Online Appendix for College Tuition and Income Inequality*

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1 Theoretical Appendix

1.1 Proof of Proposition 1

Statement: Consider any equilibrium with an equilibrium tuition schedule $\tilde{t}(q, y, a, r)$. The same allocation can be supported by an alternative tuition schedule that is given by

$$t(q, a, r) = v(q, a) - s(q, a, r)$$

where

$$v(q, a) = \max_{y,r} \{\tilde{t}(q, y, a, r) + s(q, a, r)\}$$

Proof: Fix some (q, a) . Let (y^*, r^*) denote a solution to the per student revenue maximization problem

$$(y^*, r^*) \in \arg\max_{y,r} \{\tilde{t}(q, y, a, r) + s(q, a, r)\} . \tag{1}$$

Suppose that there exists another $y^{**} \neq y^*$ such that:

$$\tilde{t}(q, y^{**}, a, r^*) < \tilde{t}(q, y^*, a, r^*)$$

Then the demand for students of type (y^{**}, a, r^*) at colleges of quality q would be zero as colleges could make strictly higher profit by admitting students of type (y^*, a, r^*) while maintaining technological feasibility. By market clearing the supply of such students at colleges of quality q must be zero as well.

Now we can reset the tuition faced by students of type (y^{**}, a, r^*) at colleges of quality q to:

$$t(q, y^{**}, a, r^*) = \tilde{t}(q, y^*, a, r^*) > \tilde{t}(q, y^{**}, a, r^*) \tag{2}$$

Given this new tuition value, the supply of students of type (y^{**}, a, r^*) to colleges of quality q would still be zero as they now face higher tuition. The demand for such students can still be zero, as the college (weakly) prefers to admit students of type (y^*, a, r^*). Thus the market clearing condition still holds. We can continue this procedure for all $y \neq y^*$ until for all y, $t(q, y, a, r^*) = \tilde{t}(q, y^*, a, r^*) = v(q, a) - s(q, a, r^*)$ where the first equality follows from equation 2 and the second equality follows from equation 1.

A similar argument can be made for residence status. In particular, suppose that for $r^{**} \neq r^*$:

$$\tilde{t}(q, y^*, a, r^{**}) + s(q, a, r^{**}) < \tilde{t}(q, y^*, a, r^*) + s(q, a, r^*)$$

Then colleges strictly prefer to admit students with residence status r^*, so if markets clear it must be the case that students with residence status r^{**} do not want to attend. But then we can raise tuition for those students to the point at which college revenue (tuition plus subsidies) is equated across r^{**} and r^{*}.

Thus, we have established that a competitive equilibrium can be supported by a tuition schedule $t(q, a, r)$ that is independent of income, and a revenue function $v(q, a)$ that is independent of both income and residence status.

1.2 Proof of Proposition 2

In this proof we show that for any equilibrium with a potentially nonlinear revenue schedule, we can construct an alternative linear revenue schedule that is consistent with the equilibrium allocation. Thus, any equilibrium can be supported by a revenue schedule that is linear with respect to ability. The proof is simple if there are no missing markets (*i.e.*, if $\eta_a(q) > 0, \forall a$), but slightly more involved when only a subset of ability types attend colleges of a given quality, in which case there may be a set of tuition schedules consistent with the same allocation. We show that there must be a linear tuition schedule within this set.

Proof. Fix a college quality q such that there is positive supply, $\chi(q) > 0$. Denote the set of active college markets of quality q by $A^+(q) = \{a : \eta_a(q) > 0\}$. This set is nonempty given that $\chi(q) > 0$. Let $a_{\max}$ and $a_{\min}$ be the maximum and minimum elements in this set.

Case 1: $A^+(q)$ is not a singleton set (so that $a_{\max} > a_{\min}$):

In this case, define

$$d(q) = -\frac{v(q, a_{\max}) - v(q, a_{\min})}{a_{\max} - a_{\min}}$$
$$b(q) = v(q, a_{\min}) + d(q)(a_{\min} - a_1).$$

Now we claim that for any a_i,

$$v(q, a_i) = b(q) - d(q)(a_i - a_1).$$

We prove this by contradiction. Suppose that for some j

$$v(q, a_j) > b(q) - d(q)(a_j - a_1).$$

Then one can show that a college can increase profits by substituting in students of type a_j and substituting out a combination of students of types $a_{\min}$ and $a_{\max}$ to maintain the same average ability level in college, and can continue doing so until either $\eta_{a_{\min}}$ or $\eta_{a_{\max}}$ is zero. But this is a contradiction to the assumption that both $a_{\max}$ and $a_{\min}$ belong to the set of active markets $A^+(q)$. Thus, it must be that

$$v(q, a_j) \leq b(q) - d(q)(a_j - a_1) \text{ for any } j.$$

Now suppose that $v(q, a_j) < b(q) + d(q)(a_j - a_1)$ for some j. It must then be that

$\eta_{a_j}(q) = 0$ in equilibrium. Otherwise, the college could shift admissions from a_j students to other ability levels, maintaining the desired average ability and making greater profit. Thus, in equilibrium it must be the case that both the supply and the demand in this particular market (q, a_j) are zero. Now replace the tuition value $v(q, a_j)$ with $\tilde{v}(q, a_j)$ defined by

$$\tilde{v}(q, a_j) = b(q) - d(q)(a_j - a_1).$$

Note that at the new level of tuition $\tilde{v}(q, a_j)$, college demand for students will still be zero (because colleges are indifferent between admitting students with ability a_j or a group of students with average ability a_j). And the supply of students is zero as well because it is now more costly for the households to pick this college $\tilde{v}(q, a_j) > v(q, a_j)$. Thus the market still clears under the new revenue level $\tilde{v}(q, a_j)$. Thus, without loss of generality we can treat $\tilde{v}(q, a_j)$ as the equilibrium revenue value. And we have finished the first part of the proof.

Case 2: $A^+(q) = \{a : \eta_a(q) > 0\}$ is a singleton set:

In this case, first let a_m be the unique element of $A^+(q)$. Define a set of discount rates $D^<(q) = \left\{d_i(q), i < m : d_i(q) = -\frac{v(q,a_i)-v(q,a_m)}{a_i-a_m}\right\}$, which is the set of revenue slopes between a_m and other ability levels lower than a_m. If this set of nonempty, pick the greatest element in this set $D^<(q)$, $d_n(q)$, and denote the associated a as a_n. Define

$$b_n(q) = v(q, a_m).$$

Now we claim that for any a_j, it must be the case that

$$v(q, a_j) \le b_n(q) - d_n(q)(a_j - a_m).$$

To see this, note that for any $a_j < a_m$, by definition the slope $d_j(q) \le d_n(q)$, and thus

$$\begin{aligned} v(q, a_j) &= v(q, a_m) - d_j(q)(a_j - a_m) \\ &= b_n(q) - d_j(q)(a_j - a_m) \\ &\ge b_n(q) - d_n(q)(a_j - a_m), \end{aligned}$$

where the last inequality holds because $a_j - a_m < 0$.

Next, for any $a_j > a_m$, we can show this by contradiction. Suppose that

$$v(q, a_j) > b_n(q) - d_n(q)(a_j - a_m).$$

Then the college can use a mix of a_j and a_n students to replicate ability level a_m (since $a_j > a_m > a_n$, such a mix is feasible). This yields greater profit for the college. But this is a contradiction to optimality. Thus, we proved that for any a_j

$$v\left(q, a_j\right) \leq b_n\left(q\right) - d_n\left(q\right)\left(a_j - a_m\right).$$

And similarly to the first part of the proof, we can replace $v\left(q, a_j\right)$ with

$$\tilde{v}\left(q, a_j\right) = b_n\left(q\right) - d_n\left(q\right)\left(a_j - a_m\right)$$

and maintain market clearing. The very last step is to show that even when the set $D^{<}\left(q\right)$ is empty, we still have a linear tuition schedule. To see this, define

$$D^{>}\left(q\right) = \left\{d_i\left(q\right), i > m : d_i\left(q\right) = -\frac{v\left(q, a_i\right) - v\left(q, a_m\right)}{a_i - a_m}\right\}.$$

This set must be nonempty given that $D^{<}\left(q\right)$ is empty. Pick the smallest element in this set and denote it $d_l\left(q\right)$ with associated ability level $a_l\left(q\right)$. Also define

$$b_l\left(q\right) = v\left(q, a_m\right).$$

Now we claim that for any a_j,

$$v\left(q, a_j\right) \leq b_l\left(q\right) - d_l\left(q\right)\left(a_j - a_m\right)$$

The alternative case in which this inequality is not satisfied would violate the fact that $d_l\left(q\right)$ is the smallest element of $D^{>}\left(q\right)$ and that $D^{<}\left(q\right)$ is empty. Thus, again similarly to the first part of the proof, we can replace $v\left(q, a_j\right)$ with

$$\tilde{v}\left(q, a_j\right) = b_n\left(q\right) - d_n\left(q\right)\left(a_j - a_m\right)$$

and all the market-clearing conditions are satisfied. This completes the proof. □

For the proofs of Propositions 3 and 4, we consider a stripped-down version of the model. We shut down all government policies and transfers so that college revenue $v(.)$ is exactly equal to tuition $t(.)$. Hence the tuition function only depends on q and a. We also abstract from dropout risk and income-based subsidies on the household side, so that the household's problem becomes:

$$\max_{c\geq 0, q\in\mathcal{Q}} \mathbb{E}\left[u(c,q)\right]$$
$$s.t.$$
$$c + t(q,a) = y - \mathbb{1}_{\{q>0\}}\omega.$$

In this environment, a competitive equilibrium is defined by a family of functions $\{\chi(q), c(y,a), q(y,a), \eta(q,a), e(q), t(q,a)\}$ such that households maximize, colleges maximize and earn zero profits, and all markets clear.

For the purpose of proving equilibrium existence, we define a notion of **quasi-equilibrium**. In a quasi-equilibrium, the households' decision rules $\{c(y,a), q(y,a)\}$ satisfy quasi-optimization: for any alternative allocations $c'(y,a), q'(y,a)$ such that $\mathbb{E}u(c'(y,a), q'(y,a)) > \mathbb{E}u(c(y,a), q(y,a))$, it must be the case that $c'(y,a) + t(q'(y,a), a) \geq y - \mathbb{1}_{\{q>0\}}\omega$.

An equilibrium is necessarily a quasi-equilibrium but not vice versa.

