

Online Appendix

Consumption Insurance Against Wage Risk: Family Labor Supply and Optimal Progressive Income Taxation

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A Consumption Insurance Decomposition

In this section, we provide details on how consumption insurance decomposition is conducted based on model-simulated data *within each row* of Table 5 and in Figure 5.¹ The idea of the decomposition is illustrated in Figure 8, which shows the transmission of a male wage shock to consumption in the model.

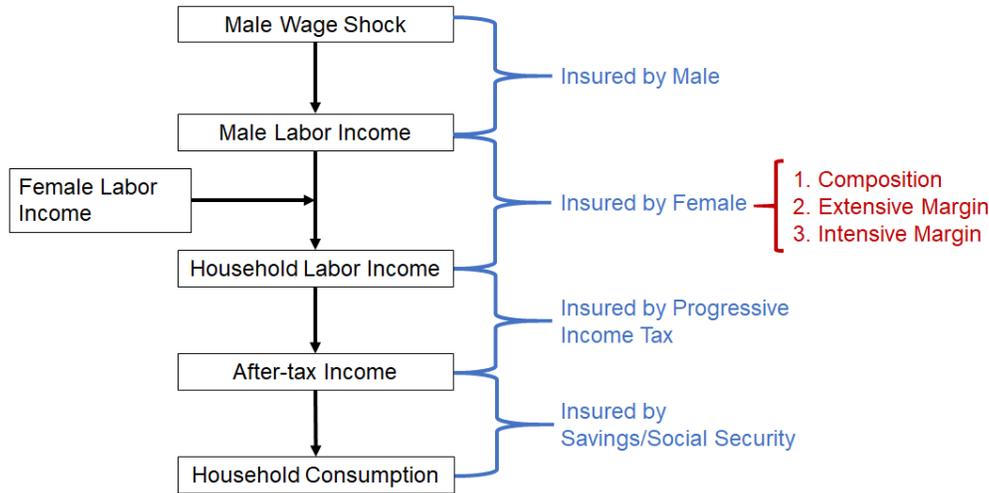


Figure 8: Transmission of Male Wage Shocks

For an age t household, the percentage change of household income in response to a wage shock x (i.e., $x = v_{j,t}$ or $x = \Delta u_{j,t}$) is approximately

$$\Delta y_t = \frac{\Delta Y_t}{Y_{t-1}} = \frac{\Delta Y_{1,t} + \Delta Y_{2,t}}{Y_{t-1}}$$

¹For the decomposition in each row, the model is only solved once, and hence changes in household behaviors in response to changes in the set of available insurance channels are not taken into account. Such information is presented *across different rows* of Table 5 because the model is solved again whenever a new insurance channel is added.

$$\approx \begin{cases} \frac{Y_{1,t-1}}{Y_{t-1}} \kappa_{y_1,x} x + \frac{Y_{2,t-1}}{Y_{t-1}} \kappa_{y_2,x} x, & \text{if } Y_{2,t-1} > 0; \\ \frac{Y_{1,t-1}}{Y_{t-1}} \kappa_{y_1,x} x + \frac{\Delta Y_{2,t}}{Y_{t-1}}, & \text{if } Y_{2,t-1} = 0, \end{cases}$$

where $\kappa_{y_1,x}$ and $\kappa_{y_2,x}$ are transmission coefficients from shock x to male and female labor income as defined in the main text, and by definition, they do not capture the responses through extensive margin. Here we are assuming that the shock may only move the female earner into the labor market, which is innocuous because we can always switch the sign of the shock to make it true. Taking the expectation of Δy_t over the distribution of households, we have the formula for the average response of household income with respect to shock x :

$$\begin{aligned} \mathbb{E}[\Delta y_t] &\approx \mathbb{E}\left[\frac{Y_{1,t-1}}{Y_{t-1}}\right] \kappa_{y_1,x} x + \underbrace{\Pr(Y_{2,t-1} > 0) \mathbb{E}\left[\frac{Y_{2,t-1}}{Y_{t-1}} \mid Y_{2,t-1} > 0\right]}_{=\mathbb{E}\left[\frac{Y_{2,t-1}}{Y_{t-1}}\right]} \kappa_{y_2,x} x \\ &+ \Pr(Y_{2,t-1} = 0) \mathbb{E}\left[\frac{\Delta Y_{2,t}}{Y_{t-1}} \mid Y_{2,t-1} = 0\right]. \end{aligned} \quad (\text{A.1})$$

By rewriting Equation (A.1), we can decompose the average response of household income $\mathbb{E}[\Delta y_t]$ with respect to male wage shock x into effects through different channels in the following way:

$$\begin{aligned} \mathbb{E}[\Delta y_t] &\approx \underbrace{\left\{ 1 + \frac{\Pr(Y_{2,t-1} = 0) \mathbb{E}\left[\frac{\Delta Y_{2,t}}{Y_{t-1}} \mid Y_{2,t-1} = 0\right]}{\left[1 + \frac{\mathbb{E}\left[\frac{Y_{2,t-1}}{Y_{t-1}}\right] \kappa_{y_2,x}}{\mathbb{E}\left[\frac{Y_{1,t-1}}{Y_{t-1}}\right] \kappa_{y_1,x}} \right]} \mathbb{E}\left[\frac{Y_{1,t-1}}{Y_{t-1}}\right] \kappa_{y_1,x} x \right\}}_{K_4} \underbrace{\left[1 + \frac{\mathbb{E}\left[\frac{Y_{2,t-1}}{Y_{t-1}}\right] \kappa_{y_2,x}}{\mathbb{E}\left[\frac{Y_{1,t-1}}{Y_{t-1}}\right] \kappa_{y_1,x}} \right]}_{K_3} \\ &\times \underbrace{\mathbb{E}\left[\frac{Y_{1,t-1}}{Y_{t-1}}\right]}_{K_2} \underbrace{\kappa_{y_1,x}}_{K_1} x. \end{aligned} \quad (\text{A.2})$$

where K_1 represents the effect of male intensive margin ($K_1 = 1$ if male labor supply does not respond to the shock), K_2 represents the composition effect of female income ($K_2 = 1$ if female labor income is zero), K_3 represents the effect of female intensive margin ($K_3 = 1$ if female labor supply does not respond to the shock through intensive margin), and K_4 represents the effect of female extensive margin ($K_4 = 1$ if females do not respond to the shock by entering the labor market).

We can calculate K_1 to K_3 easily because we can estimate $\kappa_{y_j,x}$ and $\mathbb{E}\left[\frac{Y_{j,t-1}}{Y_{t-1}}\right]$

from the model-simulated data. $\mathbb{E}[\frac{\Delta Y_{2,t}}{Y_{t-1}} \mid Y_{2,t-1} = 0]$ is more difficult to calculate directly because $\Delta Y_{2,t}$ here is supposed to be the change of income in response to *only* the shock x . Therefore, to calculate K_4 , we first regress Δy_t on all shocks using model-simulated data to estimate the transmission coefficients from shock x to *household* labor income directly, i.e., $\kappa_{y,x}$. By definition, we have

$$\mathbb{E}[\Delta y_t] = \kappa_{y,x}x. \quad (\text{A.3})$$

Combining Equation (A.1) and Equation (A.3), we have

$$\Pr(Y_{2,t-1} = 0)\mathbb{E}[\frac{Y_{2,t}}{Y_{t-1}} \mid Y_{2,t-1} = 0]x^{-1} = \kappa_{y,x} - \mathbb{E}[\frac{Y_{1,t-1}}{Y_{t-1}}]\kappa_{y_1,x} - \mathbb{E}[\frac{Y_{2,t-1}}{Y_{t-1}}]\kappa_{y_2,x}. \quad (\text{A.4})$$

And now we can calculate K_4 as well.²

By regressing Δc_t on shocks using model-simulated data, we can also estimate the transmission coefficients from shock x to consumption directly, i.e., $\kappa_{c,x}$, and hence

$$\mathbb{E}[\Delta c_t] = \kappa_{c,x}x.$$

Given the functional form of the income tax function, we know

$$\begin{aligned} \Delta y_t^{AT} &= (1 - \mu)\Delta y_t \\ \Rightarrow \mathbb{E}[\Delta y_t^{AT}] &= \underbrace{(1 - \mu)}_{K_5} \kappa_{y,x}x, \end{aligned}$$

where Δy_t^{AT} is the change of after-tax household income with respect to shock x , and K_5 represents the effect of progressive income tax ($K_5 = 1$ if income tax is flat). Let $\mathbb{E}[\Delta c_t] = K_6\mathbb{E}[\Delta y_t^{AT}]$, we have

$$\kappa_{c,x} = K_6K_5\kappa_{y,x} = \prod_{m=1}^6 K_m. \quad (\text{A.5})$$

And K_6 represents the effect of other insurance channels including household savings and social security in the model ($K_6 = 1$ if consumption responds one for one to changes in after-tax income).

