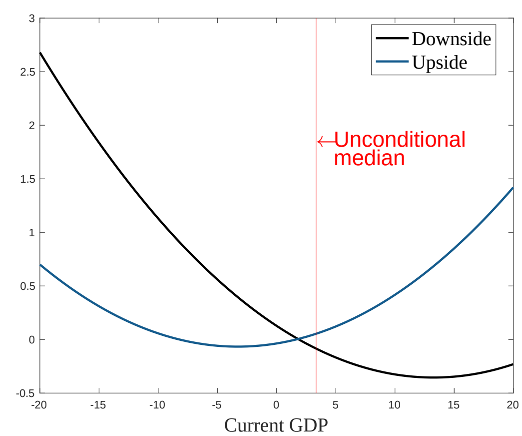
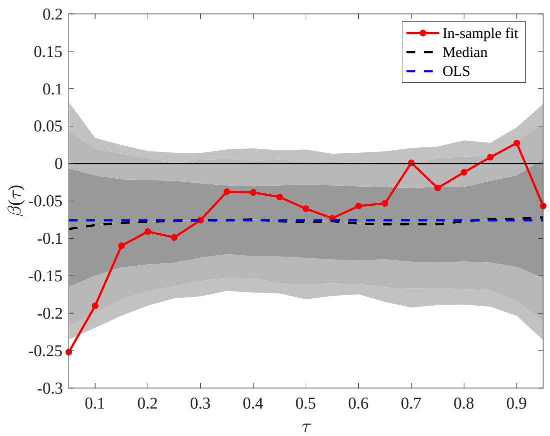
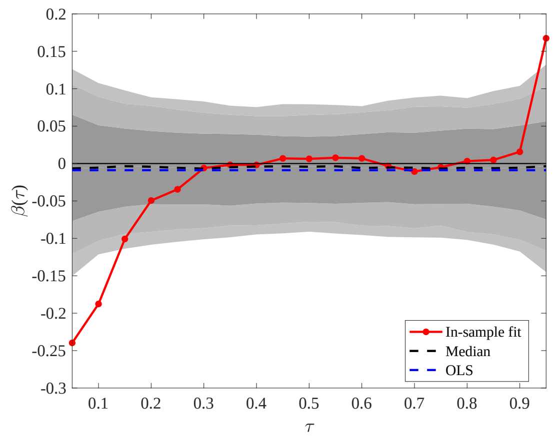


Figure A.6. GDP Growth and Other Predictors. The figure shows the estimated coefficients in quantile regressions of one quarter ahead (left column) and four quarter ahead (right column) real GDP growth on current real GDP growth and VXO; current real GDP growth and the Baa-Aaa spread; current real GDP growth and the term spread.

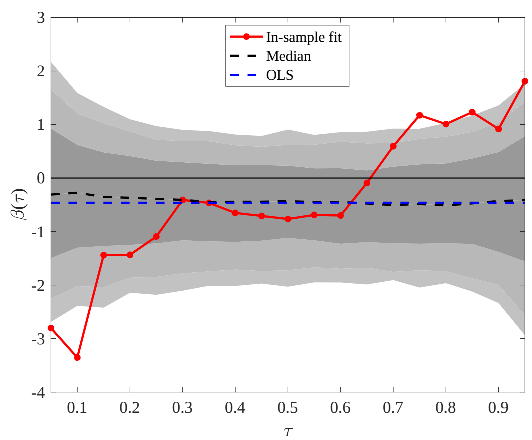
(a) **Equity Volatility**: One quarter ahead



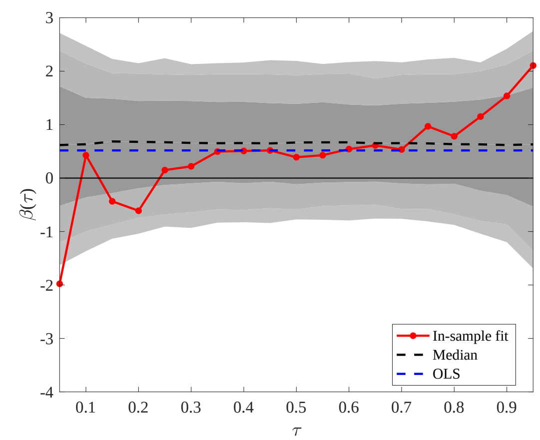
(b) **Equity Volatility**: One year ahead



