

Online Appendix for “Consumer Imperfect Information and Endogenous Price Rigidity”

Jean-Paul L’Huillier

May 2019

A.2 Example of the Firm’s Problem Using Linear Demand

The goal is to provide a less technical exposition of the forces leading to the price rigidity.

Consider the problem of a monopolist selling a good c to a unit mass of consumers, indexed by i . Demand is a linear function of the real price, p/P

$$c_i\left(\frac{p}{P}\right) = 1 - \frac{p}{P}$$

where p is the monopolist’s price and P is the price level. The price level can take two values, high (P^h) or low (P^l), $P^h > P^l$, both with equal probability $Pr(P = P^h) = Pr(P = P^l) = 1/2$. This price level is a device for modeling a monetary shock. Throughout the paper I use the terms price level, monetary shock, or aggregate state of the world interchangeably. The monopolist has zero costs.

Profit maximization yields $p^h \equiv P^h/2$ when the state is high, and $p^l \equiv P^l/2$ when the state is low. Notice that since $P^h > P^l$, $p^h > p^l$. In this example, and throughout the paper, I will use the term “price increase” to the act of posting the (high) price p^h , and to a “price decrease” to the act of posting the (low) price p^l . This terminology is used having a dynamic model in mind, in which firms increase prices in the long run in proportion to P^h after a positive monetary shock, and decrease prices in proportion to P^l after a negative monetary shock.³²

Suppose that a proportion $1 - \alpha$ of consumers are uninformed about the price level P . The complementary proportion α is informed and knows the realization

³²See A.8 for this model.

of P . Consider an uninformed consumer. Unless this consumer learns P , he is unable to compute the real price of c , p/P , and thus he is uncertain about much to buy from the monopolist. That is, he is unable to evaluate whether a price p is ‘expensive’ or ‘cheap’. As I will show this feature is key for generating the rigidity in the pricing of the monopolist.³³

Uninformed consumers form an expectation about the inverse of P , $E_i[1/P]$. This expectation depends on prior beliefs—determined by the prior distribution of P , and on the price posted by the monopolist—which can potentially provide information. Thus, the uninformed have the following demand function:

$$c_i \left(p E_i \left[\frac{1}{P} \right] \right) = 1 - p \cdot E_i \left[\frac{1}{P} \right]$$

The monopolist knows the realization of the price level, and all consumers know that the monopolist is informed. Our goal now is to analyze different pricing strategies and their implications for demand and profits. The monopolist maximizes revenues

$$p \left(\alpha \left(1 - p \frac{1}{P} \right) + (1 - \alpha) \left(1 - p \cdot E_i \left[\frac{1}{P} \right] \right) \right) \quad (19)$$

The monopolist takes into account that uninformed consumers update their beliefs about the price level upon observation of p . If, in equilibrium, the monopolist posts different prices as a function of the price level, uninformed consumers can learn the price level. If the monopolist’s price is rigid—in the sense that it does not change with the price level—then uninformed consumers keep their prior beliefs

$$E_i \left[\frac{1}{P} \right] = \frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l}$$

This fact gives rise to the following strategic tension. Notice from (19) that the monopolist is better off if uninformed consumers believe that the price level is high. The reason is that they would increase their demand for any p , and the monopolist would get higher profits. Thus, the monopolist has a motive to make

³³The uncertainty about the price level is a modeling device for introducing uncertainty about consumers’ (nominal) valuation. It should be obvious that there are other, more direct ways of producing valuation uncertainty, as assuming for instance that consumers are uncertain about the value of some parameter of their utility function.

them believe so. However, uninformed consumers understand the monopolist's strategy and use Bayes' rule when updating their beliefs, and therefore cannot be misled. Therefore, price increases are more difficult to implement than under perfect information. To understand this, suppose no consumer is informed ($\alpha = 0$). Can there be an equilibrium where the monopolist posts the same prices as under perfect information, p^h and p^l ? The answer is *no*, and the reason is as follows. Suppose that such an equilibrium is possible. When consumers see p^h , they understand the price level is high and spend more (in nominal terms). The opposite happens if consumers see p^l : they understand the price level is low and spend less. But this implies that the monopolist receives higher nominal profits when it posts p^h . Then, when the price level is low, it has a profitable deviation: to post p^h . Indeed, in this case consumers think that the price level is high, the monopolist increases nominal (and real) profits. This immediately shows that the alleged equilibrium is in fact not one.

If the proportion of informed is high enough, there exists an equilibrium where the firm posts perfect information prices. The following result establishes this fact.

Result 1 *If, and only if*

$$\alpha \geq \frac{P^l}{P^h} \tag{20}$$

there exists an equilibrium where the firm posts the same prices as under perfect information.

Proof (sketch). Optimal prices are p^h and p^l . The Incentive Compatibility (IC) constraint for the firm when the price level is low is

$$p^l \left(1 - \frac{p^l}{P^l} \right) \geq p^h \left(\alpha \left(1 - \frac{p^h}{P^l} \right) + (1 - \alpha) \left(1 - \frac{p^h}{P^h} \right) \right)$$

Solving this inequality for α yields (20).

■

The intuition for this result is that informed consumers discipline the firm by buying less when the state of the world is low and the firm posts p^h . If α is high enough, then there are enough informed consumers to discipline the

firm to the point that there is an equilibrium at p^h and p^l . Generally, even if only few consumers are informed, it turns out that equilibria with flexible prices are possible, but in these equilibria the price posted when the state is high is strictly higher than under perfect information (at a price \bar{p} such that $\bar{p} > p^h$). In other words, there is a distortion at the top. This distortion at the top implies that the monopolist gets strictly lower average (real) profits than in the perfect information benchmark. As such, the model endogenously generates a cost to adjusting prices when there is imperfect information among consumers, and the firm is superiorly informed. Here, this loss is necessary for information transmission.

The firm can be better off not transmitting this information. To see this, suppose that the monopolist charges the same price independently of the price level. In this case uninformed consumers do not update their beliefs, and their demand is determined by their ex-ante belief of the price level. Compute the (ex-post) profit maximizing price under these conditions. If the monopolist posts this price in both states, there are no distortions, and by risk neutrality this implies that real average profits in this equilibrium are the same as under perfect information. In particular, they are higher than in all equilibria where the monopolist has flexible prices and α is low. More generally, if the proportion of informed consumers is low enough, the monopolist is better off having a rigid price, as stated in the next result.

Result 2 (Price rigidity) *There is α^* and p^* such that if $\alpha \leq \alpha^*$, the firm gets higher average real profits by posting the rigid price p^* than by posting a price that reacts to the price level P .*

This is a particular case of Proposition 1 and therefore the proof is here omitted. This result provides a rationale for price rigidity, and is stated formally in the body.

A.3 Further Price Rigidity Results

This section complements the results of the static model. It goes in detail over a characterization of the game, of the benchmark equilibria, and briefly discusses refinements. The discussion and presentation of the main results also attempts to provide intuition.

The signaling game in fact belongs to the well-known class of monotonic signaling games. To show this, it is first necessary to define the following well-known property for a function of two variables.

Definition 4 (Increasing Differences Property) *A function $f(x, y)$ has strict increasing differences in (x, y) if, for $x' > x$ and $y' > y$,*

$$f(x, y') - f(x, y) < f(x', y') - f(x', y)$$

The following assumption is crucial for tractability.

Assumption 5 *The revenue function $pc_i(p/P)$ has strict increasing differences in (p, P) .*

The following lemma shows that this is a monotonic signaling game.

Lemma 5 (Characterization of the Game) *If $\alpha > 0$ and $pc_i(p \cdot 1/P)$ has strict increasing differences in (p, P) , this is a monotonic signaling game. It satisfies:*

1. *Monotonicity.*

Let $\mu'_i(p)$ and $\mu_i(p)$ be two possible beliefs of the uninformed. If $\mu'_i(p) > \mu_i(p)$, then, for all p , $pc(p, P, \mu'_i(p)) > pc(p, P, \mu_i(p))$.

2. *Single-crossing.*

For any $p' > p$, we have that, for arbitrary demand of the uninformed, $p'c(p', P^l, \mu_i(p)) \geq pc(p, P^l, \mu_i(p)) \implies p'c(p', P^h, \mu_i(p)) > pc(p, P^h, \mu_i(p))$

Proof. I first prove monotonicity, and then single-crossing.

1. Monotonicity.

By Assumption 1, $u'(c_i)$ is a strictly decreasing function. Thus the demand of the uninformed $c_i(pE_{\mu_i(p)}[1/P])$ is strictly increasing in $\mu_i(p)$. Therefore, for any $\mu'_i(p) > \mu_i(p)$, $pc(p, P, \mu'_i(p)) > pc(p, P, \mu_i(p))$.

2. Single-crossing.

Consider p, p' , such that $p < p'$, and assume

$$p'c(p', P^l, \mu_i(p)) \geq pc(p, P^l, \mu_i(p))$$

This is equivalent to

$$p'c(p', P^l, \mu_i(p)) - pc(p, P^l, \mu_i(p)) \geq 0$$

Since $c(p, P, \mu_i(p))$ has strict increasing differences in (p, P) ,

$$p'c(p', P^h, \mu_i(p)) - pc(p, P^h, \mu_i(p)) > p'c(p', P^l, \mu_i(p)) - pc(p, P^l, \mu_i(p)) \geq 0$$

and therefore

$$p'c(p', P^h, \mu_i(p)) > pc(p, P^h, \mu_i(p))$$

■

Monotonicity holds in the sense that the firm is better off if uninformed customers believe that the state of the world is high, independently of the actual realization of the state. By Assumption 5 (together with Assumption 2) the game has the single-crossing property. This means that the high type is more “at ease” posting high prices than the low type. Together, these two properties make this game tractable. For example, quadratic preferences lead to a profit function satisfying Assumption 5.

