

Optimal Inflation Target in an Economy with Menu Costs and a Zero Lower Bound

By ANDRÉS BLANCO*

I study the optimal inflation target in a quantitative menu cost model with a zero lower bound on interest rates. I find that the optimal inflation target is 3.5%, which is higher than in models commonly used for monetary policy analysis. Key to this result is that inflation has a small effect on resource misallocation when the model features firm-level shocks, which are necessary to match the empirical distribution of price changes. A higher inflation target decreases price flexibility at the zero lower bound, and through this mechanism, it reduces the severity of recessions when the monetary authority is constrained.

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Since the end of the 1980s, many countries have adopted a policy of inflation targeting whereby the central bank explicitly aims for a particular medium-term inflation rate. It is therefore crucial for monetary policy to address this question: What inflation rate should central banks target?

Following the Great Recession, some economists have argued that increasing the inflation target from the current level of 2%—a common practice across central banks—may be beneficial in the presence of a zero lower bound (ZLB) on nominal rates.¹ According to these economists, a higher inflation target gives central bankers more room to react to adverse macroeconomic shocks since it raises average nominal interest rates. Increasing the inflation target, however, is costly. In sticky-price models, inflation leads to inefficient dispersion of relative prices. Hence, productivity losses stem from the dispersion in firms' marginal product. Intuitively, a higher inflation target increases the gap between recently-adjusted prices and those that have not adjusted in a while.

This paper quantifies these trade-offs. Given that price dispersion is the main cost of inflation, I quantify this cost with a menu cost (MC from hereon) model that features idiosyncratic cost shocks. This pricing model reproduces micro-level pricing behavior. I incorporate this pricing model into a medium-scale dynamic

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¹For an early reference before the Great Recession see Phelps (1972) and Summers (1991). See also Blanchard, DellAriccia and Mauro (2010) and Ball (2013).

stochastic general equilibrium (DSGE) model with a Taylor rule subject to a ZLB constraint. The rich general equilibrium framework allows my model to reproduce U.S. business cycles and to quantify the main benefit of increasing the inflation target concerning business cycle stabilization.

My main result is that the optimal inflation target is 3.5%, two times larger than in an otherwise identical model with Calvo pricing. This quantitative result arises from the fact that in the MC model, the cost of inflation—given by inefficient price dispersion—does not vary much for low levels of inflation. Therefore, there is a substantial leeway to increase the inflation target up to the level at which the ZLB does not generate inefficient volatility of consumption and labor.

In my model, the cost of inflation is low because of the interaction between idiosyncratic shocks and menu costs. To demonstrate the significance of this interaction, I consider an alternative specification of my model where idiosyncratic shocks are not present, and firms only respond to inflation. If firms respond only to inflation, higher inflation increases the width of the adjustment triggers. Under my benchmark specification with large idiosyncratic shocks, changes in inflation do not affect the width of the adjustment triggers at low levels of inflation. Intuitively, in the latter economy, firms respond mainly to idiosyncratic shocks. A constant width of the adjustment triggers is the first of two mechanisms that imply a low cost of inflation from inefficient price dispersion.

Inflation also affects the law of motion of relative prices between price adjustments and, as a result, price dispersion within the adjustment triggers. Importantly, the level of inflation relative to the volatility of idiosyncratic shocks determines this relation. Therefore, when the volatility of idiosyncratic shocks is significant, inflation has little effect on the dispersion of relative prices through this mechanism. When I estimate my model on micro pricing behavior, I find high volatility of idiosyncratic shocks and, thus, a low effect of inflation on price dispersion through this mechanism. If there are no idiosyncratic shocks, then a MC model has higher inflation costs than an analogous Calvo model for the relevant range of optimal inflation targets.

Alternative pricing models deliver different levels of aggregate price flexibility. Therefore, they affect not only the cost of inflation, but also, they produce alternative results regarding the beneficial role of higher inflation for business cycle stabilization. For this reason, I extend an off-the-shelf MC model to deliver similar macroeconomic dynamics to the main workhorse monetary model: the New Keynesian model with Calvo pricing.

I enrich a MC model with random free price change opportunities and fat-tailed idiosyncratic shocks. As Gertler and Leahy (2008), Midrigan (2011) and Álvarez, Le Bihan and Lippi (2016) have shown, a MC model with these extensions delivers a similar slope of the Phillips curve as the Calvo pricing model under two conditions. First, the frequency and the distribution of price changes do not respond to aggregate shocks. Second, the proportion of price changes due to free adjustments and fat-tailed idiosyncratic shocks is large enough. Intuitively, if

firms adjust prices due to random price change opportunities or large idiosyncratic shocks, then price adjustment probabilities across firms and the inflation dynamics in my MC model are similar to a Calvo pricing model.

When I estimate the model to match the dispersion of price changes observed in the data, these prerequisites hold whenever the ZLB is not sufficiently binding. Indeed, at the optimal inflation target of 3.5%, my MC model replicates the same business cycle properties as the Calvo model, but with a low cost of inflation. Therefore, my MC model does not change the beneficial role of higher inflation for business cycle stabilization near the optimal inflation target.

INTERACTION BETWEEN ENDOGENOUS PRICE FLEXIBILITY AND THE INFLATION TARGET. — Price adjustment becomes responsive to aggregate shocks when the ZLB is sufficiently binding (i.e. the inflation target is less than 2%). This property leads to the following question: How does the inflation target affect this endogenous price flexibility?

The role of the inflation target in mitigating recessions at the ZLB depends on both the inflation dynamics *within* the ZLB episodes and the inflation dynamics *before* the ZLB episodes. To understand the first mechanism, note that the economy tends to be depressed at the ZLB with output gap and inflation below trend. At *low* inflation targets, inflation below trend often leads to deflation; this negative drift pushes firms toward their downward price adjustment trigger. Thus, a broader set of firms will make sizable downward price changes. Since the central bank is constrained, the output gap drops, putting more downward pressure on prices. This “domino” effect amplifies the business cycle cost of the ZLB at low inflation targets. At *high* inflation targets, inflation below the trend is still inflation. Thus, firms only lower their prices in response to idiosyncratic shocks (i.e., fat-tailed cost or free adjustment opportunity shocks) and not aggregate shocks. Therefore the level of the inflation target is a crucial determinant of price flexibility within ZLB episodes.

To understand the second mechanism, notice that inflation depends on reset prices and the current distribution of relative prices. Intuitively, if ZLB episodes arise with a large mass of firms close to the downward price adjustment trigger, then these firms decrease their prices. These adjustments make ZLB episodes more costly, with lower output and inflation. In the opposite direction, if a significant number of firms has relative prices close to the upward price adjustment trigger, then the frequency of price cuts does not increase. Thus, ZLB episodes are less costly. Because the inflation target determines the average inflation before ZLB episodes, a higher inflation target can shape the distribution of relative prices to mitigate the decline of inflation during ZLB episodes.

In summary, a higher inflation target would raise average nominal interest rates, thus relaxing the ZLB constraint. Additionally, it also mitigates the severity of recessions at the ZLB by affecting the distribution of adjusting relative prices

when the monetary authority is constrained.²

ROBUSTNESS OF THE OPTIMAL INFLATION TARGET. — I analyze the robustness of the optimal inflation target under different values of critical parameters. The optimal inflation ranges from 2.5% to 4% in all the exercises, and it is always 2–3 times higher than in the Calvo model. In all of these exercises, the following result emerges: The optimal inflation target in the MC model is whenever the incidence of the ZLB in equilibrium allocation is almost null.

CHALLENGES FOR SOLVING THE MODEL. — There are two challenges to solving, numerically, a MC model in a medium-scale DSGE New Keynesian model. First, the firm and aggregate equilibrium conditions have kinks, which renders perturbation methods inappropriate. Therefore, I develop global projection methods suitable for my model. The second challenge is that the state of the economy includes the distribution of relative prices. Consequently, I use the Krusell and Smith (1998) solution method to compute this economy. Because the standard application of this algorithm fails in this context, I develop a modified version.

In my model, a direct application of the Krusell–Smith (KS from hereon) solution approach consists of estimating the inflation policy function in the simulation and then using this function to solve aggregate and idiosyncratic equilibrium conditions. However, replacing the Phillips curve with the inflation policy function—when solving aggregate equilibrium conditions—implies an exogenous nominal interest rate and indeterminacy. To avoid this problem, I apply the KS algorithm to the components of inflation that depend only on the distribution of relative prices, but not on reset prices, when solving the equilibrium conditions.

RELATED LITERATURE. — The optimal inflation target in the presence of a ZLB constraint was first studied quantitatively by Walsh (2009), Williams (2009) and Billi (2011). The setting for all of these papers is a Calvo model linearized around a zero trend inflation steady state and a central bank that optimizes a non-micro-founded loss function. Each argues that the optimal rate of inflation is greater than 2%.

Coibion, Gorodnichenko and Wieland (2012) and the literature that follows also study a Calvo model, but one with positive trend inflation and a micro-founded welfare function.³ They find an optimal inflation rate that is significantly

²Inflation dynamics in a MC model depend on the current distribution of relative prices. This property does not imply that forward guidance is ineffective in the MC model. As I explain above, inflation dynamics depend on the distribution of adjusting relative prices *and* reset prices—a forward-looking variable. Thus, forward guidance could have an effect on inflation dynamics during ZLB periods, since inflation is also forward-looking.

³Ascari, Phaneuf and Sims (2018) emphasize that raising the inflation target is very costly in models featuring Calvo price and wage setting without a ZLB constraint. Dordal i Carreras et al. (2016) introduce a regime-switching representation of risk premium shocks to match the rare and long run lived episodes

lower, approximately 1.5%.⁴ In their Calvo pricing model, the main cost of a higher inflation target is an increment to the steady-state price dispersion. The main benefit is a reduction in inflation volatility and its impact on mean price dispersion. This property means that the welfare contribution of consumption and labor volatilities across inflation targets is negligible. Therefore, the business cycle stabilization of consumption and labor is not the primary benefit of increasing the inflation target in their model.⁵

Like Coibion, Gorodnichenko and Wieland (2012), I consider a micro-founded welfare function. Unlike them, I consider pricing frictions capable of matching large and heterogeneous price adjustments. I capture this micro-pricing behavior with idiosyncratic shocks and a rich pricing framework. This heterogeneity turns out to matter since it increases the optimal inflation target by two times as compared with the identical model with Calvo pricing. Importantly, the business cycle stabilization of consumption and labor is a primary benefit of increasing the optimal inflation target, since price dispersion has a low sensitivity to inflation.

Several papers have analyzed the slope of the Phillips curve in MC models with idiosyncratic shocks and monetary shocks as the only source of aggregate fluctuations—for example, Golosov and Lucas (2007), Gertler and Leahy (2008), and Midrigan (2011), among others. Other papers have studied the steady-state cost of inflation in the Golosov and Lucas (2007) model, for example, Burstein and Hellwig (2008), Álvarez et al. (2018), and Nakamura et al. (2018). My paper unifies these two topics within a standard DSGE model with a ZLB designed to study the trade-offs in the optimal inflation target quantitatively. To achieve this objective, I use a MC model consistent with the dispersion in price changes observed in micro-pricing behavior *and* aggregate U.S. business cycle fluctuations.

ROAD MAP. — Section I describes the model. Section II calibrates the model and discusses the solution method. Section III analyzes the optimal inflation target and the economic trade-offs for its determination. Section IV assesses the robustness of the optimal inflation target. Section V concludes.

I. Model

This section describes a model to study the optimal inflation target in the U.S. economy, which is similar to Coibion, Gorodnichenko and Wieland (2012). The

of a binding ZLB constraint. This feature duplicates the inflation target. See also Andrade et al. (2018), Kiley and Roberts (2017), and Diercks (2019). For a comprehensive description of the optimal inflation target across studies since the mid-1990's, see Schmitt-Groh and Uribe (2010) and Diercks (2017).

⁴Importantly, they find that this result is robust to alternative specifications of firm price setting, such as time-dependent (Taylor) pricing and state-dependent pricing as in Dotsey, King and Wolman (1999).

⁵Online Appendix Section G shows that in the Coibion, Gorodnichenko and Wieland (2012) Calvo model with their calibration, solution method and welfare evaluation, the optimal inflation target with inefficient fluctuations of consumption and labor is the same as if they would be constant across inflation targets.

model's main departure from the original paper is a rich pricing framework that is able to reproduce micro-pricing behavior. I present the model before discussing the empirical motivation for the pricing and general equilibrium frameworks.

There are five types of agents in the economy: a representative household, a final good competitive firm, a measure-one continuum of intermediate good firms indexed by $i \in [0, 1]$, a central bank, and a government. I now present the agent's optimization problems and define the equilibrium.

REPRESENTATIVE HOUSEHOLD. — Household's preferences are given by

$$(1) \quad U_0 = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u_t(C_t, L_t) \right], \text{ with } u_t(C_t, L_t) = \frac{\left(C_t - \eta_{tz} L_t^{1+\chi} / (1 + \chi) \right)^{1-\sigma}}{1 - \sigma}.$$

Period utility u_t follows a Greenwood–Hercowitz–Huffman (GHH from hereon) preferences specification, where C_t is aggregate consumption and L_t is labor supply. I scale the disutility of labor by aggregate productivity η_{tz} to generate a balanced growth path.

The household's budget constraint is given by

$$(2) \quad P_t C_t + B_t = \eta_{th} W_t L_t + \int \Phi_{ti} di + \eta_{t-1q} R_{t-1} B_{t-1} + T_t.$$

Here W_t and P_t are the nominal prices of labor and consumption, respectively; Φ_{ti} denotes nominal profits for the intermediate producer; T_t is lump-sum transfers from the government. The term B_{t-1} is the stock of one-period nominal bonds, and R_{t-1} is the nominal interest rate.

Two exogenous stochastic processes affect household's behavior: a risk premium shock and a cost-push shock denoted by η_{tq} and η_{th} , respectively. The risk premium shock generates a wedge between the nominal interest rate (which is controlled by the central bank) and the return on assets held by the household. The cost-push shock generates a wedge in the household's marginal rate of substitution between consumption and leisure, and the real wage. This shock is akin to exogenous variations in wage markups in an economy with employment agencies.

The representative consumer's problem is given by

$$(3) \quad \max_{\{C_t, L_t, B_t\}_t} U_0$$

subject to (2) for all periods. It follows from this problem that the time-zero

stochastic nominal discount factor Q_t is

$$(4) \quad Q_{t+1} = \beta^{t+1} \frac{u_{ct}(C_{t+1}, L_{t+1})}{u_{c0}(C_0, L_0)} \frac{P_0}{P_{t+1}}.$$

Here $u_{ct}(\cdot)$ is the marginal utility of consumption.

FINAL GOOD PRODUCER. — The final good firm produces output Y_t using intermediate firms' production y_{ti} subject to random idiosyncratic shocks A_{ti}

$$(5) \quad Y_t = \left(\int_0^1 \left(\frac{y_{ti}}{A_{ti}} \right)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}},$$

where the final output uses a Dixit–Stiglitz aggregator with elasticity γ .

The final good producer's problem is given by

$$(6) \quad \max_{\{Y_t, \{y_{ti}\}_i\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t \left(P_t Y_t - \int_0^1 p_{ti} y_{ti} di \right) \right]$$

subject to (5), where p_{ti} denotes the price of intermediate firms' production. Given constant returns to scale and the zero-profit condition, we can state the aggregate price level and firm's demand as (respectively)

$$(7) \quad P_t = \left(\int_0^1 (p_{ti} A_{ti})^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}, \quad \text{and} \quad y_t(A_{ti}, p_{ti}) = A_{ti} \left(\frac{A_{ti} p_{ti}}{P_t} \right)^{-\gamma} Y_t.$$

INTERMEDIATE GOOD PRODUCERS. — Intermediate good firms are monopolistically competitive. Intermediate good firm i produces output y_{ti} using labor l_{ti} and material n_{ti} , and that firm's productivity is a function of an idiosyncratic component A_{ti} and an aggregate component η_{tz} according to

$$(8) \quad y_{ti} = A_{ti} n_{ti}^{\alpha} (\eta_{tz} l_{ti})^{1-\alpha}.$$

Following the literature, I refer to A_{ti} as a *quality shock*.⁶ A decrease in A_{ti} increases the final good producer's marginal product, but at the same time, it reduces the intermediate good producer's marginal product. These two effects offset each other in such a way that the marginal product of labor in firm i for

⁶This formulation was first used by Woodford (2009) to keep his model tractable. It was also used by Midrigan (2011), Álvarez and Lippi (2014), and Kehoe and Midrigan (2015), among others.

the final output is independent of A_{ti} . I add this quality shock to the Dixit–Stiglitz aggregator to reduce the firm’s state (and that of the aggregate economy) from relative prices *and* idiosyncratic productivity to their product (see equation (13) in the intermediate firm problem to follow).

The quality shock growth rate follows a mixed normal distribution (fat-tailed idiosyncratic shocks) given by

$$(9) \quad \log(A_{ti}/A_{t-1i}) = \begin{cases} \eta_{ti}^1 & \text{with probability } \psi \\ \eta_{ti}^2 & \text{with probability } 1 - \psi \end{cases},$$

with $\eta_{ti}^k \sim i.i.d. N(0, \sigma_{ak})$ for $k = 1, 2$.

Firms face a stochastic physical cost of changing their price. Every time the firm changes its nominal price, it must pay a menu cost equal to θ_{ti} units of labor. The menu cost is an *i.i.d.* random variable that exhibits the following process over time:

$$(10) \quad \theta_{ti} = \begin{cases} \theta & \text{with probability } 1 - \zeta \\ 0 & \text{with probability } \zeta \end{cases},$$

where the menu cost is fixed to a constant θ , but with some *i.i.d.* probability ζ the firm has an opportunity to change the price without cost.

The intermediate firm’s problem is given by

$$(11) \quad \max_{p_{ti}} \mathbb{E} \left[\sum_{t=0}^{\infty} Q_t \Phi_{ti} \right],$$

where nominal profits Φ_{ti} follow

$$(12) \quad \Phi_{ti} = y_t(A_{ti}, p_{ti}) \left(p_{ti} - \iota(1 - \tau)(W_t/\eta_{tz})^{1-\alpha} P_t^\alpha / A_{ti} \right) - I(p_{t-1i} \neq p_{ti})W_t\theta_{ti},$$

subject to (9), (10) and the initial conditions A_{-1} and p_{-1} given. Note that the optimal choice of inputs is already included in the firm’s marginal cost (with $\iota = (1 - \alpha)^{\alpha-1} \alpha^{-\alpha}$). There is a subsidy to total cost τ that allows my model to separate the demand elasticity from the average level of markups. We can now rewrite nominal profits as

$$(13) \quad \Phi_{ti} = \left(\frac{A_{ti} p_{ti}}{P_t} \right)^{-\gamma} Y_t \left(p_{ti} A_{ti} - \iota(1 - \tau)(W_t/\eta_{tz})^{1-\alpha} P_t^\alpha \right) - I(p_{t-1i} \neq p_{ti})W_t,$$

where nominal profits are affected only by the product of nominal price and idiosyncratic productivity.

MONETARY POLICY. — The central bank sets the nominal interest rate following a Taylor rule given by

$$(14) \quad R_t^* = \mathcal{R} (R_{t-1}^*)^{\phi_r} \left[\left(\frac{P_t}{P_{t-1}(1 + \bar{\pi})} \right)^{\phi_\pi} X_t^{\phi_y} \right]^{1-\phi_r} \left(\frac{X_t}{X_{t-1}} \right)^{\phi_{dy}} \eta_{rt},$$

$$(15) \quad R_t = \max \{1, R_t^*\},$$

$$(16) \quad \mathcal{R} = \left(\frac{1 + \bar{\pi}}{\beta(1 + g)^{-\sigma}} \right)^{1-\phi_r}.$$

Here R_t is the nominal interest rate, $\bar{\pi}$ is the target inflation rate, η_{rt} is a monetary policy shock, and X_t is the output gap—that is, the ratio between current output and its natural level in an economy without price rigidities. I define R_t^* as the desired interest rate, i.e., the interest rate whenever the ZLB is not binding.

AGGREGATE FEASIBILITY AND GOVERNMENT POLICY. — Aggregate output is equal to aggregate consumption plus government expenditures:

$$(17) \quad \left(Y_t - \int n_{ti} di \right) = C_t + G_t,$$

where Y_t is gross output and $(Y_t - \int n_{ti} di)$ is gross domestic product (GDP). The government's expenditures G_t follow the stochastic process $G_t = (Y_t - \int n_{ti} di) (1 - 1/\eta_{tg})$, where η_{tg} is an exogenous process. The government follows a balanced budget each period.

Labor supply is allocated into the production of intermediate goods and the physical cost of changing their prices; thus

$$(18) \quad \int_0^1 (l_{ti} + I(p_{ti} \neq p_{t-1i})\theta_{ti}) di = L_t.$$

AGGREGATE EXOGENOUS SHOCKS. — Aggregate shocks follow a first-order autoregressive process given by

$$(19) \quad \log(\eta_{tj}) = (1 - \rho_j)\eta_j + \rho_j \log(\eta_{t-1j}) + \sigma_j \varepsilon_{tj}, \quad \varepsilon_{tj} \sim_{i.i.d.} N(0, 1),$$

here $j \in \{r, q, g, h\}$. Productivity growth ($d\eta_{tz} = \frac{\eta_{tz}}{\eta_{t-1z}}$) follows

$$(20) \quad \log(d\eta_{tz}) = (1 - \rho_z) \log(1 + g) + \rho_z \log(d\eta_{t-1z}) + \sigma_z \varepsilon_{tz}, \quad \varepsilon_{tz} \sim_{i.i.d.} N(0, 1),$$

where g is the economy's growth rate.

EQUILIBRIUM DEFINITION. — *An equilibrium is a set of stochastic processes for (i) a policy $\{C, L, B\}_t$ of the representative consumer; (ii) pricing policy functions $\{p_{ti}\}_t$ of firms and input demands $\{n_{ti}, l_{ti}\}_{ti}$ of monopolistic firms; (iii) final output and input demands $\{Y_t, \{y_{ti}\}_i\}_t$ of the final producer; and (iv) the nominal and desired interest rate $\{R_t, R_t^*\}_t$. The following statements hold in equilibrium.*

1. $\{C, L, B\}_t$ solve the consumer's problem in (3).
2. $\{Y_t, \{y_{ti}\}_i\}_t$ solve the final good producer's problem in (6).
3. The firm's policy p_{ti} solves (11) and the demand for inputs is optimal.
4. The nominal and desired interest rates satisfy the Taylor rule (14) to (16).
5. The labor and good markets clear (equations (17) and (18) are satisfied in each period).

PRICING FRAMEWORK AND STRATEGIC COMPLEMENTARITIES. — Several features in the pricing model allow my model to reproduce micro-pricing behavior. Idiosyncratic shocks yield a large size of price changes with a moderate level and volatility of inflation as in the U.S. economy. Nevertheless, a simple MC model with Gaussian idiosyncratic shocks *cannot* reproduce the significant heterogeneity in the price-change distribution that we observe in the data. To match this fact, I add two features to the pricing model. First, I use fat-tailed idiosyncratic shocks, which allows my model to generate the large size of price changes we see in the data with empirical evidence on the small cost of price adjustment. Second, I use random free cost of price adjustment to enable my model to produce small price changes.

My model is flexible enough to nest the key pricing models used heretofore in the literature: the Calvo (1983) model, the Golosov and Lucas (2007) model, the Gertler and Leahy (2008) model, and intermediate combinations of these models, such as the ones studied in Nakamura and Steinsson (2010) and Midrigan (2011). Importantly, each calibration of the pricing model generates different slopes of the Phillips curve, aggregate dynamics at the ZLB, and different cost of inflation. For this reason, I estimate the parameters of the pricing model to match micro-pricing behavior in the data.

This paper uses quality shocks in the final good aggregator. Online Appendix Section E shows that this assumption is neutral concerning the main cost of inflation given by the elasticity of price dispersion to the inflation target in the steady-state. Since price dispersion with and without business cycles are almost the same, I provide suggestive evidence that the assumption of quality shocks is not quantitatively relevant for the main cost of inflation given by price dispersion.

GENERAL EQUILIBRIUM FRAMEWORK. — The general equilibrium framework in my model is similar to the one used in Coibion, Gorodnichenko and Wieland (2012).

My model has the same structural shocks as their paper—productivity, government expenditures, risk premium, monetary and cost–push shocks—together with a similar Taylor rule.⁷ We both have strategic complementarities in our models, but from different frictions: I use intermediate inputs, while they use segmented labor markets. Qualitatively, they have the same effect on the cost of inflation, and the slope of the Phillips curve.⁸

Another departure from Coibion, Gorodnichenko and Wieland (2012) is in the specification of the household’s preferences. In the Coibion, Gorodnichenko and Wieland (2012) benchmark specification, they use CRRA preferences with habit formation. Instead, I calibrate period utility with GHH preferences. Abstracting from habit formation and *ceteris paribus* a Frisch elasticity, the GHH specification of preferences has a lower elasticity of real wages to output than the CRRA specification; in a similar spirit of sticky wages. This departure from their environment allows my model to match the volatility of real wages observed in the U.S. economy.⁹

INFLATION DYNAMICS IN MENU COST MODELS. — To analyze the solution method and the business cycle dynamics, it is useful to understand and quantify how important non-random price adjustment is for aggregate inflation dynamics. For this reason, I use the inflation decomposition described below.

Inflation can be decomposed into three components: the frequency of price changes Ω_t , the relative reset price P_t^* , and the menu cost inflation φ_t .¹⁰ Inflation is given by

$$(21) \quad \Pi_t = \left(\frac{1 - \Omega_t}{1 - \Omega_t (P_t^*)^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_t,$$

$$(22) \quad \varphi_t = \mathbb{E}_i \left[(\tilde{p}_{t-1i} \exp(\Delta \log(A_{t-1i})))^{1-\gamma} \mid \text{no price adjustment in } t \right]^{\frac{1}{1-\gamma}}.$$

The menu cost inflation reflects the relative position of price changes since the mean of relative prices is constant. Thus, it measures the component of inflation coming from the distribution of relative prices and, therefore, the magnitude of the “selection effect” after an aggregate shock, as argued in Golosov and Lucas

⁷The Taylor rule in my model depends on output gap growth (theirs, output growth) and it depends only on one period lag of the interest rate (theirs, one and two periods lags of the interest rate).

