Online Appendix

An Equilibrium Model of the International Price System

Dmitry Mukhin

d.mukhin@lse.ac.uk

A Analytical results

A.1 Baseline model

A.1.1 Equilibrium conditions

The Kimball aggregator for consumption bundle in region i is defined as

$$(1-\gamma)\int_0^1 \Upsilon\left(\frac{C_{ii}(\omega)}{(1-\gamma)C_i}\right)\mathrm{d}\omega + \gamma\int_0^1\int_0^1 \Upsilon\left(\frac{C_{ji}(\omega)}{\gamma C_i}\right)\mathrm{d}\omega\mathrm{d}j = 1, \tag{A1}$$

where $\Upsilon(1)=\Upsilon'(1)=1$, $\Upsilon'(\cdot)>0$ and $\Upsilon''(\cdot)<0$. I define $h(\cdot)\equiv\Upsilon'^{-1}(\cdot)$ and borrow expressions for price indices and demand under the Kimball aggregator from Itskhoki and Mukhin (2021) and Amiti, Itskhoki, and Konings (2019). The equilibrium system of the model consists of the following blocks:

1. Labor supply and labor demand:

$$C_i = \frac{W_i}{P_i}, \tag{A2}$$

$$L_i = (1-\phi)\left(\frac{P_i}{W_i}\right)^{\phi}\frac{Y_i}{A_i}. \tag{A3}$$

2. Market clearing condition:

$$Y_i(\omega) = \int_0^1\left[(1-\gamma)\,h\left(\frac{D_iP_{ii}(\omega)}{P_i}\right)(X_i+C_i) + \gamma\int_0^1 h\left(\frac{D_jP_{ij}(\omega)}{P_j}\right)(X_j+C_j)\,\mathrm{d}j\right]\mathrm{d}\omega, \tag{A4}$$

where intermediate demand is given by $X_i = \phi\frac{MC_iY_i}{P_i}$ and marginal costs of production are equal to

$$MC_i = \frac{W_i^{1-\phi}P_i^{\phi}}{A_i}. \tag{A5}$$

3. Price setting and currency choice:

$$P_{ji}(\omega)=\begin{cases}\mathcal{E}_{ik}\bar{P}^{k}_{ji}, & \text{w/p } \lambda\\ \tilde{P}_{ji}, & \text{w/p } 1-\lambda\end{cases},$$

where

$$\tilde{P}_{ji}=\arg\max_{P}\ (P\mathcal{E}_{ji}-MC_j)\,h\left(\frac{D_iP}{P_i}\right)(X_i+C_i),$$

$$\bar{P}^{k}_{ji}=\arg\max_{P,k}\ \mathbb{E}\,(P\mathcal{E}_{jk}-MC_j)\,h\left(\frac{D_iP\mathcal{E}_{ik}}{P_i}\right)(X_i+C_i).$$

4. Definition of price indices:

$$(1-\gamma)\int_0^1\Upsilon\left(h\left(\frac{D_iP_{ii}(\omega)}{P_i}\right)\right)\mathrm{d}\omega+\gamma\int_0^1\int_0^1\Upsilon\left(h\left(\frac{D_iP_{ji}(\omega)}{P_i}\right)\right)\mathrm{d}\omega\mathrm{d}j=1,$$

$$(1-\gamma)\int_0^1 h\left(\frac{D_iP_{ii}(\omega)}{P_i}\right)\frac{P_{ii}(\omega)}{P_i}\mathrm{d}\omega+\gamma\int_0^1\int_0^1 h\left(\frac{D_iP_{ji}(\omega)}{P_i}\right)\frac{P_{ji}(\omega)}{P_i}\mathrm{d}\omega\mathrm{d}j=1.$$

5. Risk-sharing:

$$\mathcal{E}_{i0}=\eta_i\frac{e^{\psi_i}P_iC_i}{e^{\psi_0}P_0C_0}. \tag{A6}$$

6. Country's budget constraint pins down constant η_i. The net exports expressed in dollar terms are

$$\mathbb{E}\frac{e^{\psi_i}NX_i}{e^{\psi_0}P_0C_0}=0,\quad \text{where} \tag{A7}$$

$$NX_i=\gamma\int_0^1\int_0^1\left\{\mathcal{E}_{0j}P_{ij}(\omega)\,h\left(\frac{D_jP_{ij}(\omega)}{P_j}\right)(X_j+C_j)-\mathcal{E}_{0i}P_{ji}(\omega)\,h\left(\frac{D_iP_{ji}(\omega)}{P_i}\right)(X_i+C_i)\right\}\mathrm{d}\omega\mathrm{d}j.$$

7. Monetary policy:

$$e^{m_i}=P_iC_i. \tag{A8}$$

A.1.2 Steady state

Consider symmetric steady state with zero net foreign asset positions and $\psi_i=0$. The symmetry implies that bilateral exchange rates between all countries are equal one, $\mathcal{E}_{ij}=1$ and therefore, $P_{ji}=D_i$. Given the steady-state elasticity of demand θ, the optimal markup is $\frac{\theta}{\theta-1}$ and hence, from the labor supply condition (A2), the steady-state consumption is $C_i=\frac{\theta-1}{\theta}$. From market clearing conditions (A3) and (A4), the aggregate output and employment are equal $Y_i=\frac{1}{1-\phi}C_i$ and $L_i=C_i^{1-\phi}$.

A.1.3 Prices

For the applications below, it is sufficient to focus on the case when domestic firms set prices in local currency and invoicing is symmetric across countries. Let μ^P and μ^D be dummy variables that are equal one if exporters choose respectively PCP and DCP and are zero otherwise. The bilateral price index (14) can then be written as

$$p_{ji}=(1-\lambda)\tilde{p}_{ji}+\lambda\big[(\mu^P+\mu^D)e_i-\mu^Pe_j-\mu^De_0\big],$$

where $p_{ii} = (1-\lambda)\tilde{p}_{ii}$ for domestic prices. Substitute the bilateral prices into the aggregate price index (13) and integrate using the fact that $\int_n^1 e_i \mathrm{d}i = 0$:

$$p_i = (1-\gamma)(1-\lambda)\tilde{p}_{ii} + \gamma(1-\lambda)\int_0^1 \tilde{p}_{ji}\mathrm{d}j + \gamma\lambda\big[(\mu^P+\mu^D)e_i - (n\mu^P+\mu^D)e_0\big].$$

Given that nominal wages are constant $w_i = 0$, the desired price (11) is given by

$$\tilde{p}_{ji} = (1-\alpha)(\phi p_j + e_{ij}) + \alpha p_i$$

and can be substituted into the previous expression to obtain

$$\Big[1-(1-\lambda)\big(\alpha+(1-\gamma)(1-\alpha)\phi\big)\Big]p_i = \gamma(1-\lambda)(1-\alpha)\phi\int_0^1 p_j\mathrm{d}j + \gamma\Big[(1-\lambda)(1-\alpha)+\lambda(\mu^P+\mu^D)\Big]e_i \\ -\gamma\Big[\lambda(n\mu^P+\mu^D)+(1-\lambda)(1-\alpha)n\Big]e_0.$$

Integrate p_i across countries into the global price index $p \equiv \int_0^1 p_i \mathrm{d}i$:

$$\Big[1-(1-\lambda)\big(\alpha+(1-\alpha)\phi\big)\Big]p = -\gamma\lambda(1-n)\mu^D e_0.$$

Substitute p into the previous equation to solve for p_i:

$$p_i = \chi e_i - \chi_0 e_0, \tag{A9}$$

where

$$\chi = \frac{\gamma[(1-\lambda)(1-\alpha)+\lambda(\mu^P+\mu^D)]}{1-(1-\lambda)(\alpha+(1-\gamma)(1-\alpha)\phi)},$$
$$\chi_0 = \frac{\gamma}{1-(1-\lambda)(\alpha+(1-\gamma)(1-\alpha)\phi)}\left[(1-\lambda)(1-\alpha)n+\lambda(n\mu^P+\mu^D)+\frac{\lambda(1-\lambda)(1-\alpha)\gamma\phi\mu^D(1-n)}{1-(1-\lambda)(\alpha+(1-\alpha)\phi)}\right].$$

The optimal price (15) can then be expressed as

$$\tilde{p}_{ji} + e_{ki} = e_k - (1-\alpha)(1-\phi\chi)e_j - \alpha(1-\chi)e_i - (\alpha+(1-\alpha)\phi)\chi_0 e_0. \tag{A10}$$

It is easy to verify that the aggregate pass-through coefficients (A9) are positive and no greater than one, i.e. $0 \le \chi, \chi_0 \le 1$. It follows that the coefficients before e_j, e_i and e_0 are between 0 and 1 as well.

A.2 Proofs

Proof of Lemma 1 Suppress country indices and take the second-order approximation of the profit function at price p around the state-dependent optimal price $\tilde{p}_{ji}$:

$$\Pi(p) = \Pi(\tilde{p}_{ji}) + \Pi_p(\tilde{p}_{ji})(p-\tilde{p}_{ji}) + \frac{1}{2}\Pi_{pp}(\tilde{p}_{ji})(p-\tilde{p}_{ji})^2 + \mathcal{O}(p-\tilde{p}_{ji})^3,$$

The first term on the right hand side does not depend on currency of invoicing. From the first-order condition for optimal price, $\Pi_p(\tilde{p}_{ji}) = 0$. Finally, to the zero-order approximation, $\Pi_{pp}(\tilde{p}_{ji}) = \bar{\Pi}_{pp}(\bar{\tilde{p}}_{ji}) < 0$, where $\bar{\Pi}_{pp}(0)$ denotes the derivative in the deterministic steady state and $\bar{\tilde{p}}_{ji}$ is the corresponding optimal price. Therefore, to the second-order approximation, the currency choice problem is equivalent to minimizing $\mathbb{E}(p-\tilde{p}_{ji})^2$. Note that only the first-order approximation is required for p and $\tilde{p}_{ji}$. In particular, the optimal preset price in currency k

is $\bar{p}_{ji}^{k} = \mathbb{E}\left(\tilde{p}_{ji} - e_{ik}\right)$, so that ex post price is $p = \bar{p}_{ji}^{k} + e_{ik}$. Substitute this expression into the objective function to write the currency problem as

$$\min_{k} \mathbb{V}\left(\tilde{p}_{ji} + e_{ki}\right), \tag{A11}$$

which completes the proof of the lemma. ■

Proof of Lemma 2 An equal increase in all exchange rates $\{e_i\}$ leaves the bilateral exchange rates unchanged and hence, has no effect on desired prices $\tilde{p}_{ji}$. It follows that $\tilde{p}_{ji}$ is homogeneous of degree zero in exchange rates and that $\tilde{p}_{ji} - e_i$ is homogeneous of degree one. Therefore, it is always possible to contract a basket of currencies with the sum of currency weights equal one that perfectly replicates the desired price. The equilibrium prices then coincide with the case of flexible prices, i.e. $\chi = \frac{\gamma}{1-(1-\gamma)\phi}$ and $\chi_0 = \frac{\gamma n}{1-(1-\gamma)\phi}$. It follows from equation (A10) that the share of dollars is $(\alpha+(1-\alpha)\phi)\chi_0$ for trade flows between non-U.S. economies, $\alpha(1-\chi)+(\alpha+(1-\alpha)\phi)\chi_0$ for U.S. imports, and $(1-\alpha)(1-\phi\chi)+(\alpha+(1-\alpha)\phi)\chi_0$ for U.S. exports. Excluding within-U.S. trade from international flows, the weights of the corresponding flows are $\frac{(1-n)^2}{1-n^2}$, $\frac{(1-n)n}{1-n^2}$ and $\frac{(1-n)n}{1-n^2}$. Integrate across all flows to get the share of DCP in international trade equal

$$(\alpha+(1-\alpha)\phi)\chi_0+\Big[\alpha(1-\chi)+(1-\alpha)(1-\phi\chi)\Big]\frac{(1-n)n}{1-n^2} = \frac{n}{1+n}\left[1+\frac{\gamma(\alpha+(1-\alpha)\phi)}{1-(1-\gamma)\phi}n\right] \leq \frac{n}{1+n}(1+n) = n.$$

Note that the dollar share in trade would be exactly n if the flows between U.S. islands $i \in [0, n]$ were included in international trade. ■

Proof of Proposition 1 Consider for example the limit $\gamma, \alpha \to 1$, so that $\chi \to \mu^P + \mu^D$, $\chi_0 \to n\mu^P + \mu^D$ and $\tilde{p}_{ji} + e_{ki} \to e_k - (1-\chi)e_i - \chi_0 e_0$. Conjecture that other firms choose DCP, so that $\mu^D = 1$ and $\tilde{p}_{ji} + e_{ki} \to e_k - e_0$. It follows that the firm finds it optimal to choose $k = 0$ and the DCP equilibrium can be sustained in the neighbourhood of $\gamma = \alpha = 1$.

Note that both χ and χ_0 are increasing in γ and ϕ. In addition, given χ and χ_0, the coefficient before e_j in equation (A10) is decreasing in ϕ, while the coefficient before e_0 is increasing in ϕ. It follows that higher γ and ϕ decrease the weights of e_j and e_i and increase the weight of e_0 in equation (A10), which makes PCP and LCP less likely and raises the chances of DCP. Figure 2 shows that the effect of α can be not monotonic. ■

Lemma A1 *In the flexible-price limit $\lambda \to 0$, the equilibrium exists and is generically unique. The invoicing is symmetric across small countries.*

Proof In the flexible-price limit $\lambda \to 0$, the pass-through coefficients from (A9) converge to $\chi \to \frac{\gamma}{1-(1-\gamma)\phi}$ and $\chi_0 \to \frac{\gamma n}{1-(1-\gamma)\phi}$ and do not depend on invoicing decisions of firms. The currency choice problem then has a unique solution except for some borderline values of parameters. Finally, since coefficients before exchange rates are the same for exporters from all small economies and the volatility of exchange rates is also the same, the equilibrium invoicing is symmetric across them. ■

Proof of Lemma 4 When $n = 0$, the desired price of exporters is

$$\tilde{p}_{ji} + e_{ki} = e_k - \frac{1-\phi}{1-(1-\gamma)\phi}\Big[(1-\alpha)e_j + \alpha(1-\gamma)e_i\Big]. \tag{A12}$$

Since volatility of all exchange rates is the same when $\rho = 1$, the exporter chooses between producer and local currency based on their weights in (A12): $k = j$ when $1-\alpha \geq \alpha(1-\gamma) \Leftrightarrow \alpha \leq \frac{1}{2-\gamma}$ and $k = i$ otherwise. ■

Proof of Proposition 2 The desired price in the flexible-price limit with $n > 0$ is

$$\tilde{p}_{ji} + e_{ki} = e_k - \frac{1-\phi}{1-(1-\gamma)\phi}\Big[(1-\alpha)e_j + \alpha(1-\gamma)e_i\Big] - \frac{\gamma(\alpha+(1-\alpha)\phi)}{1-(1-\gamma)\phi}ne_0. \tag{A13}$$

As long as $n > 0$, choosing $k = 0$ is optimal for example in the limit $\phi \to 1$. Moreover, keeping the values of other parameters fixed, higher n increases the relative weight of e_0 in the optimal price, and therefore makes DCP more likely. ■

Proof of Proposition 3 Rewrite expression (A10) as $\tilde{p}_{ji} + e_{ki} = e_k - ae_j - be_i - ce_0$. From Lemma 1, exporters choose PCP, LCP or DCP depending on whether respectively a, b or $\rho(c - 0.5) + 0.5$ is greater. If $n \leq 0.5$, it follows that $c - 0.5 \geq 0$ and hence, lower values of ρ unambiguously increase the chances of DCP. Note that in the limit $\phi \to 1$, we have $a = b = 0$ and under $\rho < 1$ DCP strictly dominates both PCP and LCP. ■