I now characterize separating equilibria. The following lemma characterizes the benchmark separating equilibrium, the one where both types get the highest (ex-post) profits possible, also called the “Least Cost Separating Equilibrium”.

Lemma 6 (Separating Equilibrium) *The following is the (best) Separating Equilibrium. Define $\underline{\alpha}$ by*

$$p^l c_i \left(p^l \frac{1}{P^l} \right) = p^h \left(\underline{\alpha} c_i \left(p^h \frac{1}{P^l} \right) + (1 - \underline{\alpha}) c_i \left(p^h \frac{1}{P^h} \right) \right) \quad (21)$$

where

$$p^h = \arg \max_p pc_i \left(p \frac{1}{P^h} \right) \quad (22)$$

$$p^l = \arg \max_p pc_i \left(p \frac{1}{P^l} \right) \quad (23)$$

Then, $\underline{\alpha} < 1$ and,

- if $\alpha \geq \underline{\alpha}$:
 - The firm posts the same prices as in the perfect information benchmark, p^h and p^l . Moreover, for a given equilibrium set of prices $p(P)$, define ex-ante real profits as

$$\Pi(p(P)) = \frac{1}{2} \frac{1}{P^h} \pi(P^h) + \frac{1}{2} \frac{1}{P^l} \pi(P^l)$$

where $\pi(P) = pc(p, P, \mu_i(p))$. In this case, ex-ante real profits $\Pi(p(P))$ are equal to ex-ante real profits in the perfect information benchmark:

$$\Pi^* = \frac{1}{2} \frac{1}{P^h} \pi(P^h) + \frac{1}{2} \frac{1}{P^l} \pi(P^l) \quad (24)$$

where $\pi(P^h) = \max_p pc_i(p \cdot 1/P^h)$ and $\pi(P^l) = \max_p pc_i(p \cdot 1/P^l)$.

- If $\alpha < \underline{\alpha}$:
 - The firm posts p^l and $\bar{p} > p^h$ such that

$$p^l c_i \left(p^l \frac{1}{P^l} \right) = \bar{p} \left(\alpha c_i \left(\bar{p} \frac{1}{P^l} \right) + (1 - \alpha) c_i \left(\bar{p} \frac{1}{P^h} \right) \right) \quad (25)$$

In this case, \bar{p} is strictly decreasing and $\Pi(p(P))$ is strictly increasing in α .

Proof. The cutoff $\underline{\alpha}$ is obtained using the Incentive Compatibility (IC) constraint for the low type (21). This inequality states that if the low type imitates the high type, the $1 - \alpha$ proportion of uninformed consumers believe that he is the high type (and have beliefs $\mu_i(p) = 1$). However, informed consumers know that he is the low type (and their beliefs are fixed at $\mu_i = 0$).

Because of Assumptions 1 and 5, the game has the single crossing property (strictly) and therefore $\underline{\alpha} < 1$. Indeed, in equation (21) we have that

$$p^h c_i \left(p^h \frac{1}{P^h} \right) > p^h c_i \left(p^h \frac{1}{P^l} \right)$$

and therefore $\underline{\alpha} < 1$.

Once this cutoff obtained, there are two cases:

- $\alpha \geq \underline{\alpha}$.

In this case, (21) is satisfied at p^h and p^l defined by (22) and (23). Firms optimization in each state yields p^h and p^l . Therefore, this is the Separating Equilibrium, and if off equilibrium path beliefs are $\mu_i(p) = 0$ (pessimistic), there are no deviations for either type. Since consumers uses Bayes' rule and have beliefs $\mu_i(p) = 1$ when facing the high type and $\mu_i = 0$ when facing the low type, ex-ante profits are Π^* .

- $\alpha < \underline{\alpha}$.

In this case, the IC constraint for the low type is satisfied for a price \bar{p} defined by (25). Because the game has the single-crossing property \bar{p} always exists and is s.t. $\bar{p} > p^h$. The low type posts p^l and gets the highest profits possible. The high type posts \bar{p} and gets the highest profits possible ensuring he is not imitated by the low type. If off equilibrium path beliefs are $\mu_i(p) = 0$ (pessimistic), then there are no profitable deviations for either type. The low type does not deviate from its perfect information optimal price. For the high type, write the optimal deviation

$$\tilde{p} = \arg \max \left\{ p \left(\alpha c_i \left(p \frac{1}{P^h} \right) + (1 - \alpha) c_i \left(p \frac{1}{P^l} \right) \right) \right\}$$

We need to check that

$$\bar{p} c_i \left(\bar{p} \frac{1}{P^h} \right) \geq \tilde{p} \left(\alpha c_i \left(\tilde{p} \frac{1}{P^h} \right) + (1 - \alpha) c_i \left(\tilde{p} \frac{1}{P^l} \right) \right) \quad (26)$$

The LHS of (26) can be written

$$\bar{p} c_i \left(\bar{p} \frac{1}{P^h} \right) = (1 - \alpha) \bar{p} c_i \left(\bar{p} \frac{1}{P^h} \right) + \alpha \bar{p} c_i \left(\bar{p} \frac{1}{P^h} \right) \pm \alpha \bar{p} c_i \left(\bar{p} \frac{1}{P^l} \right) \quad (27)$$

From (25) we know that

$$\bar{p} \left(\alpha c_i \left(\bar{p} \frac{1}{P^l} \right) + (1 - \alpha) c_i \left(\bar{p} \frac{1}{P^h} \right) \right) = p^l c_i \left(p^l \frac{1}{P^l} \right)$$

and thus (27) is

$$= \alpha \left[\bar{p} c_i \left(\bar{p} \frac{1}{P^h} \right) - \bar{p} c_i \left(\bar{p} \frac{1}{P^l} \right) \right] + p^l c_i \left(p^l \frac{1}{P^l} \right)$$

that by single-crossing revenue function and strict increasing differences

$$> \alpha \left[\tilde{p}c_i \left(\tilde{p} \frac{1}{P^h} \right) - \tilde{p} \left(\tilde{p} \frac{1}{P^l} \right) \right] + \tilde{p}c_i \left(\tilde{p} \frac{1}{P^l} \right)$$

showing that (26) holds.

From (21), \bar{p} is strictly decreasing in α , and therefore $\Pi(p(P))$ is strictly increasing in α , as claimed.

This completes the proof.

■

This lemma shows that when the proportion of informed consumers is high enough, the high type can separate from the low type by posting the perfect information prices p^h and p^l . The reason is that, in this case, the proportion of informed consumers is high enough to discourage the low type from imitating him: if the low type posts p^h , the informed know that his price is too high and they reduce their demand, thereby decreasing the low type's profits. In other words, informed consumers discipline the monopolist. Formally this is expressed by the IC constraint for the low type (21). When the proportion of informed is lower, the only way a separating equilibrium is possible is by having the high type post a price strictly higher than p^h , so that the low type does not imitate.³⁴ Figure 1 is a graphical illustration of this lemma. On the right panel I plot real average ex-ante profits in this equilibrium. The plot shows that ex-ante profits are increasing in α , and reach Π^* when $\alpha \geq \underline{\alpha}$.

Having characterized the separating equilibrium, I will now characterize the benchmark pooling equilibrium. Pooling equilibria are interesting for the study of nominal rigidities since in these equilibria the firm sets the same price independently of the state of the world. Pooling equilibria exist when the proportion of informed is low. The price in the benchmark pooling equilibrium delivers profit maximization when α is equal to zero (no consumer knows the state of the world.)³⁵ I also show that when α is equal to zero, this equilibrium reaches the perfect information level of ex-ante profits Π^* .

Lemma 7 (Pooling Equilibrium) *Consider p^* such that*

³⁴Notice that this means that price changes are asymmetric.

³⁵In a dynamic cash in advance model, this equilibrium corresponds to keeping the price unchanged after a monetary shock, as explained in detail in A.8.

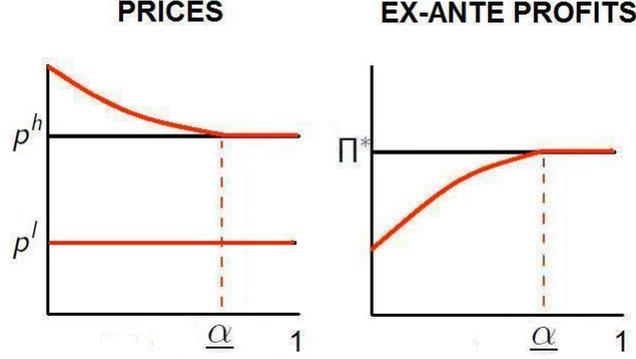


Figure 6: Separating Equilibrium (Lemma 6).

$$p^* = \arg \max p c_i \left(p \left[\frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) \quad (28)$$

and consider the lowest $\bar{\alpha}$ such that

$$p^* \left(\bar{\alpha} c_i \left(p^* \frac{1}{P^h} \right) + (1 - \bar{\alpha}) c_i \left(p^* \left[\frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) \right) \geq \max_p \left\{ \bar{\alpha} p c_i \left(p \frac{1}{P^h} \right) + (1 - \bar{\alpha}) p c_i \left(p \frac{1}{P^l} \right) \right\} \quad (29)$$

and

$$p^* \left(\bar{\alpha} c_i \left(p^* \frac{1}{P^l} \right) + (1 - \bar{\alpha}) c_i \left(p^* \left[\frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) \right) \geq p^l c_i \left(p^l \frac{1}{P^l} \right) \quad (30)$$

For all $\alpha \leq \bar{\alpha}$, there exists a pooling equilibrium at p^* . If $\alpha = 0$, ex-ante profits reach Π^* . Moreover, ex-ante profits $\Pi(p^*)$ are strictly decreasing in α .

Proof. Off equilibrium path beliefs are $\mu_i(p) = 0$ (pessimistic). Given these beliefs, the cutoff $\bar{\alpha}$ is the lowest α for which both the IC constraint of the high and low types ((29) and (30)) hold and thus this is an equilibrium.

