

Non-Neutrality of Open-Market Operations On-Line Appendix

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This Appendix describes the features of the general model used for the numerical simulations in the paper “Non-Neutrality of Open-Market Operations”, and discusses some additional simulations not reported in the main text.

A. General Model

We assume that preferences are of the form

$$(1) \quad E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\frac{C_t^{1-\rho}}{1-\rho} - \int_0^1 \frac{(L_t(j))^{1+\eta}}{1+\eta} dj \right]$$

where C is a consumption bundle of the form

$$C \equiv \left[\int_0^1 C(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}};$$

$C(j)$ is the consumption of a generic good j produced in the economy and θ , with $\theta > 1$, is the intratemporal elasticity of substitution between goods; $L(j)$ is hours worked of variety j which is only used by firm j to produce good j while η is the inverse of the Frisch elasticity of labor supply, with $\eta > 0$. Each household supplies all the varieties of labor used in the production. The asset markets now change to

$$(2) \quad M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t \leq B_{t-1} + X_{t-1} + (1 - \varkappa_t)(1 + \delta Q_t) D_{t-1} + \int_0^1 W_{t-1}(j) L_{t-1}(j) dj - \tilde{T}_t^F + \Phi_{t-1} + (M_{t-1} - P_{t-1} C_{t-1}).$$

In the budget constraint (2), $W(j)$ denotes wage specific to labor of quality j . Wage income for each variety of labor j , $W_{t-1}(j)L_{t-1}(j)$, and firms' profits, Φ_{t-1} , of period $t-1$ are deposited in the financial account at the beginning of period t ; \tilde{T}_t^F are lump-sum taxes levied by the treasury.

Given that in this general model labor supply is endogenous first-order conditions of the household's problem imply that the marginal rate of substitution

between labor and consumption, for each variety j , is given by

$$(3) \quad \frac{(L_t(j))^\eta}{C_t^{-\rho}} = \frac{1}{1+i_t} \frac{W_t(j)}{P_t},$$

which is shifted by movements in the nominal interest rate, reflecting the financial friction. Wage income, indeed, can be used to purchase goods only with one-period delay.

We now turn to the supply of goods. We assume that there is a continuum of firms of measure one, each producing one of the goods in the economy. The production function is linear in labor $Y(j) = A_t L(j)$, in which A is a stochastic productivity disturbance which is assumed to follow a Markov process, with transition density $\pi_a(A_{t+1}|A_t)$ and initial distribution f_a . We assume that (π_a, f_a) is such that $A \in [A_{\min}, A_{\max}]$. Given preferences, each firm faces a demand of the form $Y(i) = (P(i)/P)^{-\theta} Y$ where in equilibrium aggregate output is equal to consumption

$$(4) \quad Y_t = C_t.$$

Firms are subject to price rigidities as in the Calvo model. A fraction of measure $(1 - \alpha)$ of firms with $0 < \alpha < 1$ is allowed to change its price. The remaining fraction α of firms indexes their previously-adjusted prices to the inflation target $\bar{\Pi}$. Adjusting firms choose prices to maximize the presented discounted value of profits under the circumstances that the prices chosen, appropriately indexed to the inflation target, will remain in place until period T with probability α^{T-t} :

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left[\bar{\Pi}^{T-t} P_t(j) Y_T(j) - (1 - \varrho_T) \frac{W_T(j)}{A_T} Y_T(j) \right],$$

where ϱ_t is a subsidy on firms' labor costs. We assume that ϱ_t is a stochastic disturbance which is assumed to follow a Markov process, with transition density $\pi_\varrho(\varrho_{t+1}|\varrho_t)$ and initial distribution f_ϱ . We assume that (π_ϱ, f_ϱ) is such that $\varrho \in [\varrho_{\min}, \varrho_{\max}]$. The optimality condition implies

$$(5) \quad \frac{P_t^*(j)}{P_t} = \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^\theta \mu_T \frac{W_T(j)}{A_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T P_t \bar{\Pi}^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^\theta Y_T \right\}}$$

in which we have used the demand function $Y(i) = (P(i)/P)^{-\theta} Y$ and have defined $\mu_t \equiv \theta(1 - \varrho_t)/(\theta - 1)$.¹ We can also replace in the previous equation $\lambda_t = C_t^{-\rho} \xi_t / P_t$

¹An interesting result is that the efficient steady state of the model can be implemented by setting

and $W_t(j)/P_t$ from (3) together with the demand function, $Y(i) = (P(i)/P)^{-\theta}Y$, to obtain

$$\left(\frac{P_t^*}{P_t}\right)^{1+\theta\eta} = \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta(1+\eta)} (1+i_T)\mu_T \left(\frac{Y_T}{A_T}\right)^{1+\eta} \xi_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta-1} Y_T^{1-\rho} \xi_T \right\}}$$

where P_t^* is the common price chosen by the firms that can adjust it at time t .

Calvo's model further implies the following law of motion of the general price index

$$(6) \quad P_t^{1-\theta} = (1-\alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \bar{\Pi}^{1-\theta},$$

through which we can write the aggregate supply equation as

$$(7) \quad \left(\frac{1 - \alpha \bar{\Pi}_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha}\right)^{\frac{1+\theta\eta}{1-\theta}} = \frac{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta(1+\eta)} (1+i_T)\mu_T \left(\frac{Y_T}{A_T}\right)^{1+\eta} \xi_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}}\right)^{\theta-1} Y_T^{1-\rho} \xi_T \right\}}.$$

The additional difference with respect to the model of Section I is now in the flow budget constraint of the government which is given by

$$Q_t D_t^G + \frac{B_t^G}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^G + B_{t-1}^G - T_t^F - T_t^C$$

where

$$T_t^F \equiv \tilde{T}_t^F - \varrho_t \int_0^1 W_t(j)L_t(j).$$

the steady-state employment subsidy to $\varrho \equiv 1 - (1-1/\theta)/(1+\bar{\nu})$ where $\bar{\nu}$ is the steady-state level of the nominal interest rate. One needs to use only one instrument of policy to offset both the monopolistic distortion and the financial friction, since both create an inefficient wedge between the marginal rate of substitution between leisure and consumption and the marginal product of labor. Moreover, given this result, the steady-state level of the nominal interest rate can be different from zero, while the inflation rate can be set at the target $\bar{\Pi}$. The steady-state version of equation (8) relates the nominal interest rate to the inflation rate $\beta(1+\bar{\nu}) = \bar{\Pi}$. This result crucially depends on the assumption that all consumption requires cash. It would fail in a model with cash and credit goods.