Proof of Corollary 1 Adopt the following notation: two currency unions have masses n_1 and n_2 with $n \equiv n_1 + n_2$, the relative exchange rate volatility of pound is $\rho \equiv \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}$, μ_i^k denotes the share of country i imports invoiced in currency k ($\mu_i^1 + \mu_i^2 = 1$), and the aggregate price index in country i is $p_i = \chi_0^i e_i - \chi_1^i e_1 - \chi_2^i e_2$, where $i = 0$ for a representative country from the RoW. Vehicle currency 1 dominates vehicle currency 2 for exporter from j to i iff

$$(1-\alpha)\frac{\text{cov}\left(\phi p_j + e_1 - e_j, e_1 - e_2\right)}{\text{var}\left(e_1 - e_2\right)} + \alpha\frac{\text{cov}\left(p_i + e_1 - e_i, e_1 - e_2\right)}{\text{var}\left(e_1 - e_2\right)} < \frac{1}{2}.$$

Applying this formula for each bilateral trade flow, we get:

- RoW exports to RoW:

$$\left(\alpha + (1-\alpha)\,\phi\right)\chi_2^0 + \left[1 - \left(\chi_1^0 + \chi_2^0\right)\left(\alpha + (1-\alpha)\,\phi\right)\right]\rho < \frac{1}{2},$$

- RoW exports to currency unions:

$$(1-\alpha)\,\phi\chi_2^0 + \alpha\chi_2^1 + \left[(1-\alpha)\left(1 - \phi\chi_1^0 - \phi\chi_2^0\right) + \alpha\left(\chi_0^1 - \chi_1^1 - \chi_2^1\right)\right]\rho < \frac{1}{2},$$
$$(1-\alpha)\,\phi\chi_2^0 + \alpha\left(1 + \chi_2^2 - \chi_0^2\right) + \left[(1-\alpha)\left(1 - \phi\chi_1^0 - \phi\chi_2^0\right) + \alpha\left(\chi_0^2 - \chi_1^2 - \chi_2^2\right)\right]\rho < \frac{1}{2},$$

- Currency union exporting to RoW:

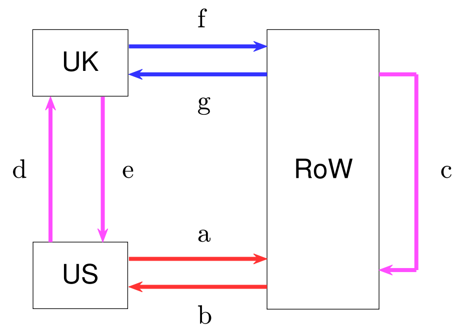
$$(1-\alpha)\,\phi\chi_2^1 + \alpha\chi_2^0 + \left[(1-\alpha)\,\phi\left(\chi_0^1 - \chi_1^1 - \chi_2^1\right) + \alpha\left(1 - \chi_1^0 - \chi_2^0\right)\right]\rho < \frac{1}{2},$$
$$(1-\alpha)\left(1 + \phi\chi_2^2 - \phi\chi_0^2\right) + \alpha\chi_2^0 + \left[(1-\alpha)\,\phi\left(\chi_0^2 - \chi_1^2 - \chi_2^2\right) + \alpha\left(1 - \chi_1^0 - \chi_2^0\right)\right]\rho < \frac{1}{2},$$

- One currency union exporting to the other:

$$(1-\alpha)\,\phi\chi_2^1 + \alpha\left(1 + \chi_2^2 - \chi_0^2\right) + \left[(1-\alpha)\,\phi\left(\chi_0^1 - \chi_1^1 - \chi_2^1\right) + \alpha\left(\chi_0^2 - \chi_1^2 - \chi_2^2\right)\right]\rho < \frac{1}{2},$$
$$(1-\alpha)\left(1 + \phi\chi_2^2 - \phi\chi_0^2\right) + \alpha\chi_2^1 + \left[(1-\alpha)\,\phi\left(\chi_0^2 - \chi_1^2 - \chi_2^2\right) + \alpha\left(\chi_0^1 - \chi_1^1 - \chi_2^1\right)\right]\rho < \frac{1}{2}.$$

I next argue that the share of DCP is monotonically increasing along the transition path focusing separately on two cases when the dynamics is driven by ρ and by n_2. Parameter ρ is present only in the currency choice

Figure A1: Transition



block:

$$(1-\alpha)\left[\phi\chi_2^j+\left(1-\phi\chi_1^j-\phi\chi_2^j\right)\rho-\left(1-\phi\chi_0^j\right)\frac{\mathrm{cov}\left(e_j,e_1-e_2\right)}{\mathrm{var}\left(e_1-e_2\right)}\right]$$
$$+\alpha\left[\chi_2^i+\left(1-\chi_1^i-\chi_2^i\right)\rho-\left(1-\chi_0^i\right)\frac{\mathrm{cov}\left(e_i,e_1-e_2\right)}{\mathrm{var}\left(e_1-e_2\right)}\right]<\frac{1}{2}.$$

The derivative of each term with respect to ρ is clearly positive for all countries except for country 1, for which it is proportional to $\chi_0^1-\chi_1^1-\chi_2^1$. This term, however, is non-negative as well:

$$\gamma(1-\lambda)\left(1-\alpha\right)\left(1-n\right)\left[\frac{\left(1-\lambda\right)\left(1-\alpha\right)\left(1-\phi\right)+\lambda\left(1-\gamma\phi\right)}{1-\left(1-\lambda\right)\left(\alpha+\left(1-\alpha\right)\phi\right)}\right].$$

Thus, as ρ goes up, all constraints become more binding and everything else equal, can only decrease the use of the pound. Hence, μ_i^1 falls and μ_i^2 rises, which leaves χ_0^i unaffected, decreases χ_1^i and increases χ_2^i. This tightens the constraint for currency 1 even further in a monotonic way.

Consider next an increase in n_2, assuming that n remains unchanged. Country sizes n_i are present only in price indices, but not directly in the currency choice inequalities. The second part of the proposition (proven below) implies that the share of dollar denominated imports from RoW to the first country is not smaller than the one to the second country. From the inequalities for the trade flows between the currency unions we get $\mu_1^1-\mu_2^1\geq n_1-n_1=0$. This inequality ensures that for a given currency choice, χ_1^i is monotonic in n_1 and χ_2^i is monotonic in n_2. This implies χ_1^i decreases and χ_2^i increases as n_2 goes up. The currency choice inequalities then tighten with n_2. The argument from above shows that endogenous change in invoicing patterns amplifies the fall in the global share of the pound.

Consider next the order, in which the trade flows change the currency of invoicing. Suppose n_2 goes up leaving n unchanged. First, note that price index for any country consists of three terms:

$$p_i\propto(1-\lambda)\gamma\left(1-\alpha\right)\phi\int p_j\mathrm{d}j+(1-\lambda)\gamma\left(1-\alpha\right)\int\left(e_i-e_j\right)\mathrm{d}j+\lambda\gamma\left[e_i-\mu_i^1e_1-\mu_i^2e_2\right]$$

The first term is the same for all countries, while the second one does not depend on currency of invoicing. The last term, however, implies that in the initial equilibrium with all global trade denominated in currency 1, μ_i^2 is positive only for $i=2$. Therefore, χ_2^j is higher and χ_1^j is lower for country 2. Denote with $T(x)$ the threshold of n_2/n_1 or ρ when trade flow x switches from the pound to the dollar and denote trade flows as in Figure A1. The currency choice inequalities from above imply then $T\left(b\right)\leq T\left(c\right)$, $T\left(e\right)\leq T\left(f\right)$ and $T\left(a\right)\leq T\left(c\right)$, $T\left(d\right)\leq T\left(g\right)$. This in turn implies $\chi_2^2\geq\chi_2^j$ for any j, which confirms that the previous inequalities hold and the ordering of switches is correct. The symmetric argument can be made for country 1 with higher χ_1^j and lower

χ_2^j implying $T(c) \leq T(f), T(b) \leq T(e)$ and $T(c) \leq T(g), T(a) \leq T(d)$. The comparative statics for ρ can be made in the similar way: the derivative of the LHS of currency choice inequality with respect to ρ is the same for all countries, so that only levels of χ_k^j matter. ■

A.3 Additional results

A.3.1 Multiple equilibria

Definition 3 *An equilibrium is symmetric if all exporters in the world use either PCP, LCP or the same vehicle currency. The equilibrium is unstable if exogenous perturbation of currency choice of an arbitrarily small fraction of exporters makes a positive mass of other firms change their invoicing decisions.*

Proposition A1 (Multiple equilibria) *Assume that $n = 0$ and $\rho = 1$. Then*

1. *at least one symmetric equilibrium always exists,*
2. *if symmetric equilibrium is unique, then no other equilibria exist,*
3. *all non-pure-strategy equilibria are unstable.*

The proof of Proposition A1 requires a few additional lemmas. When $n = 0$ and $\rho = 1$, the currency choice of exporters is based on the following inequalities:

$$PCP \succ LCP \quad \Leftrightarrow \quad (1-\alpha)\,\phi\chi + \alpha\,(2-\chi) < 1, \tag{A14a}$$
$$PCP \succ DCP \quad \Leftrightarrow \quad (1-\alpha)\,\phi\,(\chi+\chi_0) + \alpha\,(1+\chi_0) < 1, \tag{A14b}$$
$$DCP \succ LCP \quad \Leftrightarrow \quad (1-\alpha)\,(1-\phi\chi_0) + \alpha\,[2-(\chi+\chi_0)] < 1. \tag{A14c}$$

where $\succ$ stays for "prefered to". I use χ^X and χ_0^X to denote the values of the corresponding pass-through coefficients in (A9) under symmetric invoicing X.

Lemma A2 *If DCP is prefered to PCP (LCP) under PCP (LCP) price index, then this ordering holds under DCP price index as well. Symmetrically, if PCP (LCP) dominates DCP under DCP price index, then this ordering holds under PCP (LCP) price index as well.*

Proof Since condition (A14b) gets tighter with χ and χ_0, and $\chi^P = \chi^D$, $\chi_0^P < \chi_0^D$, the relation $DCP \succ PCP$ under χ^P and χ_0^P implies the same ordering under χ^D and χ_0^D. Since condition (A14c) is relaxed by higher χ and χ_0 and $\chi^L < \chi^D$, $\chi_0^L < \chi_0^D$, the relation $DCP \succ LCP$ for χ^L and χ_0^L implies the same ordering for χ^D and χ_0^L. ■

Lemma A3 *It is impossible that for given parameter values, an exporter (i) chooses PCP when all others choose LCP, and (ii) chooses LCP when all others choose PCP.*

Proof Suppose that was the case. Then from (A14a) $\frac{1-\phi\chi^P}{2-\chi^P(1+\phi)} < \alpha < \frac{1-\phi\chi^L}{2-\chi^L(1+\phi)}$. But this requires $\chi^L > \chi^P$, which can not be the case. ■

Lemma A4 *Consider a pure-strategy NE with a choice only between PCP and LCP. If the symmetric LCP equilibrium does not exist, the only possible pure-strategy NE is the symmetric PCP.*

Proof Pure-strategy equilibria can be parametrized by cdf $F(\cdot)$ for $\mu_i^P \in [0,1]$ across countries. PCP is chosen by exporter from country j to country i iff

$$(1-\alpha)\,\phi\chi_j + \alpha\,(2-\chi_i) < 1 \quad \Rightarrow \quad \mu_j < a + b\mu_i$$

for some positive constants a and b. Integrating across importers, we then derive the equilibrium condition: $\mu_i = \int_j \mathbb{I}\{\mu_j < a + b\mu_i\}\, dj$, or equivalently

$$\int_0^1 \mathbb{I}\{z < a + bx\}\, \mathrm{d}F(z) = F(a+bx) = x$$

for any x with positive density. Suppose next that symmetric LCP equilibrium does not exist, i.e. $F(a) = 0$ is unattainable. This is possible only if $a > 1$. But then for any $x > 0$ with positive density we have $x = F(a+bx) \geq F(a) = 1$, i.e. symmetric PCP is the only PSE. ■

Proof of Proposition A1 (1) Suppose there are no symmetric equilibria for some combination of parameters. Note that since $\chi^P = \chi^D$, it follows from (A14a) that the preferences between PCP and LCP should be the same under PCP and DCP price indices. First, suppose that $PCP \succ LCP$ under DCP and PCP. Since there is no PCP equilibrium, we must have $DCP \succ PCP$ under PCP price index. But by Lemma A2, we have $DCP \succ PCP$ under DCP price index as well and hence, DCP equilibrium exists. Second, suppose that $LCP \succ PCP$ under DCP and PCP. Then from Lemma A3, we have $LCP \succ PCP$ under LCP price index. Non-existence of LCP equilibrium requires then $DCP \succ LCP$ under LCP price index. By Lemma A2, $DCP \succ LCP$ under DCP price index as well and hence, we obtain DCP equilibrium. In both cases we arrive to contradiction.

(2) First, suppose that DCP is a unique symmetric equilibrium. Then $DCP \succ LCP$ under LCP and $DCP \succ PCP$ under PCP price index. Since χ^i and χ_0^i can get only higher as one deviates from symmetric LCP, constraint (A14c) implies that DCP dominates LCP in any PSNE. But then χ^i stays the same and χ_0^i can only increase relative to symmetric PCP and constraint (A14b) implies that DCP dominates PCP in any PSNE as well. Second, suppose that LCP is a unique symmetric equilibrium. Since χ^i and χ_0^i can only get lower as one deviates from symmetric DCP, constraint (A14c) implies that LCP dominates DCP in any PSE as well. The existence of symmetric LCP requires according to constraint (A14a) that $\alpha > \frac{1-\phi\chi^L}{2-\chi^L(1+\phi)} > \frac{1}{2}$. This implies $\alpha > (1-\alpha)\,\phi$, so that constraint (A14a) relaxes as χ^i decreases. Therefore, there can be no PSNE with PCP. Finally, suppose that PCP is a unique symmetric NE. Since χ^i and χ_0^i can get only lower than under symmetric DCP, constraint (A14b) implies that DCP is dominated by PCP in PSNE. According to Lemma A4, there can be no PSE with positive measure of LCP.

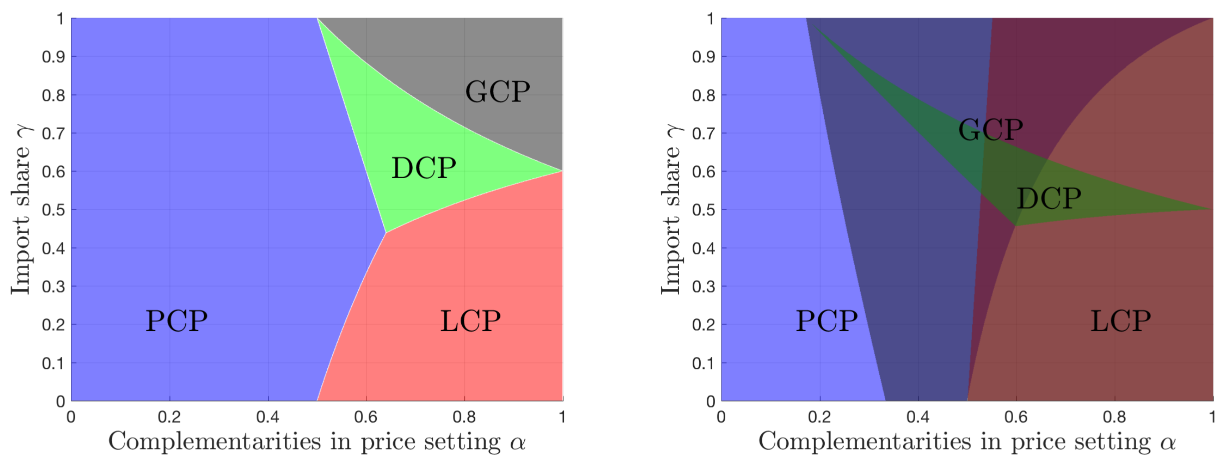
(3) Suppose there is market i, in which a positive mass of importers are indifferent between PCP and DCP and play mixed strategies. Take an arbitrary small share of firms pricing in the producer currency and exogenously switch their invoicing into dollars. The coefficient χ^i does not change, while χ_0^i increases. Condition (A14b) implies that the firms that were indifferent now strictly prefer DCP, while condition (A14c) implies that the share of LCP can only fall. Since firms (endogenously) switch to dollar in response to the perturbation, the initial equilibrium is not stable. Note there are no indirect effects coming from other markets: as country i is infinitely small, the changes in invoicing of its imports or exports has no impact on other countries. A symmetric argument applies for other types of mixed equilibria. ■

A.3.2 Domestic dollarization

In contrast to the assumption of the baseline model, it is not uncommon for *local* firms in developing countries to set prices in dollars (see e.g. Drenik and Perez 2021). I therefore extend the model allowing domestic producers to choose optimally the currency of invoicing and define the global currency pricing (GCP) equilibrium, in which

Figure A2: The optimal invoicing of domestic firms

(a) Flexible price limit (b) Sticky prices



Note: figure (a) shows equilibria in the flexible price limit $\lambda \to 0$ and $\rho = 0.5$, while figure (b) shows symmetric equilibria under sticky prices $\lambda = 0.5$ and $\rho = 1$. The grey area is the region of the global currency pricing (GCP) equilibrium with all firms including domestic ones using the dollar for invoicing. Other parameters: $\phi = 0.5$, $n = 0$.

all firms in the world (including domestic ones) use dollars for invoicing. In contrast, in DCP equilibrium only exporters price in dollars, while domestic firms use local currency.