Based on Equation (A.5), we can decompose total consumption insurance based on the additional insurance generated through each insurance channel such that $\text{Total Insurance} = \sum_m \text{Insurance}(m)$.

²Since $\kappa_{y,x} = \prod_{m=1}^4 K_m$, we can simply calculate K_4 as $\kappa_{y,x}/(\prod_{m=1}^3 K_m)$. But we will still need Equation (A.4) for the decomposition of consumption insurance against female shocks.

1. Insurance provided by male labor supply: $1 - K_1$.
2. Insurance provided by the female earner: $(1 - \Pi_{m=1}^4 K_m) - (1 - K_1)$.
 - (a) Composition effect: $(1 - \Pi_{m=1}^2) - (1 - K_1)$;
 - (b) Intensive margin: $(1 - \Pi_{m=1}^3) - (1 - \Pi_{m=1}^2)$;
 - (c) Extensive margin: $(1 - \Pi_{m=1}^4) - (1 - \Pi_{m=1}^3)$.
3. Insurance from progressive tax: $(1 - \Pi_{m=1}^5 K_m) - (1 - \Pi_{m=1}^4 K_m)$.
4. Insurance from other channels (savings and social security): $(1 - \Pi_{m=1}^6 K_m) - (1 - \Pi_{m=1}^5 K_m)$.

For a female wage shock, we can decompose the effects of different channels in a similar way:

$$\mathbb{E}[\Delta y_t] \approx \underbrace{\left[1 + \frac{\mathbb{E}\left[\frac{Y_{1,t-1}}{Y_{t-1}}\right] \kappa_{y_1,x}}{\mathbb{E}\left[\frac{Y_{2,t-1}}{Y_{t-1}}\right] \left\{ 1 + \frac{\Pr(Y_{2,t-1}=0) \mathbb{E}\left[\frac{\Delta Y_{2,t}}{Y_{t-1}} \mid Y_{2,t-1}=0\right]}{\mathbb{E}\left[\frac{Y_{2,t-1}}{Y_{t-1}}\right] \kappa_{y_2,x} x} \right\} \kappa_{y_2,x}} \right]}_{K_4} \times \underbrace{\mathbb{E}\left[\frac{Y_{2,t-1}}{Y_{t-1}}\right]}_{K_3} \underbrace{\left\{ 1 + \frac{\Pr(Y_{2,t-1}=0) \mathbb{E}\left[\frac{\Delta Y_{2,t}}{Y_{t-1}} \mid Y_{2,t-1}=0\right]}{\mathbb{E}\left[\frac{Y_{2,t-1}}{Y_{t-1}}\right] \kappa_{y_2,x} x} \right\}}_{K_2} \underbrace{\kappa_{y_2,x} x}_{K_1},$$

where K_1 represents the effect of female intensive margin, K_2 represents the effect of female extensive margin, K_3 represents the composition effect of male labor income, and K_4 represents the effect of male intensive margin. We can define the effect of progressive income tax K_5 and the effect of other channels K_6 in the same way as for male shocks.

B Decomposition of Welfare Changes

In this section, we explain how welfare changes in consumption-equivalent variations (CEV) and the decomposition of welfare changes into level and distribution effects of consumption and labor supply are calculated in the main text, for which we follow the method in Conesa, Kitao, and Krueger (2009).

Consider, for example, an income tax change in the model economy. Let \mathbf{c}^0 and $\{\mathbf{h}_j^0\}_{j=1}^2$ denote the state-contingent plan of household consumption and labor supply for a new-born household before the tax change, and let $W(\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0)$ denote the expected lifetime utility of this new-born household under this state-contingent plan. After the tax change, the corresponding state-contingent plan is denoted by \mathbf{c}^1 and $\{\mathbf{h}_j^1\}_{j=1}^2$, and the lifetime utility is $W(\mathbf{c}^1, \mathbf{h}_1^1, \mathbf{h}_2^1)$.

The welfare effect of this tax change in consumption-equivalent variation, CEV , is defined by the following equation:

$$W((1 + CEV)\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \mathbf{h}_1^1, \mathbf{h}_2^1).$$

Hence, CEV is the percentage change of lifetime consumption that is required to generate a change of lifetime utility equal to that induced by the tax change. If CEV is positive (negative), the tax change is welfare-improving (welfare-reducing).

We can decompose CEV into components stemming from the change in consumption and the change in labor supply. The welfare change due to consumption change, CEV_C , is defined by the following equation:

$$W((1 + CEV_C)\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \mathbf{h}_1^0, \mathbf{h}_2^0).$$

And the welfare change due to changes in male and female labor supply, CEV_{H_1} and CEV_{H_2} , are defined by:

$$W((1 + CEV_{H_1})(1 + CEV_C)\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \mathbf{h}_1^1, \mathbf{h}_2^0),$$

$$W((1 + CEV_{H_2})(1 + CEV_{H_1})(1 + CEV_C)\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \mathbf{h}_1^1, \mathbf{h}_2^1).$$

Therefore,

$$(1 + CEV) = (1 + CEV_C)(1 + CEV_{H_1})(1 + CEV_{H_2}).$$

Furthermore, the consumption impact on welfare can itself be divided into a part that captures the change in average consumption, and a part that reflects the change in the distribution of consumption across the life cycle and different states. Let $\bar{\mathbf{c}}^0$ and $\bar{\mathbf{c}}^1$ denote the average household consumption before and after the tax change, then the welfare change due to the change in consumption level, CEV_{CL} , is defined by

$$W((1 + CEV_{CL})\bar{\mathbf{c}}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\frac{\bar{\mathbf{c}}^1}{\bar{\mathbf{c}}^0}\bar{\mathbf{c}}^0, \mathbf{h}_1^0, \mathbf{h}_2^0),$$

i.e., $CEV_{CL} = \bar{\mathbf{c}}^1/\bar{\mathbf{c}}^0 - 1$, which is the percentage change of average household

consumption due to the tax change. The welfare change due to the change in the distribution of consumption, CEV_{CD} , is defined by

$$W((1 + CEV_{CD})(1 + CEV_{CL})\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \mathbf{h}_1^0, \mathbf{h}_2^0).$$

And hence we have,

$$(1 + CEV_C) = (1 + CEV_{CD})(1 + CEV_{CL}).$$

Similarly, for male and female labor supply changes, we can define CEV_{H_1L} , CEV_{H_1D} , CEV_{H_2L} , and CEV_{H_2D} by

$$W((1 + CEV_{H_1L})(1 + CEV_C)\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \frac{\bar{\mathbf{h}}_1^1}{\bar{\mathbf{h}}_1^0}\mathbf{h}_1^0, \mathbf{h}_2^0),$$

$$W((1 + CEV_{H_1D})(1 + CEV_{H_1L})(1 + CEV_C)\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \mathbf{h}_1^1, \mathbf{h}_2^0),$$

$$W((1 + CEV_{H_2L})(1 + CEV_{H_1})(1 + CEV_C)\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \mathbf{h}_1^1, \frac{\bar{\mathbf{h}}_2^1}{\bar{\mathbf{h}}_2^0}\mathbf{h}_2^0),$$

$W((1 + CEV_{H_2D})(1 + CEV_{H_2L})(1 + CEV_{H_1})(1 + CEV_C)\mathbf{c}^0, \mathbf{h}_1^0, \mathbf{h}_2^0) = W(\mathbf{c}^1, \mathbf{h}_1^1, \mathbf{h}_2^1)$, where $\bar{\mathbf{h}}_1^0$, $\bar{\mathbf{h}}_1^1$, $\bar{\mathbf{h}}_2^0$, and $\bar{\mathbf{h}}_2^1$ are average male and female labor supply before and after the tax change, and we have

$$(1 + CEV_{H_1}) = (1 + CEV_{H_1D})(1 + CEV_{H_1L}),$$

$$(1 + CEV_{H_2}) = (1 + CEV_{H_2D})(1 + CEV_{H_2L}).$$

C Supplementary Results for the Benchmark Model

C.1 Wage Profiles

The life-cycle male wage trend is interpolated and extrapolated from Rupert and Zanella (2015) and plotted in Figure 9. The scale of it is normalized such that the average male trend wage is 1. The female wage trend is rescaled from the male wage trend to match the ratio of the average earnings between working males and females in the BPS data set.