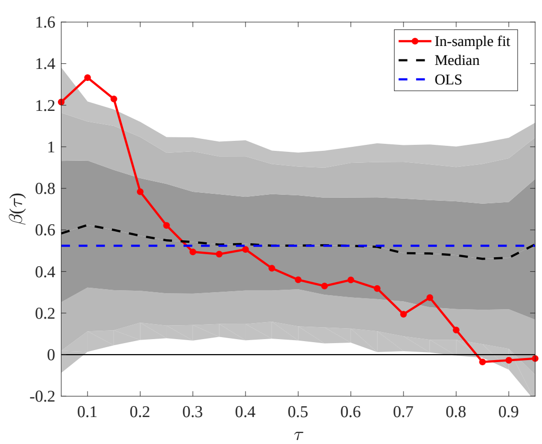
(c) **Credit Spread**: One quarter ahead



(d) **Credit Spread**: One year ahead



(e) **Term Spread**: One quarter ahead



(f) **Term Spread**: One year ahead

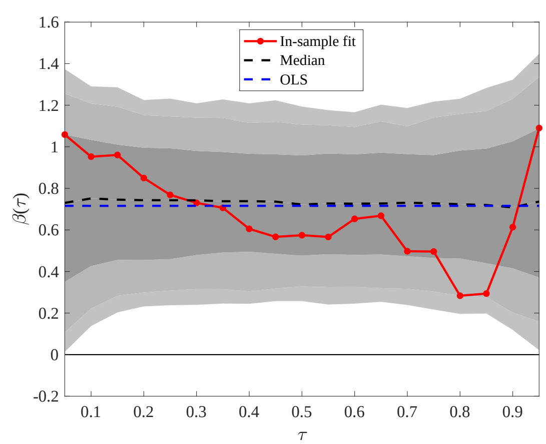
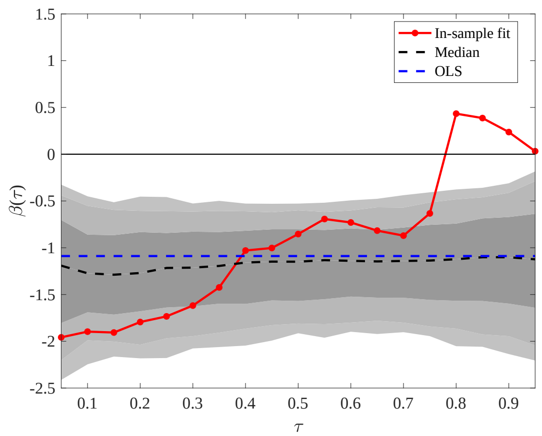
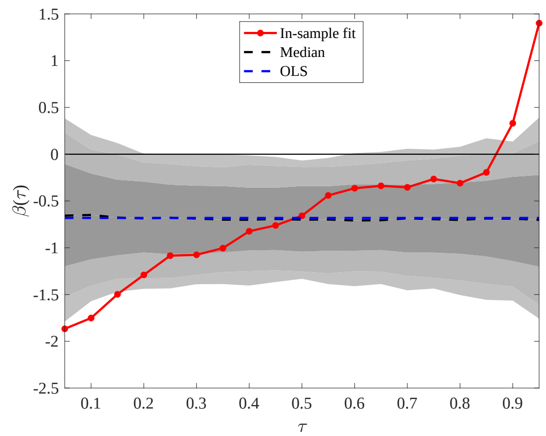


Figure A.7. GDP Growth and Subcomponents of NFCI. The figure shows the estimated coefficients in quantile regressions of one quarter ahead (left column) and four quarter ahead (right column) real GDP growth on current real GDP growth and the risk subindex of the NFCI; current real GDP growth and the credit subindex of the NFCI; current real GDP growth and the leverage subindex of the NFCI.