⁸Quantitatively, the level of strategic complementarities in my calibration is lower than theirs. We can measure this property in the slope of the Phillips curve coming from strategic complementarities. In Coibion, Gorodnichenko and Wieland (2012) the slope of the Phillips curve is proportional to $1/(1 + \epsilon_{frisch}\gamma)$, where ϵ_{frisch} stands for the Frisch elasticity; while in my model it is proportional to $1 - \alpha$. With their calibration of $\gamma = 10$ and $\epsilon_{frisch} = 1$, the proportionality factor is approximately 0.1 while mine is close to 0.5.

⁹Increasing the elasticity of real wages to output (e.g., with CRRA preferences) increases the optimal inflation target since it magnifies the deflationary spiral.

¹⁰The relative reset price is equal to $P_t^* = p_{ti}^* A_{ti} / P_t$ where p_{ti}^* is the nominal reset price. See Online Appendix D for a proof of equations (21) and (22).

(2007). The menu cost inflation has two properties: (i) It is equal to one in the Calvo model; and (ii) it is bigger than one if the average price change comes from low relative prices.

This decomposition exposes a critical property of the inflation process. While the menu cost inflation relies heavily on the distribution of past relative prices, the reset price depends only on current and future real marginal cost. Therefore, the menu cost inflation is a primarily backward-looking variable, while the reset price is a strictly forward-looking variable.

II. Calibration and Solution Method

A. Calibration

To calibrate the model, I first divide the parameters into three separate sets related to (i) preferences and technology, (ii) menu costs and idiosyncratic shocks, and (iii) the Taylor rule and aggregate shocks. I then externally calibrate the parameters for preferences and technology using micro evidence on their empirical counterparts. Lastly, I use the simulated method of moments (SMM) to estimate the rest of the parameters. Table I details the parameters used in the MC model.

PREFERENCES AND TECHNOLOGY. — A period in the model is a month. Therefore, I choose a discount factor of $\beta = 0.96^{1/12}(1+g)^\sigma$ and calibrate $g = 0.0017$ to match the U.S. annual growth rate of 2%. The GHH preference parameters are set to $\sigma = 2$ and $\chi = 0.8$, following Greenwood, Hercowitz and Huffman (1988).¹¹ I set $\eta_g = 4/3$ to match the average U.S. ratio of government expenditures to output of 25%.

For the production function, I set the between-inputs elasticity γ equal to 3. This calibration choice falls within the range of estimates of the price to quantity elasticity used in industrial organization and international trade. This elasticity, as it determines the cost of price dispersion, is an important dimension in my model.¹² I set the elasticity output to materials equal to 0.53. This value matches the ratio of intermediate inputs to total output in the U.S. economy—intermediate inputs are 45% of output when markups are 17%. I calibrate τ to match an aggregate markup of 17%.

RANDOM MENU COST AND QUALITY SHOCK STOCHASTIC PROCESSES. — I estimate the random menu cost and quality shock stochastic processes to match moments

¹¹Greenwood, Hercowitz and Huffman (1988) calibrate $\chi = 0.6$. I use $\chi = 0.8$ to match the volatility of wages relative to output.

¹²Macroeconomic estimates tend to be fairly large (about 10); industrial organization and international trade estimates tend to be much smaller. Estimates of the elasticity of substitution equal to about 2 were given by Chevalier, Kashyap and Rossi (2003), Nevo (2001), and Barsky et al. (2003); Burstein and Hellwig (2007) estimated a value between 1.55 and 4.64.

TABLE I—PARAMETERS VALUE AND TARGETS

Parameter	Value	Target
Preferences and technology		
β : Discount factor	$0.96^{1/12}(1+g)^\sigma$	Standard
g : Growth rate	0.0017	2% GDP growth rate
σ : Intertemporal-consumption elasticity	2	Greenwood, Hercowitz and Huffman (1988)
χ : Labor supply elasticity	0.8	Greenwood, Hercowitz and Huffman (1988)
γ : Demand elasticity	3	Micro-estimates
α : Share of intermediate inputs	0.53	Intermediate inputs/total output of 45%
τ : Cost subsidy	0.2	Aggregate markup 17%
Random menu cost and quality shocks processes		
$(\sigma_{a1}, \sigma_{a2})$: Idiosyncratic shock innovations	(0.235, 0.018)	SMM—micro price statistics
ψ : Prob. of large idiosyncratic shock	0.070	SMM—micro price statistics
ζ : Prob. of free price adjustment	0.044	SMM—micro price statistics
θ : Menu cost	0.135	SMM—micro price statistics
Taylor rule and aggregate shocks		
$(\phi_r, \phi_\pi, \phi_y, \phi_{dy})$: Taylor rule	(0.89, 2.62, 0.32, 0.001)	SMM—US business cycles
$(\rho_z, \sigma_z 100)$: Productivity shocks	(0.980, 0.010)	SMM—US business cycles
$(\rho_g, \sigma_g 100)$: Gov. expenditure shocks	(0.979, 0.139)	SMM—US business cycles
$(\rho_r, \sigma_r 100)$: Taylor rule shocks	(0, 0.010)	SMM—US business cycles
$(\rho_q, \sigma_q 100)$: Risk premium shocks	(0.979, 0.030)	SMM—US business cycles
$(\rho_h, \sigma_h 100)$: Cost push shocks	(0.960, 0.322)	SMM—US business cycles

Note: The table presents the parameter values assigned in the MC model. The coefficient of the Taylor rule for the output gap is multiplied by four whenever I use the Taylor rule as a function of real marginal cost since the elasticity of the output gap to the real marginal cost is 4.

Source: Author's calculations

of the price-change distribution and the physical cost of price changes.¹³ I use the steady-state moments of the MC model and verify ex-post that the model incorporating business cycles reproduces similar moments of the price-change distribution. This approach is valid since the price-change distribution depends mainly on idiosyncratic shocks, and not aggregate shocks, given their relative magnitude. Thus, the model with business cycle dynamics generates almost the same price-change distribution as the steady-state economy.

Table II describes some selected moments in the data and the model. I set the average resources spent on price adjustment equal to 0.4% of revenue computed in Zbaracki et al. (2004) in the data. In the model, the physical cost of price changes is the menu cost multiplied by the number of costly price adjustments and then normalized by total revenue: $W_t \theta \frac{\Omega_t - \zeta}{P_t Y_t}$. The rest of the moments involve micro price statistics from the U.K. CPI. Next, I describe these data and the main steps used to compute the moments in Table II.

I use monthly price quotes collected for the consumer price index micro dataset of the United Kingdom's Office for National Statistics (ONS). This dataset offers

¹³Estimated parameters are robust to excluding the physical cost of price adjustment; see Table B.II column 4 in the Online Appendix.

TABLE II—MICRO-LEVEL PRICING MOMENTS: MODEL AND DATA

Moments	Data		Model	
	Raw	With filters	Steady state	Business cycle
Absolute value of price changes				
Mean	0.154	0.153	0.153	0.154
Standar Deviation	0.159	0.137	0.143	0.139
Skewness	1.538	1.314	1.188	1.249
10th Percentile	0.014	0.020	0.010	0.012
25th Percentile	0.034	0.049	0.032	0.037
50th Percentile	0.096	0.114	0.130	0.130
75th Percentile	0.223	0.217	0.229	0.224
90th Percentile	0.386	0.356	0.360	0.353
Price changes				
Standar Deviation	0.222	0.205	0.209	0.207
Kurtosis	4.672	3.809	3.810	3.841
Mean frequency of price changes	0.169	0.126	—	—
Frequency with implied duration	0.119	0.097	0.097	0.097
Percentage of free to total price adjustments	—	—	48.228	46.032

Note: The table presents selected moments of the micro price statistics in the SMM estimation. The first column describes the price statistics with standard filters, and the second column describes the same moments computed with the filters explained in Online Appendix Section B.B.2. Columns 3 and 4 show the price statistics in the model with and without business cycles.

Source: Columns II (i.e., Raw) and III (i.e., With filters) use data from United Kingdom's Office for National Statistics (ONS) micro-level price data. The micro-level price dataset for the period 1996-2005 is available online in United Kingdom's Office for National Statistics (1996-2005) and for the period 2006-2017 is available online in United Kingdom's Office for National Statistics (2006-2017). The rest of the columns are author's calculations

several advantages. First, it is representative of the whole economy because it reflects all prices in the consumer consumption basket. Second, data from January 1996 to the present is publicly available with a lag of only two months. Third, when I treat the UK data in the same way Klenow and Kryvtsov (2008); Nakamura and Steinsson (2008), and Midrigan (2011) treat their US data, I find micro price moments similar to theirs (see Table B.I in the Online Appendix).¹⁴

The distribution of price changes is critical to my calibration, and therefore, to the cost of inflation. For that reason, I apply several filters to render the data compatible with the model. First, I employ standard filters used in previous studies (e.g., Klenow and Kryvtsov 2008; Nakamura and Steinsson 2008).¹⁵ Second,

¹⁴See Alvarez et al. (2006) and Klenow and Malin (2010) for cross-country studies. For a comparison between the U.K. and the U.S., see Table B.I in the Online Appendix.

¹⁵I complete price quotes for temporary missing observations (less than a year) and out-of-season observations with the last available price, and I redefine a product as a new product whenever there

since my model abstracts from sales and heterogeneity across sectors, I compute micro-price moments filtering out unmeasured sales and controlling for sectoral heterogeneity. I briefly describe these steps below and leave further details to the Online Appendix Section B.

The sale filter drops price changes preceded *and* followed by the same (base) price, even if there is no “sales and recovery” flag. Around 20% of price changes have this property (see columns 1 and 2 in Table II). A mismeasurement of sales using only sales flags increases the frequency of price changes and the magnitude of the volatility of idiosyncratic shocks. Higher volatility of idiosyncratic shocks reduces the cost of inflation.

The pervasive heterogeneity of pricing behaviors across sectors is well known. For this reason, following the “Classification of Individual Consumption According to Purpose” (COICOP), I control for heterogeneity at class-level in two ways.

First, I target the inverse of the average duration of price at class-level, instead of the average frequency of price changes. Thus, I target a frequency of price changes equal to 0.097 instead of 0.126. Álvarez, Le Bihan and Lippi (2016), Baley and Blanco (2018), and Blanco and Cravino (2019) have shown that this is the correct method to map class-level heterogeneity in pricing data onto a single-sector model.¹⁶

Second, I target the aggregate standardized price-change distribution at class-level (cf. Klenow and Kryvtsov 2008). Intuitively, targeting the standardized distribution of price changes homogenizes this variable for the mean and the variance across classes, lowering the dispersion in the price-change distribution. This filter decreases the amount of small and large price changes reflected in the kurtosis of price changes, as we can see in Table II. The standardized price change distribution affects the estimated ratio of the frequency of free price adjustments to total price changes: that ratio is 62% without this filter, but 48% with it. Online Appendix Section B.B.4 shows theoretical results that support this aggregation in models with sectorial heterogeneity. Table B.II in the Online Appendix shows the estimates of the structural parameters when I use different filters.

The final calibration specifies a model that is neither a Calvo model or a standard MC model, but a hybrid of the two. Thus, how close is the calibration of my model to a benchmark MC model like the one in Golosov and Lucas (2007) vis-à-vis to a Calvo model regarding the slope of the Phillips curve?

I answer this question by looking at the proportion of price changes due to zero

are more than twelve consecutive missing observations. I exclude price changes flagged as involving noncomparable product substitutions, or “sales and recovery,” as well as outliers and observations in months during which the value-added tax rate changed.

¹⁶The average frequency and the inverse of the average duration of price changes are given by $\sum_i \omega_i \Omega_i$ and $\left(\sum_i \omega_i \Omega_i^{-1}\right)^{-1}$, respectively, where ω_i and Ω_i are CPI weights and the frequency of price changes at the class level. Álvarez, Le Bihan and Lippi (2016) in Online Appendix E and Baley and Blanco (2018) show that if shocks are i.i.d and there are no strategic complementarities, then the effects of monetary shocks in a single-sector vs. a multi-sector model are the same if the two models are calibrated to match the same average duration of price spells. Blanco and Cravino (2019) extend this result quantitatively in a model with persistent aggregate shocks and strategic complementarities.

menu cost plus price changes conditional on fat-tailed idiosyncratic shocks. From the work of Gertler and Leahy (2008), Midrigan (2011) and Álvarez, Le Bihan and Lippi (2016), we can use this number as a metric for the difference in the slope of the Phillips curves between my model and the Calvo model. The intuition behind this result is the following: if the frequency and distribution of price changes do not respond to aggregate shocks, then firms change their prices if and only if they receive a fat-tailed idiosyncratic shock or a zero menu cost. Since these two events are independent of the aggregate state and *i.i.d.* across time and firms, inflation dynamics are similar to the Calvo model.

When my model is consistent with micro-data, around 50% of price changes are due to zero menu cost (see model's columns in Table II, last row). Additionally, if we sum up the price changes after fat-tailed idiosyncratic shocks, this quantity increases to 93%. This quantitative result suggests that my MC model has similar business cycle properties than an otherwise equivalent Calvo model if the frequency and distribution of price changes do not fluctuate. Online Appendix Section C shows that my MC model generates similar aggregate dynamics to the Calvo model without a ZLB constraint and at a 2% inflation target.¹⁷

TAYLOR RULE AND AGGREGATE SHOCKS. — For the estimation of the Taylor rule and aggregate exogenous shocks, I use SMM with business cycle moments in the U.S. economy and the Calvo model at 2% inflation and no-ZLB. I verify ex-post that the model with menu costs reproduces similar moments at a 3.5% inflation with ZLB. This methodology gives a good approximation of the best fit with MC and ZLB for two reasons. First, the Calvo model's business cycle properties are similar to those of the MC model with a low inflation target and no ZLB. Second, the business cycle moments of the MC model with ZLB at 3.5% inflation target are similar to those with no ZLB.

Table III presents the moments in the model and the U.S. data from 1960 to 2017, specifically the standard deviations, autocorrelations, and correlations with output of each HP-filtered variable. My model yields a frequency of binding ZLB of 3.5% at 3.5% inflation target and 12% at 3% inflation target. This frequency is a conservative target concerning the U.S. historical frequency of binding ZLB (i.e., 10% with average inflation of 3.4%), and closer to Coibion, Gorodnichenko and Wieland (2012) (i.e., 3.5% with average inflation of 3.5%). I construct output in the data as the sum of total consumption plus government expenditures instead of gross domestic product to be consistent with my model.

The model reproduces the standard deviations of output and consumption, the main inputs for welfare and the main target of my estimation, together with the standard deviations of interest rate, real wages, and government expenditures. Additionally, due to cost-push and productivity shocks, the model generates the

¹⁷Free adjustments due to zero menu cost and fat-tailed idiosyncratic shocks reduce the volatility of inflation even when the ZLB is binding in comparison to an analogous MC model without these two features.

TABLE III—BUSINESS CYCLE MOMENTS: MODEL AND DATA

	Model			Model		
	Data	Median	[2,98]	Data	Median	[2,98]
	Standard deviation			Autocorrelation		
Output	0.95	1.05	[0.84,1.30]	0.83	0.78	[0.67,0.85]
Consumption	1.14	1.07	[0.86,1.31]	0.85	0.78	[0.68,0.85]
Interest rate	0.35	0.41	[0.32,0.51]	0.83	0.86	[0.81,0.90]
Real wage	0.86	0.67	[0.49,0.89]	0.71	0.87	[0.78,0.92]
Inflation	0.26	0.13	[0.11,0.15]	0.49	0.58	[0.45,0.68]
Gov. expenditure	1.42	1.40	[1.12,1.70]	0.82	0.77	[0.67,0.84]
	Correlation with output					
Output	1.00	1.00	[1.00,1.00]			
Consumption	0.92	0.96	[0.94,0.98]			
Interest rate	0.19	0.28	[0.00,0.50]			
Real wage	0.25	0.65	[0.45,0.79]			
Inflation	0.09	0.25	[0.05,0.44]			
Gov. expenditure	0.42	0.80	[0.65,0.88]			

Note: The table presents business cycle moments from the U.S. data and the simulated series of the model at a 3.5% inflation target. Online Appendix Section A describes the variables in the U.S. data. Model and data series are detrended with Hodrick-Prescott filter ($\lambda = 1600$) to remove the trend component. The period in the data is from 1960 first quarter to 2017 fourth quarter. The moments in the MC model are the median and a [2,98] percent confidence interval across simulations. I compute the statistics in the model for over 5000 simulations with the same length as in the data.

Source: Author's calculations

low to zero correlation between the interest rate and inflation with output. The model does a relatively good job matching the persistence of the variables on average, but it doesn't match the relatively high (resp. low) persistence of real (resp. nominal) variables.

The model overpredicts the persistence and the correlation with output of real wages. Qualitatively, cost-push shocks generate these properties over real wages, since they produce a wedge between real wages and the marginal rate of substitution of consumption and labor. Nevertheless, quantitatively, these shocks affect mainly output and not real wages; therefore, the model cannot use these shocks to "disconnect" the stochastic process of output and real wages.

Under this estimation, the model understates the relation between inflation and real wages, i.e., the slope of the Phillips curve. To show this property, I compare the stochastic process of inflation relative to real wages. The ratio of the standard deviations of inflation to real wages in the data is 0.30 (resp. 0.19 in the model). This property suggests that my model understates the slope of the Phillips curve, and therefore also the optimal inflation target. Nevertheless, missing the standard deviation of inflation alone does not necessarily imply that my model misses a key statistic for the optimal inflation target. Intuitively, if my model misses *i.i.d.* fluctuations of inflation that are not correlated with consumption and labor, then

those fluctuations should not affect equilibrium dynamics. I analyze this insight in Section IV.

Two observations are important. First, I do not use aggregate data on inflation to estimate the slope of the Phillips curve. I use orthogonal evidence on micro-pricing behavior, and the ratio of intermediate inputs to total output observed in the U.S. economy. Second, the property by which my model understates the frequency of the ZLB implies that the model's optimal inflation is a lower bound for the U.S. economy.

B. Solution Method

The two main challenges associated with the computation of the equilibrium are that (i) the combination of a Taylor rule for monetary policy and an infinite-dimensional aggregate state requires a nonstandard application of (and evaluation of results from) the Krusell–Smith algorithm and (ii) both the aggregate and idiosyncratic policy functions have kinks. Here, I briefly describe the solution method regarding (i). Interested readers can find more details in the Online Numerical Appendix, Sections J, K, and L.

MODIFICATION OF KRUSELL–SMITH ALGORITHM. — Given that the distribution of relative prices is a part of the state, I use the KS algorithm to solve this problem. However, the standard method of implementing that algorithm does not work in my model.

To understand why KS algorithm fails in this environment, consider the uniqueness of the equilibrium with price rigidities and a Taylor rule. Assume a deviation from the equilibrium with an increase in consumption. This increase in consumption raises the output gap and also (because of the Phillips curve) inflation. If the Taylor principle is satisfied, then this change in inflation affects the real rate, feeding back to consumption and undoing the original increase. The general equilibrium effect involving households, firms, and the central bank makes the equilibrium unique.

In my model, the KS algorithm would ideally obtain the aggregate inflation policy from the simulation and then use it to solve the equilibrium conditions. However, replacing the Phillips curve—which captures the relation between inflation and real marginal cost—with an approximation of the inflation policy function generates indeterminacy, as in a standard New Keynesian model with exogenous inflation.

The problem is that there is no information on the relationship between inflation and real marginal cost (i.e., the Phillips curve) whenever we obtain the inflation policy from the simulation. My proposed solution consists of applying the KS algorithm to the components of inflation that depend on the distribution of relative prices (the frequency of price changes and the menu cost inflation

components in equation (21)), and then solving *jointly* the aggregate and idiosyncratic equilibrium conditions. Despite generating some (solvable) numerical challenges, this method yields a cross-equation restriction of the intensive margin of the Phillips curve (i.e., the relation between the relative reset price and the real marginal cost) at the moment of solving the equilibrium policies, thereby breaking the indeterminacy I mention above. Such numerical computation seems to be reliable in that it provides a unique solution whenever solving for the equilibrium conditions.¹⁸

ACCURACY OF THE MODIFIED KRUSELL–SMITH ALGORITHM. — I verify the accuracy of the modified KS algorithm by checking the equilibrium conditions with the simulated values of inflation, price dispersion, and frequency of price changes.¹⁹ For each variable in the model, I construct the solution with the KS projections and the simulated version of the projected variables. I then compute the differences between these two ways of calculating the equilibrium to verify the error in the KS approximation. The Numerical Online Appendix L describes the construction of the errors for each variable, and plots the time series. Table L.I in Numerical Online Appendix L reports the Krusell-Smith errors for inflation targets 1.3%, 3% and 5%. Under the simulated level of inflation, the standard deviation of the error in the nominal interest rate *divided by* the standard deviation of nominal rate is 0.038% at a 1.3 inflation target with no ZLB; that ratio increases to 0.624% in the model with a ZLB. At a 5% inflation target, the respective errors increase to 0.084% and 0.133%. Thus, the KS algorithm yields a good fit, losing some of it whenever the ZLB is binding by a large amount, but not at the optimal inflation target or its neighborhood.

AGGREGATE STATE. — The assumption of idiosyncratic quality shocks implies that the idiosyncratic state variable for the firm is $\tilde{p}_{ti} = \frac{p_{ti}A_{ti}}{P_t}$, not $\tilde{p}_{ti} = \frac{p_{ti}}{P_t}$ and A_{ti} separately. An additional implication is that the aggregate state consists of the distribution of adjusted relative prices $f(\tilde{p}_{ti})$ or markups that I approximate with the first two moments.

III. The Optimal Inflation Target

This section analyzes the optimal inflation target in my MC model. To achieve this goal, I proceed in three steps. In the first step, I show the optimal inflation

¹⁸Aggregate kinks precludes the use of methods employed by Reiter (2009) and implemented in MC models by Costain and Nakov (2011).

¹⁹I did not use the R^2 statistics, as most authors do. The main reason is that, if a variable has a small effect on the equilibrium conditions, then the projection's fit does not in itself inform us about whether or not the KS approximation is valid. The only projected endogenous state variable is price dispersion, but since that variable exhibits low volatility, there is no significant law of motion of the aggregate state estimated in the simulation.

target with and without a ZLB constraint in my MC model, alongside key variables to characterize the trade-offs mentioned in the introduction for the optimal inflation target. In the second step, I analyze the importance of the pricing model by computing the optimal inflation target in the same general equilibrium framework and a Calvo pricing model. This exercise provides a quantitative description of the main trade-offs in my MC model, and it compares my results relative to Coibion, Gorodnichenko and Wieland (2012) and related literature. In the last step, I explain how the primary mechanisms in my model generate a different inflation target than that of a Calvo model.

A. The Optimal Inflation Target in a Menu Cost Model

The optimal inflation target in my MC model with a ZLB constraint is 3.5%. This result presupposes a ZLB constraint on nominal interest rates, otherwise the optimal level of inflation would be 0%. Figure I-Panel A describes welfare using the consumption equivalent. Each point on the y -axis marks the percentage increase in consumption needed to achieve the same welfare as under the optimal inflation target. Formally, let $\tilde{U}_t^{\bar{\pi}}$ be the value of the household normalized by productivity,

$$(23) \quad \tilde{U}_t^{\bar{\pi}}(\Psi) = \mathbb{E} \left[\sum_{j=0}^{\infty} \beta^j u_{t+j} \left(C_{t+j}^{\bar{\pi}} \left(1 + \frac{\Psi}{100} \right), L_{t+j}^{\bar{\pi}} \right) / \eta_{t+jz}^{1-\sigma} \right].$$

Here $C_t^{\bar{\pi}}$, and $L_t^{\bar{\pi}}$ are consumption and labor at the inflation target $\bar{\pi}$ whenever there is an increase in consumption by $\Psi\%$. The consumption equivalent at inflation target $\bar{\pi}$, $\Psi(\bar{\pi})$, is given by $\mathbb{E}[\tilde{U}_t^{\bar{\pi}}(\Psi(\bar{\pi}))] = \mathbb{E}[\tilde{U}_t^{\bar{\pi}^*}(0)]$, where $\bar{\pi}^*$ is the optimal inflation target.

Without a ZLB constraint, increasing the inflation target from 0 to 6% in the MC model is equivalent to decreasing consumption by 0.15%. The consumption equivalent changes by 0.04% following an increase in inflation from 0% to 3%. In this case, the consumption equivalent is increasing for all levels of inflation. Notice the consumption equivalent has the same order of magnitude as the cost of the business cycles by Lucas Jr (2003) for levels of inflation less than 4%, i.e., 0.05% of consumption equivalent. With ZLB, the consumption equivalent is decreasing at low levels of inflation and then increasing. The consumption equivalent between the optimal inflation target and a 1.3% inflation target is 0.03%.

