

Liquidity Traps and Monetary Policy: Managing a Credit Crunch Online Appendix

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‡The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Chicago, the Federal Reserve Bank of Minneapolis, or the Federal Reserve System.

A Solution for Inactive Entrepreneurs at the ZLB

The solution for inactive entrepreneurs in periods in which the nominal interest rate is zero, $(1 + r_t) p_{t+1}/p_t - 1 = 0$, is

$$a_{t+1}(z) + \frac{m_{t+1}(z)}{p_t} - \frac{m_{t+1}^T(z)}{p_t} = \beta \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}^e}{Q_{t+j}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}^e}{Q_{t+j}(z)},$$

where

$$\frac{m_{t+1}^T(z)}{p_t} = \frac{\nu(1 - \beta)\beta}{1 - \nu(1 - \beta)} \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}^e}{Q_{t+j}(z)} \right] \quad (1)$$

are the real money balances that will be used for transaction purposes in period $t + 1$. Thus, $m_{t+1}/p_t - m_{t+1}^T/p_t \geq 0$ are the excess real money balances, hoarded from period t to $t + 1$.

B Deflation Follows Passive Policy: Analytics

We want to discuss the behavior of the price level following a credit crunch such that the zero bound constraint binds given a constant money supply. As before, we maintain the net supply of bonds equal to zero, so there are no taxes or transfers in any period.

Given the one-period credit crunch considered and under certain assumptions regarding parameters, the zero bound is binding for only one period, so $i_1 = 0$, but $i_t > 0$ for all $t \geq 2$.¹ In addition, we focus on the cashless limit case, because the real allocation does not depend on the evolution of the price level. If this were not the case, there would be an interaction between nominal and real variables, and we could not obtain closed-form solutions.² Under these conditions, we then explain why deflation would follow a credit crunch if policy does not respond.

We consider the limiting case of the cashless economy (i.e., $\nu \rightarrow 0$). In taking the limit, though, we also let nominal money balances shrink at the same rate, so we can still meaningfully determine the equilibrium price level. The details follow.

¹ Nothing relevant changes if the nominal interest is zero for more periods, but the characterization is simpler in this case. We will assume that parameters satisfy those properties.

²In the general case, non negligible money balances crowd out capital and ameliorate the drop in the real interest rate and in total factor productivity.

When the cash-in-advance constraint is binding, the first-order condition (??) is

$$p_t c_t^1(z) = \frac{\nu}{1-\nu} c_t^2(z) p_{t-1} \frac{1}{R_t(z)}.$$

We define $\bar{m}_t(z) = \frac{m_t(z)}{\nu}$, so $p_t c_t^1(z) = m_t(z) = \bar{m}_t(z) \nu$. Replacing this condition on the equation above and taking the limit when $\nu \rightarrow 0$, we obtain

$$\bar{m}_t(z) = c_t^2(z) p_{t-1} \frac{1}{R_t(z)}.$$

Finally, using the optimal rule for the credit good (??), specialized for the limiting case

$$c_t^2(z) = (1 - \beta) R_t(z) k_t(z)$$

and aggregating over all agents, we obtain

$$\bar{M}_t = (1 - \beta) K_t p_{t-1}, \tag{2}$$

where $\bar{M}_t = \frac{M_t}{\nu}$ represents aggregate money balances relative to the preference parameter ν . This equation determines the price level in the economy.

Because of the cashless limit and since debt and transfers are all zero, the real variables follow the solution described in (23) and (24), irrespectively of the evolution of the price level.

As we mentioned, we will consider a configuration of parameters such that the real interest rate is positive in the steady state, becomes negative at time 1 during the credit crunch, and becomes positive again from time 2 onward. Thus, we assume that

$$\frac{2\theta_{ss}}{(1 + \theta_{ss})} > \beta, \tag{3}$$

which implies that the real interest rate is positive in the steady state.³ During the credit crunch, at $t = 1$ the real interest rate is

$$r_1 = (\rho + \delta) \frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha - \delta,$$

³The necessary condition for positive interest rates in the steady state, $\frac{2\theta_{ss}}{(1+\theta_{ss})} > \frac{\delta}{\rho+\delta}$, is weaker. The stronger condition that we assume will also imply that the zero bound on nominal interest rates binds one period at most and simplifies the example.

which is negative as long as

$$\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}} \right)^\alpha < \frac{\delta}{(\rho+\delta)}. \quad (4)$$

Clearly, there exists a value for $\theta_l \in (0, \theta_{ss})$ such that this constraint is satisfied. As $\theta_t = \theta_{ss}$ for $t \geq 2$, the real interest rate becomes positive from time 2 onward.