1.3 Proof of Proposition 3

This proof consists of two steps. In the first step, we establish that a competitive equilibrium exists when there are finitely many types of colleges, i.e. when the set $\Omega = \{q_1, q_2....q_n\}_{n<\infty}$ contains only a finite number of discrete elements. This part of the proof draws on Ellickson et al. (1999), which establishes equilibrium existence in a general club model setting. However, there are two subtle differences between our model and that of Ellickson et al. (1999). First, Ellickson et al. (1999) only allow for a finite number of types of clubs, while we are ultimately interested in a situation where colleges can enter at any quality level desired, and the equilibrium quality distribution is potentially continuous. Second, Ellickson et al. (1999) assume multiple private consumption goods, while we only have one type of consumption good.

Step 1: Prove existence with finite types of clubs

We start by verifying that Theorem 6.1 of Ellickson et al. (1999) holds in our environment when Ω is a finite discrete set:

If the agents' endowments are desirable and uniformly bounded from above, then a quasi-equilibrium exists. (Theorem 6.1, page 1201)

Uniform boundedness of endowments is satisfied, since the income distribution is defined over a compact set $[y_{\min}, y_{\max}]$. Next we check the desirability of endowment requirement.

The endowments are desirable if for every household, consuming his endowment and no club membership gives strictly higher utility than consuming no private consumption and any membership:

$$\mathbb{E}\left[u(y,0)\right] > \mathbb{E}\left[u(0,q)\right], \forall y \in [y_{min}, y_{max}] > 0, q \in \Omega$$

This condition is satisfied in our setting because of the reservation utility of no college $\kappa > 0$, while consuming no private consumption goods give negative infinite utility.

We then need to show that a quasi-equilibrium is indeed an equilibrium. In Ellickson et al. (1999) this is done through a "club irreducibility condition", which depends on there being more than one type of private consumption good. Given that our model only has one type of consumption good, we cannot use their condition to verify that an equilibrium exists. Instead we exploit the local non-satiation property of our utility function to directly establish existence. To see this, suppose that the quasi-equilibrium is not an equilibrium. Then it must be the case that there exists an alternative consumption bundle $c'(y,a), q'(y,a)$ such that $\mathbb{E}u(c'(y,a), q'(y,a)) > \mathbb{E}u(c(y,a), q(y,a))$ and

$$c'(y,a) + t(q'(y,a), a) = y - \mathbb{1}_{\{q'(y,a)>0\}}\omega.$$

Suppose that $c'(y,a) > 0$ (to be verified later). By continuity we can reduce $c'(y,a)$ a little bit and the resulting allocation (call it) (c'', q') lies strictly inside the budget constraint:

$$c''(y,a) + t(q'(y,a), a) < y - \mathbb{1}_{\{q'(y,a)>0\}}\omega$$

And is strictly preferred to the competitive allocation: $\mathbb{E}u(c''(y,a), q'(y,a)) > \mathbb{E}u(c(y,a), q(y,a))$. This contradicts the quasi-optimization condition that the competitive allocation is strictly preferred to any allocations strictly inside the budget constraint.

The last thing we need to verify is that $c'(y,a)$ is indeed strictly positive. This can be guaranteed with the assumption that the household income distribution has a strictly positive support $y_{\min} > 0$. Suppose that $c'(y,a) = 0$. Then by the form of utility function we know that $\mathbb{E}u(c'(y,a), q'(y,a)) = -\infty$. Now given that $y \geq y_{\min} > 0$, the consumption bundle $(y,0)$ is feasible. Therefore $\mathbb{E}u(c(y,a), q(y,a)) \geq \mathbb{E}u(y,0) > -\infty$. This contradicts the assumption that $\mathbb{E}u(c'(y,a), q'(y,a)) > \mathbb{E}u(c(y,a), q(y,a))$. Thus $c'(y,a)$

must be strictly positive. This concludes the existence proof when the set Ω has finitely many elements.

Step 2: Extending the existence result to a continuum of college types

Next, we extend existence to a continuum of college types, i.e., $\Omega = [0, q_{\max}]$. Note that Caucutt (2001) shows that an equilibrium exists by introducing lotteries to convexify individual consumption sets. However, the step in her proof going from finitely many college types to infinitely many college types does not depend on the assumption of convex consumption sets and can therefore be adapted here.

We first show that an equilibrium exists if the aggregate technology set is restricted so that only a finite number of school types are permitted to operate. There the first step of the proof applies and we can show that an equilibrium exists. These economies are referred to as r^{th} approximate economies, where r denotes elements in this restricted set Ω_r. We then show a convergent subsequence of these economies exists and that the limit is an equilibrium for an economy with a continuum of college types.

More precisely, we pick the r^{th} approximate economy as follows. We set the college quality set $\Omega_r \subset \Omega$ such that all points in Ω lies in a $\frac{1}{r}-$neighborhood of at least one point in Ω_r . For the r^{th} approximate economy, we know that an equilibrium exists. Denote it by $t^r(q,a)$ and x^r, where x^r is abbreviated notation for the allocation associated with the competitive equilibrium $\{\chi(q), c(y,a), q(y,a), \eta(q,a), e(q)\}_r$. Then our goal is to show that the sequence $\{t^r, x^r\}$ converges.

Step 2.1: Uniform Convergence of the Tuition Function

We first show that the tuition function converges uniformly. This involves showing that the tuition functions are bounded uniformly and are Lipschitz continuous.

To show that the tuition functions are bounded uniformly, it is useful to note from Proposition 2 that the tuition functions take the following form:

$$t^r(q,a) = b^r(q) - d^r(q)(a - a_{\min})$$

Given that the ability space is bounded, it suffices to show that the functions b^r and d^r are bounded uniformly. We first show that $d^r(q)$ is bounded. Note from the college's FOC (equation **??**):

$$d^r(q) = \frac{\theta}{1-\theta}\frac{e(q)}{\bar{a}(q)}$$

Given that the (a,q) space is bounded with $a_{min} > 0$, the expenditure $e(q)$ is also implicitly bounded according to the production function $q = \bar{a}^{\theta} e^{1-\theta}$. This shows that $d^r(q)$ is bounded.

Now we show that $b^r(q)$ is also bounded uniformly. We divide it into two cases. Suppose that $\chi(q) > 0$. Then zero profit condition must hold:

$$b^r(q) - d^r(q)(\bar{a} - a_{\min}) - e - \phi = 0$$

$$b^r(q) = d^r(q)(\bar{a} - a_{\min}) + e + \phi$$

Since all objects on the right-hand-side are uniform bounded, $b^r(q)$ is uniform bounded.

Next we check the case $\chi(q) = 0$. In this case the college makes non-positive profit:

$$b^r(q) - d^r(q)(\bar{a} - a_{\min}) - e - \phi \leq 0$$

This provides an upper bound on $b^r(q)$. To establish a lower bound of $b^r(q)$, we focus on the $(a_{\min}, y_{\min})$ household. We impose that the opportunity cost of going to college ω is larger than $y_{\min}$, which implies that the household at y_{min} will optimally choose to consume his endowment $c = y_{\min}$. Otherwise his utility would be negative infinity as he would consume negative consumption. Thus

$$\log(y_{\min}) + \varphi \log(\kappa) \geq \log(y_{\min} - b^r(q)) + \varphi \log(\kappa + q)$$

$$y_{\min}\left(\frac{\kappa}{\kappa + q}\right)^{\varphi} \geq y_{\min} - b^r(q)$$

$$b^r(q) \geq y_{\min}\left(1 - \left(\frac{\kappa}{\kappa + q}\right)^{\varphi}\right) \geq 0 \text{ as } q \geq 0$$

Thus we have found a uniform lower bound for $b^r(q)$. This concludes the proof that the tuition function is uniformly bounded.

We next show that the tuition function $t^r(q, a)$ is Lipschitz continuous. That is, there exists a k independent of r such that for any q, q', a, a'

$$\left|t^r(q, a) - t^r(q', a')\right| \leq k\left|(q, a) - (q', a')\right|$$

where the metric $|(q, a) - (q', a')|$ is given by $|q - q'| + |a - a'|$. By triangular inequality, it suffices to show that the tuition function is Lipschitz continuous with respect to q and a respectively:

$$\begin{aligned}
\left|t^r(q, a) - t^r(q', a')\right| &= \left|t^r(q, a) - t^r(q, a') + t^r(q, a') - t^r(q', a')\right| \\
&\leq \left|t^r(q, a) - t^r(q, a')\right| + \left|t^r(q, a') - t^r(q', a')\right|
\end{aligned}$$

For the first part, we have by the linearity property of the tuition schedule:

$$\left|t^r(q,a) - t^r\left(q,a'\right)\right| = |d^r(q)|\left|a - a'\right|$$

By the uniform boundedness of $d^r(q)$, we know that t^r is Lipschitz continuous with respect to a.

We next show that the tuition function is Lipschitz continuous with respect to q. We invoke the household's indifference condition between q and q'. There must exist a level of income $\bar{y}$ such that:

$$\log(\bar{y} - t^r(q,a)) + \varphi \log(\kappa + q) = \log\left(\bar{y} - t^r\left(q',a\right)\right) + \varphi \log\left(\kappa + q'\right)$$

$$\bar{y}(\kappa + q)^{\varphi} - t^r(q,a)(\kappa + q)^{\varphi} = \bar{y}\left(\kappa + q'\right)^{\varphi} - t^r\left(q',a\right)\left(\kappa + q'\right)^{\varphi}$$

$$t^r\left(q',a\right)\left(\kappa + q'\right)^{\varphi} - t^r\left(q',a\right)(\kappa + q)^{\varphi} + t^r\left(q',a\right)(\kappa + q)^{\varphi} - t^r(q,a)(\kappa + q)^{\varphi} = \bar{y}\left(\left(\kappa + q'\right)^{\varphi} - (\kappa + q)^{\varphi}\right)$$

Thus

$$\left[t^r(q,a) - t^r\left(q',a\right)\right](\kappa + q)^{\varphi} = \left(\bar{y} - t^r\left(q',a\right)\right)\left(\left(\kappa + q'\right)^{\varphi} - (\kappa + q)^{\varphi}\right)$$

Or

$$\left|t^r(q,a) - t^r\left(q',a\right)\right| = \left|\left(\bar{y} - t^r\left(q',a\right)\right)\right|\left|\frac{\left(\kappa + q'\right)^{\varphi} - (\kappa + q)^{\varphi}}{(\kappa + q)^{\varphi}}\right|$$

Given that income $\bar{y}$ is drawn from a compact set $[y_{\min}, y_{\max}]$ and $t^r\left(q',a\right)$ is bounded uniformly in r, we have

$$\left|t^r(q,a) - t^r\left(q',a\right)\right| \leq K\left|\frac{\left(\kappa + q'\right)^{\varphi}}{(\kappa + q)^{\varphi}} - 1\right|$$

for some constant K. Now given that the function $\frac{(\kappa+q')^{\varphi}}{(\kappa+q)^{\varphi}} - 1$ is continuously differentiable and hence Lipschitz continuous. This concludes the proof that tuition function $t^r(q,a)$ is Lipschitz continuous with respect to q.