Proposition A2 *Assume that domestic firms optimally choose the currency of invoicing. Then*

1. *in the flexible price limit $\lambda \to 0$, the region of GCP is the subset of DCP and is increasing in γ, ϕ, α, n and is decreasing in ρ if $n \leq 1/2$,*

2. *in the limit of fully rigid prices $\lambda \to 1$, the region of DCP is a subset of GCP.*

Proof Note that PCP, LCP and DCP coincide for U.S. local firms. Therefore, it is sufficient to focus on the decisions of domestic firms in non-U.S. economies. In the flexible price limit, the currency of invoicing of both exporters and domestic firms has no effect on equilibrium prices and the optimal currency choice is determined by equation (A13), where $i = j$ for domestic firms. It follows immediately that if local firms choose DCP, then so do the exporters. Moreover, domestic firms prefer dollar pricing iff

$$\left[1 - \frac{2\gamma(\alpha + (1-\alpha)\phi)}{1-(1-\gamma)\phi} n\right] \rho < 1 - \frac{2(1-\phi)(1-\gamma\alpha)}{1-(1-\gamma)\phi}.$$

This inequality is more likely to be satisfied when n, γ, α, and ϕ are high, and if $n \leq 0.5$, when ρ is low.

Consider next the case of fully sticky prices. Assume that all exporters set dollar prices and denote with μ_D^D a dummy that is equal one if domestic firms use DCP. It follows that $p_i = [\gamma + (1-\gamma)\mu_D^D](e_i - e_0)$ and hence, adoption of DCP by local firms lowers the weight of PCP and LCP and raises the weight of DCP in the desired price of exporters (A10), making it easier to sustain the DCP equilibrium. ■

To see the intuition, consider first the flexible-price limit when equilibrium prices are independent from firms' invoicing decisions (Figure A2a). Because producer and local currencies coincide for domestic firms, their total weight in the optimal price is higher than the share of producer currency or local currency for exporters. As a result, domestic firms are less likely to use dollar invoicing and the GCP equilibrium is a subset of the DCP equilibrium. On the other hand, when prices are fully sticky and strategic complementarities in currency choice are strong, it is easier to support the equilibrium where all firms invoice in dollars than the equilibrium where only exporters use dollars and domestic firms set prices in local currency (Figure A2b). Thus, the model predicts that while domestic firms might be less likely to switch to dollar invoicing than exporters, once they do so — e.g. because of the unstable monetary policy discussed above — the DCP equilibrium can be sustained more easily and can persist even after fundamental factors turn against the dollar.

B Quantitative analysis

B.1 Full model

Consider an infinite horizon model with discrete time. There are N economies, S sectors (industries), and K currencies. I use subscripts j and i to denote respectively the countries of origin and destination and superscripts r and s for the sectors of origin and destination. Index k is reserved for currency of invoicing. Firms are free to set different prices across markets with the latter defined by type of good (industry of production) r and country of destination i.

General equilibrium block in each country i is described by households maximizing expected utility

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big(\log C_{it}-L_{it}\Big)$$

subject to the budget constraint

$$P_{it}C_{it}+e^{-\psi_{it}}\mathcal{E}_{i0t}\,\Big(\mathbb{E}_t\left[Q_{t+1}B_{it+1}\right]-B_{it}\Big)=W_{it}L_{it}+\Pi_{it}-T_{it}+\Omega_{it},$$

where T_{it} includes fixed costs of currency adjustment. As before, the optimal risk sharing under complete markets and log-linear preferences implies

$$\mathcal{E}_{i0t}=\frac{e^{\psi_{it}}P_{it}C_{it}}{e^{\psi_{0t}}P_{0t}C_{0t}}=\frac{e^{\psi_{it}}W_{it}}{e^{\psi_{0t}}W_{0t}}.$$

Without loss of generality, the bilateral exchange rates can be decomposed into country-specific series given by $\mathcal{E}_{it}=e^{\psi_{it}}W_{it}$. I assume that the monetary authorities make nominal demand and nominal wages grow at a constant rate μ_i, while the financial shocks are a martingale $\Delta\psi_{it}=\varepsilon_{it}$ with innovations potentially correlated across economies $\varepsilon_t\sim$ i.i.d.$(0,\Sigma)$. Therefore, nominal exchange rates follow a random walk process with a drift μ and covariance matrix Σ across countries: $\Delta e_t\sim$ i.i.d.(μ,Σ).

Production of a representative firm in sector s of country i is described by a Cobb-Douglas technology that combines labor and intermediates from sectors $r=1,..S$:

$$Y^s_{it}=A^s_{it}\left(\frac{L^s_{it}}{\phi^{Ls}_i}\right)^{\phi^{Ls}_i}\prod_{r=1}^{S}\left(\frac{X^{rs}_{it}}{\phi^{rs}_i}\right)^{\phi^{rs}_i},\qquad \phi^{Ls}_i+\sum_{r=1}^{S}\phi^{rs}_i=1,$$

where X^{rs}_{it} is the amount of good r used in production of sector s in country i in period t. The sum of input shares is equal to one ensuring a constant returns to scale. It follows that the marginal costs of production are

$$MC^s_{it}=\frac{1}{A^s_{it}}\Big(W_{it}\Big)^{\phi^{Ls}_i}\prod_{r=1}^{S}\Big(P^r_{it}\Big)^{\phi^{rs}_i}.$$

Similar to production function, the final consumption bundle is a Cobb-Douglas aggregator of different goods:

$$C_{it} = \prod_{r=1}^{S} \left(\frac{X_{it}^{rC}}{\phi_i^{rC}} \right)^{\phi_i^{rC}}, \qquad \sum_{r=1}^{S} \phi_i^{rC} = 1.$$

The individual products within each market are combined via the Kimball aggregator:

$$\sum_{j=1}^{N} \gamma_{ji}^r \int_0^1 \Upsilon \left(\frac{Y_{jit}^r(\omega)}{\gamma_{ji}^r X_{it}^r} \right) \mathrm{d}\omega = 1, \qquad \sum_{j=1}^{N} \gamma_{ji}^r = 1,$$

where $Y_{jit}^r(\omega)$ are the quantities sold by firm ω from country j and sector r in country i in period t and γ_{ji}^r are demand shifters, which determine the steady-state trade flows across countries and sectors. As before, for each market, the aggregate price index P_{it}^r is implicitly defined by the system of equations:

$$\sum_{j=1}^{N} \gamma_{ji}^r \int_0^1 \Upsilon \left(h \left(\frac{D_{it}^r P_{jit}^r(\omega)}{P_{it}^r} \right) \right) \mathrm{d}\omega = 1 \quad \text{and} \quad \sum_{j=1}^{N} \gamma_{ji}^r \int_0^1 h \left(\frac{D_{it}^r P_{jit}^r(\omega)}{P_{it}^r} \right) \frac{P_{jit}^r(\omega)}{P_{it}^r} \mathrm{d}\omega = 1.$$

The market clearing condition can then be written as

$$Y_{jt}^r = \sum_{i=1}^{N} \int_0^1 Y_{jit}^r(\omega) \mathrm{d}\omega = \sum_{i=1}^{N} \int_0^1 h \left(\frac{D_{it}^r P_{jit}^r(\omega)}{P_{it}^r} \right) \mathrm{d}\omega \left(\sum_{s=1}^{N} X_{it}^{rs} + X_{it}^{rC} \right),$$

where the last bracket is the sum of intermediate demand and final demand for good r in country i given by

$$X_{it}^{rs} = \frac{\phi_i^{rs} MC_{it}^s Y_{it}^s}{P_{it}^r} \quad \text{and} \quad X_{it}^{rC} = \frac{\phi_i^{rC} P_{it} C_{it}}{P_{it}^r}.$$

Currency choice and proof of Proposition 4 I solve for the optimal currency choice under the Calvo friction taking the approximation around the steady-state with non-zero inflation and flexible prices.[1] Firms choose the currency of invoicing before the realization of shocks and are free to adjust the price in that currency subject to the Calvo friction. As before, to the second order of approximation, a firm chooses the currency of invoicing to minimize expected deviations of the preset price $\bar{p}_{ji}^r(k)$ from the state-dependent desired price $\tilde{p}_{jit}^r + e_{kit}$:

$$\min_k \mathbb{E} \sum_{t=0}^{\infty} (\beta\lambda^r)^t \left(\tilde{p}_{jit}^r + e_{kit} - \bar{p}_{ji}^r(k) \right)^2,$$

where the time subscript $t = 0$ of the reset price is suppressed for brevity. Dropping the terms that are invariant to the currency k, the problem can be rewritten as

$$\max_k \mathbb{E} \sum_{t=0}^{\infty} (\beta\lambda^r)^t \left\{ -2 \left(\tilde{p}_{jit}^r - e_{it} \right) e_{kt} + 2 \left(\tilde{p}_{jit}^r - e_{it} \right) \bar{p}_{ji}^r(k) - \left(\bar{p}_{ji}^r(k) - e_{kt} \right)^2 \right\}. \tag{A15}$$

Consider separately each term of this expression and focus on expectations conditional on information in period $t = 0$. Using the stationarity properties of the model, which imply that $\mathbb{E}_0 \left(\tilde{p}_{ji\tau+t}^r - e_{i\tau+t} \right) \Delta e_{k\tau}$ is

[1] While this approach is less accurate than the approximation around the steady-state with Calvo price adjustment (see e.g. Ascari and Sbordone 2014), it is much more tractable and has similar quantitative implications for low values of inflation.

independent of τ, the first term can be expressed as

$$\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left(\tilde{p}^r_{jit}-e_{it}\right)e_{kt}=\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left(\tilde{p}^r_{jit}-e_{it}\right)\sum_{\tau=0}^{t}\Delta e_{k\tau}=\mathbb{E}_0\sum_{\tau=0}^{\infty}\sum_{t=\tau}^{\infty}(\beta\lambda^r)^t\left(\tilde{p}^r_{jit}-e_{it}\right)\Delta e_{k\tau}$$

$$=\sum_{\tau=0}^{\infty}(\beta\lambda^r)^\tau\sum_{t=0}^{\infty}(\beta\lambda^r)^t\,\mathbb{E}_0\left(\tilde{p}^r_{ji\tau+t}-e_{i\tau+t}\right)\Delta e_{k\tau}=\frac{1}{1-\beta\lambda^r}\sum_{t=0}^{\infty}(\beta\lambda^r)^t\,\mathbb{E}_0\left(\tilde{p}^r_{jit}-e_{it}\right)\Delta e_{k0}.$$

For the second term of (A15), note that the optimal reset price is equal to a discounted sum of expected desired prices expressed in currency of invoicing k:

$$\bar{p}^r_{ji}(k)=(1-\beta\lambda^r)\,\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left(\tilde{p}^r_{jit}+e_{kit}\right)=(1-\beta\lambda^r)\,\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left(\tilde{p}^r_{jit}-e_{it}\right)+e_{k0}+\frac{\beta\lambda^r}{1-\beta\lambda^r}\mu_k,$$

where the second equality uses the fact that $\sum_{t=0}^{\infty}x^t t=x\left(\sum_{t=0}^{\infty}x^t\right)'_x=\frac{x}{(1-x)^2}$. It follows that

$$\bar{p}^r_{ji}(k)\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left(\tilde{p}^r_{jit}-e_{it}\right)=\bar{p}^r_{ji}(k)\left[\frac{\bar{p}^r_{ji}(k)}{1-\beta\lambda^r}-\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left(e_{k0}+t\mu_k\right)\right]$$

$$=\frac{1}{1-\beta\lambda^r}\left(\bar{p}^r_{ji}(k)\right)^2-\frac{\beta\lambda^r}{(1-\beta\lambda^r)^2}\mu_k\bar{p}^r_{ji}(k)-\frac{1}{1-\beta\lambda^r}\bar{p}^r_{ji}(k)e_{k0}.$$

Finally, the last term is equal

$$\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left(\bar{p}^r_{ji}(k)-e_{kt}\right)^2=\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\Bigg[\left(\bar{p}^r_{ji}(k)-\mu_k t-e_{k0}\right)^2-2\left(\bar{p}^r_{ji}(k)-\mu_k t-e_{k0}\right)\left(e_{kt}-\mu_k t-e_{k0}\right)$$

$$+\left(e_{kt}-\mu_k t-e_{k0}\right)^2\Bigg]=\frac{1}{1-\beta\lambda^r}\left(\bar{p}^r_{ji}(k)-e_{k0}\right)^2+\left[\sigma_k^2+2\mu_k e_{k0}-2\mu_k\bar{p}^r_{ji}(k)\right]\frac{\beta\lambda^r}{(1-\beta\lambda^r)^2}+\frac{\beta\lambda^r\left(1+\beta\lambda^r\right)}{(1-\beta\lambda^r)^3}\mu_k^2,$$

where the last equality follows from $\sum_{t=0}^{\infty}x^t t^2=x\left[x\left(\sum_{t=0}^{\infty}x^t\right)'_x\right]'_x=x\left[\frac{x}{(1-x)^2}\right]'_x=\frac{(1+x)x}{(1-x)^3}$.

Combine all three pieces of (A15) together, use the optimal reset price and simplify to obtain

$$\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left\{-2\left(\tilde{p}^r_{jit}-e_{it}\right)e_{kt}+2\left(\tilde{p}^r_{jit}-e_{it}\right)\bar{p}^r_{ji}(k)-\left(\bar{p}^r_{ji}(k)-e_{kt}\right)^2\right\}=(1-\beta\lambda^r)\left[\mathbb{E}_0\sum_{t=0}^{\infty}(\beta\lambda^r)^t\left(\tilde{p}^r_{jit}-e_{it}\right)\right]^2$$

$$-\frac{\beta\lambda^r}{(1-\beta\lambda^r)^2}\left[2\left(1-\beta\lambda^r\right)\sum_{t=0}^{\infty}(\beta\lambda^r)^t\,\mathbb{E}_0\left(\tilde{p}^r_{jit}-e_{it}\right)\left(\Delta e_{k0}-\mu_k\right)+\sigma_k^2\right]-\frac{\beta\lambda^r}{(1-\beta\lambda^r)^3}\mu_k^2.$$

Apply the ex-ante expectation operator $\mathbb{E}(\cdot)$ and use the law of iterated expectation. The first term is invariant to currency of invoicing and can be dropped. Rescaling the remaining terms, one gets the expression from Proposition 4.

