Online Appendix for 'Demographic Structure and Macroeconomic Trends'

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Appendix A. Data

This provides a description of the data used in the empirical study.

- World Population Prospects: The 2015 Revision File: Annual total population (both sexes combined) by five-year age group, major area, region and country, 1950-2100 (thousands) Estimates;
  Code - POP/DB/WPP/Rev.2015/POP/F15-1. (The data is the de facto population as of 1 July of the year indicated and in the age group indicated and the percentage it represents with respect to the total population.) United Nations, Population Division.
- Death Rates by Age: The Human Mortality Database, http://www.mortality.org/.
- Age-specific Fertility: The Human Fertility Database, http://www.humanfertility.org/cgi-bin/main.php
- Residential Patent Applications (annual): World Bank, World Development Indicators.
- Trademark Applications (annual): World Bank, World Development Indicators.
- Nominal Short Term Interest Rates: Central Bank Discount Rates (Percent per annum): Austria, Belgium, Finland, Greece, Iceland, Ireland, Italy, Japan, Netherlands (Discount rate for Netherlands discontinues between 1994-1998; Fixed Advance Rate available at the DNB replaces the Discount Rate for this period), New Zealand (1970-1998), Portugal, Spain, Sweden (Discount rate discontinues after 2002 and replaced by the Reference rate available at the Riksbanken), United States: International Financial Statistics/IMF; Central Bank Borrowing Facility Rate (Percent per annum): Canada: International Financial Statistics/IMF; Monetary Policy-Related Interest Rate (Percent per annum) Australia, Canada, Denmark, Euro Area

(1999-2015), New Zealand (1999-2015), Norway, Switzerland, United Kingdom: International Financial Statistics/IMF; France: 1970-1998 Money Market Rate: International Financial Statistics/IMF. Euro area discount rate replaces national discount rates after the establishment of the Euro area: International Financial Statistics/IMF.

- Nominal Long term Interest Rates (Percent per annum): Unless stated otherwise AMECO, Annual macro-economic database, European Commission. Government Bond Yield (long term) Australia, Canada, Iceland, Japan, Norway, Switzerland: International Financial Statistics/IMF.
- Consumer Price Index (annual): International Financial Statistics/IMF.
- National Savings Rate (annual): National Accounts, OECD.
- Hours worked (annual): Productivity Statistics, OECD.
- Gross Fixed Capital Formation (annual): National Accounts, OECD.
- Gross Domestic Product (annual): National Accounts, OECD.
- GDP per capita (annual): Penn World Tables.
- Spot Oil Price, West Texas Intermediate (Dollars per Barrel, annual, average): Dow Jones & Company retrieved from FRED.
- Net Foreign Assets: Updated and Extended "External Wealth of Nations" Dataset, 1970-2011, http://www.philiplane.org/EWN.html.

The 21 countries covered by our dataset are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States. For some countries data is not available over the whole period, so the panel is unbalanced. Data on hours are only available for Austria from 1995-2014 and for Greece from 1983-2014. Savings rates for Switzerland are only available from 1990-2014 and for France for 1978-2014. All other countries have full datasets. Though it would also be desirable to include Germany as a mature OECD economy, we exclude it due to reunification. However, we include predictions for Germany in the tables.

Appendix B. Additional Estimation Results

This appendix provides additional results on the estimations discussed in the main body of the paper.

B.1. Benchmark Panel VAR

	Growth (g)		Investment (I)		Savings (S)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.23	0.06	0.10	0.02	-0.02	0.03
I_{t-1}	-0.04	0.06	0.82	0.03	-0.09	0.03
S_{t-1}	0.06	0.06	0.08	0.03	0.87	0.02
H_{t-1}	-0.06	0.02	-0.01	0.01	-0.03	0.01
rr_{t-1}	-0.06	0.04	-0.04	0.01	-0.07	0.02
π_{t-1}	-0.14	0.04	-0.03	0.01	-0.08	0.02
P^{oil}_{t-1}	-0.02	0.00	-0.00	0.00	-0.01	0.00
P^{oil}_{t-2}	0.01	0.00	0.00	0.00	0.01	0.00
Δpop	0.72	0.60	0.18	0.28	0.25	0.31
Δpop_{t-1}	-0.12	0.62	-0.01	0.22	-0.48	0.13
β_1	0.09	0.04	0.02	0.01	0.08	0.02
β_2	0.03	0.02	0.00	0.02	0.04	0.02
R^2	0.21		0.85		0.82	
$\Pr(\delta_j = 0)$	0.01		0.35		0.00	
N	832		832		832	

Table A.1—Results for Growth, Investment and Savings

	Hours (H)		Real Rates (rr)		Inflation (π)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.19	0.03	0.09	0.03	0.01	0.02
I_{t-1}	0.01	0.05	-0.16	0.08	0.25	0.06
S_{t-1}	0.08	0.03	-0.10	0.05	0.08	0.06
H_{t-1}	0.89	0.01	-0.00	0.04	-0.02	0.02
rr_{t-1}	-0.07	0.03	0.85	0.05	-0.14	0.03
π_{t-1}	-0.02	0.02	0.24	0.07	0.56	0.03
P^{oil}_{t-1}	-0.01	0.00	0.00	0.00	-0.02	0.00
P^{oil}_{t-2}	0.01	0.00	0.00	0.00	0.01	0.01
Δpop	-0.16	0.50	-0.98	0.21	0.98	0.15
Δpop_{t-1}	-0.27	0.49	0.26	0.19	-0.79	0.19
β_1	-0.07	0.03	-0.14	0.05	0.24	0.03
β_2	0.11	0.03	0.28	0.08	-0.28	0.07
R^2	0.91		0.65		0.78	
$\Pr(\delta_j = 0)$	0.00		0.00		0.00	
N	832		832		832	

Table A.2—Results for Hours, Interest Rate, and Inflation

	g	I	S	H	rr	π
g	1.000	0.323	0.560	0.436	-0.114	0.237
I	0.323	1.000	0.149	0.487	-0.042	0.178
S	0.560	0.149	1.000	0.286	-0.069	0.049
H	0.436	0.487	0.286	1.000	0.047	0.104
rr	-0.114	-0.042	-0.069	0.047	1.000	-0.746
π	0.237	0.178	0.049	0.104	-0.746	1.000