I now show that if $\alpha = 0$, $\Pi(p^*) = \Pi^*$. For $\alpha = 0$,

$$\Pi(p^*) = \frac{1}{2} \frac{1}{P^h} p^* c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) + \frac{1}{2} \frac{1}{P^l} p^* c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) \quad (31)$$

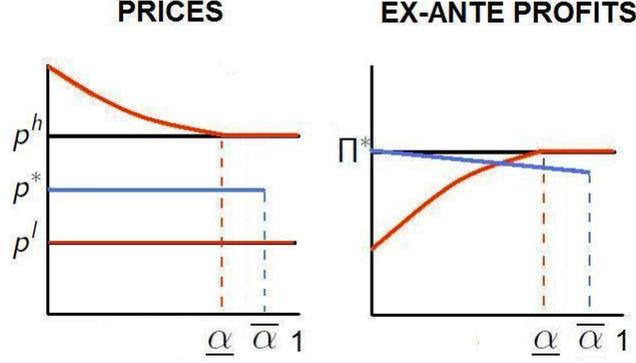


Figure 7: Separating Equilibrium and Pooling Equilibrium (Lemmas 6 and 7).

From prices (28), (22) and (23), notice that

$$p^* \left[\frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] = p^h \cdot \frac{1}{P^h} = p^l \cdot \frac{1}{P^l} \quad (32)$$

Also,

$$c_i \left(p^* \left[\frac{1}{2} \cdot \frac{1}{P^h} + \frac{1}{2} \cdot \frac{1}{P^l} \right] \right) = c_i \left(p^h \frac{1}{P^h} \right) = c_i \left(p^l \frac{1}{P^l} \right) \equiv c^{ss} \quad (33)$$

Together, (32) and (33) imply that the right hand side of (31) is equal to Π^* .

More generally,

$$\Pi(p^*) = \alpha \left(\frac{1}{2} \frac{1}{P^h} p^* c_i \left(p^* \frac{1}{P^h} \right) + \frac{1}{2} \frac{1}{P^l} p^* c_i \left(p^* \frac{1}{P^l} \right) \right) + (1 - \alpha) \Pi^* \quad (34)$$

Since the revenue function is single-peaked, this is strictly decreasing in α .

■

When the proportion of informed consumers is low enough this equilibrium exists because, according to (29) and (30), both types do not want to deviate. Figure 2 is a graphical illustration of this lemma. The right panel illustrates that ex-ante profits are decreasing in α , and there is a unique α^* where the ex-ante profit functions under pooling and separating cross.

A comparison of the Separating Equilibrium and the Pooling Equilibrium in terms of ex-ante real profits delivers that the latter ex-ante dominates the former when the proportion of informed consumers is low enough. The next lemma

develops this result in the case of any utility function satisfying Assumptions 1 and 5.

Lemma 8 (Price Rigidity) *There is $\alpha^P > 0$ s.t., if $\alpha \leq \alpha^P$, the Pooling Equilibrium delivers higher ex-ante profits than any separating equilibrium.*

Proof. Consider first the Separating Equilibrium, $\Pi(p(P))$ is continuous and strictly increasing in α . From (34), $\Pi(p^*)$ is continuous at $\alpha = 0$, reaches Π^* at $\alpha = 0$, and it is strictly decreasing thereafter. Thus, there is a boundary $[0, \alpha^P]$ away from $\alpha = 0$ where $\Pi(p(P))$ is strictly higher in the Pooling Equilibrium than in the Separating Equilibrium. Since the Separating Equilibrium is the least cost separating equilibrium, in all other separating equilibria the price posted by the high type is greater than \bar{p} . Thus, in all other separating equilibrium, ex-ante profits $\Pi(p(P))$ are lower than in the Separating Equilibrium. All other separating equilibria are therefore ex-ante dominated.

■

The intuition for this result is the following. There is an ex-ante tradeoff between two possible distortions. The first distortion arises in separating equilibria: in any separating equilibrium, when the proportion of informed consumers is low enough, there is a distortion at the top, because the firm needs to post a higher price than under perfect information to be able to signal the state of the world to uninformed consumers. This distortion hurts ex-ante profits. On the other hand, another type of distortion arises in pooling equilibria: in any pooling equilibrium the price posted does not correspond to beliefs of the informed in all states of nature, making them buy a different quantity than under perfect information, creating a distortion that hurts ex-ante profits. The first type of distortion is bigger the lower the proportion of informed consumers. The opposite happens in the second type of distortion, which is bigger the higher the proportion of informed consumers. Thus, each of these distortions varies monotonically with α , but in opposite directions. As shown below this holds even in the presence of marginal costs proportional to the price level P .

A symmetric result follows when the proportion of informed consumers is high enough.

Lemma 9 *There is $\alpha^S > 0$ s.t., if $\alpha \geq \alpha^S$, the Separating Equilibrium delivers higher ex-ante profits than any pooling equilibrium.*

Proof. Pick $\alpha^S = \underline{\alpha}$. In the Separating Equilibrium $\Pi(p(P)) = \Pi^*$ for $\alpha \geq \alpha^S$. Because the revenue function is single-peaked, in any potential pooling equilibrium at \tilde{p} , if $\alpha \geq \underline{\alpha}$, $\Pi(\tilde{p}) < \Pi^*$.

■

The intuition for this result is the same as for Lemma 8. There is an ex-ante tradeoff between two types of distortions. The distortion arising in the Separating Equilibrium is small when the proportion of informed consumers α is high, and therefore this equilibrium ex-ante dominates any pooling equilibrium.³⁶

I now prove the main result on rigidity, Proposition 1. It shows that there is a unique cutoff α^* that balances out this ex-ante tradeoff.

Proof of Proposition 1. The proof consists in comparing ex-ante profits in the Separating and in the Pooling Equilibrium, $\Pi(p(P))$ and $\Pi(p^*)$ (equations (24) and (34)).

Consider α^C such that

$$\alpha^C \left(\frac{1}{2} \frac{1}{P^h} p^* c_i \left(p^* \frac{1}{P^h} \right) + \frac{1}{2} \frac{1}{P^l} p^* c_i \left(p^* \frac{1}{P^l} \right) \right) + (1 - \alpha^C) \Pi^* = \frac{1}{2} \frac{1}{\bar{P}^h} \left(\bar{p} c_i \left(\bar{p} \frac{1}{\bar{P}^h} \right) \right) + \frac{1}{2} \frac{1}{\bar{P}^l} \left(\bar{p}^l c_i \left(\bar{p}^l \frac{1}{\bar{P}^l} \right) \right) \quad (35)$$

Because to the left of $\underline{\alpha}$ $\Pi(p(P))$ is strictly monotonically increasing in α and $\Pi(p^*)$ is strictly monotonically decreasing, α^C is unique. There are two cases.

- $\bar{\alpha} \geq \alpha^C$

In this case, both the Separating Equilibrium and the Pooling Equilibrium exist at α^C . Then, $\alpha^* = \alpha^C$.

- $\bar{\alpha} < \alpha^C$

³⁶One may wonder why, in the pooling equilibrium, informed consumers do not transmit their information to uninformed consumers. Taken literally, my wording implicitly assumes that such communication is not possible. However, one may instead imagine that the firm faces a unit mass of either informed or uninformed consumers, with probabilities α and $1 - \alpha$ respectively. In this case this problem of information transmission among consumers does not arise.

In this case, only the Separating Equilibrium exists at α^C . But because $\Pi(p^*)$ is continuous monotonically decreasing in α and $\Pi(p(P))$ is strictly monotonically increasing in α ,

$$\bar{\alpha} \left(\frac{1}{2} \frac{1}{P^h} p^* c_i \left(p^* \frac{1}{P^h} \right) + \frac{1}{2} \frac{1}{P^l} p^* c_i \left(p^* \frac{1}{P^l} \right) \right) + (1 - \bar{\alpha}) \Pi^* >$$

$$\frac{1}{2} \frac{1}{P^h} \left(\bar{p} c_i \left(\bar{p} \frac{1}{P^h} \right) \right) + \frac{1}{2} \frac{1}{P^l} \left(p^l c_i \left(p^l \frac{1}{P^l} \right) \right)$$

and so $\alpha^* = \bar{\alpha}$.

This completes the proof.

■

This proof provides a gist of what happens in the mechanism presented in L'Huillier and Zame (2017). There, the mechanism maximizes firm's expected profits over *all* equilibria, establishing generality of these rigidity results.

A discussion of popular equilibrium refinements seems in order. In the literature, there is no consensus on how to select equilibria in signaling games. A popular criterion is the intuitive criterion. Unfortunately, the intuitive criterion is not useful in this paper. The reason is that in model with more than two states of the world (clearly a relevant extension of this model for the analysis of monetary policy) this criterion loses its bite: it fails to select a unique equilibrium (Cho and Kreps 1987, p. 212). As I show in A.7, the cutoff I use selects a unique equilibrium for more than two types.

A.4 Comparative Statics in the Presence of Marginal Costs: Linear Demand Case

In this section I show numerically how the cutoff of price adjustment varies with the marginal cost of the firm in the linear demand case (the reader can refer to the quantitative section in the body for similar results in the case of the generalized constant-elasticity demand.)

In a more general model, all the cutoffs presented above should depend on firm specific characteristics. To illustrate this point, let me consider the case where the monopolist has a linear marginal cost of production kP . For tractability, I assume k is known by both the firm and consumers. I analyze which equilibrium

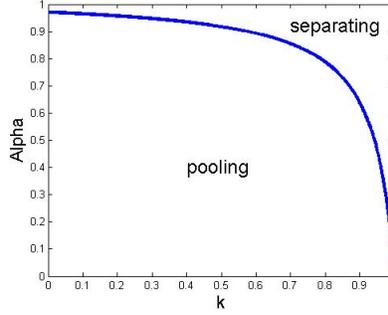


Figure 8: Cutoff $\alpha^*(k)$, and regions where Pooling/Separating Equilibrium delivers highest ex-ante profits ($P^h/P^l = 1.03$).

among the Separating vs. the Pooling Equilibrium is ex-ante optimal. The following numerical result follows.

