Online Appendix

Micro-level Misallocation and Selection

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To save on notation, I supress the sectoral index s in all following calculations.

1 Derivation of Aggregate Productivity

Profit maximization and Optimal size

Profits are given by

$$\begin{aligned} \max_{\{K(\omega),L(\omega)\}} \Pi(\omega) &= [1-\tau_Y(\omega)]\cdot py(\omega) - wL(\omega) - [1+\tau_K(\omega)]\cdot R \cdot K(\omega) \\ \text{subject to: } & y(\omega) = A(\omega)\cdot \left[K(\omega)^{\alpha}L(\omega)^{1-\alpha}\right]^{\gamma} \end{aligned} \tag{1}$$

Taking first order conditions and solving for optimal size gives

$$\begin{aligned} \frac{wL(\omega)}{1-\alpha} &= [\gamma p(1-\tau_Y(\omega))A(\omega)]^{\frac{1}{1-\gamma}} \left[\left(\frac{(1+\tau_K(\omega))R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right]^{-\frac{\gamma}{1-\gamma}} \\ \frac{(1+\tau_K(\omega))RK(\omega)}{\alpha} &= [\gamma p(1-\tau_Y(\omega))A(\omega)]^{\frac{1}{1-\gamma}} \left[\left(\frac{(1+\tau_K(\omega))R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right]^{-\frac{\gamma}{1-\gamma}} \\ py(\omega) &= \left(\frac{1}{\gamma}(pA(\omega))^{\frac{1}{1-\gamma}}\right)\left[\left(\frac{(1+\tau_K(\omega))R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right]^{-\frac{\gamma}{1-\gamma}} \end{aligned} \tag{2}$$

Aggregation I: factor payments

Labor market clearing can be written as

$$(1-s_e)wL = \int_{\Pi(\omega)\geq w} wL(\omega)d\omega \tag{3}$$

where L is a given labor supply. Equation can also be rewritten as

$$pY = \frac{1}{\gamma(1-\alpha)}\frac{1}{1-\bar{\tau}_Y}w(1-s_e)L \tag{4}$$

with the average output distortion is defined by

$$(1-\bar{\tau}_Y)=\int_{\Pi(\omega)\geq w}(1-\tau_Y(\omega))\left(\frac{py(\omega)}{pY}\right)d\omega \tag{5}$$

and aggregate output in (4) is given by

$$Y=\int_{\Pi(\omega)\geq w}y(\omega)d\omega \tag{6}$$

Using (2) in (1) one obtains

$$\frac{w}{1-\alpha}=(1-s_e)^{-\frac{1-\gamma}{1-\alpha\gamma}}(\gamma p)^{\frac{1}{1-\alpha\gamma}}\left(\frac{R}{\alpha}\right)^{-\frac{\alpha\gamma}{1-\alpha\gamma}}\Sigma_L^{\frac{1-\gamma}{1-\alpha\gamma}} \tag{7}$$

with

$$\Sigma_L=E\left[A(\omega)^{\frac{1}{1-\gamma}}\left(\frac{1}{1-\tau_Y(\omega)}\right)^{-\frac{1}{1-\gamma}}(1+\tau_K(\omega))^{-\alpha\frac{\gamma}{1-\gamma}}\middle|\Pi(\omega)\geq w\right]s_e \tag{8}$$

Similarily, capital market clear is given by

$$RK=\int_{\Pi(\omega)\geq w}RK(\omega)d\omega \tag{9}$$

which can be rewritten as

$$pY=\frac{1}{\gamma\alpha}\left(\frac{1+\bar{\tau}_K}{1-\bar{\tau}_Y}\right)RK \tag{10}$$

where

$$\frac{1-\bar{\tau}_Y}{1+\bar{\tau}_K}=\int_{\Pi(\omega)\geq w}\left(\frac{1-\tau_Y(\omega)}{1+\tau_K(\omega)}\right)\left(\frac{py(\omega)}{pY}\right)d\omega \tag{11}$$

Combining (2) in (9) implies

$$\frac{R}{\alpha}=(\gamma p)^{\frac{1}{1-\gamma+\alpha\gamma}}\left(\frac{w}{1-\alpha}\right)^{-\frac{(1-\alpha)\gamma}{1-\gamma-\alpha\gamma}}\left(\frac{L}{K}\right)^{\frac{1-\gamma}{1-\alpha+\alpha\gamma}}\Sigma_K^{\frac{1-\gamma}{1-\alpha+\alpha\gamma}} \tag{12}$$

with

$$\Sigma_K=E\left[A(\omega)^{\frac{1}{1-\gamma}}\left(\frac{1}{1-\tau_Y(\omega)}\right)^{-\frac{1}{1-\gamma}}(1+\tau_K(\omega))^{-\frac{1-\gamma+\alpha\gamma}{1-\gamma}}\middle|\Pi(\omega)\geq w\right]s_e \tag{13}$$