TABLE A.3—RESIDUAL CORRELATION MATRIX

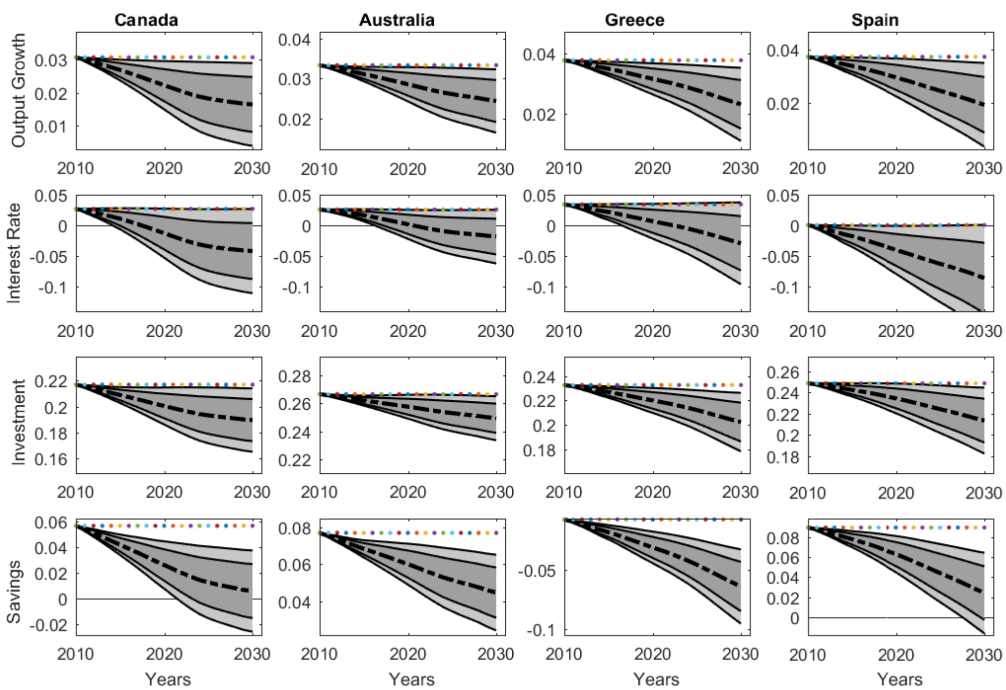

FIGURE A.1. IMPACT OF PREDICTED FUTURE DEMOGRAPHIC STRUCTURE - ADDITIONAL COUNTRIES

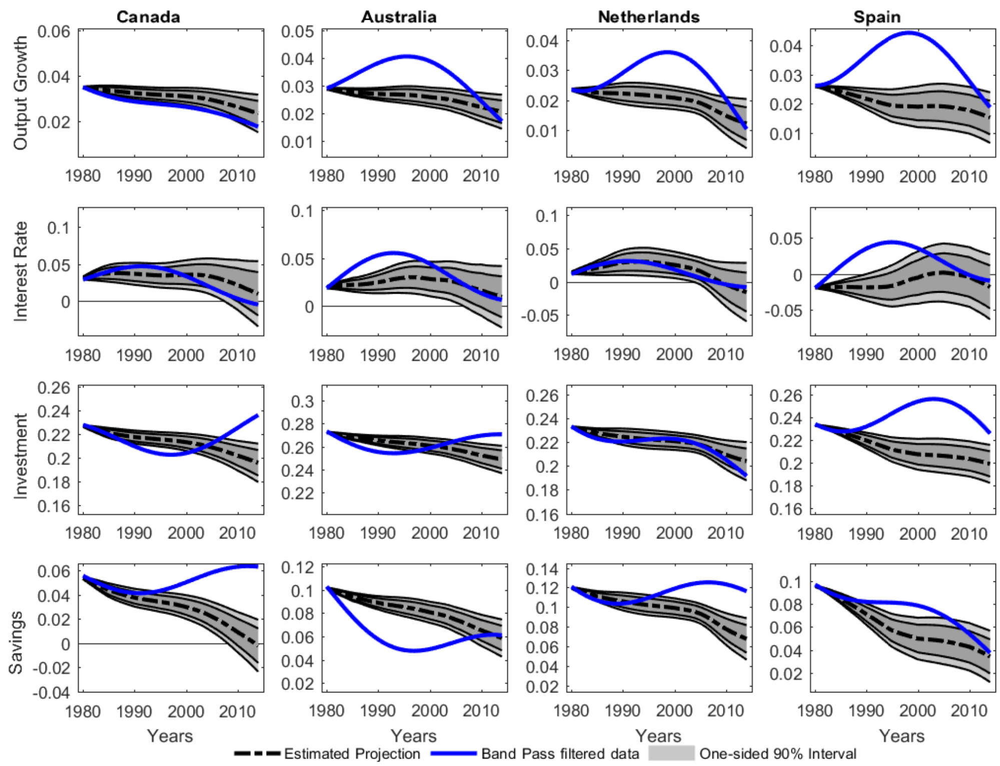

FIGURE A.2. DEMOGRAPHICS AND TRENDS - IN SAMPLE PROJECTION - ADDITIONAL COUNTRIES

B.2. Robustness Exercises

Lee-Carter model

The Lee-Carter model specifies a system of equations for log mortality (m) (and fertility (f)) rates for age cohort x at time t: $m_{x,t}$ ($f_{x,t}$) and a time-series specification for an unobservable time-varying mortality (fertility) index k_t^m (k_t^f): it captures an age-period surface of log-mortality (fertility) rates in terms of an age profile vector (a), a vector (k) of mortality (fertility) changes over time and and a vector (b) tracks how much each age group changes when vector k changes along age and time dimensions. The system is given by

$$\begin{aligned} m_{x,t} &= a_x + b_x k_t + \epsilon_{x,t}, \\ k_t &= c_0 + c_1 k_{t-1} + e_t, \\ \epsilon_{x,t} &\sim NID(0, \sigma_\epsilon^2), \\ e_t &\sim NID(0, \sigma_e^2) \end{aligned}$$

where a_x describes the general shape of the age specific death rates, k_t is an index of the general level of mortality at all ages. The b_x coefficients describe the tendency of mortality at age x to change when the general level of mortality (k_t) changes. The error term $\epsilon_{x,t}$ reflects unobserved heterogeneity not captured by the model, the error term e random fluctuations in the time series of the common factor. The model restricts b coefficients to sum to unity and the k's sum to zero, so the a's are average log rates. This unobservable mortality (fertility) index, k, evolves over time as an autoregressive process. Preferred Lee-Carter specification assumes a random walk with drift process by setting $c_1 = 1$. The model allows to identify the unobservable stochastic mortality and fertility trends common to all cohorts. The a_x, b_x and k are estimated with single value decompositions. For further details on the Lee-Carter model see for instance Favero and Galasso (2015) and Lee and Miller (2001).

	Benchmark			Lee-Carter			Time Effects			Without Inflation		
	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3
g	0.04	0.06	-0.10	0.13	0.04	-0.16	0.09	0.05	-0.14	0.04	0.04	-0.09
	(0.15)	(0.28)	(0.04)	(0.06)	(0.57)	(0.01)	(0.00)	(0.37)	(0.01)	(0.08)	(0.45)	(0.09)
I	0.13	0.08	-0.22	0.22	0.11	-0.33	0.08	0.10	-0.18	0.17	-0.05	-0.13
	(0.01)	(0.48)	(0.02)	(0.10)	(0.39)	(0.00)	(0.27)	(0.34)	(0.13)	(0.00)	(0.63)	(0.12)
S	0.24	0.16	-0.40	0.85	-0.13	-0.72	0.69	0.22	-0.90	0.26	0.09	-0.35
	(0.00)	(0.29)	(0.00)	(0.00)	(0.38)	(0.00)	(0.00)	(0.14)	(0.00)	(0.00)	(0.56)	(0.01)
H	-0.54	1.08	-0.54	0.18	0.61	-0.79	-0.16	0.81	-0.65	-0.45	0.76	-0.31
	(0.00)	(0.00)	(0.05)	(0.63)	(0.09)	(0.02)	(0.44)	(0.02)	(0.07)	(0.00)	(0.00)	(0.18)
rr	-0.05	0.46	-0.42	-0.50	0.50	0.00	-0.22	0.20	0.02	-0.15	0.80	-0.65
	(0.72)	(0.15)	(0.10)	(0.12)	(0.09)	(1.00)	(0.09)	(0.27)	(0.92)	(0.12)	(0.00)	(0.00)
π	0.70	-0.75	0.05	0.66	-0.60	-0.05	0.50	-0.51	0.01			
	(0.00)	(0.00)	(0.68)	(0.00)	(0.00)	(0.67)	(0.00)	(0.00)	(0.95)			