Result 3 (Comparative Statics of α^*) Assume $u(c_i) = c_i - \frac{1}{2}c_i^2$, and consider the Separating Equilibrium and the Pooling Equilibrium. For $k \leq \hat{k}$, there is $\alpha \leq \hat{\alpha}$ where both equilibria exist. In this region there is $\alpha^*(k)$ such that:

- for $\alpha > \alpha^*(k)$, ex-ante profits are higher in the Separating Equilibrium,
- for $\alpha < \alpha^*(k)$, ex-ante profits are higher in the Pooling Equilibrium.

Moreover, $\alpha^*(k)$ is decreasing with k .

As this result shows, which equilibrium delivers higher ex-ante profits depends on firms' marginal cost kP . The higher k , the lower the critical value α^* . Figure 3 plots this cutoff as a function of k and shows that it is decreasing. The region below the curve is where the Pooling Equilibrium delivers higher ex-ante profits, then region above the curve is where the Separating Equilibrium delivers higher ex-ante profits.

This result has an interesting application in a macroeconomic model, which is the focus of Section II in the body. Indeed, there I write a model where firms are heterogeneous and thus have different cutoffs for adjusting prices. The presence of firms playing the Separating Equilibrium allows for the possibility of consumer learning. This, in turn, has implications for the proportion of firms playing the Separating Equilibrium. Thus, that dynamic model can deliver interesting feedback effects between consumer learning and the proportion of firms playing separating equilibria. See the body for more details.

A.5 Supplementary Proofs

A.5.1 Proof of Lemma 3

Under laissez-faire, firms always play the Pooling Equilibrium. Given that $\alpha_0 > 0$, there is a distortion in the consumption bundle of informed at every instant $t \in [0, T]$. Define the perfect information consumption of good c

$$c^{ss} = c_i \left(p^h \frac{1}{P^h} \right)$$

Because of the distortion, informed consumers instantaneous ex-ante utility

$$\begin{aligned} \frac{1}{2}u \left(c_{it} \left(p^* \frac{1}{P^h} \right) \right) + \frac{1}{2}u \left(c_{it} \left(p^* \frac{1}{P^l} \right) \right) < \\ u \left(\frac{1}{2}c_{it} \left(p^* \frac{1}{P^h} \right) + \frac{1}{2}c_{it} \left(p^* \frac{1}{P^l} \right) \right) = c^{ss} \end{aligned}$$

where in the last step I used the linearity of $c_i(\cdot)$.

Under regulation, after \bar{T} instantaneous utility of all consumers is $u(c^{ss})$. Thus, if $T - \bar{T}$ is large enough, welfare under regulation is strictly higher than welfare under laissez-faire.

■

A.5.2 Lemma 4

I first prove the following preliminary lemma.

Lemma 10 *Consider the equation in x*

$$x^\varepsilon + a_1 x k^{\varepsilon-1} + a_0 k^\varepsilon = 0$$

where $\varepsilon, k, a_0, a_1 \in \mathbb{R}$, $\varepsilon > 1$, $k > 0$. For any real root x^* , x^*/k does not depend on k .

Proof. Dividing by k^ε , one can re-write the equation as

$$(x/k)^\varepsilon + a_1(x/k) + a_0 = 0$$

Call the roots of this x^* , which must be the same roots as the original expression. Let $y = x/k$, then this is really

$$y^\varepsilon + a_1 y + a_0 = 0$$

Clearly the roots of this (call these y^*) only depend on ε , a_0 , and a_1 , and not k at all. Therefore since $y = x/k$, then $y^* = x^*/k$ does not depend on k , as claimed.

■

Armed with this result, I can now prove the lemma in the body.

Proof of Lemma 4. The proof consists in showing that profit functions in the Pooling Equilibrium and in the Separating Equilibrium are proportional to $1/k_j^{\varepsilon-1}$.

Consider a firm with marginal cost k_j . First, perfect information prices and the price in the Pooling Equilibrium are all proportional to k_j :

$$\begin{aligned} p_j^h &= \mathcal{M}k_j P^h \\ p_j^l &= \mathcal{M}k_j P^l \\ p_j^* &= \mathcal{M}k_j \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right]^{-1} \end{aligned}$$

where $\mathcal{M} = \varepsilon/(\varepsilon - 1)$.

Second, the price \bar{p} in the Separating Equilibrium is given by the IC constraint

$$(p_j^l - k_j P^l) c_i \left(p_j^l \frac{1}{P^l} \right) = \alpha \left[(\bar{p}_j - k_j P^l) c_i \left(\bar{p}_j \frac{1}{P^l} \right) \right] + (1-\alpha) \left[(\bar{p}_j - k_j P^l) c_i \left(\bar{p}_j \frac{1}{P^h} \right) \right]$$

where the demand function is (dropping time indexes)

$$c_i(p_j/P) = (p_j/P)^{-\varepsilon}$$

The IC is equivalent to

$$\bar{p}_j^\varepsilon + a_1 \bar{p}_j k_j^{\varepsilon-1} + a_0 k_j^\varepsilon = 0$$

where

$$a_1 = -\frac{\alpha(P^l)^\varepsilon + (1 - \alpha)(P^h)^\varepsilon}{\frac{\mathcal{M}-1}{\mathcal{M}^\varepsilon} P^l}$$

and

$$a_0 = a_1 P^l$$

Thus, by Lemma 10, \bar{p}_j is proportional to k_j .

Hence, all equilibrium prices are proportional to k_j . When p_j is proportional to k_j , the profit function

$$(p_j/P)^{-\varepsilon} (p_j - k_j) \propto 1/k_j^{\varepsilon-1}$$

Because of the proportionality of all profit functions to $1/k_j^{\varepsilon-1}$, marginal costs cancel out in the comparison of profits in equation (35). Thus, for $k_j \neq k_{\hat{j}}$, $\alpha^*(k_j) = \alpha^*(k_{\hat{j}})$: the cutoff α^* is constant, as claimed.

■

A.6 Results in the Presence of Marginal Costs

As argued in the body of the text, all results of Section I can be extended to the case of marginal costs proportional to the price level P . In this appendix I prove this claim.

Suppose the monopolist's cost function is of the form $k(c_i(p/P)) \cdot P$. For tractability, I assume the function $k(\cdot)$ is known by both the firm and consumers. I make the following assumption about this function and the implied profit function.

Assumption 6 *The profit function $\pi(p, P) = pc_i(p/P) - k(c_i(p/P))P$ is twice continuously differentiable on \mathbb{R}_{++} , single-peaked at a maximum, and has strict increasing differences in (p, P) .*

The following lemma states that, under perfect information, the optimal price of the monopolist is proportional to the price level P .

Lemma 11 *When all consumers know the value of the price level, the monopolist's price is proportional to the price level.*

Proof. Under perfect information the monopolist's problem is

$$\max_p \left\{ p c_i \left(p \frac{1}{P} \right) - k \left(c_i \left(p \frac{1}{P} \right) \right) P \right\}$$

Taking the first order condition delivers

$$c \left(p \frac{1}{P} \right) + p \frac{1}{P} c'_i \left(p \frac{1}{P} \right) - k' \left(c_i \left(p \frac{1}{P} \right) \right) c'_i \left(p \frac{1}{P} \right) = 0$$

From this equation it is clear that p is proportional to P . ■

The following lemma characterizes the best Separating Equilibrium and ex-ante real profits in this equilibrium. It generalizes Lemma 6.

Lemma 12 (Separating Equilibrium) *The following is the (Best) Separating Equilibrium. Define $\underline{\alpha}$ by*

$$\begin{aligned} p^l c_i \left(p^l \frac{1}{P^l} \right) - k \left(c_i \left(p^l \frac{1}{P^l} \right) \right) P^l &= \underline{\alpha} \left[p^h c_i \left(p^h \frac{1}{P^l} \right) - k \left(c_i \left(p^h \frac{1}{P^l} \right) \right) P^l \right] \\ &+ (1 - \underline{\alpha}) \left[p^h c_i \left(p^h \frac{1}{P^h} \right) - k \left(c_i \left(p^h \frac{1}{P^h} \right) \right) P^l \right] \end{aligned}$$

where

$$p^h = \arg \max_p \left\{ p c_i \left(p \frac{1}{P^h} \right) - k \left(c_i \left(p \frac{1}{P^h} \right) \right) P^h \right\} \quad (36)$$

$$p^l = \arg \max_p \left\{ p c_i \left(p \frac{1}{P^l} \right) - k \left(c_i \left(p \frac{1}{P^l} \right) \right) P^l \right\} \quad (37)$$

Then, $\alpha < 1$ and,

- if $\alpha \geq \underline{\alpha}$:
 - The firm posts the same prices as in the perfect information benchmark, p^h and p^l . Moreover, for a given equilibrium set of prices $p(P)$, define ex-ante real profits as

$$\Pi(p(P)) = \frac{1}{2} \frac{1}{P^h} \pi(P^h) + \frac{1}{2} \frac{1}{P^l} \pi(P^l)$$

where $\pi(P) = pc(p, P, \mu_i(p))$: in this case, ex-ante real profits $\Pi(p(P))$ are equal to ex-ante real profits in the perfect information benchmark:

$$\Pi^* = \frac{1}{2} \frac{1}{P^h} \pi^*(P^h) + \frac{1}{2} \frac{1}{P^l} \pi^*(P^l)$$

where $\pi^*(P_h) = \max_p pc(p/P^h) - k(c(p/P^h))P^h$ and $\pi^*(P_l) = \max_p pc(p/P^l) - k(c(p/P^l))P^l$.