The conditions that determine the price level Since we assume that policy is passive, we let $\bar{M}_{t+1} = \bar{M}$. Using (2), we obtain that

$$1 + i_t = (1 + r_t) \frac{p_t}{p_{t-1}} = (1 + r_t) \frac{K_t}{K_{t+1}} \text{ for all } t \geq 2.$$

From $t = 2$ onward, the real interest rate is positive, but there is deflation: capital is growing back to the steady state. It is possible to show, however, that under assumption (3), the deflation is not enough to make the nominal interest rate zero from time 2 onward.

Lemma 1: *Given assumption (3), $i_t > 0$ for $t \geq 2$.*

Proof: First, it is useful to write explicitly the solutions for capital and the real interest rate in the steady state. If we let $\beta \equiv (1 + \rho)^{-1}$, $\rho > 0$, they are

$$K_{ss} = \left(\frac{\alpha}{\rho + \delta} \right)^{\frac{1}{1-\alpha}} \left(\frac{1 + \theta_{ss}}{2} \right)^{\frac{\alpha}{1-\alpha}} \quad (5)$$

and

$$\left(\frac{2\theta_{ss}}{1 + \theta_{ss}} \right) (\rho + \delta) = r_{ss} + \delta. \quad (6)$$

Now, in order to prove Lemma 1, recall equation (18) in the paper, rewritten here for convenience:

$$1 + i_t = (1 + r_t) \frac{p_t}{p_{t-1}}.$$

Assume the lemma is true, so $i_t > 0$ for $t \geq 2$. Using the solution for the price level

from equation (18) in the paper, the solutions for the real interest rate and capital, equations (23) and (24) in the paper, and noting that $\theta_t = \theta^{ss}$ for $t \geq 2$, we can write it as

$$1 + i_t = \left[\alpha \theta_{ss} \left(\frac{1 + \theta_{ss}}{2} K_t \right)^{\alpha-1} + (1 - \delta) \right] \frac{K_t}{K_{t+1}} = \frac{\left[\alpha \theta_{ss} \left(\frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t \right]}{\beta \left[\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right]},$$

for all $t \geq 2$. Assume now, by contradiction, that

$$1 + i_t = \frac{\left[\alpha \theta_{ss} \left(\frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t \right]}{\beta \left[\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right]} \leq 1.$$

Then,

$$\alpha \theta_{ss} \left(\frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t \leq \beta \left[\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right],$$

which can be written as

$$\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha \left(\frac{2\theta_{ss}}{1 + \theta_{ss}} - \beta \right) + (1 - \delta) K_t (1 - \beta) \leq 0.$$

The assumption in equation (3) implies that the first term on the left-hand side is positive. As δ and $\beta \in (0, 1)$, this is a contradiction. \square

The previous lemma implies that

$$p_t = \frac{\bar{M}}{(1 - \beta)K_{t+1}} \text{ for } t \geq 1. \tag{7}$$

We now show that under certain conditions on the parameters, the zero bound is binding at $t = 1$. Assume, toward a contradiction, that $i_1 > 0$. Then, the price level and the inflation rate at time zero are given by

$$\bar{M} = (1 - \beta)K_1 p_0 \text{ and } \frac{p_1}{p_0} = \frac{K_1}{K_2},$$

and the solution for i_1 is given by

$$1 + i_1 = (1 + r_1) \frac{p_1}{p_0} = \left[\frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta) \right] \frac{K_1}{K_2}.$$

Using the law of motion for capital, equation (24) in the paper, we have

$$1 + i_1 = \left[\frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta) \right] \left[\frac{\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha + (1 - \delta) K_{ss}^{1-\alpha}}{\alpha \left(\frac{1 + \theta_l}{2} \right)^\alpha + (1 - \delta) K_{ss}^{1-\alpha}} \right]. \quad (8)$$

Because the r_1 is negative, it is clear that the first term on brackets in the right-hand side is less than one. However, because $\theta_l < \theta_{ss}$, the gross inflation rate, which is the second term in brackets on the right-hand side, is larger than one. To reach a contradiction (so the zero lower bound binds at time 1), it must be the case that the first term effect dominates.