The uniform boundedness and Lipschitz continuity guarantees that the family of tuition functions $\{t^r(q,a)\}_r$ converges uniformly to some tuition function $\{t^*(q,a)\}$ as $r \to \infty$.

Step 2.2: Convergence of allocation

We then show that the allocation converges. Note that the consumption sets for each household are closed and bounded (note that convexity is not required here). Hence the aggregate consumption set is closed and bounded. The aggregate production set is also closed and bounded, as quality and ability are closed and bounded sets. Therefore there exists a subsequence of $\{x^r\}_r$ that converges to some limit $\{x^*\}$ as $r \to \infty$.

Step 2.3: Verify that the limit equilibrium satisfies equilibrium conditions

Having established the existence of the limit equilibrium$\{t^*, x^*\}$, as a last step we need to verify that the limit equilibrium satisfies the equilibrium conditions. We first verify that it satisfies household optimality. Because x^r is an allocation associated with t^r, it must satisfy budget feasibility:

$$c^r(y,a) + t^r(q^r(y,a), a) = y - \mathbb{1}_{\{q'(y,a)>0\}}\omega$$

Taking the limit we have that x^* is feasible under t^* :

$$c^*(y,a) + t^*(q^*(y,a), a) = y - \mathbb{1}_{\{q'(y,a)>0\}}\omega$$

$c^*(y,a)$, $q^*(y,a)$ are also optimal given $t^*(q,a)$. Otherwise the household optimality condition would be violated for some r sufficiently large.

We next verify college optimality. This can be done easily by noticing that colleges' profit function is continuous with respect to the tuition function and choices. Likewise we can also verify the zero profit condition and market clearing conditions.

1.4 Proof of Proposition 4

We first define a feasible allocation in the benchmark economy.

Definition. *A feasible allocation is a set of functions* $\{\chi(q), c(y,a), q(y,a), \eta(q,a), e(q)\}$ *such that*:

1. The allocation of final output is feasible:

$$\sum_a \mu_a \int_0^\infty c(y,a)dF_a(y) + \int_0^\infty e(q)d\chi(q) + (1-\chi(0))(\omega+\phi) = \sum_a \mu_a \int_0^\infty y dF_a(y).$$

2. The allocation of students is feasible. For all a and $Q \subset \mathbb{R}^+$,

$$\mu_a \int_0^\infty \mathbb{1}_{\{q(y,a)\in Q\}} dF_a(y) = \int_Q \eta(q,a)d\chi(q),$$

   where $\mathbb{1}_{\{.\}}$ is an indicator function.

3. The allocation of quality is feasible. For all Q,

$$\int_Q \sum_a \mu_a \int_0^\infty \mathbb{1}_{\{q(y,a)\in Q\}} q(y,a)dF_a(y)d\chi(q) = \int_Q \left(\sum_a \eta(q,a)a\right)^\theta e(q)^{1-\theta}d\chi(q)$$

Definition. *An allocation* $\{\chi(q), c(y,a), q(y,a), \eta(q,a), e(q)\}$ *is Pareto optimal if 1) it is a feasible allocation and 2) there does not exist a feasible allocation* $\{\chi'(q), c'(y,a), q'(y,a), \eta'(q,a), e'(q)\}$ *such that* $u\left(c'\left(y,a\right), q'\left(y,a\right)\right) \geq u\left(c\left(y,a\right), q\left(y,a\right)\right)$ *for almost every* (y,a) and $u\left(c'\left(y,a\right), q'\left(y,a\right)\right) > u\left(c\left(y,a\right), q\left(y,a\right)\right)$ *for some set* (y,a) *of positive measure.*

We now prove the First Welfare Theorem. The proof here closely mirrors the standard proof of the Welfare Theorem.

Theorem. *Assume that u exhibits local nonsatiation. If* $\{\chi(q), c(y,a), q(y,a), \eta(q,a), e(q)\}$ *is a competitive equilibrium allocation, then it is Pareto efficient.*

Proof. Suppose to the contrary that there exists an alternative feasible allocation $\{\chi'(q), c'(y,a), q'(y,a), \eta'(q,a), e'(q)\}$ such that $u\left(c'\left(y,a\right), q'\left(y,a\right)\right) \geq u\left(c\left(y,a\right), q\left(y,a\right)\right)$ for almost every (y,a) and $u\left(c'\left(y,a\right), q'\left(y,a\right)\right) > u\left(c\left(y,a\right), q\left(y,a\right)\right)$ for some positive measure set of (y,a). Denote $t\left(q,a\right)$ the equilibrium tuition function associated with the competitive equilibrium allocation. Then, from the local non-satiation assumption, we know that the alternative allocation must lie outside households' budget set:

$$c'\left(y,a\right) + t\left(q'\left(y,a\right), a\right) \geq y - \mathbb{1}_{\{q'(y,a)>0\}}\omega \text{ for almost every } \left(y,a\right).$$

Otherwise, $c(y,a), q(y,a)$ would not be individually rational given the tuition functions. In addition,

$$c'(y,a) + t\left(q'(y,a), a\right) > y - \mathbb{1}_{\{q'(y,a)>0\}}\omega \text{ for some positive measure set.}$$

Summing up the above equations across households of different abilities and income, we get

$$\sum_a \mu_a \int c'(y,a)\, dF_a(y) + \sum_a \mu_a \int t\left(q'(y,a), a\right) dF_a(y) > \sum_a \mu_a \int y dF_a(y) - (1-\chi'(0))\omega. \tag{3}$$

Note that under the alternative feasible allocation, aggregate enrollment is given by $1 - \chi'(0)$.

Now turn to the college sector. Since the equilibrium allocation maximizes the colleges' profit under the competitive price vector, the alternative allocation must be (weakly) inferior to the competitive equilibrium allocation under the competitive tuition schedule. Thus, the colleges must make nonpositive profit:

$$\sum_a \eta'(q,a)t(q,a) - e'(q) - \phi \leq 0.$$

Since this equation holds for all colleges, the aggregate profit made by the college sector must be nonpositive, which in turn implies that the aggregate tuition revenue is no greater than the college expenditure:

$$\sum_a \int t\left(q'(y,a), a\right) dF_a(y) \leq \int_0^{q_{\max}} e'(q) d\chi'(q) + (1-\chi'(0))\phi. \tag{4}$$

But eqs. 3 and 4 together imply that the final goods resource constraint is violated under the alternative allocation:

$$\begin{aligned}
& \sum_a \mu_a \int c'(y,a)\, dF_a(y) + \int_0^{q_{\max}} e'(q) d\chi'(q) + (1-\chi'(0))(\omega+\phi) \\
\geq\ & \sum_a \mu_a \int c'(y,a)\, dF_a(y) + \sum_a \int t\left(q'(y,a), a\right) dF_a(y) + (1-\chi'(0))\omega \\
>\ & \sum_a \mu_a \int y dF_a(y),
\end{aligned}$$

where the first weak inequality follows from equation 4 and the second strict inequality follows from equation 3. Thus we have a contradiction. This establishes the Pareto-optimality of the competitive allocation.

1.5 Proof of Proposition 5

Theorem. Suppose education is a pure club good ($\theta = 1$) and there are only two ability levels a_h and a_l and household utility is given by

$$\log(c) + \log(\kappa + q),$$

and the income distribution for both high and low ability is uniform in some interval $\left[\mu_y - \frac{1}{2}\Delta_y, \mu_y + \frac{1}{2}\Delta_y\right]$. Then the college distribution is given by

$$\begin{aligned} \chi(Q) &= \frac{2}{a_h - a_l}\frac{2}{4+\pi}\int_Q \left[(1-\eta(q))^2 + \eta(q)^2\right]^{-2} dq \;\; \forall Q \subset (a_l, a_h) \\ \chi(a_h) &= \chi(a_l) = \frac{2}{4+\pi}, \end{aligned}$$

and the tuition function is given by

$$t(q,a) = \mu_y \frac{q-a}{\kappa+q}\left[1 - \frac{2}{4+\pi}\frac{\Delta_y}{\mu_y}\arctan(1 - 2\eta(q))\right], i = h, l.$$

Proof. As a first step, the household problem given income y is

$$\max_{c,q} \log(c) + \log(\kappa + q)$$

$$c + t(q,a) \leq y.$$

Assuming that $t(q,a)$ is differentiable with respect to q (verified later), the first-order condition with respect to q for households of ability a is

$$t'(q,a) = \frac{y - t(q,a)}{\kappa + q}.$$

Denoting by $y(q)$ the income of the households attending colleges of quality q in equilibrium, we have

$$\begin{aligned} t'(q,a) &= \frac{y(q) - t(q,a)}{\kappa+q} \\ &= -\frac{1}{\kappa+q}t(q,a) + \frac{y(q)}{\kappa+q}. \end{aligned}$$

This is a linear ordinary differential equation that can be solved using the integrating factor method. Define the integrating factor for low ability tuition $v^l(q)$ as

$$\begin{aligned} v^l(q) &= \int_{a_l}^{q} \frac{1}{\kappa+q'}dq' \\ &= \log\frac{\kappa+q}{\kappa+a_l}. \end{aligned}$$

Thus,

$$\exp\left(v^l(q)\right) = \frac{\kappa+q}{\kappa+a_l}$$

$$\begin{aligned} \exp\left(v^l(q)\right)t'(q,a) + \frac{1}{\kappa+q}\exp\left(v^l(q)\right)t(q,a_l) &= \exp\left(v^l(q)\right)\frac{y(q)}{\kappa+q} \\ \left[\exp\left(v^l(q)\right)t(q,a_l)\right]' &= \exp\left(v^l(q)\right)\frac{y(q)}{\kappa+q} \\ \int_{a_l}^{q}\left[\exp\left(v^l(q)\right)t(q,a_l)\right]' dq &= \int_{a_l}^{q}\exp\left(v^l(q)\right)\frac{y(q)}{\kappa+q}dq \\ \exp\left(v^l(q)\right)t(q,a_l) - \ exp(a_l)\,t(q,a_l) &= \int_{a_l}^{q}\exp\left(v^l(q)\right)\frac{y(q)}{\kappa+q}dq \\ &= \int_{a_l}^{q}\frac{y(q)}{\kappa+a_l}dq. \end{aligned}$$