C.2 Age Profiles of Transmission Coefficients

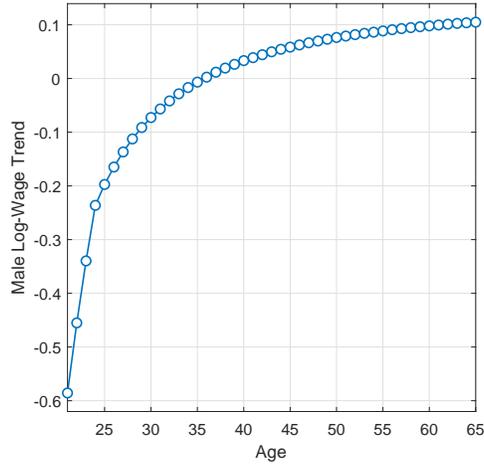


Figure 9: Male Log-Wage Trend

Notes: This figure shows the male log-wage trend used in the model. The female log-wage trend has the same shape but a different level that is calibrated to match the data.

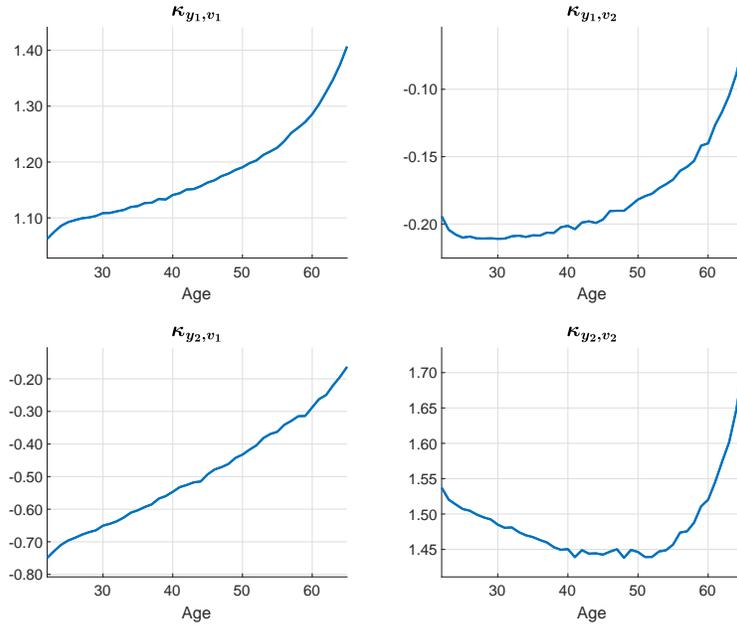


Figure 10: Age Profiles of Transmission Coefficients to Labor Income

Notes: This figure plots transmission coefficients to labor income from permanent wage shocks over the life cycle in the benchmark model with additively separable preferences.

C.3 Performance of the BPS Method

Table 10: Estimation of Transmission Coefficients (Extended)

	Model	Model BPS		
	True	Baseline ^a	SS ^b	Outside ^c
κ_{c,u_1}	0.01	-0.01	0.00	0.00
κ_{c,u_2}	0.01	0.02	0.02	0.02
κ_{c,v_1}	0.35	0.42	0.38	0.38
κ_{c,v_2}	0.18	0.22	0.20	0.20
κ_{y_1,u_1}	1.44	1.47	1.46	1.46
κ_{y_1,u_2}	-0.05	-0.03	-0.04	-0.04
κ_{y_1,v_1}	1.16	1.10	1.13	1.13
κ_{y_1,v_2}	-0.19	-0.20	-0.19	-0.19
κ_{y_2,u_1}	-0.12	-0.10	-0.12	-0.12
κ_{y_2,u_2}	1.76	1.76	1.76	1.76
κ_{y_2,v_1}	-0.51	-0.61	-0.58	-0.58
κ_{y_2,v_2}	1.46	1.53	1.54	1.54

Notes: Only households aged 30-57 are included.

^a BPS baseline method.

^b Modified BPS method accounting for the social security system explicitly. ^c BPS method allowing unspecified outside insurance. The outside insurance coefficient estimated here is $\beta = 0.1444$.

Table 11: Estimation of Frisch Elasticities

	Model	Model BPS		
	True	Baseline ^a	SS ^b	Outside ^c
$\eta_{c,p}$	0.578	0.614	0.589	0.588
η_{c,w_1}	0	-0.013	-0.002	-0.002
η_{c,w_2}	0	0.020	0.022	0.021
$\eta_{h_1,p}$	0	0.015	0.002	0.002
η_{h_1,w_1}	0.528	0.539	0.527	0.526
η_{h_1,w_2}	0	0.002	-0.005	-0.005
$\eta_{h_2,p}$	0	-0.047	-0.050	-0.049
η_{h_2,w_1}	0	0.004	-0.011	-0.011
η_{h_2,w_2}	0.850	0.817	0.809	0.810

Notes: Only households aged 30-57 are included.

^a BPS baseline method.

^b Modified BPS method accounting for the social security system explicitly. ^c BPS method allowing unspecified outside insurance. The outside insurance coefficient estimated here is $\beta = 0.1444$.

Table 12: BPS Method in Small Samples

	Data	Model		Model (Small Sample)	
	BPS	BPS	True	BPS	True
κ_{c,u_1}	-0.14(0.07)	-0.01	0.01	-0.01(0.009)	0.01(0.002)
κ_{c,u_2}	-0.04(0.07)	0.02	0.01	0.02(0.011)	0.01(0.002)
κ_{c,v_1}	0.32(0.05)	0.42	0.35	0.42(0.018)	0.34(0.007)
κ_{c,v_2}	0.19(0.03)	0.22	0.18	0.22(0.014)	0.19(0.007)
κ_{y_1,u_1}	1.58(0.16)	1.47	1.44	1.47(0.008)	1.44(0.003)
κ_{y_1,u_2}	0.11(0.06)	-0.03	-0.05	-0.03(0.008)	-0.05(0.004)
κ_{y_1,v_1}	0.92(0.08)	1.10	1.16	1.10(0.017)	1.16(0.008)
κ_{y_1,v_2}	-0.22(0.04)	-0.20	-0.19	-0.21(0.014)	-0.19(0.008)
κ_{y_2,u_1}	0.17(0.11)	-0.10	-0.12	-0.10(0.012)	-0.12(0.006)
κ_{y_2,u_2}	1.88(0.23)	1.76	1.76	1.76(0.042)	1.75(0.009)
κ_{y_2,v_1}	-0.75(0.14)	-0.61	-0.51	-0.61(0.034)	-0.51(0.013)
κ_{y_2,v_2}	1.42(0.08)	1.53	1.46	1.52(0.026)	1.45(0.012)

Notes: The numbers inside parentheses are standard errors: for “Data BPS”, they are estimated by BPS using a data set with 10479 household-year observations; For “Model (Small Sample)”, they are computed based on 100 independent model-simulated samples, each of which has 10500 household-year observations. Only households aged 30-57 are included.

D The BPS Method in This Paper

D.1 Formulas for the Transmission Coefficients

We follow closely the approximation method proposed by Blundell, Pistaferri, and Saporta-Eksten (2016) (BPS) and try to use notations consistent with the original paper. Here we only report the formulas we use for the transmission coefficients, and the details of the derivation are available in Online Appendix 1 of BPS.