Price indices in local and foreign markets determine the pass-through of exchange rates into desired prices and hence, the currency choice of exporters. As Proposition 4 makes clear, the main difference of the Calvo model from the baseline static setup is that the currency choice is shaped not only by the contemporaneous pass-through, but also by the lagged ones, i.e. by the reaction of $\tilde{p}^r_{jit}$ to innovations e_{k0}. I next derive the system of dynamic equations that characterizes the impulse responses of aggregate prices to exchange rate shocks. To this end, rewrite the optimal reset price in a recursive form

$$\tilde{p}^r_{jit}(k) = (1-\beta\lambda^r)\left(\tilde{p}^r_{jit} + e_{kit}\right) + \beta\lambda^r \mathbb{E}_t \bar{p}^r_{jit+1}(k)$$

and combine it with the price index law of motion under the Calvo friction

$$p^r_{jit}(k) = (1-\lambda^r)\,\bar{p}^r_{jit}(k) + \lambda^r p^r_{jit-1}(k)$$

to obtain the NKPC for bilateral trade flows:

$$[1+\beta+\kappa^r]\,p^r_{jit}(k) = p^r_{jit-1}(k) + \kappa^r\left(\tilde{p}^r_{jit} + e_{kit}\right) + \beta\mathbb{E}_t p^r_{jit+1}(k), \qquad \kappa^r \equiv \frac{(1-\beta\lambda^r)(1-\lambda^r)}{\lambda^r}.$$

Convert prices into the currency of destination and aggregate into the market-level price index:

$$[1+\beta+\kappa^r]\,p^r_{it} = p^r_{it-1} + \beta\mathbb{E}_t p^r_{it+1} + \kappa^r \sum_{j=1}^{N}\gamma^r_{ji}\tilde{p}^r_{jit} + \sum_{j=1}^{N}\sum_{k=1}^{K}\gamma^r_{ji}\pi^r_{ji}(k)\,\triangle e_{ikt} - \beta\mathbb{E}_t\sum_{j=1}^{N}\sum_{k=1}^{K}\gamma^r_{ji}\pi^r_{ji}(k)\,\triangle e_{ikt+1},$$

where $\pi^r_{ji}(k)$ is the share of currency k in imports of country i from sector r of country j. Combine this expression with the desired price

$$\tilde{p}^r_{jit} = (1-\alpha^r)\left(\sum_{r'=1}^{S}\phi^{r'r}_j p^{r'}_{jt} + \left(1-\phi^r_j\right)w_{jt} + e_{ijt}\right) + \alpha^r p^r_{it} \tag{A16}$$

to get

$$[1+\beta+\kappa^r(1-\alpha^r)]\,p^r_{it} = p^r_{it-1} + \beta\mathbb{E}_t p^r_{it+1} - \beta\mathbb{E}_t\sum_{j=1}^{N}\sum_{k=1}^{K}\gamma^r_{ji}\pi^r_{ji}(k)\,\Delta e_{ikt+1}$$

$$+\sum_{j=1}^{N}\sum_{k=1}^{K}\gamma^r_{ji}\pi^r_{ji}(k)\,\Delta e_{ikt} + (1-\alpha^r)\,\kappa^r\sum_{j=1}^{N}\gamma^r_{ji}\left(\sum_{r'=1}^{S}\phi^{r'r}_j p^{r'}_{jt} + \left(1-\phi^r_j\right)w_{jt} + e_{ijt}\right).$$

Finally, rewrite the system in terms of deviations from the trend inflation $\hat{p}^r_{it} \equiv p^r_{it} - w_{it}$ and $\hat{e}_{it} \equiv e_{it} - w_{it}$:

$$\begin{aligned}&[1+\beta+\kappa^r(1-\alpha^r)]\hat{p}^r_{it} - \hat{p}^r_{it-1} - \beta\mathbb{E}_t\hat{p}^r_{it+1} - (1-\alpha^r)\,\kappa^r\sum_{j=1}^{N}\gamma^r_{ji}\sum_{r'=1}^{S}\phi^{r'r}_j\hat{p}^{r'}_{jt}\\ &= (1-\alpha^r)\,\kappa^r\left[\hat{e}_{it} - \sum_{j=1}^{N}\gamma^r_{ji}\hat{e}_{jt}\right] + \left[\Delta\hat{e}_{it} - \sum_{j=1}^{N}\sum_{k=1}^{K}\gamma^r_{ji}\pi^r_{ji}(k)\,\Delta\hat{e}_{kt}\right] - (1-\beta)\sum_{j=1}^{N}\sum_{k=1}^{K}\gamma^r_{ji}\pi^r_{ji}(k)\,\mu_k.\end{aligned} \tag{A17}$$

Intuitively, the first three terms on left-hand side are the standard terms of the NKPC representing a sluggish price adjustment. The fourth term is due to the fact that the optimal reset price depends through input-output linkages and pricing-to-market on prices set by other firms. The levels of exchange rates on the right-hand side of the equation convert firms' marginal costs into local currency, while the first differences of exchange rates represent the automatic change in prices that remain sticky in foreign currency. Finally, the last term is a standard source of monetary non-neutrality in models with the New-Keynesian Phillips curve (see e.g. Mankiw and Reis 2002), which is, however, quantitatively small under standard calibration with $\beta \approx 1$ and therefore, can be ignored when solving for the currency choice.

The second-order system of linear dynamic equations (A17) can then be solved using the Blanchard-Kahn method to express current prices in terms of lagged values, exchange rates and innovations to exchange rates:

$$\hat{p}_t = M_1\hat{p}_{t-1} + M_2\hat{e}_t + M_3\Delta\hat{e}_t.$$

Although straightforward to implement, this step can only be done numerically. To compute the sufficient statistic for currency choice, denote with Λ a $NS \times NS$ matrix with $\beta\lambda^r$ on the main diagonal and zero off-diagonal elements. The discounted sum of the pass-through coefficients (in matrix form) can be calculated as follows:

$$(I-\Lambda)\sum_{t=0}^{\infty}\Lambda^t\mathbb{E}\hat{p}_t\Delta\hat{e}_0^T = (I-\Lambda)\sum_{t=0}^{\infty}\Lambda^t\mathbb{E}\sum_{j=0}^{t}M_1^j\big(M_2\hat{e}_{t-j} + M_3\Delta\hat{e}_{t-j}\big)\Delta\hat{e}_0^T$$

$$= (I-\Lambda)\sum_{t=0}^{\infty}\Lambda^t\left[\sum_{j=0}^{t}M_1^jM_2 + M_1^tM_3\right]\Sigma = (I-\Lambda)\sum_{t=0}^{\infty}\Lambda^t\Big[(I-M_1)^{-1}(I-M_1^{t+1})M_2 + M_1^tM_3\Big]\Sigma,$$

$$= (I-M_1)^{-1}M_2\Sigma - (I-\Lambda)\left[\sum_{t=0}^{\infty}\Lambda^t(I-M_1)^{-1}M_1^t\right]M_1M_2\Sigma + (I-\Lambda)\left[\sum_{t=0}^{\infty}\Lambda^tM_1^t\right]M_3\Sigma, \tag{A18}$$

where I is an identity matrix. Note that the infinite sums can be expressed as solutions to the Sylvester equation:

$$X \equiv \sum_{t=0}^{\infty}\Lambda^tM_1^t = I + \Lambda XM_1 \quad\Rightarrow\quad \Lambda^{-1}X - XM_1 = \Lambda^{-1},$$

$$Y \equiv \sum_{t=0}^{\infty}\Lambda^t(I-M_1)^{-1}M_1^t = (I-M_1)^{-1} + \Lambda YM_1 \quad\Rightarrow\quad \Lambda^{-1}Y - YM_1 = \Lambda^{-1}(I-M_1)^{-1}.$$

Numerical algorithm to solve the model includes the following steps:

1. Make initial guess about invoicing shares $\pi^r_{ji}(k)$ for each bilateral trade flow jir and currency k.
2. Solve the system of dynamic equations (A17) using the Blanchard-Kahn method:
   (a) rewrite the equations in a matrix form as a first-order dynamic system,
   (b) factorize the matrix on the left-hand side by finding its eigenvalues and eigenvectors,
   (c) find the cointegration relationship and solve for M_1, M_2 and M_3.
3. Compute discounted sum of pass-through coefficients into aggregate prices using (A18).
4. Calculate discounted sum of pass-through into bilateral desired prices (A16) and the sufficient statistic for currency choice from Proposition 4.
5. Solve for the optimal currency choice making sure that
   (a) domestic flows are in local currency,
   (b) firms can only choose among selected vehicle currencies,
   (c) firms change invoicing only if benefits are higher than fixed costs.
6. Update $\pi^r_{ji}(k)$ and iterate until convergence.

Computationally, the most costly step in this algorithm is factorizing system (A17), which involves finding $N \times S$ eigenvalues. Fortunately, there is no need to do this at every iteration of the algorithm because the left-hand side of system (A17) is independent of firms' currency choice. As a result, one can compute matrices M_1 and M_2 and most covariance terms in (A18) just one time before searching for the fixed point.

B.2 Data and calibration

B.2.1 Trade flows

There are three main sources of global input-output tables that have been extensively used in the previous literature: world input-output database (WIOD), OECD inter-country input-output (ICIO) tables, and the Eora multi-region input-output (MRIO) table. While any of these datasets can be used to calibrate the model, I adopt the trade flows from the ICIO tables in the quantitative analysis. The database covers all bilateral flows between 65 economies (including all OECD, European Union, and most East Asian countries plus the RoW) and 35 two-digit ISIC industries. According to the IMF database of bilateral trade flows in 2015, about 80% of international trade is accounted for by flows between ICIO countries (excluding the RoW), 18% are the flows between ICIO countries and the RoW, and only 2% are the flows between the RoW countries. Out of 21 datasets available from 1995–2015, I use three input-output tables for 1995, 2005 and 2015.

The raw data is adjusted in several ways. First, the tables split both China and Mexico into two economies, which roughly corresponds to services and commodities plus manufacturing. I aggregate these flows to the same sectorial level as the one used for other economies. Second, I drop Kazakhstan and Brunei from the sample and include their trade flows in the RoW. The former economy appears as a separate entity only after 2005, while Brunei is the smallest economy in the sample with no data on invoicing and a fixed exchange rate to Singapore. Third, I drop sector "private households with employed persons", which uses no intermediates and produces goods exclusively for local final consumption. Finally, because of changes in industry classification, the sector codes for 1995 are from ISIC rev.3, while the codes for 2005 and 2015 are from ISIC rev.4. To mitigate differences in industry classifications across years, I aggregate all mining sectors into one sector in tables that use ISIC rev.4 and combine "renting of machinery and equipment" with "R&D and other business activities" in tables with ISIC rev.3. In sum, there are $N = 63$ countries and $S = 32$ industries left (see Tables A1 and A2 for the full list).

Additional adjustment of trade flows is required because the market is defined by exporting sector r and country of destination i, while in the data there is also variation across importing sectors s. Therefore, I aggregate all trade flows from country j and sector r to country i across sectors s and compute the corresponding trade shares of exporters from j in market ir. Similarly, I compute the country-sector is cost shares on intermediates from each sector r. Multiplying these two matrices of trade shares and input shares, I get the adjusted input-output table that is used for the rest of the analysis. Note that by construction, the new matrix perfectly matches the original cross-country trade flows and sectoral input shares, but is different in terms of sector-to-sector flows. While there are substantial differences in some individual trade shares γ_{ji}^{rs} relative to the original table, the aggregate statistics, such as sectoral import intensities, barely change. At the same time, the aggregation brings down the number of markets and price indices from $N \times S^2 = 64,512$ to $N \times S = 2,016$ significantly reducing the computational burden of the model.

Finally, to capture the fact that there are *global* markets for many commodities with prices highly correlated across economies, I also adjust the international trade flows of commodities in such a way that countries' market shares are the same across all destinations. Thus, while the exports and imports of commodities remain the same as in the original input-output table, there is significant change in bilateral flows. As a result, the model has only two markets for commodities corresponding to agriculture and mining.

B.2.2 Exchange rates

The covariance matrix of exchange rates is computed using monthly series of bilateral exchange rates from the IMF IFS database. I use monthly averages, which are more robust to outliers and mistakes than the end-of-period values, and focus on log changes in exchange rates. The sample period is 1980–2015, although the series starts later for some economies (e.g. post-soviet countries). All members of the Eurozone are assumed to be on hard peg to the euro, while Saudi Arabia and Hong Kong are assumed to have a permanent peg to the dollar given almost no variation in their exchange rates against the U.S. Table A2 summarizes which European countries adopted the euro in each subperiod.

Since there is no readily available exchange rate for the RoW, some extrapolations are required to compute its covariances with the exchange rates of other economies. Taking the cross-country averages does not solve the issue as it eliminates the idiosyncratic component and results in highly unrealistic moments. Using more complicated transformations, on the other hand, can result in a covariance matrix that is not positive-semidefinite. Therefore, I take a different route and assume instead that the RoW pegs its nominal exchange rate to a bundle of dollars and euros. Indeed, according to the classification of Ilzetzki, Reinhart, and Rogoff (2019) for 124 countries not included in ICIO table from 1995–2015, about 50% of economies have a fixed exchange rate, 47% implement a "crawling peg" or a "managed float", and only 0.3% allow for a free floating exchange rate with the remaining currencies freely falling. I use the trade-weighted share of countries with the dollar as an anchor currency equal to 95.4% to calibrate the share of the dollar in the bundle targeted by the RoW.

B.2.3 Other parameters

Markups would be irrelevant if the data included labor costs in addition to the spendings on intermediate goods. Instead, the ICIO table reports only the total value added of a sector, which is the sum of labor costs and profits. To make the necessary adjustment, I use the markup estimates based on firms' accounting profits for U.S. publicly listed firms in Compustat from Baqaee and Farhi (2020). Using constructed mapping between NAICS codes and ICIO classification, I aggregate firm-level estimates into sector-level markups via sales-weighted harmonic average and take a simple average across years from 1995–2015. The resulting markups are positive for all sectors and are close to the alternative measures based on user costs. Given the limitations of the data, I extrapolate these estimates to all countries and periods. The sectoral labor costs are calculated as the difference between value added and profits with the latter inferred from total revenues and average markups. Applying this procedure to the ICIO table of 2015, one gets negative labor expenses for 1.8% country-sector pairs, which account for about 0.7% of global GDP and are mostly concentrated in the petroleum industry. The labor inputs are truncated at zero in this case, which implicitly implies negative profits in these country-sector pairs. Relative to the case with no markup adjustment, the sales-weighted average labor share in world production falls from 48% to 39%, while the labor share in manufacturing falls from 29% to 20%.

Price complementarities are calibrated using the recent estimates (for manufacturing industries) from Amiti, Itskhoki, and Konings (2019). I focus on the estimates for large firms, which account for most of international trade, and assume the same value of $\alpha^r = 0.5$ for all manufacturing and service sectors consistent with the fact that there are no systematic differences between differentiated versus homogeneous industries in the data. At the same time, I use a much higher value of $\alpha^r = 0.99$ for commodities in order to capture the price-taking behavior of firms in these sectors.

Price stickiness is known to exhibit large variation across sectors. The Calvo parameter λ^r is calibrated based on the median frequency of price adjustment (including product substitution and sales) from Nakamura and Steinsson (2008). I use the subcategories of PPI from Table 6 for commodities and manufacturing and the subcategories of CPI from Table 2 for services, interpreting "farm products" as agriculture, "crude materials" as mining, "fuel and related products" as petroleum, "miscellaneous products" as other manufacturing, "transportation equipment" as motor vehicles and other transportation equipment, "furniture and household durables" as computers and electronics. The obtained values for the U.S. are then used for all economies. While this is likely to provide a poor approximation to more inflationary economies, which have a higher frequency of price adjustment (Alvarez, Beraja, Gonzalez-Rozada, and Neumeyer 2019), this fact does not alter the key trade-off for currency choice: it is more costly to set prices in currencies that quickly lose their value independently whether prices are adjusted infrequently, resulting in suboptimal markups, or whether prices are adjusted frequently with firms paying higher menu costs.