EQUILIBRIUM. — Here, we describe in a compact way the equations that characterize the equilibrium allocation in the general model:

$$(8) \quad \frac{1}{1+i_t} = E_t \left\{ \beta \frac{\xi_{t+1} Y_{t+1}^{-\rho}}{\xi_t Y_t^{-\rho}} \frac{1}{\Pi_{t+1}} \right\},$$

$$(9) \quad \left(\frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha} \right)^{\frac{1+\theta\eta}{1-\theta}} = \frac{F_t}{K_t},$$

$$(10) \quad F_t = \mu_t (1+i_t) \xi_t \left(\frac{Y_t}{A_t} \right)^{1+\eta} + \alpha \beta E_t \left\{ \Pi_{t+1}^{\theta(1+\eta)} \bar{\Pi}^{-\theta(1+\eta)} F_{t+1} \right\},$$

$$(11) \quad K_t = \xi_t Y_t^{1-\rho} + \alpha \beta E_t \left\{ \Pi_{t+1}^{\theta-1} \bar{\Pi}^{1-\theta} K_{t+1} \right\},$$

$$(12) \quad \Delta_t = \Delta \left(\frac{\Pi_t}{\bar{\Pi}}, \Delta_{t-1} \right) \equiv \alpha \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\theta(1+\eta)} \Delta_{t-1} + (1-\alpha) \left(\frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha} \right)^{\frac{\theta(1+\eta)}{\theta-1}},$$

$$(13) \quad Q_t = E_t \left\{ \beta \frac{\xi_{t+1} Y_{t+1}^{-\rho} (1 - \alpha_{t+1}) (1 + \delta Q_{t+1})}{\xi_t Y_t^{-\rho} \Pi_{t+1}} \right\},$$

$$(14) \quad M_t \geq P_t Y_t,$$

$$(15) \quad i_t (M_t - P_t Y_t) = 0,$$

$$(16) \quad E_t \left\{ \sum_{T=t}^{\infty} \beta^{T+1-t} \xi_{T+1} Y_{T+1}^{-\rho} \left[Y_T + \frac{i_T}{1+i_T} \frac{M_T}{P_T} \right] \right\} < \infty,$$

$$(17) \quad \lim_{T \rightarrow \infty} E_t \left[\beta^{T-t} \frac{\xi_T Y_T^{-\rho}}{P_T} \left(M_T + \frac{B_T + X_T}{1+i_T} + Q_T D_T \right) \right] = 0,$$

$$(18) \quad Q_t D_t^F + \frac{B_t^F}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^F + B_{t-1}^F - T_t^F - T_t^C,$$

$$(19) \quad Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t)D_{t-1}^C + B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C,$$

$$(20) \quad B_t^F = B_t + B_t^C,$$

$$(21) \quad D_t^F - D_t = D_t^C.$$

A rational expectations equilibrium is a collection of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C\}$, satisfying each of the conditions in equations (8) to (21) at each time $t \geq t_0$ (and in each contingency at t) consistently with the specification of a monetary/fiscal policy regime and given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t, A_t, \mu_t\}$ and initial conditions given by the vector \mathbf{w}_{t_0-1} which at least includes $\Delta_{t_0-1}, M_{t_0-1}, X_{t_0-1}, B_{t_0-1}^C, B_{t_0-1}^F, D_{t_0-1}^C, D_{t_0-1}^F$.

OPTIMAL POLICY. — Optimal policy maximizes the utility of the consumers, the welfare metric can be written as

$$(22) \quad E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\frac{Y_t^{1-\rho}}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta} \frac{\Delta_t}{A_t^{1+\eta}} \right].$$

We consider the following –partial– specification of the monetary/fiscal policy regime: a *transfer policy* $T_t^F = \mathcal{T}^F(T_t^C, D_{t-1}^F, B_{t-1}^F, P_t, Q_t, \zeta_t)$ and $T_t^C = \mathcal{T}^C(N_{t-1}^C, \Psi_t^C, \zeta_t)$ and a *balance-sheet policy* $B_t^C = \bar{B}_t^C, D_t^C = \bar{D}_t^C, D_t^F = \bar{D}_t^F$ which includes all the cases we are going to consider in our numerical exercises. This specification leaves one degree of freedom along which we choose the optimal policy. The optimal policy is a collection of stochastic processes $\{Y_t, \Pi_t, i_t, Q_t, F_t, K_t, M_t, \Delta_t, X_t, B_t, B_t^C, B_t^F, D_t, D_t^C, D_t^F, T_t^F, T_t^C\}$ satisfying each of the conditions in equations (8) to (21) at each time $t \geq t_0$ (and in each contingency at t) consistently with $T_t^F = \mathcal{T}^F(T_t^C, D_{t-1}^F, B_{t-1}^F, P_t, Q_t, \zeta_t)$, $T_t^C = \mathcal{T}^C(N_{t-1}^C, \Psi_t^C, \zeta_t)$ and $B_t^C = \bar{B}_t^C, D_t^C = \bar{D}_t^C, D_t^F = \bar{D}_t^F$ that maximizes (22) given the definition $\Pi_t \equiv P_t/P_{t-1}$, the non-negativity constraint on the nominal interest rate $i_t \geq 0$, the stochastic processes for the exogenous disturbances $\{\xi_t, \varkappa_t, A_t, \mu_t\}$ and initial conditions \mathbf{w}_{t_0-1} .

