ONLINE APPENDIX for "Inversions in US Presidential Elections: 1836-2016"

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A Background

A.1 The Party Systems in the 19th and 20th Centuries

Our earliest sampling frame consists of Antebellum elections from 1836 to 1852. This range includes all years in which Democrats and Whigs were the predominant political parties in national politics. Political scientists typically classify the range 1828 to 1854 as the Second Party System and consider 1852 to be the last presidential election year prior to the Civil War in which the parties were stable. The 1832 election does not easily fit with our two-major-party procedure, nor does it fit with the rest of the Antebellum period, as the presidential candidates earned Electoral College votes from parties other than the Whigs and Democrats (National Republican, Nullifier, Anti-Masonic). We start in 1836, when the major parties were Whigs and Democrats and after all states (other than South Carolina) began allowing their citizens to vote in presidential elections.

Our second sampling frame consists of the post-Reconstruction Era, 1872-1888. Political scientists typically classify the range 1854 to 1892 as the Third Party System. We drop the Civil War years and elections before 1872 as these were characterized by a multiplicity of competing parties that earned EC ballots as well as Republican landslide victories. There was also a changing roster of states gradually rejoining the union in this postwar period. We end our post-Reconstruction sample period at 1892 because the next election in 1896 represented a major political realignment. The realignment is typically recognized as the end of the Third Party System and the beginning of the Fourth Party System.

Other time periods are less useful in providing identifying variation in electoral outcomes. For example, the period 1896 to 1932—the Fourth Party System—mostly yielded presidential landslide victories. These create less useful variation for the purposes of understanding inversion probabilities in close elections. We nonetheless present results for the early twentieth century in Appendix Section E.5 and Figure A2.

A.2 Related Systems Around the World

Inversion are possible in the US because of the Electoral College's tiered system of voting, in which citizens cast votes for electors, who in turn elect the President. Even absent the possibility of faithless electors, the national popular vote (NPV) and the EC outcome can diverge for a host of reasons that we detail in Section 5, where we examine the aggregation mechanics of the EC.

A useful way to understand EC-NPV mismatch is that it can occur when electoral ballots at the second tier can be captured by different numbers of citizen votes at the first tier. In the US, states are heterogeneous in EC representation, which results in elector ballots cast that are disproportionate to citizen votes. Further, that heterogeneity can be (and often is) correlated with partisan alignment across states. In Westminster-style Parliamentary systems, voting units that elect the Prime Minister (Parliamentary districts) are typically similar to each other in representation because they tend to be similarly-sized. All else equal, this tends towards fewer inversions. But some countries, including India and Norway, intentionally introduce malapportionment when electing MPs, who then serve as electors in the second tier that elects the Prime Minister. For example, Norway's system upweights rural counties and India weights regional votes using the regional populations of 1971, which were very different from the geographic distribution of population today.

Even without intentional malapportionment, inversions still occur. For example, Canada's 1979 federal election resulted in a loss for the Liberals and for their incumbent Prime Minister Pierre Trudeau: Despite capturing more citizen votes than the Progressive Conservatives (L: 40% v. PC: 36%), the Liberals were elected to fewer seats in parliament (L: 114 v. PC: 136) and Trudeau was therefore defeated.[42]

B Further Discussion of the Related Literature

Because of the considerable importance of the EC to US politics—as well as the importance of multi-tiered elections to democratic systems worldwide—the EC has received extended attention in the literature. However, because no prior study has investigated the same question we ask here, which is about the fundamental statistical nature of EC inversions, no prior paper has used the same materials and methods. Here we detail how our approach is distinguished from prior literature that: (i) studies empirical facts about the EC other than about inversions in the EC (B.1) or (ii) studies properties of inversions other than their conditional and unconditional probabilities across historical periods (B.4).

In one striking example of the richness of the EC literature, political scientists and historians have even debated which elections should count as an inversion—a debate that is possible because of the complexity, and therefore ambiguity, of the implementation of the EC in practice across states, parties, and centuries (Kallina, 1985; Rakove, 2004; Estes, 2011). Gaines (2001), for example, argues that the 1960 election should be counted as an inversion because over 175,000 popular votes in Alabama (a number in excess of Kennedy's national popular vote margin of victory) were for Democratic electors who were opposed to Kennedy.

A more recent literature considers potential advantages and disadvantages of a national popular vote compact (DeWitt and Schwartz, 2016; Koza, 2016; de Mouzon et al., 2019). Because these studies often either take a normative or legal focus or do not use empirical data, and because they consider aspects of presidential elections other than the probability of inversion (such as the probability or difficulty of a recount, or incentives for strategic voting), we do not consider them further here.

B.1 Empirical facts about the EC, but not about inversions

One of the oldest empirical literatures about the EC documents empirical facts about the distribution of electoral votes across states. In particular, much of this literature describes the allocation of average electoral influence (in the sense of EC ballots per popular vote or EC ballots per person) across states or across population groups. For example, Blair (1979) computes that, by such metrics, whites have more average voting power than blacks. Warf (2009) maps differences in average voting power across states.

Another category of descriptive analysis computes facts about average "voting power" in a way that is distinct from the mere probability of being pivotal, which is the focus of the next

[42] Miller (2012) further discusses inversions in Westminster-modeled parliamentary systems, including in the United Kingdom, New Zealand, and Canada.

section. Banzhaf III (1968), for example, makes computations that compare the size of each state with its number of electoral votes, in order to compute a state-specific index of voting power. A follow-up literature has considered properties of Banzhaf's index and proposed alternatives (Owen, 1975; Dubey and Shapley, 1979).

B.2 Pivotal voters and the EC

A long literature in political science and economics considers the relative costs and benefits of voting, in particular focusing on the probability of being pivotal in deciding the election (Riker and Ordeshook, 1968; Gelman, Katz and Tuerlinckx, 2002). Several papers have applied these ideas to the Electoral College context, including Gelman, King and Boscardin (1998); Gelman, Silver and Edlin (2012); Miller (2013). Our paper is not concerned with the probability that a voter, or a voter in a particular state, or a voter in an election of particular closeness, will be pivotal.

B.3 Inversion analyses using "uniform partisan swing" method

An important feature of our analysis is the modeling of election uncertainty. This distinguishes our work from the many studies in political science that characterize presidential elections deterministically, such as via "uniform partisan swing analysis," and therefore cannot assess the *probability* of an inversion (e.g., Garand and Parent, 1991; Grofman, Koetzle and Brunell, 1997).

Uniform partisan swing analysis—originating in Butler (1951) and Gudgin and Taylor (1979)—has become a standard tool for understanding the relationship between electorate votes and election outcomes, such as congressional seats. The method takes an observed election outcome and, in the classic application, "swings" all legislative districts by the same common vote share. By varying the vote share in a deterministic way in small increments, the method can trace when seats flip and so can trace the relationship between swings in the common, across-district component of votes and the aggregated election outcomes. Primarily applied to estimating seats-votes curves in legislative elections such as for the US Congress (e.g., in Gelman and King, 1990), the method has been ported to analyzing EC. In particular, several studies map the relationship between electorate votes and EC ballots (Garand and Parent, 1991; Miller, 2012). The important differentiator of our study is the incorporation of uncertainty. In uniform swing analysis, there is no probability distribution over the aggregate vote share. In addition, there is no uncertainty in the way that contests across states (or legislative districts) resolve differently. They are assumed to comove perfectly. Therefore, these studies—which do not estimate probability distributions—do not address the goals of our paper, which are the computation of a set of important conditional and unconditional probabilities.

B.4 Inversions: Theoretical computations and election-specific predictions

Our paper uses data from many elections in the 19th through 21st centuries to estimate the unconditional and conditional probability of an inversion, abstracting away from the features of any particular pair of parties or candidates. The wide set of methods that we employ has not previously been applied to this question, and no prior set of estimates of these probabilities exists in the literature.

One of the richest existing literatures about the EC, from the game theory and formal political science literatures, theoretically computes the probability of inversions in mathematical models that abstract away from any data about the actual EC (Kikuchi, 2017; de Mouzon et al., 2018). Many of these papers, like ours, are focused on the stochastic properties of electoral systems. But unlike ours they are not grounded in voting data—for example, how partisan alignment and voting patterns in New York differ from those in Texas.

Another set of papers considers the probability of an inversion in one or more particular elections. Here, we have been able to build upon the methods of prior studies focused on single-election predictions or postdictions. For example, our M1 model is structurally analogous to the model that Katz, Gelman and King (2004) apply to specific years,[43] and our M2 model is similar to the model that Silver (2016) used to predict the distribution of potential outcomes prior to the 2016 presidential election. Some papers in this election-specific literature consider counterfactual policies, as in our Figure 3, but without a probabilistic approach. Cervas and Grofman (2019), for example, apply a set of counterfactuals to determine whether they would have yielded an inversion in several actual historical inversions, assuming that vote totals were the same as what historically occurred.

Among the literature that considers the statistical properties of presidential elections in particular time periods, two of the papers closest to ours in methodology are Merrill (1978) and Ball and Leuthold (1991), which are in dialogue with one another. Neither paper computes or discusses the probability of a close election, which plays a central role in our analysis. The interpretation of these papers is somewhat limited by the details and specificity of their modeling choices. Their sample selection differs from ours and from one another: Merrill, in the mathematics literature, pools elections from 1900 to 1976 (which ignores the mid-20th-century partisan realignment, and therefore ignores the fact that a vote for a Democrat in the time of Wilson had different economic, geographic, and racial correlates than a vote for a Democrat in the time of Carter); Ball and Leuthold (1991) (like Katz, Gelman and King, 2004) compute statistics for each of a series of years from 1920 to 1984, but also pool problematically across distinct periods of partisan realignment (e.g., their 1984 estimates pool data from 1944 to 1984). Methodologically, each paper makes analytic computations, assuming a single parametric form which specifies that each state shares the same distribution: a symmetric normal distribution in the case of Merrill (1978) and a parameterized beta distribution in the case of Ball and Leuthold. Neither paper explores robustness to these assumptions—Ball and Leuthold suggest that a non-parametric approach, such as we use in M3 and M4, would be "difficult to conceptualize." Despite these limitations, these papers are important for their early anticipation that state-indexed models could be used to describe statistical properties of presidential elections.

Finally, Bakthavachalam and Fuentes (2017) in a short note report results on inversion probabilities for a period overlapping with our Modern period. Similar to our results, they conclude that inversion probabilities are high in close elections. In contrast with our results, they conclude that there is no partisan asymmetry. The note does not provide enough technical detail to compare and contrast methods or findings in depth.

[43] Thomas et al. (2013) use essentially the same model as Katz, Gelman and King (2004) to estimate partisan bias in the EC in 14 specific elections but do not estimate the probability of inversions. Partisan bias is indeed important to quantify but is distinct from the probability of an inversion: for example, a two-tiered system that added high-variance, mean-zero noise to election outcomes would generate zero ex ante partisan bias but would yield a high probability of inversion.

C Data and Estimation

C.1 Data

C.1.1 Data Sources

For data on state populations, we use IPUMS extracts from decennial Censuses (Manson et al., 2020). For intercensal election years, we follow the standard practice of exponentially interpolating state populations.

The key inputs to our analysis are the historical election returns by state for each presidential election year. For each presidential election, we assemble data on vote tallies for each candidate in each state, as well as data on EC elector ballots cast for each candidate by the EC delegation from each state. Data on state-level election returns and on EC ballots cast come from the Leip (2018) compilation of state returns.

We use state $\times$ year data on education and race in some models for the Modern period; these are from published summary tables of the American Community Survey. Race data by state from the 19th century come from published Census reports.

C.1.2 Third Parties

For most simulations, we retain information on only the two major parties—Democrats and Whigs from 1836 to 1852 and Democrats and Republicans for the later periods we examine. This normalization, which is standard in the literature (see, e.g., Gelman and King, 1994) does not substantively impact our analysis of inversion probabilities, as third-party candidates won no EC ballots over our study periods.[44,45] When we scale popular vote outcomes by turnout, we include third-party voters in our measure of total state-level turnout.

Of course, a third-party candidate could be pivotal in determining which major party candidate wins a state $\times$ year. The building blocks of our estimation and Monte Carlo exercise are actual state $\times$ year election outcomes. We primarily take these outcomes as basic data and make no assumptions on how a state return might have differed if not for a third-party candidate. Thus, most of our statistics describe a typical election outcome over our sample period, rather than elections in which we counterfactually remove or change the influence of third parties. However, in Figure A6 we assess sensitivity to two extreme and opposite assumptions on the impact of third parties. First we reestimate our baseline model reassigning all third-party votes in each state $\times$ year to the Democratic candidate. Then we reestimate our baseline model reassigning all third-party votes in each state $\times$ year to the Republican candidate.

C.2 Data Cleaning and Restrictions

Here we catalogue our handling of various special cases and anomalies that arise in the election data:

[44] In particular, one could relabel the horizontal axes in our figures below to center on the state $\times$ year specific threshold, with ticks on the axis indicating distance from that state-election-specific threshold. In the Florida 2000 example, Bush and Gore won 48.847% and 48.838% of votes respectively, with 2.315% going to other candidates. The state $\times$ year specific threshold for a Republican victory in this case would be 0.488425 ($= (1 - 0.02315)/2$).

[45] The last third-party candidate to win a single pledged Electoral College vote was Wallace in 1968, which predates our primary modern sampling frame. The Antebellum and post-Reconstruction elections produced no third-party EC ballots, other than via faithless electors.

- We ignore the few historical instances of faithless electors, who cast EC ballots for a candidate other than the candidate to whom they were pledged. In the cases of faithless electors, we award Electoral College ballots as they would have been awarded based on state popular vote results.
- We exclude third parties from our analysis. No third-party candidate won EC ballots in any of the election years we use for our main sampling periods (except through faithless electors). See also Section C.1.2.
- In each election year, we drop states where EC ballots were allocated by state legislatures rather than by the state popular vote. This includes South Carolina in 1836 to 1852 and Colorado in 1876.
- In 1836, the Whig party ran multiple candidates across the country. All states that held a citizen vote for President (as opposed to awarding EC ballots via the state legislature) had one Whig candidate on their ballot, and no states had more than one. We treat all Whig candidates as one candidate in the 1836 election.
- We start the post-Reconstruction era in 1872 because not all of the Confederate states had rejoined the Union by 1868.
- In 1872, Horace Greeley, the Democratic candidate, died after the general election but before electors had formally cast votes. Officially, no EC ballots were allowed for Greeley. We use popular vote data from the general election and award EC ballots as if Greeley had not died.
- In 1872, the electors of Arkansas and Louisiana were not certified by Congress. We use the popular vote outcomes in these states in 1872 to award their EC ballots to Ulysses S. Grant.
- We end the post-Reconstruction era in 1888 because there was a major third party in 1892 (Populists). Additionally, not all states had both major party candidates on their ballots.
- In the Modern period, Maine (since 1972) and Nebraska (since 1992) have split their EC ballots between the state popular vote winner and congressional district popular vote winners. In practice, both states have only split their EC ballots once each. We ignore this rule and allocate Maine and Nebraska's electoral votes by a winner-takes-all rule.
- For Alabama in 1964, Leip assigns Democrats 0 votes. Leip chooses zero presumably because the Democratic electors were unpledged, rather than tied to the national candidate. For the purposes of our estimation and simulations, we assign the Democratic votes cast in Alabama to the national Democratic candidate, Johnson.

C.3 Sampling Frames

Figure 1 of the main text indicates the periods in US history that we study. Political scientists have identified several stable Party Systems, characterized by competition between a fixed pair of parties with stable political properties.[46] We take these groupings as a starting point for our

[46]See Appendix A.1 for further discussion of how our sampling frames align with conventional treatments of the historical US party systems in political science.

sample definitions. We further restrict attention to spans of years that include electoral victories for both parties because consecutive landslide victories of a single party do not generate useful variation for our purposes of studying inversion probabilities in close elections. Given these criteria, we study the Second, Third, and Sixth Party Systems, corresponding to the Antebellum, post-Reconstruction, and Modern periods, as indicated in the figure.

Our earliest study period consists of elections between 1836 and 1852. This range includes all years in which Democrats and Whigs were the predominant political parties in national politics and spans through the last presidential election year prior to the Civil War in which the parties were stable. In the post-Reconstruction Era, we study years 1872-1888. Like today, the parties during this period were Republicans and Democrats, though the political alignment of states was rather different. Democrats dominated in the Southeastern US; Republicans dominated in the North, West and Mid-West.[47]

Finally, we treat 1988 to 2016 as our baseline modern period, although model M12 (included in Figures 3 and A1) demonstrates that our estimates of the probability of an inversion conditional on a close election are robust to extending the modern sampling frame further back to the 1960s.

We do not additionally focus on elections between 1900 and 1960 because over this period there was little usable variation for our purposes. With the exception of Woodrow Wilson's terms, Republicans won landslide victories from 1900 to 1928. This was followed by consecutive Democratic landslide victories (four of them by Franklin Roosevelt) beginning in 1932, and then Republican landslide victories again in the 1950s. Sampling or estimating from periods of consecutive landslide victories of one party generates landslide counterfactuals, leading to degenerate distributions with little to no probability density around the 50% national popular vote share, which is our threshold of interest. Nonetheless, in Appendix E.5, we show results for the 1916–1956 timeframe for completeness.

D Additional Details on Methods

D.1 Parametric Analysis

Table A1 reports the maximum likelihood estimates of the parameters in Equation 1 of the main text. Estimates in the table are grouped by period. Within each period, the first model (M1) corresponds to the baseline estimate, following the Gelman and King (1994) "unified method of evaluating electoral systems." It includes a national shock and independent state shocks, with state shocks drawn from a common distribution. The parameters of particular interest are the variances of the national and state shocks, σ^2_γ and σ^2_ϕ. Either 31, 38, or 51 expected state vote share parameters, $\overline{\alpha}_s$, are also estimated, depending on the data period.[48]

Other columns compute estimates for alternative samples or model restrictions. Column 2 (M2) estimates additional state covariance terms on the basis of geographic region, race, and education. States within a region receive a common, independent shock. The race term

[47] Although the Third Party System includes the 1892 election, we exclude it from our analysis as the two major parties were not on the ballot in every state.

[48] Depending on the data period, some states were not present for all election years within the sample frame or did not use a statewide citizen vote to determine EC votes. Either 25, 37, or 51 expected state vote share parameters, $\overline{\alpha}_s$, are estimated by joint maximum likelihood. For other states, including Colorado in the post-Reconstruction frame (which was not a state in 1872 and which did not hold a popular vote in 1876), $\overline{\alpha}_s$ parameters are estimated separately as means and do not contribute to estimating variance parameters.

multiplies the fraction of each state that is nonwhite by a random, common coefficient drawn from a mean-zero t distribution. The education term multiples the fraction of each state's adult population that is college-educated by a random, common coefficient drawn from a mean-zero t distribution. Data on these demographics come from published, state-level summary statistics of the American Community Survey. The M2 model closely follows the Silver (2016) approach to modeling uncertainty in election forecasting.[49] The next column (M5) estimates the model assuming no national shocks, counter to the stylized facts from the elections literature about the importance of a common, national component to the uncertainty. Columns 7 through 9 add race and education covariance terms singly and together. M10 drops from the sampling frame the two historical instances of inversions in 2000 and 2016. The last model in the modern period (M12) extends the sample to 1964, which walks the data period backward to the partisan realignment of the North and South in the early 1960s (Kuziemko and Washington, 2018). Additional columns repeat these estimates for the Antebellum and post-Reconstruction eras. Model R10 drops the inversion instances (1876 and 1888) from the post-Reconstruction period; there were no inversions in the Antebellum period. The requisite data for estimating the demographic covariance terms (models M7, M8, M9) exist only for the Modern period.