- If $\alpha < \underline{\alpha}$ and

$$\begin{aligned} \bar{p}c_i\left(\bar{p}\frac{1}{P^h}\right) - k\left(c_i\left(\bar{p}\frac{1}{P^h}\right)\right)P^h &\geq \max_p \left\{ \alpha \left(pc_i\left(p\frac{1}{P^h}\right) - k\left(c_i\left(p\frac{1}{P^h}\right)\right)P^h \right) \right. \\ &\quad \left. + (1-\alpha) \left(pc_i\left(p\frac{1}{P^l}\right) - k\left(c_i\left(p\frac{1}{P^l}\right)\right)P^h \right) \right\} \end{aligned}$$

holds,

- The firm posts p^l and $\bar{p} > p^h$ such that

$$\begin{aligned} p^l c_i\left(p^l \frac{1}{P^l}\right) - k\left(c_i\left(p^l \frac{1}{P^l}\right)\right)P^l &= \alpha \left[\bar{p}c_i\left(\bar{p}\frac{1}{P^l}\right) - k\left(c_i\left(\bar{p}\frac{1}{P^l}\right)\right)P^l \right] \\ &\quad + (1-\alpha) \left[\bar{p}c_i\left(\bar{p}\frac{1}{P^h}\right) - k\left(c_i\left(\bar{p}\frac{1}{P^h}\right)\right)P^l \right] \end{aligned}$$

In this case, \bar{p} is strictly decreasing and $\Pi(p(P))$ is strictly increasing in α .

Proof (sketch). Given Assumptions 1 and 6, the objective function of the monopolist is single-peaked and satisfies the single-crossing property. Marginal costs are proportional to P . Using the IC constraints ensures that the equilibrium exists. The details of the proof are similar to the case of Lemma 6.

■

Lemma 13 (Pooling Equilibrium) Consider p^* such that

$$p^* = \arg \max_p \left\{ pc_i\left(p \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right]\right) - k\left(p \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right]\right) \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right]^{-1} \right\}$$

For given P^h and P^l , suppose that there is $\bar{\alpha}$ such that

$$\begin{aligned}
& \bar{\alpha} \left[p^* c_i \left(p^* \frac{1}{P^h} \right) - k \left(c_i \left(p^* \frac{1}{P^h} \right) \right) P^h \right] \\
& + (1 - \bar{\alpha}) \left[p^* c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) - k \left(c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) \right) P^h \right] \\
& \geq \max_p \left\{ \bar{\alpha} \left[c_i \left(p^* \frac{1}{P^h} \right) - k \left(c_i \left(p^* \frac{1}{P^h} \right) \right) P^h \right] \right. \\
& \left. + (1 - \bar{\alpha}) \left[p^* c_i \left(p^* \frac{1}{P^l} \right) - k \left(c_i \left(p^* \frac{1}{P^l} \right) \right) P^h \right] \right\}
\end{aligned} \tag{38}$$

and

$$\begin{aligned}
& \bar{\alpha} \left[p^* c_i \left(p^* \frac{1}{P^l} \right) - k \left(c_i \left(p^* \frac{1}{P^l} \right) \right) P^l \right] \\
& + (1 - \bar{\alpha}) \left[p^* c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) - k \left(c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) \right) P^l \right] \\
& \geq p^l c_i \left(p^l \frac{1}{P^l} \right) - k \left(c_i \left(p^l \frac{1}{P^l} \right) \right) P^l
\end{aligned} \tag{39}$$

Consider the lowest possible $\bar{\alpha}$. Then, for all $\alpha \leq \bar{\alpha}$, there exists a pooling equilibrium at p^* . If $\alpha = 0$, this equilibrium reaches ex-ante profits. Moreover, ex-ante profits $\Pi(p^*)$ are strictly decreasing in α .

Proof. Off equilibrium path beliefs are $\mu_i(p) = 0$. Given these beliefs, for all $\alpha \leq \bar{\alpha}$ the IC constraints for both the high and low types ((38) and (39)) are satisfied and thus this is an equilibrium.

I now show that if $\alpha = 0$, $\Pi(p^*) = \Pi^*$. For $\alpha = 0$,

$$\begin{aligned}
\Pi(p^*) &= \frac{1}{2} \frac{1}{P^h} \left[p^* c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) - k \left(c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) \right) P^h \right] \\
&+ \frac{1}{2} \frac{1}{P^l} \left[p^* c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) - k \left(c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) \right) P^l \right]
\end{aligned}$$

Similar to Lemma 7,

$$c_i \left(p^* \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] \right) = p^h c_i \left(p^h \frac{1}{P^h} \right) = p^l c_i \left(p^l \frac{1}{P^l} \right) \equiv c^{ss}$$

where p^h and p^l are defined by (36) and (37). Thus,

$$\Pi^*(p^*) = \left[\frac{1}{2} \frac{1}{P^h} + \frac{1}{2} \frac{1}{P^l} \right] [p^* c^{ss}] - \frac{1}{2} \frac{1}{P^h} [k(c^{ss}) P^h] - \frac{1}{2} \frac{1}{P^l} [k(c^{ss}) P^l] = \Pi^*$$

More generally,

$$\begin{aligned} \Pi(p^*) &= \alpha \left(\frac{1}{2} \frac{1}{P^h} \left[p^* c_i \left(p^* \frac{1}{P^h} \right) - k \left(c_i \left(p^* \frac{1}{P^h} \right) \right) P^h \right] \right. \\ &\quad \left. + \frac{1}{2} \frac{1}{P^l} \left[p^* c_i \left(p^* \frac{1}{P^l} \right) - k \left(c_i \left(p^* \frac{1}{P^l} \right) \right) P^l \right] \right) + (1 - \alpha) \Pi^* \end{aligned}$$

Since the profit function is single-peaked, this is strictly decreasing in α .

■

As shown in this proof the key to the result that, when $\alpha = 0$, p^* is ex-ante optimal relies on the fact that ex-ante real costs are the same as under perfect information.

Under Assumption 6, and having established Lemmas 12 and 13, it is straightforward to extend Proposition 1, and Lemmas 8 and 9 to the presence of marginal costs.

A.7 The Model with Three Types

In this appendix I show how the number of types can be augmented. This is an easy task due to three basic properties of the model: the monotonicity of the game, the single-crossing property, and the independence of demand from income. Together, these three properties ensure that a) the Separating Equilibrium is similar to the one presented in Lemma 6, b) the Pooling Equilibrium is also similar to the one in Lemma 7, and c) the rigidity results (Proposition 1, and Lemmas 8 and 9). (Here I characterize equilibria with three types, but an extension to more than three or even a continuum of types is standard.)

The Game with Three Types. The price level P is now drawn over $\mathfrak{P} = \{P^H, P^M, P^L\}$, where $P^H > P^M > P^L$ and $Pr(P = P^H) = Pr(P = P^M) = Pr(P = P^L) = 1/3$. I call “low type” to the firm that knows that the state is P^L , “medium type” to the firm that knows that the state is P^M , and “high type” to the firm that knows that the state is P^H . Uninformed consumers’ beliefs are a probability distribution over \mathfrak{P} defined by two mappings

$$\mu_i^H : R_+ \longrightarrow [0, 1]$$

and

$$\mu_i^M : R_+ \longrightarrow [0, 1]$$

that assign probabilities to the high and medium states.

All other definitions of the problem remain the same.

Lemma 14 (Separating Equilibrium) *The following is the (best) Separating Equilibrium. For a given α , the low type posts $p(P^L) = p^L$, with p^L such that*

$$p^L = \arg \max_p p c_i \left(p \frac{1}{P^L} \right)$$

Consider p^M such that

$$p^M = \arg \max_p p c_i \left(p \frac{1}{P^M} \right)$$

If

$$p^L c_i \left(p^L \frac{1}{P^L} \right) > p^M \left(\alpha c_i \left(p^M \frac{1}{P^L} \right) + (1 - \alpha) c_i \left(p^M \frac{1}{P^M} \right) \right) \quad (40)$$

then the medium type posts $p(P^M) = p^M$. Otherwise, the medium type posts $p(P^M) = \bar{p}^M$ such that

$$p^L c_i \left(p^L \frac{1}{P^L} \right) = \bar{p}^M \left(\alpha c_i \left(\bar{p}^M \frac{1}{P^L} \right) + (1 - \alpha) c_i \left(\bar{p}^M \frac{1}{P^M} \right) \right) \quad (41)$$

Consider p^H such that

$$p^H = \arg \max_p p c_i \left(p \frac{1}{P^H} \right)$$

If

$$p(P^M) c_i \left(p(P^M) \frac{1}{P^M} \right) > p^H \left(\alpha c_i \left(p^H \frac{1}{P^M} \right) + (1 - \alpha) c_i \left(p^H \frac{1}{P^H} \right) \right)$$

then the high type posts $p(P^H) = p^H$. Otherwise, the high type posts $p(P^H) = \bar{p}^H$ such that

$$p(P^M) c_i \left(p(P^M) \frac{1}{P^M} \right) = \bar{p}^H \left(\alpha c_i \left(\bar{p}^H \frac{1}{P^M} \right) + (1 - \alpha) c_i \left(\bar{p}^H \frac{1}{P^H} \right) \right)$$

Define ex-ante real profits by

$$\Pi(p(P)) = \frac{1}{3} \frac{1}{P^L} \pi(P^L) + \frac{1}{3} \frac{1}{P^M} \pi(P^M) + \frac{1}{3} \frac{1}{P^H} \pi(P^H) \quad (42)$$

where $\pi(P) = p c(p, P, \mu_i(P))$. Then, $\Pi(p(P))$ is (weakly) increasing in α .

Proof. The low type posts p^L and, if off-equilibrium path beliefs are $\mu_i^H(p) = 0$ and $\mu_i^M(p) = 0$ (pessimistic), he finds no profitable deviation. (40) (or (41)) ensures that the low type does not imitate the medium type. A fortiori, by monotonicity, he does not imitate the high type. Using the same steps as in p. 48 one can proof that there are no profitable deviations for the medium type. A similar reasoning shows that this is an equilibrium for the high type as well.