To compute the optimal policy, we consider the associated Lagrangian problem maximizing (22) and attaching Lagrange multipliers $\lambda_{j,t}$ for $j = 1 \dots 15$ to the

following constraints (which rewrite those above)

$$\begin{aligned} \xi_t Y_t^{-\rho} &= \beta(1+i_t) E_t \left\{ \frac{\xi_{t+1} Y_{t+1}^{-\rho}}{\Pi_{t+1}} \right\} \\ F_t &= \mu_t(1+i_t) \xi_t \left(\frac{Y_t}{A_t} \right)^{1+\eta} + \alpha \beta E_t \left\{ \Pi_{t+1}^{\theta(1+\eta)} \bar{\Pi}^{-\theta(1+\eta)} F_{t+1} \right\} \\ K_t &= \xi_t Y_t^{1-\rho} + \alpha \beta E_t \left\{ \Pi_{t+1}^{\theta-1} \bar{\Pi}^{1-\theta} K_{t+1} \right\} \\ &\left(\frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha} \right)^{\frac{1+\theta\eta}{1-\theta}} K_t = F_t \\ \Delta_t &= \alpha \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\theta(1+\eta)} \Delta_{t-1} + (1 - \alpha) \left(\frac{1 - \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1 - \alpha} \right)^{\frac{\theta(1+\eta)}{\theta-1}} \\ &i_t \geq 0 \\ Q_t \xi_t Y_t^{-\rho} &= E_t \left\{ \beta \xi_{t+1} Y_{t+1}^{-\rho} \frac{(1 - \varkappa_{t+1})(1 + \delta Q_{t+1})}{\Pi_{t+1}} \right\} \\ m_t &= Y_t \\ r_t Q_{t-1} &= (1 - \varkappa_t)(1 + \delta Q_t) - Q_{t-1} \\ n_t^C &= Q_t d_t^C - m_t - \tilde{x}_t \\ \xi_t Y_t^{-\rho} (t_t^C - \psi_t^C + n_t^C) &= \xi_t Y_t^{-\rho} n_{t-1}^C \Pi_t^{-1} \\ \psi_t^C \Pi_t &= i_{t-1} (n_{t-1}^C + m_{t-1}) + (r_t - i_{t-1}) Q_{t-1} d_{t-1}^C \\ t_t^C &= \bar{T}^C + \gamma_c \psi_t^C + \phi_c n_{t-1}^C \Pi_t^{-1} \\ Q_t d_t^F + \frac{1}{1+i_t} b_t^F &= (1+r_t) Q_{t-1} d_{t-1}^F \Pi_t^{-1} + b_{t-1}^F \Pi_t^{-1} - t_t^F - t_t^C \\ t_t^F &= \bar{T}^F - \gamma_f t_t^C + \phi_f [(1+r_t) Q_{t-1} d_{t-1}^F \Pi_t^{-1} + b_{t-1}^F \Pi_t^{-1}] \end{aligned}$$

where lower-case variables denote the real counterpart of the upper-case variable, while $\tilde{x}_t \equiv (X_t - B_t^C)/(P_t(1+i_t))$.

The first-order conditions with respect to the vector $(Y_t, i_t, \Pi_t, K_t, F_t, \Delta_t, m_t, Q_t, r_t, t_t^C, n_t, \tilde{x}, \psi_t^C, t_t^F, b_t^F)$ are respectively:

$$\begin{aligned} 0 &= \xi_t Y_t^{-\rho} - \xi_t Y_t^\eta \Delta_t A_t^{-(1+\eta)} - \rho \lambda_{1,t} \xi_t Y_t^{-\rho-1} + \lambda_{1,t-1} \rho (1+i_{t-1}) \xi_t Y_t^{-\rho-1} \Pi_t^{-1} \\ &- \lambda_{2,t} (1+\eta) \mu_t (1+i_t) \xi_t \frac{1}{Y_t} \left(\frac{Y_t}{A_t} \right)^{1+\eta} - \lambda_{3,t} (1-\rho) \xi_t Y_t^{-\rho} - \lambda_{7,t} \rho Q_t \xi_t Y_t^{-\rho-1} \\ &+ \lambda_{7,t-1} \rho \xi_t Y_t^{-\rho-1} \frac{(1-\varkappa_t)(1+\delta Q_t)}{\Pi_t} - \lambda_{8,t} \end{aligned}$$

$$0 = \lambda_{1,t}\beta E_t \left\{ \xi_{t+1} Y_{t+1}^{-\rho} \Pi_{t+1}^{-1} \right\} + \lambda_{2,t} \mu_t \xi_t \left(\frac{Y_t}{A_t} \right)^{1+\eta} - \lambda_{6,t} + \beta E_t \{ \lambda_{12,t+1} \} (n_t + m_t - Q_t d_t^C) + \frac{\lambda_{14,t}}{(1+i_t)^2} b_t^F$$