	No Oil Prices			GDP Per capita			Long-run Rates			Net Foreign Assets		
	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3	β_1	β_2	β_3
g	0.05	0.10	-0.15				0.05	0.07	-0.11	0.07	0.04	-0.10
	(0.08)	(0.10)	(0.00)				(0.13)	(0.27)	(0.03)	(0.05)	(0.63)	(0.09)
I	0.15	0.14	-0.30	0.13	0.08	-0.22	0.14	0.09	-0.24	0.14	0.08	-0.21
	(0.00)	(0.18)	(0.00)	(0.01)	(0.48)	(0.02)	(0.00)	(0.34)	(0.00)	(0.04)	(0.63)	(0.07)
S	0.26	0.22	-0.48	0.24	0.16	-0.40	0.19	0.24	-0.43	0.21	0.19	-0.41
	(0.00)	(0.12)	(0.00)	(0.00)	(0.29)	(0.00)	(0.02)	(0.14)	(0.00)	(0.00)	(0.24)	(0.00)
H	-0.45	1.35	-0.89	-0.54	1.08	-0.54	-0.57	1.13	-0.55	-0.52	1.05	-0.53
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.05)	(0.00)	(0.00)	(0.04)	(0.03)	(0.04)	(0.18)
rr	-0.07	0.36	-0.29	-0.05	0.47	-0.42				-0.07	0.47	-0.39
	(0.62)	(0.23)	(0.20)	(0.72)	(0.15)	(0.10)				(0.69)	(0.26)	(0.18)
π	0.74	-0.64	-0.09	0.70	-0.75	0.05	0.59	-0.50	-0.09	0.71	-0.80	0.09
	(0.00)	(0.00)	(0.31)	(0.00)	(0.00)	(0.68)	(0.00)	(0.00)	(0.19)	(0.00)	(0.00)	(0.53)
g^{pc}				0.04	0.06	-0.10						
				(0.14)	(0.30)	(0.04)						
rr^{lr}							0.01	0.33	-0.34			
							(0.92)	(0.22)	(0.17)			
nfa										2.68	-5.57	2.89
										(0.27)	(0.32)	(0.46)

Note: The p-values of the non-linear Wald Test (See footnote 4 in the paper) with $H_0 : D_{LR}(i,j) = 0$ is reported within brackets.

TABLE A.4—ROBUSTNESS EXERCISES

8 Generations

	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g	-0.10	0.06	0.16	-0.07	0.24	-0.11	-0.08	-0.10
	(0.52)	(0.67)	(0.21)	(0.57)	(0.09)	(0.45)	(0.65)	(0.56)
I	-0.14	0.13	0.18	0.01	0.11	0.16	0.31	-0.76
	(0.68)	(0.66)	(0.45)	(0.97)	(0.71)	(0.59)	(0.34)	(0.02)
S	0.57	-0.10	-0.29	-0.13	0.52	0.85	0.40	-1.82
	(0.09)	(0.76)	(0.31)	(0.64)	(0.08)	(0.01)	(0.32)	(0.00)
H	-1.95	0.32	0.18	1.65	0.33	0.85	-0.50	-0.87
	(0.08)	(0.75)	(0.83)	(0.05)	(0.73)	(0.38)	(0.64)	(0.42)
rr	-0.99	0.21	0.59	0.95	-0.13	-0.12	-0.42	-0.09
	(0.18)	(0.75)	(0.24)	(0.08)	(0.84)	(0.85)	(0.50)	(0.89)
π	1.34	0.37	-0.31	-0.92	-0.66	0.11	0.00	0.07
	(0.00)	(0.29)	(0.28)	(0.00)	(0.05)	(0.74)	(0.99)	(0.85)

Note: The p-values of the non-linear Wald Test (See footnote 4 in the paper) with $H_0 : D_{LR}(i,j) = 0$ is reported within brackets.

TABLE A.5—LONG-RUN DEMOGRAPHIC IMPACT - D_{LR} - 8 GENERATIONS

	Workers	Dependants	Difference
g	$p(\sum_{j=2}^{5}\delta_j = 0)$=0.044	$p(\delta_1 + \sum_{j=7}^{8}\delta_j = 0)$=0.184	$p(\sum_{j=2}^{5}\delta_j = \delta_1 + \sum_{j=7}^{8})$=0.075
I	$p(\sum_{j=2}^{6}\delta_j = 0)$=0.166	$p(\delta_1 + \delta_8 = 0)$=0.056	$p(\sum_{j=2}^{6}\delta_j = \delta_1 + \delta_8)$=0.076
S	$p(\sum_{j=4}^{6}\delta_j = 0)$=0.001	$p(\delta_1 + \delta_8 = 0)$=0.0132	$p(\sum_{j=4}^{6}\delta_j = \delta_1 + \delta_8)$=0.000
H	$p(\sum_{j=2}^{5}\delta_j = 0)$=0.057	$p(\delta_1 + \sum_{j=7}^{8}\delta_j = 0)$=0.021	$p(\sum_{j=2}^{5}\delta_j = \delta_1 + \sum_{j=7}^{8}\delta_j)$=0.023
rr	$p(\sum_{j=2}^{5}\delta_j = 0)$=0.048	$p(\delta_1 + \sum_{j=7}^{8}\delta_j = 0)$=0.105	$p(\sum_{j=2}^{5}\delta_j = \delta_1 + \sum_{j=7}^{8}\delta_j)$=0.055
π	$p(\sum_{j=2}^{5}\delta_j = 0)$=0.001	$p(\delta_1 + \delta_8 = 0)$=0.009	$p(\sum_{j=2}^{5}\delta_j = \delta_1 + \delta_8)$=0.001

TABLE A.6—JOINT TESTS - P-VALUES OF NONLINEAR WALD TEST

Weak Exogeneity Test

Our estimation procedure assumes the demographic structure is exogenous to the dynamics of the main macroeconomic variables and thus a VARX is the appropriate specification. In order to test for the validity of the benchmark VARX, we run a VAR with both vectors Y_{it} and W_{it} treated as endogenous. Thus, we estimate

$$\begin{bmatrix} Y_{it} \\ W_{it} \end{bmatrix} = a_i + \begin{bmatrix} A^{endo} & D^{endo} \\ B_1 & B_2 \end{bmatrix} \begin{bmatrix} Y_{i,t-1} \\ W_{i,t-1} \end{bmatrix} + u_{it} \tag{A.1}$$

where A^{endo} and D^{endo} are the counterpart of A and D when demographics is considered endogenous, B_2 is the parameter matrix of lagged demographic weights and B_1 is the parameter matrix that links past macroeconomic variables and demographic weights. The weak exogeneity test verifies whether matrix B_1 is equal to zero.