We know from the zero profit condition that

$$t(a_l,a_l) = 0$$

which means that a college of quality a_l (which must consists of a_l students only) must charge those a_l kids zero tuition. Thus

$$\begin{aligned} \exp\left(v^l(q)\right)t(q,a_l) &= \int_{a_l}^{q}\frac{y(q')}{\kappa+a_l}dq' \\ t(q,a_l) &= \exp\left(-v^l(q)\right)\int_{a_l}^{q}\frac{y(q')}{\kappa+a_l}dq' \\ &= \frac{\kappa+a_l}{\kappa+q}\int_{a_l}^{q}\frac{y(q')}{\kappa+a_l}dq' \\ &= \int_{a_l}^{q}\frac{y^l(q')}{\kappa+q}dq'. \end{aligned}$$

Likewise, the integrating factor for the high-ability type is given by

$$\begin{aligned} v^h(q) &= \int_{a_h}^{q} \frac{1}{\kappa + q'} dq' \\ &= \log \frac{\kappa + q}{\kappa + a_h}. \end{aligned}$$

And we can follow the same procedure and obtain an expression for the high-ability tuition function:

$$\exp\left(v^h(q)\right) t(q, a_h) - \exp\left(v^h(a_h)\right) t(a_h, a_h) = \int_{a_h}^{q} \exp\left(v^h(q)\right) \frac{y(q)}{\kappa + q} dq.$$

The zero profit condition for the $q = a_h$ college implies

$$t(a_h, a_h) = 0.$$

Thus,

$$\begin{aligned} t(q, a_h) &= \int_{a_h}^{q} \frac{y(q)}{\kappa + q} dq \\ &= -\int_{q}^{a_h} \frac{y^h(q)}{\kappa + q} dq. \end{aligned}$$

Now we derive the income function $y^a(q)$, given uniformly distributed income and any college distribution function $\chi(q)$:

$$\begin{aligned} y^h(q) &= \mu_y + \frac{1}{2}\Delta_y - \Delta_y \int_{q}^{a_h} \chi(q') \frac{q' - a_l}{a_h - a_l} dq' \\ y^l(q) &= \mu_y + \frac{1}{2}\Delta_y - \Delta_y \int_{q}^{a_h} \chi(q') \frac{a_h - q'}{a_h - a_l} dq'. \end{aligned}$$

Now, we would like to solve for $\chi(q)$ from the zero profit condition $\pi(q) = 0$. We conjecture that there is a strictly positive measure of high-ability students going to colleges of quality $q = a_h$. Denote that mass $\chi(a_h)$. Thus,

$$y^h(q) = \mu_y + \frac{1}{2}\Delta_y - \Delta_y \left(\int_{q}^{a_h} \chi(q') \frac{q' - a_l}{a_h - a_l} dq' + \chi(a_h) \right).$$

Write out the expression for $\pi(q)$:

$$\begin{aligned}
\pi(q) &= \frac{q-a_l}{a_h-a_l} t^h(q) + \frac{a_h-q}{a_h-a_l} t^l(q) \\
&= -\frac{q-a_l}{a_h-a_l} \int_q^{a_h} \frac{y^h(q')}{\kappa+q} dq + \frac{a_h-q}{a_h-a_l} \int_{a_l}^q \frac{y^l(q')}{\kappa+q} dq' \\
&= 0.
\end{aligned}$$

Canceling out some terms, we have that for any q

$$(a_h-q) \int_{a_l}^q y^l\left(q'\right) dq' - (q-a_l) \int_q^{a_h} y^h\left(q'\right) dq' = 0$$

Substitute in expressions for $y^l(q)$ and $y^h(q)$:

$$(a_h-q)\left(\int_{a_l}^q \left(\mu_y + \frac{1}{2}\Delta_y - \Delta_y \int_{q'}^{a_h} \chi(x) \frac{a_h-x}{a_h-a_l} dx\right) dq'\right) -$$

$$(q-a_l)\left(\int_q^{a_h} \left(\mu_y + \frac{1}{2}\Delta_y - \Delta_y \left(\int_{q'}^{a_h} \chi(x) \frac{x-a_l}{a_h-a_l} dx + \chi(a_h)\right)\right) dq'\right) = 0.$$

Differentiate with respect to q :

$$\begin{aligned}
&-\left(\int_{a_l}^q \left(\mu_y + \frac{1}{2}\Delta_y - \Delta_y \int_x^{a_h} \chi(x) \frac{a_h-x}{a_h-a_l} dx\right) dq'\right) \\
&+ (a_h-q)\left(\mu_y + \frac{1}{2}\Delta_y - \Delta_y \int_q^{a_h} \chi(x) \frac{a_h-x}{a_h-a_l} dx\right) \\
&-\left(\int_q^{a_h} \left(\mu_y + \frac{1}{2}\Delta_y - \Delta_y \left(\int_x^{a_h} \chi(x) \frac{x-a_l}{a_h-a_l} dx + \chi(a_h)\right)\right) dq\right) \\
&+ (q-a_l)\left(\mu_y + \frac{1}{2}\Delta_y - \Delta_y \left(\int_q^{a_h} \chi(x) \frac{x-a_l}{a_h-a_l} dx + \chi(a_h)\right)\right) = 0.
\end{aligned}$$

Differentiate again with respect to q :

$$
\begin{aligned}
& -\left(\mu_y+\frac{1}{2}\Delta_y-\Delta_y\int_q^{a_h}\chi\left(x\right)\frac{a_h-x}{a_h-a_l}dx\right)\\
& -\left(\mu_y+\frac{1}{2}\Delta_y-\Delta_y\int_q^{a_h}\chi\left(x\right)\frac{a_h-x}{a_h-a_l}dx\right)\\
& +\left(a_h-q\right)\Delta_y\chi\left(q\right)\frac{a_h-q}{a_h-a_l}+\left(\mu_y+\frac{1}{2}\Delta_y-\Delta_y\left(\int_x^{a_h}\chi\left(x\right)\frac{x-a_l}{a_h-a_l}dx+\chi\left(a_h\right)\right)\right)\\
& +\left(\mu_y+\frac{1}{2}\Delta_y-\Delta_y\left(\int_q^{a_h}\chi\left(x\right)\frac{x-a_l}{a_h-a_l}dx+\chi\left(a_h\right)\right)\right)+\left(q-a_l\right)\Delta_y\left(\chi\left(q\right)\frac{q-a_l}{a_h-a_l}\right)=0.
\end{aligned}
$$

Collect terms:

$$
\begin{aligned}
& -2\left(-\Delta_y\int_q^{a_h}\chi\left(x\right)\frac{a_h-x}{a_h-a_l}dx\right)+\left(a_h-q\right)\Delta_y\chi\left(q\right)\frac{a_h-q}{a_h-a_l}\\
& 2\left(-\Delta_y\left(\int_q^{a_h}\chi\left(x\right)\frac{x-a_l}{a_h-a_l}dx+\chi\left(a_h\right)\right)\right)+\left(q-a_l\right)\Delta_y\left(\chi\left(q\right)\frac{q-a_l}{a_h-a_l}\right)=0.
\end{aligned}
$$

Note that Δ_y can be factored out, and we arrive at a functional equation $\chi\left(q\right)$ that is independent of the income distribution parameters:

$$
\begin{aligned}
& 2\int_q^{a_h}\chi\left(x\right)\left(a_h-x\right)dx+\left(a_h-q\right)^2\chi\left(q\right)\\
& -2\left(\int_q^{a_h}\chi\left(x\right)\left(x-a_l\right)dx+\chi\left(a_h\right)\left[a_h-a_l\right]\right)+\left(q-a_l\right)^2\chi\left(q\right)\\
= & \ 0
\end{aligned}
$$

Thus, we have the following integral equation:

$$
\left[\left(a_h-q\right)^2+\left(q-a_l\right)^2\right]\chi\left(q\right)+2\int_q^{a_h}\chi\left(x\right)\left(a_h+a_l-2x\right)dx=2\chi\left(a_h\right)\left[a_h-a_l\right]
$$

$$
\chi\left(q\right)=\frac{-2}{\left[\left(a_h-q\right)^2+\left(q-a_l\right)^2\right]}\int_{a_h}^{q}\chi\left(x\right)\left(2x-a_h-a_l\right)dx+\frac{2\chi\left(a_h\right)\left[a_h-a_l\right]}{\left(a_h-q\right)^2+\left(q-a_l\right)^2}.
$$

This is a **Volterra equation of the second type with degenerate kernels**, which happens to have an analytical solution. Define the following objects:

$$\begin{aligned} g(q) &= \frac{-2}{\left[(a_h - q)^2 + (q - a_l)^2\right]} \\ h(x) &= (2x - a_h - a_l) \\ f(q) &= \frac{2\chi(a_h)\left[a_h - a_l\right]}{(a_h - q)^2 + (q - a_l)^2}. \end{aligned}$$

and we have

$$\chi(q) = f(q) + \int_{a_h}^{q} R(q, x)\, f(x)\, dx$$

where

$$\begin{aligned} R(q, x) &= g(q)\, h(x) \exp\left[\int_x^q g(s)\, h(s)\, ds\right] \\ &= \frac{-2(2x - a_h - a_l)}{\left[(a_h - q)^2 + (q - a_l)^2\right]} \exp\left[\int_x^q \frac{-2(2s - a_h - a_l)}{\left[(a_h - s)^2 + (s - a_l)^2\right]} ds\right] \\ &= \frac{-2(2x - a_h - a_l)}{\left[(a_h - q)^2 + (q - a_l)^2\right]} \exp\left[-\int_x^q \frac{1}{\left[(a_h - s)^2 + (s - a_l)^2\right]} d\left[(a_h - s)^2 + (s - a_l)^2\right]\right] \\ &= \frac{-2(2x - a_h - a_l)}{\left[(a_h - q)^2 + (q - a_l)^2\right]} \exp\left[-\log\frac{(a_h - q)^2 + (q - a_l)^2}{(a_h - x)^2 + (x - a_l)^2}\right] \\ &= \frac{-2(2x - a_h - a_l)\left((a_h - x)^2 + (x - a_l)^2\right)}{\left[(a_h - q)^2 + (q - a_l)^2\right]^2}. \end{aligned}$$

Now,

$$\begin{aligned}
& \int_{a_h}^{q} R(q,x)\, f(x)\, dx \\
= & \int_{a_h}^{q} \frac{-2\,(2x-a_h-a_l)\left((a_h-x)^2+(x-a_l)^2\right)}{\left[(a_h-q)^2+(q-a_l)^2\right]^2} \frac{2\chi(a_h)\,[a_h-a_l]}{(a_h-x)^2+(x-a_l)^2} dx \\
= & \frac{-2\chi(a_h)\,[a_h-a_l]}{\left[(a_h-q)^2+(q-a_l)^2\right]^2} \left[(a_h-q)^2+(q-a_l)^2\right] |_{a_h}^{q} \\
= & \frac{-2\chi(a_h)\,[a_h-a_l]}{\left[(a_h-q)^2+(q-a_l)^2\right]^2} \left[(a_h-q)^2+(q-a_l)^2-(a_h-a_l)^2\right].
\end{aligned}$$