$$\begin{aligned} 0 &= \lambda_{1,t-1} (1+i_{t-1}) \xi_t Y_t^{-\rho} \Pi_t^{-2} + \lambda_{4,t} K_t \alpha \frac{1+\theta\eta}{1-\alpha} \left(\frac{1-\alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1-\alpha} \right)^{\frac{\theta(1+\eta)}{1-\theta}} \Pi_t^{\theta-2} \bar{\Pi}^{1-\theta} \\ &\quad - \lambda_{2,t-1} F_t \alpha \theta (1+\eta) \Pi_t^{\theta(1+\eta)-1} \bar{\Pi}^{-\theta(1+\eta)} - \lambda_{3,t-1} K_t \alpha (\theta-1) \Pi_t^{\theta-2} \bar{\Pi}^{1-\theta} \\ &\quad - \lambda_{5,t} \Delta_{t-1} \alpha \theta (1+\eta) \Pi_t^{\theta(1+\eta)-1} \bar{\Pi}^{-\theta(1+\eta)} + \lambda_{5,t} \alpha \theta (1+\eta) \left(\frac{1-\alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1-\alpha} \right)^{\frac{1+\theta\eta}{\theta-1}} \Pi_t^{\theta-2} \bar{\Pi}^{1-\theta} \\ &\quad + \lambda_{7,t-1} \xi_t Y_t^{-\rho} \frac{(1-\varkappa_t)(1+\delta Q_t)}{\Pi_t^2} + \lambda_{11,t} \xi_t \frac{Y_t^{-\rho}}{\Pi_t^2} n_{t-1}^C + \lambda_{12,t} \psi_t^C + \lambda_{13,t} \phi_c n_{t-1}^C \Pi_t^{-2} \\ &\quad + (\lambda_{14,t} + \phi_f \lambda_{15,t}) [(1+r_t) Q_{t-1} d_{t-1}^F + b_{t-1}^F] \Pi_t^{-2} \end{aligned}$$

$$0 = \lambda_{4,t} \left(\frac{1-\alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}}{1-\alpha} \right)^{\frac{1+\theta\eta}{1-\theta}} + \lambda_{3,t} - \lambda_{3,t-1} \alpha \Pi_t^{\theta-1} \bar{\Pi}^{1-\theta}$$

$$0 = -\lambda_{4,t} + \lambda_{2,t} - \lambda_{2,t-1} \alpha \Pi_t^{\theta(1+\eta)} \bar{\Pi}^{-\theta(1+\eta)}$$

$$0 = -\xi_t \frac{Y_t^{1+\eta}}{1+\eta} A_t^{-(1+\eta)} + \lambda_{5,t} - \alpha \beta E_t \left\{ \lambda_{5,t+1} \Pi_{t+1}^{\theta(1+\eta)} \bar{\Pi}^{-\theta(1+\eta)} \right\}$$

$$0 = \lambda_{8,t} + \lambda_{10,t} - \beta i_t E_t \lambda_{12,t+1}$$

$$\begin{aligned} 0 &= \lambda_{7,t} \xi_t Y_t^{-\rho} - \lambda_{7,t-1} \delta (1-\varkappa_t) \xi_t Y_t^{-\rho} \Pi_t^{-1} + \beta E_t \{ \lambda_{9,t+1} (1+r_{t+1}) \} - (1-\varkappa_t) \delta \lambda_{9,t} \\ &\quad - \beta E_t \{ (\lambda_{14,t+1} + \phi_f \lambda_{15,t+1}) (1+r_{t+1}) d_{t+1}^F \Pi_{t+1}^{-1} \} - \lambda_{10,t} d_t^C - \beta d_t^C E_t \{ \lambda_{12,t+1} (r_{t+1} - i_t) \} \\ &\quad + \lambda_{14,t} d_t^F \end{aligned}$$

$$\lambda_{9,t} Q_{t-1} - \lambda_{12,t} Q_{t-1} d_{t-1}^C - (\lambda_{14,t} + \phi_f \lambda_{15,t}) Q_{t-1} d_{t-1}^F \Pi_t^{-1} = 0$$

$$0 = \lambda_{11,t} \xi_t Y_t^{-\rho} + \lambda_{13,t} + \lambda_{14,t} + \gamma_f \lambda_{15,t}$$

$$0 = \lambda_{10,t} - \beta E_t \left\{ \lambda_{11,t+1} \xi_{t+1} Y_{t+1}^{-\rho} \Pi_{t+1}^{-1} \right\} + \lambda_{11,t} \xi_t Y_t^{-\rho} - \beta i_t E_t \{ \lambda_{12,t+1} \} - \beta \phi_c E_t \{ \Pi_{t+1}^{-1} \lambda_{13,t+1} \}$$

$$\lambda_{10,t} = 0$$

$$\lambda_{12,t} \Pi_t - \lambda_{11,t} \xi_t Y_t^{-\rho} - \gamma_c \lambda_{13,t} = 0$$

$$\lambda_{14,t} + \lambda_{15,t} = 0$$

$$\frac{\lambda_{14,t}}{1+i_t} - \beta E_t \{ (\lambda_{14,t+1} + \phi_f \lambda_{15,t+1}) \Pi_{t+1}^{-1} \} = 0.$$

SOLUTION METHOD, CALIBRATION AND SIMULATED EXPERIMENTS. — We study the optimal policy problem using linear-quadratic methods. We approximate, around a non-stochastic steady state, the objective welfare function to second order, and the models equilibrium conditions to first order. We solve and simulate the model using the piecewise-linear algorithm developed by Guerrieri and Iacoviello (2015): the approximated system of linear equations is treated as a regime-switching model, where the alternative regimes depend on whether specific constraints are binding or not. In particular, in our model there are two distinct constraints that may occasionally bind. The first one is the familiar zero-lower bound on the nominal interest rate, while the second is a non-negativity constraint that may affect central bank’s remittances under some specifications of the transfer policies.