Table A.7 shows the estimated coefficients of matrices A^{endo} (top left partition), B^{endo} (top right partition), B_1 (bottom left partition) and B_2 (bottom right partition). Changes in $Y_{i,t-1}$ do not significantly (economically) affect W_{it} supporting the VARX specification. We also find that the diagonal elements of B_2 are smaller but close to one, as demographic weights are very persistent and that matrices A^{endo} and B^{endo} are similar to matrices A and D obtained in the VARX estimation

	g	I	S	H	rr	π	β_1	β_2
g	0.22	-0.04	0.06	-0.05	-0.06	-0.14	0.08	0.02
	(0.04)	(0.07)	(0.04)	(0.02)	(0.04)	(0.05)	(0.03)	(0.05)
I	0.10	0.82	0.08	0.00	-0.04	-0.03	0.02	-0.01
	(0.02)	(0.03)	(0.02)	(0.01)	(0.02)	(0.02)	(0.01)	(0.02)
S	-0.02	-0.09	0.87	-0.03	-0.07	-0.07	0.08	0.04
	(0.03)	(0.04)	(0.03)	(0.01)	(0.02)	(0.03)	(0.02)	(0.03)
H	0.20	0.00	0.08	0.89	-0.07	-0.03	-0.06	0.09
	(0.02)	(0.04)	(0.02)	(0.01)	(0.02)	(0.03)	(0.02)	(0.03)
rr	0.10	-0.17	-0.11	0.00	0.85	0.24	-0.13	0.25
	(0.07)	(0.12)	(0.06)	(0.03)	(0.08)	(0.10)	(0.05)	(0.06)
π	0.00	0.24	0.07	-0.01	-0.14	0.56	0.23	-0.26
	(0.06)	(0.11)	(0.06)	(0.03)	(0.07)	(0.09)	(0.05)	(0.06)
β_1	0.00	-0.01	-0.02	0.01	-0.008	0.00	1.00	0.01
	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
β_2	0.01	0.01	-0.01	-0.00	0.01	0.01	0.02	1.01
	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: Standard errors are reported within brackets.

Table A.7—Weak Exogeneity Test

B.3. Demographics and Innovation

We now show the estimation results of the augmented model that links demographics and innovation.

	Three Generations			Eight Generations							
	β_1	β_2	β_3	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
g	0.02	0.07	-0.09	-0.10	0.06	0.17	-0.07	0.24	-0.12	-0.10	-0.08
	(0.52)	(0.26)	(0.09)	(0.51)	(0.67)	(0.21)	(0.58)	(0.09)	(0.43)	(0.63)	(0.65)
I	0.15	0.05	-0.20	-0.07	0.12	0.05	-0.02	0.04	0.30	0.61	-1.03
	(0.02)	(0.74)	(0.06)	(0.84)	(0.68)	(0.85)	(0.94)	(0.88)	(0.29)	(0.11)	(0.00)
S	0.24	0.22	-0.45	0.64	-0.11	-0.42	-0.16	0.47	0.97	0.67	-2.06
	(0.01)	(0.20)	(0.00)	(0.08)	(0.74)	(0.19)	(0.60)	(0.14)	(0.01)	(0.16)	(0.00)
H	-0.48	0.95	-0.47	-1.69	0.26	-0.32	1.55	0.10	1.35	0.60	-1.85
	(0.02)	(0.02)	(0.15)	(0.11)	(0.77)	(0.69)	(0.04)	(0.91)	(0.13)	(0.61)	(0.09)
rr	-0.12	0.53	-0.42	-1.08	0.23	0.76	0.99	-0.05	-0.29	-0.80	0.24
	(0.47)	(0.12)	(0.10)	(0.14)	(0.71)	(0.14)	(0.06)	(0.93)	(0.61)	(0.29)	(0.73)
$R\&D$	-3.70	4.50	-0.80	-9.26	2.45	3.56	2.43	4.84	0.30	-14.96	10.63
	(0.00)	(0.03)	(0.60)	(0.10)	(0.60)	(0.38)	(0.53)	(0.27)	(0.95)	(0.02)	(0.06)
π	0.72	-0.81	0.08	1.46	0.34	-0.53	-0.97	-0.76	0.33	0.49	-0.37
	(0.00)	(0.00)	(0.52)	(0.00)	(0.35)	(0.10)	(0.00)	(0.03)	(0.35)	(0.31)	(0.40)

Note: The p-values of the non-linear Wald Test (See footnote 4 in the paper) with $H_0 : D_{LR}(i, j) = 0$ is reported within brackets.

Table A.8—Long-Run Demographic Impact - Innovation

	Workers	Dependants	Difference
$R\&D$	$p(\sum_{j=2}^{5} \delta_j = 0)$=0.051	$p(\delta_1 + \delta_7 = 0)$=0.006	$p(\sum_{j=2}^{5} \delta_j = \delta_1 + \delta_7)$=0.010
$R\&D$	$p(\sum_{j=2}^{5} \delta_j = 0)$=0.051	$p(\delta_1 + \sum_{j=7}^{8} \delta_j = 0)$=0.129	$p(\sum_{j=2}^{5} \delta_j = \delta_1 + \sum_{j=7}^{8} \delta_j)$=0.068

Table A.9—Joint Tests - p-values of Nonlinear Wald Test

	Growth (g)		Investment (I)		Savings (S)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.22	0.07	0.10	0.02	-0.01	0.03
I_{t-1}	-0.04	0.06	0.83	0.03	-0.09	0.03
S_{t-1}	0.04	0.06	0.07	0.02	0.86	0.02
H_{t-1}	-0.04	0.02	-0.00	0.01	-0.02	0.01
rr_{t-1}	-0.11	0.03	-0.06	0.01	-0.10	0.02
$R\&D^{PA}{}_{t-1}$	-0.01	0.00	-0.00	0.00	-0.01	0.00
π_{t-1}	-0.19	0.05	-0.05	0.01	-0.11	0.02
P^{oil}_{t-1}	-0.02	0.00	-0.00	0.00	-0.01	0.00
P^{oil}_{t-2}	0.01	0.00	0.00	0.00	0.01	0.00
Δpop_t	0.39	0.48	0.01	0.23	-0.00	0.29
Δpop_{t-1}	-0.28	0.63	-0.04	0.20	-0.65	0.11
β_1	0.09	0.04	0.02	0.01	0.08	0.02
β_2	0.03	0.02	-0.00	0.02	0.05	0.02
R^2	0.22		0.86		0.81	
$\Pr(\delta_j = 0)$	0.01		0.32		0.00	

Table A.10—Results for Growth, Investment and Savings: Augmented Model (3G)

	Hours (H)		Real Rates (rr)		$R\&D^{PA}$		Inflation (π)	
	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
g_{t-1}	0.19	0.04	0.10	0.03	-0.01	0.13	-0.00	0.02
I_{t-1}	-0.00	0.05	-0.16	0.09	0.17	0.16	0.26	0.06
S_{t-1}	0.06	0.03	-0.11	0.06	-0.54	0.26	0.07	0.07
H_{t-1}	0.90	0.01	-0.00	0.05	0.25	0.15	-0.01	0.03
rr_{t-1}	-0.10	0.04	0.83	0.06	-0.13	0.24	-0.15	0.04
$R\&D^{PA}{}_{t-1}$	-0.01	0.00	0.00	0.00	0.90	0.04	-0.01	0.00
π_{t-1}	-0.05	0.03	0.22	0.07	-0.05	0.15	0.55	0.04
P^{oil}_{t-1}	-0.01	0.00	0.01	0.00	0.02	0.03	-0.02	0.01
P^{oil}_{t-2}	0.01	0.00	-0.00	0.00	-0.03	0.02	0.01	0.01
Δpop_t	-0.43	0.42	-1.05	0.24	-0.97	2.38	0.81	0.20
Δpop_{t-1}	-0.33	0.47	0.20	0.14	-2.60	0.76	-0.80	0.19
β_1	-0.06	0.03	-0.11	0.05	-0.14	0.19	0.23	0.04
β_2	0.11	0.02	0.28	0.08	0.37	0.38	-0.28	0.08
R^2	0.92		0.64		0.85		0.77	
$\Pr(\delta_j = 0)$	0.00		0.00		0.20		0.00	
N	716		716		716		716	

Table A.11—Results for Hours, Interest Rate, R&D and Inflation: Augmented Model (3G)

Appendix C. Theoretical Model

In this section we present the equilibrium conditions in more detail, the extension that incorporates pension and health expenditure and some additional simulation results.