In order to convert state vote shares, V_{st}, into a national popular vote tally, it is necessary to scale V_{st} by voter turnout. Although the national level of turnout is irrelevant to our statistics of interest, the *relative* turnout across states could meaningfully impact simulated election results. Additionally, EC representation changed within each sample period. For example, Florida had 21 EC ballots in 1988 and 29 in 2016. Unless otherwise noted, we use the actual turnout and EC apportionment from the last election of each sample period. For example, we use 2016 turnout for the modern period. In practice, choices around which numbers to use for turnout and EC apportionment have little bearing on our results. In Figure A7, we rerun the M1 model eight times, in each case assigning different turnout and EC apportionment. Lines in the overlay plot correspond to setting turnout and EC representation to 1988, 1992, 1996, 2000, 2004, 2008, 2012, and 2016. Across these specifications, the probability of an inversion conditional on a margin of victory within one percentage point varies only slightly—from 41% to 42%.

D.2 Bootstrap Monte Carlo: Turnout and the Varying State Roster

One practical consideration that arises when sampling and combining state election outcomes from different years is that the raw vote counts of later years tend to be larger, reflecting population growth. This creates a problem when summing citizen votes across states to yield a national popular vote. We address this by scaling the each party's vote tally in a state by that state's turnout in some common reference year before summing across states. Unless otherwise noted, we use turnout from the last election of each sample period. For example, we use 2016 turnout for the modern period. In practice, using alternative reference years for the turnout weights make almost no difference to the simulation results. See Figure A7. A related consideration is that EC apportionment can vary across election years. As with turnout, we assign EC apportionment according to the last election year of the relevant sample period.

[49] FiveThirtyEight's probability distributions over elections account for three potential types of error and uncertainty, relative to the best mean predicted vote share in each state: a common national error, a set of demographic and regional errors, and independent state-specific errors. For demographic and regional errors: "The following characteristics are considered in the simulations: religion (Catholic, mainline Protestant, evangelical, Mormon, other, none); race (white, black, Hispanic, Asian, other); region (Northeast, South, Midwest, West); party (Democrat, Republican, independent); and education (college graduate or not)." (Silver, 2016)

Finally, in our earliest historical period, the Union itself was changing: There were 25 states in 1836 but 31 states by 1852. Therefore, when performing bootstrap Monte Carlo simulations in this period, the sampling procedure generates some draws of state $\times$ years for which the state did not exist. These null draws do not contribute to the simulated NPV or to the simulated EC outcome in these simulated elections.

D.3 Bootstrap Monte Carlo: Generating Correlation Between State Outcomes

A downside of the independent sampling in our baseline bootstrap is that the lack of a common election-year component to the variation leads to under-dispersion relative to the actual span of election outcomes. To better capture the fact that national sentiment (or the characteristics of a particular pair of candidates) tends to moves states together in a given election year, we generate a variant in which we sample state outcomes with probability weights that attach extra probability mass to being drawn from the same election year. In particular, for each simulation we first draw a focal year, y^*, uniformly, independently, with replacement. Let M denote the excess probability that the outcome from the y^* election is sampled for each state in a given simulated national election, t. Increasing M increases the within-year, across-state correlation in voting patterns without imposing parametric assumptions on the distribution of the shocks.

This addition to the bootstrap procedure brings the dispersion closer to the actual dispersion of observed elections. For model M4 in Figure A1, we set the excess probability of drawing from the same focal year at 50%. In Figure 3, we vary M in 0.05 steps from 0.15 to 0.50. In our baseline sample, which contains 8 elections from 1988 to 2016, an equal-probability draw would put 0.125 weight on each year.

D.4 Bootstrap Monte Carlo: Wild Pooled Error Sampling

Models M6, R6, A6 create a larger pool of empirical error terms for bootstrap drawing, consisting of all state deviations from their period means over all elections in the period. Each state is first assigned its empirical sample-period mean two-party vote share. Then for each state, there is an independent wild bootstrap draw from this common pool, so that the ϕ_{st} term is drawn identically across states. The "wild" here is in the sense of Cameron, Gelbach and Miller (2008). It refers to multiplying each draw by a random 1 or -1, effectively doubling the sampling frame and imposing symmetry on the empirical distribution.

D.5 Bootstrap Monte Carlo: Swing State and Safe State Correlations

In Section 4.3, we discuss how we include models in Figure 3 that vary the implied correlation structure of the bootstrap procedure, tuning the excess probability that state draws come from the same election in 5% steps from 15% up to 50%. We do this overall, as well as for swing states separately and "safe" states separately. We define swing states following recent convention: Colorado, Florida, Iowa, Michigan, Minnesota, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Virginia, and Wisconsin.[50] If, for example, the 1992 outcome is drawn with excess probability mass for Colorado, then the 1992 outcomes are also drawn with the same excess mass for the other 11 swing states. In this approach, simulated elections are meant to come

[50] *Politico* published a swing state list leading up to the 2016 election that included: Colorado, Florida, Iowa, Michigan, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Virginia and Wisconsin. FiveThirtyEight adds Minnesota to this list to generate a list of "traditional swing states."

closer to the true equilibrium processes by which campaigns are making joint decisions on allocating investments across swing states as they anticipate factors like voter responsiveness to advertising and candidate visits. The primary source of variation in this set of simulations is the margin by which reliably red or reliably blue states are won (according to state-specific historical variability).

Conversely, when we sample "safe" states from the same election with excess probability, we primarily vary the state victor and the margin by which swing states and other potentially contestable states are won. Safe states, in the context of this analysis include the top quartile of states (12) in terms of the average Democrat or Republic margin of the victory over the sample period.

E Supplementary Results and Robustness

E.1 Extended results for 25 models

Figure A1 reports key summary statistics from several parametric and bootstrap models, each with different assumptions and constraints on the data-generating process. The first row in the figure corresponds to M1 (Panels A and B from Figure 2). Subsequent rows correspond to alternative models. The middle graphical panel displays the main findings: inversion probabilities conditional on a close race occurring.

The $x1$ and $x2$ models and their variants ($x7$ through $x10$) track the main approaches to modeling election uncertainty from the political science literature and election forecasting professionals, though such models have generally not been applied backwards to the periods of US history we study. The NPV distributions implied from these models' estimates are wider than in the ad hoc bootstraps of $x3$. This is because the assumed structure in $x1$ and $x2$ allows for a common, national component to deviations from state-level expectations. The vote share distribution is more diffuse in the $x2$ models (which align with Silver, 2016) than in $x1$ (which align with the Gelman and King, 1994 "unified method") because $x2$ incorporates additional correlated shocks linked across states within the same region and across states with similar demographics. The $x2$ set also draws from a fatter-tailed distribution, as described in Section 3.2. The independent sampling inherent in the bootstrap models ($x3$) tends to generate NPV distributions that are under-dispersed relative to historical data. This is true also for the parametric models ($x5$) that include no national common component to the shock. Other models are described elsewhere in the text in detail. Despite these significant differences in the predicted NPV distributions, Figure A1 shows that the conditional probability of an inversion in a close election is stable.

E.2 Asymmetry in Inversion Probabilities

Table A8 reports, conditional on an inversion occurring, which party was likely to have won the EC (and to have lost the popular vote). Table A7 reports the probability that an inversion accounts for the expected wins of each party. All models agree that, for the Modern period, inversion favored Republicans. Across all 12 Modern-period models, the probability that an inversion was won by a Republican ranges from 62% to 93%. For the post-Reconstruction period, Democrats were favored in inversions according to the standard, parametric models—though models based on bootstrap draws disagree. In the Antebellum period, there is no consensus across models as to whether Whigs or Democrats were favored.

In Section 4.4, we describe how the asymmetry that favored Democrats during the post-Reconstruction era arose in large part from the suppression of black votes in the US South, where Democrats were dominant. Here we present the analysis supporting that claim. At the core of our claim is the observation that blacks were numerous in the South and counted towards apportionment but were kept from voting in commensurate numbers. This is a consequence of the widely documented suppression of black rights, including voting rights, in the South at the turn of the twentieth century (e.g., Epperly et al., 2019 and Lichtman and Kazin, 2010).

The left panels of Figure A11 plot citizen votes per EC ballot against the Republican share of the two-party vote total for the post-Reconstruction period. The positive slope indicates that states likely to vote for Democrats (to the left within each plot) tended to have lower turnout-to-EC elector ratios. Voters in such states could control an EC ballot with fewer citizen votes. For comparison, we create analogous plots for the Modern period in the right panels of Figure A11. In the Modern period, Republicans were advantaged in the sense of EC ballots per citizen votes, so the slope is negative, opposite to the post-Reconstruction period. But the sources of imbalance were different over time. In the Modern period, part of the Republican advantage was tied to over-representation of small-population states due to the two senator-linked electors. To show this, in panels C and D we subtract two EC ballots from each state?s apportionment and recompute the plots. This adjustment negates any relationship in the Modern period between the partisan alignment of states and EC representation (Panel D). But the same is not true for the post-Reconstruction period where subtracting two Electors from each state does not neutralize the relationship (Panel C).

Figure A12 completes the picture: Whether examining all states or restricting attention to former Confederate states with large black populations, voter turnout per population and voter turnout per EC ballot were both strongly negatively correlated with the black share of the state population. Therefore, states with the largest black populations could control an EC ballot with fewer white votes. Winning low-turnout states, all else equal, helps a party win EC inversions because electoral votes are won with few popular votes.

In the Antebellum period, the disproportionate electoral ballots per citizen voter allocated to the South did not confer a partisan advantage to any party because Whigs and Democrats split the South.

These results illustrate that heterogeneity in turnout across states is an important source of potential inversions. In the post-Reconstruction, this heterogeneity was the result of deliberate policy. But even random variation in turnout—such as due to weather or to differences in the intercensal rate of population growth–could cause inversions, especially in the close elections that we study. This turnout-to-representation ratio is important across the world as well. Notably, India has what has been called a “crisis of representation,”[51] because the apportionment of Parliamentary seats today is based on population counts in the 1971 Census. As population growth has trended differentially in the north and south of India over the last 50 years, the votes (or registered voters or even residents) that can elect a member of Parliament have diverged dramatically across regions of India, leading to skewed representation that favors populations in low-growth states. (In the US context, India’s situation would be as if Texas had not gained in representation in the EC or US Congress over the last several decades despite its explosive population growth over that period).

[51]See Carnegie Endowment for International Peace: “India’s Emerging Crisis of Representation” https://carnegieendowment.org/2019/03/14/india-s-emerging-crisis-of-representation-pub-78588

E.3 Robustness to Gridded Parameter Values

With only a few elections per party system, it is impossible to be confident that estimates of the true parameter values underlying the data-generating process are precise. To examine the extent to which our main results could be sensitive to errors in these estimates, we calculate our outcomes of interest under a set of exogenously specified variances and correlations. In these simulations, we take only the state historical means of vote shares as data. Uncertainty around these means is assumed to follow $\gamma_t \sim N(0, \sigma_\gamma)$ and $\phi_{st} \sim N(0, \sigma_\phi)$ as in our baseline models (M1, R1, A1). But here we cycle over a grid of values for σ_γ^2 and σ_ϕ^2, rather than relying on estimates.

Figure A4 presents results from this procedure. The procedure generates many unique, assumed parameter combinations in each period. In addition to iterating over national and state variances, each combination is used while including or omitting a shared shock by geographic region.

In Figure A5, we present supplementary detail for a subset of the assumed parameter combinations in Figure A4. These simulations include state and national shocks. The variance of the national shock increases along the horizontal axis in each panel. The variance of the state shocks are traced in several contour lines in each panel, as indicated. In the panels on the left, we report the probability of close elections within a 2 percentage point margin. In the panels on the right, we report inversion probabilities, conditional on close elections within the same margins.

The slopes of contours in Panels A, C and E indicate that the probability of a close election outcome is sensitive in each period to the gridded parameter values. In particular, it is sensitive to the variance of the common, national shock. However, in all cases the inversion probabilities (Panels B, D, and F) remain high. In the Modern period, the probability of an inversion—conditional on a margin less than 2 percent—remains high regardless of the parameters exogenously set. The graph thus traces the same lower envelope on inversion probabilities for the Modern period as Figure A1.

We can summarize Figures A1, 3, A4, and A5 as indicating that our finding of high inversion probabilities in close elections is robust to: (i) parametric approaches that vary the assumptions on the data-generating process across those adopted by the political science literature and election forecasting practitioners, (ii) non-parametric bootstrap approaches that include both independent and highly correlated sampling of state outcomes, (iii) approaches that omit from estimation or bootstrap sampling the actual historical instances of electoral inversions, and (iv) searching over a wide grid of potential parameters, including parameters that are likely to be outside of the true parameter space.

E.4 Alternative Parameterizations State Demographic Characteristics

In Section 4.3, we note that models with shocks linked by election year, region, racial composition, and educational characteristics produce similar inversion probabilities to models that assume that state shocks are completely independent, and it is therefore unlikely that smaller tweaks will affect our main findings. Here we demonstrate this.

For the model plotted in Figure A10, we allow for race-linked shocks to multiply an **X** vector that includes region indicators, % non-hispanic white, % non-hispanic black, % hispanic, % college degree, and % high school completion in the state. This contrasts with M2, where **X** includes only % non-white and % college degree. In the figure we overlay a plot of this more flexible model with M2. The two are statistically indistinguishable in terms of the conditional probability of an inversion they imply (right panel).

E.5 Results for the Fourth and Fifth Party Systems (1916-1956)

Our main analysis samples do not include elections in the first half of the twentieth century, which was characterized by landslide victories for both Democrats and Republicans. For completeness, we estimate inversion probabilities for this time period here. We divide the timeframe according to a standard typology of party systems. We analyze separately elections in the Fourth Party System (1896–1932) and the Fifth Party System (1936–1956). For the Fifth Party System, we do not include 1960, because doing so would add the complication that it would be the *only* election in this span during which Alaska or Hawaii were states. We also omit 1948, when Strom Thurmond received 39 electoral votes for the States' Rights Democratic Party. For the Fourth Party System, we begin in 1916 in order to generate a stable set of states over the sample period, and we drop 1912 and 1924 because a third party won EC ballots in each of these election years.

Figure A2 presents results for the $x1$ and $x2$ class of models over the Fourth and Fifth Party System periods. These models apply the same structural assumptions and estimation procedures used for M1 and M2 in the Modern period (see Figure A1). The characteristic $\text{Win}(\text{NPV})$ curves are similar to other periods. Further, the ex ante probabilities of an inversion in a close election are high in these models. An important difference between these results and results from the Antebellum, post-Reconstruction, and Modern periods is that the probability of a close election was much lower, making the unconditional probability of an inversion lower. It is notable that the conditional inversion probabilities in Figure A2—the primary results of interest—are very similar to the corresponding statistics for our main sample periods, even though the other statistical properties of these times are so different.

F Counterfactuals that Account For Endogenous Behavior

In Section 5, we reference a modification to our statistical model that incorporates endogenous responses to counterfactual EC aggregation rules. Here we describe the procedure in detail. The counterfactual rule sets we consider are the same as in Figure 4—minus two ballots, awarding ballots proportionally, or both changes simultaneously.

Our approach begins with identifying how the set of potentially pivotal states (i.e., "swing" or "battleground" states) would change under the counterfactuals. Denote the probability that state s is pivotal in an election with $\hat{Q}_s$, which we sometime refer to as "swingy-ness" below. Whereas a state like CA has essentially no chance of being pivotal under the status quo and present political alignment towards Democratic candidates, a counterfactual in which its 55 elector ballots are distributed in proportion to the state vote tally brings CA into play and opens the possibility that an EC ballot from CA could be decisive in the election. Call the status-quo swingy-ness of a state $\hat{Q}_s^0$, and call the corresponding quantity under the counterfactual $\hat{Q}_s^{CF}$. After estimating $\hat{Q}_s^{CF}$ for each state, we allow for the behavior of campaigns and voters to influence the data-generating process—in a stylized manner precisely described below.

The approach is grounded in the idea that changes to campaign investment, voter attentiveness, and other political inputs influencing the election are likely to track the changes in the set of potentially pivotal states. Beyond the consistency of this approach with the folk wisdom that *only swing states matter* in a presidential contest in terms of investments like campaign spending, our focus on the changing electoral map aligns with the expert consensus. In economics for example, the model in Strömberg (2008) shows that in equilibrium, campaign resources will be spent (symmetrically by both parties) exactly in proportion to the probability that a state is pivotal in swinging the election ($\hat{Q}_s$). Likewise, statisticians and political scientists have long been focused on the empirical, state-specific probability that a vote cast is decisive in swinging the election (e.g., Gelman, Katz and Tuerlinckx, 2002 and Gelman, Silver and Edlin, 2012). The consensus view is that these probabilities are focal for campaigns assessing where to invest in turning out or persuading voters.

Despite wide consensus around $\hat{Q}_s$ as an object of interest, the literature is mixed on exactly how a change in $\hat{Q}_s$ could alter a race. Would an exogenous shock that increased $\hat{Q}_s$ tighten the race in the state? Or cause turnout to climb in the state? Or reduce election-day uncertainty around the expectation, as parties lock in their voters and convert the undecided ahead of election day? The literature offers no singular guidance.

We therefore model three plausible but substantively different types of endogenous responses as states gain or lose battleground status. Our intent is to span the range of plausible endogenous responses with stylized mechanisms, without taking a position on the correct behavioral model, which is unknown to social science. We assume, in turn, either that (i) the margins will tighten in new battleground states, (ii) variances of potential voting outcomes will shrink in new battleground states, or (iii) turnout will increase in new battleground states. Each of these reduced-form adjustments makes no assumption about the exact mechanisms underlying the net effects. For example, Enos and Fowler (2018) show that an aggregate effect of large-scale campaigning in 2012 was to increase voter turnout by several percentage points relative to the counterfactual in the most highly targeted states. Our turnout counterfactual nests that phenomenon. But our reduced-form turnout adjustment would also nest the case in which turnout effects were instead arising from the (correct) perception among voters in newly minted battleground states that they have an increased potential to impact the election outcome. We model the turnout change (for example), not the ultimate cause of it.