The main cost of increasing inflation is a lower mean consumption-labor ratio. Moreover, the mean consumption-labor ratio is the primary variable that explains the consumption equivalent. Figure I-Panel C plots the mean consumption-labor ratio business cycles alongside the steady-state consumption-labor ratio. I normalize this variable with its value at the optimal inflation target. The consumption equivalent is identical to the negative of the mean consumption-labor ratio without a ZLB constraint. They both change by 0.15% when I increase the infla-

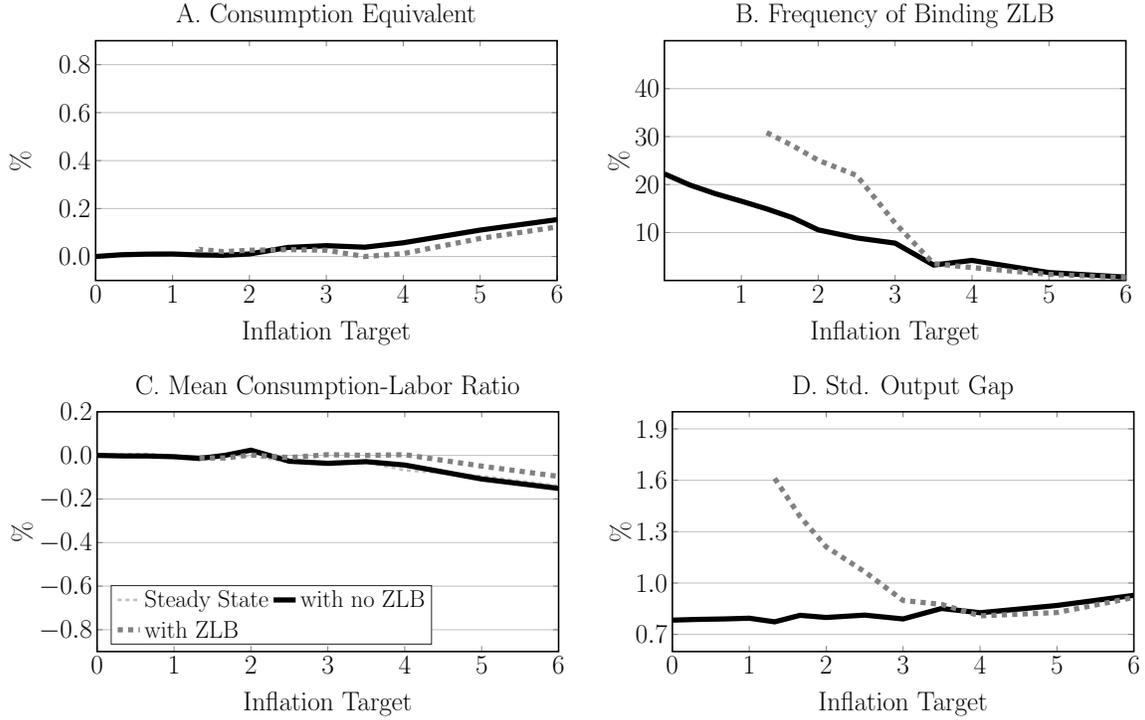


FIGURE I. MENU COST MODEL: MAIN VARIABLES FOR OPTIMAL INFLATION

Note: Panel A describes the consumption equivalent in the MC model normalized by the optimal inflation target. Panel B describes the frequency of a binding ZLB. The frequency of a binding ZLB without a ZLB constraint refers to frequency of negative values of the interest rate. Panel C describes the mean consumption-labor ratio given by $(\mathbb{E}^{\bar{\pi}}[C_t/(L_t\eta_{tz})]/\mathbb{E}^{\bar{\pi}^*}[C_t/(L_t\eta_{tz})] - 1) \times 100$. Panel D describes the standard deviation of output gap given by $Std^{\bar{\pi}}[\log(mc_t)] \times \frac{100}{\chi(1-\alpha)}$. The light grey dashed line describes the steady state moments, the solid black lines describe the moments without a ZLB and the dotted grey lines describe the moments with a ZLB. The scale of this figure coincide with Figure II.
Source: Author's calculations

tion target from 0% to 6%. Intuitively, changes in the inflation target only affect this variable without a ZLB constraint.

With a ZLB constraint, the mean consumption-labor ratio is relatively constant for levels of inflation less than 4%. Intuitively, a higher inflation target increases the mean of inflation and decreases the volatility of inflation, since the economy is avoiding the ZLB constraint. These two effects almost cancel each other out. After a 4% inflation target, the change in the mean consumption-labor ratio with and without ZLB is the same.

There are two reasons why the consumption-labor ratio decreases with inflation. First, higher levels of inflation increase inefficient distortions in the relative prices across firms, decreasing aggregate productivity. Second, higher inflation raises the resources allocated for price changes since labor is not used for production

and consumption, but for price adjustment. I ignore the physical cost of repricing because it is quantitatively small. Next, I use the steady-state consumption-labor ratio to explain the low cost of inflation, since it remarkably describes the mean consumption-labor ratio across the business cycle.

Under a ZLB constraint and low inflation targets, welfare is increasing in inflation. Intuitively, since the consumption-labor ratio does not change much for low inflation targets, there are only benefits to increasing inflation as it reduces inefficient fluctuations of consumption and labor due to the ZLB constraint. Figure I-Panels B and D describe the frequency of a binding ZLB and the standard deviation of the output gap. The standard deviation of the output gap measures the fluctuations of labor and consumption resulting from inefficient movements of the price-marginal cost ratio due to nominal frictions. The output gap is independent of efficient variations due to real shocks or price dispersion. The standard deviation of the output gap with a ZLB constraint coincides with the model without a ZLB for levels of inflation target more than 3.5%.

Figure I describes the central intuition for the optimal inflation target in my MC model. At the optimal inflation target, the equilibrium allocation of consumption and labor are the same with or without a ZLB constraint. Since the mean consumption-labor ratio is almost constant for levels of inflation less than the optimal, the optimal inflation target is the level at which the ZLB does not generate inefficient volatility of consumption and labor.

The stochastic processes of consumption and labor are the same with or without a ZLB constraint at the optimal inflation target, even if the frequency of the ZLB is positive. Intuitively, for the ZLB to have an incidence in equilibrium allocation, this restriction has to be binding enough. If not, then interest rate smoothing of the desired interest rate in the Taylor rule (14) and (16) is enough to generate equilibrium allocations similar to those in the model without a ZLB constraint.

B. The Optimal Inflation Target in a Calvo Model

I compare my MC model with a Calvo model with no idiosyncratic shocks and identical preferences and technology. The parameters for the Calvo model are the same as in the MC model, except for $\sigma_{a,1} = \sigma_{a,2} = 0$, $\xi = 0.097$, and $\theta \rightarrow \infty$. This calibration without idiosyncratic shocks yields a benchmark against which I compare my model with the standard assumptions in the literature. A Calvo model with idiosyncratic shocks has a higher cost of inflation than without them, making the differences in the optimal inflation targets across models even larger (see Subsection III.C). Figure II reproduces Figure I for a similarly specified Calvo model.²⁰

²⁰I use indexation of prices for business cycle fluctuations in inflation in the Calvo model whenever the inflation target exceeds 3%. I use this form of indexation only in the Calvo model and not in the MC model. Absent this assumption, the model's global and local solutions are not stable for high inflation levels.

The optimal inflation in the Calvo model without idiosyncratic shocks is 1.3%. The optimal inflation target is close to the 1.5% optimal inflation in Coibion, Gorodnichenko and Wieland (2012).²¹ The striking property we see in the Calvo model is the significant cost of inflation reflected in the consumption equivalent. Increasing the inflation target from 0% to 6% in the Calvo model is equivalent to decreasing consumption by approximately 0.84% with and without a ZLB constraint. The cost of inflation is high also at low inflation targets: increasing the inflation target from 0% to 3% is equivalent to decreasing consumption by approximately 0.17%. The consumption equivalent of the same policy change is 0.04% in the MC model.

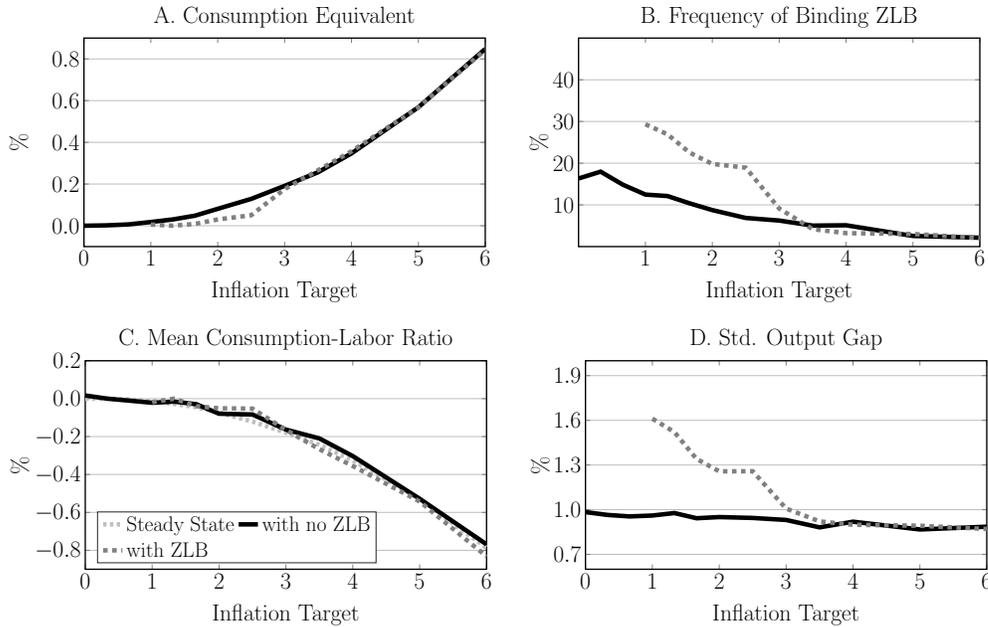


FIGURE II. CALVO MODEL: MAIN VARIABLES FOR OPTIMAL INFLATION

Note: Panel A describes the consumption equivalent in the Calvo model normalized by the optimal inflation target. Panel B describes the frequency of a binding ZLB. The frequency of a binding ZLB without a ZLB constraint refers to frequency of negative values of the interest rate. Panel C describes the mean consumption-labor ratio given by $(\mathbb{E}^{\pi}[C_t/(L_t\eta_{tz})]/\mathbb{E}^{\pi^*}[C_t/(L_t\eta_{tz})] - 1) \times 100$. Panel D describes the standard deviation of output gap given by $Std^{\pi}[\log(mc_t)] \times \frac{100}{\chi(1-\alpha)}$. The light grey dashed line describes the steady state moments, the solid black lines describe the moments without a ZLB and the dotted grey lines describe the moments with a ZLB. The scale of this figure coincide with Figure I.
Source: Author's calculations

²¹The optimal mean inflation in the Calvo (resp. MC) model is 0.23% (resp. 3.5%). The ZLB has a second-order effect on mean inflation, thereby generating a wedge between average inflation and the target inflation, i.e., the average inflation with *no* zero lower bound.

Similarly, as in the MC model, the consumption equivalent across inflation targets mimics the mean consumption-labor ratio at different levels of inflation with and without a ZLB at all levels of inflation. The elasticity of price dispersion to mean inflation implies this quantitative relation since there is no physical cost of price adjustment. Without a ZLB constraint, the consumption-labor ratio decreases by 0.19% between 0% and 3%, and 0.83% between 0% and 6%. With a ZLB, the mean consumption-labor is increasing between 1.0% and 1.3% inflation targets and starts falling after a 1.6% inflation target.

There is another quantitative difference between the MC model and the Calvo model: the ZLB constraint increases the volatility of the output gap more in the MC model than in the Calvo model. The increase in the volatility of the output gap in the MC (resp. Calvo) model is by 0.84% (resp. 0.52%) at the optimal inflation target in the Calvo model. While this mechanism is important to quantify the cost of the ZLB at low inflation targets, it is not crucial for understanding the optimal inflation target in the MC model. The argument is simple. Since the mean consumption-labor ratio is constant for levels of inflation less than the optimal, there are only benefits of increasing the inflation target for the stabilization of business cycles.

This paper focuses on the main consequence of changing the pricing model for the optimal inflation target: a low elasticity of the consumption-labor ratio to the mean inflation. Nevertheless, departing from a MC model to a Calvo model adds a new cost of the ZLB constraint. This constraint increases the volatility of inflation, therefore raising the mean of the dispersion of relative prices. If this mechanism is strong enough, then the ZLB is costly because the central bank cannot stabilize inflation and its effect on price dispersion. Thus, it is not because a central bank cannot stabilize aggregate output. This is the main benefit in the trade-off for the optimal inflation target in Coibion, Gorodnichenko and Wieland (2012) (see Online Appendix Section G for a further discussion). This mechanism is not quantitatively important in my model, as Figure I-Panel C shows.

C. Understanding the Optimal Inflation Target in the MC model

To understand the optimal inflation target in my model, one must first understand how the interaction between menu costs and idiosyncratic shocks reduces the cost of inflation given by price dispersion. Additionally, my MC model increases the business cycle cost of the ZLB constraint at a low inflation target compared to a Calvo model. In this section, I explain why the mean consumption-labor ratio and the volatility of the output gap are different in my MC model.

UNDERSTANDING THE MEAN CONSUMPTION-LABOR RATIO. — My MC model with 93% of price changes due to fat-tailed idiosyncratic shocks or zero menu cost of price adjustment generates a cost of inflation five times lower than a Calvo model. I now demonstrate this result is the outcome of the interaction between idiosyncratic shocks and menu costs.

To this achieve this goal, I compute the consumption-labor ratio at different levels of inflation in the Calvo and MC models with and without idiosyncratic shocks. I show that the MC model with idiosyncratic shocks generates the lowest cost of inflation, followed by the Calvo model without idiosyncratic shocks. These two models generate a lower cost of inflation than the MC model without idiosyncratic shocks and the Calvo model with idiosyncratic shocks. Thus, a low cost of inflation results from the *interaction* between menu cost and idiosyncratic shocks.

To maximize the clarity of the results reported here, I analyze how the cost of inflation—i.e., the consumption–labor ratio—changes in the steady-state across pricing models. As established above, the steady–state is a good approximation of the mean with business cycles.²²

Panels A and B of Figure III show the distribution of relative prices \tilde{p}_{ti} at different inflation targets in the MC model with and without idiosyncratic shocks. Panels C and D plot the consumption–labor ratio, and the frequency of price changes. Figure III-Panel E shows the width of the Ss bands in the MC model with and without idiosyncratic shocks: the (log) percentage difference between the adjustment triggers and the reset price.

At low levels of inflation, the consumption–labor ratio decreases faster in the MC model without idiosyncratic shocks than in the two versions of the Calvo model. This difference can be seen in panel C of Figure III. Three mechanisms explain this significant difference. First, price dispersion is discontinuous at zero in the MC model with no idiosyncratic shocks while it is continuous in the Calvo models—with and without idiosyncratic shocks. Price dispersion in the Calvo model is differentiable at zero (see the Online Appendix Section D for a formal proof), and therefore continuous. In the MC model with no idiosyncratic shocks, the width of the Ss bands is positive at all inflation levels. It follows that the distribution of relative prices jumps from a probability atom at zero inflation—since all the prices are at the optimal reset price—to a uniform probability distribution at any positive level of inflation (see Figure III-Panel A). The jump from a probability atom to a uniform distribution increases the cost of inflation because of the sudden steep increase in the dispersion of relative prices.

Two more mechanisms increase the cost of inflation in a MC model without idiosyncratic shocks. First, the width of the Ss bands is strictly increasing with inflation. Intuitively, firms optimally choose to increase the width of the Ss bands to save on the cost of price adjustment. Given that the mass of firms in the Ss bands is positive, there is a first-order effect of increasing inflation on price dispersion for the change within the width of the Ss bands. Second, an increase in inflation raises the frequency of price changes and the associated physical cost

²²I set the random menu cost sufficiently small ($\zeta = 0.001$) to generate a unique ergodic distribution in the MC model with no idiosyncratic shocks. I calibrate this version of the model with the same menu cost as the MC model with idiosyncratic shocks. I calibrate the variance of the idiosyncratic shocks in the Calvo model as 10% of the MC model. I keep the level of markups constant across inflation targets to focus on price dispersion and the physical cost of adjustments.

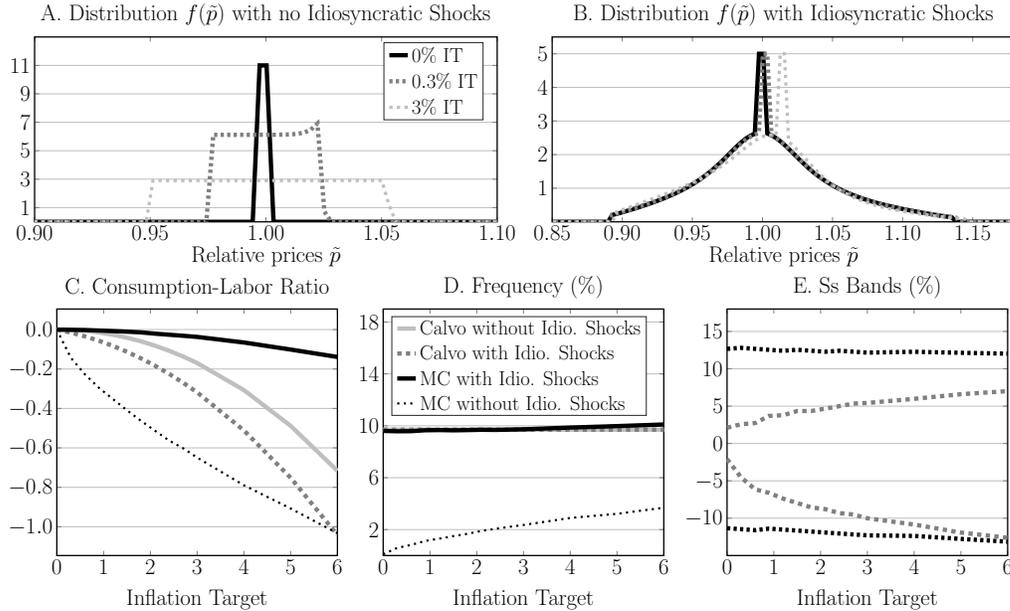


FIGURE III. STEADY STATE COST OF INFLATION

Note: Panels A and B describe the distribution of relative prices at 0%, 0.3%, and 3% inflation targets. Panel C describes the normalized consumption–labor ratio in the Calvo and MC models with and without idiosyncratic shocks. Panel D describes the frequency of prices across models. Panel E describes the (log) percentage difference between the upper and lower adjustment triggers and the reset price in the menu cost models. The distributions are rescaled by 100, and the probability atoms in these distributions are rescaled to fit in the graph.

Source: Author’s calculations

of changing prices (see Figure III-Panel D).

The MC model with idiosyncratic shocks has a high level of price dispersion (see Figure III-Panel B), and low elasticity of the consumption–labor ratio to inflation. The presence of idiosyncratic shocks mitigates the mechanisms mentioned above that create a high cost of inflation in a MC model without idiosyncratic shocks. Whenever idiosyncratic shocks are present, the width of the Ss bands, the distribution of relative prices, and frequency of price changes are almost constant for low levels of inflation. To understand why the width of the Ss bands is practically constant with inflation, note that due to large idiosyncratic shocks, firms respond mainly to idiosyncratic shocks and not inflation. Therefore, changes in inflation do not affect the width of the Ss bands at low levels of inflation.

Inflation also affects the law of motion of relative prices between price changes. Consequently, the dispersion of relative prices within the Ss bands and the frequency of price changes. The level of inflation relative to the volatility of idiosyncratic shocks determines these relations. Therefore, when the volatility of idiosyncratic shocks is significant, inflation has little effect on the dispersion of

relative prices.²³

Large idiosyncratic shocks decrease the cost of inflation in the MC model, while they increase the cost of inflation in the Calvo model. In this model, the presence of idiosyncratic shocks moves relative prices further away from the mean at a zero inflation target. Thus, changes in inflation have a considerable effect on the variance of relative prices. Figure III-Panel C shows this property.²⁴

UNDERSTANDING THE VOLATILITY OF OUTPUT GAP. — The volatility of the output gap in my MC model depends on the frequency of the ZLB. If the frequency of the ZLB is not sufficiently binding, then the frequency and the distribution of price changes do not respond to aggregate shocks. In this case, business cycle moments of my MC model are similar to the Calvo model. Table IV shows business cycle moments in the Calvo and MC models at the optimal inflation targets of the Calvo and MC models. As we can see, business cycle fluctuations in the Calvo and MC models at a 3.5% inflation target are similar. Below, I explain the volatility of the output gap when the ZLB does bind enough.

In the MC and Calvo models, when the ZLB is binding, the real interest rate is too high. A high real interest rate leads to excessive saving. Since the nominal interest rate cannot decrease due to the ZLB, this exacerbates the depression of spending and output, which in turn creates more deflationary pressure. Next, I show how the response of inflation to the output gap at the ZLB depends on the endogenous price flexibility in the MC model. I define the two mechanisms as: “Price flexibility due to inflation *during* ZLB periods,” and “Price flexibility due to inflation *before* ZLB periods.” Importantly, the inflation target affects the magnitude of these two mechanisms. Table IV shows the relevant moments to explain the mechanisms.

MECHANISM 1: PRICE FLEXIBILITY DUE TO INFLATION DURING ZLB PERIODS. —

At a 1.3% inflation target, depressed inflation implies a deflation. Thus, there is an *increase* in the fraction of repricing firms at the ZLB. The key feature in MC models is that this increase is not random across firms. A new mass of repricing firms hits the downward price adjustment trigger with a significant downward price adjustment (about 13%). These newly repricing firms reduce

²³Formally, Álvarez et al. (2018) show that price dispersion and the frequency of price changes depend on the ratio between inflation and variance of idiosyncratic shocks in a MC model with idiosyncratic shocks.

²⁴Formally, define Δ_{ss} as the misallocation of labor due to inefficient price dispersion in the steady-state. Δ_{ss} satisfies

$$\Delta_{ss} = \Omega(P_{ss}^*)^{-\gamma} + (1 - \Omega)\Pi^\gamma \mathbb{E} \left[\left(\frac{A_{ti}}{A_{t-1i}} \right)^{-\gamma} \right] \Delta_{ss}.$$

Here $\Pi^\gamma \mathbb{E} \left[\left(\frac{A_{ti}}{A_{t-1i}} \right)^{-\gamma} \right] = \Pi^\gamma e^{\gamma^2/2(\psi(\sigma_a^1)^2 + (1-\psi)(\sigma_a^2)^2)}$ is the accumulated dispersion between price changes. Therefore, the elasticity of price dispersion to inflation is increasing in the volatility of idiosyncratic shocks.

TABLE IV— BUSINESS CYCLE MOMENTS WITH ZLB IN CALVO AND MENU COST MODELS

Inflation Target	Menu Cost		Calvo	
	1.3 %	3.5 %	1.3%	3.5 %
Frequency of binding ZLB	0.309	0.035	0.270	0.042
Statistics for output gap and Inflation				
Output gap				
Unconditional std	1.539	0.808	1.456	0.864
Std conditional on binding ZLB	1.884	0.651	1.579	0.943
Std conditional on no binding ZLB	0.718	0.757	0.815	0.796
Inflation				
Unconditional mean	-0.57	3.50	0.25	3.65
Mean deviation conditional on binding ZLB	-1.18	-0.53	-0.47	-0.50
Mean deviation conditional on no binding ZLB	0.52	0.02	0.17	0.02
Unconditional std	1.08	0.26	0.36	0.25
Std conditional on binding ZLB	1.23	0.11	0.30	0.14
Std conditional on no binding ZLB	0.34	0.24	0.20	0.23
Inflation Components				
Frequency of price changes				
Unconditional mean	9.846	9.846		
Mean conditional on binding ZLB	10.225	9.672		
Mean conditional on no binding ZLB	9.676	9.852		
Menu cost inflation				
Unconditional mean	-0.07	0.39		
Mean deviation conditional on binding ZLB	-0.53	-0.25		
Mean deviation conditional on no binding ZLB	0.24	0.01		

Note: The table presents business cycle moments in the Calvo and MC models at 1.3% and 3.5% inflation targets at a quarterly frequency. The statistics for output and inflation are (log) percentage points. “Mean deviation” describes the log-deviation of the variable for the mean conditional on zero or positive rates. “Std conditional” describes the standard deviation of the variables in logs conditional on zero or positive rates.

Source: Author’s calculations

inflation. Since the ZLB is binding, this initial drop in inflation increases real interest rates, thereby depressing the output gap and inflation even further.

We can see this mechanism in Table IV. Since inflation at the ZLB is negative (i.e., -1.70%), there is a negative drift with the increase in the frequency of price changes, from an average of 9.84% to 10.22%. Moreover, this increment is asymmetric across the distribution of price changes. The variable that reflects this mechanism is the menu cost inflation. As we can see, the menu cost inflation at the ZLB is -0.60% during ZLB periods. Therefore, the new price changes are price drops, and these changes explain half of the decrease in inflation (-1.18% decrease in inflation vs. -0.53% decrease in menu cost inflation).

At a higher inflation target, the MC model yields a much weaker deflationary spiral. First, as shown in Table IV, there is a *decline* in the frequency of price

changes. The intuition for this result is clear: with positive inflation on average, if an economy hits the ZLB then there is a low inflation *but not deflation*. Even more, the menu cost inflation is close to zero when the ZLB is active. Hence, price changes are driven mostly by fat-tailed shocks and zero menu cost—not aggregate shocks. For this reason, business cycles in the Calvo and the MC models are similar at 3.5% inflation target.

MECHANISM 2: PRICE FLEXIBILITY DUE TO INFLATION BEFORE ZLB PERIODS. — The distribution of relative prices at the moment of entering a ZLB period affects the reaction of inflation to the output gap. The inflation target affects the average inflation, and therefore it can shape this distribution to mitigate deflation during ZLB periods. To show this mechanism, Figure IV plots the average distribution of relative prices at 1.3% and 3.5% inflation targets across simulations. The figure reveals that at a 1.3% inflation target, the distribution of relative prices is asymmetric with more (resp. less) mass of firms close to the downward (resp. upward) adjustment trigger than at a 3.5% inflation target. Thus, at a low inflation target, there is a relatively large set of firms near the upper Ss bands that explains the high elasticity of menu cost inflation to negative aggregate shocks when the ZLB is binding.