As in BPS, the wage of earner j in household i at age t is determined by

$$\ln W_{i,j,t} = Z_t^{W_j'} \beta^{W_j} + F_{i,j,t} + u_{i,j,t},$$

where $Z_t^{W_j}$ is a group of observable characteristics affecting wages such as age, and

$$F_{i,j,t} = F_{i,j,t-1} + v_{i,j,t}.$$

This implies

$$\Delta \ln W_{i,j,t} - \Delta Z_t^{W_j'} \beta^{W_j} = \Delta u_{i,j,t} + v_{i,j,t}.$$

Define $\Delta w_{i,j,t}$ as the unexpected growth of wage that is not explained by the ob-

servables, i.e.,

$$\Delta w_{i,j,t} = \Delta u_{i,j,t} + v_{i,j,t}.$$

For simplicity, we omit the household index i and write it as $\Delta w_{j,t}$. Similarly, we can define unexpected the growth of consumption, labor supply, and labor income as Δc_t , $\Delta h_{j,t}$, and $\Delta y_{j,t}$.

Assuming that the after-tax income $\tilde{T}(Y)$ is given by $(1 - \chi)Y^{1-\mu}$, BPS show that by log-linearizing the first-order conditions and the budget constraints of the two-earner household problem, we have

$$\begin{bmatrix} \Delta c_t \\ \Delta y_{1,t} \\ \Delta y_{2,t} \end{bmatrix} = \begin{bmatrix} \kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \\ \kappa_{y_1,u_1} & \kappa_{y_1,u_2} & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\ \kappa_{y_2,u_1} & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2} \end{bmatrix} \begin{bmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{bmatrix},$$

where

$$\begin{aligned} \kappa_{c,u_j} &= \psi_{c,w_j}, \\ \kappa_{c,v_j} &= \psi_{c,w_j} + \frac{\psi_{c,\lambda}[(1-\mu)(1-\pi_t)(s_{j,t} + \tilde{\psi}_{h,w_j}) - \psi_{c,w_j}]}{\psi_{c,\lambda} - (1-\mu)(1-\pi_t)\tilde{\psi}_{h,\lambda}}, \\ \kappa_{y_j,u_j} &= 1 + \psi_{h_j,w_j}, \\ \kappa_{y_j,u_{-j}} &= \psi_{h_j,w_{-j}}, \\ \kappa_{y_j,v_j} &= 1 + \psi_{h_j,w_j} + \frac{\psi_{h_j,\lambda}[(1-\mu)(1-\pi_t)(s_{j,t} + \tilde{\psi}_{h,w_j}) - \psi_{c,w_j}]}{\psi_{c,\lambda} - (1-\mu)(1-\pi_t)\tilde{\psi}_{h,\lambda}}, \\ \kappa_{y_j,v_{-j}} &= \psi_{h_j,w_{-j}} + \frac{\psi_{h_j,\lambda}[(1-\mu)(1-\pi_t)(s_{-j,t} + \tilde{\psi}_{h,w_{-j}}) - \psi_{c,w_{-j}}]}{\psi_{c,\lambda} - (1-\mu)(1-\pi_t)\tilde{\psi}_{h,\lambda}}, \end{aligned}$$

where π_t is approximately the share of asset in the total discounted wealth for the household at age t , $s_{j,t}$ is approximately the share of earner j 's discounted labor income in the total discounted labor income of the household, and

$$\begin{aligned} \tilde{\psi}_{h,\lambda} &= \sum_{j=1}^2 s_{j,t} \psi_{h_j,\lambda}, \\ \tilde{\psi}_{h,w_1} &= \sum_{j=1}^2 s_{j,t} \psi_{h_j,w_1}, \\ \tilde{\psi}_{h,w_2} &= \sum_{j=1}^2 s_{j,t} \psi_{h_j,w_2}, \end{aligned}$$

and

$$\begin{aligned}
& \begin{pmatrix} \psi_{c,\lambda} & \psi_{c,w_1} & \psi_{c,w_2} \\ \psi_{h_1,\lambda} & \psi_{h_1,w_1} & \psi_{h_1,w_2} \\ \psi_{h_2,\lambda} & \psi_{h_2,w_1} & \psi_{h_2,w_2} \end{pmatrix} \\
& = \begin{pmatrix} 1 & \mu q_{1,t-1}(\eta_{c,w_1} + \eta_{c,w_2}) & \mu q_{2,t-1}(\eta_{c,w_1} + \eta_{c,w_2}) \\ 0 & 1 + \mu q_{1,t-1}(\eta_{h_1,w_1} + \eta_{h_1,w_2}) & \mu q_{2,t-1}(\eta_{h_1,w_1} + \eta_{h_1,w_2}) \\ 0 & \mu q_{1,t-1}(\eta_{h_2,w_1} + \eta_{h_2,w_2}) & 1 + \mu q_{2,t-1}(\eta_{h_2,w_1} + \eta_{h_2,w_2}) \end{pmatrix}^{-1} \\
& \times \begin{pmatrix} -\eta_{c,p} + \eta_{c,w_1} + \eta_{c,w_2} & \eta_{c,w_1} - \mu q_{1,t-1}(\eta_{c,w_1} + \eta_{c,w_2}) & \eta_{c,w_2} - \mu q_{2,t-1}(\eta_{c,w_1} + \eta_{c,w_2}) \\ \eta_{h_1,p} + \eta_{h_1,w_1} + \eta_{h_1,w_2} & \eta_{h_1,w_1} - \mu q_{1,t-1}(\eta_{h_1,w_1} + \eta_{h_1,w_2}) & \eta_{h_1,w_2} - \mu q_{2,t-1}(\eta_{h_1,w_1} + \eta_{h_1,w_2}) \\ \eta_{h_2,p} + \eta_{h_2,w_1} + \eta_{h_2,w_2} & \eta_{h_2,w_1} - \mu q_{1,t-1}(\eta_{h_2,w_1} + \eta_{h_2,w_2}) & \eta_{h_2,w_2} - \mu q_{2,t-1}(\eta_{h_2,w_1} + \eta_{h_2,w_2}) \end{pmatrix}
\end{aligned}$$

where $q_{j,t-1} = Y_{j,t-1}/Y_{t-1}$ is the share of labor income from the earner j at age $t-1$.

The formulas when the separable preference assumption is imposed can be obtained by assuming the values of all the cross Frisch elasticities to be zero. To estimate the outside insurance coefficient β , one only needs to multiply all the $(1 - \pi_t)$ in the formulas by $(1 - \beta)$.

When taking into account the social security system explicitly, we define the human wealth as the sum of the discounted after-payroll-tax labor income and the discounted retirement benefits, and multiply all the $(1 - \pi_t)$ in the formulas for κ by one minus the share of retirement benefits in human wealth.

D.2 Estimation

The estimation method in this paper follows the empirical strategy in the original BPS paper. To apply the method, we first need the data on the unexpected wage growth $\Delta w_{j,t}$, unexpected consumption growth Δc_t , and unexpected labor income growth $\Delta y_{j,t}$ at household level. These can be obtained by regressing the log-differences of the corresponding variables on observable characteristics and constructing the residuals, i.e.,

$$\Delta \log X_t = Z\hat{\beta} + \Delta x_t,$$

where Z represents the observable characteristics, and $\hat{\beta}$ is the estimated coefficients. For the simulated data, because all the households are ex ante identical, Z contains only a group of age dummies.

D.2.1 Wage Covariances

From the wage process, we know

$$\Delta w_{j,t} = \Delta u_{j,t} + v_{j,t}.$$

Following BPS, we estimate the six wage process parameters, $\sigma_{u_1}^2$, $\sigma_{u_2}^2$, σ_{u_1, u_2} , $\sigma_{v_1}^2$, $\sigma_{v_2}^2$, and σ_{v_1, v_2} , by GMM with an identity weighting matrix using 7 moment conditions: $E[\Delta w_{1,t}^2]$, $E[\Delta w_{2,t}^2]$, $E[\Delta w_{1,t} \Delta w_{2,t}]$, $E[\Delta w_{1,t} \Delta w_{1,t-1}]$, $E[\Delta w_{2,t} \Delta w_{2,t-1}]$, $E[\Delta w_{1,t} \Delta w_{2,t-1}]$, and $E[\Delta w_{2,t} \Delta w_{1,t-1}]$. This step requires only wage data.