The model is calibrated (quarterly) as follows. We set the steady-state inflation rate and nominal interest rate on short-term bonds to 2% and 3.5%, respectively and in annualized terms; accordingly, we set $\beta = (1 + \bar{\pi}) / (1 + \bar{r})$. We calibrate the composition of central bank’s balance sheet considering as initial steady state the situation in 2009Q3, when the economy had already been in a liquidity trap for about three quarters. Accordingly we set the share of money to total liabilities equal to 53%, the share of net worth to total liabilities to 1%, and the share of long-term asset to total assets to 72%. This calibration implies that the steady-state quarterly remittances to the treasury are equal to about 0.6% of the central bank’s assets and that the central bank’s position on short-term interest-bearing liabilities (central bank reserves) amounts to 46% of the central bank’s balance sheet. The duration of long-term assets is set to ten years (accordingly, $\delta = .9896$). Moreover, we set the ratio of long-term public debt to GDP in the initial steady state equal to $\bar{Q}\bar{D}^G / (4\bar{Y}\bar{P}) = 0.35$, in annual terms, as reported by the US Bureau of Public Debt for 2009Q3. In particular, we consider the stock of publicly-held marketable government debt including securities with maturity above one year. Finally, following Benigno et al. (2016), we set the relative risk-aversion coefficient to $\rho = 1/.66$, the inverse of the Frisch-elasticity of labor supply to $\eta = 1$, the elasticity of substitution across goods to $\theta = 7.88$, the parameter α capturing the degree of nominal rigidity in the model implies an average duration of consumer prices of four quarters ($\alpha = 0.75$). As a result, the slope of the Phillips Curve is $\kappa = .024$. To calibrate the initial level of the natural interest rate, we follow Benigno et al. (2016), who show that the extent of households’ debt deleveraging observed since 2008 in the U.S. is consistent with a fall of the natural interest rate to about -6% from a steady-state level of 1.5%. See also Gust et al. (2016), who provide consistent empirical evidence.

We evaluate the implications for Neutrality of interest-rate and credit risks.

With respect to interest-rate risk, we run the following experiment. We simulate an economy which at time $t_0 - 1$ is already in a liquidity trap, because of a preference shock (ξ) that hit sometime in the past and turned the natural interest rate negative. At time t_0 the central bank first chooses whether to stick to its past

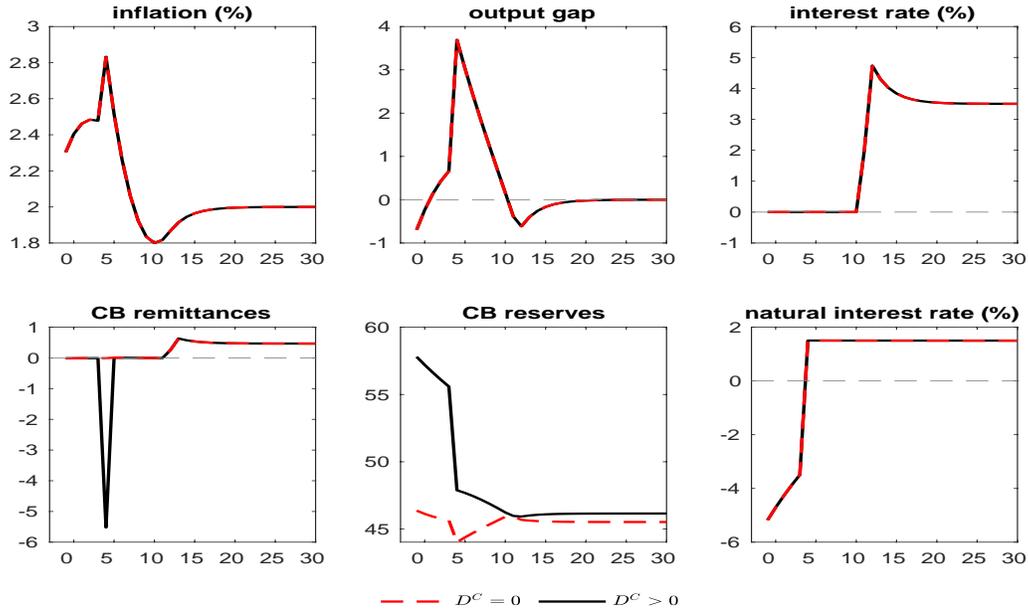


FIGURE 1. EQUILIBRIUM DYNAMICS OF SELECTED VARIABLES UNDER OPTIMAL MONETARY POLICY FACING INTEREST-RATE RISK UNDER *passive transfer policies*. THE ECONOMY STARTS IN A LIQUIDITY TRAP WITH A NEGATIVE NATURAL RATE OF INTEREST; THE LATTER TURNS POSITIVE UNEXPECTEDLY AFTER ONE YEAR. RED SOLID LINE: CENTRAL BANK HOLDS ONLY SHORT-TERM ASSETS. BLACK DASHED LINE: CENTRAL BANK HOLDS ALSO LONG-TERM ASSETS. X-AXIS DISPLAYS QUARTERS.

balance-sheet policy ($D^C = 0$) or to engage in large-scale asset purchases ($D^C > 0$) and then commits to a state-contingent path for the endogenous variables from t_0 onward, conditional on the chosen balance-sheet policy. One year later (at time $t_0 + 4$) an unexpected preference shock hits, turning the natural interest rate positive again. At this time, the path of current and future short-term rates changes, producing an unexpected fall in the price of long-term securities and therefore implying income losses for the central bank, in the case it holds long-term assets.

With respect to credit risk, we consider an economy starting at steady state, and a credit event hitting at time t_0 , which implies default on a share α of long-term debt. After period t_0 no other credit event or other shocks are either expected or actually occur. As clear from equation (14), when a credit event occurs, the central bank might experience a loss on its balance sheet if it holds long-term securities. To simulate the optimal response to an increase in the probability of future credit events, we use the result of the above experiment to compute the response of the economy in the contingency in which the credit event occurs. We then use the respective equilibrium decision rules and the probability of a credit event to characterize the one-period-ahead expectations for the relevant

variables, and solve a linear-quadratic approximation of the optimal monetary-policy problem at time 0.

B. Additional Simulations

Here we discuss some additional simulations not reported in the main text.

Consider a regime with *passive transfer policies*, combining a *passive fiscal policy* and a *passive remittance policy*. The top panels of Figure 1 display the path of inflation, the output gap and the nominal interest rate, and show the familiar result, already discussed in Eggertsson and Woodford (2003), that committing to a higher inflation for the periods after the liftoff of the natural rate of interest allows to limit the deflationary impact of the negative shock, despite the nominal interest rate cannot be cut as much as needed because of the zero floor. This commitment translates into maintaining the policy rate at the zero bound for several periods after the natural rate has turned back positive (in the specific case of Figures 1, for six quarters more).