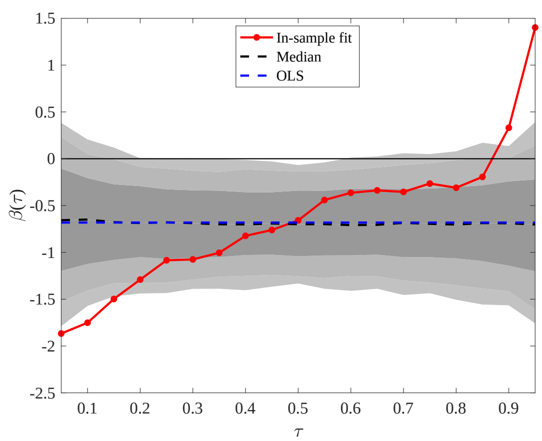
(a) **Risk**: One quarter ahead



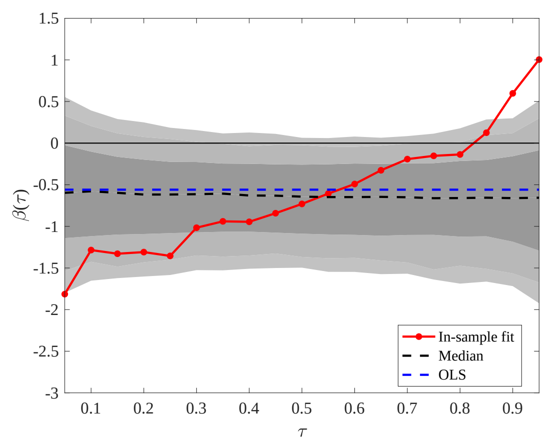
(b) **Risk**: One year ahead



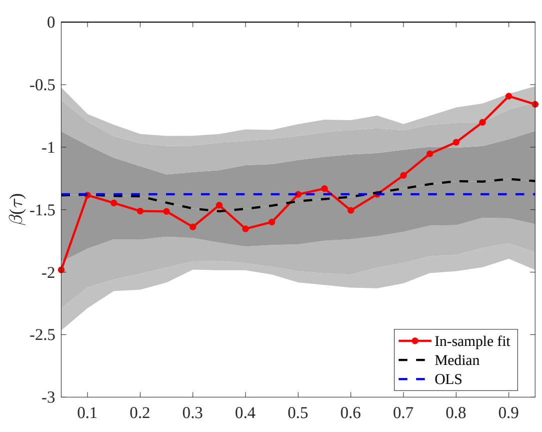
(c) **Credit**: One quarter ahead



(d) **Credit**: One year ahead



(e) **Leverage**: One quarter ahead



(f) **Leverage**: One year ahead

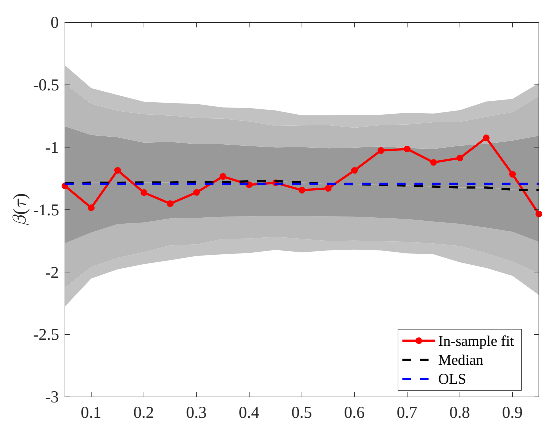
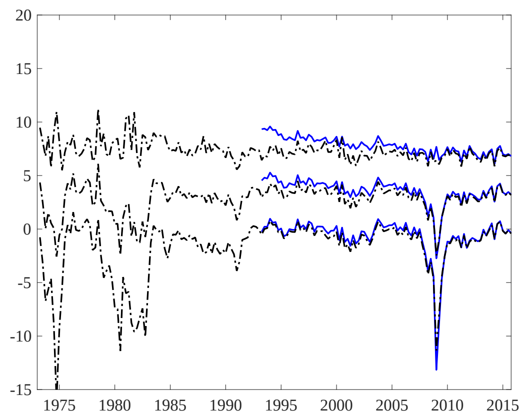
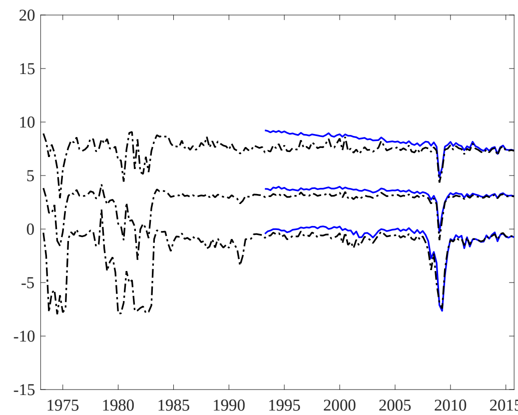