F.1 Methods

Separately, for each counterfactual rule set, we perform the following steps:

1. **Determine Q_s (Swingy-ness) Under Counterfactual Rules.** We begin by using the main simulation results to calculate a probability that each state would be pivotal under the counterfactual EC rules. For each state, separately for each of the original 100,000 simulation runs in Model M1, we re-assign 0.5% of statewide votes from the state winner to the state loser. We calculate the fraction of simulated elections in which this reassignment of votes would flip the EC outcome under the counterfactual EC aggregation rules being considered. Even without changing the simulated voting outcomes, different EC rules will produce different probabilities of each state being pivotal. For example, in the status quo, a high $\hat{Q}^0_s$ would require both that the simulated state outcome has the losing candidate within 0.5% and that the national EC margin of victory is no larger than two times the state's EC ballots. But for other rule sets, including proportional allocation of a state's EC ballots, this is not the case. Call the normalized probability that a state is pivotal under current rules $Q^0_s \left(= \frac{\hat{Q}^0_s}{\sum_{j=1}^{51} \hat{Q}^0_j} \right)$ and the analogous normalized probability that a state is

pivotal under counterfactual rules Q_s^{CF}. For each state and each counterfactual CF, we compute the ratio $\Lambda_s^{CF} \equiv Q_s^{CF}/Q_s^0$, which is greater than one if a state gains battleground importance and less than one if it loses relative importance.

2. **Adjust Data-Generating Process.** We next introduce an endogenous response, altering the state mean, variance, or turnout as a function of Λ_s^{CF} calculated in step 1. In three separate counterfactuals, we:

   (a) Shrink the expected partisan alignment in proportion to Λ_s^{CF}. This shifts expectations toward 50/50 in states that become more swingy and away from 50/50 in states that become safer, where the trailing party loses incentive to compete. In these simulations, we adjust the log-odds ratio of a Republican victory at the state level by setting a new state mean: $\alpha_s^{CF} = \alpha_s^0 \cdot (\Lambda_s^{CF})^{-1}$, where α^0 is the original estimate of the log-odds of Republican victory in the state. The shifted α_s^{CF} becomes the new constant in the log-odds-transformed vote share process: $V_s = \alpha_s + \epsilon_s$ from Equation 1. Note that $(\Lambda_s^{CF})^{-1}$ is less than one when a state's battleground importance increases under counterfactual rules. For example, as California goes from being a safe state under the status quo to having some of its elector ballots in play under the proportional rules, the adjustment to α_s^0 will move the expectation of the log-odds ratio $\left(ln\left(\frac{\text{Pr.(R)}}{\text{Pr.(D)}}\right)\right)$ closer to zero. In other words, it moves California closer to a 50/50 vote share.[52] In this way, the race endogenously tightens in newly generated battlegrounds and loosens in newly generated safe states, where $(\Lambda_s^{CF})^{-1}$ is greater than one.

   (b) Shrink the variance term in the state's data-generating process in proportion to Λ_s^{CF}. This reduces uncertainty in the new battleground states and increases it in states that become safer. In these simulations, we set $\sigma_s^{CF} = \sigma_s^0 \cdot \Lambda_s^{CF}$. Recall that σ_s^2 is the variance of the state uncertainty term.

   (c) Inflate voter turnout in proportion to changes in $\hat{Q}_s^{CF}$. This increases turnout in the new battleground states and reduces it in states that become safer. In these simulations, we multiply the turnout that would otherwise occur in a state by $\left(1 + 0.1 \times \frac{\hat{Q}_s^{CF} - \hat{Q}_s^0}{\max_r\left(\hat{Q}_r^{CF} - \hat{Q}_r^0\right)}\right)$, where the denominator is the maximal difference across states for a specified counterfactual set of rules, so that turnout increases by 10% for the state with the greatest increase in swingy-ness and increases or decreases by other amounts for other states, depending on their relative change in swingy-ness.

   In each case above, we bottom-code Q_s at the 33rd centile across the states prior to calculating Λ_s^{CF}. This is to avoid large ratios due to tiny probabilities in the denominator. Intuitively, there is little practical difference in exactly how safe Kansas and Massachusetts are in the Modern period, but noise in small values of Q_s among extremely safe states could explode the ratio Λ_s^{CF}.

3. **Rerun the Simulations Using the Adjusted DGP.** Starting from the adjusted parameters to the data-generating process, we generate 100,000 new simulation draws. From these counterfactual-specific simulations, we calculate inversion rates under each counterfactual rule set. This generates probabilistic voting outcomes that incorporate endogenous responses to the changing electoral map.

[52] In the logit equation $V_s = \alpha_s \cdot (\Lambda_s^{CF})^{-1} + \epsilon_s$, and the Republican vote share is $e^{V_s}/(1 + e^{V_s})$.

As a check on our process, we first calculate the Q_s^0 probabilities (swingy-ness) assuming the status quo EC rules, rather than a counterfactual. The validation check produces a familiar list. In descending order of Q_s^0, the pivotal states over the 1988–2016 period are FL, PA, OH, MI, and VA. We then calculate analogous probabilities under each of the counterfactual rule sets, Q_s^{CF}. We show these plotted against Q_s^0 in Figure A14. Panel A shows that removing two elector ballots from each state changes little in the list of swing and safe states. For example, PA gains in relative importance compared to OH, but the effect is small. In contrast, under the proportional ballot counterfactual in Panel B, the relative importance of PA, OH, MI, and VA fall, while CA, TX, and NY rise to join FL among the most important battleground states.[53] Note that under the counterfactuals that include proportional allocation (Panels B and C), states' probabilities of swinging the election are less differentiated. Everywhere becomes more in-play, so the distribution of Q_s^{CF} (across the vertical axis) is tighter.

An important caveat here is that the exercise is most likely to be informative for counterfactuals that are closest to the kinds of marginal changes to states' battleground importance that exist in the estimation sample. For example, removing the two electors tied to each state's Senators is a counterfactual similar in both kind and magnitude to the effects of decennial reapportionment. Consider the 1990 re-apportionment, which our Modern period spans. Following re-apportionment, New York lost three EC ballots; Pennsylvania, Ohio, Illinois, and Michigan each lost two ballots; New Jersey, Massachusetts, Louisiana, Kentucky, Iowa, Montana, Kansas each lost one ballot; while California, Texas, and Florida each gained several EC ballots.[54] In contrast, proportional ballot allocation in place of a winner-takes-all rule at the state level is a counterfactual that is far out of sample.

Further, we view these counterfactuals primarily as directionally informative. The appropriate size of the adjustment—e.g., by just how much does turnout increase?—involves parameters for which there exist no credible estimates to our knowledge. Therefore, while the exercise potentially forms bounds by revealing whether accounting for endogeneity increases or decreases the estimated probabilities of inversion in a counterfactual, we do not claim it to be informative of the precise magnitudes of effects.

F.2 Results

With those caveats, we present results in Figure A15 and Table A9. Panel A repeats results from Figure 4 for reference. In Panel A, there is no adjustment to the data-generating process for possible endogenous responses. The simulations behind Panel B assume that the race tightens in states that increase in battleground importance and loosens in states that become safer. To put the magnitude of the changes this procedure generates in context, when considering the counterfactual of proportional ballot allocation, states in the top quintile of movement towards Republicans move an average of 6.7 percentage points in expectation. States in the top quintile of movement towards Democrats move an average of 7.0 percentage points. Interestingly, comparing Panel B to Panel A indicates that the effect of accounting for the endogenous

[53] A "battleground" state is conceptually different when EC ballots are awarded proportionally rather than as winner-takes-all. Under proportional allocation, CA, TX, and NY gain in relative importance because swaying half a percent of votes in these states (e.g., by campaigns appealing to state-specific concerns) corresponds to more voters.

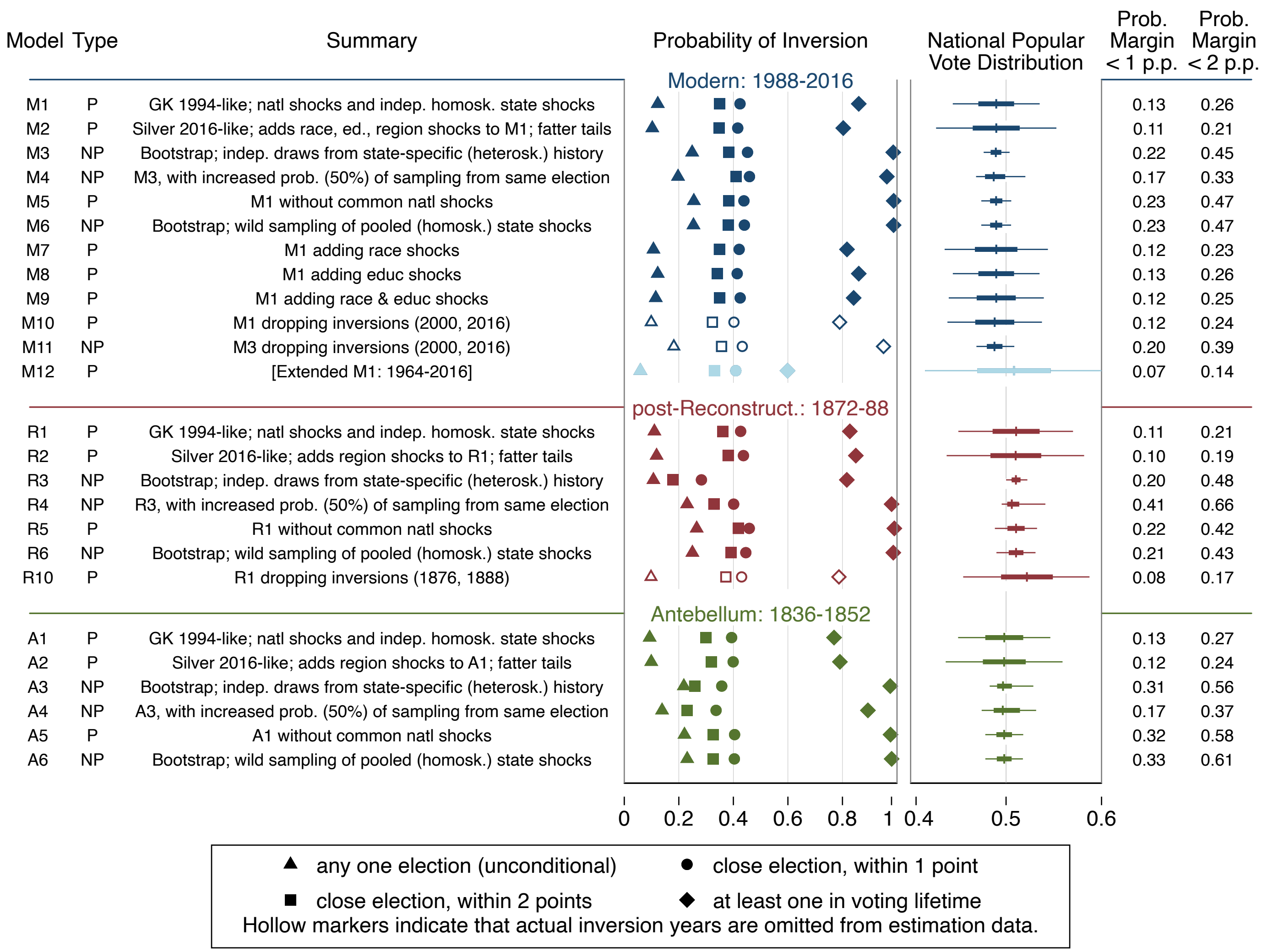
[54] Likewise, inflating the size of the House of Representative to reduce the rounding errors inherent in allocating a small number of US House Districts (today 435 with voting members) across states is similar in kind (though not magnitude, depending on the particular inflation factor) to changes that have occurred in US history: The size of the US House has grown through various Apportionment Acts of Congress.

responses of voters and campaigns in this way is to amplify the resulting partisan imbalances under the counterfactuals and to increase the probability of inversions under the counterfactuals.

The remaining panels in Figure A15 report the analogous results for the assumptions that gaining battleground status reduces variance (Panel C) or increases turnout (Panel D). Again, the simulated effects of the electoral map shifting are large. In the simulations behind Panel D, the national composition of turnout across states shifts towards the most politically salient states.

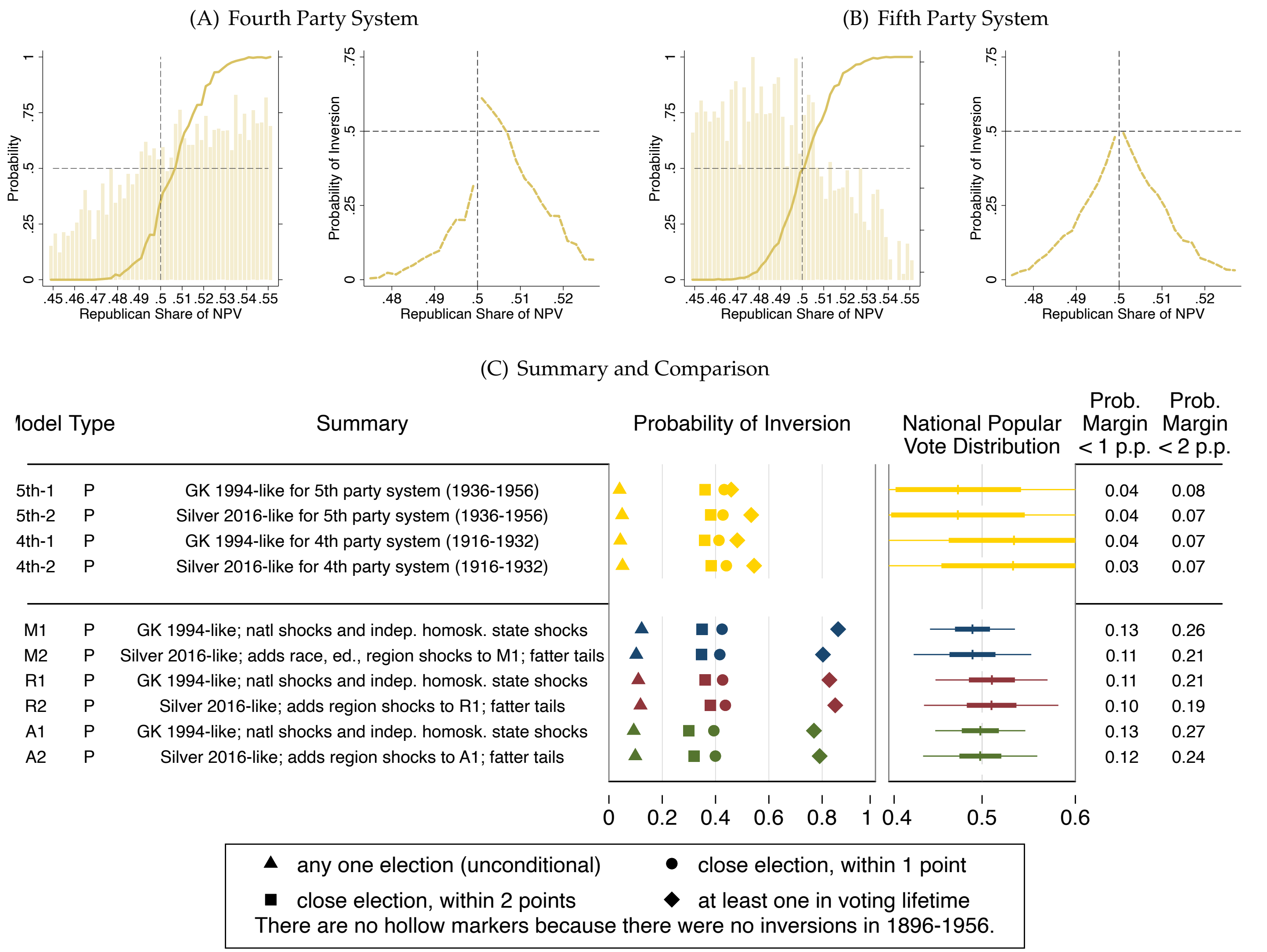
The impacts of endogenizing turnout and variance are less stark than the impacts of endogenizing tightness. But it remains the case, as we show in Table A9, that each of these endogenous-response counterfactuals either has almost no impact on the probability of an inversion in a close race or increases it relative to the baseline assumption of no endogenous response. On this basis, we conclude that the baseline decompositions in Figure 4, which hold the statistical model fixed when calculating the impact of the policy changes, are likely to represent a lower bound on inversion probabilities under the counterfactuals.

Figure A1: Inversion Probabilities Across Models, Methodologies, and Time Periods



Note: Figure reports summary statistics from 25 election models. Repeated numerals in model names indicate that the same specification is used across time periods—as in M1, R1, A1. The popular vote distribution presents 5th, 25th, 50th, 75th, and 95th centiles of the simulated elections. P/NP denotes parametric/non-parametric. Parametric models are estimated by maximum likelihood; non-parametric (i.e., "bootstrap") models resample past election outcomes for each state to generate a national outcome. P(mar. <1pp) and P(mar. <2pp) report the probability that the popular vote margin is within 1 and 2 percentage points, respectively. "At least one in voting lifetime" indicates the probability of experiencing an inversion for a voter who votes in 15 presidential elections over a 60-year voting lifetime and faces the unconditional distribution of voting outcomes described by the model.

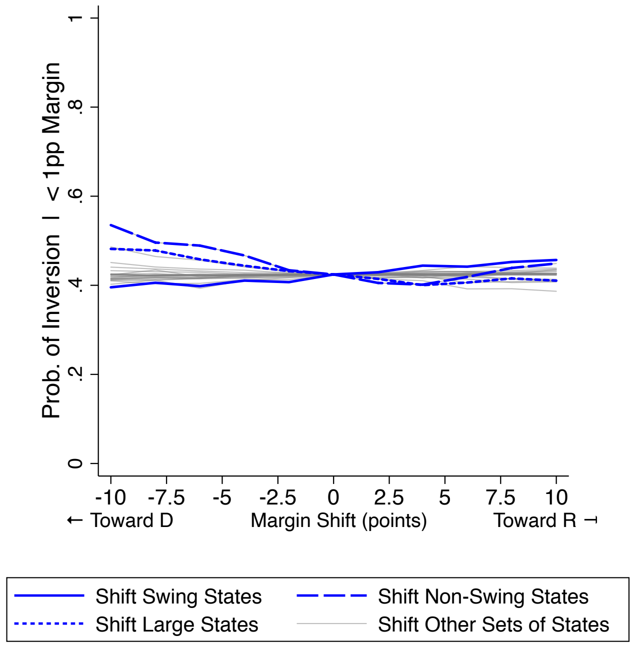
Figure A2: Results for the Fourth and Fifth Party Systems: 1916–1956



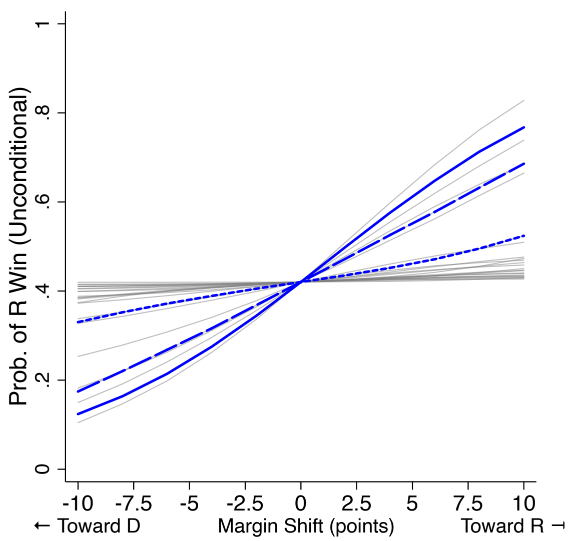
Note: Figure shows inversion probabilities and probability distributions over national popular vote outcomes implied by the parametric estimates of the baseline $x1$ and $x2$ family of models estimated for sample periods in the first half of the twentieth century—the sand-colored regions of the party system history described in Figure 1. Each model simulation consists of 100,000 simulated election draws. The Republican share of the national popular vote runs along the horizontal axis in the top panels. See Figure 2 and Figure A1 for additional notes on Figure elements. See Appendix E.5 for a full description of the exercise.