The bottom panels show instead the evolution of two key variables related to the balance sheet of the central bank – as well as the path of the natural interest rate: the quarterly real remittances to the treasury T_t^C/P_t and the central bank's real reserves X_t/P_t , all expressed as a share of the steady-state balance sheet of the central bank. Consistently with Proposition 1, the central bank's real net worth remains constant at its initial level of 1% (not shown) and the dynamics of profits (and remittances) reflect the specific composition of the central bank's balance sheet. When the central bank has only short-term assets, remittances are non-negative while with long-term assets they mainly follow their return. As the natural rate unexpectedly turns positive, the expectation that the nominal interest rate will jump up a few periods later is enough to bring down long-term asset prices and their return, thereby implying negative profits for the central bank. Under *passive remittance policy*, negative profits trigger a transfer of resources from the treasury to the central bank (negative remittances), so that net worth does not move. Central bank's reserves instead fall as a consequence of the lower valuation of the long-term assets.

In Figure 2, under the same calibration, we consider a mild and a strong credit event with default rate respectively of 40% and 80% (i.e. $\varkappa = 0.40$ or $\varkappa = 0.80$, displayed by the continuous and dashed lines in Figure 2). The top panels show that the optimal monetary policy requires to completely stabilize inflation, output and interest rate at their targets. Indeed, the shock \varkappa does not appear in either the objective function (22) or the constraints that are relevant under Neutrality (8)–(12). Given the *transfer policy* assumed, the optimal monetary policy is also not affected by the alternative *balance-sheet policy*. The difference is in the remittances to the treasury. In the case of a standard composition of the balance-sheet ($D_t^C = 0$), profits and remittances are always positive while when the central bank holds long-term securities losses are covered by the treasury, given *passive remittance policy*, and the more so the higher the default rate.

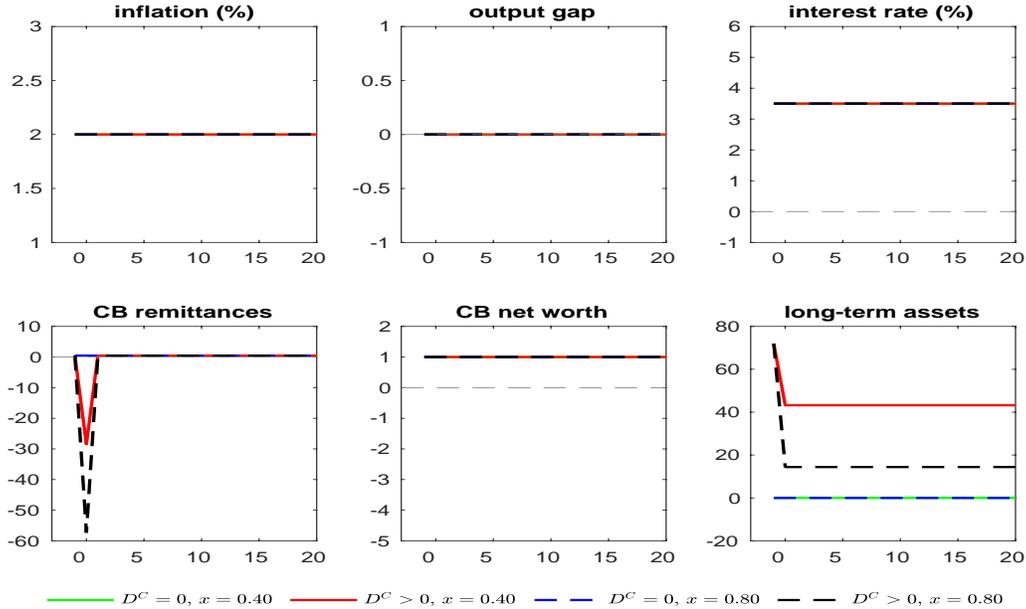


FIGURE 2. RESPONSE OF SELECTED VARIABLES, UNDER OPTIMAL MONETARY POLICY, TO A ONE-PERIOD CREDIT EVENT OF ALTERNATIVE SIZES, UNDER ALTERNATIVE BALANCE-SHEET POLICIES AND *passive transfer policies*. GREEN SOLID LINE: CREDIT EVENT IMPLIES DEFAULT ON 40% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ONLY SHORT-TERM ASSETS. RED SOLID LINE: CREDIT EVENT IMPLIES DEFAULT ON 40% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ALSO LONG-TERM ASSETS. BLUE DASHED LINE: CREDIT EVENT IMPLIES DEFAULT ON 80% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ONLY SHORT-TERM ASSETS. BLACK DASHED LINE: CREDIT EVENT IMPLIES DEFAULT ON 80% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ALSO LONG-TERM ASSETS. X-AXIS DISPLAYS QUARTERS.

We consider now the “*deferred-asset*” regime. Figures 3 through 5 analyze the same scenarios as Figures 1 and 2, respectively, maintaining the assumption of *passive fiscal policy*, the same *balance-sheet policies* but changing the remittance policy to a “*deferred-asset*” regime analogous to the one specified in Definition 6.²

With only interest-rate risk, as shown in Figure 3, the responses of inflation, output and interest rate do not change across the two alternative balance-sheet policies. This case is indeed consistent with the necessary and sufficient conditions for neutrality of Proposition 5. Indeed, losses are not large enough to impair the profitability of the central bank and violate condition (38) under the optimal monetary policy. As central bank’s profits turn negative, remittances to the treasury fall to zero and stay at this level even when central bank’s profits start to be positive as long as real net worth is below its long-run level, thereby allowing

²In particular, since we simulate a linear approximation of the model, we adapt the rules introduced in the previous section to ensure a stationary real net worth (rather than nominal). This adjustment will also apply later when we deal with the case of financial independence.