As it turns out, if the rate of depreciation δ is higher than the discount rate in preferences ρ , the real interest rate effect dominates and the zero bound will bind at time 1. Larger values of δ make the derivative of the real interest rate with respect to θ_l higher than its derivative on the deflation. The technical details are provided in the following lemma.

Lemma 2: *If assumption (3) holds and $\delta > \rho$, then there exists a $\tilde{\theta}_l > 0$ such that $i_1 = 0$ for all $\theta_l \in (0, \tilde{\theta}_l]$.*

Proof: Assume, toward a contradiction, that $i_1 > 0$. Then

$$\bar{M} = (1 - \beta)K_1 p_0$$

so

$$\frac{p_1}{p_0} = \frac{K_1}{K_2},$$

and the solution for the nominal interest rate is given by

$$1 + i_1 = (1 + r_1) \frac{p_1}{p_0} = \left[\frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta) \right] \frac{K_1}{K_2},$$

but

$$\frac{K_1}{K_2} = \frac{\alpha \left(\frac{1+\theta_{ss}}{2}\right)^\alpha K_{ss}^\alpha + (1-\delta) K_{ss}}{\alpha \left(\frac{1+\theta_l}{2}\right)^\alpha K_{ss}^\alpha + (1-\delta) K_{ss}} = \frac{\alpha \left(\frac{1+\theta_{ss}}{2}\right)^\alpha + (1-\delta) K_{ss}^{1-\alpha}}{\alpha \left(\frac{1+\theta_l}{2}\right)^\alpha + (1-\delta) K_{ss}^{1-\alpha}}.$$

Replacing the solution for K_{ss} , we obtain

$$\frac{K_1}{K_2} = \frac{\alpha \left(\frac{1+\theta_{ss}}{2}\right)^\alpha + (1-\delta) \frac{\alpha}{\frac{1}{\beta}-1+\delta} \left(\frac{1+\theta_{ss}}{2}\right)^\alpha}{\alpha \left(\frac{1+\theta_l}{2}\right)^\alpha + (1-\delta) \frac{\alpha}{\frac{1}{\beta}-1+\delta} \left(\frac{1+\theta_{ss}}{2}\right)^\alpha} = \frac{1/\beta}{\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) + (1-\delta)}.$$

Then

$$1 + i_1 = \frac{1}{\beta} \frac{\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) + (1-\delta)}{\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) + (1-\delta)}. \quad (9)$$

We assumed the interest rate to be positive, which implies

$$\frac{1}{\beta} \frac{\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) + (1-\delta)}{\left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) + (1-\delta)} > 1, \quad (10)$$

which implies that

$$\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) + (1-\delta) > \beta \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) + \beta(1-\delta)$$

or

$$(1-\delta)(1-\beta) > \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) \left[\beta - \frac{2\theta_l}{(1+\theta_l)}\right].$$

We now briefly characterize the right-hand side as a function of θ_l :

$$f(\theta_l) \equiv \left(\frac{1+\theta_l}{1+\theta_{ss}}\right)^\alpha (\rho + \delta) \left[\beta - \frac{2\theta_l}{(1+\theta_l)}\right].$$

Equation (3) implies that $f(\theta_{ss}) < 0$, so the inequality is satisfied for θ_l close enough to θ_{ss} , and no contradiction arises in this case.

On the other hand,

$$f(0) = \left(\frac{1}{1 + \theta_{ss}} \right)^\alpha \frac{(\rho + \delta)}{1 + \rho}.$$

We show now that in this case, condition (10) is violated. As $\delta > \rho$,

$$\frac{\delta}{1 - \delta} > \frac{\rho}{1 + \rho}.$$

But $\theta_{ss} < 1$, so

$$\theta_{ss} < 1 < \frac{\delta}{1 - \delta} \frac{1 + \rho}{\rho} = \frac{\delta + \rho}{(1 - \delta)\rho} - 1,$$

and therefore

$$1 + \theta_{ss} < \frac{\delta + \rho}{(1 - \delta)\rho}.$$

As $\alpha < 1$, it follows that

$$(1 + \theta_{ss})^\alpha < 1 + \theta_{ss} < \frac{\delta + \rho}{(1 - \delta)\rho},$$

which, rearranging and dividing both sides by $1 + \rho$, can be written as

$$(1 - \delta) \frac{\rho}{1 + \rho} < \left(\frac{1}{1 + \theta_{ss}} \right)^\alpha \frac{\delta + \rho}{1 + \rho}.$$