Thus,

$$\begin{aligned}
\chi(q) &= \frac{2\chi(a_h)\,[a_h-a_l]}{(a_h-q)^2+(q-a_l)^2} + \frac{-2\chi(a_h)\,[a_h-a_l]}{\left[(a_h-q)^2+(q-a_l)^2\right]^2} \left[(a_h-q)^2+(q-a_l)^2-(a_h-a_l)^2\right] \\
&= \frac{2\chi(a_h)\,[a_h-a_l]^3}{\left[(a_h-q)^2+(q-a_l)^2\right]^2} \\
&= M(q)\,\chi(a_h),
\end{aligned}$$

where

$$M(q) = \frac{2\,[a_h-a_l]^3}{\left[(a_h-q)^2+(q-a_l)^2\right]^2}.$$

Thus, we can derive the value of $\chi(a_h)$ from

$$\begin{aligned}
\int_{a_l}^{a_h} \chi(q)\, \frac{q-a_l}{a_h-a_l} dq + \chi(a_h) &= 1 \\
\chi(a_h) &= \frac{1}{1+\int_{a_l}^{a_h} M(q)\, \frac{q-a_l}{a_h-a_l} dq}.
\end{aligned}$$

Now,

$$
\begin{aligned}
& \int_{a_l}^{a_h} M(q) \frac{q-a_l}{a_h-a_l} dq \\
= \; & 2\left[a_h-a_l\right]^2 \int_{a_l}^{a_h} \frac{(q-a_l)}{\left[(a_h-q)^2+(q-a_l)^2\right]^2} dq \\
= \; & 2\left[a_h-a_l\right]^2 \frac{\frac{(a_h-a_l)(q-a_h)}{(a_h-q)^2+(q-a_l)^2}+\arctan \frac{2q-a_l-a_h}{a_h-a_l}}{2\left(a_h-a_l\right)^2} \Big|_{a_l}^{a_h} \\
= \; & \frac{(a_h-a_l)(q-a_h)}{(a_h-q)^2+(q-a_l)^2}+\arctan \frac{2q-a_l-a_h}{a_h-a_l} \Big|_{a_l}^{a_h} \\
= \; & 0+\arctan(1)+1-\arctan(-1) \\
= \; & 1+2\arctan(1) \\
= \; & 1+\frac{\pi}{2}.
\end{aligned}
$$

Thus,

$$
\chi(a_h) = \frac{1}{2+\frac{\pi}{2}}
$$

$$
\chi(q) = \frac{2\left[a_h-a_l\right]^3}{\left[(a_h-q)^2+(q-a_l)^2\right]^2} \frac{1}{2+\frac{\pi}{2}}.
$$

Now we know that all low-ability types not going to $q > a_l$ will go to $q = a_l$ college. That is given by

$$
\begin{aligned}
\chi(a_l) &= 1-\int_{a_l}^{a_h} \chi(q') \frac{a_h-q'}{a_h-a_l} dq' \\
&= 1-\chi(a_h) \int_{a_l}^{a_h} \frac{2\left[a_h-a_l\right]^2 (a_h-q')}{\left[(a_h-q')^2+(q'-a_l)^2\right]^2} dq' \\
&= 1-\chi(a_h)\left[-\arctan \frac{2x-a_l-a_h}{a_l-a_h}-\frac{(a_l-a_h)(x-a_l)}{(a_h-x)^2+(x-a_l)^2}\right] \Big|_{a_l}^{a_h} \\
&= 1-\chi(a_h)\left(2\arctan(1)+1\right) \\
&= 1-\frac{1}{2+\frac{\pi}{2}}\left(\frac{\pi}{2}+1\right) \\
&= \frac{1}{2+\frac{\pi}{2}}.
\end{aligned}
$$

Now we would like to derive closed forms for tuition functions:

$$
\begin{aligned}
t\left(q,a_l\right) &= \int_{a_l}^{q}\frac{y^l\left(q'\right)}{\kappa+q}dq' \\
&= \frac{1}{\kappa+q}\int_{a_l}^{q}\left(\mu_y+\frac{1}{2}\Delta_y-\Delta_y\int_{q'}^{a_h}\chi\left(x\right)\frac{a_h-x}{a_h-a_l}dx\right)dq' \\
&= \frac{1}{\kappa+q}\int_{a_l}^{q}\left(\mu_y+\frac{1}{2}\Delta_y-\frac{\Delta_y}{2+\frac{\pi}{2}}\int_{q'}^{a_h}\frac{2\left[a_h-a_l\right]^2\left(a_h-x\right)}{\left[\left(a_h-x\right)^2+\left(x-a_l\right)^2\right]^2}dx\right)dq' \\
&= \frac{1}{\kappa+q}\int_{a_l}^{q}\left(\mu_y+\frac{1}{2}\Delta_y-\frac{\Delta_y}{2+\frac{\pi}{2}}\left(\begin{array}{c}-\arctan\frac{2x-a_l-a_h}{a_l-a_h}\\ -\frac{\left(a_l-a_h\right)\left(x-l\right)}{\left(a_h-x\right)^2+\left(x-a_l\right)^2}\end{array}\right)|_{q'}^{a_h}\right)dq' \\
&= \frac{1}{\kappa+q}\int_{a_l}^{q}\left(\mu_y+\frac{1}{2}\Delta_y+\frac{\Delta_y}{2+\frac{\pi}{2}}\left(\begin{array}{c}\arctan\frac{2a_h-a_l-a_h}{a_l-a_h}+\frac{\left(a_l-a_h\right)\left(a_h-a_l\right)}{\left(a_h-a_h\right)^2+\left(a_h-a_l\right)^2}\\ -\arctan\frac{2q'-a_l-a_h}{a_l-a_h}-\frac{\left(a_l-a_h\right)\left(q'-l\right)}{\left(a_h-x\right)^2+\left(x-a_l\right)^2}\end{array}\right)\right)dq' \\
&= \frac{1}{\kappa+q}\int_{a_l}^{q}\left(\mu_y+\frac{1}{2}\Delta_y+\frac{\Delta_y}{2+\frac{\pi}{2}}\left(\begin{array}{c}-\frac{\pi}{4}-1-\arctan\frac{2q'-a_l-a_h}{a_l-a_h}\\ -\frac{\left(a_l-a_h\right)\left(q'-l\right)}{\left(a_h-q'\right)^2+\left(q'-a_l\right)^2}\end{array}\right)\right)dq' \\
&= \frac{1}{\kappa+q}\int_{a_l}^{q}\left(\mu_y+\frac{\Delta_y}{2+\frac{\pi}{2}}\left(-\arctan\frac{2q'-a_l-a_h}{a_l-a_h}-\frac{\left(a_l-a_h\right)\left(q'-l\right)}{\left(a_h-q'\right)^2+\left(q'-a_l\right)^2}\right)\right)dq' \\
&= \frac{1}{\kappa+q}\left[\int_{a_l}^{q}adq'-\frac{\Delta_y}{2+\frac{\pi}{2}}\int_{a_l}^{q}\arctan\frac{2x-a_l-a_h}{a_l-a_h}+\frac{\left(a_l-a_h\right)\left(x-l\right)}{\left(a_h-x\right)^2+\left(x-a_l\right)^2}dx\right] \\
&= \frac{1}{\kappa+q}\left[\mu_y\left(q-a_l\right)-\frac{\Delta_y}{2+\frac{\pi}{2}}\left(x-a_l\right)\arctan\frac{2x-a_l-a_h}{a_l-a_h}|_{a_l}^{q}\right] \\
&= \frac{\left(q-a_l\right)}{\kappa+q}\left[\mu_y-\frac{\Delta_y}{2+\frac{\pi}{2}}\arctan\frac{2q-a_l-a_h}{a_l-a_h}\right].
\end{aligned}
$$

Following similar steps:

$$
t\left(q,a_h\right)=-\frac{\left(a_h-q\right)}{\kappa+q}\left[\mu_y-\frac{\Delta_y}{2+\frac{\pi}{2}}\arctan\frac{2q-a_l-a_h}{a_l-a_h}\right].
$$

Rearranging and plugging in the expressions for $\eta\left(q\right)$, we obtain the theorem.

1.6 Proof of Proposition 6

This proposition establishes efficiency of equilibrium when education is treated as an investment good and there is a perfect credit market. Suppose, to the contrary, that the equilibrium allocation $\{c_1(a,y), c_2(a,y), q(a,y), \chi(q), \eta_a(q), e(q)\}$ is not efficient. Then there exists an alternative allocation
$\{c_1'(a,y), c_2'(a,y), q'(a,y), \chi'(q), \eta_a'(q), e'(q)\}$ that is feasible and that Pareto dominates the equilibrium allocation. Then the household decision rules $\{c_1'(a,y), c_2'(a,y), q'(a,y)\}$ must lie outside households' budget constraints under the equilibrium price function $\{t(q,a), R\}$:

$$c_1'(a,y) + t\left(q'(a,y), a\right) \geq y + \frac{y_2\left(q'(a,y), a\right) - c_2'(a,y)}{R} - \mathbb{1}_{\{q'(a,y)>0\}}\omega \tag{5}$$

with strict inequality for a positive measure of households. Summing up across all types of household, we have

$$\begin{aligned}
& \sum_a \int_y c_1'(a,y)\, dF_a(y) + \sum_a \int_y t\left(q'(a,y), a\right) dF_a(y) \qquad (6) \\
> \;& \sum_a \int_y y dF_a(y) + \frac{\sum_a \int_y y_2\left(q'(a,y), a\right) dF_a(y) - \sum_a \int_y c_2'(a,y)\, dF_a(y)}{R} - \left(1 - \chi'(0)\right)\omega \\
\geq \;& \sum_a \int_y y dF_a(y) - \left(1 - \chi'(0)\right)\omega
\end{aligned}$$

where the second equality follows from the second period resource constraint.

On the other hand the college's decision rule must also be inferior under the competitive price function

$$\sum_a \eta_a'(q)\, t(q,a) - e'(q) - \phi \leq 0$$

Summing up across all the colleges, we have that under the alternative allocation, aggregate tuition revenue cannot exceed aggregate college expenditure:

$$\sum_a \int_y t\left(q'(a,y), a\right) dF_a(y) \leq \int e'(q)\, d\chi'(q) + \left(1 - \chi'(0)\right)\phi$$

Thus we can show that the aggregate first period resource constraint is violated:

$$\begin{aligned}
& \sum_a \int_y c_1' (a,y)\, dF_a (y) + \int e' (q)\, d\chi' (q) + \left(1 - \chi' (0)\right) (\omega + \phi) \\
\geq \;& \sum_a \int_y c_1' (a,y)\, dF_a (y) + \sum_a \int_y t \left(q' (a,y), a\right) dF_a (y) + \left(1 - \chi' (0)\right) \omega \\
> \;& \sum_a \int_y y dF_a (y) - \left(1 - \chi' (0)\right) \omega + \left(1 - \chi' (0)\right) \omega \\
= \;& \sum_a \int_y y dF_a (y)
\end{aligned}$$

Thus, the alternative allocation is infeasible. This contradicts the original assertion. Thus, the competitive equilibrium is efficient.