C.1. Equilibrium Conditions

We start by looking at the factor markets with the final and input firms decisions.

Production Sector

Intermediate good firms select capital, its utilisation, labour and intermediate goods demand to minimise total costs, $TC_t^j = W_t \xi_t L_t^j + (r_t^k + \delta(U_t^j))K_t^j + P_t^M M_t^j$ given a level of production $Y_{c,t}^j = \left[(U_t^j K_t^j)^{\alpha}(\xi_t L_t^j)^{(1-\alpha)}\right]^{(1-\gamma_I)} \left[M_t^j\right]^{\gamma_I}$.

Labour allocation is such that

$$(1-\alpha)(1-\gamma_I)Y_{c,t} = \mu_t W_t \xi_t L_t. \tag{A.2}$$

Capital stock and utilisation are such that

$$\alpha(1-\gamma_I)Y_{c,t} = \mu_t[r_t^k + \delta(U_t)]K_t, \tag{A.3}$$
$$\alpha(1-\gamma_I)Y_{c,t} = \mu_t \delta'(U_t)K_t U_t. \tag{A.4}$$

Intermediate goods are set such that

$$\mu_t M_t P_t^M = \gamma_I Y_{c,t} \tag{A.5}$$

where P_t^M is the price of intermediate goods.

In order to obtain this price one can minimise total cost of intermediary goods $\int_0^A \tilde{P}^M M^i di$ subject to (6) getting

$$P_t^M = \vartheta A_t^{1-\vartheta} \tag{A.6}$$

Combining (4) and (5) and defining total labour supply as $L_t \equiv \int_0^{N_t^f} L_t^j dj$ and total intermediate composite demand as $M_t \equiv \int_0^{N_t^f} M_t^j dj$, then[1]

$$Y_{c,t} = (N_t^f)^{\mu_t - 1} \left[(U_t \frac{K_t}{\xi_t L_t})^{\alpha} (\xi_t L_t) \right]^{(1-\gamma_I)} [M_t]^{\gamma_I} \tag{A.7}$$

Due to free entry the number of intermediate good firms is such that their profits are equal to the operating costs. Using (4) total output per firm is given by $Y_{c,t}(N_t^f)^{-\mu_t}$, while their mark-up is given by $\frac{\mu_t - 1}{\mu_t}$, thus

$$\frac{\mu_t - 1}{\mu_t} Y_{c,t}(N_t^f)^{-\mu_t} = \Omega \tilde{\Psi}_t \tag{A.8}$$

Finally, let Y_t denote aggregate value added output. Y_t is equal to the total output net intermediate goods and operating costs. Thus, using (A.6)[2],

$$Y_t = Y_{c,t} - A_t^{1-\vartheta} M_t - N_t^f \Omega \tilde{\Psi}_t. \tag{A.9}$$

On the expenditure side, output must be equal to consumption, investment and costs of R&D and adoption. Thus,

$$Y_t = C_t + I_t + S_t + \Xi_t (Z_t - A_t) + \tau_t. \tag{A.10}$$

Innovation Process

From conditions (10) and (14) one can easily determine the flow of the stock of invented (prototypes) and adopted goods, which are given by

[1] Note that all firms select the same capital labour ratio.

[2] In order to net out intermediate goods one has to compute total expenditure on intermediate goods ($\int_0^A \tilde{P}^M M^i di$) minus the markup on intermediate goods ($\int_0^A (\tilde{P}^M - 1) M^i di$).

$$\text{(A.11)} \qquad \frac{Z_{t+1}}{Z_t} = (\gamma^{yw})^{\rho^{yw}} \chi \left(\frac{S_t}{\tilde{\Psi}_t}\right)^{\rho} + \phi, \text{ and}$$

$$\text{(A.12)} \qquad \frac{A_{t+1}}{A_t} = \lambda \left(\frac{A_t \Xi_t}{\tilde{\Psi}_t}\right) \phi[Z_t/A_t - 1] + \phi$$

Investment in R&D (S_t) is determined by (11) which using (10) becomes

$$\text{(A.13)} \qquad S_t = R_{t+1}^{-1} \phi E_t J_{t+1}(Z_{t+1} - \phi Z_t).$$

Profits are given by the total gain in seeling the right to goods invented as a result of the previous period investment S_{t-1} to adopters minus the cost of borrowing for that investment. Thus,

$$\Pi_t^{RD} = \phi J_t (Z_t - \phi Z_{t-1}) + S_{t-1} R_t$$

In a perfect foresight equilibrium $\Pi_t^{RD} = 0$.

Investment in adoption (Ξ_t) is determined by solving (13). We thus obtain the following condition

$$\text{(A.14)} \qquad \frac{A_t}{\tilde{\Psi}_t} \lambda' R_t^{-1} \phi [V_{t+1} - J_{t+1}] = 1$$

where $\frac{A_t}{\tilde{\Psi}_t}\lambda' = \frac{\partial \lambda\left(\frac{A_t \Xi_t}{\tilde{\Psi}_t}\right)}{\partial \Xi_t}$. Assuming the elasticity of λ_t to changes in Ξ_t is constant, thus $\epsilon_\lambda = \frac{\lambda'}{\lambda_t} \frac{A_t \Xi_t}{\tilde{\Psi}_t}$, then we obtain

$$\text{(A.15)} \qquad \Xi_t = \epsilon_\lambda \lambda_t R_t^{-1} \phi [V_{t+1} - J_{t+1}]$$

Finally, the value of an invented good and an adopted good are given by

$$\text{(A.16)} \qquad J_t = -\Xi_t + (R_{t+1})^{-1} \phi E_t [\lambda_t V_{t+1} + (1 - \lambda_t) J_{t+1}], \text{ and}$$

$$\text{(A.17)} \qquad V_t = (1 - 1/\vartheta) \gamma_I \frac{Y_{c,t}}{\mu_t A_t} + (R_{t+1})^{-1} \phi E_t V_{t+1}$$

where $\lambda_t = \lambda \left(\frac{A_t \Xi_t}{\tilde{\Psi}_t}\right)$ and $\Pi_{m,t} = (1 - 1/\vartheta) P_t^M M_t = (1 - 1/\vartheta) \gamma_I \frac{Y_{c,t}}{\mu_t A_t}$.