Figure A.8. Out-of-Sample Performance of Alternative Approaches. The figure reports the out-of-sample predictive density, the predictive scores and the cumulative distribution of the probability integral transform, generated with a conditionally Gaussian model (left column) and with a non-parametric model (right column). Figures A.8a and A.8b show the 5, 50, and 95 percent quantiles. Figures A.8c and A.8d compare the out- of-sample predictive scores of the predictive distribution conditional on both NFCI and real GDP growth, and the (semi-parametric) predictive distribution conditional on real GDP growth only. Figures A.8e and A.8f report the empirical cumulative distribution of the probability integral transform (PITs). Critical values are obtained as in Rossi and Sekhposyan (2017).

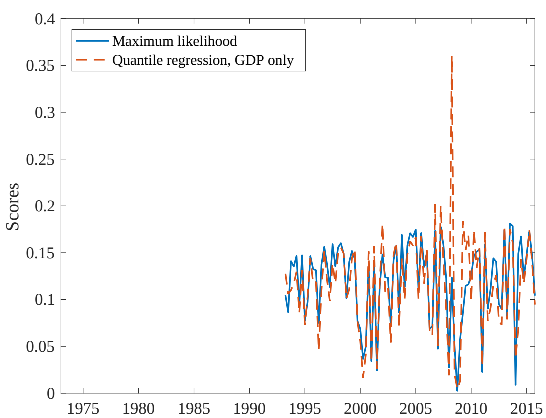
(a) **Quantiles**: MLE



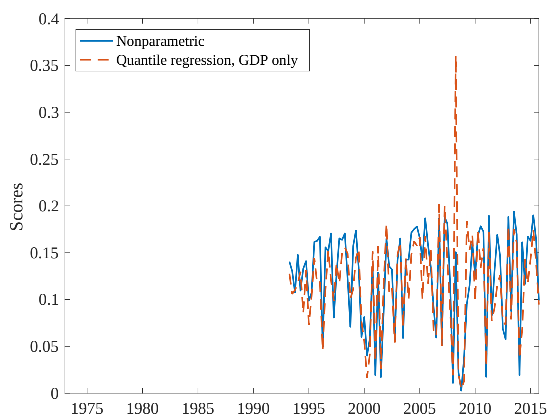
(b) **Quantiles**: Non-parametric



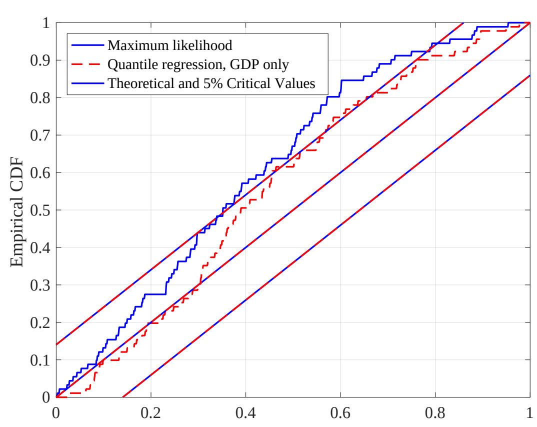
(c) **Scores**: MLE



(d) **Scores**: Non-parametric



(e) **PITs**: MLE



(f) **PITs**: Non-parametric