D.2.2 Smoothing Parameters

The smoothing parameters π_t and $s_{j,t}$ are calculated directly from the data. The human wealth of household at age t is calculated as

$$Human\ Wealth_t = (1 - \chi)Y_t^{1-\mu} + E_t \sum_{k=1}^{R-t} \frac{(1 - \chi)Y_{t+k}^{1-\mu}}{(1 + r)^k}.$$

Note that the expected future income levels should technically depend on the current states of the households. However, in practice, it is difficult to calculate such conditional expectations exactly, so following BPS, the expected future income levels are only conditional on characteristics that either do not change over time (e.g. education) or change in a perfectly forecastable way (e.g. age).

The smoothing parameter π_t is then

$$\pi_t = \frac{Assets_t}{Assets_t + Human\ Wealth_t}.$$

And $s_{j,t}$ is simply

$$s_{j,t} = \frac{Human\ Wealth_{j,t}}{\sum_{j=1}^2 Human\ Wealth_{j,t}}.$$

To be exact, the human wealth here should be the discounted after-tax labor income of each member, but with non-linear income tax at the household level, it is unclear how to divide the tax between the two members. Therefore, when calculating $s_{j,t}$, we use before-tax labor income of each member.

D.2.3 Frisch Elasticities and Outside Insurance

For the Frisch elasticities and the ‘‘outside insurance’’ coefficient, we follow BPS and use the 31 moment conditions in Figure 8 of the original BPS paper (and Table 1 of the BPS online Appendix) to conduct a GMM estimation with an identity weighting matrix. The moment conditions include a set of second-order moments

of Δc_t , $\Delta y_{j,t}$, $\Delta w_{j,t}$, and the lag of them. The formulas for these moment conditions are derived based on the BPS formulas for Δc_t , $\Delta y_{j,t}$ and $\Delta w_{j,t}$. For example,

$$\begin{aligned}
E(\Delta c_t^2) &= E[(\kappa_{c,u_1} \Delta u_{1,t} + \kappa_{c,u_2} \Delta u_{2,t} + \kappa_{c,v_1} v_{1,t} + \kappa_{c,v_2} v_{2,t})^2] \\
&= E[\kappa_{c,u_1}^2 (2\sigma_{u_1}^2) + \kappa_{c,u_2}^2 (2\sigma_{u_2}^2) + 2(\kappa_{c,u_1} \kappa_{c,u_2}) (2\sigma_{u_1 u_2}) \\
&\quad + \kappa_{c,v_1}^2 (\sigma_{v_1}^2) + \kappa_{c,v_2}^2 E(\sigma_{v_2}^2) + 2(\kappa_{c,v_1} \kappa_{c,v_2}) (\sigma_{v_1 v_2})] \\
&= p \lim \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \{ \kappa_{c,u_1}^2 (i, t) (2\sigma_{u_1}^2) + \kappa_{c,u_2}^2 (i, t) (2\sigma_{u_2}^2) + 2[\kappa_{c,u_1} (i, t) \kappa_{c,u_2} (i, t)] (2\sigma_{u_1 u_2}) \\
&\quad + \kappa_{c,v_1}^2 (i, t) (\sigma_{v_1}^2) + \kappa_{c,v_2}^2 (i, t) (\sigma_{v_2}^2) + 2[\kappa_{c,v_1} (i, t) \kappa_{c,v_2} (i, t)] (\sigma_{v_1 v_2}) \}.
\end{aligned}$$

The results for other moment conditions can be derived in a similar way. We also impose the symmetry assumptions $\eta_{h_j, p} = -\eta_{c, w_j} \frac{1}{(1-\chi)(1-\mu)Y^{-\mu}} \frac{p c}{w_j h_j}$, $j = 1, 2$, and $\eta_{h_2, w_1} = \eta_{h_1, w_2} \frac{w_1 h_1}{w_2 h_2}$ as the original BPS paper.

D.2.4 Transmission Coefficients

Collecting the estimation results from previous steps, the transmission coefficients for each household at each age are calculated using the formulas in Appendix D.1. The reported transmission coefficients are the sample averages of them.

E Supplementary Results For Non-separable Preferences

This section reports the calibration of the model with non-separable preferences and presents all the figures and tables for the model with non-separable preferences that are not included in the main text, each of which corresponds to one figure or table for the benchmark model with additively separable preferences.

E.1 Parameters for Non-separable Preferences

With non-separable preferences, Frisch elasticities are no longer deep parameters. Given the functional form in this paper, we are able to derive the Frisch elasticities as functions of preference parameters and allocations. The formulas are shown in Appendix F. The preference parameters are then calibrated jointly to match the Frisch elasticities estimated by BPS in the absence of the separability assumption, and average hours worked by males and females in the BPS data. The parameters γ , θ and σ mainly affect the Frisch elasticities, whereas the parameters

α and ξ mainly affect labor supply. In particular, for a given set of parameters, the model is first solved and a panel of household data are simulated. Then, based on the simulated data and the formulas for Frisch elasticities, the sample averages of Frisch elasticities and labor supply are calculated and compared with the calibration targets.³

For the non-separable preferences, we add an additional parameter Ψ , which is a constant that scales the marginal utility of consumption after retirement. Ψ is chosen such that the age profile of consumption is smooth at retirement. Consumption typically falls upon retirement in the data, but the literature on the retirement consumption puzzle shows that it is mainly due to work-related consumption expenditures not modeled here.⁴

The values of calibrated parameters of the model with non-separable preferences and the implied Frisch elasticities are summarized in Tables 13 and 14. The moments matched in calibration and their values from data are in Table 15.

³Note that, given the functional form of the utility function, there are not enough degrees of freedom to match all Frisch elasticities perfectly, and we focus on matching the upper triangular part of the matrix of Frisch elasticities, in particular, $\eta_{c,p}$, η_{c,w_1} , η_{c,w_2} , η_{h_1,w_1} , η_{h_1,w_2} and η_{h_2,w_2} . In fact, there might be no utility function that can match exactly all Frisch elasticities estimated by BPS, due to the theoretical restrictions between Frisch elasticities imposed by their definitions. In the calibration of the non-separable utility function we then need to choose between a good fit of own-price elasticities ($\eta_{c,p}$, η_{h_1,w_1} and η_{h_2,w_2}) and cross-elasticities (η_{c,w_1} , η_{c,w_2} and η_{h_1,w_2}). The chosen parameterization is a compromise between both.

⁴See Hurst (2008) for a survey of this literature.

Table 13: Calibrated Model Parameters
(Non-separable Preferences)

Parameter	Governing	Value
<i>A. Preferences</i>		
δ	discount rate of utility	1.006×10^{-2}
α	weight of consumption	0.124
ξ	weight of male labor supply	0.413
σ	consumption Frisch elasticity	2.24
γ	substitution between consumption and leisure	-3.00
θ	substitution between male and female labor supply	3.00
Ψ	consumption level after retirement	0.695
f	fixed utility cost of female participation	0.0155
<i>B. Wage Process</i>		
$e^{g_{2,t} - g_{1,t}}$	female-male wage trend ratio	0.499
<i>C. Financial Market</i>		
\underline{A}	borrowing constraints	-0.124

Notes: This table reports the values of parameters in the model with non-separable preferences that are different from the benchmark model with additively separable preferences. Parameters not included in this table share the same values as in the benchmark model.

Table 14: Calibrated Frisch Elasticities
(Non-separable Preferences)

	Data BPS	Model True
$\eta_{c,p}$	0.417(0.122)	0.413
η_{c,w_1}	-0.162(0.074)	-0.220
η_{c,w_2}	-0.050(0.077)	-0.094
$\eta_{h_1,p}$	0.126(0.057)	0.326
η_{h_1,w_1}	0.681(0.189)	0.940
η_{h_1,w_2}	0.159(0.071)	0.188
$\eta_{h_2,p}$	0.079(0.121)	0.326
η_{h_2,w_1}	0.325(0.140)	0.440
η_{h_2,w_2}	0.958(0.267)	0.688

Notes: The numbers inside parentheses are standard errors from BPS. Only households aged 30-57 are included.