Profits for adopters are given by the gain from marketing specialised intermediated goods net the amount paid to inventors to gain access to new goods and the expenditures on loans to pay for adoption intensity.

$$\Pi_t^A = (1 - 1/\vartheta)\gamma_I \frac{Y_{c,t}}{\mu_t} - J_t(Z_t - \phi Z_{t-1}) - \Xi_{t-1}(Z_{t-1} - A_{t-1})R_t$$

Household Sector

Retiree j decision problem is

$$\max V_t^{jr} = \left\{(C_t^{jr})^{\rho_U} + \beta\omega_{t,t+1}^r([V_{t+1}^{jr}]^{\rho_U})\right\}^{1/\rho_U} \tag{A.18}$$

subject to

$$C_t^{jr} + FA_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr} + d_t^{jr}. \tag{A.19}$$

The first order condition and envelop theorem are

$$(C_t^{jr})^{\rho_U - 1} = \beta\omega_{t,t+1}^r \frac{\partial V_{t+1}^{jr}}{\partial FA_{t+1}^{jr}} (V_{t+1}^{jr})^{\rho_U - 1}, \tag{A.20}$$

$$\frac{\partial V_t^{jr}}{\partial FA_t^{jr}} = (V_{t+1}^{jr})^{1-\rho_U} (C_t^{jr})^{\rho_U - 1} \frac{R_t}{\omega_{t-1,t}^r}. \tag{A.21}$$

Combining these conditions above gives the Euler equation

$$C_{t+1}^{jr} = (\beta R_{t+1})^{1/(1-\rho_U)} C_t^{jr} \tag{A.22}$$

We conjecture that retirees consume a fraction of all assets (including financial assets, profits from financial intermediaries), such that

$$C_t^{jr} = \varepsilon_t \varsigma_t \left[\frac{R_t}{\omega_{t-1,t}^r} FA_t^{rj} + D_t^{rj}\right]. \tag{A.23}$$

Combining these and the budget constraint gives

$$FA_{t+1}^{jr} = \frac{R_t}{\omega_{t-1,t}^r} FA_t^{jr}(1 - \varepsilon_t\varsigma_t) + d_t^{jr} - \varepsilon_t\varsigma_t(D_t^{rj}).$$

Using the condition above, the Euler equation and the solution for consumption gives

$$(\text{A.24})\quad (\beta R_{t+1})^{1/(1-\rho_U)}\varepsilon_t\varsigma_t[\frac{R_t}{\omega^r_{t-1,t}}FA^{rj}_t + D^{rj}_t] =$$
$$\varepsilon_{t+1}\varsigma_{t+1}\left[\frac{R_{t+1}}{\omega^r_{t,t+1}}\left(\frac{R_t}{\omega^r_{t-1,t}}FA^{jr}_t(1-\varepsilon_t\varsigma_t) + d^{jr}_t - \varepsilon_t\varsigma_t D^{rj}_t\right) + D^{jr}_{t+1}\right].$$

Collecting terms we have that

$$(\text{A.25})\quad 1-\varepsilon_t\varsigma_t = \frac{(\beta R_{t+1})^{1/(1-\rho_U)}\omega^r_{t,t+1}}{R_{t+1}}\frac{\varepsilon_t\varsigma_t}{\varepsilon_{t+1}\varsigma_{t+1}},$$
$$(\text{A.26})\quad D^{jr}_t = d^{jr}_t + \frac{\omega^r_{t,t+1}}{R_{t+1}}D^{jr}_{t+1}.$$

One can also show that $V^{jr}_t = (\varepsilon_t\varsigma_t)^{-1/\rho_U}C^{jr}_t$.

Worker j decision problem is

$$(\text{A.27})\quad \max V^{jw}_t = \left\{(C^{jw}_t)^{\rho_U} + \beta[\omega^w V^{jw}_{t+1} + (1-\omega^w)V^{jr}_{t+1}]^{\rho_U}\right\}^{1/\rho_U}$$

subject to

$$(\text{A.28})\quad C^{jw}_t + FA^{jw}_{t+1} = R_t FA^{jw}_t + W_t\xi_t + d^{jw}_t - \tau^{jw}_t.$$

First order conditions and envelop theorem are

$$(\text{A.29})\quad (C^{jw}_t)^{\rho_U-1} = \beta[\omega^w V^{jw}_{t+1} + (1-\omega^w)V^{jr}_{t+1}]^{\rho_U-1}\left[\omega^w\frac{\partial V^{jw}_{t+1}}{\partial FA^{jw}_{t+1}} + (1-\omega^w)\frac{\partial V^{jr}_{t+1}}{\partial FA^{jw}_{t+1}}\right],$$
$$(\text{A.30})\quad \frac{\partial V^{jw}_t}{\partial FA^{jw}_t} = (V^{jw}_{t+1})^{1-\rho_U}(C^{jw}_t)^{\rho_U-1}R_t,\ \text{and}$$
$$(\text{A.31})\quad \frac{\partial V^{jr}_t}{\partial FA^{jw}_t} = \frac{\partial V^{jr}_t}{\partial FA^{jr}_t}\frac{\partial FA^{jr}_t}{\partial FA^{jw}_t} = \frac{\partial V^{jr}_t}{\partial FA^{jr}_t}\frac{1}{\omega^r_{t-1,t}} = (V^{jr}_{t+1})^{1-\rho_U}(C^{jr}_t)^{\rho_U-1}R_t.$$

$\frac{\partial FA^{jr}_t}{\partial FA^{jw}_t} = \frac{1}{\omega^r_{t-1,t}}$ since as individuals are risk neutral with respect to labour income they select the same asset profile independent of their worker/retiree status, adjusting only for expected return due to the probability of death.

Combining these conditions above, and using the conjecture that $V_t^{jw} = (\varsigma_t)^{-1/\rho_U} C_t^{jw}$, gives the Euler equation

$$
\begin{aligned}
\text{(A.32)}\quad C_t^{jw} \quad &= \quad \left((\beta R_{t+1} \mathfrak{z}_{t+1})^{1/(1-\rho_U)} \right)^{-1} [\omega^w C_{t+1}^{jw} + (1-\omega^w) \varepsilon_{t+1}^{\frac{-1}{\rho_U}} C_{t+1}^{jr}] \\
&\quad \text{where } \mathfrak{z}_{t+1} = (\omega^w + (1-\omega^w)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}).
\end{aligned}
$$

We conjecture that retirees consume a fraction of all assets (including financial assets, human capital and profits from financial intermediaries), such that

$$
\text{(A.33)} \qquad C_t^{jw} = \varsigma_t [R_t F A_t^{jw} + H_t^{jw} + D_t^{jw} - T_t^{jw}].
$$

Following the same procedure as before we have that

$$
\begin{aligned}
\text{(A.34)}\quad & \varsigma_t [R_t F A_t^{jw} + H_t^{jw} + D_t^{jw}] (\beta R_{t+1} \mathfrak{z}_{t+1})^{1/(1-\rho_U)} = \\
& \omega^w \varsigma_{t+1} \big[R_{t+1} \big(R_t F A_t^{jw} (1-\varsigma_t) + W_t \xi_t + d_t^{jw} - \tau_t^{jw} - \varsigma_t (H_t^{jw} + D_t^{jw} - T_t^{jw}) \big) + H_{t+1}^{jw} + D_{t+1}^{jw} - T_{t+1}^{jw} \big] + \\
& \varepsilon_{t+1}^{\frac{-1}{\rho_U}} (1-\omega^w) \varepsilon_{t+1} \varsigma_{t+1} \big[R_{t+1} \big(R_t F A_t^{jw} (1-\varsigma_t) + W_t \xi_t + d_t^{jw} - \tau_t^{jw} - \varsigma_t (H_t^{jw} + D_t^{jw} - T_t^{jw}) \big) + D_{t+1}^{jr} \big].
\end{aligned}
$$