I now show that (42) is weakly increasing in α . Consider $\alpha' > \alpha$. If types post p^H , p^M and p^L , then there are no distortions and $\Pi(\alpha') = \Pi(\alpha)$. If for α either $p(P^M) \neq P^M$ or $p(P^H) \neq P^H$, then:

- If $p(P^H) \neq P^H$, $p(P^H)$ is strictly decreasing in α , and therefore $\Pi(\alpha)$ is strictly increasing,
- Similarly, if $p(P^M) \neq P^M$, $p(P^M)$ is strictly decreasing in α , and therefore $\Pi(\alpha)$ is strictly increasing.

■

Lemma 15 (Pooling Equilibrium) Consider p^* such that

$$p^* = \arg \max_p pc_i(p \cdot P^m)$$

where $P^m = \left[\frac{1}{3} \cdot \frac{1}{P^H} + \frac{1}{3} \cdot \frac{1}{P^M} + \frac{1}{3} \cdot \frac{1}{P^L}\right]$, and consider the highest $\bar{\alpha}$ such that

$$p^* \left(\bar{\alpha} c_i \left(p^* \frac{1}{P^H} \right) + (1 - \bar{\alpha}) c_i (p^* \cdot P^m) \right) \geq \max_p \left\{ \bar{\alpha} p c_i \left(p \frac{1}{P^H} \right) + (1 - \bar{\alpha}) p c_i \left(p \frac{1}{P^L} \right) \right\}$$

and

$$p^* \left(\bar{\alpha} c_i \left(p^* \frac{1}{P^M} \right) + (1 - \bar{\alpha}) c_i (p^* \cdot P^m) \right) \geq \max_p \left\{ \bar{\alpha} p c_i \left(p \frac{1}{P^M} \right) + (1 - \bar{\alpha}) p c_i \left(p \frac{1}{P^L} \right) \right\}$$

and

$$p^* \left(\bar{\alpha} c_i \left(p^* \frac{1}{P^L} \right) + (1 - \bar{\alpha}) c_i (p^* \cdot P^m) \right) \geq p^L c_i \left(p^L \frac{1}{P^L} \right)$$

For all $\alpha \leq \bar{\alpha}$, there exists a pooling equilibrium at p^* . If $\alpha = 0$, ex-ante profits reach Π^* . Moreover, ex-ante profits $\Pi(p^*)$ are strictly decreasing in α .

The proof is similar to the proof of Lemma 7. Given that ex-ante profits are increasing in the Separating Equilibrium, and decreasing in the Pooling Equilibrium, all results concerning ex-ante profits follow through. Moreover, because of single-crossing the Separating Equilibrium always exists. Therefore, a result similar to Proposition 1 can be used to apply this game in a monetary framework.

A.8 The Cash in Advance General Equilibrium Framework

The goal of this Section is to show that the simple dynamic model of Section II is compatible with a cash in advance general equilibrium framework.

The setup is fairly involved and therefore I start its description with an overview of the key economic interactions and main technical pieces. Subsequently I fully describe every piece of the model.

Preview of the Setup. The population of the economy is composed by a continuum of households. These households own a continuum of firms, which operate in different geographic locations called islands. There is a unit mass of islands, and in each island there is a single firm.

The aggregate state of the economy is the supply of money M . Firms, by assumption, are informed about this quantity.³⁷ Consumers are imperfectly informed and learn M by looking at firms' prices.³⁸ Workers learn M by looking at the wage in a centralized economy-wide labor market.

I now describe the main dynamic elements of the model. At every time period firms and consumers randomly meet. Notice that in order to allow for gradual learning from prices among consumers I need to move away from commonly used structures of goods markets, such as monopolistic competition. The reason is that in such structures typically all consumers observe all prices at every period, and therefore, they would learn the aggregate state right away. In my environment, instead, consumers observe one price at a time, which allows for gradual learning. Moreover, consumers become informed by seeing a price that has adjusted to the aggregate amount of money M . Firms adjust prices as a function of how many consumers are informed. Thus, there is a two-way key interaction between the proportion of firms adjusting prices, and the proportion of consumers that are informed. My goal is to analyze the dynamic properties of this interaction, and its implications for the aggregate adjustment of the economy.

My setup borrows tools from two important pieces of the literature: Lagos and Wright (2005) and Lucas and Stokey (1987). As Lagos and Wright (2005), I exploit quasilinearity and periods that are divided in subperiods to be able to handle heterogeneity. As Lucas and Stokey (1987), I use a cash in advance model with credit and cash goods. The quasilinearity of preferences in my model, together with a time structure including periods and subperiods, allow me to handle the heterogeneity implied by dispersed information in a simple way, and to model game theory interactions preserving compatibility with general equilib-

³⁷It is possible to relax this assumption and letting firms learn M from their interactions with consumers, as long as an arbitrary small proportion of consumers know M and – in contrast to Lucas (1972) – each firm sells to a representative sample of consumers. To simplify the exposition, here I assume that firms are informed right from the start.

³⁸One can think about this assumption as representing the fact that – for at least a portion of the consumer population – gathering precise information directly about money supply is a costly and complex process. But prices may convey this information more readily, as it is the case in my model.

rium. Importantly, I focus on the events happening within a period – i.e. during the subperiods, which is when consumer learning happens. At the end of every period trading takes place in centralized markets and under perfect information. As it will be clear, the infinite recurrence of periods in the model is only used as a technical device to introduce money in a standard cash in advance framework. Regarding the use of both credit and cash goods, I will focus on the transactions of credit goods, which will allow consumers to buy from firms without knowing the supply of money. Trade of the cash good happens at the end of each period, and is used simply as a way of “closing” the model.

A.8.1 Model Setup

Population and Geography. There is a continuum of households indexed by i . Each of these households is divided into a worker and a consumer-shopper, called for brevity ‘consumer’. There is a (unit mass) continuum of islands, indexed by j .

Time Structure. Similar to Lagos and Wright (2005), periods are divided in subperiods. The number of subperiods is $N \in \mathbb{N}_{++}$. This appendix proceeds by presenting the model in discrete time; the continuous time limit corresponds to $N \rightarrow \infty$. Periods are indexed by τ and run from $\tau = 0$ to infinity. Subperiods are indexed by t and run from $t = 1$ to N . Since the length of a period is T , the length of a subperiod is denoted by Δ , with $\Delta = T/N$.

The focus of the paper is on the subperiods, which is when learning happens. For this purpose, I will consider N large (although the equilibrium definition of the economy is valid for all $N \in \mathbb{N}_{++}$.)

Monetary Shocks. Money supply evolves as

$$\log M_\tau = \log M_{\tau-1} + \nu_\tau \tag{43}$$

where ν_τ is a monetary shock that hits *at the beginning* of period τ . ν_τ is drawn from a binary probability distribution over $\mathfrak{V} = \{\nu^h, \nu^l\}$, with $\nu^h > 0$ and $\nu^l < 0$. The shock is independent across periods. I refer to ν^h as the “high” state, and to ν^l as the “low” state. Both states are equally likely.

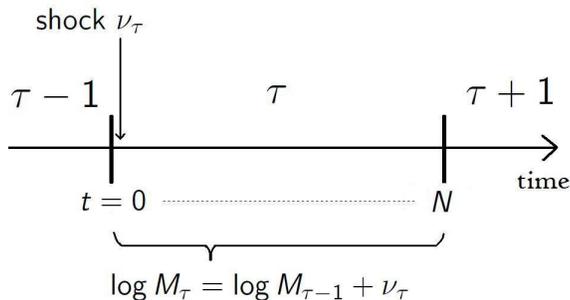


Figure 9: Time structure and evolution of money.

Notice that (43) implies that the amount of money is the same within a period. Figure 9 summarizes the time structure of the model together with the assumptions on the evolution of money.

Information Structure. Firms are informed about the state of the world, i.e. they know the realization of ν_τ from the beginning of period τ , and the implied value of M_τ . At the beginning of every period, there is an exogenous proportion α_0 of consumers who are informed. Within period τ this proportion evolves *endogenously* as the result of meetings between consumers and firms, to be specified later. The proportion α_0 can be arbitrarily small, and serves only the purpose of initial condition for the dynamic characterization of learning among consumers. Workers become informed when they supply labor in the centralized economy-wide labor market, to be fully described below.

Goods Markets. Within period τ , that is at every subperiod t , with $t \leq N$, trade of goods happens in a decentralized market. These goods are bought on credit. Specifically, consumers are sent randomly to an island. The random assignment of consumers is such that every island receives a representative sample of consumers.

On island j there is a firm. This firm is a monopolist and sets a price p for a good c . Throughout the paper I refer to this firm as “firm j ” or “monopolist j ” interchangeably. The group of consumers sent to island j at subperiod t , period τ , is a subset denoted $\hat{I}(j, t, \tau)$. Thus, all consumers i such that $i \in \hat{I}(j, t, \tau)$

buy good c from firm j . At subperiod $t + 1$ this process is repeated, for $t < N$. In this sense, from the perspective of consumers, good c is bought sequentially, from one monopolist at a time, repeating this process within the period.

At the end of period τ (end of subperiod $t = N$), consumers go to a centralized competitive markets to buy a good C on cash from a competitive firm. The price of this good is P . I now comment on the role of good C in the model. This good is simply a way of “closing” the model, in the following sense. In equilibrium, the price P will be proportional to the money supply M , and therefore this good is a device to model the idea that in the “long run” prices are flexible and proportional to money supply. The “long run” is represented in this stylized model as the end of each period by having N large enough, and thus a high enough sequence of meetings between firms and consumers.

Labor and Financial Markets. At the end of every period τ , a number of events happen together with the opening of the centralized market for good C described earlier. First, workers sell labor in an economy-wide competitive labor market at a wage W_τ . At this point, production of all goods bought in the period takes place, and these goods are delivered to households and consumed.³⁹ Moreover, as in Lucas and Stokey (1987), workers bring home labor income, credit goods are paid, and profits from firms are received. Only then financial markets open and bonds and cash for period $\tau + 1$ are traded.