Table 15: Empirical Targets Matched
(Non-separable Preferences)

Empirical Targets	Data	Model
average male labor income	1	1.000
average female labor income work	0.491	0.490
female-male ratio of average labor supply work	0.733	0.733
average female non-participation rate	0.20	0.200
average net worth	4.188	4.190
median debt-to-income ratio debt (age 21-30)	0.163	0.163

Notes: This table reports the empirical moments matched by the model with non-separable preferences. Moments are for age 30-57 households unless specified otherwise.

E.2 Life Cycle Profiles

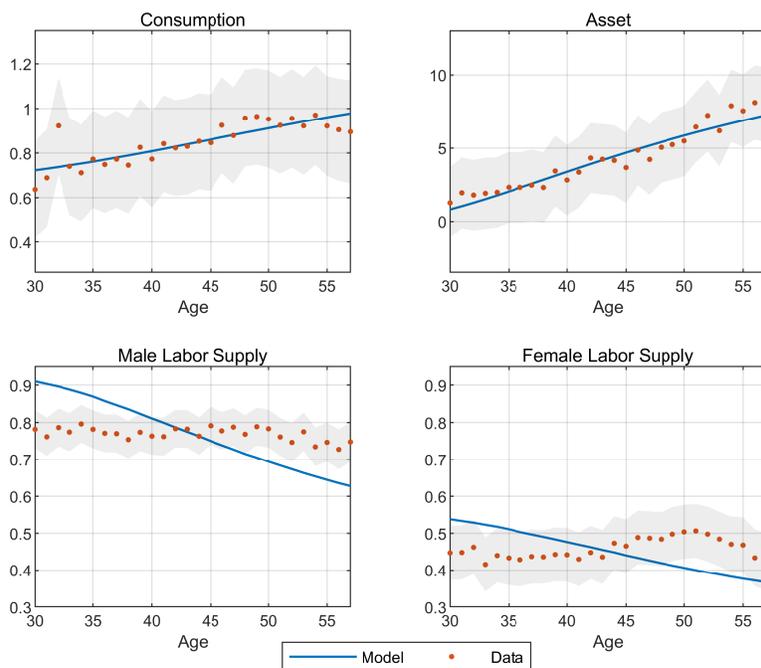


Figure 11: Life Cycles of Cross-sectional Means
(Non-separable Preferences)

Notes: This figure shows the life cycles of cross-sectional means in the model with non-separable preferences (solid lines) and in the data (dotted lines) together with the 95% confidence interval (grey bands). The data are from the BPS data set including age 30-57 households. The consumption life cycle from the data is scaled up to match the life-cycle average of consumption in the model. (Note that due to the normalization choice, the unit of labor supply here is different from that in the benchmark model.)

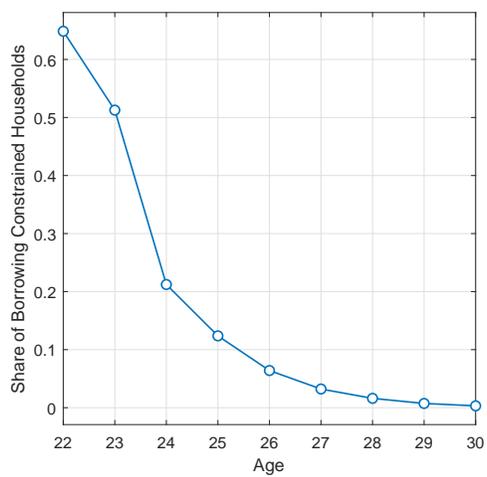


Figure 12: Share of Borrowing Constrained Households
(Non-separable Preferences)

Notes: This figure plots the share of young households on the borrowing constraints in the model with non-separable preferences.

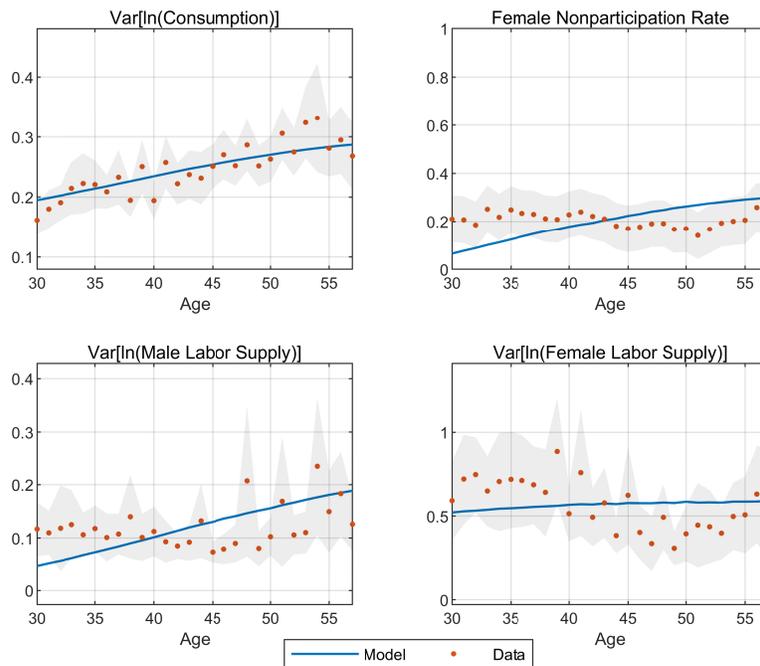


Figure 13: Life Cycles of Cross-sectional Variances and Female Non-participation (Non-separable Preferences)

Notes: This figure shows the life cycles of cross-sectional variances and female non-participation rate in the model with non-separable preferences (solid lines) and in the data (dotted lines) together with the 95% confidence interval (grey bands). The data are from the BPS data set including age 30-57 households. The life cycles of variances in the model are shifted to match the life-cycle averages of variances in the data.

E.3 Age Profiles of Consumption Insurance

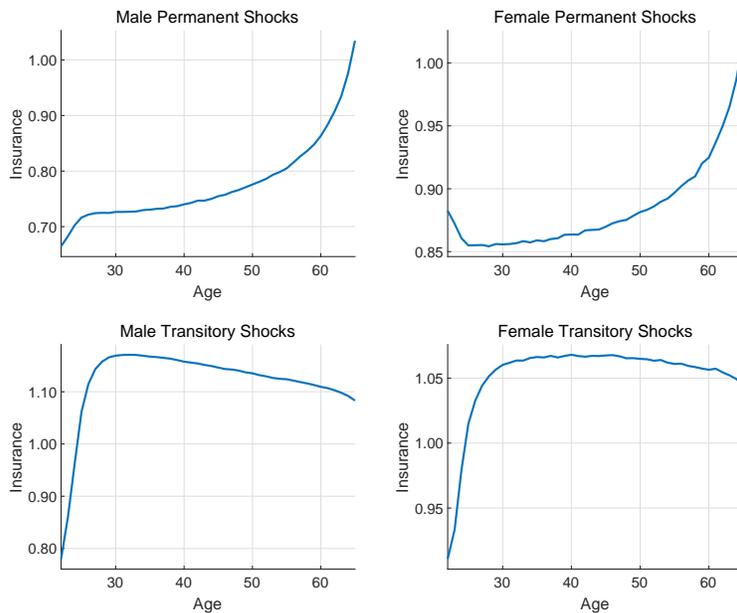


Figure 14: Age Profiles of Consumption Insurance
(Non-separable Preferences)

Notes: This figure plots consumption insurance over the life cycle against male (left) and female (right) permanent (top) and transitory (bottom) wage shocks in the model with non-separable preferences.