But the left-hand side can be written as

$$(1 - \beta)(1 - \delta) = \frac{\rho}{1 + \rho} (1 - \delta) < \left(\frac{1}{1 + \theta_{ss}} \right)^\alpha \frac{\delta + \rho}{1 + \rho}.$$

Thus, by the intermediate value theorem, there exists a $\tilde{\theta}_l \in (0, \theta_{ss})$ such that

$$(1 - \delta)(1 - \beta) = \left(\frac{1 + \tilde{\theta}_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) \left[\beta - \frac{2\tilde{\theta}_l}{(1 + \tilde{\theta}_l)} \right].$$

Since $f(\theta_l)$ is decreasing, the zero bound will bind for all $\theta_l \in (0, \tilde{\theta}_l]$. \square

Finally, we show that if the economy starts at the steady state, when agents learn there is a credit crunch at time zero, the equilibrium price level must be strictly below its steady state value.

Lemma 3: *Under the assumptions of lemmas 1 and 2, $p_0 < p_{ss}$ for all $\theta_t \in (0, \tilde{\theta}_t)$.*

Proof: The ratio of the price level at $t = 0$ to the price level in the steady state p_{ss} is given by

$$\frac{p_0}{p_{ss}} = (1 + r_1) \frac{p_1}{p_{ss}} = (1 + r_1) \frac{K_{ss}}{K_2},$$

where the second equality follows from (2) plus the fact that policy is passive, so $\overline{M}_t = \overline{M}$. But the law of motion of capital, equation (24) in the paper, implies that $K_{ss} = K_1$, so $\frac{p_0}{p_{ss}}$ is equal to the right-hand side of equation (8), which, under the conditions of lemmas 1 and 2, is lower than one. \square

The credit crunch drives the real interest rate below zero to the point at which the zero bound is reached. At this point, there is an excess demand for money as a “store of value.” That excess demand is, of course, real rather than nominal. Because the nominal quantity is fixed by policy, the demand pressure results in deflation. The excess demand for money as a store of value will be positive until future inflation is high enough such that the return on money is as negative as the return on bonds. The initial deflation allows for future inflation along the path, required for the arbitrage condition to hold, with zero “long-run” inflation. This zero-long run inflation is the natural consequence of a constant nominal money supply.

C The Effect of Public Debt Around $B = 0$

In this appendix we characterize the effect of public debt on GDP for two limiting cases. First, we consider the example presented in Section 3.3.2, where only entrepreneurs pay taxes and receive subsidies associated with the temporary one-period increase in government debt. For this case, we show condition for GDP to be an increasing function of the level of public debt in the neighborhood of $B = 0$. Second, we consider the polar case in which only workers pay taxes and receive subsidies associated with

the temporary one-period increase in government debt. In this case, we show that GDP is a decreasing function of the level of public debt in the neighborhood of $B = 0$. These examples illustrate that the net effect of government debt on aggregate output depends on the particular implementation of the debt policy and on the relative size of workers and entrepreneurs in the population.

C.1 Taxing/Subsidizing Only Entrepreneurs

Differentiating equation (26) in the paper around $B_1 = 0$,

$$\left. \frac{\partial K_1}{\partial B_1} \right|_{B_1=0} = -(1 - \beta) \left[1 - \int \frac{(1 + r_{ss})}{R_{ss}(z)} dz \right]. \quad (11)$$

Similarly, differentiating equation (21) in the paper around $B_1 = 0$,

$$\left. \frac{\partial Z_1}{\partial B_1} \right|_{B_1=0} = \alpha Z_{ss} K_{ss}^{-1} \frac{1 - \theta}{1 + \theta}. \quad (12)$$

Thus, the net effect on GDP around $B_1 = 0$ is as follows:

$$\begin{aligned} \left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} &= \alpha Z_{ss} K_{ss}^{\alpha-1} \frac{1 - \theta}{1 + \theta} - \alpha Z_{ss} K_{ss}^{\alpha-1} (1 - \beta) \left[1 - \int \frac{(1 + r_{ss})}{R_{ss}(z)} dz \right] \\ &= \alpha Z_{ss} K_{ss}^{\alpha-1} \left[\frac{1 - \theta}{1 + \theta} - (1 - \beta) \left[1 - \int \frac{(1 + r_{ss})}{R_{ss}(z)} dz \right] \right]. \end{aligned}$$

Finally, using the expressions for $R_1(z)$ and solving the integral, we have

$$\left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} = \alpha Z_{ss} K_{ss}^{\alpha-1} (1 - \theta) \left[\frac{1}{1 + \theta} - (1 - \beta) \left[1 - \frac{1 + r_{ss}}{r_{ss} + \delta} \theta \log \left(\frac{r_{ss} + \delta}{1 + r_{ss}} \frac{1}{\theta} + 1 \right) \right] \right],$$

where around $B_1 = 0$ the real interest rate $r_{ss} = (\rho + \delta)2\theta/(1 + \theta) - \delta$. It is straightforward to show that this expression is positive for β close to 1 or θ close to 0.