1.7 Proof of Proposition 7

First, observe that we can combine the first- and second-period budget constraints by substituting out the credit variable b to obtain the following life-time household budget constraint:

$$c_1 + \frac{c_2}{R} \leq y + \frac{y_2(q,a)}{R} - t(q,a) - \mathbb{1}_{\{q>0\}}\omega$$

Households face a college enrollment choice and an income-smoothing problem, but these two problems can be cleanly separated. Specifically, we can define

$$Y_2(a) = \max_q \left\{ \frac{y_2(q,a)}{R} - t(q,a) - \mathbb{1}_{\{q>0\}}\omega \right\} \tag{7}$$

as the maximum net income the household can generate by going to college. Then the income smoothing problem simplifies to:

$$\max_{c_1,c_2} \{\log(c_1) + \beta \log(c_2)\}$$

$$s.t.$$

$$c_1 + \frac{c_2}{R} \leq y + Y_2(a)$$

Since equation 7 does not involve first period income y, the optimal college enrollment choice $q(y,a)$ is independent of y.[1] Hence the investment model with a frictionless credit market features *no sorting by income.* The intuition is that the returns to saving in the credit market or investing in education must always be equated, irrespective of family income.

Next we explore how the college enrollment choice varies with ability. With complementarity between education and child ability in the production function (eq. **??**), any efficient allocation must feature positive sorting by ability in enrollment. Formally, consider two different levels of ability, with $a_1 > a_2$. Suppose a child with a_1 does not enroll, but one with with a_2 does. Now flip the enrollment pattern, letting the a_1 child take the college spot of the a_2 child. This is a profitable perturbation from the social planner's perspective, since the ability-education complementarity means it yields higher aggregate output. Since any competitive equilibrium is Pareto efficient (Proposition 6), this concludes the proof of Proposition 7.

Lastly, we note that the proof of the welfare theorem (Proposition 6) fails when there is a positive credit wedge $\rho > 0$. In that case, borrowers and lenders face different effective

[1]Note that we have implicitly assumed that tuition does not depend on income. This property can be proved using a similar logic as in Proposition 1, since the college side of the model is unchanged.

interest rates. Therefore, in their life-time budget constraints (eq. 5), different households (y, a) use different interest rates $R(y, a)$ to discount future consumption:

$$c_1'(a,y) + t\left(q'(a,y), a\right) \geq y + \frac{y_2\left(q'(a,y), a\right) - c_2'(a,y)}{R(y,a)} - \mathbb{1}_{\{q'(a,y)>0\}}\omega$$

This means that the derivation in eq. 6 fails, as one can no longer use the second-period resource constraint to deduce the second inequality.

1.8 Proof of Proposition 8

In this proof we will construct a $\bar{\rho}$ such that if the credit market friction $\rho > \bar{\rho}$, then the equilibrium of the investment model corresponds to that of the consumption model.

To start, we know by Proposition 3 that a competitive equilibrium exists when education is a consumption good. In that equilibrium, given the tuition function $t(q, a)$, a (y, a)-type household chooses his optimal allocations $q(y, a)$ and $c(y, a)$. For this particular type household, we can derive an interest rate $\hat{R}(y, a)$ at which he is unwilling to either borrow or save. We divide it into two cases:

Case 1: he enrolls in college $q(y, a) > 0$. In this case, the interest rate $\hat{R}(y, a)$ is implicitly defined by the first order condition:

$$\frac{1}{y - \omega - t(q(y, a), a)} = \frac{\beta \hat{R}(y, a)}{y_2(q(y, a), a)}$$

Or

$$\hat{R}(y, a) = \frac{y_2(q(y, a), a)}{\beta [y - \omega - t(q(y, a), a)]}$$

Case 2: he does not go to college $q(y, a) = 0$. In this condition a similar condition implies that

$$\hat{R}(y, a) = \frac{y_2(0, a)}{\beta y}$$

Given the constructed function $\hat{R}(y, a)$, we can define $\bar{\rho}$ as

$$\bar{\rho} = \max_{y,a} \hat{R}(y, a) - \min_{y,a} \hat{R}(y, a)$$

Now we need to verify two things: first $\bar{\rho} < \infty$; second, for any $\rho > \bar{\rho}$, there exists an equilibrium with no borrowing and lending, and thus the education-as-investment model is isomorphic to the education-as-consumption model.

To see $\bar{\rho}$ is finite, we need to show that the maximum and the minimum of $\hat{R}(y, a)$ is finite. Observe that (y, a) lies in a closed and bounded set, and we also know that the tuition function $t(.)$ is bounded (from Proposition 3) and that the second-period income function $y_2(.)$ is bounded as well (due to the assumed functional form and because the quality support is bounded). These imply that any element $\hat{R}(y, a)$ must be finite and therefore the maximum and the minimum must be finite (note that the extrema are always attainable as the domain is a closed set). As a result $\bar{\rho} < \infty$.

To see the second property, fix any $\rho > \bar{\rho}$. Now suppose, to the contrary, that in equilibrium there is strictly positive borrowing and lending at some equilibrium interest rate R. Let (y_1, a_1) be the household with positive lending. Then it must be the case that $R > \hat{R}(y_1, a_1)$, thus the borrowing rate in this economy must be

$$\begin{aligned}
R + \rho &> \hat{R}(y_1, a_1) + \rho \\
&> \hat{R}(y_1, a_1) + \bar{\rho} \\
&= \hat{R}(y_1, a_1) + \max_{y,a} \hat{R}(y, a) - \min_{y,a} \hat{R}(y, a) \\
&\geq \hat{R}(y_1, a_1) + \max_{y,a} \hat{R}(y, a) - \hat{R}(y_1, a_1) \\
&= \max_{y,a} \hat{R}(y, a)
\end{aligned}$$

As the borrowing rate exceeds the maximum interest rate at which *any* households would like to borrow, there will be no demand for borrowing in this economy. A contradiction. Thus, there will be no credit in this equilibrium.

2 Computation Appendix

2.1 Algorithm With Two Ability Types

This section explains the computational algorithm used to solve the quantitative model with two ability types. The key equilibrium object to solve for is the college distribution function $\chi(.)$ defined over a discrete grid on college quality q. Note that this equilibrium exists by step 1 in the proof of Proposition 3. Hence we are computing an approximation of an exact equilibrium, rather than an approximate equilibrium. The algorithm uses extensively the sorting property of the model, i.e., richer households are always more likely to go to college and, in case they do, always prefer better college quality. However, a complication arises regarding the need-based aid: households with income level just above the threshold y^* might have less incentive to attend college than households with income level just below. The way we deal with it is to assume that households with and without need-based aid are two different types. Then within each type, sorting by income holds. To highlight the working of the algorithm, in the following we lay out an algorithm with just two types (high and low ability). Details on computation with more types including different residence status and pell eligibility are available on request.

1. Construct a grid on college quality q with values $q(1), q(2), ..., q(N)$ where $q(1) > 0$ and $q(N) = q_{max}$.

2. Make an initial guess of the share of high-ability students not entering college: $\eta(0)$. By definition, the fraction of low-ability students not entering college is $1 - \eta(0)$.

3. Solve for $\chi(0)$ from the zero profit condition of colleges of quality $q(1)$.

   (a) Starting with a conjecture for $\chi(0)$, compute the income of the "marginal" household attending colleges of quality $q(1)$:

$$\begin{aligned} y\left(i^h(1)\right) &= y(\eta(0)\chi(0)) \\ y\left(i^l(1)\right) &= y((1-\eta(0))\chi(0)). \end{aligned}$$

   Next, pin down the college tuition $\left(t^h(1), t^l(1)\right)$ of the $q(1)$ college by the marginal household's indifference condition:

$$\begin{aligned} \log\left(y\left(i^h(1)\right)\right) + \varphi\log(\kappa) &= \log\left(y\left(i^h(1)\right) - t^h(1)\right) + \varphi\log(\kappa + q(1)) \\ \log\left(y\left(i^l(1)\right)\right) + \varphi\log(\kappa) &= \log\left(y\left(i^l(1)\right) - t^l(1)\right) + \varphi\log(\kappa + q(1)). \end{aligned}$$

Given the prevailing market tuition $t^h(1)$ and $t^l(1)$, solve the $q(1)$ college optimization problem and obtain its profit $\pi(1)$ as well as its optimal decision rules $\eta(1), e(1)$.

(b) Use the mapping described in part (a) to solve for $\chi(0)$ such that $\pi(1) = 0$.

i. Check the value of $\pi(1)$ at boundaries $\left[\chi_{lb}(0) = 0; \chi_{ub}(0) = \min\left\{\frac{1}{\eta(0)}, \frac{1}{1-\eta(0)}\right\}\right]$. The upper bound arises because the total mass of high(low) ability is 1. Note that the profit $\pi(1)$ should be increasing in $\chi(0)$, as the market tuition rates $t^h(1)$ and $t^l(1)$ are both increasing in $\chi(0)$.

A. If $\pi(1) > 0$ at $\chi_{lb}(0) = 0$, or $\pi(1) < 0$ at $\chi_{ub}(0) = \min\left\{\frac{1}{\eta(0)}, \frac{1}{1-\eta(0)}\right\}$, zero profits cannot be obtained at grid $q(1)$. Thus we delete $q(1)$ and set $q(1) = q(2)$ and go back to step 3, else go to step ii.

ii. As $\pi(1) < 0$ at $\chi_{lb}(0) = 0$ and $\pi(1) > 0$ at $\chi_{ub}(0)$, one can solve for $\chi(0)$ from $\pi(1) = 0$ using a simple one-dimensional nonlinear solver.

4. Having solved for $\{\chi(i)\}_{i=1}^{n-1}$, along with $\{\eta(i), e(i)\}_{i=1}^{n}$ we now solve for $\chi(n)$ from $\pi(n+1) = 0$.