Fixed costs of currency change are calibrated as follows. The second-order approximation of firm's profits is

$$\sum_{t=0}^{\infty}(\beta\lambda^r)^t\Pi^r_{jit} = \frac{\Pi}{1-\beta\lambda^r} - \frac{\Pi_{pp}}{2}\sum_{t=0}^{\infty}(\beta\lambda^r)^t\big(\tilde{p}^r_{jit} + e_{kit} - \bar{p}^r_{ji}(k)\big)^2,$$

where suppressing market indices, Π is the steady-state value of profits and Π_{pp} is its second derivative:

$$\Pi = (e^p - MC)h(p), \qquad \Pi_{pp} = -(\theta-1)e^p h(p),$$

where $h(p)$ is demand function, $R = e^p h(p)$ are total revenues of an exporter in a given market, and the last expression uses the optimal price-setting condition. Thus, to be compared to the firm's profits from Proposition 4, the menu costs f^r have to be rescaled as follows: $\tau^r = \frac{2}{\theta^r-1}\frac{(1-\beta\lambda^r)^2}{\beta\lambda^r}\frac{f^r}{R^r}$. Nakamura and Steinsson (2010), Golosov and Lucas (2007), Levy, Bergen, Dutta, and Venable (1997) report that the share of menu costs in revenues computed as $\frac{(1-\lambda^r)f^r}{R^r}$ is approximately equal to $0.3-0.7\%$ for sectors with sticky prices. Combining these estimates with λ^r and θ^r from Table A1, we get $\tau^r = 0.3-2\cdot 10^{-4}$. For simplicity, I use the same value of τ^r for all sectors: although the menu costs clearly vary with the probability of price adjustment, expression (17) is scaled in such a way that it is invariant to price stickiness λ^r to the first-order.

Inflation rate matters for currency choice in the model because it generates a positive trend in both nominal wages and exchange rates. On the one hand, inferring this common trend from exchange rates is complicated: the series are highly volatile and are also too short for post-Soviet states in 1995. On the other hand, the data on wage inflation is scarce, while the measures based on CPI inflation suffer from the reverse causality as non-monetary shocks driving exchange rates pass through into import prices with no direct affect on the labor costs of local producers. At the same time, there is high correlation between these two potential measures of nominal trends in the data. Acknowledging these limitations, I calibrate μ_i using harmonic averages of CPI inflation and exchange rate depreciation in each country in a given period. This ensures that countries with hyperinflation, but noisy measures of exchange rate trends have high μ_i, and that countries with prices and exchange rates moving in opposite directions have μ_i close to zero (see Table A2). The three periods used in calibration are 1981–1995, 1991–2005, and 2001–2015. When calculating inflation for countries with a common currency (USD or EUR), I take a GDP-weighted average across economies. The value of inflation in the RoW is irrelevant because it does not have its own currency.

B.2.4 Currency of invoicing

The main source of information regarding currency use in international trade is the recent dataset compiled by the IMF, which provides annual shares of exports and imports invoiced in dollars, euros, and home currencies for more than 100 economies over the period from 1990–2020 (Georgiadis, Le Mezo, Mehl, Casas, Boz, Nguyen, and Gopinath 2020). The panel is not balanced with the coverage relatively sparse for the 1990s and more comprehensive for the 2000s and 2010s. Given these limitations, I adopt the following approach to calculate invoicing shares that maximizes the coverage of countries from the ICIO tables. For each economy in my sample, I first estimate the three-year averages for the periods of 1994-96, 2004-06, and 2014-16. For countries with no invoicing data for these years, I use a larger window of five years, i.e. the averages over 1992-98, 2002-08, and 2012-18.[2] I drop observations for Malaysia before the 2010s because most of invoicing is not classified. This procedure allows me to get invoicing shares for about 18% of global trade in the 1990s, 62% in the 2000s and 63% in the 2010s with the main limitation being the absence of data for China.

Aside from the aggregation issues, one important source of measurement error is the fact that while most of the numbers represent the currency of invoicing, a few countries (Cambodia, Malaysia, South Korea, Bulgaria,

[2]Note that the sum of invoicing shares might be greater than one in the case when the data on different currencies is available for different years, although this happens in only two cases in the sample.

Italy, Russia, Brazil) report, at least for some years, the currency of settlements instead. Although the data suggests that exporters use predominantly the same currency for pricing and payments (Friberg and Wilander 2008), the empirical evidence is scarce. More importantly, the invoicing data is usually collected by customs authorities and covers mostly merchandise trade, while the settlement data comes from central banks and includes payments for both goods and services. I compare empirical numbers to the model-implied invoicing shares for commodities and manufacturing, excluding services, which can result in discrepancies for the countries mentioned above.

To analyze the bilateral and sectoral invoicing I use data from Switzerland in 2015, which was generously shared with me by Philip Saure (Bonadio, Fischer, and Sauré 2020). The dataset includes the value of imports and exports invoiced in Swiss francs, euros, and dollars at the trade partner-product level. To match this data with the classification of the sectoral trade flows, I map HS 2-digit codes to ISIC rev.3 sector codes. For duplicates, I keep the ISIC sector code that is most frequently matched to the HS commodity. Given the coverage of the dataset, only manufacturing sectors are used. For consistency, I compute the currency shares at the trade partner-sector level and then aggregate to the country or sector level using the weights from the ISIC table. The trade shares are somewhat different in the dataset with invoicing, but the results are robust to using different weights.

B.3 Counterfactuals

B.3.1 Counterfactual trade flows

Following the previous international macro literature, the baseline model abstracts from the fundamentals that determine the size of the economies and the bilateral trade flows between them. Instead, the steady-state flows are fully determined by exogenous demand shifters γ^r_{ji}, which are calibrated to match exactly the world input-output table. However, once we are interested in the future of the international price system, these trade shares can no longer be taken as primitives and one needs to forecast future trade flows. For this counterfactual, I follow a standard approach in international trade and use a multisector gravity model, in which country-sector productivities, capital imbalances, and iceberg trade costs are the primitives that determine the equilibrium flows between economies (see Eaton and Kortum 2002, Caliendo and Parro 2015, Allen, Arkolakis, and Takahashi 2020). For consistency, the standard gravity model is extended to allow for pricing-to-market.

Consider the equilibrium system that fully characterizes the flexible-price steady state of the economy. Following the trade literature, all prices are expressed in the same currency, i.e. one can think of monetary policy normalizing nominal exchange rates to one. For simplicity, assume CES demand and strategic complementarities in price setting arising from strategic interactions between firms. The optimal price and the marginal costs of exporter from country j and sector r to country i is given by

$$P^r_{ji} = \left(\vartheta^r_j \tau^r_{ji} MC^r_j\right)^{1-\alpha^r} \left(P^r_i\right)^{\alpha^r}, \qquad MC^r_j = \left(\frac{W_j}{A^r_j}\right)^{1-\phi^r_j} \prod_s \left(P^s_j\right)^{\phi^{sr}_j}$$

where ϑ^r_j is a constant component of the markup and $1-\phi^r_j = 1 - \sum_n \phi^{nr}_j$ is the labor share in production. The bilateral trade shares in the market of destination in terms of total sales and quantities sold are respectively

$$S^r_{ji} = \left(\frac{\left(\tau^r_{ji}\vartheta^r_j MC^r_j\right)^{1-\alpha^r} \left(P^r_i\right)^{\alpha^r}}{P^r_i}\right)^{1-\theta^r} = \left(\tau^r_{ji}\vartheta^r_j \frac{MC^r_j}{P^r_i}\right)^{(1-\theta^r)(1-\alpha^r)} \quad \text{and} \quad s^r_{ij} = \frac{1}{\vartheta^r_i}\left(S^r_{ij}\right)^{\frac{1-\theta^r(1-\alpha^r)}{(1-\theta^r)(1-\alpha^r)}},$$

where the sectoral ideal price index P^r_i can be written as

$$\left(P^r_i\right)^{(1-\theta^r)(1-\alpha^r)} = \sum_n \left(\vartheta^r_n \tau^r_{ni} MC^r_n\right)^{(1-\theta^r)(1-\alpha^r)}.$$

As before the log-linear preferences guarantee that nominal spendings are equal to nominal wages:

$$C_i P_i = W_i, \qquad P_i = \prod_r \left(\frac{P_i^r}{\phi_i^{rc}}\right)^{\phi_i^{rc}}.$$

The general equilibrium block of the model is then summarized by the market clearing condition that determines total costs $T_i^r \equiv \sum_j MC_i^r \tau_{ij}^r Y_{ij}^r$ of sector r in country i

$$T_i^r = \sum_j s_{ij}^r \left(\sum_s \phi_j^{rs} T_j^s + \phi_j^{rc} W_j\right)$$

and the country's budget constraint that allows for trade imbalances D_i:

$$\sum_r \sum_j S_{ji}^r \left(\sum_s \phi_i^{rs} MC_i^s Y_i^s + \phi_i^{rc} W_i\right) = \sum_r \sum_j S_{ij}^r \left(\sum_s \phi_j^{rs} MC_j^s Y_j^s + \phi_j^{rc} W_j\right) + D_i.$$

Following Dekle, Eaton, and Kortum (2007), rewrite the equilibrium system using the hat algebra: for arbitrary variable Z, denote the change between the counterfactual value Z' and the original value Z with $\hat{Z} \equiv \frac{Z'}{Z}$:

1. Marginal costs:
$$\hat{MC}_j^r = \left(\frac{\hat{W}_j}{\hat{A}_j^r}\right)^{\phi_j^r} \prod_n \left(\hat{P}_r^n\right)^{\phi_j^{nr}}$$

2. Prices:
$$\left(\hat{P}_i^r\right)^{(1-\theta^r)(1-\alpha^r)} = \sum_n S_{ni}^r \left(\hat{\tau}_{ni}^r \hat{MC}_n^r\right)^{(1-\theta^r)(1-\alpha^r)}$$

3. Market shares:
$$\hat{S}_{ji}^r = \left(\hat{\tau}_{ji}^r \frac{\hat{MC}_j^r}{\hat{P}_i^r}\right)^{(1-\theta^r)(1-\alpha^r)}, \qquad \hat{s}_{ji}^r = \left(\frac{\hat{\tau}_{ji}^r \hat{MC}_j^r}{\hat{P}_i^r}\right)^{1-\theta^r(1-\alpha^r)}$$

4. Market clearing:
$$\hat{T}_i^r = \sum_j \hat{s}_{ij}^r \left(\sum_s \omega_{ij}^{rs} \hat{T}_j^s + \omega_{ij}^{rc} \hat{W}_j\right),$$
where $\omega_{ij}^{rs} \equiv \frac{s_{ij}^r \phi_j^{rs} T_j^s}{T_i^r}$ and $\omega_{ij}^{rc} \equiv \frac{s_{ij}^r \phi_j^{rc} W_j}{T_i^r}$ are the initial export shares.

5. Budget constraint:
$$\sum_r \sum_j \hat{S}_{ji}^r \left(\sum_s \varsigma_{ji}^{rs} \hat{T}_i^s + \varsigma_{ji}^{rc} \hat{W}_i\right) = \varrho_i \sum_r \sum_j \hat{S}_{ij}^r \left(\sum_s \zeta_{ij}^{rs} \hat{T}_j^s + \zeta_{ij}^{rc} \hat{W}_j\right) + (1-\varrho_i)\hat{D}_i,$$
where $\varsigma_{ji}^{rs} \equiv \frac{S_{ji}^r \phi_i^{rs} T_i^s}{\sum_r \sum_j S_{ji}^r \left(\sum_s \phi_i^{rs} T_i^s + \phi_i^{rc} W_i\right)}$ is the initial share of exports from j to i in sector r in gross purchases of economy i, $\varsigma_{ji}^{rc} \equiv \frac{S_{ji}^r \phi_i^{rc} W_i}{\sum_r \sum_j S_{ji}^r \left(\sum_s \phi_i^{rs} T_i^s + \phi_i^{rc} W_i\right)}$ is a similar share of exports from j to i for final consumption,

$\zeta_{ij}^{rs} \equiv \frac{S_{ij}^r \phi_j^{rs} T_j^s}{\sum\limits_r \sum\limits_j S_{ij}^r \left(\sum\limits_s \phi_j^{rs} T_j^s + \phi_j^{rc} W_j \right)}$ and $\zeta_{ij}^{rc} \equiv \frac{S_{ij}^r \phi_j^{rc} W_j}{\sum\limits_r \sum\limits_j S_{ij}^r \left(\sum\limits_s \phi_j^{rs} T_j^s + \phi_j^{rc} W_j \right)}$ are the corresponding shares of flows from i to j in gross income of country i, and $\varrho_i \equiv \frac{\sum\limits_r \sum\limits_j S_{ij}^r \left(\sum\limits_s \phi_j^{rs} T_j^s + \phi_j^{rc} W_j \right)}{\sum\limits_r \sum\limits_j S_{ji}^r \left(\sum\limits_s \phi_i^{rs} T_j^s + \phi_i^{rc} W_i \right)}$ is the ratio of gross income to gross spendings.

Thus, given the elasticities of substitution θ^r, complementarities in price setting α^r, input shares ϕ_i^{rs}, initial trade shares S_{ji}^r, and gross changes in productivities $\hat{A}_i^r$, trade costs $\hat{\tau}_{ji}^r$ and trade imbalances $\hat{D}_i$, this system allows to solve for new equilibrium prices and quantities $\hat{MC}_i^r, \hat{P}_i^r, \hat{S}_{ji}^r, \hat{T}_i^r, \hat{W}_i$ without calculating original trade costs or productivities.

Instead, I treat changes in productivities as endogenous and calibrate the model to the predicted changes in real GDP. Abstracting from the differences between GDP and gross national income (GNI), which are small and hardly affect the results, these changes correspond to $\hat{C}_i$. Given little availability of sectoral forecasts, I assume the same productivity growth rates within each country $\hat{A}_i^r = \hat{A}_i$ and compute the fixed point using the following algorithm:

- make an initial guess for changes in endogenous variables (e.g. $\hat{Z} = 1$),
- given $\left\{ \frac{\hat{W}_i}{\hat{A}_i} \right\}$, solve the first three equations for prices and trade shares,
- given $\{\hat{P}_i\}$, compute nominal wages as $\hat{W}_i = \hat{P}_i \hat{C}_i$ and update productivities $\hat{A}_i = \frac{\hat{W}_i}{\hat{W}_i / \hat{A}_i}$,
- given $\left\{ \hat{W}_i, \hat{s}_{ij}^r \right\}$, solve the market clearing conditions for $\left\{ \hat{T}_j^r \right\}$,
- given $\left\{ \hat{T}_i^r, \hat{S}_{ij}^r, \hat{A}_i \right\}$, solve the budget constraints for $\left\{ \frac{\hat{W}_i}{\hat{A}_i} \right\}$ and iterate until convergence.

Although the implied changes in productivities are highly correlated with GDP growth rates used for calibration, the two numbers do not coincide due to international spillovers, which are especially pronounced for more open economies and the countries with growth rates substantially different from the average growth of global economy.

B.3.2 Calibration

The baseline model predicts that the counterfactual currency choice depends on changes in trade flows and exchange rate behavior. Estimating the new input-output table and the covariance matrix of exchange rates requires the calibration of additional parameters. First, from the gravity model it follows that changes in trade shares depends crucially on the elasticity of substitution between products. I borrow the sectoral estimates from Caliendo and Parro (2015) (see Table A1). Second, one needs to calibrate exogenous shocks $\{\hat{A}_i^r, \hat{\tau}_{ji}^r, \hat{D}_i\}$ transforming the structure of global trade. For simplicity, I assume no changes in current account imbalances $\hat{D}_i = 1$ or trade costs $\hat{\tau}_{ji}^r = 1$ and focus instead on long-run economic growth $\hat{A}_i^r$. Because of data limitations, the growth rates are assumed to be the same across all sectors within each country. The changes in real consumption $\{\hat{C}_i\}$ are calibrated to match the growth rates of real GDP between 2015–2025 using the actual values from 2016–2019 and the forecasts from the IMF World Economic Outlook for the rest of the years. The global averages are used for the RoW. The mean annual growth rates reported in Table A2 are then applied to the whole forecast horizon of 20 years. Note that input shares ϕ_i^r, elasticities θ^r, complementarities in price-setting α^r, and price stickiness λ^r are assumed to stay the same across years.