Figure A3: Alternative Partisan Alignment Scenarios

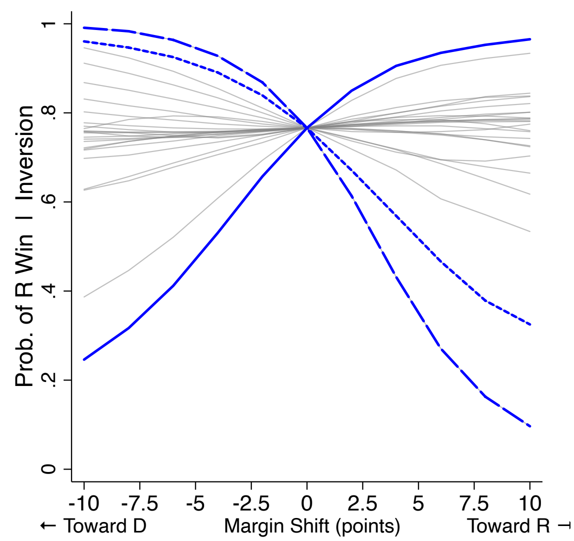
(A) Prob. of Inv., Cond'l on Election Within 1 pp



(B) Prob. of Republican EC Win

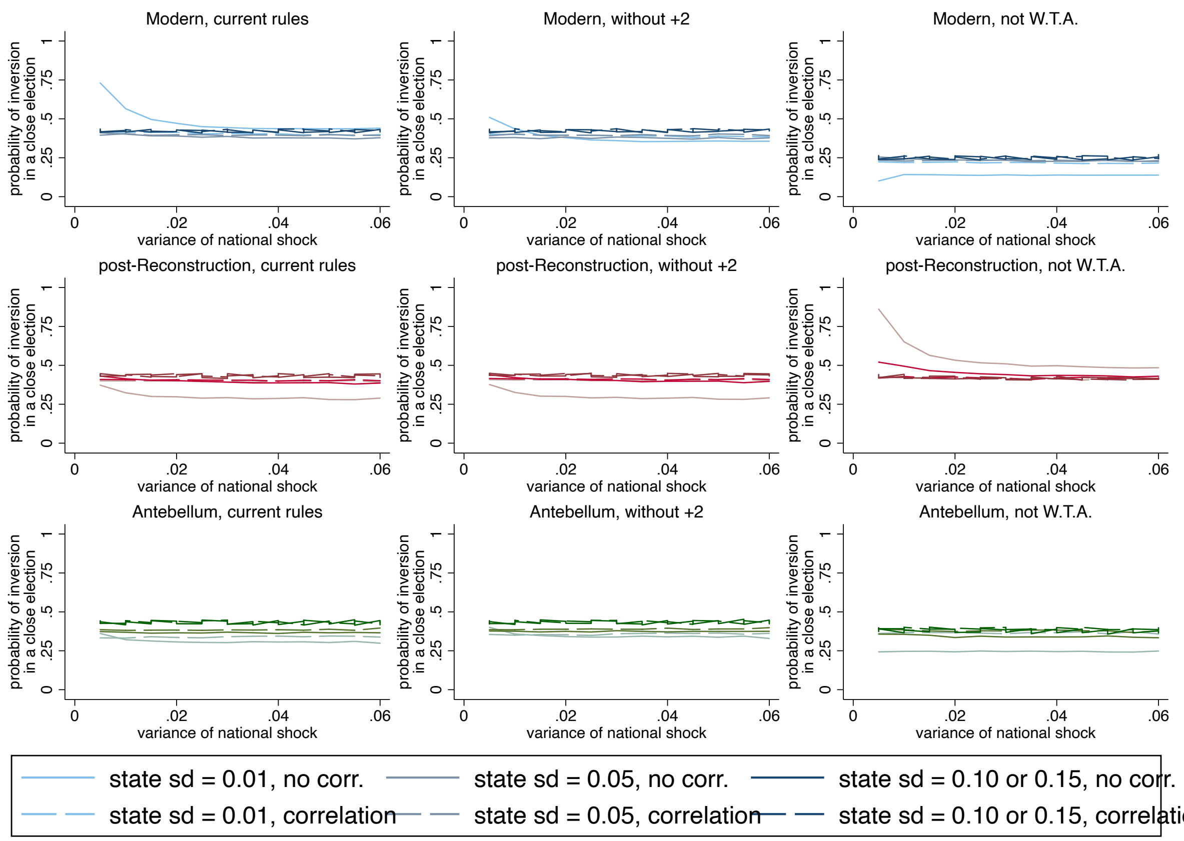


(C) Prob. of Republican EC Win, Cond'l on Inversion



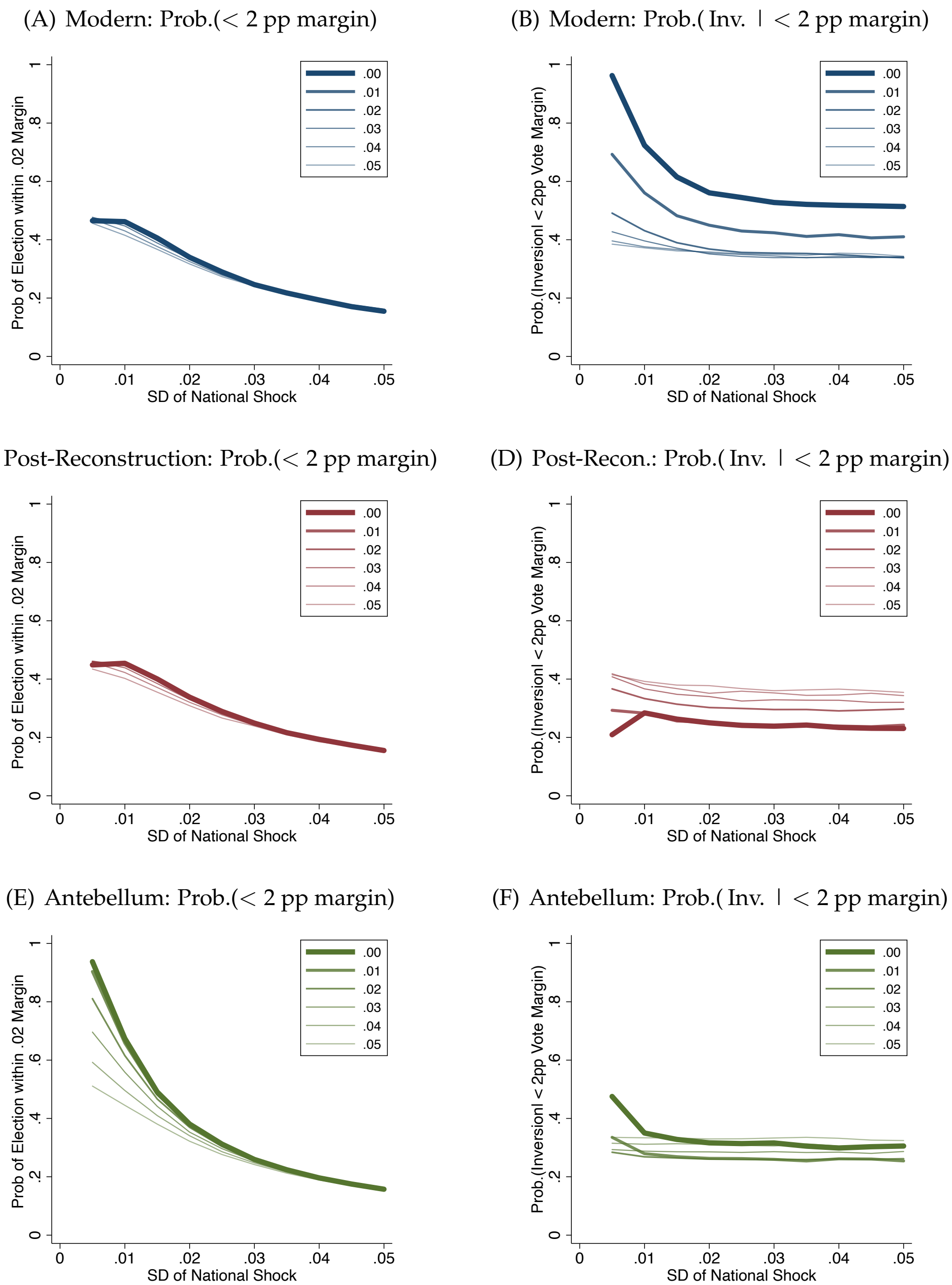
Note: Figure shows the impacts on key statistics of simulating a range of potential changes to the partisan alignment of states. Each point in each panel corresponds to a partisan shift relative to the $M1$ model estimates for the indicated set of states. Moving left to right across the horizontal axis steps through 10 such counterfactuals, where the partisan alignment of the indicated states is shifted in 2 percentage point increments from a 10 point margin shift toward Republican to a 10 point margin shift toward the Democrat. Panel A shows the impacts of these shifts on the conditional probability of an inversion in an election decided by within one percentage point, $\pi(.01)$. Panel B shows the impacts on the unconditional probability of a Republican EC victory. Panel C shows the impacts on the probability of a Republican EC victory, conditional on an inversion occurring. All corresponding estimates used to generate the figure are displayed in Tables A2, A4, and A3.

Figure A4: Robustness: Iterating Over a Grid of Exogenously Set Parameters



Note: Figure calculates inversion probabilities under a set of exogenously specified variances and correlations. The error terms from the data-generating process in Equation 1 of the main text are assumed to follow $\gamma_t \sim N(0, \sigma_\gamma)$ and $\phi_{st} \sim N(0, \sigma_\phi)$ as in the baseline models (M1, R1, A1). Specifically, these models implement Eq (1) as a random national shock (with magnitude as indicated along the horizontal axes) plus a random state-specific shock (at a standard deviation of 1, 5, 10, or 15 percentage points as indicated), plus (for the models labelled with "correlation") a random regionally correlated shock at a standard deviation of 5 percentage points. Each panel plots the probability of an inversion conditional on a 1.55 percentage point popular vote margin or less (which corresponds to 2 million popular votes at 2016 turnout). Note that these 288 models use only state-specific means from past election data. The variances are exogenously specified as hyperparameters. The second and third columns of plots expand on the decomposition described in Section 5: "Without +2 Senators" allocates each state a number of electors equal to its number of Representatives, without two electors for Senators. "Proportional" divides the whole number of electors per state between parties.

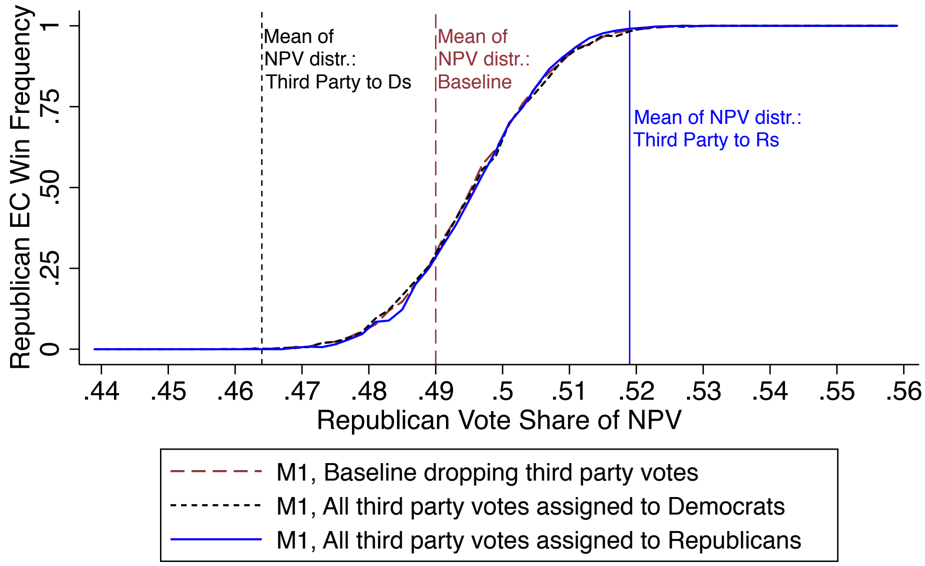
Figure A5: Robustness: Iterating Over a Grid of Exogenously Set Parameters (further detail)



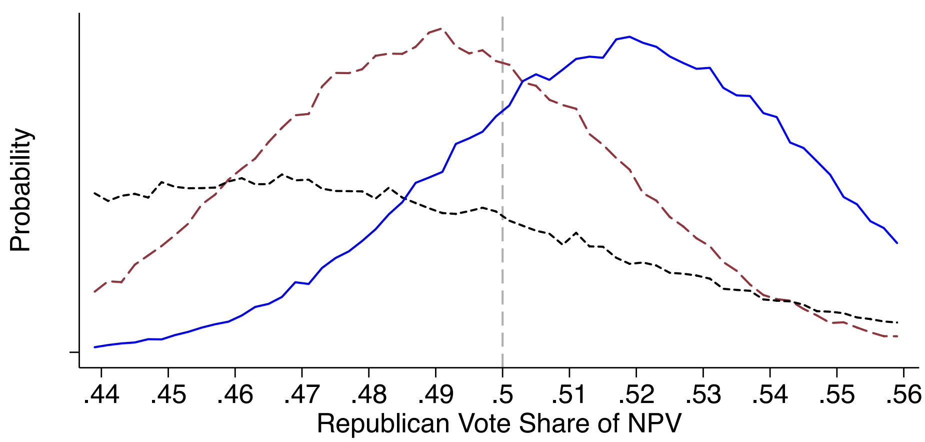
Note: Figure calculates inversion probabilities under a set of exogenously specified variances and correlations. The error terms from the data-generating process in Equation 1 of the main text are assumed to follow $\gamma_t \sim N(0, \sigma_\gamma)$ and $\phi_{st} \sim N(0, \sigma_\phi)$ as in the baseline models (M1, R1, A1). We cycle over a grid of values for σ^2_γ and σ^2_ϕ, rather than relying on estimates. The variance of the national shock increases along the horizontal axis in each panel. The variance of the state shocks are traced in several contour lines in each panel, as indicated. In the panels on the left, we report the probability of close elections. In the panels on the right, we report inversion probabilities, conditional on close elections within the same margins.

Figure A6: Results Are Robust to Any Treatment of Third-Party Votes

(A) Probability of Republican EC Win at Each NPV Under Various Third-Party Treatments

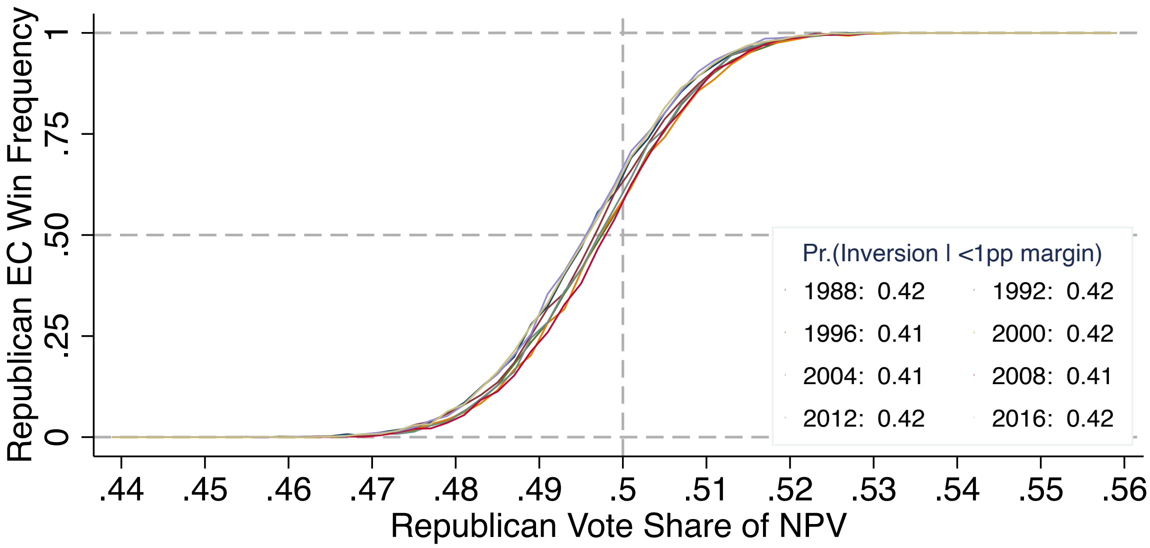


(B) NPV Distribution Under Various Third-Party Treatments



Note: Figure demonstrates robustness of estimates to extreme treatments of third-party votes, plotting the conditional win function for three models that treat third-party votes differently. The results here estimate the M1 model after either assigning all third-party votes to the Democratic candidate or after assigning all third-party votes to the Republican candidate. The original M1 model, which ignores third-party votes, is also plotted for reference. As the vertical lines show, this counterfactual assignment makes a large difference to the central tendency of the distribution of popular votes. However, it does not change the object of interest: the conditional probability of winning the EC as a function of the popular vote outcome.