Households’ Preferences. Household i faces the problem

$$\max E_i \left[\sum_{\tau=0}^{\infty} \beta^\tau \left(\sum_{t=1}^N u(c_{it\tau})\Delta + V(C_{i\tau}) - L_{i\tau} \right) \right] \quad (44)$$

where $c_{it\tau}$ is consumption of good c at subperiod t time τ , produced by a randomly matched firm \hat{j} of island \hat{j} , $C_{i\tau}$ is consumption of the cash good, and $L_{i\tau}$ is labor supplied by the worker. $\hat{j}(i, t, \tau)$ is a function that designates firm \hat{j} that is randomly matched to consumer i at subperiod t time τ . E_i is the expectation operator conditional on the information set at each subperiod. This

³⁹I could have avoided production taking place at the end of the period by introducing another type of labor supplied within the period. Not wanting to complicate the environment even further, I use here only one type of labor which is supplied at the end of every period.

maximization is subject to the budget constraint

$$\sum_{t=1}^N p_{\hat{j}(i,t,\tau)} c_{it\tau} \Delta + P_{\tau} C_{i\tau} + M_{i\tau} + B_{i\tau} = (1 + R_{\tau}) B_{i\tau-1} + M_{i\tau-1} + T_{\tau} + W_{\tau} L_{i\tau} + \pi_{i\tau} \quad (45)$$

where T_{τ} is a lump-sum transfer⁴⁰, $B_{i\tau}$ are nominal bond holdings, R_{τ} is the nominal interest rate, and $\pi_{i\tau}$ are profits of firms owned by household i .

The cash-in-advance constraint for good C is

$$P_{\tau} C_{i\tau} \leq M_{i\tau-1} + T_{\tau} \quad (46)$$

A salient feature of households' preferences is the quasilinearity in labor. It implies an absence of income effects in the demand of goods c and C which is the key for tractability in the model.

I make the following assumptions concerning utility functions $u(\cdot)$ and $V(\cdot)$.

Assumption 7 *The utility functions $u(\cdot)$ and $V(\cdot)$ are twice continuously differentiable on \mathbb{R}_{++} , strictly increasing, and strictly concave.*

Production. All firms in the economy have a linear technology and produce using only labor. Within every period, monopolist j of the decentralized market produces according to the production function

$$c_j = A_j L_j$$

For simplicity, I assume that all A_j s are common knowledge. The competitive firm produces C according to the production function

$$C = L$$

where productivity has been normalized to one.

Signaling Game Played in Island j at subperiod t period τ . In island j at subperiod t period τ , firm j meets consumers i such that $i \in \hat{I}(j, t, \tau)$. Some

⁴⁰More specifically, T_{τ} is such that $T_{\tau} = M_{\tau} - M_{\tau-1}$. Due to quasilinearity, all agents have the same money holdings and therefore I can write this transfer in this way.

of this consumers are informed, others uninformed, determined endogenously by previous interactions. Let the proportion of informed consumers at this point be $\alpha_{t\tau}$, the complement $1 - \alpha_{t\tau}$ being the proportion of uninformed consumers. As the body shows, $\alpha_{t\tau}$ evolves deterministically and therefore it is common knowledge. To simplify the analysis, it is assumed that the monopolist cannot discriminate between informed and uninformed consumers.

Informed consumers maximize (44) subject to (45) and (46) under perfect information. These consumers know M_τ and maximize their utility without any uncertainty. Uninformed consumers makes inferences about M_τ based on their observation of the posted price $p_{jt\tau}$. Conversely, firms understand that uninformed consumers will make an inference based on the price they set, and take this into account when choosing their price. This interaction is formally described as a signaling game: Firm j and uninformed consumers $i \in \hat{I}(j, t, \tau)$ play the following one-shot⁴¹ game. First, knowing the realization of M_τ , firm j posts a price $p_{jt\tau}$. Then, uninformed consumers observe $p_{jt\tau}$, form beliefs $\mu_{it\tau}$, $i \in \hat{I}(j, t, \tau)$, about M_τ , and decide how much to demand.

Formally, the sender of the signaling game is monopolist j . The type of the sender is defined by referring to different possible information sets this monopolist could have access to.⁴² Therefore, there are two possible types of monopolist j : the “high type” – the monopolist who observed a high realization of the monetary shock, ν^h , and the “low type” – the monopolist who observed a low realization of the monetary shock, ν^l .⁴³ The message of the sender is the price $p_{jt\tau}$. The receiver is the set of uninformed consumers, whose action is $c_{it\tau}(\cdot)$, where uninformed $i \in \hat{I}(j, t, \tau)$. This action depends on beliefs $\mu_{it\tau}$, $i \in \hat{I}(j, t, \tau)$.

Monopolists’ Problem. At subperiod t period τ , monopolist j chooses a nominal price $p_{jt\tau}$ to maximize profits:

$$\max_{p_{jt\tau}} (p_{jt\tau} - k_j W_\tau) c(p_{jt\tau}, M_\tau, \mu_{it\tau}) \quad (47)$$

⁴¹This is a one-shot game because, for every consumer, the probability of returning to the same island in the future is a zero probability event.

⁴²This is the standard definition of “type” in game theory.

⁴³To be clear, all firms in the economy are of the same type, given that the type is given by the aggregate state ν_τ .

where $i \in \hat{I}(j, t, \tau)$, $k_j \equiv 1/A_j$, and $c(\cdot)$ is total demand of good c , to be derived below. As it will become clear, total demand $c(p_{jt\tau}, M_\tau, \mu_{it\tau})$ depends on three objects. First, it depends directly on the price $p_{jt\tau}$. Second, it depends on the money supply M_τ . Given that this is a nominal price, the demand of informed consumers depends on M_τ . Third, it depends on beliefs of the uninformed $\mu_{it\tau}$, which in turn depend on the monopolist's price $p_{jt\tau}$.

Equilibrium Definition for the Signaling Game. I now define a perfect Bayesian equilibrium of the game played in island j at subperiod t period τ . I first describe the strategy of monopolist j . I focus on pure strategies. A pure strategy for the monopolist $p_{jt\tau}$ is a mapping

$$p_{jt\tau} : \mathfrak{V} \longrightarrow \mathbb{R}_+ \quad (48)$$

that assigns a price $p_{jt\tau}$ to each state of nature $\nu_\tau \in \mathfrak{V}$. Next, I describe beliefs $\mu_{it\tau}(p_{jt\tau})$ of uninformed consumer $i \in \hat{I}(j, t, \tau)$. I focus on symmetric beliefs. Beliefs are a probability distribution over \mathfrak{V} defined by a mapping

$$\mu_{it\tau} : \mathbb{R}_+ \longrightarrow [0, 1] \quad (49)$$

that assigns a probability $\mu_{it\tau}(p)$ to the high state of nature ν^h . Mapping (49) is consistent with Bayes' rule on the path of equilibrium play. Because I focus on pure strategies for the monopolist, the requirement is simply that, for any equilibrium prices (48), denoted $p_{jt\tau}(\nu^h)$ and $p_{jt\tau}(\nu^l)$ for the high and low states respectively, if $p_{jt\tau}(\nu^h) \neq p_{jt\tau}(\nu^l)$ (a separating equilibrium), then $\mu_{it\tau}(p_{jt\tau}(\nu^h)) = 1$ and $\mu_{it\tau}(p_{jt\tau}(\nu^l)) = 0$, $i \in \hat{I}(j, t, \tau)$. If instead $p_{jt\tau}(\nu^h) = p_{jt\tau}(\nu^l)$ (a pooling equilibrium), then $\mu_{it\tau}(p_{jt\tau}(\nu^h)) = \mu_{it\tau}(p_{jt\tau}(\nu^l)) = 1/2$, uninformed $i \in \hat{I}(j, t, \tau)$. Beliefs $\mu_{it\tau}(p_{jt\tau})$, uninformed $i \in \hat{I}(j, t, \tau)$, are unrestricted for other prices.

I now describe the strategy of uninformed consumers. I focus on symmetric pure strategies. A symmetric pure strategy $c_{it\tau}$ for a given uninformed consumer i , $i \in \hat{I}(j, t, \tau)$, is a mapping

$$c_{it\tau} : \mathbb{R}_+ \times [0, 1] \longrightarrow \mathbb{R}_{++}$$

that assigns a demand $c_{it\tau}$ to each price $p_{jt\tau}$ and beliefs $\mu_{it\tau}(p_{jt\tau})$. A perfect Bayesian equilibrium requires that both the firm and the uninformed consumers

play a best response. Given these definitions, I can now define an equilibrium formally.

Definition 5 *A Perfect Bayesian Equilibrium (PBE) in island j is a list $(p_{jt\tau}(\nu_\tau), \mu_{it\tau}(p_{jt\tau}), c_{it\tau})$, for all $i \in \hat{I}(j, t, \tau)$, such that*

1. *There is no profitable deviation from posting $p_{jt\tau}$, given consumers' play,*
2. *$\mu_{it\tau}(p_{jt\tau})$ are derived using Bayes' rule on the equilibrium path,*
3. *there are no profitable deviations from consumption decisions $c_{it\tau}$ given firm's play.*

Definition of Equilibrium for the Economy. Having defined an equilibrium for the signaling games played by firms and consumers, I can now define a general equilibrium.