Collecting terms and simplifying we have that

$$
\begin{aligned}
\text{(A.35)}\qquad \varsigma_t \quad &= \quad 1 - \frac{\varsigma_t}{\varsigma_{t+1}} \frac{(\beta R_{t+1} \mathfrak{z}_{t+1})^{1/(1-\rho_U)}}{R_{t+1} \mathfrak{z}_{t,t+1}} \\
\text{(A.36)}\qquad H_t^{jw} \quad &= \quad W_t \xi_t + \frac{\omega^w}{R_{t+1} \mathfrak{z}_{t,t+1}} H_{t+1}^{jw} \\
\text{(A.37)}\qquad T_t^{jw} \quad &= \quad \tau_t^{jw} + \frac{\omega^w}{R_{t+1} \mathfrak{z}_{t,t+1}} T_{t+1}^{jw} \text{ and} \\
\text{(A.38)}\qquad D_t^{jw} \quad &= \quad d_t^{jw} + \frac{\omega^w}{R_{t+1} \mathfrak{z}_{t,t+1}} D_{t+1}^{jw} + \frac{(1-\omega^w)\varepsilon_{t+1}^{(\rho_U-1)/\rho_U}}{R_{t+1} \mathfrak{z}_{t,t+1}} D_{t+1}^{jr}.
\end{aligned}
$$

Aggregation across households

Assume that for any variable X_t^{jz} we have that $X_t^z = \int_0^{N_t^z} X_t^{jz}$ for $z = \{w, r\}$, then

$$L_t = N_t^w, \tag{A.39}$$

$$H_t^w = W_t \xi_t L_t + \frac{\omega^w}{R_{t+1} \mathfrak{z}_{t,t+1}} H_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, \tag{A.40}$$

$$T_t^w = \tau_t + \frac{\omega^w}{R_{t+1} \mathfrak{z}_{t,t+1}} T_{t+1}^w \frac{N_t^w}{N_{t+1}^w}, \tag{A.41}$$

$$D_t^w = d_t^w + \frac{\omega^w}{R_{t+1} \mathfrak{z}_{t,t+1}} D_{t+1}^w \frac{N_t^w}{N_{t+1}^w} + \frac{(1-\omega^w)\varepsilon_{t+1}^{(\rho_U - 1)/\rho_U}}{R_{t+1} \mathfrak{z}_{t,t+1}} D_{t+1}^r \frac{N_t^w}{N_{t+1}^r}, \tag{A.42}$$

$$C_t^w = \varsigma_t [R_t F A_t^w + H_t^w + D_t^w - T_t^w], \tag{A.43}$$

$$D_t^r = d_t^r + \frac{\omega_{t,t+1}^r}{R_{t+1}} D_{t+1}^r \frac{N_t^r}{N_{t+1}^r}, \tag{A.44}$$

$$C_t^r = \varepsilon_t \varsigma_t [R_t F A_t^r + D_t^r]. \tag{A.45}$$

Note that $\omega_{t,t+1}^r$ is not shown in the last equation due to the perfect annuity market for retirees, allowing for the redistribution of assets of retirees who died at the end of the period.

Decision of Investment in Labour Skill

Society maximizes the net gains from investing in education. The net gain is given by the increase in the utility of the young when she becomes a worker minus the cost of funding education for a given worker. The gain for education is denoted by the change in the present value of the utility for a young individual when (s)he becomes a worker (in the next period) until her death. The transition from young to worker occurs with probability $(1-\omega^y)$. The cost of funding is denoted by the variation in the current value of utility of a worker. The net gain is given by

$$NV^E = \int_{N_t^y} \beta(1-\omega^y) V_{t+1}^{wh} - \int_{N_t^w} V_t^{wj} \tag{A.46}$$

Society then selects τ_t^{wj}, or implicitly the investment in education, to maximise NV^E. This effectively entails equating the marginal cost and marginal benefit of education investment. The marginal cost of increasing lump-sum taxes for worker

j today to finance higher investment in young's education is given by

$$MC_t^{Ej} = -\frac{\partial V_t^{wj}}{\partial \tau_t^{wj}} = \frac{\partial V_t^{wj}}{\partial C_t^{wj}} = \varsigma_t^{-1/\rho_U} \tag{A.47}$$

The marginal benefit of increasing lump-sum taxes at time t for a young h who becomes a worker next period is

$$MB_t^{Eh} = \beta(1-\omega^y)\frac{\partial V_{t+1}^{wh}}{\partial \tau_t^{wj}} = \beta(1-\omega^y)\frac{\partial V_{t+1}^{wh}}{\partial \xi_{t+1}^y}\frac{\partial \xi_{t+1}^y}{\partial I_t^y}\frac{\partial I_t^y}{\partial \tau_t}\frac{\partial \tau_t}{\partial \tau_t^{wj}} \tag{A.48}$$

$$= \beta(1-\omega^y)\varsigma_{t+1}^{-1/\rho_U}\frac{W_{t+1}}{W_t}\chi_E\frac{I_t^y}{\xi_t} \tag{A.49}$$

Adding costs across all workers and benefits across all young at time t gives the condition that determines I_t^y. That is

$$\varsigma_t^{-1/\rho_U} = \beta(1-\omega^y)\varsigma_{t+1}^{-1/\rho_U}\zeta_t^y\frac{W_{t+1}}{W_t}\chi_E\frac{I_t^y}{\xi_t} \tag{A.50}$$

Financial Intermediary

The profits of the financial intermediary are

$$\Pi_t^F = [r_t^k+1]K_t+R_tB_t-R_t(FA_t^w+FA_t^r)-K_{t+1}-B_{t+1}+FA_{t+1}^w+FA_{t+1}^r+\sum_x(\Pi_t^{RD}+\Pi_t^A), \tag{A.51}$$

where $B_{t+1} = S_t + \Xi_t(Z_t - A_t)$ and $FA_t = FA_t^w + FA_t^r$.

The financial intermediaries selects capital and bonds such that it maximize profits and thus we obtain the standard arbitrage conditions whereby all assets must pay the same expected return, thus

$$E_t\left[r_{t+1}^k + 1\right] = R_t. \tag{A.52}$$

Also note that under a perfect foresight solution, by ensuring the financial intermediary behaves under perfect competition, this equality holds without expectations, $\Pi_t^F = 0$ and thus $d_t^r = d_t^w = 0$. If $\Pi_t^F \neq 0$, then we assume profits are divided based on the ratio of assets. As such, $d_t^r = \Pi_t^F\frac{FA_t^r}{FA_t^r+FA_t^w}$ and $d_t^w = \Pi_t^F\frac{FA_t^w}{FA_t^r+FA_t^w}$.

The flow of capital is then given by

$$K_{t+1} = K_t(1-\delta(U_t)) + I_t. \tag{A.53}$$

Where I_t is the investment in capital made by the financial intermediary.

Asset Markets

Asset Market clearing implies

$$FA_{t+1} = FA^w_{t+1} + FA^r_{t+1} = K_{t+1} + B_{t+1} \tag{A.54}$$

Finally, the flow of assets are given by

$$FA^w_{t+1} = \omega^w(R_t FA^w_t + W_t\xi_t L_t + d^w_t - C^w_t - \tau_t) \tag{A.55}$$

$$FA^r_{t+1} = R_t FA^r_t + d^r_t - C^r_t + (1-\omega^w)(R_t FA^w_t + W_t\xi_t L_t + d^w_t - C^w_t - \tau_t) \tag{A.56}$$

C.2. Extension - Including Pension and Health Expenditures

In this extension we include a pay-as-you-go pension scheme and a society funded health expenditure to retirees. These expenditures are funded by lump-sum taxes paid by workers.