A switch of China to a floating regime is modelled by assuming that China has the same exchange rate volatility as the dollar in 2001–2015 and a zero correlation with other exchange rates once they are expressed against an unweighted bundle of floating currencies (Australia, Canada, Germany, Japan, New Zealand, Norway, South Africa, Sweden, Switzerland, U.K., U.S.).

B.4 Additional figures and tables

Table A1: Sectors: classification and parameter values

Sector	ISIC codes rev.3	ISIC codes rev.4	Markup	$1-\lambda^s$	α^s	θ^s
Commodities				.		
Agriculture and forestry	C01T05AGR	D01T03	1.14	0.88	0.99	8.1
Mining	C10T14MIN	D05T06, D07T08, D09	1.17	0.99		15.7
Manufacturing						
Food, beverages and tobacco	C15T16FOD	D10T12	1.15	0.27		2.6
Textiles and wearing apparel	C17T19TEX	D13T15	1.07	0.04		5.6
Wood	C20WOD	D16	1.10	0.04		10.8
Paper and printing	C21T22PAP	D17T18	1.09	0.09		9.1
Coke and refined petroleum	C23PET	D19	1.12	0.49		51.1
Chemicals and pharmaceuticals	C24CHM	D20T21	1.25	0.11		4.8
Rubber and plastic	C25RBP	D22	1.12	0.04		1.7
Other non-metallic products	C26NMM	D23	1.09	0.06	0.50	2.8
Basic metals	C27MET	D24	1.10	0.05		8.0
Fabricated metal products	C28FBM	D25	1.12	0.05		8.0
Computer and electronics	C30T33XCEQ	D26	1.11	0.06		10.6
Electrical equipment	C31ELQ	D27	1.07	0.06		10.6
Machinery and equipment	C29MEQ	D28	1.12	0.05		1.5
Motor vehicles	C34MTR	D29	1.06	0.45		0.4
Other transport equipment	C35TRQ	D30	1.12	0.45		0.4
Other manufacturing	C36T37OTM	D31T33	1.13	0.17		5.0
Services						
Utilities	C40T41EGW	D35T39	1.18			
Construction	C45CON	D41T43	1.08			
Wholesale and retail	C50T52WRT	D45T47	1.05			
Transportation and storage	C60T63TRN	D49T53	1.10			
Accomodation and food services	C55HTR	D55T56	1.12			
Post and telecommunications	C64PTL	D58T60, D61	1.18			
IT services	C72ITS	D62T63	1.12	0.07	0.50	5.0
Finance and insurance	C65T67FIN	D64T66	1.50			
Real estate	C70REA	D68	2.68			
Other services	C71RMQ, C73T74OBZ	D69T82	1.13			
Public administration	C75GOV	D84	1.11			
Education	C80EDU	D85	1.15			
Health	C85HTH	D86T88	1.10			
Arts and entertainment	C90T93OTS	D90T96	1.10			

Note: the table shows the sector classification used in the analysis, the mapping into ISIC codes from ICIO tables, the sectoral values of markups, monthly probability of price adjustment $1-\lambda^s$, degree of complementarities in price setting α^s, and the elasticity of substitution between individual products θ^s.

Table A2: Country characteristics

Country	Inflation rate			Eurozone		Growth rate
	1995	2005	2015	2005	2015	
Australia	4.0	1.0	0.7			1.9
Austria	-0.1	1.2	0.7	✓	✓	1.4
Belgium	0.9	1.2	0.7	✓	✓	1.0
Canada	2.1	0.6	0.9			1.5
Chile	16.3	4.6	2.6			1.7
Czech Republic	6.0	2.7	0.0			2.4
Denmark	1.3	0.9	0.7			1.6
Estonia	23.2	8.5	0.7		✓	2.8
Finland	2.3	1.2	0.7	✓	✓	1.4
France	2.0	1.2	0.7	✓	✓	1.1
Germany	-0.3	1.2	0.7	✓	✓	1.1
Greece	13.3	1.2	0.7	✓	✓	1.2
Hungary	12.1	11.1	2.6			2.7
Iceland	19.4	1.8	4.3			2.1
Ireland	2.9	1.2	0.7	✓	✓	4.3
Israel	59.5	5.8	1.0			3.0
Italy	5.1	1.2	0.7	✓	✓	0.4
Japan	-2.1	-0.6	0.4			0.5
Korea	3.0	3.0	1.4			2.3
Latvia	94.1	35.6	0.7		✓	2.8
Lithuania	130.2	45.4	0.7		✓	2.8
Luxembourg	1.0	1.2	0.7	✓	✓	2.1
Mexico	44.6	10.9	4.3			1.0
Netherlands	-0.4	1.2	0.7	✓	✓	1.5
New Zealand	4.5	0.0	0.0			2.0
Norway	2.9	1.1	1.2			1.5
Poland	62.5	13.5	1.3			3.2
Portugal	9.7	1.2	0.7	✓	✓	1.0
Slovak Republic	8.7	4.8	0.7	✓	✓	2.6
Slovenia	88.2	19.3	0.7	✓	✓	2.4
Spain	4.7	1.2	0.7	✓	✓	1.5
Sweden	3.9	1.8	0.5			1.6
Switzerland	-0.6	0.5	-1.2			1.2
Turkey	49.2	49.9	12.5			2.9
United Kingdom	3.5	1.2	1.3			1.0
United States	1.2	0.9	1.3			1.7
Argentina	266.6	12.2	18.2			-0.3
Brazil	377.4	180.4	5.9			0.6
Bulgaria	69.1	79.3	1.9			2.7
Cambodia	17.3	8.1	2.8			6.1
China	11.2	3.7	0.6			5.9
Colombia	21.3	11.9	3.9			2.0
Costa Rica	20.6	11.9	6.1			2.2
Croatia	239.7	83.8	1.0			2.3

Table A2: Country characteristics (continued)

Country	Inflation rate			Eurozone		Growth rate
	1995	2005	2015	2005	2015	
Cyprus	2.5	1.7	0.7		✓	2.6
India	9.0	6.3	4.9			5.3
Indonesia	8.1	11.3	5.3			4.5
Hong Kong	1.2	0.9	1.3			1.5
Malaysia	1.5	2.2	1.8			4.2
Malta	0.6	1.6	0.7		✓	3.7
Morocco	5.0	1.6	0.8			2.2
Peru	341.1	25.4	1.5			2.2
Philippines	10.2	5.3	2.0			5.0
Romania	90.7	64.5	6.3			3.4
Russia	125.7	62.5	8.9			1.1
Saudi Arabia	1.2	0.9	1.3			1.2
Singapore	-0.8	0.1	0.5			1.9
South Africa	11.4	6.4	5.1			0.5
Taiwan	-0.4	1.2	0.8			1.7
Thailand	2.3	3.1	0.9			2.7
Tunisia	5.4	3.1	3.3			1.5
Vietnam	90.0	3.6	5.5			6.4
RoW	-	-	-	-	-	2.6

Note: the table shows the estimated rates of inflation across countries and years, the members of the Eurozone in a given period, and the productivity growth rates used in the counterfactual analysis.

Table A3: Import share and exchange rate pass-through

Country	Import share (%)				Pass-through (%)			
	total	manufacturing	weighted	w/o pegs	home	EUR	USD	RMB
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Australia	10	45	57	57	25	10	19	17
Austria	23	48	55	26	58	–	11	7
Belgium	27	49	55	28	50	–	15	7
Canada	18	43	51	51	26	7	34	13
Chile	16	43	58	58	22	10	21	19
Czech Republic	28	51	55	55	20	32	12	10
Denmark	23	50	57	57	25	24	12	8
Estonia	30	62	71	42	42	–	13	12
Finland	18	36	45	26	52	–	12	8
France	16	45	58	35	52	–	15	9
Germany	17	32	38	26	57	–	12	8
Greece	18	42	60	44	44	–	14	10
Hungary	37	62	68	68	12	40	12	9
Iceland	22	55	72	72	15	21	19	10
Ireland	44	44	63	38	46	–	19	6
Israel	16	34	47	47	30	17	17	10
Italy	14	28	35	18	60	–	12	7
Japan	9	15	25	25	56	5	11	13
Korea	17	23	25	25	38	7	17	15
Latvia	21	56	68	34	48	–	14	8
Lithuania	31	52	60	32	48	–	15	7
Luxembourg	52	54	62	42	52	–	20	5
Mexico	21	43	56	56	23	8	30	16
Netherlands	22	37	44	25	52	–	16	8
New Zealand	13	38	51	51	29	10	17	15
Norway	18	45	56	56	25	18	13	10
Poland	20	41	55	55	24	29	12	11
Portugal	21	47	59	16	61	–	12	7
Slovak Republic	32	62	67	46	41	–	14	11
Slovenia	28	58	62	31	52	–	14	9
Spain	15	32	42	24	56	–	13	9
Sweden	19	42	49	49	32	26	11	7
Switzerland	22	48	58	58	25	34	13	8
Turkey	13	30	43	43	36	17	15	10
United Kingdom	15	45	52	52	32	24	14	10
United States	8	25	33	32	59	7	–	12
Argentina	7	13	28	28	49	6	14	11
Brazil	8	16	23	23	48	9	16	10
Bulgaria	27	52	62	62	14	31	15	8
Cambodia	28	46	58	58	23	5	16	25
China	7	9	15	15	51	6	17	–
Colombia	13	34	51	51	30	10	23	14
Costa Rica	18	48	67	67	21	8	34	12
Croatia	20	45	55	55	25	35	13	7
Cyprus	29	65	75	35	43	–	17	10
India	12	18	27	27	34	7	21	14
Indonesia	11	20	31	31	36	6	18	14
Hong Kong	24	35	57	48	44	7	–	18

Table A3: Import share and exchange rate pass-through (continued)

Country	Import share (%)				Pass-through (%)			
	total	manufacturing	weighted	w/o pegs	home	EUR	USD	RMB
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Malaysia	24	36	45	45	16	8	22	18
Malta	46	75	81	40	45	–	16	7
Morocco	25	50	64	64	16	29	19	11
Peru	13	29	46	46	30	8	20	16
Philippines	17	28	48	48	28	8	18	14
Romania	18	37	50	50	29	32	10	7
Russia	11	23	35	35	38	14	15	13
Saudi Arabia	25	53	67	53	42	14	–	13
Singapore	32	39	54	54	16	9	27	12
South Africa	15	36	50	50	21	14	21	18
Taiwan	23	30	34	34	30	7	20	14
Thailand	25	36	55	55	18	8	19	19
Tunisia	24	45	54	54	22	29	16	10
Vietnam	26	40	54	54	11	7	19	24
RoW	14	31	38	38	–	14	46	13
Mean	15	31	39	35	44	11	21	12

Note: columns 1 and 2 show the import-to-sales ratio respectively at the aggregate level and exclusively for markets of manufacturing goods. The remaining columns focus on statistics computed at market level and then aggregated across manufacturing goods using imports as weights. Column 4 excludes imports from economies with a hard peg to the currency of the home country. The remaining columns report the pass-through of respectively home, euro, dollar and renminbi exchange rates into markets of manufacturing goods under flexible prices taking into account the whole global input-output production structure. The last row shows the import-weighted average across countries with pass-throughs averaged only across countries that are not issuers of the respective currency.

Table A4: Foreign inputs and exchange rates in exports

Country	Share of foreign inputs (%)			Pass-through (%)			
	total	manufacturing	w/o pegs	home	EUR	USD	RMB
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Australia	23	41	41	47	5	19	10
Austria	23	30	14	73	—	8	4
Belgium	28	39	23	60	—	15	6
Canada	27	39	39	48	4	25	8
Chile	21	33	33	54	5	15	10
Czech Republic	32	37	37	43	23	9	6
Denmark	27	26	26	63	11	7	4
Estonia	34	45	29	58	—	11	8
Finland	23	29	20	65	—	11	6
France	20	29	17	72	—	10	4
Germany	17	21	13	76	—	7	4
Greece	24	44	38	53	—	19	8
Hungary	41	49	49	35	29	10	6
Iceland	31	42	42	44	9	16	9
Ireland	38	30	21	67	—	14	2
Israel	16	24	24	60	9	12	5
Italy	16	21	12	74	—	9	4
Japan	8	10	10	78	2	7	5
Korea	21	23	23	53	5	14	10
Latvia	23	34	23	64	—	10	6
Lithuania	31	44	35	57	—	15	7
Luxembourg	55	40	21	57	—	15	2
Mexico	29	36	36	51	5	20	10
Netherlands	24	35	22	61	—	15	6
New Zealand	25	38	38	50	5	15	11
Norway	21	30	30	55	9	10	6
Poland	23	31	31	52	17	9	6
Portugal	26	36	16	71	—	10	5
Slovak Republic	38	45	31	56	—	11	7
Slovenia	28	35	17	71	—	9	5
Spain	19	27	17	68	—	11	6
Sweden	19	26	26	62	12	8	4
Switzerland	21	28	28	58	18	9	4
Turkey	16	22	22	64	7	11	6
United Kingdom	14	25	25	64	12	9	4
United States	11	19	15	79	3	—	5
Argentina	22	32	32	56	4	15	9
Brazil	16	24	24	59	5	14	8
Bulgaria	32	45	45	37	16	16	7
Cambodia	28	33	33	57	3	9	14
China	12	13	13	62	4	15	—
Colombia	18	29	29	57	4	17	7
Costa Rica	17	27	27	62	4	17	6
Croatia	21	31	31	55	17	10	5
Cyprus	26	38	25	61	—	12	6
India	19	28	28	48	5	19	10
Indonesia	23	30	30	53	4	15	10
Hong Kong	22	39	33	61	5	—	11

Table A4: Foreign inputs and exchange rates in exports (continued)

Country	Share of foreign inputs (%)			Pass-through (%)			
	total	manufacturing	w/o pegs	home	EUR	USD	RMB
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Malaysia	36	43	43	30	7	20	14
Malta	55	51	28	57	–	13	4
Morocco	31	41	41	48	14	14	8
Peru	24	39	39	49	4	19	10
Philippines	21	30	30	57	4	12	8
Romania	19	25	25	59	17	7	4
Russia	18	29	29	54	6	18	8
Saudi Arabia	11	35	21	72	5	–	7
Singapore	31	39	39	34	7	23	6
South Africa	29	42	42	39	7	22	11
Taiwan	21	25	25	51	4	15	9
Thailand	31	40	40	40	5	16	13
Tunisia	32	37	37	50	16	12	7
Vietnam	41	45	45	28	6	18	19
RoW	24	30	30	–	9	61	10
Mean	19	24	21	62	6	15	7

Note: columns 1 and 2 show the share of foreign inputs in country's exports computed respectively at the aggregate level and exclusively for manufacturing exports. The remaining columns focus on statistics computed at sectoral level and then aggregated across manufacturing sectors using exports as weights. Column 3 excludes imports and exports to economies with a hard peg to the currency of the home country. The remaining columns report the pass-through of respectively home, euro, dollar and renminbi exchange rates into exports of manufacturing goods under flexible prices taking into account the whole global input-output production structure. The last row shows the export-weighted average across countries with pass-throughs averaged only across countries that are not issuers of the respective currency.