E.4 Performance of the BPS Method

Table 16: Estimation of Transmission Coefficients
(Non-separable Preferences)

	Model	Model BPS		
	True	Baseline ^a	SS ^b	Outside ^c
κ_{c,u_1}	-0.15	-0.16	-0.16	-0.15
κ_{c,u_2}	-0.07	-0.07	-0.07	-0.07
κ_{c,v_1}	0.24	0.32	0.29	0.27
κ_{c,v_2}	0.13	0.14	0.12	0.11
κ_{y_1,u_1}	1.70	1.73	1.72	1.72
κ_{y_1,u_2}	0.09	0.10	0.10	0.10
κ_{y_1,v_1}	0.98	0.91	0.94	0.95
κ_{y_1,v_2}	-0.27	-0.26	-0.25	-0.25
κ_{y_2,u_1}	0.18	0.19	0.19	0.19
κ_{y_2,u_2}	1.62	1.59	1.59	1.59
κ_{y_2,v_1}	-0.47	-0.57	-0.55	-0.55
κ_{y_2,v_2}	1.16	1.26	1.26	1.27

Notes: Only households aged 30-57 are included.

^a BPS baseline method.

^b Modified BPS method accounting for the social security system explicitly. ^c BPS method allowing unspecified outside insurance. The outside insurance coefficient estimated here is $\beta = 0.2217$.

Table 17: Estimation of Frisch Elasticities
(Non-separable Preferences)

	Model	Model BPS		
	True	Baseline ^a	SS ^b	Outside ^a
$\eta_{c,p}$	0.413	0.431	0.399	0.376
η_{c,w_1}	-0.220	-0.213	-0.206	-0.202
η_{c,w_2}	-0.094	-0.091	-0.093	-0.094
$\eta_{h_1,p}$	0.326	0.254	0.246	0.242
η_{h_1,w_1}	0.940	0.914	0.905	0.901
η_{h_1,w_2}	0.188	0.177	0.175	0.174
$\eta_{h_2,p}$	0.326	0.221	0.227	0.230
η_{h_2,w_1}	0.440	0.362	0.357	0.356
η_{h_2,w_2}	0.688	0.663	0.663	0.665

Notes: Only households aged 30-57 are included.

^a BPS baseline method.

^b Modified BPS method accounting for the social security system explicitly. ^c BPS method allowing unspecified outside insurance. The outside insurance coefficient estimated here is $\beta = 0.2217$.

Table 18: BPS Method in Small Samples
(Non-separable Preferences)

	Data	Model		Model (Small Sample)	
	BPS	BPS	True	BPS	True
κ_{c,u_1}	-0.14(0.07)	-0.16	-0.15	-0.16(0.009)	-0.15(0.003)
κ_{c,u_2}	-0.04(0.07)	-0.07	-0.07	-0.07(0.010)	-0.07(0.003)
κ_{c,v_1}	0.32(0.05)	0.32	0.24	0.32(0.012)	0.24(0.004)
κ_{c,v_2}	0.19(0.03)	0.14	0.13	0.14(0.008)	0.13(0.004)
κ_{y_1,u_1}	1.58(0.16)	1.73	1.70	1.73(0.014)	1.70(0.005)
κ_{y_1,u_2}	0.11(0.06)	0.10	0.09	0.10(0.011)	0.09(0.005)
κ_{y_1,v_1}	0.92(0.08)	0.91	0.98	0.91(0.018)	0.98(0.009)
κ_{y_1,v_2}	-0.22(0.04)	-0.26	-0.27	-0.26(0.018)	-0.27(0.010)
κ_{y_2,u_1}	0.17(0.11)	0.19	0.18	0.19(0.016)	0.18(0.006)
κ_{y_2,u_2}	1.88(0.23)	1.59	1.62	1.59(0.032)	1.61(0.009)
κ_{y_2,v_1}	-0.75(0.14)	-0.57	-0.47	-0.57(0.025)	-0.47(0.010)
κ_{y_2,v_2}	1.42(0.08)	1.26	1.16	1.26(0.019)	1.16(0.009)

Notes: The numbers inside parentheses are standard errors: for “Data BPS”, they are estimated by BPS using a data set with 10479 household-year observations; For “Model (Small Sample)”, they are computed based on 100 independent model-simulated samples, each of which has 10500 household-year observations. Only households aged 30-57 are included.

E.5 Consumption Insurance Decomposition

Table 19: Consumption Insurance Decomposition (Male Shocks)
(Non-separable Preferences)

Economy	Insurance Provided by						Total Insurance
	Male Earner	Female Earner			Income Tax	Savings+ Social Security	
		Composition	Extensive	Intensive			
<i>A. Permanent Shock</i>							
1-Earner, exogenous income	–	–	–	–	13.3%	34.7%	48.0%
+ male intensive margin	18.5%	–	–	–	10.8%	39.4%	68.8%
+ female exogenous income	8.9%	27.7%	–	–	8.4%	28.0%	72.9%
+ female extensive margin	6.5%	22.9%	7.2%	–	8.4%	29.0%	74.1%
+ female intensive margin	–0.05%	29.6%	2.6%	13.9%	7.2%	23.9%	77.0%
<i>B. Transitory Shock</i>							
1-Earner, exogenous income	–	–	–	–	13.3%	84.5%	97.7%
+ male intensive margin	–74.2%	–	–	–	23.1%	169.7%	118.6%
+ female exogenous income	–62.8%	43.9%	–	–	15.8%	113.9%	110.8%
+ female extensive margin	–67.1%	41.0%	–2.0%	–	17.0%	124.5%	113.3%
+ female intensive margin	–65.6%	48.9%	–0.5%	–3.8%	16.0%	117.1%	112.3%

Notes: This table reports the decomposition results of consumption insurance against male permanent and transitory wage shocks in a sequence of economies with different sets of insurance channels available. Households aged 21-65 are included. Total Insurance = $\sum_m \text{Insurance}(m)$. Details about the decomposition method are in Appendix A

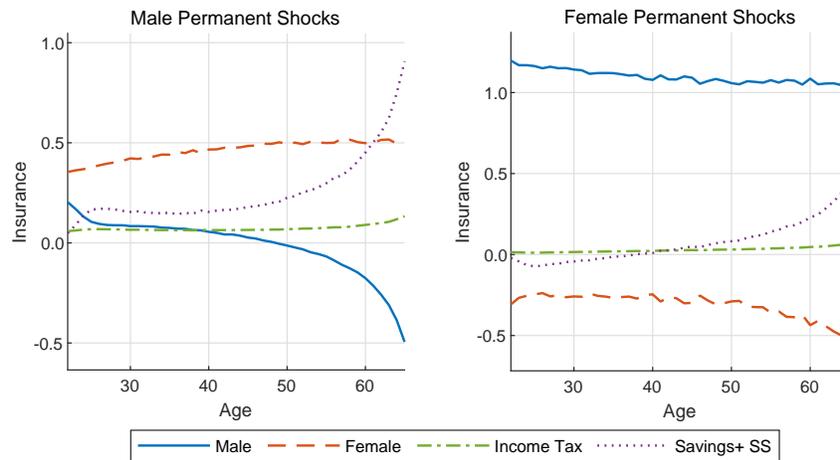


Figure 15: Age Profiles of Consumption Insurance Decomposition
(Non-separable Preferences)

Notes: This figure plots consumption insurance by source over the life cycle against male (left panel) and female (right panel) permanent wage shocks in the model with non-separable preferences. The sources are the male earner (solid line), the female earner (dash line), progressive income tax (dash-dot line), and savings plus social security (dotted line).

F Frisch Elasticities for Non-separable Preferences

In this section, we derive the formulas for the Frisch elasticities with the non-separable preferences. They are generally functions of household allocations, i.e., they are not deep parameters. So we use the sample averages of them as the approximated true values of the Frisch elasticities, i.e., the “Model True” results.