To obtain aggregate production, note that $pY = (pY)^{\alpha}(pY)^{1-\alpha}$ and use (4) and (18) to obatain

$$\begin{aligned} \frac{Y}{L} =& \left(E\left[\Sigma_K \middle| \Pi(\omega) \geq w\right]^{\alpha} E\left[\Sigma_L \middle| \Pi(\omega) \geq w\right]^{1-\alpha} \right)^{1-\gamma} \left(\frac{(1+\bar{\tau}_K)^{\alpha}}{1-\bar{\tau}_Y} \right) \\ & \times \left(\frac{K}{L}\right)^{\alpha\gamma} s_e^{1-\gamma}(1-s_e)^{\gamma(1-\alpha)} \end{aligned} \tag{14}$$

Aggregation II: output

Combine equation (6) and (2) gives

$$Y = \left[\frac{1}{\gamma p} \left(\frac{R}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha} \right]^{-\frac{\gamma}{1-\gamma}} E\left[\Sigma_Y \middle| \Pi(\omega) \geq w\right] s_e L \tag{15}$$

with

$$E\left[\Sigma_Y \middle| \Pi(\omega) \geq w\right] = E\left[A(\omega)^{\frac{1}{1-\gamma}} \left[\frac{(1+\tau_K(\omega))^{\alpha}}{1-\tau_Y(\omega)} \right]^{-\frac{\gamma}{1-\gamma}} \middle| \Pi(\omega) \geq w \right] \tag{16}$$

Using (7) and (12) in (15) to get

$$\frac{Y}{L} = \frac{E\left[\Sigma_Y \middle| \Pi(\omega) \geq w\right]}{\left(E\left[\Sigma_K \middle| \Pi(\omega) \geq w\right]^{\alpha} E\left[\Sigma_L \middle| \Pi(\omega) \geq w\right]^{1-\alpha} \right)^{\gamma}} \left(\frac{K}{L}\right)^{\alpha\gamma} s_e^{1-\gamma}(1-s_e)^{\gamma(1-\alpha)} \tag{17}$$

Matching coefficients of (14) and (17) gives

$$\left(\frac{(1+\bar{\tau}_K)^{\alpha}}{1-\bar{\tau}_Y} \right) = \frac{\Sigma_Y}{\Sigma_K^{\alpha} \Sigma_L^{1-\alpha}} \tag{18}$$

Then, using (18) in (17) gives

$$\frac{Y}{L} = E\left[A(\omega)^{\frac{1}{1-\gamma}} \left[\frac{1-\bar{\tau}_Y}{1-\tau_Y(\omega)} \right]^{-\frac{1}{1-\gamma}} \left[\frac{1+\tau_K(\omega)}{1+\bar{\tau}_K} \right]^{-\alpha\frac{\gamma}{1-\gamma}} \middle| \Pi(\omega) \geq w \right]^{1-\gamma} \left(\frac{K}{L}\right)^{\alpha\gamma} s_e^{1-\gamma}(1-s_e)^{\gamma(1-\alpha)} \tag{19}$$

2 Derivation of MPEC estimator

This section builds on the previous section to derive the MPEC estimator used in the paper. As before, I supress sector subscripts s to simplify notation.

MLE Objective

From (2) it follows that

$$\begin{aligned} D_1(\omega) &= py(\omega) \\ &= \left(\frac{1}{\gamma}(pA(\omega))^{\frac{1}{1-\gamma}}\right)\left[\left(\frac{(1+\tau_K(\omega))R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right]^{-\frac{\gamma}{1-\gamma}} \end{aligned} \tag{20}$$

Combining the expressions for factor demands in (2) it also follows that

$$\begin{aligned} D_2(\omega) &= \left[\left(\frac{RK(\omega)}{\alpha}\right)^{\alpha}\left(\frac{wL(\omega)}{1-\alpha}\right)^{1-\alpha}\right] \\ &= (p\gamma)^{\frac{1}{1-\gamma}}\left[\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right]^{-\frac{\gamma}{1-\gamma}} A(\omega)^{\frac{1}{1-\gamma}}\left(\frac{1}{1-\tau_Y(\omega)}\right)^{-\frac{1}{1-\gamma}}(1+\tau_K(\omega))^{-\frac{\alpha}{1-\gamma}} \end{aligned} \tag{21}$$

as well as

$$\begin{aligned} D_3(\omega) &= \ln\left(\frac{wL(\omega)/(1-\alpha)}{RK(\omega)/\alpha}\right) \\ &= (1+\tau_K(\omega)) \end{aligned} \tag{22}$$