C.2 Taxing/Subsidizing Only Workers

In this case,

$$\left. \frac{\partial K_1}{\partial B_1} \right|_{B_1=0} = -1, \quad (13)$$

and the effect on TFP is also given by (12). Thus,

$$\left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} = -\alpha Z_{ss} K_{ss}^{\alpha-1} \frac{2\theta}{1+\theta} < 0.$$

D Distribution of Welfare Impacts

In the previous section, we focused on the impact of policies on aggregate outcomes and factor prices. The aggregate figures suggest a relatively simple trade-off at the aggregate level. These dynamics, though, hide very disparate effects of a credit crunch and alternative policies among different agents. Although workers are hurt by the drop in wages, the profitability of active entrepreneurs and their welfare can increase as a result of lower factor prices. Similarly, unproductive entrepreneurs are bondholders in equilibrium, and therefore their welfare depends on the behavior of the real interest rate.

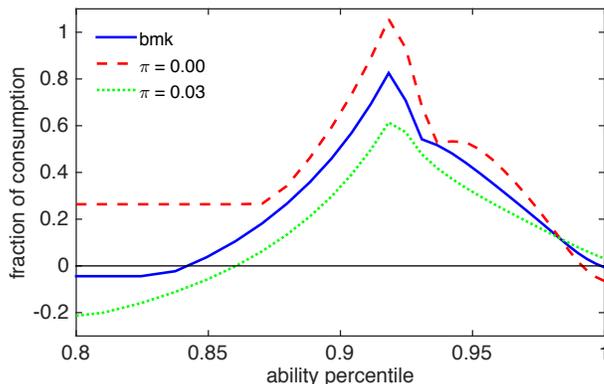


Figure A1: Distribution of welfare gains among entrepreneurs. The solid line corresponds to the benchmark case shown in Figure 3 in the main paper. The dashed and dotted lines are for the cases with alternative inflation targets, $\pi = 0.00$ and $\pi = 0.03$, reported in Figure 7 in the main paper. The welfare gains for workers are -0.03 , -0.05 , and -0.02 , and in the benchmark, $\pi = 0.00$ and $\pi = 0.03$, respectively.

Figure D presents the impact of a credit crunch on the welfare of workers and entrepreneurs of different abilities under alternative inflation targets for the bailout case. We measure the welfare impact of a credit crunch in terms of the fraction of consumption that an individual is willing to permanently forgo in order to experience a credit crunch.⁴ If positive (negative), we refer to this measure as the welfare gains

⁴For entrepreneurs, we consider the welfare of individuals that at the time of the shock have wealth

(losses) from a credit crunch and alternative policy responses.

The dotted line shows the welfare gains for entrepreneurs from a policy that implements a 3% inflation rate as a function of the percentile of their ability distribution. This level of inflation is implemented with a negligible increase in the net supply of outside liquidity, and therefore, the economy in the long run returns to the initial steady state. Unproductive entrepreneurs are clearly hurt by a credit crunch, since the return on the bonds they hold becomes negative for over 31 quarters and only gradually returns to the original steady state. Their losses amount to over 20% of permanent consumption. On the contrary, entrepreneurs who become active as the credit crunch lowers factor prices, and who increase their profitability, benefit the most. The same effect increases welfare for previously active entrepreneurs, but they are hurt by the tightening of collateral constraints, which limit their ability to leverage their high productivity. Clearly, workers are hurt by experiencing a credit crunch, since the wages drop for a number of periods. The credit crunch amounts to a permanent drop of 2 percentage points in their consumption.