(a) Starting from a conjecture for $\chi(n)$, compute the income of the marginal household attending colleges of quality $q(n+1)$:

$$
\begin{aligned}
y\left(i^h(n+1)\right) &= y\left(\sum_{i=0}^{n} \eta(i)\chi(i)\right) \\
y\left(i^l(n+1)\right) &= y\left(\sum_{i=0}^{n} (1-\eta(i))\chi(i)\right).
\end{aligned}
$$

Next, pin down the college tuition $\left(t^h(n+1), t^l(n+1)\right)$ of the $q(n+1)$ college from the following household first-order conditions:

$$
\begin{aligned}
t^h(n+1) &= \left[1 - \left(\frac{\kappa + q(n)}{\kappa + q(n+1)}\right)^{\varphi}\right] y\left(i^h(n+1)\right) + \left(\frac{\kappa + q(n)}{\kappa + q(n+1)}\right)^{\varphi} t^h(n) \\
t^l(n+1) &= \left[1 - \left(\frac{\kappa + q(n)}{\kappa + q(n+1)}\right)^{\varphi}\right] y\left(i^l(n+1)\right) + \left(\frac{\kappa + q(n)}{\kappa + q(n+1)}\right)^{\varphi} t^l(n).
\end{aligned}
$$

Given the prevailing market tuition $t^h(n+1)$ and $t^l(n+1)$, solve the $q(n+1)$ college optimization problem (procedure outlined below) and obtain its profit $\pi(n+1)$ as well as the optimal decision rules $\eta(n+1), e(n+1)$.

(b) Given the mapping described in part (a), solve for $\chi(n)$ such that $\pi(n+1) = 0$.

i. Check the value of $\pi(n)$ at boundaries $\left[\chi_{lb}(n) = 0; \chi_{ub}(n) = \min\left(\frac{1-i^h(n)}{\eta(n)}, \frac{1-i^l(n)}{(1-\eta(n))}\right)\right]$.

A. If $\pi(n+1) > 0$ at $\chi_{lb}(n)$, this implies that $q(n+1)$ college would always make strictly positive profits and keep growing, squeezing $q(n)$ college out of the market. Thus, we can delete the grid point $q(n)$. Set $q(n) = q(n+1)$ and go back to step 3 with the new grid on q.

B. If $\pi(n+1) < 0$ at $\chi_{ub}(n)$, this implies that $q(n+1)$ college always makes negative profits and thus is driven out of the market. Thus, we delete the grid point $q(n+1)$. Set $q(n+1) = q(n+2)$ and go back to step 3 with the new grid on q.

C. If $\pi(n+1) < 0$ at $\chi_{lb}(n)$ and $\pi(n+1) > 0$ at $\chi_{ub}(n)$, then we can solve for $\chi(n)$ such that $\pi(n+1) = 0$.

5. Having solved for $\{\chi(i)\}_{i=1}^{N-1}$, along with $\{\eta(i), e(i)\}_{i=1}^{N}$, we still have $\chi(N)$ undetermined. We pin it down using the consistency requirement for high ability spots at $q(N)$ colleges:
$$\chi(N)\,\eta(N) = 1 - i^h(N).$$
Lastly, check the consistency requirement for low ability spots at $q(N)$ college:
$$1 - i^l(N) - \chi(N)(1-\eta(N)) = 0.$$
If this requirement is satisfied (to desired numerical accuracy), stop. If not, go back to step 2 and adjust $\eta(0)$.

2.2 Algorithm With More Ability Types: A Comparison

We first present a computational algorithm that can be used to solve a model with more than two ability types.[2] We then apply the algorithm to solve a model with 10 ability types and compare the resulting equilibrium college distribution with our baseline calibration with 2 ability types. We find that varying the number of grid points has negligible effects on the equilibrium quality distribution and on key moments of the enrollment and tuition distributions.

[2] More details are available upon request.

With the college problem reformulated as in eq. **??**, we can simplify the college market-clearing condition, replacing eq. **??** with the following two conditions:

$$\begin{aligned}
\chi(Q) &= \sum_r \mu_r \sum_a \mu_a \int \mathbb{1}_{\{q(y,a,r)\in Q\}} dF_a(y) \qquad \forall Q \subset \Omega, \\
\int_Q \bar{a}(q) d\chi(q) &= \sum_r \mu_r \sum_a \mu_a \int \mathbb{1}_{\{q(y,a,r)\in Q\}} a \, dF_a(y) \qquad \forall Q \subset \Omega.
\end{aligned}$$

The first condition states that the measure of students in any quality set Q is consistent with student attendance choices. The second equates the average student ability demanded by colleges producing in quality set Q to the average ability of the students choosing to supply to quality set Q. This greatly simplifies the set of market clearing conditions that needs to be checked. The general strategy is to solve for a college distribution over a quality grid where we know with certainty that colleges are active, and to then check for profitable entry at the bottom.

1. Set up a grid of college quality $(q_1, q_2, ..., q_N)$ where we know with certainty that colleges enter.

2. Make an initial guess of the vector of corresponding discount rates $D_0 = (d_1, d_2, ..., d_N)$.

3. Given the discount rates, use the college first-order conditions and zero profit conditions to compute the set of implied baseline tuition $(b_1, b_2, ..., b_N)$. Thus, we obtain a full set of tuition schedules.

4. Given the tuition schedules, use the household's indifference condition to pin down a set of income thresholds for each ability type $(y_1^a, y_2^a, ..., y_N^2)$, where y_i^a is the income of a household indifferent between quality $i-1$ and quality i colleges.

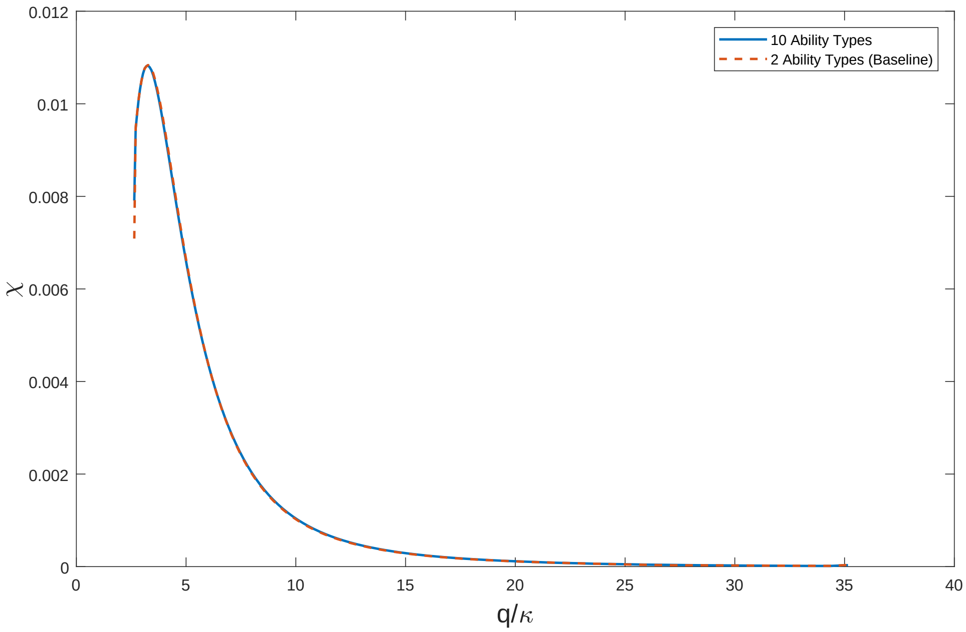
5. Given the income thresholds, compute the supply of average ability to each college $(a_1^s, a_2^s, ..., a_N^s)$.

6. Given demand for ability by each college i (pinned down given d_i from the college first-order conditions) $(a_1^d, a_2^d, ..., a_N^d)$ check market clearing: $a_n^d - a_n^s = 0 \; \forall n$.

7. If markets do not clear, go back to step 2 and adjust the discounts d_n.

8. Check for profitable entry at the bottom. For instance, suppose a college of quality $q_0 < q_1$ enters. To charge the maximum tuition, it has to appeal to the marginal household with income y_1^a. Thus, we can use the household's indifference condition

Figure 1: College Distribution



to pin down tuition t_0^a. We then solve the college problem and check its profit. If profit is positive, go back to step 1 and add q_0 to the grid of college quality. Otherwise, we stop.

We now solve the college model with 10 points in the grid on ability and compare the results to our benchmark two-ability-types calibration. We use the same parameterization as in our baseline. We discretize the 10 grid points such that 1) the 10-grid-point model has the same variance of ability as the baseline, and 2) the conditional mean of income distribution varies linearly with ability with the same slope as the baseline. Figure 1 plots the equilibrium college distribution (density) with different number of ability types.

3 Data Appendix

In this data appendix, we first explain how we construct first moments for the 2016 and 1990 calibrations. We then explain how we construct the second moments in Table 2 from micro college-level data.

Enrollment and graduation rates

We target observed 4 year college enrollment rates and impose observed dropout rates by ability. The graduation rate from the bottom half of the AFQT distribution is 0.52 (**?**) while the graduation rate from the top half of the AFQT distribution is 0.78. From the NLSY97, we computed that 12.5% of below median ability children have graduated with a college degree. That suggests $0.125/0.52 = 24.0\%$ of low ability children enrolled in 4 year colleges. Similarly, 52.8% of above median ability children have a college degree, suggesting $0.528/0.78 = 67.7\%$ enrolled. The total enrollment rate for the economy is therefore $0.5 \times 0.240 + 0.5 \times 0.677 = 45.85\%$.

But note that while the NLSY graduation rates suggest an overall graduation rate of $0.5 * 0.528 + 0.5 * 0.125 = 32.7\%$, the March CPS, a standard reference for educational attainment, suggests a graduation rate of 36.1% in 2016.[3] We decided to target the CPS graduation rate number, so we scaled up the enrollment rate numbers by a factor of $36.1/32.7 = 1.106$, so that enrollment rates become $1.106 * 67.7 = 74.9\%$ and $1.106 * 24.0 = 26.5\%$ for an average of 50.7%.

Shares in public vs private colleges, and shares in-state vs out-of-state

In 2016, 78% of public 4 year college students were from in-state (Trends in College Pricing (TICP) 2018, Fig 23). Enrollment in 4 year schools in public and private colleges in 2016 was, respectively, $4,994,668 + 1,185,002$, and $2,187,122 + 466,900$, giving a public share of

$$\frac{4994668 + 1185002}{4994668 + 1185002 + 2187122 + 466900} = 70.0\%.$$

(Trends in College Pricing 2018, Fig 21). So the share of in-state students as a share of all students in 4 year colleges is $0.78 \times 0.70 = 54.6\%$.