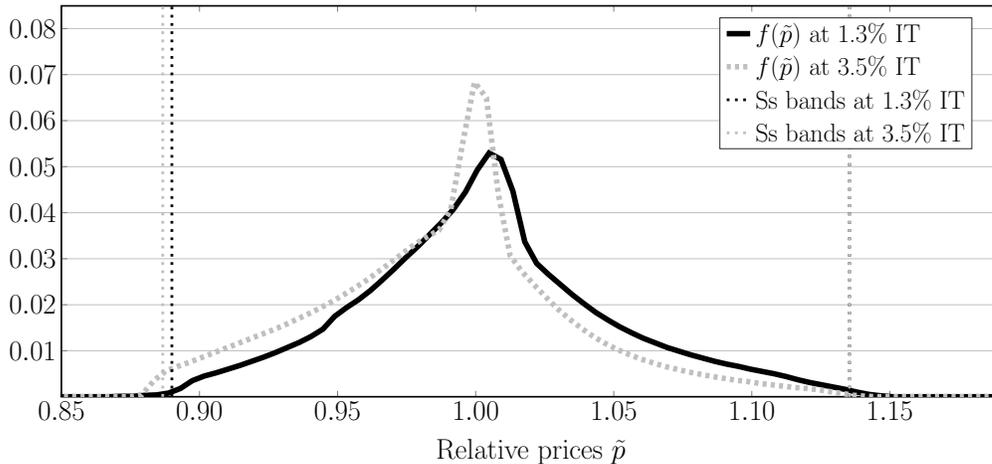


FIGURE IV. DISTRIBUTION OF RELATIVE PRICES AT 1.3% AND 3.5% INFLATION TARGET

Note: This figure describes the mean distribution of relative prices conditional together with the average Ss bands at 1.3% and 3.5% inflation targets. I shift the distribution of relative prices at 3.5% to make the right Ss band equal at both levels of inflation targets.

Source: Author's calculations

In conclusion, a higher inflation target reduces the business cycle volatility of consumption and labor for three reasons. First, it reduces the likelihood of hitting

the ZLB. Second, the inflation target determines the *level* of inflation whenever the ZLB is active. The level of inflation (together with a depressed output gap) implies the dynamics of the menu cost inflation at the ZLB, and therefore the feedback from the output gap to inflation. Third, it moves prices further away from the downward price adjustment trigger before entering periods with a binding ZLB. Consequently, it determines how much the menu cost inflation responds to aggregate shocks.

IV. Robustness

This section analyzes the sensitivity of the optimal inflation target to different calibrations of preferences and technologies. For each of these variant calibrations, I solve and analyze my MC model and the analogous Calvo model and reproduce Figures I and II in Online Appendix Section F. I perform this exercise in service of two objectives. First, I demonstrate the robustness of an optimal inflation target equal to 3.5% by changing and explaining critical parameters for the trade-offs of the optimal inflation target. Second, I situate my results within the literature that analyzes the optimal inflation target in the Calvo model. I show that while the cost of inflation in the Calvo model is sensitive to alternative parameterizations, the cost of inflation in my MC model is robust.

ARE MY RESULTS ROBUST TO THE ALTERNATIVE SPECIFICATION OF AN ANALOGOUS GOLOSOV AND LUCAS (2007) MODEL?. — I analyze the importance of random free price change opportunities and fat-tailed idiosyncratic shocks by calibrating the model without these two frictions. The pricing model analyzed here is similar to Golosov and Lucas (2007). I set $\xi = \psi = \sigma_{a1} = 0$ and calibrate the menu cost and the variance of idiosyncratic shocks to match the frequency and the size of price change. This calibration matches the physical cost of price adjustment.

Table F.I reproduces the micro price statistics of Table II. As we can see in the table, this calibration cannot reproduce the heterogeneity in the price-change distribution. For example, the kurtosis of price changes is 1.15. Table F.II shows aggregate moments found in Table II. As we can see in the table, this calibration increases the volatility of inflation by two. The increment does not have much of an effect on the rest of the business cycle statistics, since this increment is almost *i.i.d.*. As the table shows, from the benchmark calibration to this new calibration, the persistence of inflation falls from 0.58 to 0.44, and there is no change in the correlation with output.²⁵

The optimal inflation target in the Golosov and Lucas (2007) version of my model is 4%. This calibration reduces significantly the cost inflation (see Figure F.I.b). In this version of my model, increasing the inflation target from 0% to

²⁵Inflation affects equilibrium dynamics through the Euler equation and the Taylor rule. Since the Euler equation depends on expected inflation and the Taylor rule has a high smoothing parameter, *i.i.d.* fluctuations of inflation uncorrelated with the output gap do not affect equilibrium dynamics.

6% decreases the mean consumption-labor ratio by 0.02%. For this reason, the consumption equivalent is almost constant for levels of inflation greater than the optimal inflation target. Intuitively, reducing the randomness in the price adjustment policies, not only increases the slope of the Phillips curve but also decreases the cost of inflation, as firms can respond more efficiently to their state.²⁶

The characteristics of the optimal inflation target under this calibration support two results. First, the central intuition I provide for the optimal inflation target for my benchmark calibration in Section III.A holds in this case, even if inflation is much less costly. Second, increasing inflation volatility without affecting other business cycle moments, specifically by keeping the frequency of the ZLB constant, does not raise the optimal inflation target by much.

WHICH PARAMETERS ARE IMPORTANT FOR THE COST OF INFLATION?. — I now analyze the elasticity of the consumption-labor ratio to inflation. This elasticity consists of two other elasticities: (1) the elasticity of inefficient price dispersion to inflation determined by the pricing model, and (2) the elasticity of the consumption-labor ratio to inefficient price dispersion determined by the degree of strategic complementarities and the demand elasticity.

The elasticity of the consumption-labor ratio to inefficient price dispersion is important for calculating the cost of inflation. I show this result with two new calibrations. First, I increase the demand elasticity from 3 to 7. In this exercise, I adjust the labor subsidy to generate the same levels of markups. Since the objective of this exercise is to increase the cost of inflation *ceteris paribus* the frequency of the ZLB, I decrease the standard deviation of the innovations by 5%. Second, I change the level of strategic complementarities by increasing the share of intermediate inputs from 0.53 to 0.63.

To understand the role of the demand elasticity for the cost of inflation, consider an economy without intermediate inputs and government expenditures. In this economy, the consumption–labor ratio is given by $1/\Delta_t$, where $\Delta_t = \int_0^1 \tilde{p}_{ti}^{-\gamma} di$ and γ is the demand elasticity. Under a second–order approximation, it is easy to show that $-\log(\Delta_t) = -\frac{\gamma}{2}\text{Var}[\tilde{p}_{ti}]$. Therefore, for a given inefficient dispersion of relative prices, a larger demand elasticity increases the misallocation of inputs of production across firms since firms’ output is more responsive to the price.

The optimal inflation target is 3% in my MC model with a demand elasticity of 7. To comprehend this result, we need to understand how an increase in the demand elasticity affects the cost of inflation in my model. The direct effect of an increase in the demand elasticity is an increase in the misallocation of labor *ceteris paribus* inefficient price dispersion. We can observe the direct effect in the Calvo model. With a demand elasticity of 3 (resp. 7), an increase in inflation from 0 to 5% decreases the consumption-labor ratio by 0.6% (resp. 1.4%).

²⁶Baley and Blanco (2020) analyzes the relation between the slope of the Phillips curve and the cost of inflation.

In the MC model, there is also an indirect effect of increasing the demand elasticity. Since firms internalize the social cost of inefficient price dispersion, which is exacerbated by an increased demand elasticity, they respond by reducing the width of the Ss bands, which decreases the variance of relative prices. Intuitively, a larger demand elasticity increases the static profit function's curvature and therefore *reduces* the Ss bands by 1/3.²⁷ The indirect effect attenuates the total effect of increasing the demand elasticity on the cost of inflation. In a MC model with a demand elasticity of 7 (resp. 3), the consumption-labor ratio now decreases by 0.14% (resp. 0.10%) whenever there is an increase of the target inflation from 0 to 5%.

To understand the role of the intermediate inputs for the cost of inflation, I assume a steady-state economy where the aggregate cost of price changes is zero. Under this assumption, the (detrended) consumption labor ratio is given by

$$(24) \quad \tilde{C}_{ss}/L_{ss} = \left(\frac{\alpha \tilde{w}_{ss}}{1 - \alpha} \right)^\alpha \frac{1}{\Delta_{ss}} \left(1 - \left(\frac{\alpha \tilde{w}_{ss}}{1 - \alpha} \right)^{1-\alpha} \Delta_{ss} \right) \frac{1}{\eta_{g,ss}}, \text{ with } \tilde{w}_{ss} = \left(\frac{1}{\mathcal{M}_{ss}^\ell} \right)^{\frac{1}{1-\alpha}}.$$

Here \tilde{C}_{ss} and \tilde{w}_{ss} denote detrended consumption and real wages respectively. \mathcal{M}_{ss} denotes the level of markups. Under the observation that $\left(\frac{\alpha \tilde{w}_{ss}}{1 - \alpha} \right)^{1-\alpha} = \frac{\alpha}{\mathcal{M}_{ss}}$, the elasticity of the steady state consumption-labor ratio to price dispersion evaluated at $\Delta_{ss} = 1$ is

$$(25) \quad \left. \frac{d \log(C_{ss}/L_{ss})}{d \log(\Delta_{ss})} \right|_{\Delta_{ss}=1} = -\frac{1}{1 - \alpha/\mathcal{M}_{ss}}.$$

Therefore, the elasticity of the consumption-labor ratio to inefficient dispersion prices increases with the share of intermediate inputs.²⁸

The optimal mean inflation in the Calvo model is 1%, and in the menu cost model is 3% with a share of intermediate inputs of 0.63. In both models, there is an increase in the cost of inflation, as indicated by the consumption-labor ratio in Figures F.II.a and F.II.b. Note that the increase in the cost of inflation is still modest in the MC model: Increasing inflation from 0% to 4% decreases the mean consumption-labor ratio by 0.05%.

WHICH PARAMETERS ARE IMPORTANT FOR THE BENEFITS OF INFLATION?. — The optimal inflation target in the MC model is the minimum level at which the ZLB does not generate inefficient volatility of consumption and labor. Below, I change parameters critical for the frequency of the ZLB constraint and analyze

²⁷See the policy function in Figure F.VII in the Online Appendix

²⁸This paper features strategic complementarities with intermediate inputs since it is easy to calibrate. The finding that strategic complementarities affect the elasticity of the consumption-labor ratio to the dispersion of relative prices holds under decreasing returns to scale and the Kimball aggregator.

the change of the optimal inflation target.

I recalibrate the model to yield a lower frequency of the ZLB by changing two types of parameters. First, I reduce the standard deviation of the innovations of all the exogenous shocks by 10%. My model hits the ZLB 2.5% of the time at a 3% inflation target under this calibration. Second, I decrease the discount factor to $0.955^{1/12}(1+g)^{\sigma_{np}}$.²⁹ This calibration increases the steady state interest rate from 4% to 4.5% and reduces the frequency of the ZLB to 5% of the time at a 3% inflation target. Both exercises have the minimum frequency of binding ZLB supported by the empirical evidence in the literature.

The optimal inflation target with a lower volatility of aggregate shocks is 2.5%, and the optimal inflation target with a lower discount factor is 3.5%. These results depend on two key properties of my model. First, an increase of inflation from 0% to 3% decreases the consumption-labor ratio by at most 0.02%. Second, levels of inflation between 1% and 2% are never optimal for the reason explained in Section III.C. Since at low levels of inflation, when the economy hits the ZLB constraint, there is deflation and new price changes that magnifies the cost of the ZLB constraint.

Increasing the cost of business cycles does not change the optimal inflation target. Intuitively, as Figure I shows, in my benchmark calibration the consumption-labor ratio is almost constant for levels of inflation lower than the optimal one. Thus, there are only (small) benefits from increasing target inflation for business cycle stabilization. I show this argument by solving my model with Epstein-Zin preferences. The advantage of solving the model with Epstein-Zin preferences is that it increases the cost of business cycles without changing the properties of aggregate fluctuations (see Figures F.IV.a and F.IV.b and Table F.III with business cycle moments).

Household's preferences with Epstein-Zin utility and negative period utility are given by

$$(26) \quad U_t = u_t(C_t, L_t) - \beta \mathbb{E}_t [(-U_{t+1})^{1-\vartheta}]^{\frac{1}{1-\vartheta}}.$$

Here ϑ is the risk sensibility parameter.

To convey a simple intuition of the role of Epstein-Zin preferences, I compute the coefficient of relative risk aversion characterized by Swanson (2012) with no growth and no TFP shocks. Define the one period relative coefficient of risk aversion when labor supply is fixed as $\mathcal{R}^{ap} := -C \frac{d^2 U(C,L)}{dC^2} / \frac{dU(C,L)}{dC}$. \mathcal{R}^{ap} stands for the Arrow-Pratt coefficient of relative risk aversion. It measures the consumption equivalent of a lottery over consumption. Then, following Swanson (2012) definition of the coefficient of relative risk aversion with endogenous labor supply and

²⁹See Andrade et al. (2018) for a similar exercise with a Calvo pricing model.

Epstein-Zin preferences, we have

$$(27) \quad \mathcal{R} = \mathcal{R}^{ap} \left(1 + \vartheta \frac{1 - \sigma}{\sigma} \right).$$

Equation (27) shows how recursive preferences increases risk aversion without changing the intertemporal elasticity of substitution. I calibrate $\vartheta = -1$ to increase by 50% this measure of risk aversion. The optimal inflation target with these preferences continues to be 3.5%.

TAKEAWAY. — Across calibrations, the optimal inflation target in the MC model is the minimum level at which the ZLB does not generate inefficient volatility of consumption and labor.³⁰

V. Conclusion

In this paper I employ a menu cost model with realistic idiosyncratic shocks to identify an optimal inflation target of 3.5%. This undertaking required that I extend a menu cost model with idiosyncratic shocks to a standard New Keynesian framework with a Taylor rule subject to a ZLB constraint. The optimal inflation is pinned down by: (i) the relative insensitivity of price dispersion (which is the main cost in sticky price models) to the inflation target; (ii) the fact that likelihood of hitting the ZLB, and the magnitude of the selection effect when the ZLB is binding, are reduced with any increase in the inflation target.

Much could be learned from exploring the optimal inflation rate in different environments. For instance, many countries with inflation targets are small open economies, which may affect the optimal inflation target? Also, this paper focuses on price rigidities while assuming flexible wages. How does the optimal inflation change under sticky wages and downward wage rigidity? A framework similar to this paper's would be a suitable setting in which to answer these questions, relying on careful quantification of the micro structure of wage setting.

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³⁰In contrast, in the Calvo model the optimal inflation target is whenever the average inflation is close to zero (see Figure F.VIII in the Online Appendix).

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Online Appendix: Not for Publication

*Optimal Inflation Target in an Economy with Menu Costs and
Zero Lower Bound*

Andrés Blanco

A. DATA DESCRIPTION: U.S. MACROECONOMIC TIME SERIES

Table A.I describes the data sources for the U.S. macroeconomic time series. I construct output without investment and net exports to be consistent with my model. The following are the variables used to compute business cycle moments in the data (after HP filter): U.S. Bureau of Economic Analysis (1947-2017a)

- **Output** : $\log((GDP - I - NE)/(N \times P))$.
- **Government expenditure**: $\log(G/(N \times P))$
- **Consumption**: $\log(C/(P \times N))$
- **Real wage**: $\log(W/P)$.
- **Inflation**: $\log(P/P(-1))$.
- **Nominal Interest Rate**: $\log(1 + (DFP/100)^{1/4})$.

TABLE A.I— MACROECONOMIC TIME SERIES: DESCRIPTION AND SOURCES

Label	Short description	Source	Frequency	Seasonally Adjusted
GDP	Gross Domestic Product	BEA (1947-2017)	Q	SA
G	Gov. Cons. and Gross Investment	BEA (1947-2017)	Q	SA
C	Personal Consumption Expenditures	BEA (1947-2017)	Q	SA
NE	Net exports of goods and services	BEA (1947-2017)	Q	SA
I	Gross Private Domestic Investment	BEA (1947-2017)	Q	SA
P	GDP implicit price deflator	FRED (1947-2017)	Q	SA
DFP	Effective Federal Funds Rate, Percent	FRED (1955-2017)	Q	NSA
W	Business hourly compensation	BLS (1947-2017)	Q	SA
N	POPULATION LEVEL	BLS (1948-1976)	M	NSA
N	POPULATION LEVEL	BLS (1996-2005)	M	NSA

Note: The table describes the aggregate data I use to compute aggregate business cycle statistics in Table III. GDP, G, C, NE and I are in billions of dollars. Q and M denote frequency at monthly or quarterly level, respectively. SA and NSA denote seasonally adjusted at annual rate and not seasonally adjusted, respectively.

Source: GDP, G, C, NE, I: U.S. Bureau of Economic Analysis (1947-2017b). P: U.S. Bureau of Economic Analysis (1947-2017a). DFP: Board of Governors of the Federal Reserve System (US). W: U.S. Bureau of Labor Statistics (1947-2017b). N: U.S. Bureau of Labor Statistics (1948-1976) and U.S. Bureau of Labor Statistics (1976-2017a).

B. COMPUTATION AND DESCRIPTION OF MICRO-LEVEL PRICE STATISTICS

B.1. UK Data Description

The United Kingdom’s Office for National Statistics publishes the consumer price index (CPI) micro database, containing data from 1996 to the present day at a monthly frequency. The product-level price quotes and item-level price indexes used for the construction of the CPI were made publicly available on September 2012. The data cover all of the UK—England, Wales, Scotland, and Northern Ireland—and the data include a sample of items of the UK’s household final monetary consumption expenditures. In total, there are 31 million price quotes in the period between 1996 until the present at a monthly frequency. They cover more than 650 items. The Classification of Individual Consumption classifies each product by Purpose (COICOP) at the sectoral, group, and class levels. It excludes the housing portion of consumer prices, such as mortgage interest payments, house depreciation, insurance, and other house purchase fees.

For most item categories, the ONS collects price quotes of individual products by sampling outlets at 150 locations in the UK. Each elementary price quote collected through this method represents a unique product, sampled in a particular outlet. Prices for the remaining CPI items are collected centrally by the ONS with no fieldwork. Such items include shelter, university tuition fees, rail fares, and other services. Unfortunately, the ONS only provides item-level price indexes for these items. Since observing individual price trajectories is central for the study of micro-level price statistics, I exclude these items from my analysis.

For a small subset of items and regions, the ONS does not report outlet identifiers to comply with confidentiality guidelines. In such cases, there could be multiple price quotes with the same product-outlet identifier in a given month in the dataset. In most of these cases, there is no variation in prices that share an identifier in a given month. For the few cases in which I do observe different prices with the same identifier, I use information provided by the ONS on cumulative inflation at the unique good level and the algorithm described in Blanco and Cravino (2019) to recover: i) a unique price-trajectory associated with a product-outlet pair, and ii) a weight for each price trajectory reflecting the frequency of each price under a particular identifier.

B.2. Computation of Micro-Level Price Statistics

I apply several filters in the computation of price statistics to render the data compatible with the model. Table B.I describes the micro-level price statistics with the different filters.

Filter I: I drop outliers in the price change distribution (i.e., bottom and top 1% of price changes). I also drop price changes with a sales flag and noncomparable product substitutions, but I do include comparable product substitutions. I drop months in the sample with changes in the VAT (December 2008, January 2011, and January 2011). Following Nakamura and Steinsson (2008) and Klenow and Kryvtsov (2008), I impute price quotes for temporarily missing observations (missing for less than a year) with the last available price and out-of-season observations with the last available price. I redefine a product as a new product whenever there are more than twelve consecutive missing observations.

Filter II: As it is well known, the “sales” and “recovery” flags do not cover all the sales in the data. For this reason, I repeat the same filters described above with an additional filter on sales. The algorithm to detect sales is as follows. I identify the upper and lower bounds of periods with sales of length κ , by

$$(B.1) \quad \mathcal{F}_\kappa = \left\{ i, t : \sum_{j=0}^{\kappa} (p_{t+j}^i - p_{t-1+j}^i) = 0 \right\}, \quad \mathcal{L}_\kappa = \left\{ i, t : \sum_{j=0}^{\kappa} (p_{t-j}^i - p_{t-1-j}^i) = 0 \right\},$$

where p_t^i is the price quote of item i at time t . Importantly, $(i, t) \in \mathcal{F}_\kappa$, if and only if $(i, t+\kappa) \in \mathcal{L}_\kappa$. For any given $(i, t^*) \in \mathcal{F}_\kappa$, I drop all price changes between t^* and $t^* + \kappa$. I choose $\kappa = 3$ at monthly frequency for the implementation of this algorithm.

Filter III: I repeat the same filters described above (Filter II) with an additional filter on heterogeneity at COICOP class-level. I assign each product id to a single class. If I denote each id with the class-index i , then each id is in only one class C_j with $j = 1, 2, \dots, n_C$. I redefine the normalized

price change by item as

$$(B.2) \quad \Delta \bar{p}_t^i = \frac{\Delta p_t^i - \mathbb{E}[\Delta p_t^i | i \in C_j]}{\text{Std}[\Delta p_t^i | i \in C_j]} \text{Std}[\Delta p_t^i] + \mathbb{E}[\Delta p_t^i].$$

I then re-computed all price statistics with the normalized price changes. Online Appendix Section B.B.4 establishes the result that this aggregation “cleans” heterogeneity in the Calvo, MC and Taylor models.

Table B.I describes the price statistics with each filter using CPI weights to aggregate. As we can see in this table, there is a large dispersion in the price change distribution. The average frequency of price changes with Filter I is 0.16 at the monthly level. Computing the frequency as the inverse of the average duration yields a value of 0.97.³¹ With the additional filter on sales (Filter II, column 2), the average frequency of price changes and the frequency implied by the average duration decrease by 20%. The intuition behind this result is that there are many price changes preceded and followed by the same (base) price which the filter classified as unflagged sales. Finally, the filter concerning heterogeneity (Filter III, column 3) increases the mass of price changes near the mean price change (in absolute value), with the direct effect of decreasing the kurtosis of price change.

COMPARISON WITH THE U.S. ECONOMY AND OTHER LOW-INFLATION ECONOMIES. — Blanco and Cravino (2019) compares the average frequency and duration of price changes using the same methodology in Austria, Finland, and U.K., and find almost the same numbers in the three countries at item-level disaggregation. It is important to note that they do not find the same average frequency and duration in Chile and Mexico. Álvarez, Le Bihan and Lippi (2016) analyzes the kurtosis of price changes. My computation of the kurtosis between 4 or 5 is close to their findings.

How do U.K. statistics compare with U.S. statistics? I compare the statistics in my data to the statistics computed for U.S. by Nakamura and Steinsson (2008). I report these numbers in column US (NaSt) in Table B.I. They find an average frequency of price changes between 0.18-0.20 and a median frequency between 0.091-0.11—since they compute the frequency of price changes across two different time periods. These numbers are close to the frequency I compute for the U.K. without the additional filter on sales. For example, my average frequency is 0.169, while their average frequency is between 0.18 and 0.20. The median frequency in their data is between 0.091 and 0.11, if I compute the median frequency as they do, I find 0.11—slightly higher than the inverse of the average duration. They also find a kurtosis of price changes equal to 5.1 (see Álvarez, Le Bihan and Lippi (2016) for more information on this statistic) and a median size of price adjustment equal to 8.5%. Again, these statistics are close to the kurtosis and the median size of absolute price changes in the U.K. with only the first filter (4.7 for the kurtosis and 9% for the median). With this comparison, I can conclude that my data produce similar statistics that they report, when applying the same filters.

Klenow and Kryvtsov (2008) computes the mean and median of price changes excluding reported sales. The paper finds a mean equal to 11.3% (resp. 15% with the U.K. data) and a median of 9.7% (resp. 9.6% with U.K. data).

Midrigan (2011) computes the statistics for the absolute value of price changes for regular price changes and all price changes using U.S. supermarket data. Column US (Mi-No Sales) describes the statistics with his filter for sales. It is important to remark that his filter for sales is different from mine. While he drops all the price changes different from the modal price in a rolling window, I only drop price changes cumulated price changes equal to zero. The price change distribution I compute for the U.K. has a larger mean and standard deviation of the absolute value of price changes. The percentiles of the distribution are close except for the very large price changes. My data have a more dispersed price change distribution.

B.3. SMM Estimation with Price Statistics under Different Filters

Table B.II describes the estimated parameters with micro-level price statistics with different filters. The quantitative results are the following:

- **Filter I:** With this filter, idiosyncratic shocks are larger than with the final micro-level price statistics with filter III. Additionally, there is a 10% increase in the number of price changes due to zero menu costs.

³¹The inverse of the average duration is computed as $\left(\sum \omega_i \Omega_i^{-1}\right)^{-1}$, where ω_i is the class-level weight, and Ω_i is the class-level frequency.

TABLE B.I—U.K. MICRO-LEVEL PRICE STATISTICS WITH DIFFERENT FILTERS

Moments	Filters for UK			US (NaSt)	US (Mi-No Sales)
	I	II	III		
Absolute Value of Price Change					
Mean	0.154	0.139	0.153	–	0.110
Standar Deviation	0.159	0.151	0.137	–	0.080
Skewness	1.538	1.729	1.314	–	–
Kurtosis	5.059	5.801	4.321	–	4.020
5th percentile	0.010	0.008	0.010	–	–
10th percentile	0.014	0.014	0.020	–	0.030
25th percentile	0.034	0.031	0.049	–	0.050
50th percentile	0.096	0.080	0.114	0.085	0.090
75th percentile	0.223	0.194	0.217	–	0.013
90th percentile	0.386	0.357	0.356	–	0.210
95th percentile	0.507	0.470	0.447	–	–
Price Change					
Mean	0.004	0.005	0.005	–	–
Standar Deviation	0.222	0.205	0.205	–	–
Skewness	-0.135	-0.263	-0.050	–	–
Kurtosis	4.672	5.306	3.809	5.100	–
5th percentile	-0.402	-0.362	-0.356	–	–
10th percentile	-0.272	-0.238	-0.248	–	–
25th percentile	-0.095	-0.071	-0.113	–	–
50th percentile	0.019	0.021	0.012	–	–
75th percentile	0.098	0.085	0.115	–	–
90th percentile	0.252	0.223	0.254	–	–
95th percentile	0.372	0.336	0.355	–	–
Frequency	0.169	0.126	0.126	0.186-20.9	–
Inverse ave. duration	0.119	0.097	0.097	0.091-11.2	–

Note: The table describe micro-price statistics in U.K. and in U.S..