Table A5: Simulated currency shares by country

Country	Imports			Exports		
	LCP	DCP	ECP	PCP	DCP	ECP
Australia	25.6	48.2	9.2	5.8	83.2	1.0
Austria	81.9	17.4	81.9	78.6	19.9	78.6
Belgium	71.9	25.6	71.9	78.9	17.9	78.9
Canada	19.6	74.8	2.8	5.6	91.7	0.9
Chile	10.8	63.8	11.5	5.5	83.4	3.0
Czech Republic	20.3	7.8	68.5	17.7	4.4	72.1
Denmark	47.0	21.6	23.4	41.3	38.7	15.4
Estonia	72.2	22.2	72.2	73.5	23.0	73.5
Finland	63.3	29.5	63.3	62.6	27.9	62.6
France	65.2	32.3	65.2	66.4	29.7	66.4
Germany	77.2	22.4	77.2	76.2	22.0	76.2
Greece	47.0	38.6	47.0	45.4	44.8	45.4
Hungary	5.3	8.2	71.4	10.3	5.7	70.1
Iceland	7.0	34.7	34.2	1.2	27.2	44.5
Ireland	63.6	30.5	63.6	47.1	43.5	47.1
Israel	14.9	45.9	28.2	16.1	56.7	15.1
Italy	58.7	37.0	58.7	60.4	33.0	60.4
Japan	25.6	61.8	7.2	30.9	56.1	5.0
Korea	10.1	64.5	8.6	18.2	64.6	3.6
Latvia	68.6	21.3	68.6	42.4	33.0	42.4
Lithuania	51.7	38.9	51.7	44.5	32.3	44.5
Luxembourg	74.5	24.2	74.5	79.9	17.9	79.9
Mexico	2.4	74.4	9.4	2.3	86.1	2.4
Netherlands	62.5	36.1	62.5	69.4	27.6	69.4
New Zealand	22.3	44.4	11.2	6.9	61.5	4.0
Norway	46.3	25.4	21.7	13.0	77.2	6.0
Poland	9.1	28.8	47.2	10.4	20.3	47.8
Portugal	70.3	26.3	70.3	63.4	29.8	63.4
Slovak Republic	60.8	27.3	60.8	76.9	15.5	76.9
Slovenia	91.6	6.0	91.6	93.6	3.7	93.6
Spain	56.2	38.2	56.2	64.1	28.2	64.1
Sweden	17.4	23.6	42.4	19.7	28.2	29.3
Switzerland	11.2	13.5	62.0	16.4	27.6	46.1
Turkey	0.0	48.8	31.0	0.0	42.7	35.3
United Kingdom	21.4	35.3	35.5	21.4	44.1	24.0
United States	99.6	99.6	0.4	99.7	99.7	0.3
Argentina	0.0	78.6	0.5	0.0	79.9	0.2
Brazil	6.0	35.3	16.4	4.4	67.3	6.9
Bulgaria	0.0	31.6	63.9	0.0	34.3	60.8
Cambodia	0.0	91.2	0.0	0.1	91.7	0.3
China	1.8	84.8	7.9	2.8	88.4	5.3
Colombia	11.8	63.1	12.9	5.1	85.8	1.9
Costa Rica	0.0	99.8	0.2	0.0	99.9	0.1
Croatia	12.6	14.8	67.3	13.4	22.9	49.4
Cyprus	74.8	22.0	74.8	53.5	43.9	53.5
India	3.1	74.5	8.3	5.9	64.8	11.7
Indonesia	1.4	50.6	7.0	1.3	65.0	5.8
Hong Kong	100.0	100.0	0.0	99.0	99.0	1.0
Malaysia	56.8	20.2	5.0	52.8	31.9	3.7

Table A5: Simulated currency shares by country (continued)

Country	Imports			Exports		
	LCP	DCP	ECP	PCP	DCP	ECP
Malta	74.8	16.7	74.8	54.1	45.2	54.1
Morocco	15.5	37.6	43.4	16.8	40.7	38.2
Peru	21.1	66.0	7.5	8.1	82.6	3.8
Philippines	9.9	55.9	6.9	7.6	70.8	6.4
Romania	3.6	14.8	58.5	2.4	25.0	51.9
Russia	1.6	54.9	25.1	0.0	75.2	14.1
Saudi Arabia	99.4	99.4	0.6	100.0	100.0	0.0
Singapore	40.1	56.6	2.2	56.6	40.5	2.9
South Africa	2.9	57.6	19.0	0.8	78.5	9.4
Taiwan	29.2	62.0	4.4	25.5	69.8	3.2
Thailand	14.1	53.7	6.1	19.2	52.2	5.2
Tunisia	11.3	33.4	51.0	2.8	33.0	61.4
Vietnam	0.0	83.2	4.6	0.0	73.8	7.9
RoW	0.0	98.4	1.5	0.0	99.3	0.6

Note: the table shows for each country the model-implied share (in %) of home currency, DCP and ECP in imports and exports. The shares do not have to sum up to one as the categories are not mutually exclusive and there are other categories of invoicing.

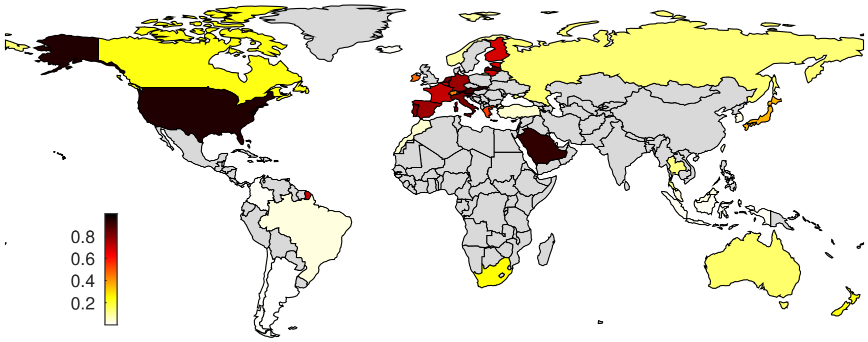
Table A6: Currency shares in world trade by sector

	Trade share	PCP	LCP	VCP	DCP	ECP
Agriculture and forestry	3.3	13	12	75	100	0
Mining	11.4	12	13	77	100	0
Food, beverages and tobacco	6.4	31	49	32	55	25
Textiles and wearing apparel	5.5	15	43	47	73	19
Wood	1.0	25	42	42	63	25
Paper and printing	1.6	48	43	23	52	32
Coke and refined petroleum	4.0	35	43	29	62	16
Chemicals and pharmaceuticals	10.7	46	38	27	53	30
Rubber and plastic	3.0	39	43	30	51	34
Other non-metallic products	1.5	29	39	42	63	25
Basic metals	6.1	30	42	39	59	24
Fabricated metal products	3.2	39	42	30	58	30
Computer and electronics	11.8	25	31	47	69	15
Electrical equipment	4.9	28	37	42	62	25
Machinery and equipment	8.1	41	35	33	56	31
Motor vehicles	9.7	47	52	13	49	35
Other transport equipment	4.1	59	32	19	59	23
Other manufacturing	3.7	27	52	27	56	23
Utilities		43	49	29	40	47
Construction		47	54	18	28	50
Wholesale and retail		46	41	23	52	30
Transportation and storage		39	39	29	56	24
Accomodation and food services		45	43	23	58	26
Post and telecommunications		57	35	17	61	26
IT services		48	57	12	39	44
Finance and insurance		60	46	11	46	40
Real estate		46	44	20	57	26
Other services		56	48	11	48	37
Public administration		39	57	21	38	42
Education		62	37	14	61	28
Health		48	46	19	52	31
Arts and entertainment		49	39	21	53	27

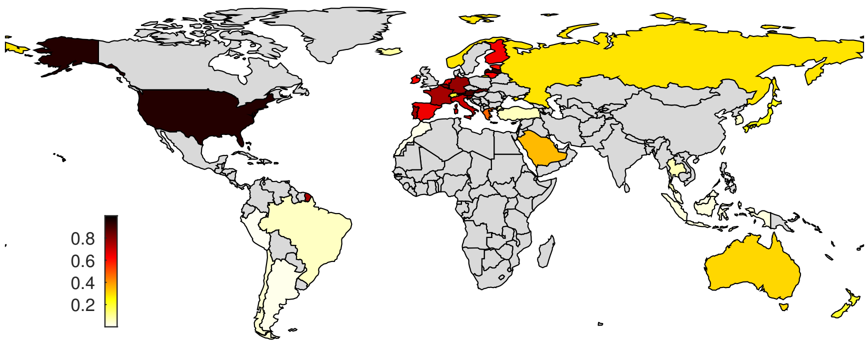
Note: the table shows for each sector the model-implied share of international trade (in %) invoiced in a given currency. The shares do not have to sum up to one as the categories are not mutually exclusive. 'Trade share' is the share of a given sector in world merchandise trade (excluding services).

Figure A3: Invoicing patterns across countries

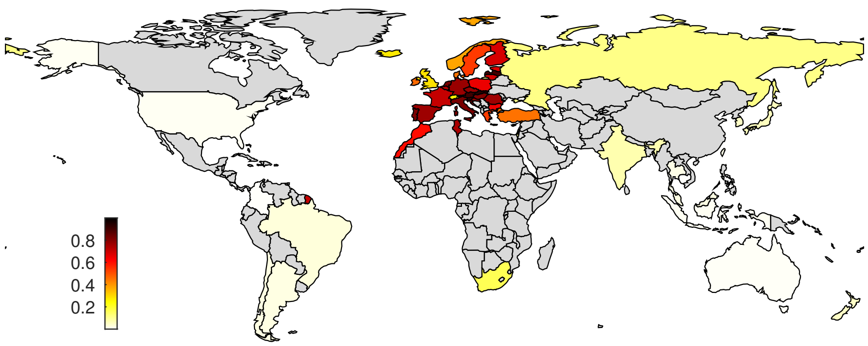
(a) PCP in exports



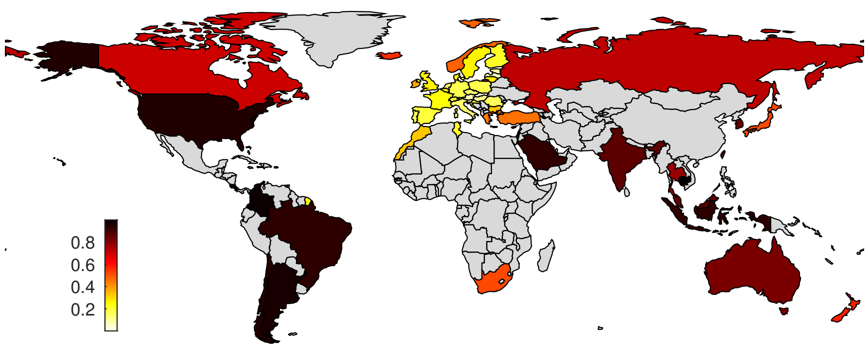
(b) LCP in imports



(c) ECP in exports



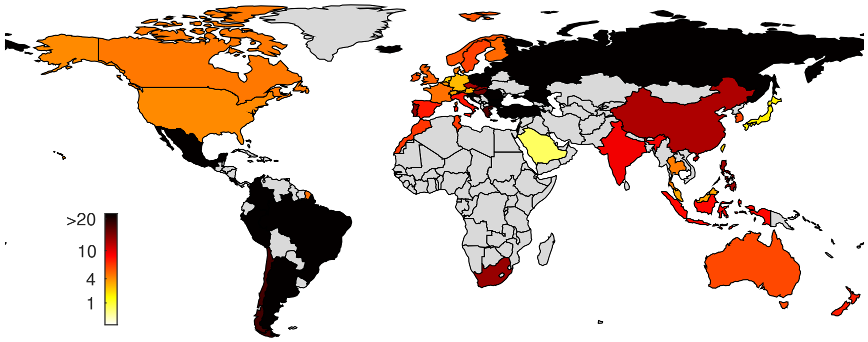
(d) DCP in exports



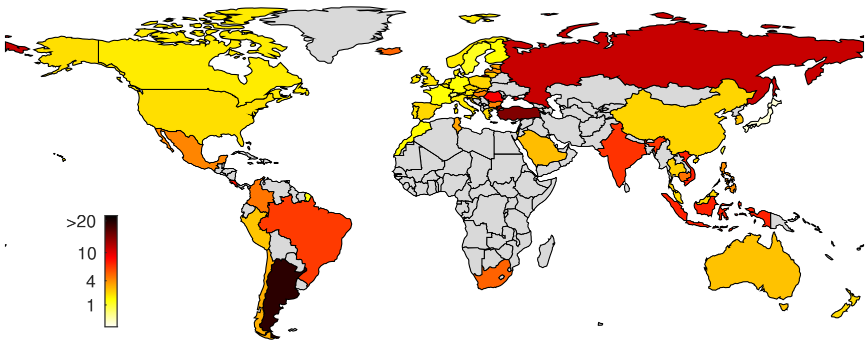
Note: the figure visualises the cross-country invoicing data from Georgiadis, Le Mezo, Mehl, Casas, Boz, Nguyen, and Gopinath (2020). By default, the currency shares are for 2010s, but are simple averages across all years when the data is not available for 2010s.

Figure A4: Inflation rates μ_i

(a) 1995



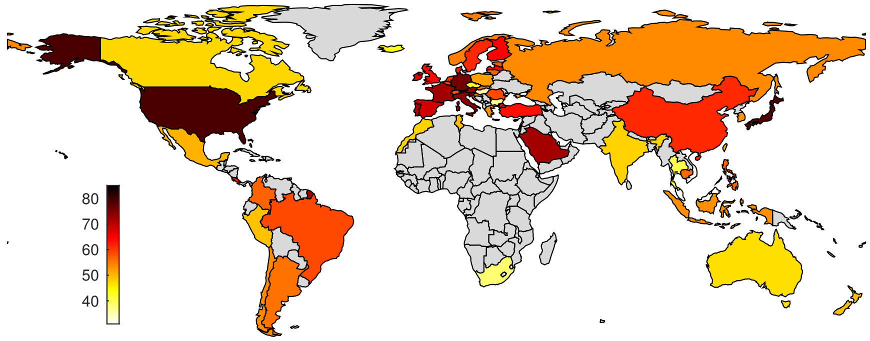
(b) 2015



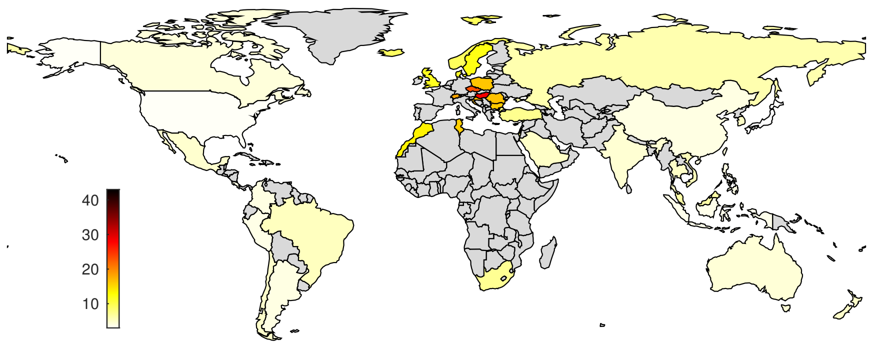
Note: the figure shows a heatmap of average annual inflation rates (%) for countries in 1981–1995 and 2001–2015. The inflation rate is defined as an average of the CPI inflation and the trend in exchange rates.