Health Expenditure

We assume society aims to keep a constant health expenditure for each retiree divided by output per capita. As such,

$$\frac{HE^{jr}_t}{Y_{c,t}/N_t} = \delta_{HE}, \tag{A.57}$$

summing across retirees,

$$HE_t = \delta_{HE} Y_{c,t}\frac{N^r_t}{N_t} = \delta_{HE} Y_{c,t}\frac{\zeta^r_t}{1+\zeta^r_t+\zeta^y_t}. \tag{A.58}$$

Pay-as-you-go Pension Scheme

Each new retiree at time t ($nr \in (1-\omega^w)N^w_{t-1}$) is promised a pension payment ($PE^{nr}_\tau = PE^{nr}_t g^\xi_\tau$) to be delivered at every period during retirement (for $\tau > t$

condition on the retiree surviving until then) that comprises a payment PE_t^{nr} at time of retirement based on her average labour income in the past n_p years before retirement, denoted (AY_t^{nr}), that is then adjusted based on societies labour productivity gain due to human capital accumulation. As such

(A.59) $PE_t^{nr} = \eta^r AY_t^{nr}$,

where η^r is the replacement ratio, and

(A.60) $AY_t^{nr} = (1/n_p)(W_t\xi_t) + (1 - 1/n_p)AY_{t-1}^{nr}$,

summing across new retirees,

(A.61) $PE_t = \eta^r AY_t$,

(A.62) $$AY_t = (1/n_p)(W_{t-1}\xi_{t-1}(1-\omega^w)N_{t-1}^w) + (1 - 1/n_p)AY_{t-1}\frac{N_{t-1}^w}{N_{t-2}^w}.$$

We can now determine the total payment of pensions (TPE_t), where the adjustments on TPE_{t-1} are made due to mortality and labour productivity growth. Thus,

(A.63) $$TPE_t = PE_t + \omega_{t-1,t}^r TPE_{t-1} g_t^{\xi}.$$

Taxes

Total taxes on workers (τ) now must fund the expenditure on education of the young, $\tau_t^E = W_t I_t^y N_t^w$, and total aging expenditure $\tau_t^A = TPE_t + HE_t$.

Retirees Problem and Good Market Clearing Condition

Retirees now have an additional source of income and thus their consumption decision now is given by

(A.64) $$C_t^r = \varepsilon_t \varsigma_t [R_t FA_t^r + D_t^r + PE_t^r], \text{ where}$$

(A.65) $$PE_t^r = TPE_t + \frac{\omega_{t,t+1}^r}{R_{t+1}} PE_{t+1}^r \frac{N_t^r}{N_{t+1}^r}.$$

Health Expenditure must be added to the goods market clearing condition and thus this condition now becomes,

(A.66) $$Y_t = C_t + I_t + S_t + \Xi_t(Z_t - A_t) + \tau_t^E + HE_t.$$

Calibration of new parameters: np, η^r, and δ_{HE}. np is set to 25 years (in some countries a measure using the full career is used but in many cases the best or last 25 years is used - see European Commission (2015)). We set the replacement ratio to be 40%, $\eta^r = 0.4$ (slightly higher than the average in Europe in 2013, although replacement ratios are expected to decrease substantially in the next decades - see European Commission (2015)). Finally, using data for Medicare in the US, we note that roughly %60 of the population above 65 are enrolled and the average payment per enrolle is around 16000 per year or roughly 20% of per capita income (see AARP Public Policy Institute (2009) and Curto, Einav, Finkelstein, Levin, and Bhattacharya (2017)). We thus set $\delta_{HE} = 0.12$. The ratio of health spending per capita and GDP per capita for individuals above 60 in countries in the European Union is also in the range 10% to 15% (see European Commission (2015)).

C.3. Additional Simulations

Figure A.3 shows the results of our theoretical projections for additional countries.

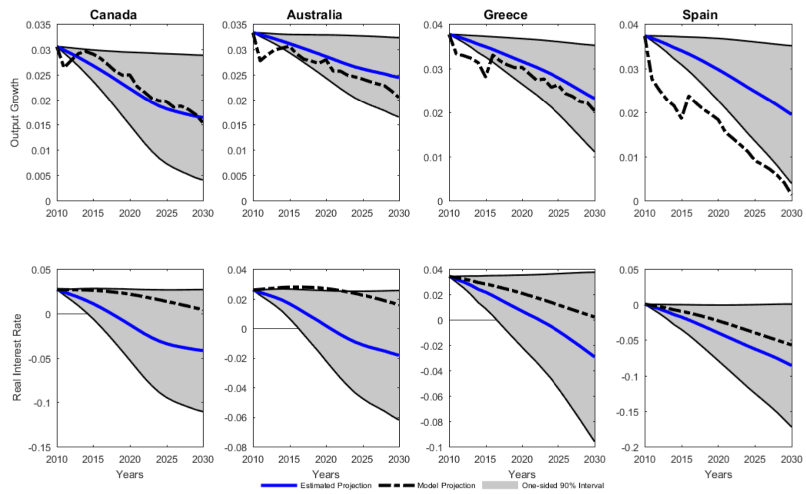

FIGURE A.3. SIMULATION: *Projection* - ADDITIONAL COUNTRIES

Figure A.4 shows the results of our theoretical projections when we consider that the congestion factor depends on the active population instead of total population.

The ageing simulation exercises consider the case where the age distribution of productivity in innovation shifts to the right. The Figure A.5 shows the benchmark and the new distribution. Under this new distribution the ageing effect on growth is partially offset.

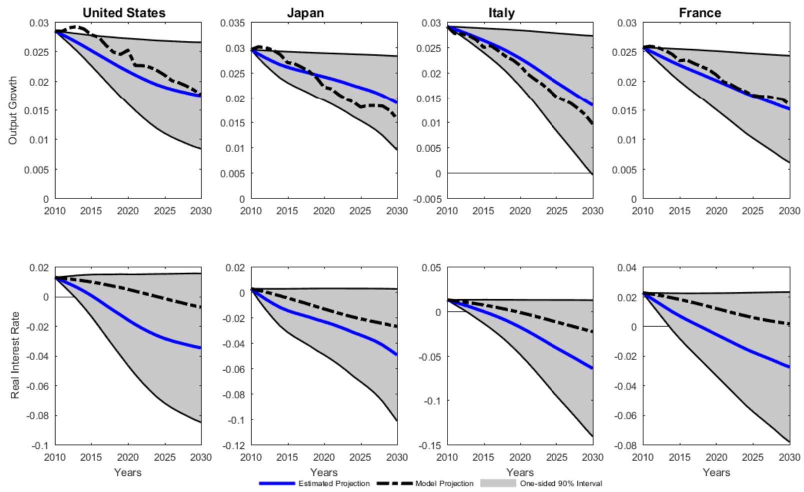

FIGURE A.4. SIMULATION: *Projection* - ROBUSTNESS - $\hat{\gamma}_{yw} = \Gamma_{yw}/N_{w,t}$

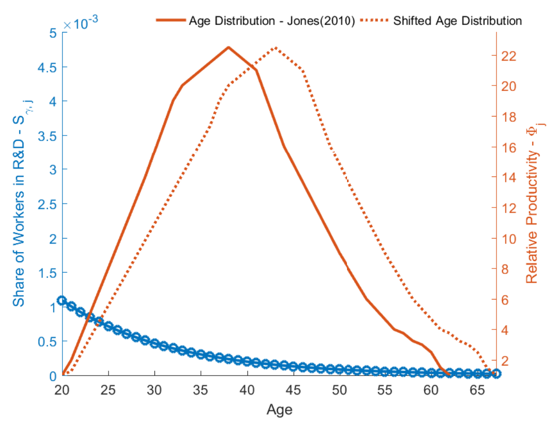

FIGURE A.5. AGE DISTRIBUTION OF IDEAS AND SHARE OF WORKERS IN INNOVATION

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