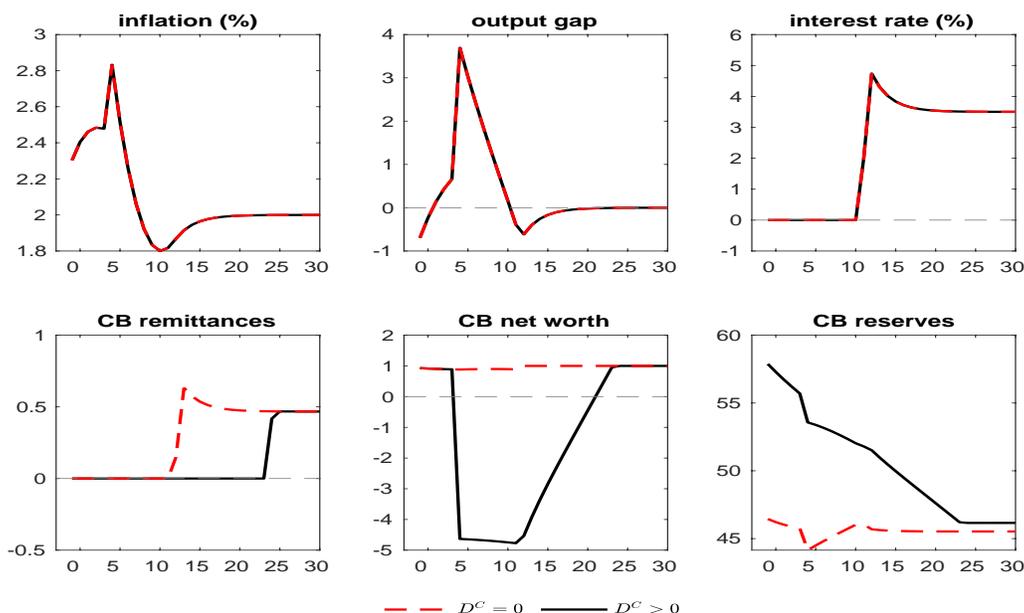


FIGURE 3. EQUILIBRIUM DYNAMICS OF SELECTED VARIABLES UNDER OPTIMAL MONETARY POLICY FACING INTEREST-RATE RISK. REGIME II): “*lack of treasury’s support*”. THE ECONOMY STARTS IN A LIQUIDITY TRAP WITH A NEGATIVE NATURAL RATE OF INTEREST; THE LATTER TURNS POSITIVE UNEXPECTEDLY AFTER ONE YEAR. RED SOLID LINE: CENTRAL BANK HOLDS ONLY SHORT-TERM ASSETS. BLACK DASHED LINE: CENTRAL BANK HOLDS ALSO LONG-TERM ASSETS. X-AXIS DISPLAYS QUARTERS.

the latter to converge back to 1% of the balance sheet within a few quarters. After net worth is back at the initial value of 1%, central bank’s profits are again rebated to the treasury. The implication is that central bank’s reserves are temporarily higher than under passive remittance policy, and are paid back by next-period profits.

Figure 4, in the case of credit risk, shows instead a non-neutrality result when the credit event is significant (i.e. $\varkappa = 0.80$) and the central bank holds long-term assets ($\tilde{D}^C > 0$). Indeed, in this case losses are strong enough to impair the profitability of the central bank: without a change in prices and output with respect to the case $D_t^C = 0$, profits would remain indefinitely negative. The conditions for neutrality of Propositions 5 and 6 are violated. Instead, if the credit event is not too strong (i.e. $\varkappa = 0.40$), neutrality emerges and the central bank is therefore able to return to the steady-state level of net worth in a finite period of time without changing equilibrium prices and output with respect to the case in which $D_t^C = 0$, as shown in the Figure.

Figure 5 further shows the path of remittances, nominal money supply and central bank’s net worth under the mild and strong credit events of Figure 4 given the two balance-sheet policies $D_t^C = 0$ and $\tilde{D}^C > 0$. The solid line, capturing