Source: The moments in column US (NaSt) are the ones computed by Nakamura and Steinsson (2008). The moments in column US (Mi-No Sale) are obtained from Midrigan (2011) with his algorithm after removing sales. Column I to III are author's calculations

- **Filter II:** With this filter the ratio of free price adjustments over total price adjustments is 62% larger than with filter III.
- **Filter IIIb:** This filter puts zero weight on the physical cost of menu cost. It finds parameters similar to the benchmark estimation.

TABLE B.II—SMM ESTIMATION WITH DIFFERENT FILTERS

Moments	(Data,model)			
	Filter I	Filter II	Filter III	Zero Weight Cost on P.A.(IIIb)
Absolute value of price change				
Mean	(0.154,0.150)	(0.139,0.126)	(0.153,0.155)	(0.153,0.155)
Standar deviation	(0.159,0.154)	(0.151,0.138)	(0.137,0.145)	(0.137,0.142)
Skewness	(1.538,1.540)	(1.729,1.668)	(1.314,1.156)	(1.314,1.208)
Kurtosis	(5.059,5.065)	(5.801,5.685)	(4.321,3.970)	(4.321,4.197)
10th percentile	(0.014,0.012)	(0.014,0.009)	(0.020,0.010)	(0.020,0.014)
25th percentile	(0.034,0.035)	(0.031,0.024)	(0.049,0.032)	(0.049,0.037)
50th percentile	(0.096,0.100)	(0.080,0.075)	(0.114,0.135)	(0.114,0.130)
75th percentile	(0.223,0.194)	(0.194,0.164)	(0.217,0.234)	(0.217,0.228)
90th percentile	(0.386,0.380)	(0.357,0.327)	(0.356,0.364)	(0.356,0.360)
Price change				
Mean	(0.004,0.012)	(0.005,0.018)	(0.005,0.015)	(0.005,0.019)
Standar deviation	(0.222,0.215)	(0.205,0.186)	(0.205,0.211)	(0.205,0.210)
Skewness	(-0.135,-0.081)	(-0.263,-0.131)	(-0.050,-0.078)	(-0.050,-0.134)
Kurtosis	(4.672,4.673)	(5.306,5.280)	(3.809,3.757)	(3.809,3.823)
10th percentile	(-0.272,-0.236)	(-0.238,-0.200)	(-0.248,-0.262)	(-0.248,-0.257)
25th percentile	(-0.095,-0.073)	(-0.071,-0.041)	(-0.113,-0.094)	(-0.113,-0.084)
50th percentile	(0.019,0.011)	(0.021,0.015)	(0.012,0.014)	(0.012,0.018)
75th percentile	(0.098,0.133)	(0.085,0.127)	(0.115,0.135)	(0.115,0.135)
90th percentile	(0.252,0.245)	(0.223,0.215)	(0.254,0.275)	(0.254,0.270)
Frequency of price change				
Cost of price adjustment × 100	(0.119,0.120)	(0.097,0.096)	(0.097,0.097)	(0.097,0.093)
Ratio free to total price adjustments	(0.004,0.004)	(0.004,0.003)	(0.004,0.004)	(—,0.004)
	(—,0.584)	(—,0.622)	(—,0.496)	(—,0.484)
Estimated parameters				
σ_{ah}	0.300	0.261	0.240	0.242
σ_{al}	0.028	0.018	0.018	0.017
ψ	0.055	0.045	0.070	0.067
θ	0.126	0.139	0.135	0.137
ξ	0.070	0.060	0.044	0.045

Note: Columns 1 to 3 show SMM price statistics and the estimated parameters using different filters. Column 4 describes the SMM estimates with a zero weight on the physical cost of price adjustments. The filters are described in Section B.B.3.
Source: Author's calculations

B.4. Heterogeneity and Aggregation in the Calvo, Menu Cost and Taylor Models

In this section of the online appendix, I show a closed-form solution for aggregating heterogeneous firms whenever their policy is described by MC, Calvo or Taylor pricing models. Assume a finite amount of firms indexed by i , with $i = 1, 2, \dots, N$. Each firm produces a share α_i of output. For simplicity, I assume continuous time.

- **Menu cost pricing model:** Each firm is described by a set of parameters $\{\sigma_i, \bar{p}_i\}$. The price gap follows a Brownian motion $dp_t^i = \sigma_i dW_t^i$ and the inaction region \mathcal{I} is the set $[-\bar{p}_i, \bar{p}_i]$. The distribution of price changes is given by

$$(B.3) \quad \Delta p_i = \begin{cases} \bar{p}_i & \text{with probability } 1/2 \\ -\bar{p}_i & \text{with probability } 1/2 \end{cases}$$

with a kurtosis of 1. The aggregate price change distribution is given by

$$(B.4) \quad \Delta p = \begin{cases} \bar{p}_i & \text{with probability } \alpha_i/2 \\ -\bar{p}_i & \text{with probability } \alpha_i/2 \end{cases}$$

with a kurtosis higher than 1. The normalized distribution of price change is defined as $\Delta p_i = \frac{\Delta p_i - \mathbb{E}[\Delta p_i]}{\text{Std}[\Delta p_i]} \text{Std}[\Delta p] + \mathbb{E}[\Delta p]$ with density

$$(B.5) \quad \Delta p = \begin{cases} \text{Std}[\Delta p] & \text{with probability } 1/2 \\ -\text{Std}[\Delta p] & \text{with probability } 1/2 \end{cases}$$

with kurtosis equal to 1.

- **Calvo pricing model:** Each firm is described by a set of parameters $\{\sigma_i, \lambda_i\}$. The price gap follows a Brownian motion $dp_t^i = \sigma_i dW_t^i$ and the firm's stopping time distribution follows an exponential distribution with density $f(\tau) = \lambda_i e^{-\tau/\lambda_i}$. The distribution of price changes is given by

$$(B.6) \quad f(\Delta p_i) = \frac{\sqrt{\lambda_i 0.5}}{\sigma_i} \exp\left(-\frac{\sigma_i}{\sqrt{\lambda_i 0.5}} |\Delta p|\right)$$

with a kurtosis of 6. The aggregate price change distribution is given by

$$(B.7) \quad f(\Delta p) = \{ f(\Delta p_i) \quad \text{with probability } \alpha_i$$

with a kurtosis higher than 6. The normalized price change defined as $\Delta p_i = \frac{\Delta p_i - \mathbb{E}[\Delta p_i]}{\text{Std}[\Delta p_i]} \text{Std}[\Delta p] + \mathbb{E}[\Delta p]$ with density

$$(B.8) \quad f(\Delta p) = \frac{1}{\sqrt{2} \text{Std}[\Delta p]} \exp\left(-\frac{\sqrt{2} |\Delta p|}{\text{Std}[\Delta p]}\right)$$

with kurtosis equal to 6.

- **Taylor pricing model:** Each firm is described by a set of parameters $\{\sigma_i, T_i\}$. The price gap follows a Brownian motion $dp_t^i = \sigma_i dW_t^i$ and the firm's stopping time distribution is a degenerate $Pr(\tau = T_i) = 1$. The distribution of price changes is given by

$$(B.9) \quad f(\Delta p_i) = \mathcal{N}(\Delta p, 0, \sigma_i)$$

with a kurtosis of 3. The aggregate price change distribution is a mixed normal distribution

$$(B.10) \quad f(\Delta p) = \{ \mathcal{N}(\Delta p, 0, \sigma_i) \quad \text{with probability } \alpha_i$$

with a kurtosis higher than 3. The normalized price change is defined as $\Delta p_i = \frac{\Delta p_i - \mathbb{E}[\Delta p_i]}{\text{Std}[\Delta p_i]} \text{Std}[\Delta p] + \mathbb{E}[\Delta p]$ with density

$$(B.11) \quad f(\Delta p) = \mathcal{N}(\Delta p, 0, \text{Std}[\Delta p])$$

with kurtosis equal to 3.

C. BUSINESS CYCLE STATISTICS WITH NO ZLB AND A 2% INFLATION TARGET

The Calvo model and my MC model have similar aggregate dynamics at a 2% inflation target when there is no ZLB constraint on the nominal interest rate. To formalize this claim, and to understand the general equilibrium effects in my model with and without a ZLB, I show that an econometrician with only aggregate data could not distinguish which model generates the data in finite samples.

To show the claim of similar aggregate dynamics, I compute two sets of statistics: impulse-response functions for structural shocks and business cycle statistics such as standard deviation, persistence, and correlations.

IMPULSE RESPONSE FUNCTION TO STRUCTURAL SHOCKS. — I compute the impulse response function to each shock as an econometrician would; I then check to see whether (and when) these two models differ over a finite sampling of 57 years—the length of the US time series in my sample. Because the impulse responses in each model are random variables with their confidence intervals, I can make statistical claims as an econometrician. Next, I describe the methodology to compute the impulse-response functions from simulated data of both models, together with their interval confidence.

Let IR_{tx}^{My} be the linear impulse response of variable y to the structural shock $x \in \{z, g, r, q, h\}$, in model M , after t periods. Let $\hat{IR}_{tx}^{MyT_s}$ be the estimate in a sample of length T_s . Next, I describe the steps to generate the random variable \hat{IR}_{tx}^{MT} using a Monte Carlo method.

1. Simulate the model for a large T . Let $\{X_t\}_{t=0}^T$ be the time series of the aggregate variables, S_t^X the vector that includes real marginal cost, price dispersion, nominal interest rate, and the exogenous variables, and S_t^Y the vector that includes output, inflation, labor and consumption.
2. Generate a random i.i.d. sequence of dates $\{t_i\}_{i=1}^N$ and draw $\{\{X_t\}_{t_i}^{t_i+T_s}\}_{i=1}^N$ samples of length T_s .
3. For each random sample $i = 1, 2, \dots, N$:

- i. Estimate the state space model:

$$(C.1) \quad S_{t+1}^X = \beta_x S_t^X + \Omega_x \epsilon_{t+1}^x \quad ; \quad S_t^Y = \beta_y S_t^X + \Omega_y \epsilon_t^y$$

- ii. Compute the impulse response to σ_x , where x denotes an aggregate exogenous variable $x \in \{z, g, r, q, h\}$. Compute the impulse-response $IR_x(t, i)^{MyT_s}$ using the model (C.1) from the simulated data.

4. $\{IR_x(t, i)^{MyT_s}\}_{i=1}^N$ is a random sample from the distribution $\hat{IR}_{tx}^{MyT_s}$.

Figure C.I plots the median impulse–response function to a government expenditure shock for the Calvo and MC models; it also shows the [2, 98] confidence interval of the difference between these two models' impulse responses. Figures C.II, C.III, and C.IV show similar graphs but for productivity, risk premium, and cost-push shocks (I omit the impulse–response function to a monetary shock, since in the final estimation they have almost no effect on any macroeconomic variable).

After a shock to government expenditure, the dynamics of all macroeconomic variables are close to each other in the MC model and the Calvo model. We can see this property in the confidence interval of the difference between the two impulse responses (marked by dashed black lines): the interval confidence for the difference between the two impulse responses always includes zero for all the variables. The only difference concerns inflation, which reacts more strongly the MC model than in the Calvo model (although the difference is significant for only one quarter). This difference is mainly in the business cycle fluctuations in the menu cost inflation and frequency of price changes, since, by construction, these variables do not respond to aggregate fluctuations in the Calvo model. The same behavior is observed after a cost-push shock, as Figure C.IV shows. For the risk premium shock and productivity shock, there is a larger response of inflation in the MC model than in the Calvo model; see (respectively) Figures C.II and C.III.

A second property we can observe from the impulse–response function across shocks is an increasing relationship between the impact of a structural shock to inflation, and the difference in inflation dynamics across both models. For example, after a cost-push shock or a government expenditure shocks, there is a small variation in inflation in both models, and also in their difference. This property does not hold for the risk premium shocks or productivity shocks. The main reason for this property is that as long as the frequency and distribution of price changes do not react much to the structural shocks, the slope of the Phillips curve is similar across models—slightly higher in the menu cost model.

That the equilibrium dynamics are similar in both models arises from two properties: i) a similar Phillips curve across models and ii) general equilibrium dynamics between households, firms, and the central bank. The main idea behind (ii) is that higher inflation volatility caused by a steeper Phillips curve is partially offset by the reaction of the nominal interest rate. Suppose, as a means of explaining this argument, that there occurs a structural shock that increases the output gap. The nominal interest rate depends on inflation, and so in the MC model, that interest rate will increase by more for a structural shock of a given size. Hence, the equilibrium output gap responds less, and so inflation does as well. We can see this effect in Figure C.II, where the nominal rate responds more in the MC model.

Business Cycle Statistics. The same properties hold for business cycle moments, as Table C.I shows. I compute these statistics using the Monte Carlo simulation. The first four rows of Table C.I describe the median business cycle statistics of the MC model, the second four rows describe the median business cycle statistics of the Calvo model, and the last four rows confirm that the [2, 98] percentiles of the difference in the standard deviations, persistences, and correlations of both models always includes zero.

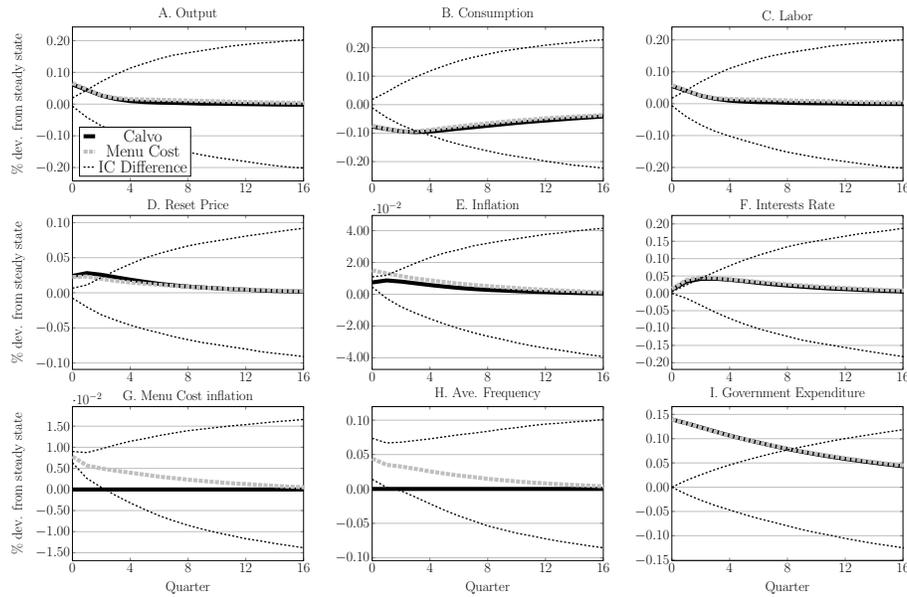


FIGURE C.I. IMPULSE-RESPONSE FUNCTION TO A GOVERNMENT EXPENDITURE SHOCK

Note: Panels A to I describe the impulse response functions of output, consumption, labor, reset price, quarterly inflation, quarterly nominal rate, average frequency in the quarter, and average menu cost inflation in the quarter to a government expenditure shock at a 2% inflation target in the models without a ZLB constraint to the nominal interest rate. All variables are percentage deviations from the steady state. The solid black lines describe the median impulse-response functions of the Calvo model. The dotted gray lines describe the median impulse-response functions of the MC model. The dashed black lines describe the 2nd and 98th percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 57 years.

Source: Author's calculations

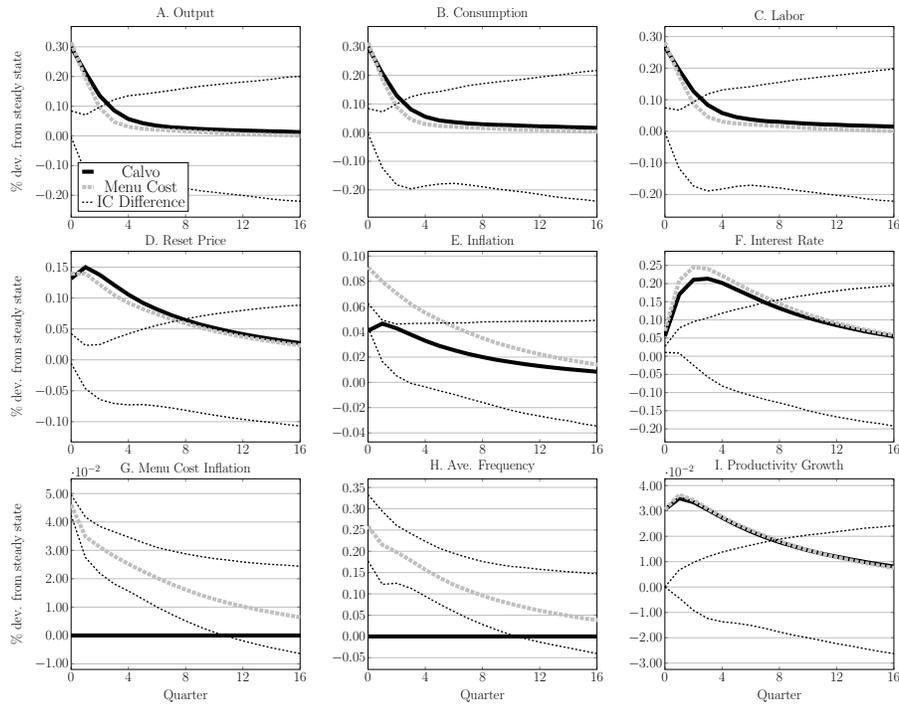


FIGURE C.II. IMPULSE-RESPONSE FUNCTION TO A PRODUCTIVITY GROWTH SHOCK

Note: Panels A to I describe the impulse response functions of output, consumption, labor, reset price, quarterly inflation, quarterly nominal rate, average frequency in the quarter, and average menu cost inflation in the quarter to a productivity shock at a 2% inflation target in the models without a ZLB constraint to the nominal interest rate. All variables are percentages deviation from the steady state. The solid black lines describe the median impulse-response functions of the Calvo model. The dotted gray lines describe the median impulse-response functions of the MC model. The dashed black lines describe the 2nd and 98th percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 57 years.

Source: Author's calculations

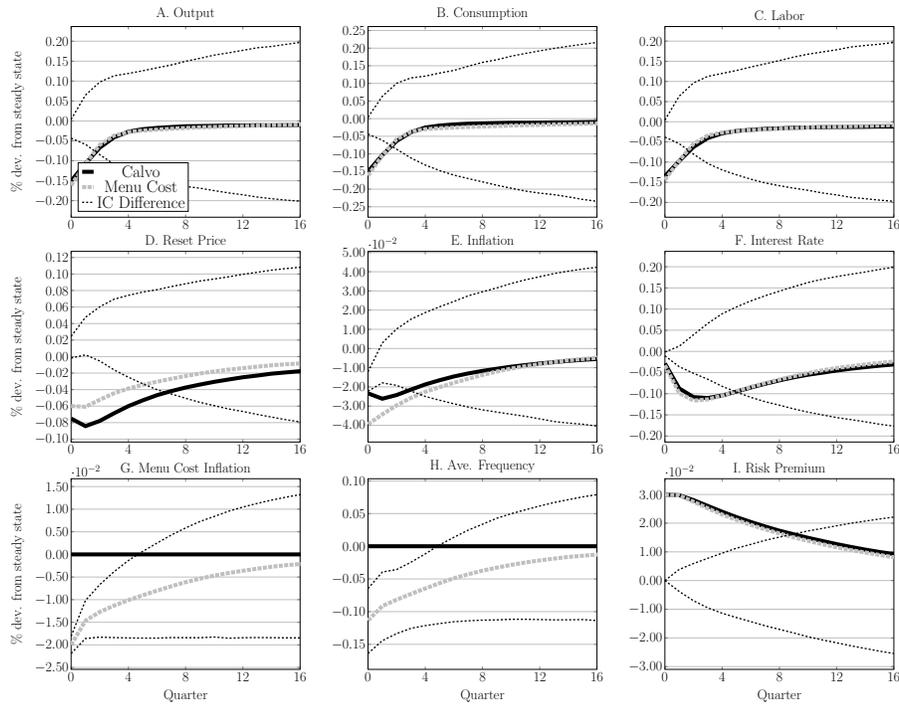


FIGURE C.III. IMPULSE-RESPONSE FUNCTION TO A RISK PREMIUM SHOCK

Note: Panels A to I describe the impulse response functions of output, consumption, labor, reset price, quarterly inflation, quarterly nominal rate, average frequency in the quarter, and average menu cost inflation in the quarter to a risk premium shock at a 0% inflation target in the models without a ZLB constraint to the nominal interest rate. All variables are percentage deviations from the steady state. The solid black lines describe the median impulse-response functions of the Calvo model. The dotted gray lines describe the median impulse-response functions of the MC model. The dashed black lines describe the 2nd and 98th percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 57 years.

Source: Author's calculations

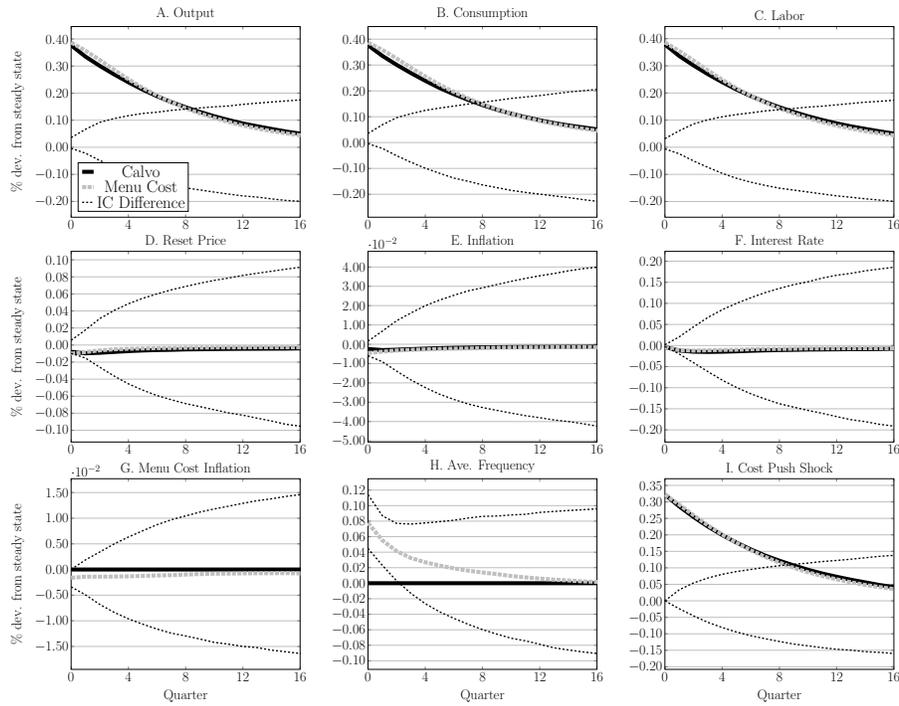


FIGURE C.IV. IMPULSE-RESPONSE FUNCTION TO A COST PUSH SHOCK

Note: Panels A to I describe the impulse response functions of output, consumption, labor, reset price, quarterly inflation, quarterly nominal rate, average frequency in the quarter, and average menu cost inflation in the quarter to a cost push shock at a 0% inflation target in the models without a ZLB constraint to the nominal interest rate. All variables are percentage deviations from the steady state. The solid black lines describe the median impulse-response functions of the Calvo model. The dotted gray lines describe the median impulse-response functions of the MC model. The dashed black lines describe the 2nd and 98th percentiles of the difference in the impulse-response functions between the two models over 5000 simulations of 57 years.

Source: Author's calculations

TABLE C.1—BUSINESS CYCLE MOMENTS AT A 2% INFLATION TARGET AND NO ZLB

Moments	Output	Consumption	Labor	Interest rate	Real wage	Inflation
Menu cost pricing model business cycle statistics—Median						
Standard Dev.	1.383	1.472	1.354	1.051	0.370	0.301
Autocorrelation	0.870	0.882	0.876	0.971	0.648	0.903
Corr. with GDP	1.000	0.909	0.999	0.165	0.332	0.227
Calvo pricing model business cycle statistics—Median						
Standard Dev.	1.411	1.502	1.398	0.991	0.427	0.186
Autocorrelation	0.877	0.889	0.885	0.974	0.730	0.933
Corr. with GDP	1.000	0.916	0.999	0.245	0.376	0.319
Difference between Calvo and menu cost models IC(2.98)						
Standard Dev.	(-0.49,0.40)	(-0.61,0.47)	(-0.50,0.39)	(-0.47,0.54)	(-0.28,0.10)	(0.00,0.22)
Autocorrelation	(-0.09,0.08)	(-0.09,0.08)	(-0.09,0.08)	(-0.04,0.03)	(-0.29,0.12)	(-0.12,0.04)
Corr. with GDP	(-0.00,0.00)	(-0.10,0.11)	(-0.00,0.00)	(-0.65,0.54)	(-0.44,0.43)	(-0.61,0.50)

Note: The table presents business cycle moments of the Calvo and MC models. Models' data are detrended with their respective productivities. The moments in the Calvo and in the MC models are the 50th percentiles of each statistic. Differences are the [2.98] percentiles confidence interval. All statistics are computed over 5000 simulations with the same length as in the data.

Source: Author's calculations

D. AUXILIARY THEOREMS

Let $\tilde{p}_{ti} = \frac{p_{ti}A_{ti}}{P_t}$ be the relative price times idiosyncratic productivity, $f_t(\tilde{p})$ the distribution of relative price, P_t^* the reset prices, $\Psi_t = \{\tilde{p}_t : \text{relative prices s.t. the firm chooses not to change the price in period } t\}$ the non-adjusting relative prices, and Ω_t the frequency of price changes. The following proposition characterizes the inflation dynamics in the terms of the frequency of price changes, the relative reset price, and the menu cost inflation.