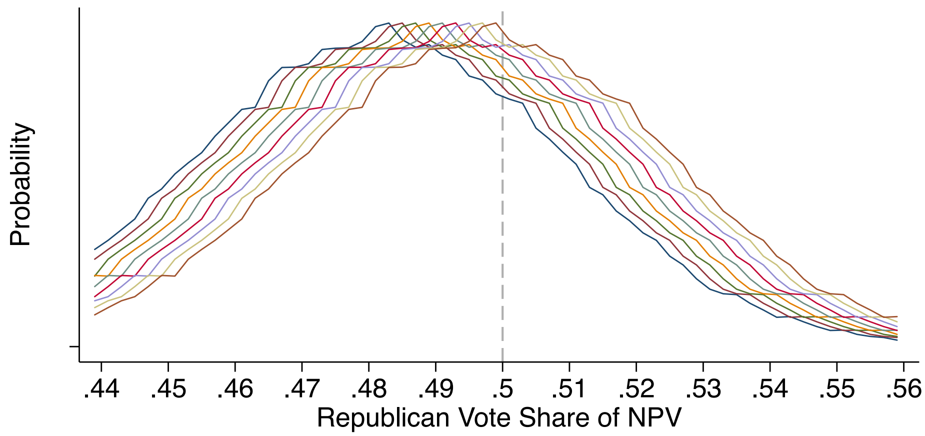
Figure A7: Results Are Robust to Alternative Turnout Weights



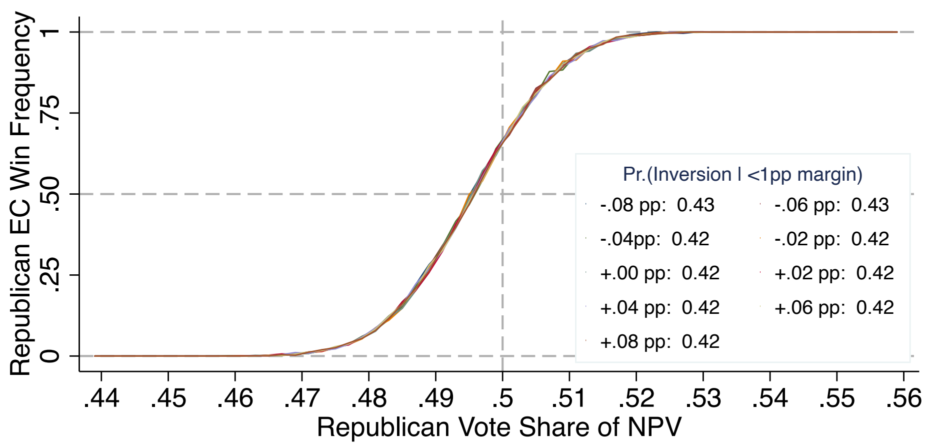
Note: Figure demonstrates robustness of estimates to various assumptions regarding turnout. In order to convert state vote shares, V_{st}, into a national popular vote tally, it is necessary to scale V_{st} by voter turnout. The figure replicates the conditional win function for the M1 model (modern period) eight times, in each case assigning different turnout and EC apportionment when tallying the popular vote and EC ballots across states. Lines in the overlay plot correspond to setting turnout and EC representation to 1988, 1992, 1996, 2000, 2004, 2008, 2012, and 2016. The box lists probabilities of an inversion in each model, conditional on an NPV victory margin within one percentage point.

Figure A8: Results Are Robust to Hybrid Models that Shift Means of Estimated Distributions

(A) Conditional Win Rate Function Under Various Shifts to the Estimated Distribution



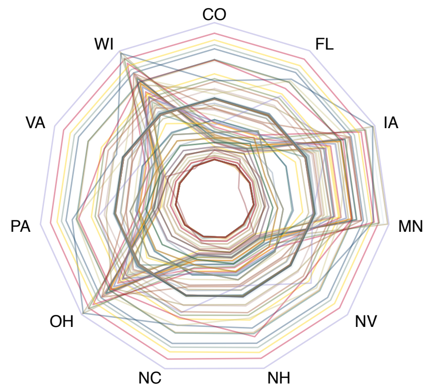
(B) NPV Distribution Under Various Shifts to the Estimated Distribution



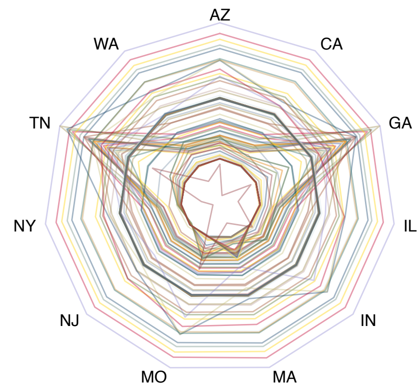
Note: Figure demonstrates robustness of estimates to hybrid models that first estimate parameters for model M1 and then mechanically shift each state mean left or right by a uniform percentage point margin. Lines in the overlay plot correspond to setting the shift at: $\{-.8, -.6 - .4, -.2, 0, +.2, +.4, +.6, +.8\}$. The distribution of the national popular vote is displayed in panel A, and the conditional win function is shown in panel B. The box lists probabilities of an inversion in each model, conditional on an NPV victory margin within one percentage point.

Figure A9: Robustness: The Conditional Probability of Inversion is Invariant to Model & Parameter Uncertainty, Despite That Other Statistics Are Not (Same 109 Models as Figure 3)

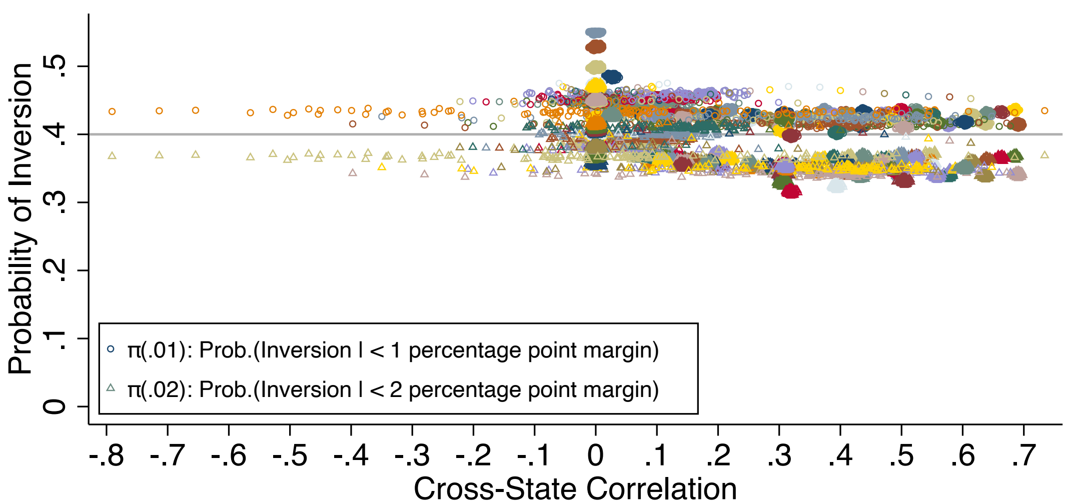
(A) Within-Election Correlation: MI with Other Swing States (Same 109 Models)



(B) Within-Election Correlation: TX with Other Large States (Same 109 Models)

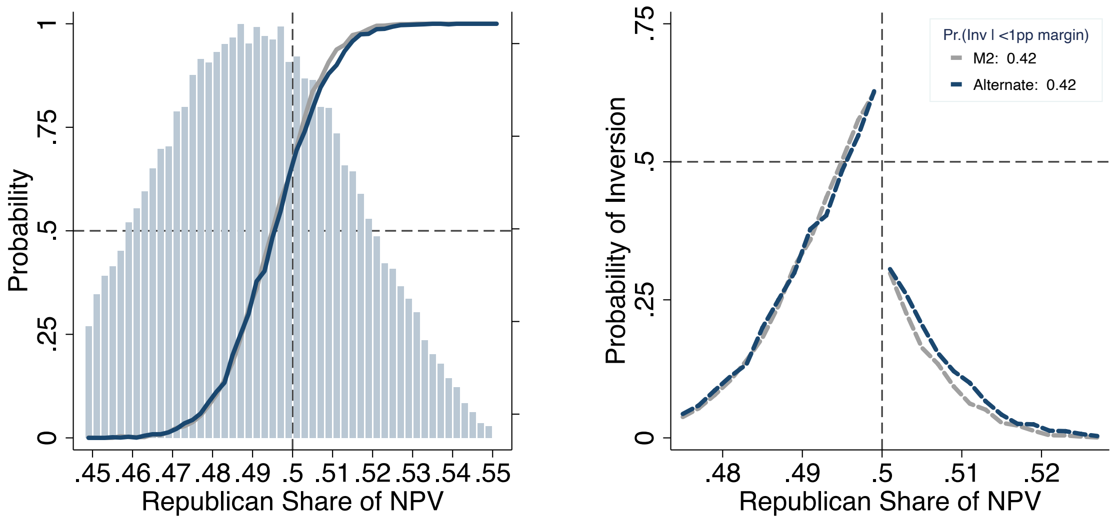


(C) Inversion Probabilities are Independent of the Cross-State Correlation Structure (Same 109 Models)



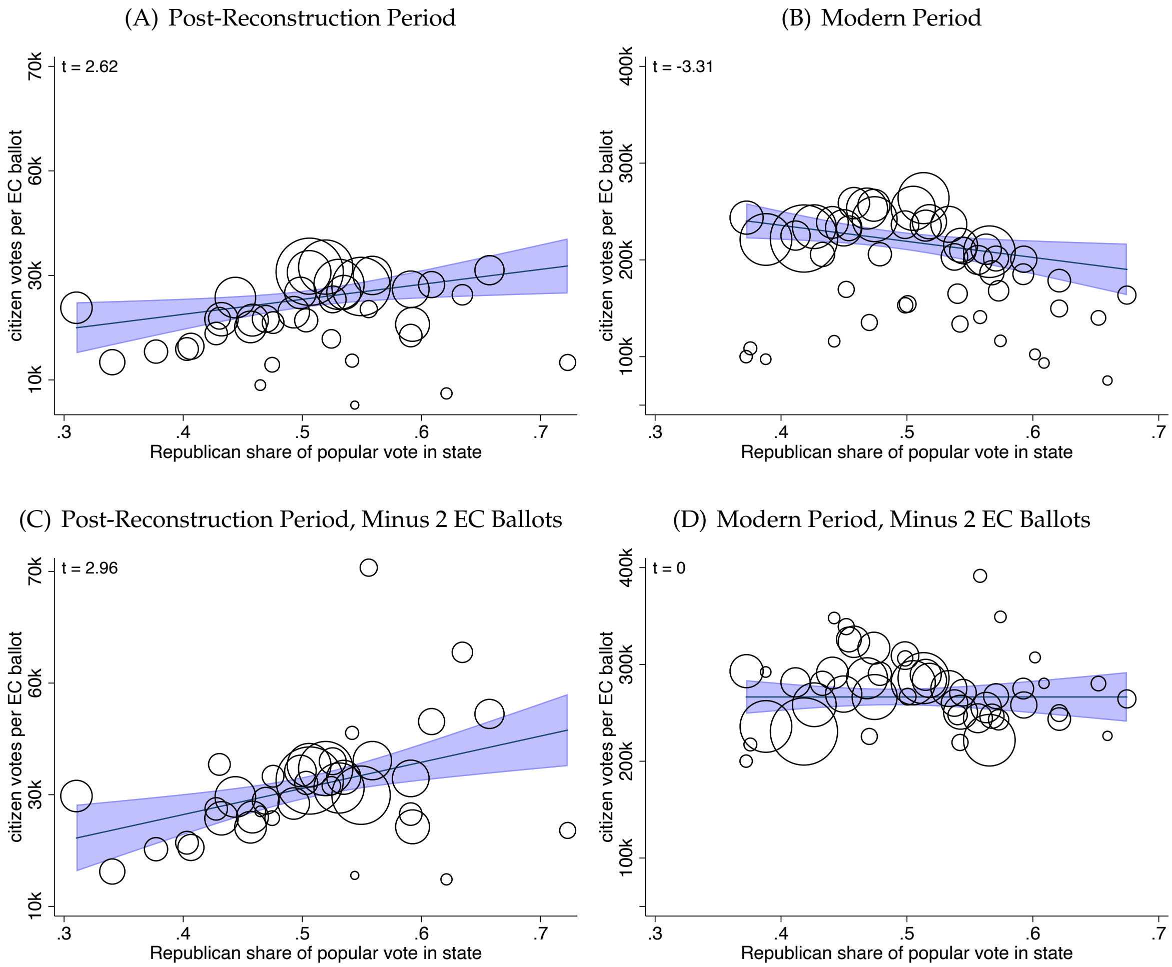
Note: Figure shows additional statistics corresponding to the models displayed in Figure 3. See Table A5 for a detailed listing of each model included. Radar plots in Panels A and B compare the within-year, across-state correlations between Michigan and 11 other swing states and between Texas and 11 other large, non-swing states. Panel C plots inversion rates against the across-state correlations in the voting outcomes for each model. Each model generates 2,550 points in Panel C: 2 statistics per model times the 51 state lower triangular correlation matrix (1275 correlations) for each model. Points in Panel C are jittered.

Figure A10: Robustness to Alternative Parameterization of Race- and Education-Linked Shocks



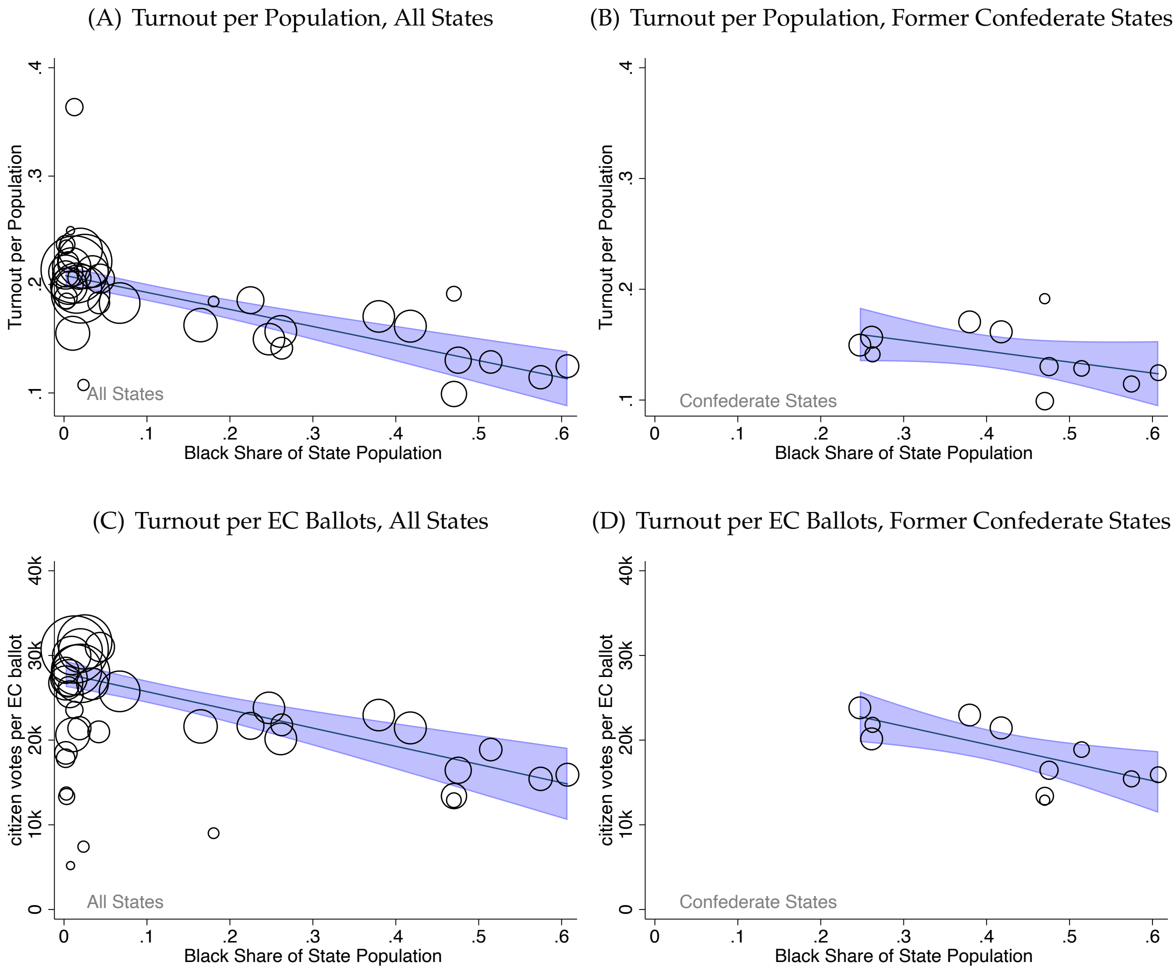
Note: Figure shows inversion probabilities and probability distributions over national popular vote outcomes implied by the parametric estimates of model M2 compared to an alternate model that changes how state characteristics are parameterized in the shock term. The alternate model allows for race-linked shocks to multiply an **X** vector that includes region indicators, % non-hispanic white, % non-hispanic black, % hispanic, % college degree, and % high school completion in the state. This contrasts with M2, where **X** includes only % non-white and % college degree. Each model simulation consists of 100,000 simulated election draws. The M2 model is plotted in gray for reference behind the alternate model in blue. The histogram corresponds to the alternate model. The Republican share of the national popular vote runs along the horizontal axis. The solid blue line is the conditional probability of a Republican electoral win at each level of the national popular vote share. The box lists probabilities of an inversion in each model, conditional on an NPV victory margin within one percentage point. See Figure 2 and Appendix E.4 for additional notes.

Figure A11: Asymmetry in the Post-Reconstruction and Modern Periods



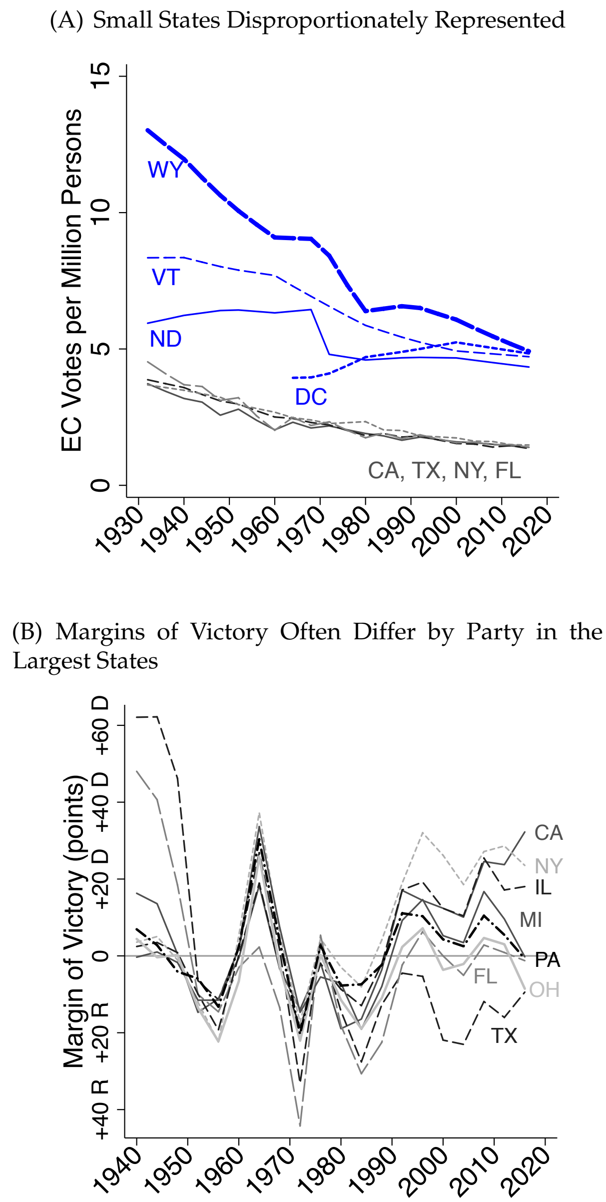
Note: Figure shows the relationship between partisan alignment and EC representation of voters in the post-Reconstruction and Modern eras. Each circle is a state, with size proportional to turnout. Voter turnout and EC electors per state are based on averages over the indicated sample periods. The Republican share (horizontal axes) is the state mean over the indicated period. OLS lines and 95% confidence intervals are displayed. The vertical axes plot the number of citizen votes in a presidential election divided by the number of EC electors apportioned to the state. In the post-Reconstruction period, Democratic alignment was correlated with EC ballots being controlled by fewer citizen votes. See Appendix E.2 for additional detail.