The other two curves in Figure D show the welfare consequences of lower inflation targets. The solid line corresponds to the benchmark economy, where the inflation is closed to 2%, and the dashed line is an economy with no inflation. The lower the inflation target, the higher the real interest rate, both during the credit crunch and in the new steady state.⁵ Unproductive entrepreneurs benefit from the highest interest rate. Similarly, productive entrepreneurs benefit from the lowest wages associated with the lowest capital during the transition and in the new steady state.⁶ Although individual entrepreneurs do not internalize it, collectively they benefit from the lower wages associated with a lower aggregate stock of capital. The lower the inflation target, the lower the capital stock and the lower the wages, so the welfare of workers goes down when the target goes down.

equal to the average wealth of their type. For workers, their welfare is calculated assuming, as is true in the steady state of the model, that they own no wealth when the credit crunch is announced.

⁵The government debt in the new steady state will be higher the lower the inflation target is. In the model, a higher level of government debt implies a lower level of capital in the new steady state.

⁶The nonmonotonic nature of the welfare effects is related to the heterogeneous impact due to the changing nature of the occupational choice of agents during the transition. For example, the entrepreneur that benefits the most is the most productive inactive entrepreneur in the steady state. As the real rate goes down, that agent becomes an entrepreneur and starts borrowing to profit from the difference between his productivity and the now low interest rate and also from the lower equilibrium wage. On the other hand, the most productive entrepreneur also benefits from the low input prices but is hurt by the reduction in her ability to borrow. Thus, although she gets a higher margin per unit of capital, she can only manage a lower amount of capital.

E Sensitivity and Robustness

We present two sensitivity and robustness analyses of our benchmark results presented in Section 4.2 of the main paper. First, we consider simulations under alternative calibrations of the collateral constraint in the initial stationary equilibrium, $\theta_0 \in \{0.59, 0.79\}$, and discuss other possible extensions of the model to capture the importance of unconstrained firms in the US economy. Secondly, we present long-run forecasts for GDP, TFP, and the capital stock under alternative assumptions about the evolution of collateral constraints beyond the sample period in which we calibrate the evolution of the collateral constraint to match the observed path of the real interest rate. Finally, we present simulations under alternative measures of the increase in the supply of government liabilities.

E.1 Alternative Values of θ_0

We calibrate the initial parameter of the collateral constraint, $\theta_0 = 0.69$, to match the average ratio of liabilities to nonfinancial assets for the US nonfinancial business sector between 1997:Q3 and 2007:Q3. On the one hand, we could argue for a smaller number given that liabilities are financing nontangible assets. On the other hand, we could argue for a larger number if we interpret that debt in the model proxies for other sources of external finance, such as equity issuance by public firms. We now present results for two alternative calibrations.

In particular, we consider a calibration with initially tighter and looser collateral constraints, $\theta_0 \in \{0.59, 0.79\}$. To match the interest rate and debt to GDP ratio in the initial steady state, we recalibrate the discount factor and the initial level of debt (and taxes).⁷ As before, we choose the evolution of the collateral constraint θ_t and the debt label to match the dynamics of the real interest rate and the debt to GDP ratio during the Great Recession.

The results for the alternative calibration are in Figure A2. The dynamics of GDP (top left panel) are mostly unchanged across alternative calibrations. The calibration with an initially looser constraint is associated with a deeper drop in TFP (top right panel) but a less pronounced drop in capital accumulation (bottom left panel). At the same time, the calibration with an initially looser constraint requires a smaller

⁷The value of the discount factor equals 0.9826 and 9898 in the low and high θ calibrations, respectively.