PROPOSITION 1: *Define*

$$\mathcal{C}_t = \left\{ (\tilde{p}_{t-1}, \Delta a_t) : \frac{\tilde{p}_{t-1}e^{\Delta a_t}}{\Pi_t} \in \Psi_t \right\}.$$

Inflation dynamics are given by

$$(D.1) \quad \begin{aligned} \Pi_t &= \left(\frac{1 - \Omega_t}{1 - \Omega_t(P_t^*)^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_t, \\ \Omega_t &= \int_{(\tilde{p}_-, \Delta a) \notin \mathcal{C}_t} f_{t-1}(d\tilde{p}_-)g(d\Delta a), \\ \varphi_t &= \left(\int_{(\tilde{p}_-, \Delta a) \in \mathcal{C}_t} \frac{(\tilde{p}_-e^{\Delta a})^{1-\gamma}}{1 - \Omega_t} f_{t-1}(d\tilde{p}_-)g(d\Delta a) \right)^{\frac{1}{1-\gamma}}, \end{aligned}$$

where $g(\Delta a)$ is the distribution of quality shock innovations and $f_{t-1}(\tilde{p}_-)$ is the distribution of relative prices in the previous period.

PROOF:

From the price aggregator

$$(D.2) \quad \begin{aligned} P_t^{1-\gamma} &= \int_i (p_{ti}A_{ti})^{1-\gamma} di \quad \Leftrightarrow \\ 1 &= \Omega_t(P_t^*)^{1-\gamma} + (1 - \Omega_t) \int \frac{(p_{t-1i}A_{ti})^{1-\gamma}}{P_t^{1-\gamma}(1 - \Omega_t)} di \quad \Leftrightarrow \\ 1 &= \Omega_t(P_t^*)^{1-\gamma} + (1 - \Omega_t) \int \int \frac{(p_{t-1i}A_{t-1i}e^{\Delta a_i})^{1-\gamma}}{P_{t-1}^{1-\gamma}(1 - \Omega_t)\Pi_t^{1-\gamma}} di g(d\Delta a) \quad \Leftrightarrow \\ 1 &= \Omega_t(P_t^*)^{1-\gamma} + (1 - \Omega_t) \int \int \frac{(\tilde{p}_-e^{\Delta a})^{1-\gamma}}{(1 - \Omega_t)\Pi_t^{1-\gamma}} f_{t-1}(d\tilde{p}_-)g(d\Delta a) \quad \Leftrightarrow \\ \Pi_t &= \left(\frac{1 - \Omega_t}{1 - \Omega_t(P_t^*)^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_t \end{aligned}$$

PROPOSITION 2: *Let $\Delta_{ss}(\Pi_{ss})$ be the price dispersion in the Calvo model at a level of inflation Π . Then $\Delta_{ss}(\Pi_{ss})$ is continuous, with $\frac{d\Delta_{ss}(\Pi_{ss})}{d\Pi_{ss}} \Big|_{\Pi_{ss}=1} = 0$ and $\frac{d^2\Delta_{ss}(\Pi_{ss})}{d\Pi_{ss}^2} \Big|_{\Pi_{ss}=1} > 0$.*

PROOF:

The price dispersion in steady state is given by

$$(D.3) \quad \Delta_{ss} = \Omega \frac{P_{ss}^{-\gamma}}{1 - (1 - \Omega)\Pi_{ss}^\gamma}.$$

Using the steady-state reset price equations,

$$(D.4) \quad \Delta_{ss}(\Pi_{ss}) = \Omega^{\frac{1}{1-\gamma}} \frac{\left(1 - (1 - \Omega)\Pi_{ss}^{\gamma-1}\right)^{-\frac{\gamma}{1-\gamma}}}{1 - (1 - \Omega)\Pi_{ss}^\gamma}.$$

It is easy to see that $\Delta_{ss}(\Pi_{ss})$ is continuous, since it is the ratio of two continuous functions. For the first-order derivative, let $N(\Pi) = \left(1 - (1 - \Omega)\Pi_{ss}^{\gamma-1}\right)^{-\frac{\gamma}{1-\gamma}}$ and $D(\Pi) = 1 - (1 - \Omega)\Pi_{ss}^\gamma$. Then we have

that

$$(D.5) \quad \frac{dN(\Pi_{ss})}{d\Pi_{ss}} = \frac{\gamma}{1-\gamma} (1 - (1-\Omega)\Pi_{ss}^{\gamma-1})^{\frac{1}{\gamma-1}} (1-\Omega)(\gamma-1)\Pi_{ss}^{\gamma-2}$$

$$(D.6) \quad \frac{dD(\Pi_{ss})}{d\Pi_{ss}} = (1-\Omega)\gamma\Pi_{ss}^{\gamma-1}$$

Thus, the first derivative is given by

$$(D.7) \quad \begin{aligned} \left. \frac{d\Delta_{ss}(\Pi_{ss})}{d\Pi_{ss}} \right|_{\Pi_{ss}=1} &= \Omega^{\frac{1}{1-\gamma}} \frac{\Omega(\frac{\gamma}{1-\gamma}\Omega^{\frac{-1}{1-\gamma}}(1-\Omega)(\gamma-1)) - (1-\Omega)\gamma\Omega^{-\frac{\gamma}{1-\gamma}}}{\Omega^2}, \\ &= \Omega^{\frac{1}{1-\gamma}-2}\gamma(1-\Omega)(\Omega^{1-\frac{1}{1-\gamma}} - \Omega^{-\frac{\gamma}{1-\gamma}}), \\ &= 0. \end{aligned}$$

To show convexity, notice that $\Delta(\Pi_{ss})$ has a minimum value of 1 at $\Pi_{ss} = 1$. Therefore, at $\Pi_{ss} = 1$, this function must be convex.

For the next proof, I follow Swanson (2012) for the definition of relative risk aversion with labor supply and generalized recursive preferences. Swanson (2012) defines the coefficient of relative risk aversion as

$$(D.8) \quad \mathcal{R} := \frac{-\frac{\partial^2 u(C,L)}{\partial C^2} + \Lambda(C,L)\frac{\partial^2 u(C,L)}{\partial C\partial L}}{\frac{\partial u(C,L)}{\partial C}} \frac{C}{1 + \Lambda(C,L)w} + \vartheta C \frac{\frac{\partial u(C,L)}{\partial C}}{u(C,L)}$$

with $\Lambda = \frac{w\frac{\partial^2 u(C,L)}{\partial C^2} + \frac{\partial^2 u(C,L)}{\partial C\partial L}}{\frac{\partial^2 u(C,L)}{\partial L^2} + w\frac{\partial^2 u(C,L)}{\partial C\partial L}}$. Here, w denotes real wages.

PROPOSITION 3: *Define the household's preferences as*

$$(D.9) \quad U_t = u(C, L) - \beta \left(\mathbb{E}_t [-U_{t+1}]^{1-\vartheta} \right)^{1/(1-\vartheta)},$$

$$(D.10) \quad u(C, L) = \frac{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]^{1-\sigma}}{1-\sigma}.$$

Then the coefficient of relative risk aversion evaluated at the steady state is given by

$$(D.11) \quad \mathcal{R} = \mathcal{R}_{ap} \left(1 + \vartheta \frac{1-\sigma}{\sigma} \right),$$

where \mathcal{R}_{ap} denotes the Arrow-Pratt coefficient of relative risk aversion given by $\mathcal{R}_{ap} = -\frac{C\frac{\partial^2 u(C,L)}{\partial C^2}}{\frac{\partial u(C,L)}{\partial C}\partial C}$

PROOF:

Departing from the definition

$$(D.12) \quad \begin{aligned} \mathcal{R} &:= \frac{-\frac{\partial^2 u(C,L)}{\partial C^2} + \Lambda(C,L)\frac{\partial^2 u(C,L)}{\partial C\partial L}}{\frac{\partial u(C,L)}{\partial C}} \frac{C}{1 + \Lambda(C,L)w} + \vartheta C \frac{\frac{\partial u(C,L)}{\partial C}}{u(C,L)} \\ &= \frac{\sigma \left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]^{-\sigma-1} + \Lambda(C,L)\sigma \left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]^{-\sigma-1} \kappa L^\chi}{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]^{-\sigma}} \frac{C}{1 + \Lambda(C,L)w} + \vartheta C \frac{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]^{-\sigma}}{\frac{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]^{1-\sigma}}{1-\sigma}} \\ &= \frac{\sigma + \sigma\Lambda(C,L)\kappa L^\chi}{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]} \frac{C}{1 + \Lambda(C,L)w} + \vartheta C \frac{1-\sigma}{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]}. \end{aligned}$$

Using the optimality condition for labor supply, $\kappa L^\chi = w$, we have that

$$\begin{aligned} \mathcal{R} &= \frac{\sigma + \sigma \Lambda(C, L)w}{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]} \frac{C}{1 + \Lambda(C, L)w} + \vartheta C \frac{1 - \sigma}{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]} \\ (D.13) \quad &= \frac{\sigma C}{C - \kappa \frac{L^{1+\chi}}{1+\chi}} \left(1 + \vartheta \frac{1 - \sigma}{\sigma} \right). \end{aligned}$$

The term $\frac{\sigma}{C - \kappa \frac{L^{1+\chi}}{1+\chi}}$ is the Arrow-Pratt coefficient of relative risk aversion since

$$(D.14) \quad \mathcal{R}_{ap} = -\frac{C \frac{\partial^2 u(C, L)}{\partial C^2}}{\partial u(C, L) \partial C} = \sigma C \frac{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]^{-\sigma-1}}{\left[C - \kappa \frac{L^{1+\chi}}{1+\chi} \right]^{-\sigma}} = \frac{\sigma C}{C - \kappa \frac{L^{1+\chi}}{1+\chi}}.$$

Assume that the physical cost of price adjustment does not enter in the demand of labor. Then the feasibility constraint in the economy at the steady state is given by

$$(D.15) \quad L_{ss} = Y_{ss} \left(\frac{1 - \alpha}{\alpha w_{ss}} \right)^\alpha \Delta_{ss},$$

$$(D.16) \quad C_{ss} = Y_{ss} \left(1 - \left(\frac{\alpha w_{ss}}{1 - \alpha} \right)^{1-\alpha} \Delta_{ss} \right) \frac{1}{\eta_{g,ss}}.$$

PROPOSITION 4: *Assume that the physical cost of price adjustment does not enter in the demand for labor. Then*

$$(D.17) \quad \frac{d \log(C_{ss}/L_{ss})}{d \log(\Delta_{ss})} = -\frac{1}{1 - \alpha/\mathcal{M}_{ss}}.$$

PROOF:

The consumption-labor ratio at the steady-state is given by

$$(D.18) \quad C_{ss}/L_{ss} = \left(\frac{\alpha w_{ss}}{1 - \alpha} \right)^\alpha \frac{1}{\Delta_{ss}} \left(1 - \left(\frac{\alpha w_{ss}}{1 - \alpha} \right)^{1-\alpha} \Delta_{ss} \right) \frac{1}{\eta_{g,ss}}.$$

Since $w_{ss} = \left(\frac{1}{\mathcal{M}_{ss}^\chi} \right)^{\frac{1}{1-\alpha}}$ with \mathcal{M}_{ss} denoting the aggregate markup at the steady state, taking logs and differentials

$$\begin{aligned} \frac{d \log(C_{ss}/L_{ss})}{d \log(\Delta_{ss})} \Big|_{\log(\Delta_{ss})=0} &= -1 + d \log \left(1 - \left(\frac{\alpha}{1 - \alpha} \right)^{1-\alpha} \frac{1}{\mathcal{M}_{ss}^\chi} e^{\log(\Delta_{ss})} \right) \\ &= -1 + d \log \left(1 - \left(\frac{\alpha}{1 - \alpha} \right)^{1-\alpha} \frac{1}{\mathcal{M}_{ss}^\chi (1 - \alpha)^{\alpha-1} \alpha^{-\alpha}} e^{\log(\Delta_{ss})} \right) \\ &= -1 + d \log \left(1 - \frac{\alpha}{\mathcal{M}_{ss}^\chi} e^{\log(\Delta_{ss})} \right) \\ &= -1 - \frac{\frac{\alpha}{\mathcal{M}_{ss}^\chi}}{1 - \frac{\alpha}{\mathcal{M}_{ss}^\chi}} \\ (D.19) \quad &= -\frac{1}{1 - \alpha/\mathcal{M}_{ss}}. \end{aligned}$$

E. COST OF INFLATION WITHOUT QUALITY SHOCKS

This section computes the steady state cost of inflation in two versions of my model: the model with quality shocks and the model without quality shocks. The two versions of the model produce a similar (if not the same) steady state cost of inflation. Since price dispersion with and without business cycles is almost the same—as I show in the main text—I provide suggestive evidence that the assumption of quality shocks is not quantitatively relevant for the main cost of inflation that is given by inefficient dispersion of relative prices.

The general equilibrium framework in the two versions of the model is the same as in Section I; thus, I skip its description. I only change the technology of the intermediate good producer and the technology for price changes, which I describe next.

MODEL WITH QUALITY SHOCKS. — The technology of the final good producer is given by

$$(E.1) \quad Y_t = \left(\int_0^1 \left(\frac{y_{ti}}{A_{ti}} \right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}},$$

where Y_t denotes output, y_{ti} denotes intermediate firms' output, and A_{ti} is the firms' quality shocks.

The intermediate good firm i produces output y_{ti} using labor l_{ti} and material n_{ti} , and that firm's productivity is a function of an idiosyncratic component A_{ti} and an aggregate component η_{tz} according to

$$(E.2) \quad y_{ti} = A_{ti} n_{ti}^\alpha (\eta_{tz} l_{ti})^{1-\alpha}.$$

Here A_{ti} follows the stochastic process described in equation (9) in the main text. The technology for price changes and the firms' problem are the same as in the main text, and they are described in equations (10) and (11), respectively.

MODEL WITHOUT QUALITY SHOCKS. — Following the same notation as before, the technology of the final good producer is given by

$$(E.3) \quad Y_t = \left(\int_0^1 y_{ti}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}.$$

The intermediate good firm i produces output using the technology described in (E.2).

The quality shock A_{ti} follows a first order auto-regressive process given by

$$(E.4) \quad \log(A_{ti}) = \begin{cases} \rho \log(A_{t-1i}) + \eta_{ti}^1 & \text{with probability } \psi \\ \rho \log(A_{t-1i}) + \eta_{ti}^2 & \text{with probability } 1 - \psi \end{cases} ; \quad \eta_{ti}^k \sim i.i.d N(0, \sigma_{ak}).$$

Firms face a stochastic physical cost of changing their price equal to $\theta W_t A_{ti}^{\gamma-1}$. I scale the adjustment cost by the factor $A_{ti}^{\gamma-1}$ to keep the firm's decision problem homogeneous as its size varies. Finally, with probability ζ the firm faces a zero menu cost.

The intermediate firm's problem is given by

$$(E.5) \quad \max_{p_{ti}} \mathbb{E} \left[\sum_{t=0}^{\infty} Q_t A_{ti}^{\gamma-1} \Phi_{ti} \right] \quad \text{subject to} \\ \Phi_{ti} = \left(\frac{A_{ti} p_{ti}}{P_t} \right)^{-\gamma} Y_t \left(p_{ti} A_{ti} - \iota(1-\tau)(W_t/\eta_{tz})^{1-\alpha} P_t^\alpha \right) - I(p_{t-1i} \neq p_{ti}) W_t \theta_{ti},$$

where I continue using the same notation as in Section I.

CALIBRATION OF THE MODELS AND MICRO-LEVEL PRICING MOMENTS. — I calibrate the model with quality shocks as in the main text. In the case of the model without quality shocks, I use the same parameters as in the model with quality shocks, and I set $\rho = 0.98$. The results are robust to values of $\rho = 0.94$ and $\rho = 0.99$ as long as there is an appropriate adjustment of the variance of idiosyncratic shocks. Table E.I shows the micro-level price moments in both models. As we can see, both models generate similar micro-level price statistics.

TABLE E.I—MICRO PRICING MOMENTS: MODELS WITH AND WITHOUT QUALITY SHOCKS AND DATA

Absolute value of price changes	Data	Model with Quality Shocks	Model with No Quality Shocks
Mean	0.153	0.152	0.143
Standard deviation	0.137	0.137	0.129
Skewness	1.314	1.137	1.232
Kurtosis	4.321	4.329	4.421
10th percentile	0.020	0.013	0.011
25th percentile	0.049	0.036	0.034
50th percentile	0.114	0.127	0.122
75th percentile	0.217	0.220	0.206
90th percentile	0.356	0.351	0.325
Price changes			
Standar deviation	0.205	0.204	0.192
Kurtosis	3.809	3.810	3.823
Frequency with implied duration	0.097	0.097	0.096

Note: The table presents selected moments of the micro-level price statistics in the models with and without quality shocks.

Source: Author's calculations

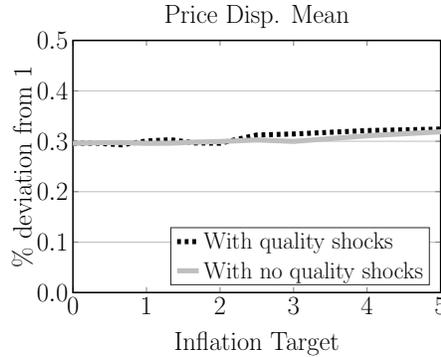


FIGURE E.I. STEADY STATE PRICE DISPERSION WITH AND WITHOUT QUALITY SHOCKS

Note: This figure describes price dispersion as a function of the inflation target in the MC models with and without quality shocks. The dotted black line plots price dispersion in the model with quality shocks given by $(\Delta_{ss}^Q - 1) \times 100$, where $\Delta_{ss}^Q = \int \tilde{p}^{-\gamma} f(\tilde{p}) d\tilde{p}$ and $f(\tilde{p})$ denotes the distribution of relative prices. The solid gray line plots price dispersion in the model with no quality shocks $(\Delta_{ss}^Q \Psi - 1) \times 100$, where $\Delta_{ss}^Q = \int \tilde{p}^{-\gamma} A^{\gamma-1} f(\tilde{p}, A) d\tilde{p} dA$ and $f(\tilde{p}, A)$ denotes the distribution of relative prices and productivities. The factor Ψ is chosen to match the level of price dispersion in both models.

Source: Author's calculations

STEADY STATE PRICE DISPERSION. — The elasticity of price dispersion to inflation is the main cost of inflation in sticky price models. Figure E.I shows steady-state price dispersion as a function of the inflation target in the models with and without quality shocks. In the case of the model with quality shocks, it shows the percentage deviation with respect to 1, as in the main text. In the case of the model with no quality shocks, I renormalize the price dispersion to match its counterpart in the model with quality shocks at a 0% inflation target. The argument for this decision is that the level of labor dispersion across models is different, since in the model without quality shocks, there is an efficient dispersion of output across firms. As we can see in Figure E.I, the cost of inflation across models is almost identical. Therefore, we can conclude that the main cost of inflation related to price dispersion is not affected by the assumption of quality shocks.

F. ROBUSTNESS EXERCISES

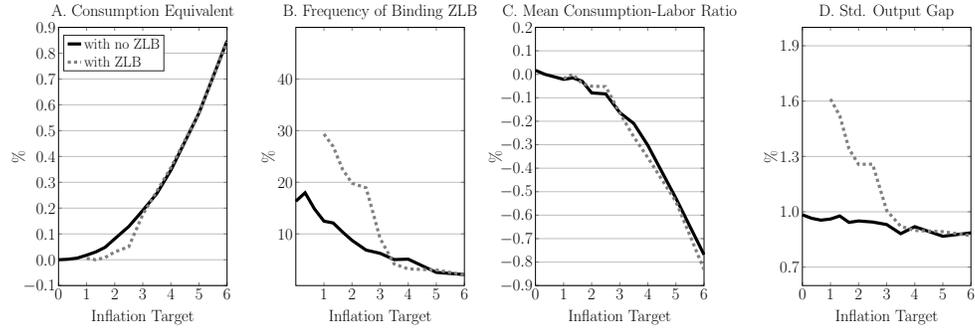


FIGURE F.I.A. OPTIMAL INFLATION TARGET: BENCHMARK CALIBRATION IN THE CALVO MODEL

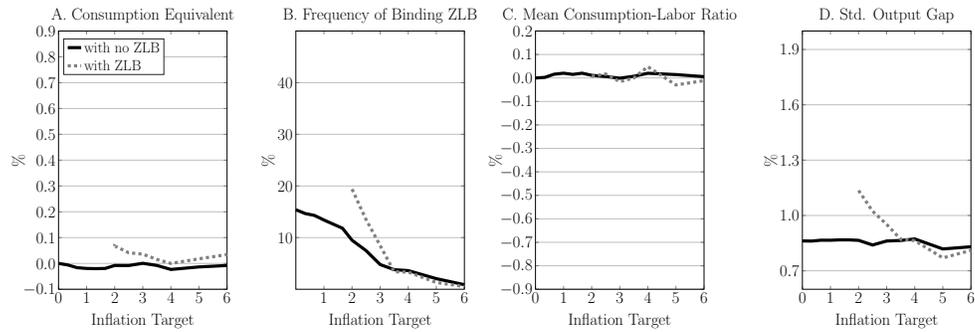


FIGURE F.I.B. OPTIMAL INFLATION TARGET: GOLOSOV AND LUCAS MENU COST MODEL

Note: Panels A describes the consumption equivalent in the MC and Calvo models normalized by the optimal inflation target. Panels B describe the frequency of a binding ZLB. The frequency of a binding ZLB without a ZLB constraint refers to frequency of negative values of the interest rate. Panels C describe the mean consumption-labor ratio given by $(\mathbb{E}^{\bar{\pi}}[C_t/(L_t\eta_{tz})]/\mathbb{E}^{\bar{\pi}^*}[C_t/(L_t\eta_{tz})] - 1) \times 100$. Panel D describes the standard deviation of the output gap given by $\text{Std}^{\bar{\pi}}[\log(m_{ct})] \times \frac{100}{\chi(1-\alpha)}$. The solid black lines describe the moments without a ZLB and the dotted gray lines describe the moments with a ZLB. The scale of figures F.I.a and F.I.b coincide.

Source: Author's calculations

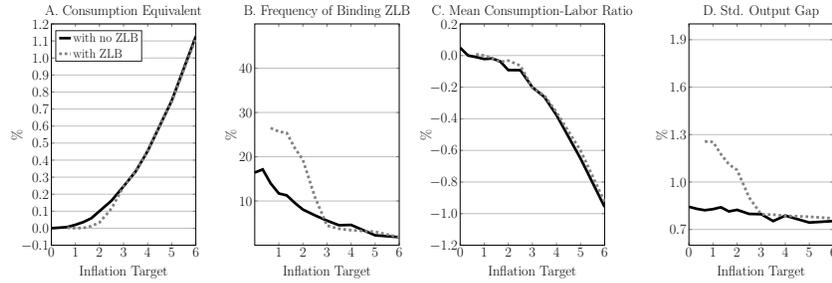


FIGURE F.II.A. OPTIMAL INFLATION TARGET: SHARE OF INTERMEDIATE INPUT OF 0.63 IN THE CALVO MODEL

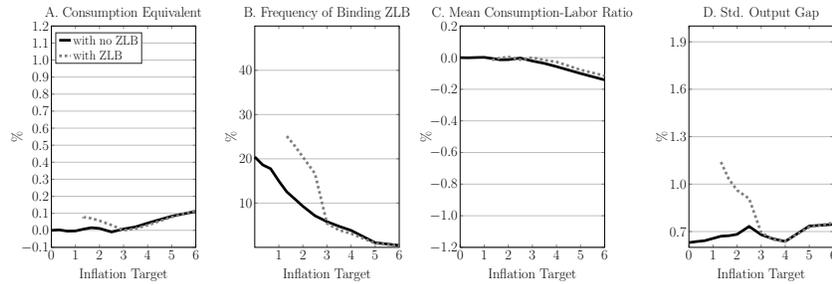


FIGURE F.II.B. OPTIMAL INFLATION TARGET: SHARE OF INTERMEDIATE INPUT OF 0.63 IN THE MENU COST MODEL

Note: Panels A describes the consumption equivalent in the MC and Calvo models normalized by the optimal inflation target. Panels B describe the frequency of a binding ZLB. The frequency of a binding ZLB without a ZLB constraint refers to frequency of negative values of the interest rate. Panels C describe the mean consumption-labor ratio given by $(\mathbb{E}^{\bar{\pi}}[C_t/(L_t\eta_{tz})]/\mathbb{E}^{\bar{\pi}^*}[C_t/(L_t\eta_{tz})] - 1) \times 100$. Panel D describes the standard deviation of the output gap given by $\text{Std}^{\bar{\pi}}[\log(mci)] \times \frac{100}{\chi(1-\alpha)}$. The solid black lines describe the moments without a ZLB and the dotted gray lines describe the moments with a ZLB. The scale of figures F.II.a and F.II.b coincide.