Equation (20), (21), (22) can be rewritten to yield

$$\begin{pmatrix} \ln D_1(\omega) \\ \ln D_2(\omega) \\ \ln D_3(\omega) \end{pmatrix} \propto \begin{bmatrix} -\frac{\gamma}{1-\gamma} & -\alpha\frac{\gamma}{1-\gamma} & \frac{1}{1-\gamma} \\ -\frac{1}{1-\gamma} & -\alpha\frac{1}{1-\gamma} & \frac{1}{1-\gamma} \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} \ln\left(\frac{1}{1-\tau_Y(\omega)}\right) \\ \ln(1+\tau_K(\omega)) \\ \ln A(\omega) \end{pmatrix} \tag{23}$$

which I assume is distributed according to a tri-variate normal distribution with parameters $\mu_i = E[\ln D_i(\omega)], \sigma_{ii} = Var[\ln D_i(\omega)]$ for $i = 1, 2, 3$ and $\sigma_{ij} = Cov(\ln D_i(\omega), \ln D_j(\omega))$ for $i, j = 1, 2, 3$ and $i \neq j$. As equation (23), shows, these parameters in turn are functions

of the underlying heterogeneity parameters $\mu_A = E[\ln A(\omega)], \mu_{\tau_Y} = E\left[\ln\left(\frac{1}{1-\tau_Y(\omega)}\right)\right], \mu_{\tau_K} = E[\ln(1+\tau_K(\omega)], \sigma_A = Var[\ln A(\omega)], \sigma_{\tau_Y} = Var\left[\ln\left(\frac{1}{1-\tau_Y(\omega)}\right)\right], \sigma_{\tau_K} = Var[\ln(1+\tau_K(\omega)], \rho_{A\tau_Y} = Corr\left(\ln A(\omega), \ln\left(\frac{1}{1-\tau_Y(\omega)}\right)\right), \rho_{A\tau_K} = Corr\left(\ln A(\omega), \ln(1+\tau_K(\omega))\right)$
$\rho_{\tau_{YK}} = Corr\left(\ln\left(\frac{1}{1-\tau_Y(\omega)}\right), \ln(1+\tau_K(\omega)\right)$

Selection is given by

$$\Pi_s(\omega) \geq w_s \tag{24}$$

which after plugging in (2) and taking the log, gives

$$\left(\frac{1+\gamma}{1-\gamma}\right)\ln\left(\frac{1}{1-\tau_Y(\omega)}\right) + \alpha\left(\frac{\gamma}{1-\gamma}\right)\ln(1+\tau_K(\omega) - \left(\frac{1}{1-\gamma}\right)\ln A(\omega) \leq \ln\kappa_Z \tag{25}$$

where the log truncation threshold $\ln\kappa_Z$ is given by

$$\ln\kappa_Z = \ln\left(\frac{1-\gamma}{w}\right) + \left(\frac{1}{1-\gamma}\right)[\ln p + \gamma\ln\gamma] - \frac{\gamma}{1-\gamma}\ln\left[\left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right] \tag{26}$$

The MLE objective with parameters $\theta = \left[\mu_A; \mu_{\tau_Y}; \mu_{\tau_K}; \sigma_A; \sigma_{\tau_Y}; \sigma_{\tau_K}; \rho_{A\tau_Y}; \rho_{A\tau_K}; \rho_{\tau_{YK}}\right]$ under truncation for a single observation can then be written as

$$\ln\left\{\frac{\phi\left(\ln D_1(\omega), \ln D_2(\omega), \ln D_3(\omega)\middle|\theta, w, R\right)}{1-\Phi\left(\kappa_Z\right)}\right\} =$$
$$\left\{\frac{3}{2}\ln(2\pi) + \frac{1}{2}\ln(|\bar{\sigma}|) - \ln\Phi\left(\frac{\ln\kappa_Z - \mu_Z}{\sigma_Z}\right) - \frac{1}{2}\begin{pmatrix}\ln D_1(\omega)-\mu_1\\ \ln D_2(\omega)-\mu_2\\ \ln D_3(\omega)-\mu_3\end{pmatrix}'\bar{\sigma}^{-1}\begin{pmatrix}\ln D_1(\omega)-\mu_1\\ \ln D_2(\omega)-\mu_2\\ \ln D_3(\omega)-\mu_3\end{pmatrix}\right\} \tag{27}$$