Figure A12: During the Post-Reconstruction Era, Turnout Was Depressed and EC Representation of White Voters Was Inflated Because Black Votes Were Suppressed



Note: Figure shows that turnout per population was strongly negatively correlated with the black share of the state population in the post-Reconstruction period. States with larger black populations could control an EC ballot with fewer citizen votes. Populations are calculated from the 1880 Census. Voter turnout and EC electors per state are based on averages over the 1872–1888 period. In Panels A and B, the vertical axes plot turnout per total population (white and black). In Panels C and D the vertical axes plot the number of citizen votes in a presidential election divided by the number of EC electors apportioned to the state. Left panels include all states; right panels include former Confederate states. OLS lines and 95% confidence intervals are displayed. See Appendix E.2 for additional detail.

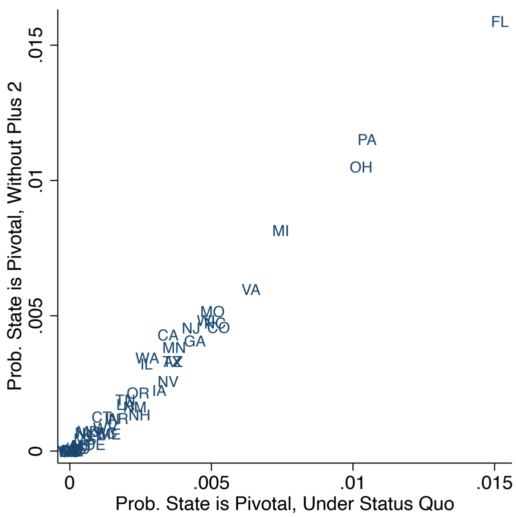
Figure A13: Possible Sources of Inversion in the Electoral College



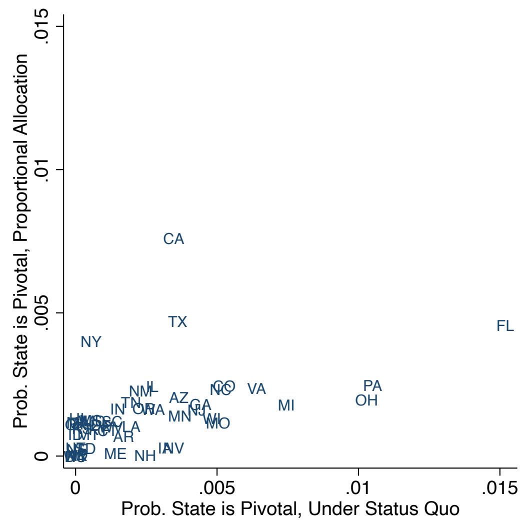
Note: Panel A plots, for the four largest and four smallest states by today's population, the state's apportionment of EC electors divided by its population. Panel B plots the average vote margins over time by Democrat and Republican candidates for the largest states.

Figure A14: New Battlegrounds: Probability of Being Pivotal Under Various EC Counterfactuals

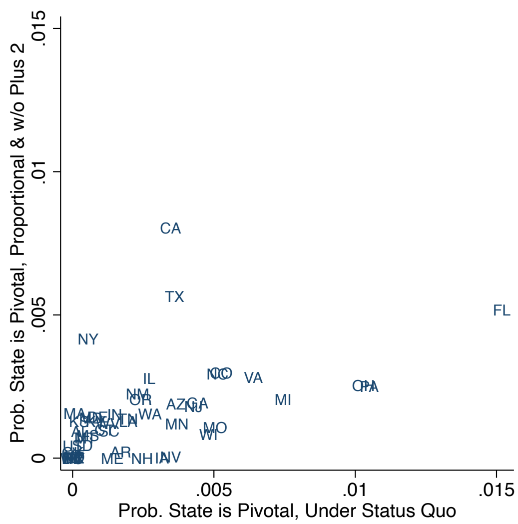
(A) Subtracting 2 EC Electors per State



(B) Awarding EC Votes Proportionally

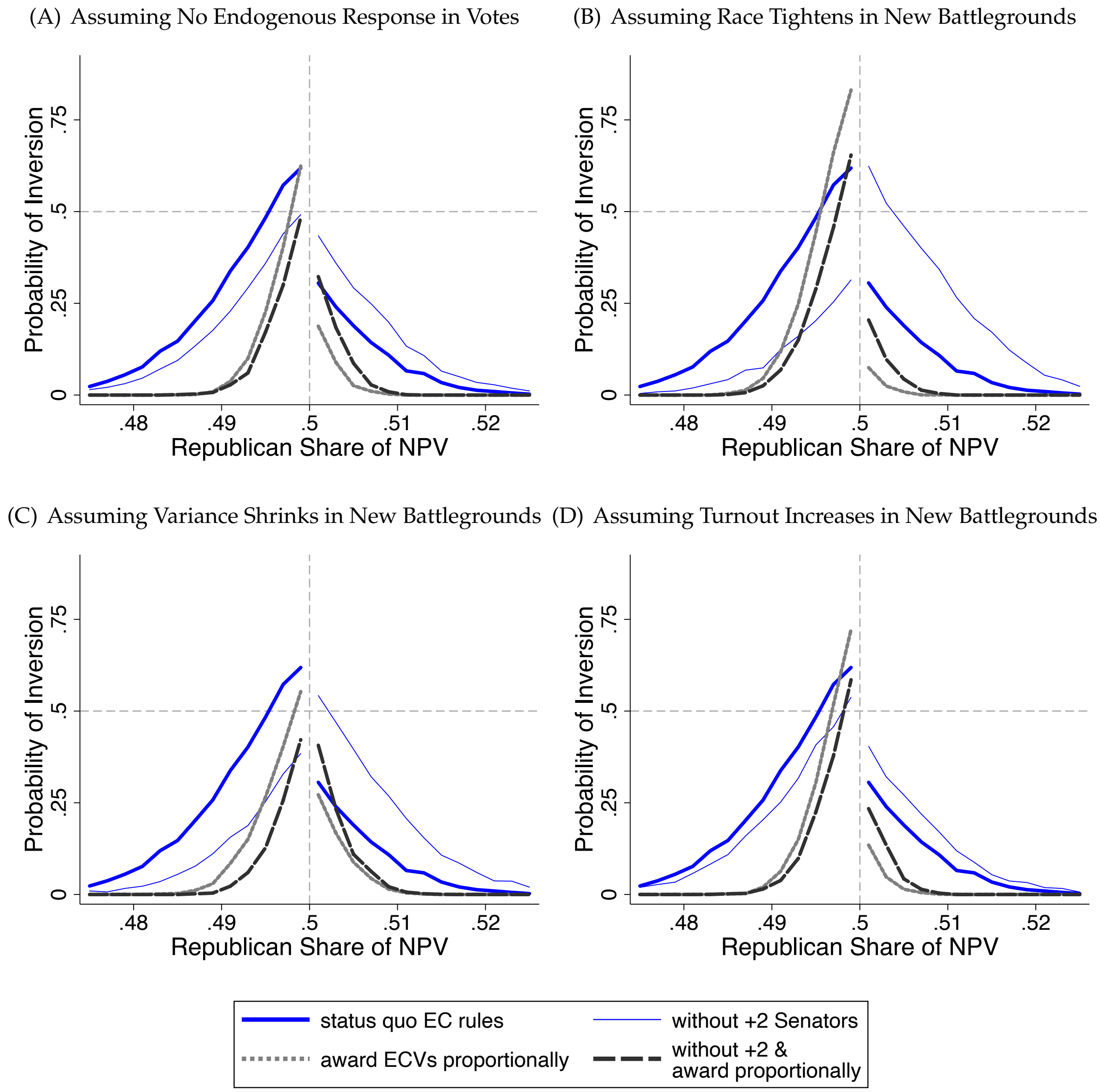


(C) Awarding Proportionally and Subtracting 2



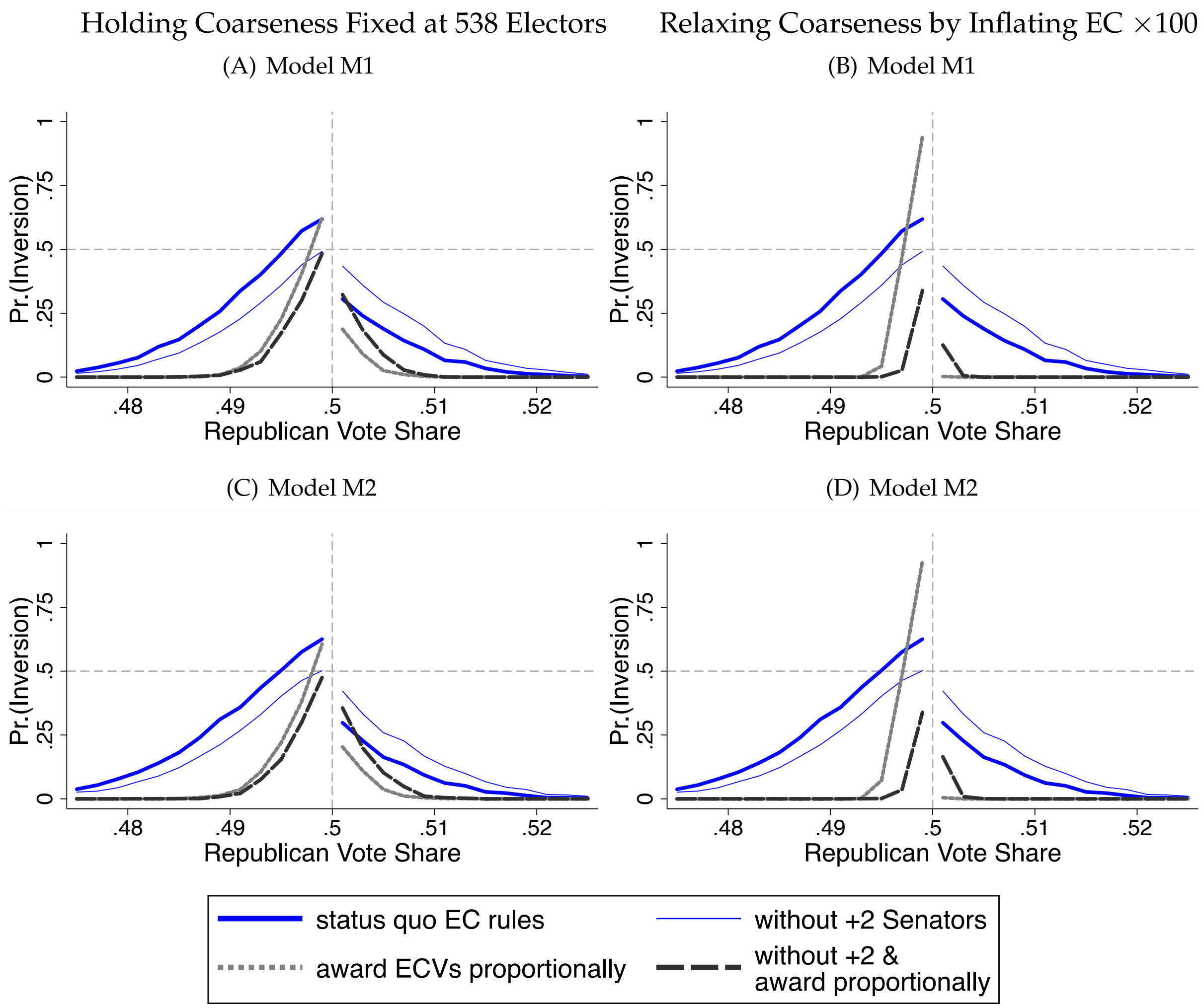
Note: Figure plots the probability that a state is pivotal under status quo EC rules ($\hat{Q}_s^0$, horizontal axes) and various counterfactual EC rules ($\hat{Q}_s^{CF}$, vertical axes). The counterfactuals considered are indicated in the panel headings. To calculate $\hat{Q}_s^{CF}$, we re-assign 0.5% of statewide votes from the state winner to the state loser for each of the original 100,000 simulation runs in Model M1. We then calculate the fraction of simulated elections in which this reassignment of votes would flip the EC outcome. See Appendix F for additional detail.

Figure A15: Counterfactuals that Allow for Endogenous Responses to a Changing Electoral Map



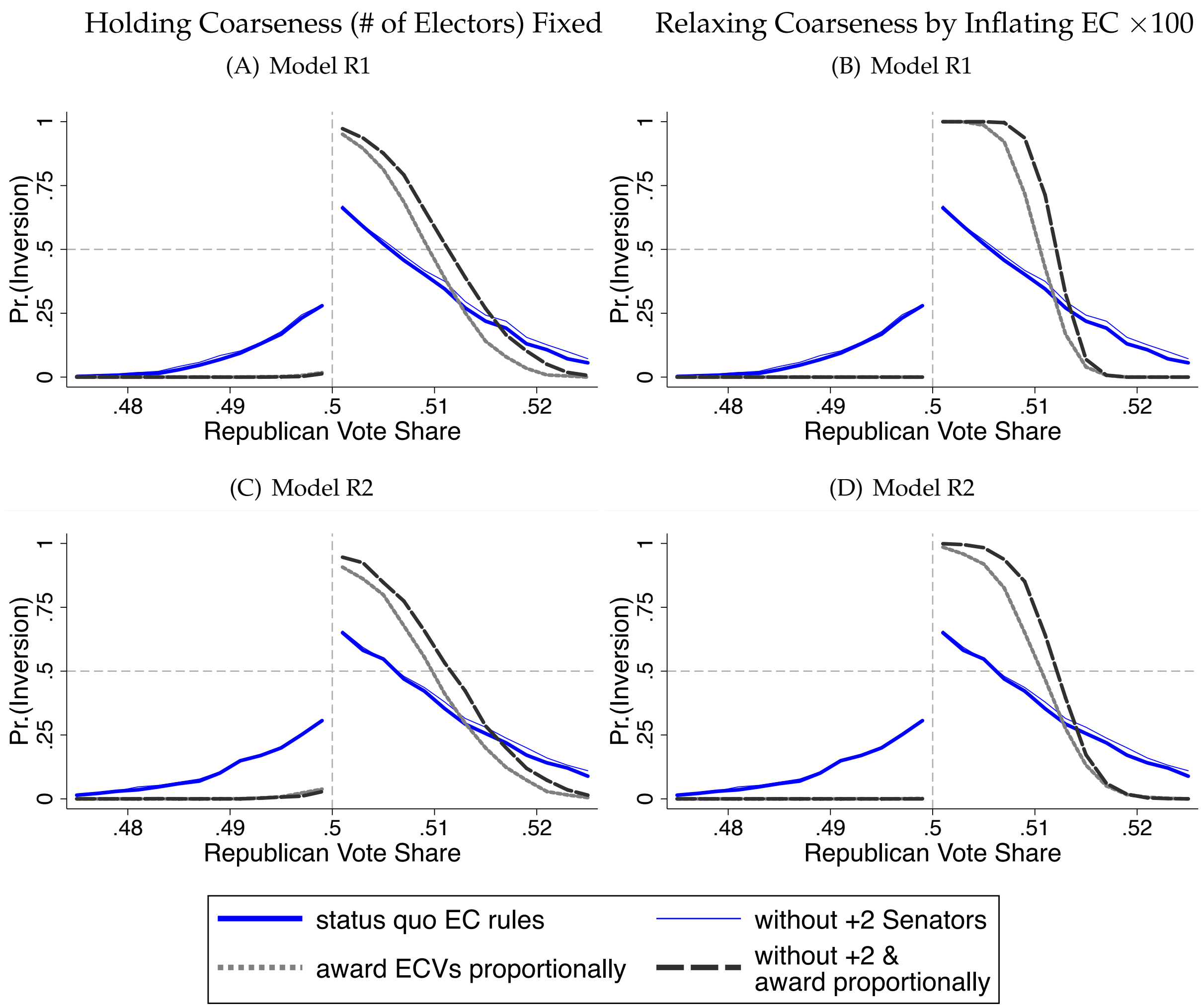
Note: Figure shows how the counterfactuals for the M1 model in Panel A (repeated from Panel A of Figure 4) change under simulations that allow for endogenous response to the counterfactuals. We incorporate a stylized, reduced-form representation of behavioral responses to the changing electoral map, as states move in or out of "battleground" and "safe" status under the counterfactual EC aggregation rules. The panels show results for different assumptions. In Panel B, we assume that margins will tighten in new battleground states. In Panel C, we assume that variances of potential voting outcomes will shrink in new battleground states. In Panel D, we assume that turnout will increase in new battleground states. See Appendix F for additional detail.