Source: Author's calculations

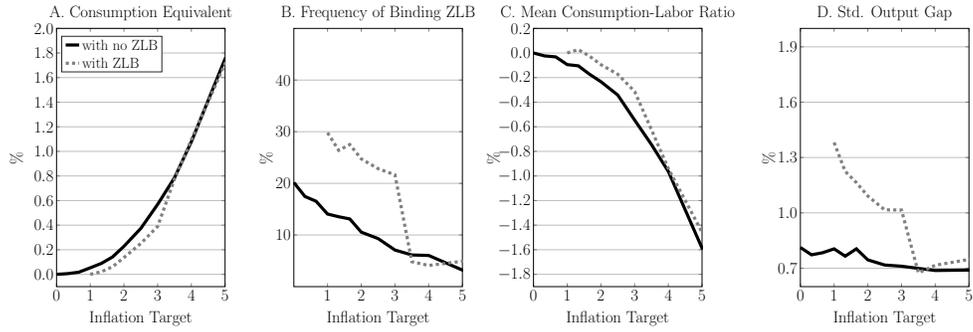


FIGURE F.III.A. OPTIMAL INFLATION TARGET: DEMAND ELASTICITY EQUAL TO 7 IN THE CALVO MODEL

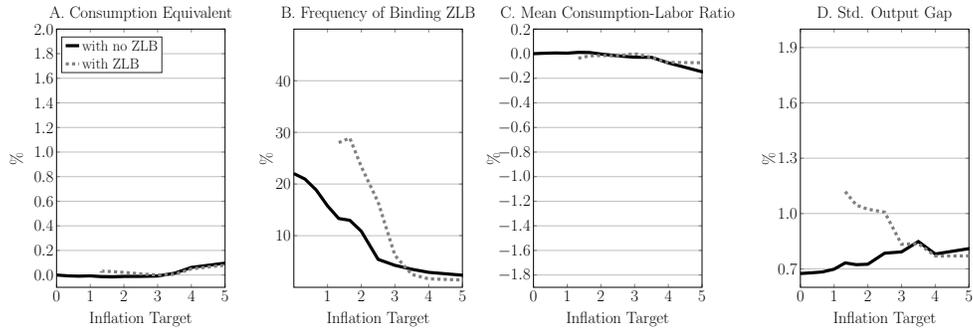


FIGURE F.III.B. OPTIMAL INFLATION TARGET: DEMAND ELASTICITY EQUAL TO 7 IN THE MENU COST MODEL

Note: Panels A describes the consumption equivalent in the MC and Calvo models normalized by the optimal inflation target. Panels B describe the frequency of a binding ZLB. The frequency of a binding ZLB without a ZLB constraint refers to frequency of negative values of the interest rate. Panels C describe the mean consumption-labor ratio given by $(\mathbb{E}^{\bar{\pi}}[C_t/(L_t\eta_{tz})]/\mathbb{E}^{\bar{\pi}^*}[C_t/(L_t\eta_{tz})] - 1) \times 100$. Panel D describes the standard deviation of the output gap given by $Std^{\bar{\pi}}[\log(mc_t)] \times \frac{100}{\chi(1-\alpha)}$. The solid black lines describe the moments without a ZLB and the dotted gray lines describe the moments with a ZLB. The scale of figures F.III.a and F.III.b coincide.

Source: Author's calculations

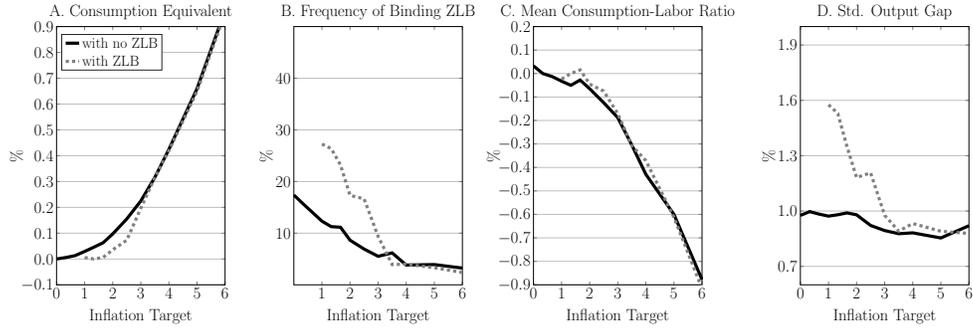


FIGURE F.IV.A. OPTIMAL INFLATION TARGET: RECURSIVE UTILITY IN THE CALVO MODEL

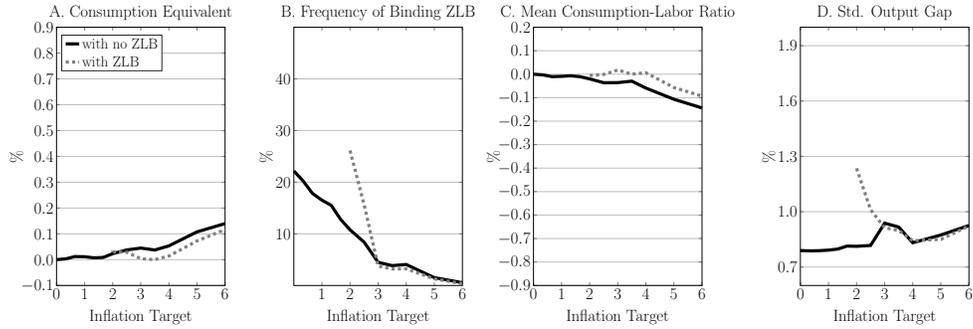


FIGURE F.IV.B. OPTIMAL INFLATION TARGET: RECURSIVE UTILITY IN THE MENU COST MODEL

Note: Panels A describes the consumption equivalent in the MC and Calvo models normalized by the optimal inflation target. Panels B describe the frequency of a binding ZLB. The frequency of a binding ZLB without a ZLB constraint refers to frequency of negative values of the interest rate. Panels C describe the mean consumption-labor ratio given by $(\mathbb{E}^{\pi}[C_t/(L_t\eta_{tz})]/\mathbb{E}^{\pi^*}[C_t/(L_t\eta_{tz})] - 1) \times 100$. Panel D describes the standard deviation of the output gap given by $\text{Std}^{\pi}[\log(mct)] \times \frac{100}{\chi(1-\alpha)}$. The solid black lines describe the moments without a ZLB and the dotted gray lines describe the moments with a ZLB. The scale of figures F.IV.a and F.IV.b coincide.

Source: Author's calculations

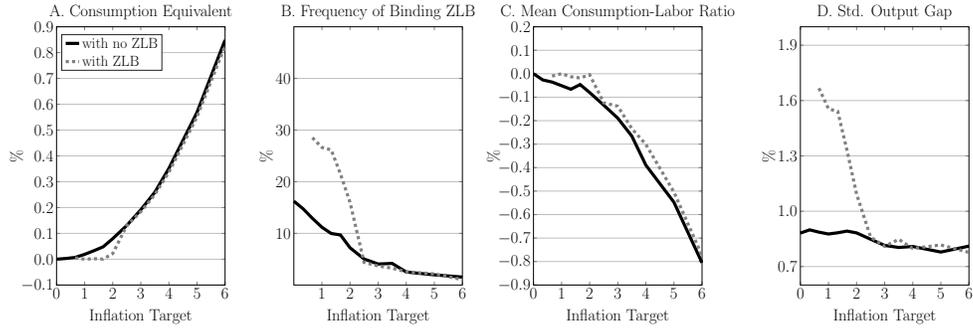


FIGURE F.V.A. OPTIMAL INFLATION TARGET: LOWER VOLATILITY OF AGGREGATE SHOCKS IN THE CALVO MODEL

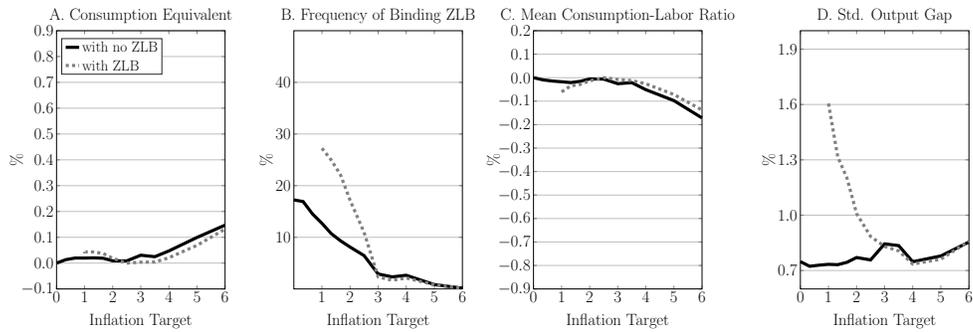


FIGURE F.V.B. OPTIMAL INFLATION TARGET: LOWER VOLATILITY OF AGGREGATE SHOCKS IN THE MENU COST MODEL

Note: Panels A describes the consumption equivalent in the MC and Calvo models normalized by the optimal inflation target. Panels B describe the frequency of a binding ZLB. The frequency of a binding ZLB without a ZLB constraint refers to frequency of negative values of the interest rate. Panels C describe the mean consumption-labor ratio given by $(\mathbb{E}^{\bar{\pi}}[C_t/(L_t\eta_{tz})]/\mathbb{E}^{\bar{\pi}^*}[C_t/(L_t\eta_{tz})] - 1) \times 100$. Panel D describes the standard deviation of the output gap given by $Std^{\bar{\pi}}[\log(mc_t)] \times \frac{100}{\chi(1-\alpha)}$. The solid black lines describe the moments without a ZLB and the dotted gray lines describe the moments with a ZLB.

Source: Author's calculations

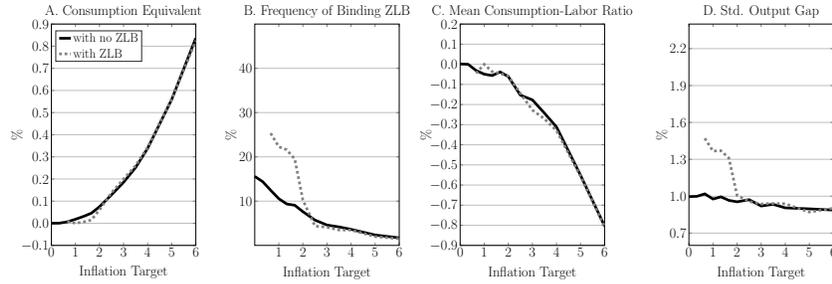


FIGURE F.VI.A. OPTIMAL INFLATION TARGET: 4.5% OF STEADY STATE REAL INTEREST RATE IN THE CALVO MODEL

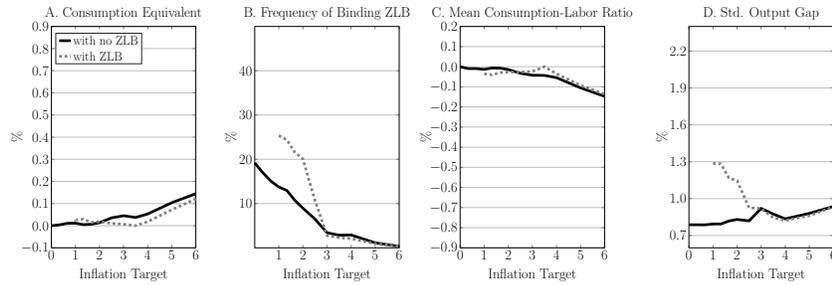


FIGURE F.VI.B. OPTIMAL INFLATION TARGET: 4.5% OF STEADY STATE REAL INTEREST RATE IN THE MENU COST MODEL

Note: Panels A describes the consumption equivalent in the MC and Calvo models normalized by the optimal inflation target. Panels B describe the frequency of a binding ZLB. The frequency of a binding ZLB without a ZLB constraint refers to frequency of negative values of the interest rate. Panels C describe the mean consumption-labor ratio given by $(\mathbb{E}^{\bar{\pi}}[C_t/(L_t\eta_{tz})]/\mathbb{E}^{\bar{\pi}^*}[C_t/(L_t\eta_{tz})] - 1) \times 100$. Panel D describes the standard deviation of the output gap given by $\text{Std}^{\bar{\pi}}[\log(mc_t)] \times \frac{100}{\chi(1-\alpha)}$. The solid black lines describe the moments without a ZLB and the dotted gray lines describe the moments with a ZLB. The scale of figures F.VI.a and F.VI.b coincide.

Source: Author's calculations

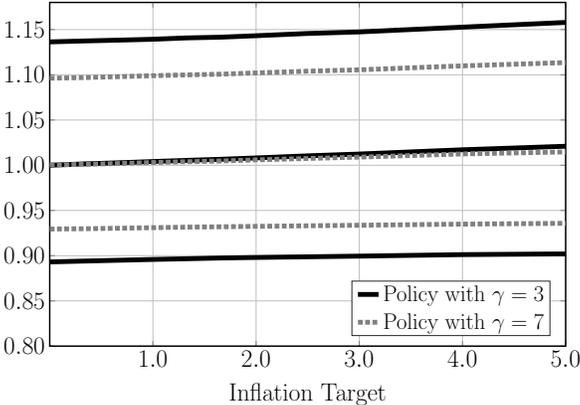


FIGURE F.VII. POLICY FUNCTION WITH DIFFERENT DEMAND ELASTICITIES

Note: The figure plots the Ss bands and reset relative prices of a firm with a demand elasticity of 3 and 7. The gray dotted lines describe the Ss bands and the reset price with a demand elasticity of 7. The black solid lines describe the same variables with a demand elasticity of 3.
Source: Author's calculations

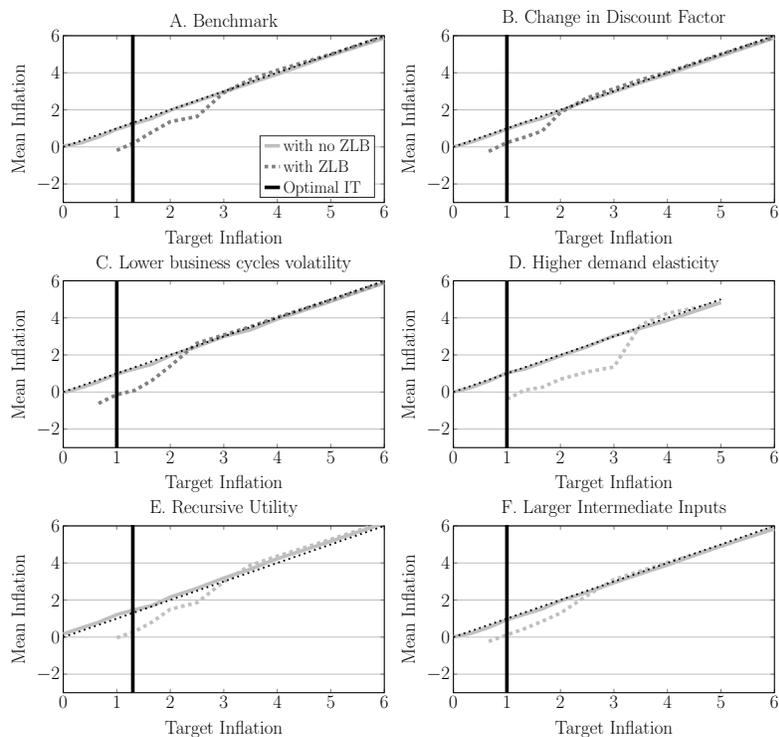


FIGURE F.VIII. MEAN AND TARGET INFLATION IN THE CALVO MODEL

Note: The light gray solid lines describe the mean inflation without a ZLB constraint, the grey dotted lines describe the same variables with a ZLB constraint and the black lines describe the optimal inflation target across all the calibration.

Source: Author's calculations

TABLE F.I—MICRO PRICING MOMENTS: GOLOSOV AND LUCAS (2007) AND DATA

Moments	Data		Model	
	Raw	With filters	Steady state	Business cycle
Absolute value of price changes				
Mean	0.154	0.153	0.155	0.156
Standar deviation	0.159	0.137	0.025	0.026
Skewness	1.538	1.314	1.264	1.203
10th percentile	0.014	0.020	0.128	0.129
25th percentile	0.034	0.049	0.138	0.137
50th percentile	0.096	0.114	0.147	0.150
75th percentile	0.223	0.217	0.166	0.170
90th percentile	0.386	0.356	0.189	0.192
Price changes				
Standar deviation	0.222	0.205	0.156	0.158
Kurtosis	4.672	3.809	1.180	1.154
Mean frequency of price change	0.169	0.126	—	—
Frequency with implied duration	0.119	0.097	0.100	0.100
Cost of price adjustment $\times 100$	0.400	0.400	0.408	0.408
Ratio free to total price adjustments	—	—	0.000	0.000

Note: The table presents selected moments of the micro price statistics in the SMM estimation. The first column describes the price statistics with standard filters, and the second column describes the same moments computed with the filters explained in Online Appendix Section B.B.2. Columns 3 and 4 show the price statistics in the model with and without business cycles.

Source: Author's calculations

TABLE F.II—BUSINESS CYCLE MOMENTS: GOLOSOV AND LUCAS (2007) AND DATA

	Standard deviation			Autocorrelation			Correlation with output		
	Model			Model			Model		
	Data	Median	[2,98]	Data	Median	[2,98]	Data	Median	[2,98]
Output	0.95	1.06	[0.82,1.34]	0.83	0.79	[0.69,0.86]	1.00	1.00	[1.00,1.00]
Consumption	1.14	1.07	[0.85,1.36]	0.85	0.79	[0.69,0.86]	0.92	0.96	[0.93,0.98]
Interest rate	0.35	0.43	[0.33,0.54]	0.83	0.84	[0.77,0.89]	0.19	0.30	[0.04,0.51]
Real wage	0.86	0.68	[0.49,0.98]	0.71	0.88	[0.80,0.93]	0.25	0.66	[0.43,0.83]
Inflation	0.26	0.28	[0.23,0.33]	0.49	0.44	[0.28,0.59]	0.09	0.22	[0.02,0.43]
Gov. expenditure	1.42	1.43	[1.15,1.79]	0.82	0.78	[0.69,0.84]	0.42	0.81	[0.66,0.88]

Note: The table presents business cycle moments from the U.S. data and the simulated series of the Golosov and Lucas (2007) model at a 3.5% inflation target. Online Appendix Section A describes the variables in the U.S. data. Model and data series are detrended with Hodrick-Prescott filter ($\lambda = 1600$) to remove the trend component. The period in the data is from 1960 first quarter to 2017 fourth quarter. The moments in the MC model are the median and a [2,98] percent confidence interval across simulations. I compute the statistics in the model for over 5000 simulations with the same length as in the data.
Source: Author's calculations

TABLE F.III—BUSINESS CYCLE MOMENTS: MODEL WITH EPSTEIN-ZIN PREFERENCES AND DATA

	Standard deviation			Autocorrelation			Correlation with output		
	Model			Model			Model		
	Data	Median	[2,98]	Data	Median	[2,98]	Data	Median	[2,98]
Output	0.95	1.07	[0.87,1.29]	0.83	0.78	[0.68,0.85]	1.00	1.00	[1.00,1.00]
Consumption	1.14	1.08	[0.88,1.32]	0.85	0.78	[0.68,0.85]	0.92	0.97	[0.94,0.98]
Interest rate	0.35	0.40	[0.31,0.50]	0.83	0.86	[0.80,0.90]	0.19	0.30	[0.06,0.53]
Real wage	0.86	0.68	[0.49,0.93]	0.71	0.88	[0.78,0.93]	0.25	0.67	[0.46,0.81]
Inflation	0.26	0.12	[0.10,0.14]	0.49	0.56	[0.40,0.68]	0.09	0.26	[0.07,0.49]
Gov. expenditure	1.42	1.42	[1.15,1.75]	0.82	0.77	[0.66,0.84]	0.42	0.80	[0.67,0.89]

Note: The table presents business cycle moments from the U.S. data and the simulated series of the model with Epstein-Zin at a 3.5% inflation target. Online Appendix Section A describes the variables in the U.S. data. Model and data series are detrended with Hodrick-Prescott filter ($\lambda = 1600$) to remove the trend component. The period in the data is from 1960 first quarter to 2017 fourth quarter. The moments in the MC model are the median and a [2,98] percent confidence interval across simulations. I compute the statistics in the model for over 5000 simulations with the same length as in the data.
Source: Author's calculations

G. OPTIMAL INFLATION TARGET WITH COIBION, GORODNICHENKO AND WIELAND (2012) WELFARE APPROXIMATION IN THE CALVO MODEL

This section uses Coibion, Gorodnichenko and Wieland (2012) calibration of the Calvo model and their welfare approximation to show the following result: In the Calvo model, the optimal inflation target is determined by a trade-off between the mean and volatility of inflation, given their respective impact on the mean of price dispersion. Thus, the main benefit of a higher inflation target in the Calvo model is not a reduction of inefficient fluctuations of consumption and labor implied by the ZLB constraint, but the reduction of inflation volatility and its effect on the dispersion of relative prices.

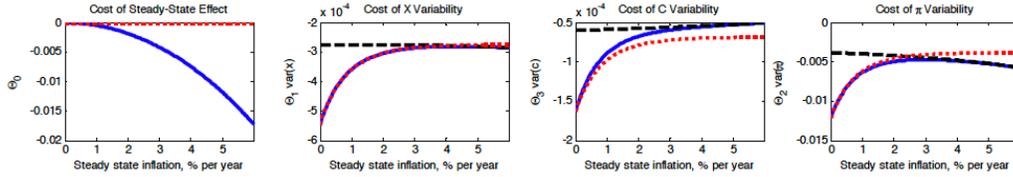
Coibion, Gorodnichenko and Wieland (2012) provides an excellent tool to analyze welfare in the Calvo model with positive trend inflation. This paper shows that welfare in the New Keynesian model with producers setting prices á la Calvo can be approximated as:

$$(G.1) \quad W = \Theta_0 + \Theta_1 \mathcal{V}ar(\hat{y}_t) + \Theta_2 \mathcal{V}ar(\hat{\pi}_t) + \Theta_3 \mathcal{V}ar(\hat{c}_t).$$

Here Θ_0 represents the steady-state cost of inflation in the form of higher markups and price dispersion, and $\Theta_1 \mathcal{V}ar(\hat{y}_t)$ and $\Theta_3 \mathcal{V}ar(\hat{c}_t)$ represent the business cycle cost associated with inefficient fluctuations of consumption and labor. $\Theta_2 \mathcal{V}ar(\hat{\pi}_t)$ represents the cost due to inflation volatility coming from the following mechanism: A rise in the volatility of inflation increases the mean of price dispersion, and therefore it reduces average labor productivity.³²

Each term in (G.1) is a function of the inflation target. If the term given by $\Theta_1 \mathcal{V}ar(\hat{y}_t) + \Theta_3 \mathcal{V}ar(\hat{c}_t)$ is almost insensitive to the inflation target, while the terms Θ_0 and $\Theta_2 \mathcal{V}ar(\hat{\pi}_t)$ are sensitive to the inflation target, then business cycle stabilization of consumption and labor is not the primary force for the optimal inflation target.

FIGURE G.I. FIGURE 3 IN COIBION, GORODNICHENKO AND WIELAND (2012)



Note: The figure plots the different components of welfare for the optimal inflation target.

Source: Coibion, Gorodnichenko and Wieland (2012)

Figure G.I shows each term from Coibion, Gorodnichenko and Wieland (2012). The steady cost of inflation goes from 0 to -0.017, the cost of inflation volatility goes from -0.012 to -0.005, and the terms representing the business cycle cost go from -0.00035 to -0.00065. These orders of magnitude for the costs and benefits of inflation suggest that the main trade-offs are related to the stochastic process of inflation. Next, I define two welfare evaluations to show this result formally.

Let W^{bc} denote the welfare function with inefficient business cycle fluctuations, and let W^{nbc} denote the welfare function without inefficient business cycle fluctuations. Formally, W^{bc} and W^{nbc} are given by

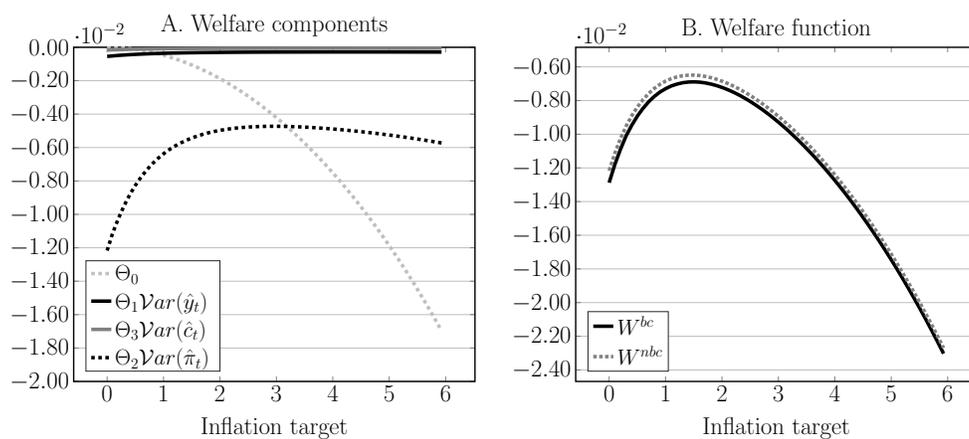
$$(G.2) \quad W^{bc} = \Theta_0 + \Theta_1 \mathcal{V}ar(\hat{y}_t) + \Theta_2 \mathcal{V}ar(\hat{\pi}_t) + \Theta_3 \mathcal{V}ar(\hat{c}_t),$$

$$(G.3) \quad W^{nbc} = \Theta_0 + \Theta_2 \mathcal{V}ar(\hat{\pi}_t).$$

Figure G.II Panel A shows Θ_0 , $\Theta_1 \mathcal{V}ar(\hat{y}_t)$, $\Theta_2 \mathcal{V}ar(\hat{\pi}_t)$, and $\Theta_3 \mathcal{V}ar(\hat{c}_t)$ in the same scale, and Figure G.II Panel B shows W^{bc} and W^{nbc} . This figure shows that the effect of the inflation target on welfare through inefficient business cycles is quantitatively insignificant, and it does not affect the optimal inflation target.

³²For a proof with zero trend inflation, see Woodford (2002); for the proof with positive trend inflation, see Coibion, Gorodnichenko and Wieland (2012).

FIGURE G.II. COIBION, GORODNICHENKO AND WIELAND (2012) WELFARE DECOMPOSITION



Note: Panel A plots the four component of welfare in equation (G.1) from their benchmark calibration. Panel B plots W^{bc} defined in equation (G.2) and W^{nbc} defined in equation (G.3).

Source: Author's calculations using original codes in Coibion, Gorodnichenko and Wieland (2012)

H. NUMERICAL ALGORITHM FOR THE COMPUTATION OF THE CALVO MODEL

An important problem in the Calvo model with positive trend inflation is that the model does not satisfy the Blanchard and Khan conditions for large inflation targets. For this reason, I add business cycle indexation: nominal prices adjust automatically to business cycle fluctuations of inflation. If p_{ti} is the nominal price of the firm i at time t , then

$$(H.1) \quad p_{it+1} = \begin{cases} \left(\frac{\Pi_{t+1}}{1+\bar{\pi}} \right)^\lambda p_{it} & \text{if no price change with prob. } 1 - \Omega \\ P_t^* & \text{if price change with prob. } \Omega \end{cases}$$

Notice that this form of indexation does not affect the steady state of the Calvo economy. Therefore I can describe the steady-state cost of inflation in this model accurately.