The utility function for the non-separable preferences is

$$u(C, H_1, H_2) = \frac{\{\alpha C^\gamma + (1 - \alpha)[\xi H_1^\theta + (1 - \xi)H_2^\theta]^{-\frac{\gamma}{\theta}}\}^{\frac{1-\sigma}{\gamma}} - 1}{1 - \sigma}.$$

The intertemporal budget constraint is

$$PC + PA' = P(1 + r)A + W_1H_1 + W_2H_2,$$

where P , W_1 , and W_2 are the price of the consumption good and the wages for male and female earners. From the recursive formulation of the household problem, the first-order conditions are

$$\begin{aligned} u_C &= \Delta^{\frac{1-\sigma}{\gamma}-1} \alpha C^{\gamma-1} = \lambda P, \\ u_{H_1} &= -\Delta^{\frac{1-\sigma}{\gamma}-1} (1 - \alpha) \Gamma^{-\frac{\gamma}{\theta}-1} \xi H_1^{\theta-1} = -\lambda W_1, \\ u_{H_2} &= -\Delta^{\frac{1-\sigma}{\gamma}-1} (1 - \alpha) \Gamma^{-\frac{\gamma}{\theta}-1} (1 - \xi) H_2^{\theta-1} = -\lambda W_2, \end{aligned}$$

where λ is the lagrangian multiplier on the budget constraint, $\Delta \equiv \alpha C^\gamma + (1 - \alpha)[\xi H_1^\theta + (1 - \xi)H_2^\theta]^{-\frac{\gamma}{\theta}}$, and $\Gamma \equiv \xi H_1^\theta + (1 - \xi)H_2^\theta$. Taking log difference for both sides of these equations, we get

$$\begin{aligned} \left(\frac{1-\sigma}{\gamma} - 1\right) d \ln \Delta + (\gamma - 1) d \ln C &= d \ln \lambda + d \ln P, \\ \left(\frac{1-\sigma}{\gamma} - 1\right) d \ln \Delta + \left(-\frac{\gamma}{\theta} - 1\right) d \ln \Gamma + (\theta - 1) d \ln H_1 &= d \ln \lambda + d \ln W_1, \\ \left(\frac{1-\sigma}{\gamma} - 1\right) d \ln \Delta + \left(-\frac{\gamma}{\theta} - 1\right) d \ln \Gamma + (\theta - 1) d \ln H_2 &= d \ln \lambda + d \ln W_2, \end{aligned}$$

and

$$\begin{aligned} d \ln \Gamma &= \theta \mathbb{B} d \ln H_1 + \theta(1 - \mathbb{B}) d \ln H_2, \\ d \ln \Delta &= \gamma \mathbb{A} d \ln C - \frac{\gamma}{\theta} (1 - \mathbb{A}) d \ln \Gamma \\ &= \gamma \mathbb{A} d \ln C - \gamma(1 - \mathbb{A}) \mathbb{B} d \ln H_1 - \gamma(1 - \mathbb{A})(1 - \mathbb{B}) d \ln H_2, \end{aligned}$$

where $\mathbb{A} \equiv \frac{\alpha C^\gamma}{\Delta}$ and $\mathbb{B} \equiv \frac{\xi H_1^\theta}{\Gamma}$. Substitute $d \ln \Delta$ and $d \ln \Gamma$ with the formulas above,

the system of equations becomes

$$G \times \begin{bmatrix} d \ln C \\ d \ln H_1 \\ d \ln H_2 \end{bmatrix} = \begin{bmatrix} d \ln \lambda + d \ln P \\ d \ln \lambda + d \ln W_1 \\ d \ln \lambda + d \ln W_2 \end{bmatrix},$$

where

$$G = \begin{bmatrix} (\gamma - 1)(1 - \mathbb{A}) - \sigma \mathbb{A} & (\gamma - 1 + \sigma)(1 - \mathbb{A})\mathbb{B} & (\gamma - 1 + \sigma)(1 - \mathbb{A})(1 - \mathbb{B}) \\ (1 - \gamma - \sigma)\mathbb{A} & [(\gamma - 1 + \sigma)(1 - \mathbb{A}) - (\gamma + \theta)]\mathbb{B} + (\theta - 1) & [(\gamma - 1 + \sigma)(1 - \mathbb{A}) - (\gamma + \theta)](1 - \mathbb{B}) \\ (1 - \gamma - \sigma)\mathbb{A} & [(\gamma - 1 + \sigma)(1 - \mathbb{A}) - (\gamma + \theta)]\mathbb{B} & [(\gamma - 1 + \sigma)(1 - \mathbb{A}) - (\gamma + \theta)](1 - \mathbb{B}) + (\theta - 1) \end{bmatrix}.$$

By the definition of Frisch elasticities, we have

$$G^{-1} = \begin{bmatrix} -\eta_{c,p} & \eta_{c,w_1} & \eta_{c,w_2} \\ \eta_{h_1,p} & \eta_{h_1,w_1} & \eta_{h_1,w_2} \\ \eta_{h_2,p} & \eta_{h_2,w_1} & \eta_{h_2,w_2} \end{bmatrix}.$$

Note that because the values of \mathbb{A} and \mathbb{B} depend on the allocations chosen by households, G and the Frisch elasticities all depend on the allocations and are not deep parameters.

If we want the Frisch elasticities to be deep parameters with such utility function, we must have \mathbb{A} and \mathbb{B} as constants. From the FOC's, this would require

$$\begin{aligned} \Delta^{\frac{1-\sigma}{\gamma}} \mathbb{A} &= \lambda PC, \\ \Delta^{\frac{1-\sigma}{\gamma}} (1 - \mathbb{A})\mathbb{B} &= \lambda W_1 H_1, \\ \Delta^{\frac{1-\sigma}{\gamma}} (1 - \mathbb{A})(1 - \mathbb{B}) &= \lambda W_2 H_2. \end{aligned}$$

\Rightarrow

$$\begin{aligned} \frac{\mathbb{A}}{(1 - \mathbb{A})\mathbb{B}} &= \frac{PC}{W_1 H_1} = \text{Constant}, \\ \frac{\mathbb{A}}{(1 - \mathbb{A})(1 - \mathbb{B})} &= \frac{PC}{W_2 H_2} = \text{Constant}. \end{aligned}$$

This implies the utility function needs to take the Cobb-Douglas form, i.e., $\gamma = 0$ and $\theta = 0$. In that case, the utility function becomes

$$U(C, H_1, H_2) = \frac{[C^\alpha (H_1^\xi H_2^{1-\xi})^{-(1-\alpha)}]^{1-\sigma} - 1}{1 - \sigma}.$$

Follow the same method, we can derive that

$$G = \begin{bmatrix} \alpha - 1 - \alpha\sigma & (1 - \sigma)(\alpha - 1)\xi & (1 - \sigma)(\alpha - 1)(1 - \xi) \\ (1 - \sigma)\alpha & -\sigma(\alpha - 1)\xi + (\alpha - 2)\xi + (\xi - 1) & -\sigma(\alpha - 1)(1 - \xi) + (\alpha - 2)(1 - \xi) + (1 - \xi) \\ (1 - \sigma)\alpha & -\sigma(\alpha - 1)\xi + (\alpha - 2)\xi + \xi & -\sigma(\alpha - 1)(1 - \xi) + (\alpha - 2)(1 - \xi) - \xi \end{bmatrix},$$

and the Frisch elasticity matrix is just G^{-1} . However, the Cobb-Douglas form is not

a good choice because it implies that the ratios between male labor income, female labor income and consumption expenditures are all constants independent of the price of consumption and wages, which is counterfactual.

G Computation Method

The household optimization problem is solved backwards using the endogenous grid method proposed by Carroll (2006). With the extensive margin of female labor supply, for each iteration and each household state, the optimization problem is solved twice under two alternative scenarios: the current period female labor supply is strictly positive or zero. The final optimal policy is obtained by comparing the discounted utility achieved in these two scenarios.

The grid for asset has 100 grid points, and the distance between two adjacent grid points increases with the asset level such that the grid points are denser around the low asset levels where borrowing constraints are more likely to bind. The range of the asset grid is age-dependent and eventually endogenously determined by the model to have a better coverage of the more relevant state space.

The joint process of the two earners' permanent wage components is approximated by a discrete Markov process with age-dependent sets of states and transition matrices, and each state corresponds to one possible realization of the two permanent components. The number of states is fixed, but the values of them vary across ages to match the unconditional dispersion of the joint distribution over the life cycle. The grid points and transition matrices are constructed in the same spirit as Tauchen (1986), and try to mimic the joint unit-root process. The grid for the two permanent components has 11 points in each dimension, so there are in total 121 grid points at each age. The discretization of the transitory components is similar and simpler. Since the transitory components are *iid* across ages, the grid no longer needs to be age-dependent, and no transition matrix is required. The grid for the transitory components has 5 points in each dimension, so there are in total 25 grid points.

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