Definition 6 *A general equilibrium of this economy is given by allocations $\{c_{it\tau}, C_{i\tau}\}$, beliefs $\mu_{it\tau}(p_{jt\tau})$, labor supply $\{L_{i\tau}\}$, labor demand $\{L_{jt\tau}, L_\tau\}$, nominal prices $\{p_{jt\tau}, P_\tau\}$, nominal wage $\{W_\tau\}$, nominal interest rate $\{1 + R_\tau\}$, for all i, j, t, τ , such that*

1. *Households' conditions for optimality and corresponding constraints are satisfied;*
2. *Equilibrium strategies for the games played between monopolists and shoppers satisfy Bayesian Perfection:*
 - *there are no profitable deviations for prices posted by monopolists, given consumers' play,*
 - *uninformed shoppers use Bayes' rule on the path of equilibrium play,*
 - *there are no profitable deviations from consumers' demand decisions;*
3. *The representative firm maximizes profits taking the price as given;*
4. *Goods, labor, bonds, and money markets clear.*

A.8.2 General Equilibrium

Here I solve for a general equilibrium (GE) of the economy, conditional on PBEs being played in all islands and at all times. Indeed, a nice property of the model

outlined above is that any set of PBEs is compatible with a GE, for reasons I detail here. Therefore, it is possible to first solve for aggregate prices and quantities in centralized markets, and then for prices and quantities in decentralized markets.

Households' Optimality Conditions. The conditions for optimality are computed as follows. Each time consumer i is matched with a monopolist, he computes the first order condition:

$$\beta^\tau u'(c_{it\tau}) = p_{jt\tau} E_{\mu_{it\tau}}[\lambda_{i\tau}] \quad (50)$$

$E_{\mu_{it\tau}}[\cdot]$ is an expectation taken using the consumer's information set at subperiod t period τ . This information set contains information previously collected plus the information revealed by the price of the monopolist at subperiod t period τ .

When the shopper buys the cash good C , he computes a first order condition for consumption of this cash good after observing its price. This good is sold in a centralized market, and therefore its price reveals the realization of the monetary shock to the shopper in case he did not know it already. Therefore, at this point the shopper does not face any uncertainty, and the first order condition is:

$$\beta^\tau V'(C_{i\tau}) = P_\tau(\lambda_{i\tau} + \psi_{i\tau}) \quad (51)$$

The worker computes a first order condition for labor supply after observing the equilibrium wage. This is a centralized market, and therefore this wage reveals the realization of the monetary shock to the worker. Therefore, the worker does not face any uncertainty, and the first order condition is:

$$\beta^\tau = W_\tau \lambda_{i\tau} \quad (52)$$

The first order condition for money holdings is computed at a financial market at the end of every period, and therefore under perfect information:

$$\lambda_{i\tau} = E_\tau[\lambda_{i\tau+1} + \psi_{i\tau+1}] \quad (53)$$

The first order condition for bond holdings is – for the same reason – computed under perfect information:

$$\lambda_{i\tau} = (1 + R_{\tau+1})E_{\tau}[\lambda_{i\tau+1}] \quad (54)$$

General Equilibrium. The following assumption on monetary shocks is here useful.

Assumption 8 *The space of realizations of monetary shocks \mathfrak{V} is such that*

$$E[e^{-\nu_{\tau}}] = 1$$

First, I conjecture that in equilibrium C is a constant. If so, then the price of this good is pinned down by the cash in advance constraint, and therefore it is proportional to money supply. Profit maximization for the representative firm immediately implies that the wage W_{τ} is also proportional to money supply M_{τ} . Since I have normalized productivity of the competitive firm to 1, all of these three quantities are equal:

$$P_{\tau} = W_{\tau} = M_{\tau} \quad (55)$$

Then, (52) gives the value of the multiplier $\lambda_{i\tau}$. Manipulating expressions (51), (53) and (54) and using Assumption 8 gives the other equilibrium values for choices of the household as $R_{\tau} = 1/\beta - 1$, $V'(C) = 1/\beta$, $M_{i\tau} = M_{\tau}$, and $B_{\tau} = 0$.⁴⁴ Notice that because of quasilinearity none of these depend on subperiods' choices.

It remains to check that the labor market clears. Because of quasilinearity, labor supply is set to satisfy the budget constraint. Aggregating the budget constraint gives the economy's resource constraint, and from this one can establish that the labor market clears. In other words, any set of PBEs and information dynamics is compatible with GE.

Demand for Credit Good $c_{it\tau}$ by Household i . Substituting (52) and (55) into (50):

$$u'(c_{it\tau}) = p_{jt\tau} E_{\mu_{it\tau}} \left[\frac{1}{M_{\tau}} \right]$$

⁴⁴To obtain that $V'(C) = 1/\beta$, substitute for the multipliers in (53) using (51) one period ahead. Then, combine the other expressions to get the result.

From this equation I get the demand function:

$$c_{it\tau} \left(p_{jt\tau} E_{\mu_{it\tau}} \left[\frac{1}{M_\tau} \right] \right)$$

Notice that the absence of income effects (visible in this equation) rules out inter-subperiod considerations in the demand for credit goods. This implies an absence of an option value when buying credit goods – which could arise for informational reasons. This is another virtue of quasilinearity.

Total Demand for Credit Good $c_{jt\tau}$. At every subperiod t period τ a proportion $\alpha_{t\tau}$ of shoppers know the monetary aggregate. Therefore, demand on island j at subperiod t period τ is

$$c_{t\tau}(p_{jt\tau}, M_\tau, \mu_{it\tau}) = \alpha_{t\tau} c_{it\tau} \left(p_{jt\tau} \cdot \frac{1}{M_\tau} \right) + (1 - \alpha_{t\tau}) c_{it\tau} \left(p_{jt\tau} \cdot E_{\mu_{it\tau}(p_{jt\tau})} \left[\frac{1}{M_\tau} \right] \right) \quad (56)$$

At this point, notice that the total demand (56) that every firm faces is the same as (8) in Section I. Also, firms' production functions are linear, which implies that profit functions satisfy Assumption 6. Firms meet every consumer only once and therefore play the one-shot game described in Section I. Thus, as claimed, it is possible to write a cash in advance general equilibrium framework compatible with all the results of the paper.

A.9 Robustness to Other Types of Information Structure

In the model presented in the body, both firms and informed consumers receive perfect signals about the state. In this section I sketch the robustness to two generalizations of this information structure: the case in which firms receive imperfect signals about the state, and the case in which both firms and informed consumers receive imperfect signals about the state. The discussion focuses on arguing that optimally rigid prices remain insensitive to firms' information in these more general cases. The discussion here is admittedly partial, but it suggests that a full analysis may be promising.

Firms receiving imperfect signals. Suppose that each firm receives a binary signal s_F about the state P , with precision $q_F = Pr(s_F = h|P^h) = Pr(s_F =$

$l|P^l) < 1$. (The case analyzed in the body corresponds to $q_F = 1$.)⁴⁵ The key is to recall that the price rigidity argument is based on a tradeoff between the costs of signaling the firm’s information and the benefits of adjusting the price. This is achieved by computing which equilibrium, among the Pooling Equilibrium and the Separating Equilibrium, maximizes ex-ante profits. I will show that this calculation does not change with imperfect signals. This is a direct implication of the law of iterated expectations.

To this end (and somewhat simplifying the previous notation in this appendix) write expected profits conditional on a signal s_F as $E[\pi(p, P)|s_F]$ and ex-ante profits as $E[E[\pi(p, P)|s_F]]$.

Lemma 16

$$E[E[\pi(p, P)|s_F]] = E[\pi(p, P)]$$

The proof of the lemma is immediate, but it clarifies that—since $E[\pi(p, P)]$ are ex-ante profits in the perfect signal case—imperfect signals do not modify the computation of ex-ante profits. Therefore, the ex-ante optimality of the *same* Pooling price for α below a cutoff is still valid in this case. To see this, first notice that any equilibrium where the price depends on the firm’s information s_F is—by definition—a separating equilibrium. In any separating equilibrium, the firm clearly gets strictly less profits than under perfect information. By the calculation in Lemmas 7 and 13, the Pooling Equilibrium attains maximum (perfect information benchmark) profits Π^* for $\alpha = 0$. Therefore, by a similar continuity argument as in Lemma 8, for small α , the Pooling Equilibrium delivers profits arbitrarily close to maximum (perfect information benchmark) profits Π^* , therefore clearly dominating any separating equilibrium. Therefore, it *remains ex-ante optimal* below a cutoff.⁴⁶ Prices in the Pooling Equilibrium do not depend on the signal s_F . Therefore, optimally rigid prices do not reflect the firm’s information, which is what I aimed to argue.⁴⁷

⁴⁵To simplify the notation, I drop the firm index j .

⁴⁶Moreover, by continuity, for signals of high precision, an analog to the main rigidity result (Proposition 1) still holds (the case of low precision signals being harder to characterize because it is not entirely obvious how the Separating Equilibrium may be modified: If signals are very imprecise, it seems that the Separating Equilibrium will either not exist or deliver suboptimal profits for all α .)

⁴⁷A minor subtlety is that there may be other equilibria that yield even higher profits than the benchmark Pooling Equilibrium for small, but strictly positive, α . However, since the Pooling Equilibrium is preferred by the firm to *any* separating equilibrium, these other candidate equilibria have to be pooling, as well (for

Both firms and consumers receiving imperfect signals. Suppose now that also informed consumers observe an imperfect signal s_C about the state P , with precision $q_C = Pr(s_C = h|P^h) = Pr(s_C = l|P^l) < 1$.⁴⁸ We know once again by Lemmas 7 and 13 that the Pooling equilibrium reaches maximum profits Π^* for $\alpha = 0$. Again, by continuity of profits (equation (34)) over consumer beliefs, when α is small, the Pooling Equilibrium must reach almost Π^* . Thus, below a cutoff, it again remains ex-ante optimal. Optimally (rigid) prices do not depend on the firm's information in this case, either.

Conclusion. In conclusion, this discussion has shown that the ex-ante optimality of the Pooling Equilibrium for α below a cutoff is maintained in the presence of imperfectly informative signals. Therefore, even in this case, optimally rigid prices do not reflect the firm's information.

a more general presentation of this issue, see L'Huillier and Zame 2017, p. 20.)

⁴⁸Once again, I drop the consumer index i .