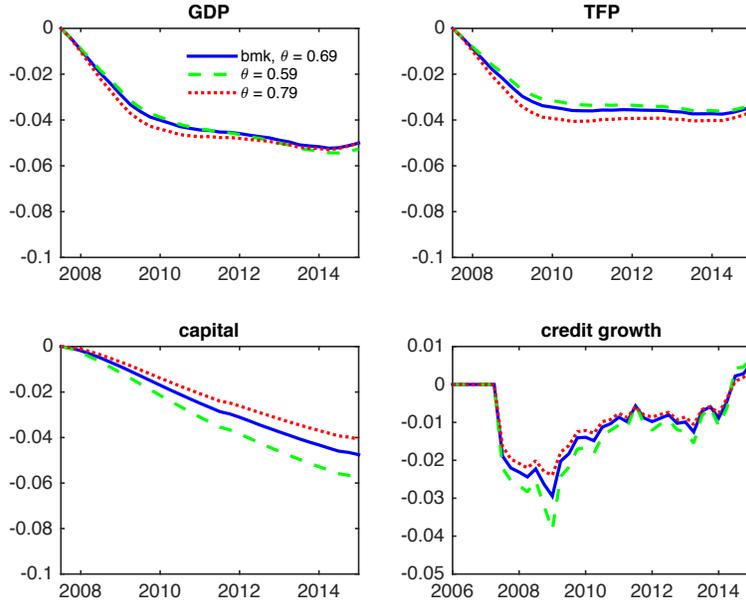


Figure A2: Simulations with alternative values of θ_0 . The solid line correspond to $\theta_0 = 0.69$, the benchmark case shown in Figure 3 of the main paper. The dashed and dotted lines correspond to calibrations using lower and higher initial values for the collateral constraint, $\theta_0 = 0.59$ and $\theta_0 = 0.79$.

percentage drop in the collateral constraint to match the drop in the real interest rate. This, together with the smaller drop in aggregate capital, explains the smaller drop in the growth rate of credit (bottom right panel).

An alternative approach to evaluate the robustness of the results to the tightness of the constraints would be to consider extensions of the model with unconstrained, active entrepreneurs.⁸ There are two natural ways to extend the model to have unconstrained, active entrepreneurs.

One alternative is to assume diminishing returns at the individual level and introduce idiosyncratic productivity shocks (?). In this case, entrepreneurs who remain productive for a long enough time would accumulate enough net worth to run their business at the unconstrained (finite) scale and save some of their net worth in bonds. An inconvenient feature of this alternative is that we would lose the tractability that allows us to illustrate the model mechanisms.

A more tractable alternative is to assume that there is an exogenous subset of

⁸In our model, all active entrepreneurs (i.e., those with $z \geq \hat{z}$) are constrained, whereas all inactive entrepreneurs (i.e., those with $z < \hat{z}$) are unconstrained.

the economy that is unconstrained, and this subsector is modeled as a representative firm operating a Cobb-Douglas production function. This sector is often referred to as the corporate sector, in juxtaposition to the constrained, entrepreneurial sector (see ?). We conjecture that in this case, the calibration would require a larger drop in the collateral constraint for the entrepreneurial sector but a relatively similar decline in the growth rate of overall credit, since some of the credit and resources would be reallocated from the entrepreneurial sector to the corporate sector. This reallocation is conceptually similar to the reallocation captured in our model between the active, constrained entrepreneurs and the inactive, unconstrained entrepreneurs.

E.2 Long-run Forecasts

In this section we present long-run forecasts for GDP, TFP, and the capital stock under alternative assumptions about the evolution of the collateral constraints beyond the sample period in which we calibrate the evolution of the collateral constraint to match the observed path of the real interest rate. Beyond the sample period, we assume that the collateral constraint mean-reverts according to the simple recursion $\lambda_t \equiv 1/(1 - \theta_t) = \rho\lambda_{t-1} + (1 - \rho)\lambda_{2007Q3}$, for $t \geq 2015Q2$.

In Figure A3 we present two alternative forecasts. The solid line corresponds to the evolution of the economy for the case $\rho = 0.95$. This is the assumption we make in our benchmark exercise. The dashed line corresponds to the case in which the collateral constraint remains fixed at the last calibrated value (i.e., $\rho = 1$). Naturally, in this second case the economy remains stagnant.

E.3 Alternative Definition of Government Liabilities

In the final sensitivity analysis, we consider an alternative definition of government liabilities. In our benchmark calibration, we assume that the total liabilities of the government equal the sum of the total public federal debt and the Federal Reserve Banks' balance sheet net of their holdings of Treasury bonds. Here we consider a narrower notion, which nets out the liabilities of the Fed that are backed by the holding of mortgage-backed securities. In this narrower definition, the total government liabilities equal the total public federal debt.

In Figure A4 we present the evolution of GDP, TFP, capital, and the government debt to GDP ratio under the alternative notions of the total supply of government