Ability discount (institutional aid)

Average institutional aid at public schools in 2015-16 was \$2,274 (Trends in College

[3] The source for the CPS number is
CPS Historical Time Series Tables
Table A-2. Percent of People 25 Years and Over Who Have Completed High School of College, by Race, Hispanic Origin and Sex: Selected Years 1940 to 2010.
We focus on the age group 25-29 and the category "Completed Four Years of College or More."
https://www.census.gov/data/tables/time-series/demo/educational-attainment/cps-historical-time-series.html

Pricing 2018, Fig 18) and $\$14,055$ at private schools (Trends in Student Aid (TISA) 2018, Fig 19). So average per student institutional aid is $0.7 \times 2,274 + (1 - 0.7) \times 14,055 = \$5,808$.

Need based aid

In 2016-17, 32% of students were receiving a Pell grant (Trends in Student Aid 2018, Fig 20A), and the average grant was $\$3,800$ (Trends in Student Aid 2018, Fig 21A). So the unconditional average was $0.32 \times 3,800 = \$1,216$.

In addition, there are need-based state grants. Of all state grant aid, 76% is need based (Trends in Student Aid 2018, Fig 23A), and state grant aid is $1,442$ at public 4-year colleges (TISA 2018 Fig 18 for 15-16) and 944 at private 4-year colleges (TISA 2018 Fig 19 for 15-16). So a good target number for need-based state grant aid is $0.76 \times (0.7 \times 1,442 + (1 - 0.7) \times 944) = 982$. So total need-based aid per student (Pell grants + state need-based aid) is $1,216 + 982 = \$2,198$.

General subsidy to students p_0

The general subsidy is average sticker tuition minus average net tuition minus institutional aid minus public need-based aid $= \$19,152 - \$9,249 - \$5,808 - \$2,198 = \$1,896$.

In-state vs. out-of-state tuition

Sticker tuition numbers for 2016-2017 are $\$9,650$ for public in-state and $\$24,930$ for public out-of-state (TICP 2016, Table 1A). So in-state tuition is 38.7% of out-of-state tuition. The in-state subsidy per subsidized student is \$15,280, which translates to \$8,343 per student overall.

Fixed costs net of subsidies directly to colleges

We have dealt separately with all subsidies that go to students directly. There are separate subsidies that go to colleges and benefit students indirectly. In the model, a fixed subsidy per student that goes to schools is the same (in terms of allocations and welfare) as a subsidy that goes to students. But whereas net tuition is the same in both cases, sticker tuition will be different: a subsidy that goes to schools will reduce sticker tuition, while a subsidy that goes to students will not.

We know that in aggregate the following aggregate budget constraint for the university sector must be satisfied

$$E + \phi = \text{Tuition Revenue} + \text{Other Revenue}$$

where ϕ is the fixed administration cost per student, and E is per student variable spending, which we define as instructional spending + student services spending.

College revenue from tuition is

$$\text{Tuition Rev.} = \text{Sticker Tuition} - \text{Ability Discounts (Inst. Aid)} = \text{Net Tuition} + \text{Public Student Aid}$$

where here sticker tuition is already net of the in-state discount for in-state students, and public student aid $p(y)$ is general plus need-based public student aid.

Other revenue is

$$\text{Other Revenue} = \bar{s} + \text{ In-state Transfers}$$

where in-state transfers are the transfers from the state for taking in-state students, and $\bar{s}$ are other sources of subsidies that go straight to colleges (rather than to students). Thus,

$$E + \phi = \bar{s} + \text{ In-state Transfers} + \text{Net Tuition} + \text{ Public Student Aid.}$$

Now in the data we can measure E, In-state Transfer, Net Tuition, and Public Student Aid. From these we can get

$$\phi - \bar{s} = \text{ In-state transfers} + \text{Net Tuition} + \text{Public student aid} - E$$

In particular, in 2016,

- Expenditure $E = 0.7 \times (12,539 + 2107) + 0.3 \times (17,996 + 4,753) = 17,077$ (NCES 334.10 and 334.30).
- Public student aid $= 1,896 + 2,198 = 4,094$.
- In-state transfers $= 0.78 \times 0.7 \times (24,930 - 9,650) = 8,343$.
- Net Tuition $= 9,249$.

Hence:

$$\begin{aligned} \phi - \bar{s} &= 8,343 + 9,249 + 4,094 - 17,077 \\ &= 4,610 \end{aligned}$$

Thus, fixed costs net of general subsidies to colleges are positive.

Forgone Earnings

From the CPS (Series ID: LEU0252886300) we take median usual weekly earnings for full-time wage and salary workers aged 16-24. In current dollars, this was \$259 in 1990 and \$501 in 2016. Adjusted by the CPI and assuming 20 weeks of college per year, forgone earnings from attending college is \$10,020 in 2016 dollars in both 2016 and 1990.

We now describe how we construct all the data moments reported in Table 3 for 1990.

Enrollment and Tuition in 1990

The share of 25-29 year-olds with a college degree was 23.2% in the CPS in 1990.[4] Assuming the same graduation to enrollment ratio as 2016, the implied enrollment rate for 1990 is 32.7%.

Of the students enrolled, we assume 70% were in public colleges, as in 2016 (according to NCES Table 303.70, the 1990 public share was 70.7%). Of the students in public colleges, we assume 83% were in-state students in 1990 (the earliest estimate for this number we found was 83% for 2004, TICP 2016, Fig 22). This implies a share of in-state students of $0.7 \times 0.83 = 0.581$.

We estimate average net tuition in 1990 to be \$6,034. Average net tuition at private colleges was \$11,750, and average net tuition for in-state students at public colleges was \$2,000. Our estimate for net tuition for out-of-state students at public colleges is sticker tuition for this group (\$12,837, see below) minus the gap between sticker and net tuition for in-state students (i.e., $\$3,520 - \$2,000 = \$1,520$) which implies a net tuition value of \$11,317. Overall average net tuition is a weighted average of these three groups of students:

Net Tuition $= 0.7 * 0.83 * 2,000 + 0.7 * (1 - 0.83) * 11,317 + 0.3 * 11,750 = \$6,034$

Subsidies in 1990

Need-based aid: The average Pell grant for 1990-1991 was $\$2,720$ per recipient (Trends in Student Aid 2018, Fig 21A). We use the 1990-91 Federal Pell Grant Program End of Year Report to estimate the share of students receiving Pell grants in 1990. Applications were 63% of students, of which 63.1% were found eligible, of which 75.5% actually ended up receiving money, suggesting 30% of students were getting grants. Equivalently 3.4 million out of 11.4 million students = 30%. Thus the unconditional Pell grant average for 1990-91 is $0.30 \times 2,720 = \$816$. We next construct a state need-based grant estimate for 1990-1991. From 1990-91 total state grant aid per student has grown by 105%, and need-based aid has grown by 75.2% (TISA 2018, Fig 23A). Thus, our state need-based grant estimate for 1990-91 is $982/1.752 = 561$ per student. Total need-based aid in 1990 is thus

[4] Table A-2 is available here: https://www.census.gov/data/tables/time-series/demo/educational-attainment/cps-historical-time-series.html

$816 + 561 = \$1,377$.

General subsidies to students, p_0: We estimate this as 12% of state grant aid in 1990 (88% of such aid is need-based) which translates to \$76.

Total public aid to students: Need-based aid plus the lump-sum component p_0. Thus, total public student aid was $\$1,377 + \$76 = \$1,453$.

In-state transfers: We estimate sticker tuition for out-of-state students at public colleges in 1990 (for which we could not find data) as sticker tuition for private college students in 1990 (\$17,240) times the ratio of sticker tuition for out-of-state public relative to private students in 2016, which is \$24,930 divided by \$33,480, which translates to an estimate for out-of-state sticker tuition in 1990 of \$12,837. Thus, in-state transfers, per in-state student are $\$12,837 - \$3,520 = \$9,317$, where \$3,520 is public in-state sticker tuition in 1990. Thus, on a per-student basis, the value of transfers to support in-state students was In-state Transfers $= 0.83 \times 0.7 \times \$9,317 = \$5,413$.

General subsidies to colleges $\bar{s}$: To estimate general per student subsidies to colleges in 1990, we use the aggregate budget constraint for the college sector, as for 2016. To do so we require an estimate for variable expenditure per student in 1990.

We measure variable expenditure (defined as instructional expenses+student services) in 1990 in the same way as in 2016. The difficulty is that the NCES changed its reporting standards twice during late 1990s and early 2000s, making numbers not directly comparable across 1990 and 2016. We instead compute growth rates in each subperiod during which reporting standards remained consistent, and use these growth rates to infer 1990 expenditure. For public colleges, the cumulative growth of variable expenditure was 17.9 percent from 1990 to 2001. The growth rate was 16 percent from 2003 to 2014. Given that the variable expenditure for public colleges was \$11,881 in 2014-2015, we infer that consistently defined variable expenditure in 1990 was \$8,680. For private nonprofit colleges, the growth rate was 11.3 percent between 1990 and 1996. The growth rate was 10.4 percent between 1996 and 1999. The growth rate was 7.4 percent between 1999 and 2003. The growth rate between 2003 and 2014 was 13.5 percent. Given these growth rates and variable expenditure for private colleges of \$22,120 in 2014-2015, the estimated variable expenditure for private nonprofit colleges in 1990 is \$14,757. Putting all this together, we estimate per student expenditure in 1990 of $E = 0.7 \times 8,680 + 0.3 \times 14,757 = 10,503$.

Hence, 1990 fixed costs net of general subsidies to students are

$$\begin{aligned} \phi - \bar{s} &= 5,413 + 6,034 + 1,453 - 10,503 \\ &= 2,396 \end{aligned}$$

3.1 Statistics of Table 2

The second moments are computed from College Scorecard microdata merged with the Mobility Report Cards data set (**?**), which has higher-quality household income data. Specifically, we download the most recent data from the College Scorecard (https://collegescorecard.ed and merge it with household income data from the Mobility Report Cards online data Table 2.[5] Next, we describe how we construct each variable.

Sticker tuition: In-state tuition and fees (variable name tuitionfee_in).

Net tuition: Average net price paid (NPT4_pub for public colleges and NPT4_priv for private colleges). Note this measure includes the full cost of attendance (including living expenses). We construct a measure of living expenses by subtracting tuition and fees (tuitionfee_in) from the full cost of attendance (costt4_a). Net tuition is then obtained by subtracting the living expenses from the average net price paid.

Household income: From Mobility Report Cards online data Table 2 (variable name par_mean). Mean income is $87,335.

Fraction of high ability: We collect data on national averages of the SAT score (whenever the SAT score is not available, we substitute the ACT score.) We then assume that the score is normally distributed at each college and use the college-specific 25th percentile and 75th percentile SAT score (satmt25, satvr25, satwr25, satmt75, satvr75, satwr75) to back out the mean and variance of the distribution at each college. Then we compute the fraction of high-ability students at each college as the fraction with a score higher than the national average.

[5] Note that to link online data Table 2 to College Scorecard data, we need to use online data Table 11.