Figure A16: Electoral Inversions Under Alternative Aggregation: Modern Period



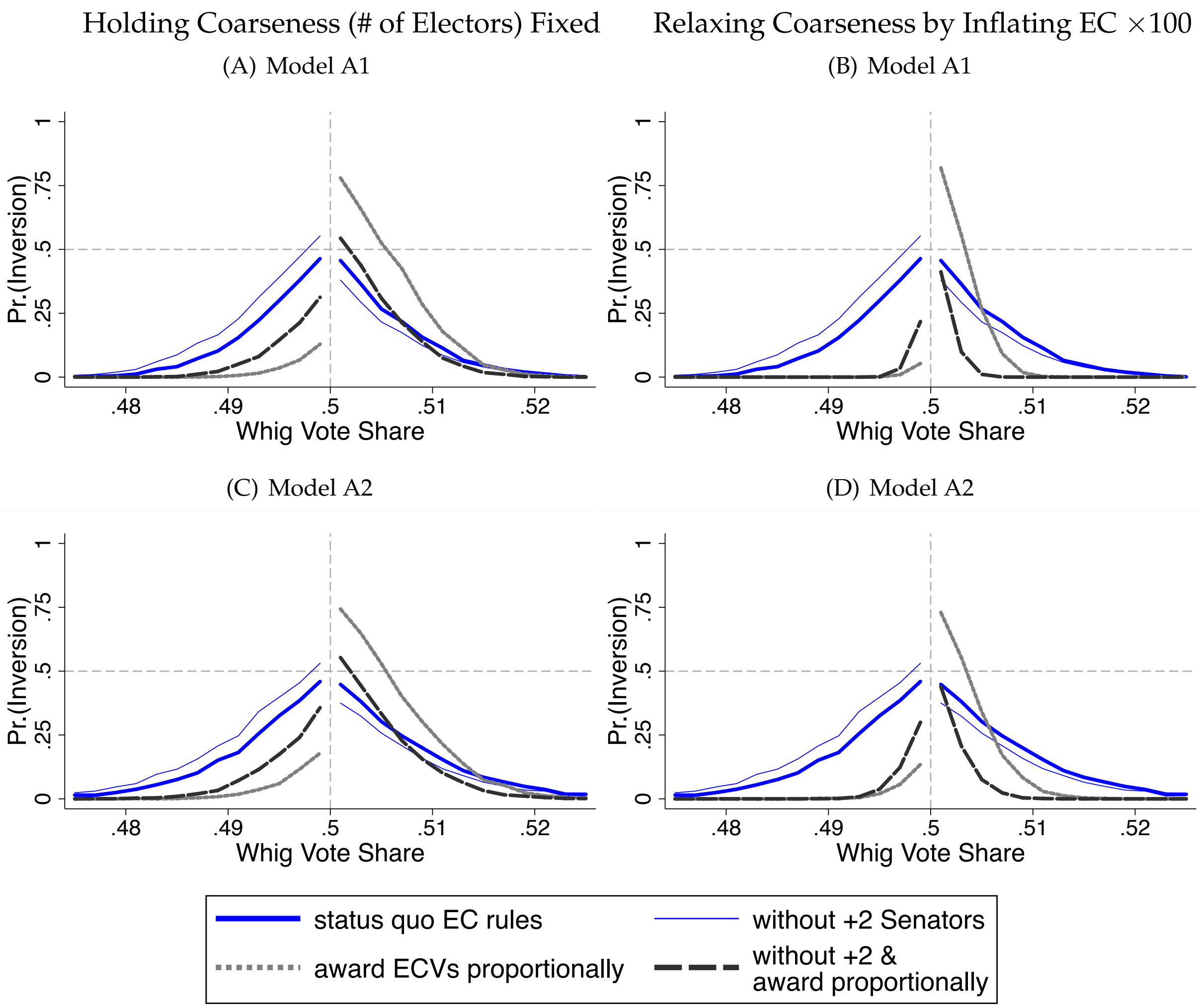
Note: Figure illustrates inversions under alternative EC aggregation rules that translate citizen votes into a presidential winner. The alternative that removes the two Senator-derived EC electors assigns each state electors equal in number to the size of the state's US House delegation. The alternative that removes the winner-takes-all condition awards state EC votes (ECVs) according to each candidate's popular vote share in the state, up to a rounding error. The left panels holds the congressional apportionment fixed at 435 House members and 2 Senators per state. The right panel inflates the congressional delegation size by 100 times to examine the impact of relaxing the rounding error ("coarseness") constraint. In these, each state receives 200 electors corresponding to Senators and these 200 are removed in the "without Senators" alternatives.

Figure A17: Electoral Inversions Under Alternative Aggregation: Post-Reconstruction Period



Note: Figure illustrates inversions under alternative EC aggregation rules that translate citizen votes into a presidential winner. The alternative that removes the two Senator-derived EC electors assigns each state electors equal in number to the size of the state's US House delegation. The alternative that removes the winner-takes-all condition awards state EC votes (ECVs) according to each candidate's popular vote share in the state, up to a rounding error. The left panels holds the congressional apportionment fixed at 435 House members and 2 Senators per state. The right panel inflates the congressional delegation size by 100 times to examine the impact of relaxing the rounding error ("coarseness") constraint. In these, each state receives 200 electors corresponding to Senators and these 200 are removed in the "without Senators" alternatives.

Figure A18: Electoral Inversions Under Alternative Aggregation: Antebellum Period



Note: Figure illustrates inversions under alternative EC aggregation rules that translate citizen votes into a presidential winner. The alternative that removes the two Senator-derived EC electors assigns each state electors equal in number to the size of the state's US House delegation. The alternative that removes the winner-takes-all condition awards state EC votes (ECVs) according to each candidate's popular vote share in the state, up to a rounding error. The left panels holds the congressional apportionment fixed at 435 House members and 2 Senators per state. The right panel inflates the congressional delegation size by 100 times to examine the impact of relaxing the rounding error ("coarseness") constraint. In these, each state receives 200 electors corresponding to Senators and these 200 are removed in the "without Senators" alternatives.

Table A1: ML Parameter Estimates for Variance Terms

	Modern Period: 1988-2016								post-Reconstruction: 1872-1888				Antebellum: 1836-1852		
	State and National Shocks M1 (1)	State, National, & Other Shocks M2 (2)	State Shocks Only M5 (3)	State, National, & Other Shocks M7 (4)	State, National, & Other Shocks M8 (5)	State, National, & Other Shocks M9 (6)	Omitting Inversion s 2000 & 2016 M10 (7)	Extended M1: 1964-2016 M12 (8)	State and National Shocks R1 (9)	State, National, & Other Shocks R2 (10)	State Shocks Only R5 (11)	Omitting Inversion s 1876 & 1888 R10 (12)	State and National Shocks A1 (13)	State, National, & Other Shocks A2 (14)	State Shocks Only A5 (15)
σ_y	0.012	0.014		0.015	0.012	0.013	0.014	0.053	0.021	0.016		0.026	0.013	0.011	
σ_s	0.027	0.013	0.039	0.022	0.018	0.016	0.022	0.051	0.041	0.038	0.062	0.038	0.020	0.018	0.032
Other shocks															
region		0.005								0.004				0.002	
race		0.198		0.240		0.129									
education		1.396			2.208	1.737									
# of α_s estimates (states)	51	51	51	51	51	51	51	51	37	37	37	37	25	25	25
										(estimation omits CO)				(omits CA, FL, IA, TX, WI)	

Note: Table reports maximum likelihood estimates of the parameters in Equation (1) of the main text under various model specifications, as indicated in the column headers. Model estimates in the table are grouped by period. Either 25 (because we only use states that were present throughout the Antebellum period for estimating variances), 37, or 51 expected state vote share parameters, $\bar{\alpha}_s$, are also estimated by joint maximum likelihood but not reported here. Depending on the data period, some states were not present for all election years within the sample frame or did not use a statewide citizen vote to determine EC votes. These, including Colorado in the post-Reconstruction frame, are simply assigned their mean for $\bar{\alpha}_s$ and do not contribute to estimating variance parameters.

Table A2: Stability: Probability of Inversion in Races Decided by <1pp Margin ($\pi(.01)$), After Imposing Alignment Shifts

	Cells Report π(.01): Probability of Inversion in Elections Decided by Within 1 Point										
	Assumed Shift, in Vote Margin Points										
Set of shifted states	Shift to More Democratic-Leaning							Shift to More Republican-Leaning			
	-10	-8	-6	-4	-2	Baseline	2	4	6	8	10
All swing states	0.40	0.41	0.40	0.41	0.41	0.42	0.43	0.44	0.44	0.45	0.46
All non-swing states	0.54	0.50	0.49	0.47	0.43	0.42	0.41	0.40	0.42	0.44	0.45
Other large states	0.48	0.48	0.46	0.44	0.43	0.42	0.41	0.40	0.41	0.42	0.41
States won by Trump 2016	0.40	0.41	0.39	0.41	0.42	0.42	0.43	0.42	0.43	0.43	0.44
States won by Clinton 2016	0.49	0.46	0.46	0.44	0.43	0.42	0.41	0.41	0.39	0.39	0.39
Swing states won by Trump 2016	0.41	0.40	0.40	0.41	0.41	0.42	0.43	0.43	0.44	0.44	0.44
Swing states won by Clinton 2016	0.41	0.41	0.41	0.42	0.42	0.42	0.43	0.43	0.43	0.43	0.44
Swing county fraction	0.42	0.43	0.42	0.43	0.42	0.42	0.42	0.42	0.41	0.41	0.41
Swing states (individually)											
Colorado	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.42	0.42	0.43
Florida	0.41	0.41	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.43	0.43
Iowa	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.43
Michigan	0.43	0.43	0.42	0.42	0.42	0.42	0.43	0.43	0.42	0.43	0.43
Minnesota	0.43	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43
Nevada	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.43	0.43	0.43
New Hampshire	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.43	0.43	0.43
North Carolina	0.41	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.42
Ohio	0.41	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.43	0.43
Pennsylvania	0.42	0.42	0.43	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.43
Virginia	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42
Wisconsin	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43
Other large states (individually)											
California	0.45	0.44	0.44	0.43	0.43	0.42	0.42	0.42	0.41	0.41	0.41
Texas	0.42	0.42	0.42	0.43	0.42	0.42	0.42	0.42	0.42	0.41	0.41
New York	0.44	0.44	0.43	0.43	0.43	0.42	0.42	0.42	0.42	0.41	0.41
Illinois	0.43	0.43	0.43	0.42	0.43	0.42	0.42	0.42	0.42	0.42	0.42
Georgia	0.42	0.42	0.42	0.42	0.43	0.42	0.42	0.42	0.42	0.42	0.42

Note: Rows specify the sets of states that is shifted for the exercise. Column headers indicate the size of the imposed shift in the states' vote share distribution. For example, row 1 shifts the state-specific sampling distribution for all swing states left (Democratic) or right (Republican). The shift spans -10 points to $+10$ points in two-point increments. Other rows repeat the exercise for all non-swing states, other large states, states won by Trump in 2016, states won by Clinton in 2016, etc., as indicated. Swing county fraction, from (Cullen, Turner and Washington, 2018), is the fraction of counties in a state that changed presidential party in elections from 1988 to 2008. (Alaska, which Cullen, Turner and Washington, 2018 omit, is assigned the national median.) This fraction multiplies the indicated shift, so that a state made up entirely of swing counties receives the full shift, and a state with no swing counties receives no shift. Each permutation cell in the tables represents 100,000 simulation draws.

Table A3: Stability: Probability of Republican EC Win, After Imposing Alignment Shifts

	Cells Report Unconditional Probability of Republican EC Victory										
	Assumed Shift, in Vote Margin Points										
Set of shifted states	Shift to More Democratic-Leaning										Shift to More Republican-Leaning
	-10	-8	-6	-4	-2	Baseline	2	4	6	8	10
All swing states	0.12	0.16	0.21	0.28	0.34	0.42	0.50	0.58	0.65	0.71	0.77
All non-swing states	0.17	0.22	0.27	0.32	0.37	0.42	0.47	0.53	0.58	0.63	0.69
Other large states	0.33	0.35	0.37	0.39	0.41	0.42	0.44	0.45	0.47	0.50	0.52
States won by Trump 2016	0.10	0.15	0.20	0.26	0.34	0.42	0.51	0.60	0.68	0.76	0.83
States won by Clinton 2016	0.25	0.28	0.31	0.34	0.38	0.42	0.47	0.51	0.56	0.61	0.66
Swing states won by Trump 2016	0.18	0.22	0.26	0.31	0.36	0.42	0.48	0.54	0.59	0.64	0.68
Swing states won by Clinton 2016	0.33	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50	0.51
Swing county fraction	0.15	0.19	0.24	0.30	0.36	0.42	0.49	0.56	0.62	0.68	0.74
Swing states (individually)											
Colorado	0.40	0.40	0.41	0.41	0.42	0.42	0.43	0.43	0.43	0.44	0.44
Florida	0.34	0.35	0.37	0.39	0.40	0.42	0.43	0.45	0.46	0.46	0.47
Iowa	0.41	0.41	0.41	0.41	0.42	0.42	0.42	0.43	0.43	0.43	0.44
Michigan	0.40	0.40	0.40	0.41	0.41	0.42	0.43	0.44	0.45	0.46	0.46
Minnesota	0.41	0.41	0.41	0.41	0.42	0.42	0.42	0.43	0.43	0.44	0.45
Nevada	0.41	0.41	0.41	0.41	0.42	0.42	0.42	0.43	0.43	0.43	0.43
New Hampshire	0.41	0.41	0.41	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.43
North Carolina	0.38	0.39	0.40	0.41	0.41	0.42	0.43	0.43	0.43	0.43	0.43
Ohio	0.37	0.38	0.39	0.40	0.41	0.42	0.43	0.44	0.45	0.45	0.46
Pennsylvania	0.39	0.39	0.40	0.40	0.41	0.42	0.43	0.44	0.45	0.47	0.48
Virginia	0.39	0.39	0.40	0.41	0.41	0.42	0.43	0.43	0.43	0.44	0.44
Wisconsin	0.40	0.41	0.41	0.41	0.42	0.42	0.43	0.43	0.44	0.44	0.45
Other large states (individually)											
California	0.41	0.41	0.42	0.42	0.42	0.42	0.43	0.43	0.44	0.46	0.47
Texas	0.37	0.39	0.40	0.41	0.42	0.42	0.42	0.42	0.43	0.43	0.43
New York	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43
Illinois	0.42	0.42	0.42	0.42	0.42	0.42	0.42	0.43	0.43	0.44	0.45
Georgia	0.38	0.39	0.40	0.41	0.42	0.42	0.42	0.43	0.43	0.43	0.43

Note: Rows specify the sets of states that is shifted for the exercise. Column headers indicate the size of the imposed shift in the states' vote share distribution. For example, row 1 shifts the state-specific sampling distribution for all swing states left (Democratic) or right (Republican). The shift spans -10 points to $+10$ points in two-point increments. Other rows repeat the exercise for all non-swing states, other large states, states won by Trump in 2016, states won by Clinton in 2016, etc., as indicated. Swing county fraction, from (Cullen, Turner and Washington, 2018), is the fraction of counties in a state that changed presidential party in elections from 1988 to 2008. (Alaska, which Cullen, Turner and Washington, 2018 omit, is assigned the national median.) This fraction multiplies the indicated shift, so that a state made up entirely of swing counties receives the full shift, and a state with no swing counties receives no shift. Each permutation cell in the tables represents 100,000 simulation draws.

Table A4: Stability: Probability of Republican EC Win Conditional on Inversion Occurring, After Imposing Alignment Shifts

	Cells Report Probability of Republican EC Victory, Conditional on Inversion Occurring										
	Assumed Shift, in Vote Margin Points										
Set of shifted states	Shift to More Democratic-Leaning							Shift to More Republican-Leaning			
	-10	-8	-6	-4	-2	Baseline	2	4	6	8	10
All swing states	0.25	0.32	0.41	0.53	0.66	0.77	0.85	0.91	0.93	0.95	0.97
All non-swing states	0.99	0.98	0.96	0.93	0.87	0.77	0.61	0.43	0.27	0.16	0.10
Other large states	0.96	0.95	0.93	0.89	0.84	0.77	0.67	0.57	0.47	0.38	0.33
States won by Trump 2016	0.76	0.76	0.75	0.74	0.75	0.77	0.79	0.80	0.82	0.84	0.84
States won by Clinton 2016	0.95	0.92	0.89	0.85	0.81	0.77	0.72	0.67	0.61	0.57	0.53
Swing states won by Trump 2016	0.39	0.45	0.52	0.61	0.69	0.77	0.83	0.88	0.91	0.92	0.93
Swing states won by Clinton 2016	0.63	0.65	0.68	0.71	0.73	0.77	0.79	0.81	0.83	0.83	0.84
Swing county fraction	0.80	0.79	0.78	0.78	0.77	0.77	0.77	0.76	0.75	0.75	0.74
Swing states (individually)											
Colorado	0.74	0.74	0.75	0.75	0.76	0.77	0.77	0.78	0.78	0.78	0.78
Florida	0.63	0.66	0.69	0.72	0.74	0.77	0.78	0.78	0.78	0.78	0.76
Iowa	0.76	0.76	0.76	0.76	0.76	0.77	0.77	0.78	0.78	0.79	0.79
Michigan	0.77	0.76	0.76	0.76	0.76	0.77	0.78	0.79	0.80	0.81	0.82
Minnesota	0.78	0.77	0.77	0.76	0.76	0.77	0.77	0.78	0.79	0.79	0.80
Nevada	0.74	0.74	0.74	0.75	0.76	0.77	0.77	0.78	0.78	0.78	0.79
New Hampshire	0.75	0.75	0.75	0.76	0.76	0.77	0.77	0.77	0.78	0.78	0.78
North Carolina	0.72	0.74	0.75	0.76	0.76	0.77	0.76	0.76	0.75	0.74	0.72
Ohio	0.70	0.71	0.72	0.73	0.75	0.77	0.78	0.79	0.79	0.79	0.79
Pennsylvania	0.76	0.75	0.74	0.75	0.75	0.77	0.78	0.80	0.81	0.83	0.84
Virginia	0.72	0.73	0.74	0.75	0.76	0.77	0.77	0.77	0.77	0.76	0.76
Wisconsin	0.76	0.76	0.76	0.76	0.76	0.77	0.77	0.78	0.79	0.80	0.80
Other large states (individually)											
California	0.91	0.89	0.86	0.83	0.80	0.77	0.74	0.71	0.69	0.69	0.70
Texas	0.76	0.79	0.79	0.79	0.78	0.77	0.74	0.72	0.69	0.65	0.62
New York	0.87	0.85	0.83	0.81	0.79	0.77	0.74	0.72	0.70	0.68	0.66
Illinois	0.83	0.82	0.80	0.79	0.78	0.77	0.76	0.76	0.76	0.76	0.77
Georgia	0.72	0.74	0.75	0.76	0.76	0.77	0.76	0.76	0.75	0.74	0.73

Note: Rows specify the sets of states that is shifted for the exercise. Column headers indicate the size of the imposed shift in the states' vote share distribution. For example, row 1 shifts the state-specific sampling distribution for all swing states left (Democratic) or right (Republican). The shift spans -10 points to $+10$ points in two-point increments. Other rows repeat the exercise for all non-swing states, other large states, states won by Trump in 2016, states won by Clinton in 2016, etc., as indicated. Swing county fraction, from (Cullen, Turner and Washington, 2018), is the fraction of counties in a state that changed presidential party in elections from 1988 to 2008. (Alaska, which Cullen, Turner and Washington, 2018 omit, is assigned the national median.) This fraction multiplies the indicated shift, so that a state made up entirely of swing counties receives the full shift, and a state with no swing counties receives no shift. Each permutation cell in the tables represents 100,000 simulation draws.