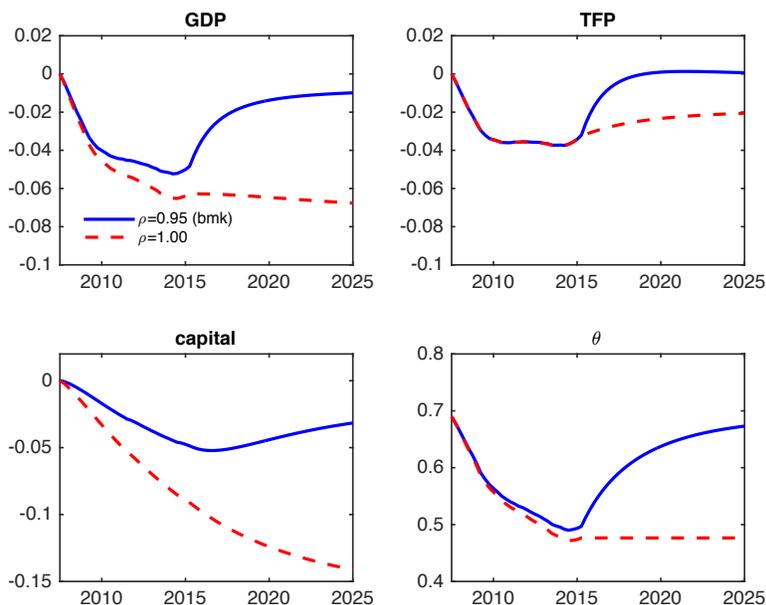


Figure A3: Long-run forecasts under alternative paths for the collateral constraint, $\lambda_t \equiv 1/(1 - \theta_t) = \rho\lambda_{t-1} + (1 - \rho)\lambda_{2007Q3}$, for $t \geq 2015Q2$. The solid line corresponds to the evolution of the economy under the assumption that corresponds to $\rho = 0.95$. This is the assumption we make in our benchmark exercise. The dashed line gives the case in which the collateral constraint remains fixed at the last calibrated value (i.e., $\rho = 1$).

liabilities. The effect on the aggregate variables is negligible.

F Monetary or Fiscal Policy?

At the zero bound, real money and bonds are perfect substitutes. Thus, standard open market operations in which the central bank exchanges money for short-term bonds have no impact on the economy. What is needed is an effective increase in the supply of government liabilities, which at the zero bound can be money or bonds. How can these policies be executed? Clearly, one way to do it is through bonds, taxes, and transfers. But another way is through a process described long ago: helicopter drops, whereby increases of money are directly transferred to agents. Sure enough, to satisfy the government budget constraint, these helicopter drops need to be compensated with future “vacuums” (negative helicopter drops).

Although the distinction between a central bank or the Treasury making direct

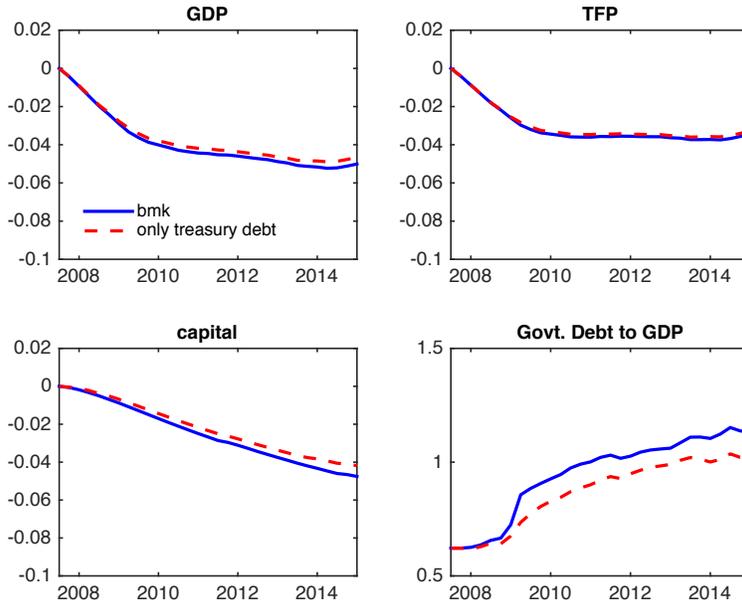


Figure A4: Benchmark simulations using alternative definitions of government liabilities. The solid line reproduces the benchmark simulations from Figure 3 in the main paper. In this case, government debt includes the total public federal debt plus the Federal Reserve Banks' balance sheet net of their holding of Treasury bonds. The dashed line corresponds to the simulations when government liabilities only include the total public federal debt.

transfers to agents may be of varying relevance in different countries because of alternative legal constraints, there is little conceptual difference in the theory. To fully control inflation during a severe credit crunch, the sum of real money plus bonds must go up at the zero bound. Otherwise, there will be an initial deflation, followed by an inflation rate that will be determined by the negative of the real interest rate. If these policies are understood as being outside the realm of central banks, then central banks should not be given tight inflation target mandates: inflation is out of their control during a severe credit crunch.