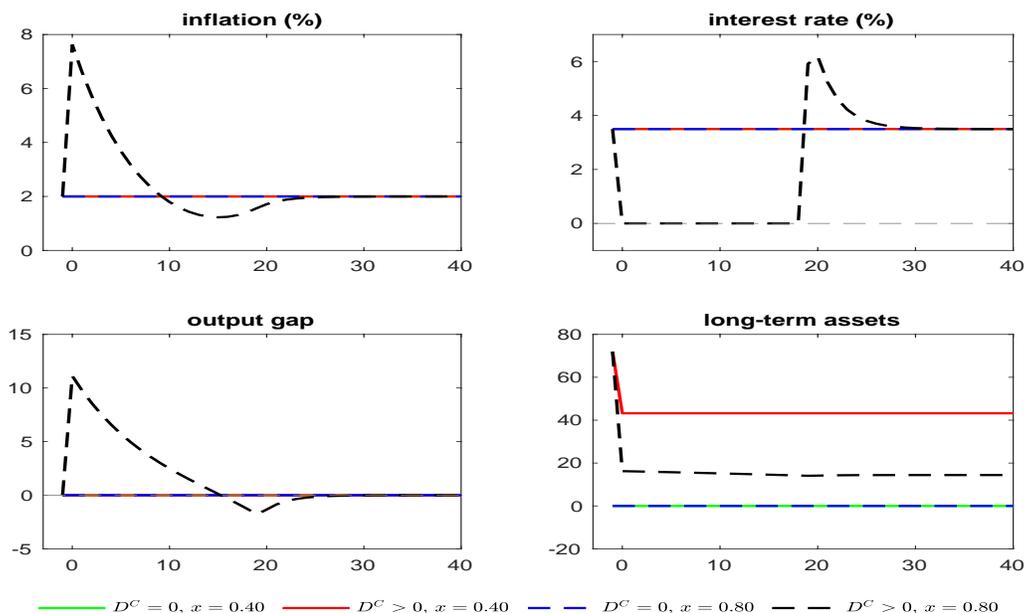


FIGURE 4. RESPONSE OF SELECTED VARIABLES, UNDER OPTIMAL MONETARY POLICY, TO A ONE-PERIOD CREDIT EVENT OF ALTERNATIVE SIZES, UNDER ALTERNATIVE BALANCE-SHEET POLICIES. REGIME II): “*lack of treasury’s support*”. GREEN SOLID LINE: CREDIT EVENT IMPLIES DEFAULT ON 40% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ONLY SHORT-TERM ASSETS. RED SOLID LINE: CREDIT EVENT IMPLIES DEFAULT ON 40% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ALSO LONG-TERM ASSETS. BLUE DASHED LINE: CREDIT EVENT IMPLIES DEFAULT ON 80% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ONLY SHORT-TERM ASSETS. BLACK DASHED LINE: CREDIT EVENT IMPLIES DEFAULT ON 80% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ALSO LONG-TERM ASSETS. X-AXIS DISPLAYS QUARTERS.

the mild-credit event (when $\tilde{D}^C > 0$), shows that the fall in net worth, as a consequence of the income loss at t_0 , is not enough to impair the ability of the central bank to produce positive gains from seigniorage in the future (i.e. $N_t^C + M_t^* > 0$ for each $t \geq \tau$). Such positive profits, therefore, will be possible without the need for the path of nominal money supply to deviate from the equilibrium associated with $D^C = 0$ (second panel of Figure 5). Moreover, these gains will be used to repay the deferred asset over a period in which remittances are zero and net worth can be rebuilt (first and third panels of Figure 5, respectively).

Results substantially change if the credit event is strong. In this case, the nominal stock of non-interest bearing liabilities, $N_t^C + M_t^*$, if evaluated at the inflation rate of the equilibrium with $D_t^C = 0$, would turn negative within the first quarters and violate afterward the solvency condition of the central bank at the initial equilibrium prices. The dashed lines in Figure 5 shows how to optimally deal with a shock of this size. The central bank should commit to substantially raise the stock of nominal money supply in the short-run – to compensate for

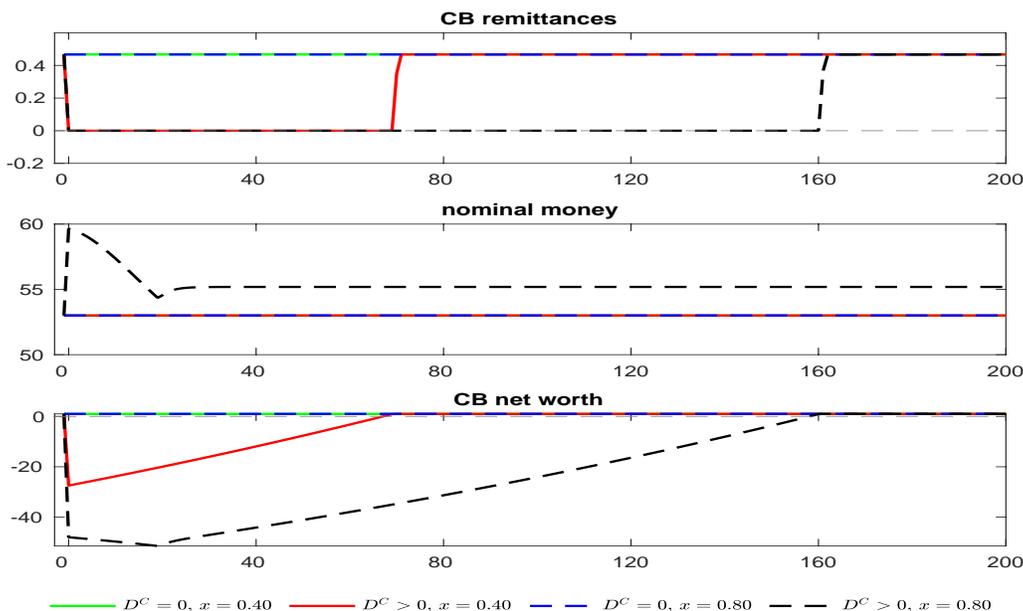


FIGURE 5. RESPONSE OF SELECTED VARIABLES, UNDER OPTIMAL MONETARY POLICY, TO A ONE-PERIOD CREDIT EVENT OF ALTERNATIVE SIZES, UNDER ALTERNATIVE BALANCE-SHEET POLICIES. REGIME II): “*lack of treasury’s support*”. GREEN SOLID LINE: CREDIT EVENT IMPLIES DEFAULT ON 40% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ONLY SHORT-TERM ASSETS. RED SOLID LINE: CREDIT EVENT IMPLIES DEFAULT ON 40% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ALSO LONG-TERM ASSETS. BLUE DASHED LINE: CREDIT EVENT IMPLIES DEFAULT ON 80% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ONLY SHORT-TERM ASSETS. BLACK DASHED LINE: CREDIT EVENT IMPLIES DEFAULT ON 80% OF LONG-TERM ASSETS, CENTRAL BANK HOLDS ALSO LONG-TERM ASSETS. X-AXIS DISPLAYS QUARTERS.

the fall in nominal net worth – and set it at a permanently higher level in the long-run. Such commitment will ensure that the stock of non-interest bearing liabilities eventually reverts to positive values and produces the profits needed to repay the deferred asset and rebuild net worth (although over an extremely long time). To generate such a path of nominal money supply, the central bank should be accommodative enough to push up prices and inflation. In particular, as the dashed line in Figure 4 shows, inflation and output should go well above their target on impact, which in turn requires the nominal interest rate to fall down to the zero-lower bound. In the specific case displayed in Figures 4 and 5, it takes about 30 quarters for real variables to converge back to the path they would follow under neutrality, and for nominal money supply to stabilize on a new, higher, level.