Table A5: Model List for Figure 3

Model Index	Parametric/ Bootstrap?	Empirical/ Hyperparam./ Hybrid	Description
1	Parametric	Empirical	M1 (see paper)
2	Parametric	Empirical	M2 (see paper)
3	Bootstrap	Empirical	M3 (see paper)
4	Bootstrap	Empirical	M4 (see paper)
5	Parametric	Empirical	M5 (see paper)
6	Bootstrap	Empirical	M6 (see paper)
7	Parametric	Empirical	M7 (see paper)
8	Parametric	Empirical	M8 (see paper)
9	Parametric	Empirical	M9 (see paper)
10	Parametric	Empirical	M10 (see paper)
11	Bootstrap	Empirical	M11 (see paper)
12	Parametric	Empirical	M12 (see paper)
13	Parametric	Empirical	M1-like; sets state turnout and EC allocations according to 1988
14	Parametric	Empirical	M1-like; sets state turnout and EC allocations according to 1992
15	Parametric	Empirical	M1-like; sets state turnout and EC allocations according to 1996
16	Parametric	Empirical	M1-like; sets state turnout and EC allocations according to 2000
17	Parametric	Empirical	M1-like; sets state turnout and EC allocations according to 2004
18	Parametric	Empirical	M1-like; sets state turnout and EC allocations according to 2008
19	Parametric	Empirical	M1-like; sets state turnout and EC allocations according to 2012
20	Parametric	Empirical	M1-like; assigns all third party votes to Republicans before estimation
21	Parametric	Empirical	M1-like; assigns all third party votes to Democrats before estimation
22	Bootstrap	Empirical	M3-like; for all states, place additional weight (15%) on bootstrap draws from same year
23	Bootstrap	Empirical	M3-like; for all states, place additional weight (20%) on bootstrap draws from same year
24	Bootstrap	Empirical	M3-like; for all states, place additional weight (25%) on bootstrap draws from same year
25	Bootstrap	Empirical	M3-like; for all states, place additional weight (30%) on bootstrap draws from same year
26	Bootstrap	Empirical	M3-like; for all states, place additional weight (35%) on bootstrap draws from same year
27	Bootstrap	Empirical	M3-like; for all states, place additional weight (40%) on bootstrap draws from same year
28	Bootstrap	Empirical	M3-like; for all states, place additional weight (45%) on bootstrap draws from same year
29	Bootstrap	Empirical	M3-like; for all states, place additional weight (50%) on bootstrap draws from same year
30	Bootstrap	Empirical	M3-like; for swing states only, place additional weight (15%) on bootstrap draws from same year
31	Bootstrap	Empirical	M3-like; for swing states only, place additional weight (20%) on bootstrap draws from same year
32	Bootstrap	Empirical	M3-like; for swing states only, place additional weight (25%) on bootstrap draws from same year
33	Bootstrap	Empirical	M3-like; for swing states only, place additional weight (30%) on bootstrap draws from same year
34	Bootstrap	Empirical	M3-like; for swing states only, place additional weight (35%) on bootstrap draws from same year
35	Bootstrap	Empirical	M3-like; for swing states only, place additional weight (40%) on bootstrap draws from same year
36	Bootstrap	Empirical	M3-like; for swing states only, place additional weight (45%) on bootstrap draws from same year
37	Bootstrap	Empirical	M3-like; for swing states only, place additional weight (50%) on bootstrap draws from same year
38	Bootstrap	Empirical	M3-like; for safe states only, place additional weight (15%) on bootstrap draws from same year
39	Bootstrap	Empirical	M3-like; for safe states only, place additional weight (20%) on bootstrap draws from same year
40	Bootstrap	Empirical	M3-like; for safe states only, place additional weight (25%) on bootstrap draws from same year
41	Bootstrap	Empirical	M3-like; for safe states only, place additional weight (30%) on bootstrap draws from same year
42	Bootstrap	Empirical	M3-like; for safe states only, place additional weight (35%) on bootstrap draws from same year
43	Bootstrap	Empirical	M3-like; for safe states only, place additional weight (40%) on bootstrap draws from same year
44	Bootstrap	Empirical	M3-like; for safe states only, place additional weight (45%) on bootstrap draws from same year
45	Bootstrap	Empirical	M3-like; for safe states only, place additional weight (50%) on bootstrap draws from same year
46	Parametric	Empirical	M1-like, but using t distribution with 7 d.o.f. instead of normal distribution for shocks
47	Parametric	Empirical	M1-like, but with state-specific coefficients multiplying the national component of the shock
48	Parametric	Hybrid	Shifts all state means in M1 left by 0.2 NPV percentage points post estimation
49	Parametric	Hybrid	Shifts all state means in M1 left by 0.4 NPV percentage points post estimation
50	Parametric	Hybrid	Shifts all state means in M1 left by 0.6 NPV percentage points post estimation

Note: Table continues on next page.

Table A6: Model List for Figure 3 (Cont.)

51	Parametric	Hybrid	Shifts all state means in M1 left by 0.8 NPV percentage points post estimation	
52	Parametric	Hybrid	Shifts all state means in M1 right by 0.2 NPV percentage points post estimation	
53	Parametric	Hybrid	Shifts all state means in M1 right by 0.4 NPV percentage points post estimation	
54	Parametric	Hybrid	Shifts all state means in M1 right by 0.6 NPV percentage points post estimation	
55	Parametric	Hybrid	Shifts all state means in M1 right by 0.8 NPV percentage points post estimation	
56	Parametric	Hybrid	Shifts all state means in M3 left by 0.2 NPV percentage points post estimation	
57	Parametric	Hybrid	Shifts all state means in M3 left by 0.4 NPV percentage points post estimation	
58	Parametric	Hybrid	Shifts all state means in M3 left by 0.6 NPV percentage points post estimation	
59	Parametric	Hybrid	Shifts all state means in M3 left by 0.8 NPV percentage points post estimation	
60	Parametric	Hybrid	Shifts all state means in M3 right by 0.2 NPV percentage points post estimation	
61	Parametric	Hybrid	Shifts all state means in M3 right by 0.4 NPV percentage points post estimation	
62	Parametric	Hybrid	Shifts all state means in M3 right by 0.6 NPV percentage points post estimation	
63	Parametric	Hybrid	Shifts all state means in M3 right by 0.8 NPV percentage points post estimation	
64	Parametric	Hyperpar.	Common national shock SD set to 0.5 pp; state shock SD set to 3.0 pp; no region shock	
65	Parametric	Hyperpar.	Common national shock SD set to 1.0 pp; state shock SD set to 3.0 pp; no region shock	
66	Parametric	Hyperpar.	Common national shock SD set to 1.5 pp; state shock SD set to 3.0 pp; no region shock	
67	Parametric	Hyperpar.	Common national shock SD set to 2.0 pp; state shock SD set to 3.0 pp; no region shock	
68	Parametric	Hyperpar.	Common national shock SD set to 2.5 pp; state shock SD set to 3.0 pp; no region shock	
69	Parametric	Hyperpar.	Common national shock SD set to 3.0 pp; state shock SD set to 3.0 pp; no region shock	
70	Parametric	Hyperpar.	Common national shock SD set to 3.5 pp; state shock SD set to 3.0 pp; no region shock	
71	Parametric	Hyperpar.	Common national shock SD set to 4.0 pp; state shock SD set to 3.0 pp; no region shock	
72	Parametric	Hyperpar.	Common national shock SD set to 4.5 pp; state shock SD set to 3.0 pp; no region shock	
73	Parametric	Hyperpar.	Common national shock SD set to 0.5 pp; state shock SD set to 4.0 pp; no region shock	
74	Parametric	Hyperpar.	Common national shock SD set to 1.0 pp; state shock SD set to 4.0 pp; no region shock	
75	Parametric	Hyperpar.	Common national shock SD set to 1.5 pp; state shock SD set to 4.0 pp; no region shock	
76	Parametric	Hyperpar.	Common national shock SD set to 2.0 pp; state shock SD set to 4.0 pp; no region shock	
77	Parametric	Hyperpar.	Common national shock SD set to 2.5 pp; state shock SD set to 4.0 pp; no region shock	
78	Parametric	Hyperpar.	Common national shock SD set to 3.0 pp; state shock SD set to 4.0 pp; no region shock	
79	Parametric	Hyperpar.	Common national shock SD set to 3.5 pp; state shock SD set to 4.0 pp; no region shock	
80	Parametric	Hyperpar.	Common national shock SD set to 4.0 pp; state shock SD set to 4.0 pp; no region shock	
81	Parametric	Hyperpar.	Common national shock SD set to 4.5 pp; state shock SD set to 4.0 pp; no region shock	
82	Parametric	Hyperpar.	National SD = 2.1 pp; state SD = 3.0 pp; region shock set to match M1 in overall NPV dispersion	
83	Parametric	Hyperpar.	National SD = 2.1 pp; state SD = 4.0 pp; region shock set to match M1 in overall NPV dispersion	
84	Parametric	Hyperpar.	National SD = 2.1 pp; state SD = 5.0 pp; region shock set to match M1 in overall NPV dispersion	
85	Parametric	Hyperpar.	National SD = 2.2 pp; state SD = 3.0 pp; region shock set to match M1 in overall NPV dispersion	
86	Parametric	Hyperpar.	National SD = 2.2 pp; state SD = 4.0 pp; region shock set to match M1 in overall NPV dispersion	
87	Parametric	Hyperpar.	National SD = 2.2 pp; state SD = 5.0 pp; region shock set to match M1 in overall NPV dispersion	
88	Parametric	Hyperpar.	National SD = 2.3 pp; state SD = 3.0 pp; region shock set to match M1 in overall NPV dispersion	
89	Parametric	Hyperpar.	National SD = 2.3 pp; state SD = 4.0 pp; region shock set to match M1 in overall NPV dispersion	
90	Parametric	Hyperpar.	National SD = 2.3 pp; state SD = 5.0 pp; region shock set to match M1 in overall NPV dispersion	
91	Parametric	Hyperpar.	National SD = 2.4 pp; state SD = 3.0 pp; region shock set to match M1 in overall NPV dispersion	
92	Parametric	Hyperpar.	National SD = 2.4 pp; state SD = 4.0 pp; region shock set to match M1 in overall NPV dispersion	
93	Parametric	Hyperpar.	National SD = 2.4 pp; state SD = 5.0 pp; region shock set to match M1 in overall NPV dispersion	
94	Parametric	Hyperpar.	National SD = 2.6 pp; state SD = 3.0 pp; region shock set to match M1 in overall NPV dispersion	
95	Parametric	Hyperpar.	National SD = 2.6 pp; state SD = 4.0 pp; region shock set to match M1 in overall NPV dispersion	
96	Parametric	Hyperpar.	National SD = 2.6 pp; state SD = 5.0 pp; region shock set to match M1 in overall NPV dispersion	
97	Parametric	Hyperpar.	National SD = 2.8 pp; state SD = 3.0 pp; region shock set to match M1 in overall NPV dispersion	
98	Parametric	Hyperpar.	National SD = 2.8 pp; state SD = 4.0 pp; region shock set to match M1 in overall NPV dispersion	
99	Parametric	Hyperpar.	National SD = 2.8 pp; state SD = 5.0 pp; region shock set to match M1 in overall NPV dispersion	
100	Parametric	Hyperpar.	National SD = 2.1 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
101	Parametric	Hyperpar.	National SD = 2.2 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
102	Parametric	Hyperpar.	National SD = 2.3 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
103	Parametric	Hyperpar.	National SD = 2.4 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
104	Parametric	Hyperpar.	National SD = 2.5 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
105	Parametric	Hyperpar.	National SD = 2.6 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
106	Parametric	Hyperpar.	National SD = 2.7 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
107	Parametric	Hyperpar.	National SD = 2.8 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
108	Parametric	Hyperpar.	National SD = 2.9 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*
109	Parametric	Hyperpar.	National SD = 3.0 pp; state SD = 4.0 pp; region shock set to match M1 NPV dispersion; all draws	*t*

Note: Table lists details for each of the variants on the models included in Figure 3.

Table A7: Asymmetry: Probability that an Observed EC Win Was Caused by an Inversion

		Probability that Inversion Occurrs, Conditional on a Republican/Whig Win								Probability that Inversion Occurrs, Conditional on a Democrat Win					
		Conditional on Margin of Victory					Any			Conditional on Margin of Victory					Any
	Model	<0.5pp	<1pp	<2pp	<3pp	<4pp	Margin		Model	<0.5pp	<1pp	<2pp	<3pp	<4pp	Margin
Republicans and Democrats	M1	0.47	0.45	0.41	0.36	0.32	0.22	Republicans and Democrats	M1	0.43	0.37	0.25	0.18	0.13	0.05
	M2	0.47	0.44	0.40	0.36	0.32	0.18		M2	0.44	0.37	0.26	0.18	0.13	0.04
	M3	0.54	0.59	0.66	0.70	0.71	0.72		M3	0.32	0.21	0.08	0.05	0.03	0.03
	M4	0.52	0.52	0.52	0.53	0.52	0.45		M4	0.43	0.36	0.23	0.15	0.09	0.05
	M5	0.52	0.56	0.62	0.65	0.67	0.68		M5	0.38	0.28	0.15	0.09	0.07	0.06
	M6	0.55	0.57	0.63	0.66	0.68	0.68		M6	0.38	0.27	0.14	0.09	0.07	0.05
	M7	0.49	0.45	0.40	0.36	0.32	0.19		M7	0.42	0.35	0.25	0.17	0.13	0.04
	M8	0.46	0.45	0.40	0.36	0.32	0.23		M8	0.43	0.35	0.24	0.17	0.12	0.05
	M9	0.48	0.46	0.41	0.36	0.32	0.21		M9	0.43	0.35	0.24	0.17	0.12	0.04
	M10	0.45	0.43	0.37	0.32	0.28	0.18		M10	0.43	0.36	0.26	0.18	0.14	0.05
	M11	0.51	0.53	0.54	0.54	0.54	0.53		M11	0.39	0.30	0.17	0.10	0.07	0.05
	M12	0.47	0.42	0.35	0.28	0.23	0.06		M12	0.45	0.40	0.31	0.24	0.19	0.05
Republicans and Democrats	R1	0.44	0.38	0.27	0.19	0.14	0.04	Republicans and Democrats	R1	0.47	0.45	0.41	0.37	0.33	0.20
	R2	0.45	0.39	0.30	0.22	0.17	0.05		R2	0.49	0.46	0.42	0.38	0.35	0.20
	R3	0.33	0.19	0.08	0.05	0.04	0.04		R3	0.60	0.67	0.77	0.82	0.83	0.83
	R4	0.42	0.35	0.25	0.22	0.21	0.16		R4	0.54	0.56	0.59	0.60	0.61	0.61
	R5	0.40	0.31	0.18	0.11	0.08	0.05		R5	0.51	0.52	0.54	0.55	0.56	0.56
	R6	0.39	0.31	0.18	0.12	0.10	0.07		R6	0.53	0.55	0.59	0.60	0.61	0.60
	R10	0.42	0.34	0.22	0.15	0.10	0.02		R10	0.47	0.46	0.43	0.40	0.37	0.23
Whigs and Democrats	A1	0.45	0.39	0.30	0.24	0.19	0.10	Whigs and Democrats	A1	0.45	0.39	0.30	0.23	0.18	0.09
	A2	0.44	0.40	0.33	0.26	0.22	0.10		A2	0.45	0.40	0.31	0.25	0.20	0.09
	A3	0.48	0.46	0.42	0.41	0.38	0.25		A3	0.39	0.29	0.18	0.15	0.17	0.20
	A4	0.45	0.41	0.32	0.23	0.18	0.11		A4	0.39	0.30	0.19	0.15	0.15	0.16
	A5	0.46	0.42	0.35	0.31	0.28	0.26		A5	0.44	0.39	0.30	0.25	0.22	0.19
	A6	0.46	0.42	0.35	0.30	0.28	0.26		A6	0.44	0.39	0.31	0.26	0.23	0.21

Note: Table reports statistics describing the probability of inversions, conditional on the Electoral College win going to the indicated party. Columns additionally condition on various two-party popular vote share margins. Model estimates in the table are grouped by period: (M)odern, (R)econstruction, (A)ntebellum.

Table A8: Asymmetry: Who Wins from an Inversion?

		Probability of Republican/Whig Win, Conditional on Inversion Occurring					
		Conditional on Margin of Victory					Any
	Model	<0.5pp	<1pp	<2pp	<3pp	<4pp	Margin
Republicans and Democrats	M1	0.68	0.70	0.73	0.75	0.76	0.77
	M2	0.68	0.71	0.74	0.76	0.78	0.79
	M3	0.79	0.84	0.90	0.92	0.92	0.93
	M4	0.69	0.72	0.78	0.82	0.84	0.85
	M5	0.66	0.72	0.80	0.84	0.85	0.85
	M6	0.68	0.73	0.81	0.84	0.85	0.86
	M7	0.71	0.72	0.75	0.77	0.78	0.79
	M8	0.67	0.71	0.74	0.76	0.78	0.79
	M9	0.72	0.73	0.76	0.78	0.79	0.80
	M10	0.62	0.63	0.66	0.68	0.68	0.69
	M11	0.67	0.72	0.78	0.81	0.82	0.82
	M12	0.61	0.60	0.61	0.61	0.62	0.62
Republicans and Democrats	R1	0.30	0.28	0.25	0.22	0.21	0.20
	R2	0.31	0.30	0.27	0.25	0.24	0.23
	R3	0.67	0.56	0.41	0.35	0.33	0.33
	R4	0.69	0.66	0.59	0.57	0.56	0.56
	R5	0.24	0.20	0.15	0.13	0.11	0.11
	R6	0.34	0.29	0.23	0.20	0.19	0.19
	R10	0.24	0.21	0.18	0.15	0.14	0.13
Whigs and Democrats	A1	0.51	0.51	0.51	0.51	0.51	0.51
	A2	0.50	0.51	0.51	0.51	0.51	0.50
	A3	0.48	0.51	0.54	0.51	0.44	0.37
	A4	0.41	0.43	0.43	0.40	0.35	0.28
	A5	0.51	0.51	0.52	0.52	0.51	0.51
	A6	0.49	0.49	0.49	0.48	0.48	0.48

Note: Table reports statistics describing the probability that inversions were won by the index party (Republican/Whig). Columns condition on various two-party popular vote share margins. Statistics for Democrats are one minus the indicated value in the table. Model estimates in the table are grouped by period: (M)odern, (R)econstruction, (A)ntebellum.

Table A9: Inversion Probabilities in Counterfactuals that Allow for Endogenous Response

	Probabiltiy of Inversion		
	If Margin < 1 p.p.	If Margin < 2 p.p.	Unconditional
	(1)	(2)	(3)
	Panel A: Baseline Decomposition from Main Paper		
status quo	0.424	0.350	0.123
without +2 Senators	0.416	0.337	0.114
award ECVs proportionally	0.297	0.177	0.047
without +2 & award ECVs proportionally	0.292	0.170	0.045
	Panel B: Adjust Tightness of Race in New Safe/Battleground States		
status quo	0.424	0.350	0.123
without +2 Senators	0.411	0.340	0.127
award ECVs proportionally	0.375	0.250	0.070
without +2 & award ECVs proportionally	0.355	0.239	0.068
	Panel C: Adjust Variance in New Safe/Battleground States		
status quo	0.424	0.350	0.123
without +2 Senators	0.410	0.327	0.109
award ECVs proportionally	0.321	0.208	0.051
without +2 & award ECVs proportionally	0.306	0.182	0.045
	Panel D: Adjust Turnout in New Safe/Battleground States		
status quo	0.424	0.350	0.123
without +2 Senators	0.417	0.340	0.115
award ECVs proportionally	0.331	0.206	0.054
without +2 & award ECVs proportionally	0.305	0.181	0.047

Note: Table reports summary statistics for the counterfactuals considered in Figure A15. See Figure A15 and Appendix F for additional detail.

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