with

$$\begin{aligned}
\mu_Z &= g\cdot\mu_{\tau_Y} + h\cdot\mu_{\tau_K} + k\cdot\mu_A\\
\sigma_Z^2 &= g^2\sigma_{\tau_Y}^2 + h^2\sigma_{\tau_K}^2 + k^2\sigma_A^2 + 2(g\cdot h\cdot\sigma_{\tau_Y,\tau_K} + h\cdot k\cdot\sigma_{\tau_K,A} + g\cdot k\cdot\sigma_{\tau_Y,A})\\
g &= \frac{1+\gamma}{1-\gamma}, h = \frac{\alpha\gamma}{1-\gamma}, k = \frac{1}{1-\gamma}
\end{aligned} \tag{28}$$

Furthermore, $\bar{\sigma}$ is the variance-covariance matrix of $\ln D_1(\omega), \ln D_2(\omega), \ln D_3(\omega)$ and the term

$|\bar{\sigma}|$ the determinant of that variance-covariance matrix. $\Phi()$ denotes the cdf of a standard normal distribution.

Equilibrium Constraints

Equilibrium constraints are given by the terms (8) and (13). To evaluate the truncated power means in these expressions, I use the following Lemmas.

Lemma 1 (Lien and Balakrishnan, 2006)

Let X and Z be two jointly log-normally distributed random variables. Define the multiplicative constraint by the set

$$1_{\{a,b,K\}} = \begin{cases} 1 \text{ if } X^a \cdot Z^b \leq K \\ 0 \text{ if else} \end{cases} \tag{29}$$

Then it follows that

$$\begin{aligned} E\left[X^m Z^n \cdot 1_{\{a,b,K\}}\right] = \exp\left\{m\mu_X + n\mu_Z + \frac{1}{2}\left(m^2\sigma_m^2 + n^2\sigma_n^2 + 2mn\sigma_{X,Z}\right)\right\} \\ \times\Phi\left(\frac{\log K - (a\mu_X + b\mu_Z) - [am\sigma_X^2 + (bm+an)\sigma_{X,Z} + bn\sigma_Z^2]}{\sqrt{a^2\sigma_X^2 + b^2\sigma_Z^2 + 2ab\sigma_{X,Z}}}\right) \end{aligned} \tag{30}$$

where $\Phi(.)$ is the cdf of a standard normal.

To apply the Lien and Balakrishnan result to the trivariate lognormal truncated moments in (8) and (13), I use the following result, based in the fact that sums of normal random variables are themselves normally distributed.

Lemma 2

Let X_1, X_2, X_3 be three jointly log-normally distributed random variables. Define the multiplicative constraint by the set

$$1_{\{\alpha,\beta,\gamma,K\}} = \begin{cases} 1 \text{ if } X_1^{\beta_1} X_2^{\beta_2} X_3^{\beta_3} \leq K \\ 0 \text{ if else} \end{cases} \tag{31}$$

Then it follows that

$$E\left[X_1^m X_2^n X_3^l \cdot 1_{\{\beta_1,\beta_2,\beta_3,K\}}\right] = E\left[X \cdot Z^c \cdot 1_{\{0,0,1,K\}}\right]$$
$$= \exp\left\{\mu_X + c\mu_Z + \frac{1}{2}\left(\sigma_X^2 + c^2\sigma_Z^2 + c\sigma_{X,Z}\right)\right\} \cdot \Phi\left(\frac{\log K - \mu_Z - [\sigma_{X,Z} + c\sigma_Z^2]}{\sigma_Z}\right) \tag{32}$$

where $\Phi(.)$ is the cdf of a standard normal and X and Z are defined by

$$\begin{aligned} \log X &= a \log X_1 + b \log X_2 \\ \log Z &= \beta_1 \log X_1 + \beta_2 \log X_2 + \beta_3 \log X_3 \end{aligned} \tag{33}$$

and the coefficients a, b, c are given by

$$a = m - \beta_1 \frac{l}{\beta_3}, b = n - \beta_2 \frac{l}{\beta_3}, c = \frac{l}{\beta_3} \tag{34}$$

Proof: apply mapping (33) and (34) to reduce the trivariate problem to the bivariate problem of Lemma 1.