Figure A5: Pass-through into export prices

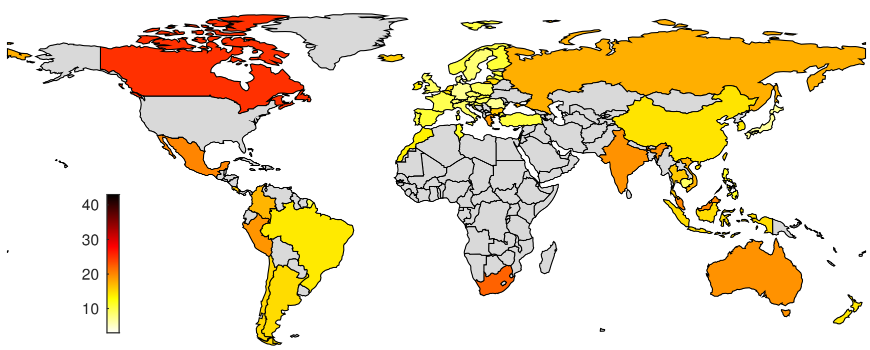
(a) Home currency



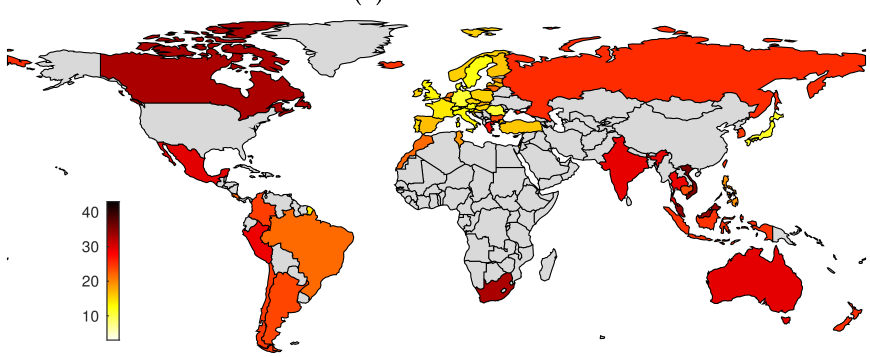
(b) EUR



(c) USD

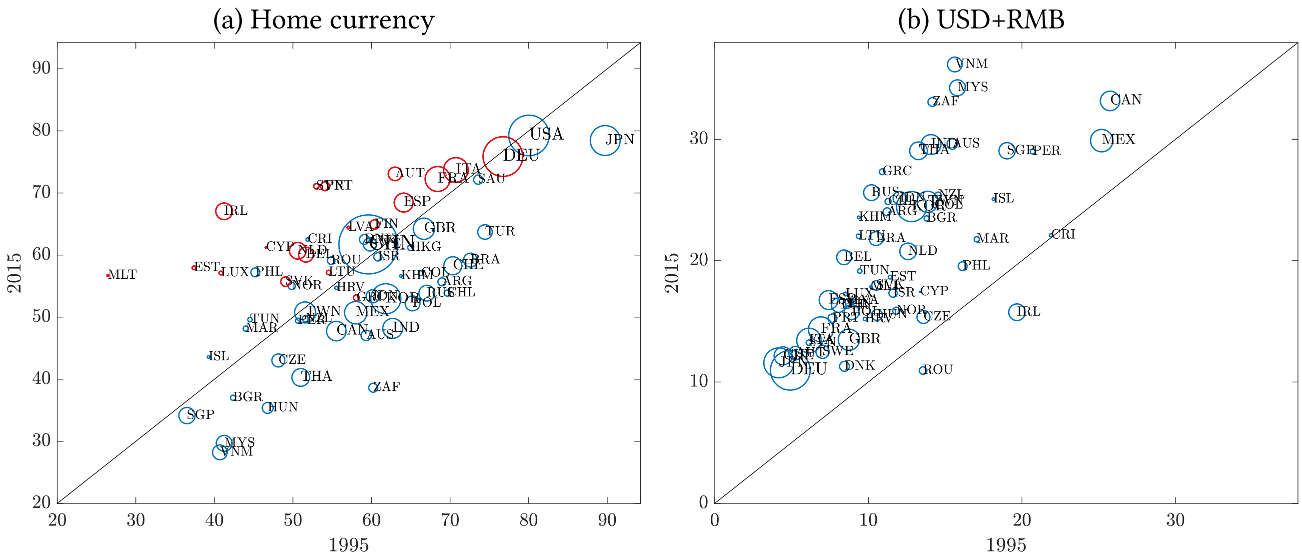


(d) USD+RMB



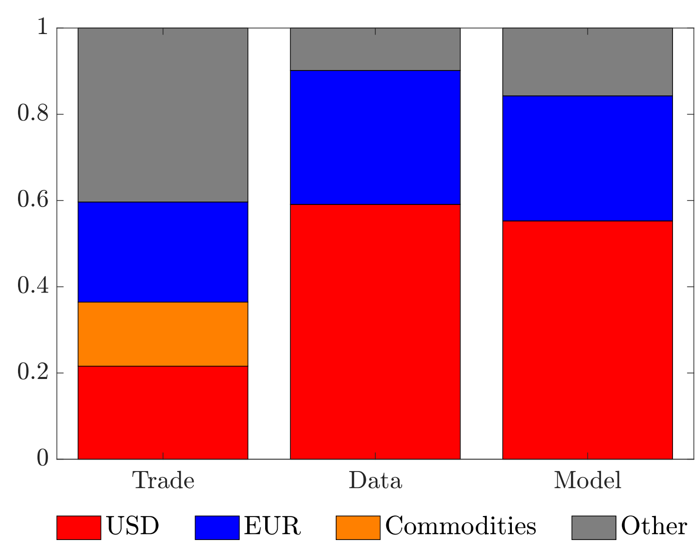
Note: the figure shows the pass-through (%) of home currency, euro, dollar and a combination of dollar and renminbi exchange rates into marginal costs of exporters. All values are computed taking into account the global input-output linkages, are aggregated across sectors using exports as weights, and include only manufacturing sectors.

Figure A6: Pass-through into export prices: 1995 vs. 2015



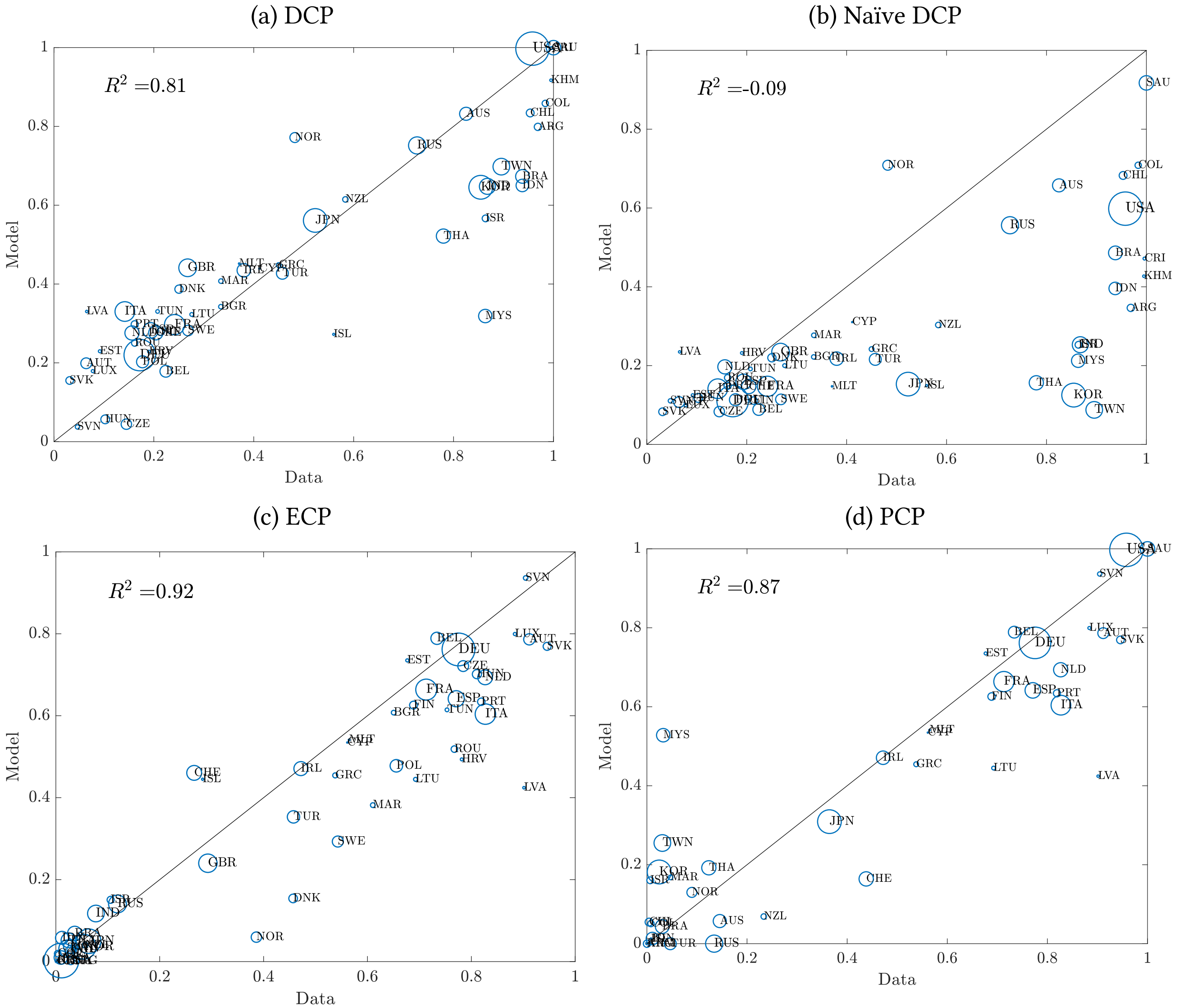
Note: the figure compares the pass-through of exchange rates from Figure A5 with the counterparts estimated for 1995. The size of each circle represents country's share in world exports. The members of the Eurozone are shown in red in panel (a).

Figure A7: Global imports and invoicing



Note: 'Trade' bars show the share of commodity trade and the share non-commodity trade with the dollarized and euroized economies in world imports. 'Data' and 'Model' bars show respectively the empirical and model-implied fraction of imports invoiced in dollars and euros. All numbers are computed for a subsample of countries with available invoicing data.

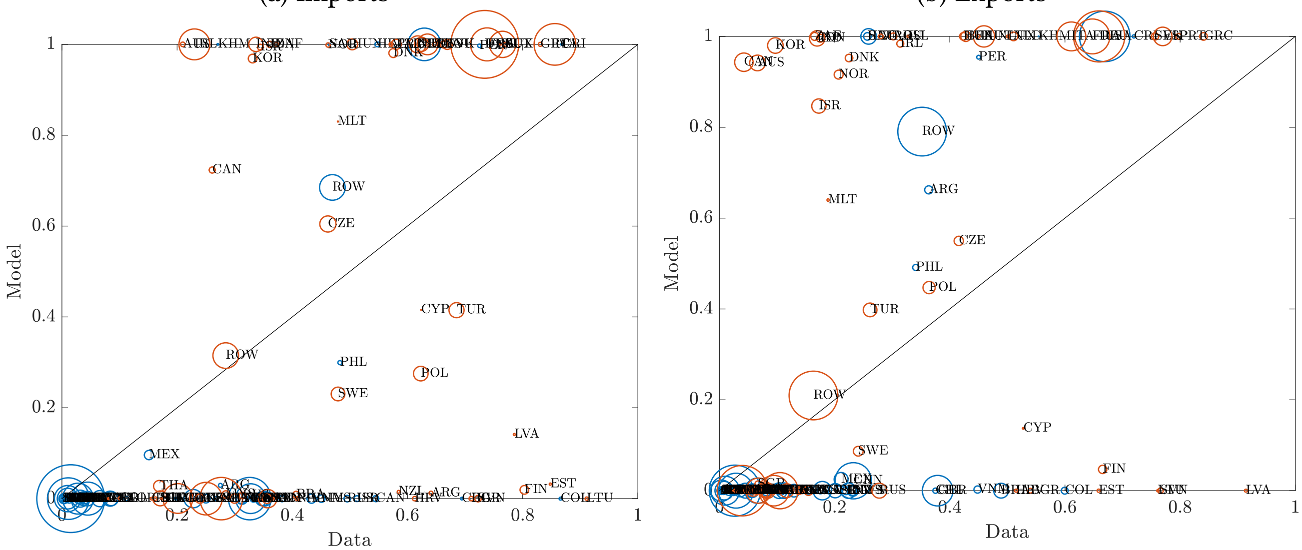
Figure A8: Cross-country export currency shares: model vs. data



Note: panels (a), (c) and (d) show empirical and model-implied shares of DCP, ECP and PCP in exports. Panel (b) shows a counterfactual share of DCP in exports if commodities were invoiced in dollars and invoicing of other goods were split equally between currencies of the buyer and the seller. The size of circles is proportional to country's share in world exports.

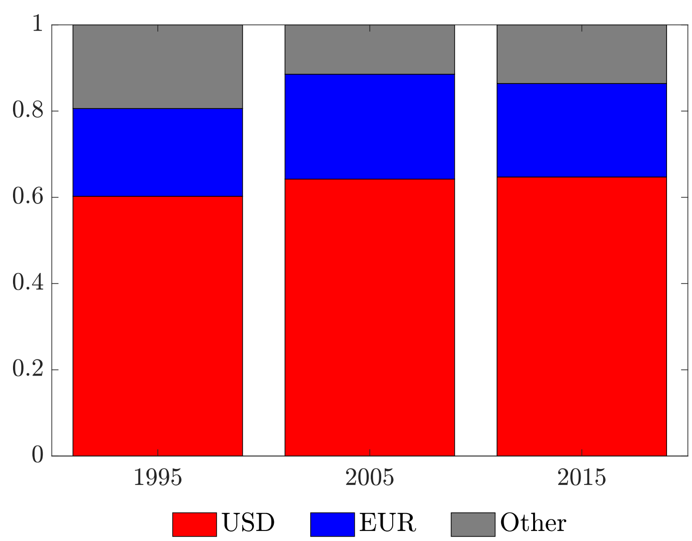
Figure A9: Currency shares in Switzerland by trade partner: model vs. data

(a) Imports (b) Exports



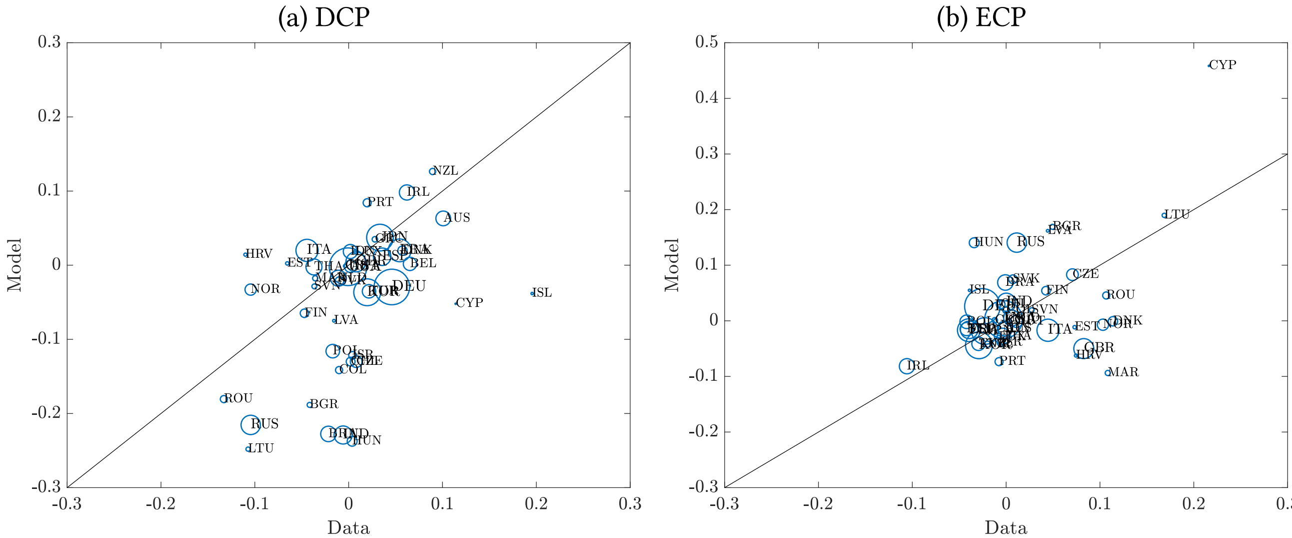
Note: the figure shows the share of DCP (blue) and ECP (red) in imports and exports of Switzerland by its trade partner. The size of circles is proportional to partner's share in Switzerland trade.

Figure A10: Simulated evolution of currency shares in world trade in 1995–2005



Note: the bars show the model-implied shares of the dollar, euro, and other currencies in global trade in 1995, 2005, 2015. In 1995, the euro corresponds to the German Mark.

Figure A11: Change in currency shares from 2005–2015: model vs. data



Note: the figures show the change in DCP and ECP shares in countries' exports between 2005 and 2015 comparing empirical numbers with the simulated values. The size of each circle represents country's share in world exports.