H.1. Equilibrium Conditions of the Calvo Model

Let X denotes a detrended variable and \tilde{X} denotes the original variable. Let S be the aggregate state of the economy, given by $S = (mc_-, \Delta_-, \tilde{R}_-, d\eta_z, \eta_g, \eta_q, \eta_h)$. Next, I describe the equilibrium conditions with and without trend.

- Household's optimality conditions

$$(H.2) \quad \tilde{m}u_t = \beta \eta_{q,t} R_t \mathbb{E}_t \left[\frac{\tilde{m}u_{t+1}}{\Pi_{t+1}} \right]$$

$$(H.3) \quad \Sigma_t = \mathbb{E}_t [U_{t+1}]$$

$$(H.4) \quad \tilde{m}u_t = \bar{u} \left(\tilde{C}_t - \kappa \eta_{z,t} \frac{\tilde{L}_t^{1+\chi}}{1+\chi} \right)^{-\sigma}$$

$$(H.5) \quad \eta_{z,t} \kappa \tilde{L}_t^\chi = \eta_{h,t} \tilde{w}_t$$

$$(H.6) \quad \tilde{u}_t = \bar{u} \frac{\left(\tilde{C}_t - \eta_{z,t} \kappa \frac{\tilde{L}_t^{1+\chi}}{1+\chi} \right)^{1-\sigma}}{1-\sigma}$$

$$(H.7) \quad \tilde{U}_t = (1-\beta)\tilde{u}_t + \beta\Sigma_t$$

- Stationary household's optimality conditions

$$(H.8) \quad mu(S) = \beta \eta_{q-}(S) R(S) \mathbb{E}_{S'} \left[\left(\frac{\eta_z(S')}{\eta_z(S)} \right)^{-\sigma} \frac{mu(S')}{\Pi(S')} \middle| S \right]$$

$$(H.9) \quad \Sigma(S) = \mathbb{E}_{S'} \left[\left(\left(\frac{\eta_z(S')}{\eta_z(S)} \right)^{1-\sigma} \frac{U(S')}{U_{ss}} \right) \middle| S \right]$$

$$(H.10) \quad mu(S) = \bar{u} \left(C(S) - \kappa \frac{L(S)^{1+\chi}}{1+\chi} \right)^{-\sigma}$$

$$(H.11) \quad \kappa L(S)^\chi = \eta_{h,-}(S) w(S)$$

$$(H.12) \quad u(S) = \bar{u} \frac{\left(C(S) - \kappa \frac{L(S)^{1+\chi}}{1+\chi} \right)^{1-\sigma}}{1-\sigma}$$

$$(H.13) \quad U(S) = (1-\beta)u(S) + \beta U_{ss} \Sigma(S)$$

– The original variables can be obtained as:

$$(H.14) \quad \begin{aligned} \tilde{C}_t &= C(S_t)\eta_{z,t} & \tilde{m}u_t &= mu(S_t)\eta_{z,t}^{-\sigma} & \tilde{L}_t &= L(S_t) & \tilde{\Pi}_t &= \Pi(S_t) \\ \tilde{u}_t &= u(S_t)\eta_{z,t}^{(1-\sigma)} & \tilde{U}_t &= U(S_t)\eta_{z,t}^{(1-\sigma)} & \tilde{w}_t &= w(S_t)\eta_{z,t} \end{aligned}$$

• Firms' optimality conditions, inflation, and price dispersion

$$\begin{aligned} \tilde{H}_t &= \frac{\gamma}{\gamma-1}(1-\beta)\tilde{m}u_t\tilde{Y}_t m c_t + \beta(1-\Omega)\mathbb{E}_t \left[\left(\Pi_{t+1}^{(1-\lambda)\gamma} (1+\bar{\pi})^{\lambda\gamma} \right) \tilde{H}_{t+1} \right] \\ \tilde{F}_t &= \tilde{m}u_t\tilde{Y}_t(1-\beta) + \beta(1-\Omega)\mathbb{E}_t \left[\left(\Pi_{t+1}^{(1-\lambda)(\gamma-1)} (1+\bar{\pi})^{\lambda(\gamma-1)} \right) \tilde{F}_{t+1} \right] \\ P_t^* &= \frac{\tilde{H}_t}{\tilde{F}_t} \\ P_t^* &= \left[\frac{1 - (1-\Omega)(1+\bar{\pi})^{\lambda(\gamma-1)} \Pi_t^{(1-\lambda)(\gamma-1)}}{\Omega} \right]^{\frac{1}{1-\gamma}} \\ \Delta_t &= \left(\Omega (P_t^*)^{-\gamma} + (1-\Omega) \left(\Pi_t^{(1-\lambda)\gamma} (1+\bar{\pi})^{\lambda\gamma} \right) \Delta_{t-1} \right) \end{aligned}$$

• Stationary firms' optimality conditions, inflation, and price dispersion

$$\begin{aligned} H(S) &= \frac{\gamma}{\gamma-1}(1-\beta)mu(S)Y(S)mc(S) + \dots \\ &\dots + \beta(1-\Omega)\mathbb{E}_{S'} \left[\left(\frac{\eta_z(S')}{\eta_z(S)} \right)^{1-\sigma} \left(\Pi(S')^{(1-\lambda)\gamma} (1+\bar{\pi})^{\lambda\gamma} \right) H(S') \middle| S \right] \\ F(S) &= mu(S)Y(S)(1-\beta) + \dots \\ &\dots + \beta(1-\Omega)\mathbb{E}_{S'} \left[\left(\frac{\eta_z(S')}{\eta_z(S)} \right)^{1-\sigma} \left(\Pi(S')^{(1-\lambda)(\gamma-1)} (1+\bar{\pi})^{\lambda(\gamma-1)} \right) F(S') \middle| S \right] \\ P^*(S) &= \frac{H(S)}{F(S)} \\ P^*(S) &= \left[\frac{1 - (1-\Omega)(1+\bar{\pi})^{\lambda(\gamma-1)} \Pi(S)^{(1-\lambda)(\gamma-1)}}{\Omega} \right]^{\frac{1}{1-\gamma}} \\ \Delta(S) &= \left(\Omega (P^*(S))^{-\gamma} + (1-\Omega) \left(\Pi(S)^{(1-\lambda)\gamma} (1+\bar{\pi})^{\lambda\gamma} \right) \Delta_-(S) \right) \end{aligned}$$

– The original variables can be obtained as:

$$(H.15) \quad \tilde{m}c_t = mc(S_t), \quad \tilde{Y}_t = Y(S_t)\eta_{z,t}, \quad \tilde{P}_t^* = P_t^*(S_t), \quad \tilde{H}_t = H(S_t)\eta_{z,t}^{(1-\sigma)}, \quad \tilde{F}_t = F(S_t)\eta_{z,t}^{(1-\sigma)}$$

- Monetary policy and aggregate feasibility with change of variables

$$(H.16) \quad mc_t = (1 - \tau_{mc})\iota \left(\frac{w_t}{\eta_{tz}} \right)^{1-\alpha}$$

$$(H.17) \quad \mathcal{M}_t = \frac{1 - \tau_{mc}}{mc_t}$$

$$(H.18) \quad R_t^* = \tilde{R}_{t-1} \mathcal{R}^{1/(1-\phi_r)} \left(\left(\frac{\Pi_t}{(1 + \bar{\pi})} \right)^{\phi_\pi} \left(\frac{mc_t}{mc_{ss}} \right)^{\tilde{\phi}_y} \right)^{1-\phi_r} \left(\frac{mc_t}{mc_{t-1}} \right)^{\tilde{\phi}_{dy}}$$

$$(H.19) \quad \tilde{R}_{t+1} = \left(\mathcal{R}^{-1/(1-\phi_r)} R_t^* \right)^{\phi_r} \eta_{r,t+1}$$

$$(H.20) \quad R_t = \max\{1, R_t^*\}$$

$$(H.21) \quad \eta_{z,t}^{1-\alpha} L_t = Y_t \left(\frac{(1-\alpha)}{\alpha w_t} \right)^\alpha \Delta_t$$

$$(H.22) \quad C_t = Y_t \left(1 - \left(\frac{w_t \alpha}{1-\alpha} \right)^{1-\alpha} \frac{\Delta_t}{\eta_{z,t}^{1-\alpha}} \right) \frac{1}{\eta_{g,t}}$$

- Stationary monetary policy and aggregate feasibility with change of variable

$$(H.23) \quad mc(S) = (1 - \tau_{mc})\iota w(S)^{1-\alpha}$$

$$(H.24) \quad \mathcal{M}(S) = \frac{1 - \tau_{mc}}{mc(S)}$$

$$(H.25) \quad R_t^*(S) = \tilde{R}_-(S) \mathcal{R}^{1/(1-\phi_r)} \left(\left(\frac{\Pi(S)}{(1 + \bar{\pi})} \right)^{\phi_\pi} \left(\frac{mc(S)}{mc_{ss}} \right)^{\tilde{\phi}_y} \right)^{1-\phi_r} \left(\frac{mc(S)}{mc_-(S)} \right)^{\tilde{\phi}_{dy}}$$

$$(H.26) \quad \tilde{R}(S') = \left(\mathcal{R}^{1/(1-\phi_r)} R^*(S) \right)^{\phi_r} \eta'_r$$

$$(H.27) \quad R(S) = \max\{1, R^*(S)\}$$

$$(H.28) \quad L(S) = Y(S) \left(\frac{(1-\alpha)}{\alpha w(S)} \right)^\alpha \Delta(S)$$

$$(H.29) \quad C(S) = Y(S) \left(1 - \left(\frac{w(S) \alpha}{1-\alpha} \right)^{1-\alpha} \Delta(S) \right) \frac{1}{\eta_{g-}(S)}$$

- Exogenous shocks

$$(H.30) \quad \log(\eta_g)(S') = (1 - \rho_g) \log(\eta_g^*) + \rho_g \log(\eta_g(S)) + \sigma_g \epsilon'_g$$

$$(H.31) \quad \log(\eta'_z/\eta_z)(S') = (1 - \rho_z) \log(1 + g) + \rho_z \log(\eta'_z/\eta_z)(S) + \sigma_z \epsilon'_z$$

$$(H.32) \quad \log(\eta_r)(S') = \sigma_r \epsilon'_r$$

$$(H.33) \quad \log(\eta_q)(S') = (1 - \rho_q) \log(\eta_q^*) + \rho_q \log(\eta_q(S)) + \sigma_q \epsilon'_q$$

$$(H.34) \quad \log(\eta_h)(S') = (1 - \rho_h) \log(\eta_h^*) + \rho_h \log(\eta_h(S)) + \sigma_h \epsilon'_h$$

H.2. Solution Method for the Calvo Model

The solution method has three steps:

- 1) Solve the model using 2nd-order perturbation methods.

- Compute the steady state.
 - Set the hypercube limits for the solution with the global solution.
 - Get initial the condition for the coefficients in the policy functions.
- 2) Solve the model using global methods without the zero lower bound.
- I use an iterative method in this step. The previous step gives the initial condition for the policy functions.
- 3) Solve the model using global methods with the zero lower bound.
- I use an iterative method in this step. The previous step gives the initial condition for the policy functions.

STEP 1: PERTURBATION METHOD AROUND THE STEADY STATE. — First, I compute the steady state of the model. It is given by:

$$\begin{aligned}
& \eta_{q,ss} = 1 \quad ; \quad \eta_{z,ss} = \eta_z \\
& \eta_{g,ss} = \eta_g \quad ; \quad \eta_{r,ss} = 1 \\
P_{ss}^* &= \left(\frac{1 - (1 - \Omega)(1 + \bar{\pi})^{\gamma-1}}{\Omega} \right)^{1/(1-\gamma)} \quad ; \quad mc_{ss} = \frac{P_{ss}^*(\gamma-1)}{\gamma} \frac{1 - \beta\eta_{q,ss}(1 - \Omega)(1 + g)^{1-\sigma_n}(1 + \bar{\pi})^\gamma}{1 - \beta\eta_{q,ss}(1 + g)^{1-\sigma_n}(1 - \Omega)(1 + \bar{\pi})^{\gamma-1}} \\
w_{ss} &= \left(\frac{mc_{ss}}{(1 - \tau_{mc})\iota} \right)^{1/(1-\alpha)} \quad ; \quad L_{ss} = \left(\frac{w_{ss}}{\kappa} \right)^{\frac{1}{\chi}} \\
\Delta_{ss} &= \frac{\Omega(P_{ss}^*)^{-\gamma}}{1 - (1 - \Omega)\Pi_{ss}^\gamma} \quad ; \quad Y_{ss} = \frac{L_{ss}}{\left(\frac{1-\alpha}{\alpha w_{ss}} \right)^\alpha \Delta_{ss}} \\
C_{ss} &= Y_{ss} \left(1 - \left(\frac{w_{ss}\alpha}{1-\alpha} \right)^{1-\alpha} \Delta_{ss} \right) (1 - 1/\eta_{g,ss}) \quad ; \quad mu_{ss} = \bar{u} \left(C_{ss} - \kappa \frac{L_{ss}^{1+\chi}}{1+\chi} \right)^{-\sigma} \\
u_{ss} &= \bar{u} \frac{(C_{ss} - \kappa \frac{L_{ss}^{1+\chi}}{1+\chi})^{1-\sigma}}{1-\sigma} \quad ; \quad U_{ss} = u_{ss} \frac{1-\beta}{1-\beta(1+g)^{1-\sigma}} \\
H_{ss} &= \frac{\gamma}{\gamma-1} \frac{(1-\beta)mu_{ss}Y_{ss}mc_{ss}}{1-\beta g^{1-\sigma_n}(1-\Omega)(1+\bar{\pi})^\gamma} \quad ; \quad F_{ss} = (1-\beta) \frac{mu_{ss}Y_{ss}}{1-\beta g^{1-\sigma_n}(1-\Omega)(1+\bar{\pi})^{\gamma-1}} \\
R_{ss} &= \frac{1+\bar{\pi}}{\beta g^{-\sigma}} \quad ; \quad \Pi_{ss} = (1+\bar{\pi}) \\
\Sigma_{ss} &= (1+g)^{1-\sigma} \quad ; \quad CCM_{ss} = (1-\tau_L)/mc
\end{aligned}$$

Then, I solve the system of equations described in the previous section without ZLB with second-order perturbation methods. I apply the log transformation $X_t = X_{ss}e^{\hat{X}_t}$ for all the positive variables. I didn't use log transformation for negative variables.

- Positive variables:

$$\{mu, \Pi, \Sigma, C, L, w, S, Y, P^*, F, mc, \Delta, R^*, R, \eta_G, \eta_Z, \eta_R, \eta_Q\}.$$

- Non-positive variables: $\{u, U\}$.

STEP 2: SOLVING THE MODEL WITHOUT ZLB. — I use an iterative method to find the equilibrium.

- **Step 2.1:** Initiate

$$(H.35) \quad mu^0(S), R^0(S), \tilde{R}^0(S), \Pi^0(S), U^0(S), H^0(S), F^0(S), \Delta^0(S)$$

using second-order perturbation methods.

- **Step 2.2:** Given $mu^i(S), \tilde{R}^i(S), \Pi^i(S), U^i(S), H^i(S), F^i(S), \Delta^i(S)$, I use the Euler equation to get

$$\begin{aligned}\Sigma^{i+1}(S) &= \mathbb{E}_{S'} \left[\left(d\eta(S')^{1-\sigma} \frac{U^i(S')}{U_{ss}} \right) \middle| S \right] \\ mu^{i+1}(S) &= \beta g^{-\sigma n} \eta_q(S) R^i(S) \mathbb{E}_{S'} \left[\frac{mu^i(S')}{\Pi^i(S')} \middle| S \right]\end{aligned}$$

- **Step 2.3:** With $mu^{i+1}(S)$ I solve the following system

$$\begin{aligned}\kappa L^{i+1}(S)^\chi &= \eta_h(S) w^{i+1}(S) \\ mu^{i+1}(S) &= \bar{u} (C^{i+1}(S) - \kappa \frac{L^{i+1}(S)^{1+\chi}}{1+\chi})^{-\sigma n} \\ L^{i+1}(S) &= Y^{i+1}(S) \left(\frac{(1-\alpha)}{\alpha w^{i+1}(S)} \right)^\alpha \Delta^i(S) \\ C^{i+1}(S) &= Y^{i+1}(S) \left(1 - \left(\frac{w^{i+1}(S)\alpha}{1-\alpha} \right)^{1-\alpha} \Delta^i(S) \right) / \eta_g(S).\end{aligned}$$

Compute the marginal cost and period utility given by

$$mc^{i+1}(S) = (1 - \tau_{mc}) \iota (w^{i+1}(S))^{1-\alpha} \quad \& \quad u^{i+1}(S) = \bar{u} \frac{(C^{i+1}(S) - \kappa \frac{L^{i+1}(S)^{1+\chi}}{1+\chi})^{1-\sigma}}{1-\sigma}$$

- **Step 2.4:** Update forward looking policies $U(S), H(S), F(S)$

$$\begin{aligned}U^{i+1}(S) &= (1 - \beta)u^{i+1}(S) + \beta U_{ss} \Sigma^{i+1}(S) \\ H^{i+1}(S) &= \frac{\gamma}{\gamma - 1} (1 - \beta) mu^{i+1}(S) Y^{i+1}(S) mc^{i+1}(S) + \dots \\ &\quad \dots + \beta g^{1-\sigma} (1 - \Omega) \mathbb{E}_{S'} \left[\left((\Pi^i(S'))^{(1-\lambda)\gamma} (1 + \bar{\pi})^{\lambda\gamma} \right) H^i(S') \middle| S \right] \\ F^{i+1}(S) &= mu^{i+1}(S) Y^{i+1}(S) (1 - \beta) + \dots \\ &\quad \dots + \beta g^{1-\sigma} (1 - \Omega) \mathbb{E}_{S'} \left[\left((\Pi^i(S'))^{(1-\lambda)(\gamma-1)} (1 + \bar{\pi})^{\lambda(\gamma-1)} \right) F^i(S') \middle| S \right]\end{aligned}$$

- **Step 2.5:** Update policies $R(S), \tilde{R}(S), \Pi(S), \Delta(S)$

$$\begin{aligned}\Pi^{i+1}(S) &= \left(\frac{1 - \Omega}{1 - \Omega((P^*)^{i+1}(S))^{1-\gamma}} \right)^{1/((1-\gamma)(1-\lambda))} (1 + \bar{\pi})^{\frac{-\lambda}{1-\lambda}} \\ \Delta^{i+1}(S) &= \left(\Omega((P^*)^{i+1}(S))^{-\gamma} + (1 - \Omega) \left(\Pi^{i+1}((S))^{(1-\lambda)\gamma} (1 + \bar{\pi})^{\lambda\gamma} \right) \right) \Delta^i(S) \\ (R^*)^{i+1}(S) &= \tilde{R}_-(S) \left(\frac{1 + \bar{\pi}}{\beta(1+g)^{-\sigma}} \right) \left(\left(\frac{\Pi^{i+1}(S)}{1 + \bar{\pi}} \right)^{\phi_\pi} \left(\frac{\tilde{m}c^{i+1}(S)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc^{i+1}(S)}{\tilde{m}c(S)} \right)^{\phi_{dy}} \\ \tilde{R}^{i+1}(S) &= \left(\left(\frac{\beta(1+g)^{-\sigma}}{1 + \bar{\pi}} \right) (R^*)^{i+1}(S) \right)^{\phi_r} \quad R^{i+1}(S) = (R^*)^{i+1}(S);\end{aligned}$$

- **Step 2.6:** If

$$(H.36) \quad error = mean_S (|mu^i(S) - mu^{i+1}(S)| + |\Pi^i(S) - \Pi^{i+1}(S)| + |U^i(S) - U^{i+1}(S)|) < \epsilon,$$

stop. Otherwise go back to step 2.2.

REMARK. — I use a Smolyak sparse grid with anisotropic construction for the projection method.

REMARK. — I use Gauss-Legendre quadrature and the inverse of the accumulated normal distribution in the nodes to recover the nodes in the normal distribution.

REMARK. — I use Golden search to obtain the labor supply in step 3. Specifically, labor supply in the i -th iteration satisfies

$$(H.37) \quad L^{i+1}(S) = \arg \min_X \left(mu^i(S) - \left(C(X) - \kappa \frac{X^{1+\chi}}{1+\chi} \right)^{-\sigma} \right)^2$$

with

$$(H.38) \quad C(X) = \frac{1}{\eta_g(S)\Delta^i(S)} X \left(\frac{\kappa X^\chi \alpha}{\eta_h(S)(1-\alpha)} \right)^\alpha \left(1 - \left(\frac{\kappa X^\chi \alpha}{(1-\alpha)\eta_h(S)} \right)^{1-\alpha} \frac{\Delta^i(S)}{1} \right)$$

REMARK. — I evaluate every function in $\min\{\max\{S', S_{min}\}, S_{max}\}$ whenever I compute the expectation. Here, S_{min} is the lower bound for the hypercube and S_{max} is the upper bound for the hypercube.

STEP 3: SOLVING THE MODEL WITH ZLB. — I repeat the same algorithm as in step 2, using the policy function of the model without ZLB with the global solution as the initial condition. The only difference comes in step 2.4, where I solve the following system:

$$\begin{aligned} \Delta^{i+1}(S) &= \left(\Omega ((P^*)^{i+1}(S))^{-\gamma} + (1-\Omega) \left(\Pi^{i+1}(S) \right)^{(1-\lambda)\gamma} (1+\bar{\pi})^{\lambda\gamma} \right) \Delta^i(S) \\ (R^*)^{i+1}(S) &= \tilde{R}_-(S) \left(\frac{1+\bar{\pi}}{\beta(1+g)^{-\sigma}} \right) \left(\left(\frac{\Pi^{i+1}(S)}{(1+\bar{\pi})} \right)^{\phi_\pi} \left(\frac{\tilde{m}c^{i+1}(S)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc^{i+1}(S)}{\tilde{m}c(S)} \right)^{\phi_{dy}} \\ \tilde{R}^{i+1}(S) &= \left(\left(\frac{\beta(1+g)^{-\sigma}}{1+\bar{\pi}} \right) (R^*)^{i+1}(S) \right)^{\phi_r} \\ R^{i+1}(S) &= \max\{1, (R^*)^{i+1}(S)\} \end{aligned}$$

For the model with the ZLB, I make sure the interest rate \tilde{R} always stays inside the hypercube. To achieve this goal, let us define \underline{R} and \underline{mc} to be the lower bounds of the hypercube for the interest rate and the real marginal cost. I add the following wedge, τ^C , into the Euler equation:

$$\begin{aligned} mu(S) &= \beta(1-\tau^C(S))(1+g)^{-\sigma} \eta_{q-}(S) R(S) \mathbb{E}_{S'} \left[\frac{mu(S')}{\Pi(S')} \middle| S \right] \\ \tau^C(S) &= \begin{cases} 0 & \text{if } R(S) \geq \underline{R} \ \& \ mc(S) \geq \underline{mc} \\ 1 - \frac{1}{\beta(1+g)^{-\sigma} \eta_{q-}(S) R(S) \mathbb{E}_{S'} \left[\frac{mu(S')}{\Pi(S')} \middle| S \right]} & \text{otherwise} \end{cases} . \end{aligned}$$

I. NUMERICAL ALGORITHM FOR THE COMPUTATION OF THE MENU COST MODEL

This section describes the computation of the menu cost model in the steady state and with business cycles.

I.1. Recursive Equilibrium Conditions

I divide the equilibrium conditions into five blocks: household optimality conditions, firms' optimality conditions, monetary policy and aggregate feasibility, Krusell-Smith cross-equation approximations and exogenous shocks. For simplicity, I denote with X the detrended version of variable of \tilde{X} and by S the aggregate state of the economy given by $S = (mc_-, \Delta_-, \tilde{R}_-, d\eta_z, \eta_g, \eta_q, \eta_h)$.

• Household optimality conditions:

$$\begin{aligned} mu(S) &= \beta\eta_{q-}(S)R(S)\mathbb{E}_{S'} \left[d\eta_z(S')^{-\sigma} \frac{mu(S')}{\Pi(S')} \middle| S \right] \\ \Sigma(S) &= \mathbb{E}_{S'} \left[\left(d\eta_z(S')^{1-\sigma} \frac{U(S')}{U_{ss}} \right) \middle| S \right] \\ mu(S) &= \bar{u} \left(C(S) - \kappa \frac{L(S)^{1+\chi}}{1+\chi} \right)^{-\sigma} \\ \kappa L(S)^\chi &= \eta_{h-}(S)w(S) \\ u(S) &= \bar{u} \frac{\left(C(S) - \kappa \frac{L(S)^{1+\chi}}{1+\chi} \right)^{1-\sigma}}{1-\sigma} \\ U(S) &= (1-\beta)u(S) + \beta U_{ss}\Sigma(S) \end{aligned}$$

– The original variables can be obtained as:

$$\begin{aligned} \tilde{C}_t &= C(S_t)\eta_{zt} \quad ; \quad \tilde{m}u_t = mu(S_t)\eta_{zt}^{-\sigma} \quad ; \quad \tilde{L}_t = L_t \quad ; \quad \tilde{\Pi}_t = \Pi(S) \\ \tilde{u}_t &= u(S_t)\eta_{zt}^{(1-\sigma)} \quad ; \quad \tilde{U}_t = U(S_t)\eta_{zt}^{(1-\sigma)} \quad ; \quad \tilde{w}_t = w(S_t)\eta_{zt} \end{aligned}$$

• Firm's optimality conditions:

$$\begin{aligned} v(\tilde{p}, S) &= \mathbb{E}_{S,x} \left[\left((1-\zeta) \max_{c,nc} \{v^c(S) - \bar{\theta}(S'), v(\tilde{p}'(\tilde{p}), S')\} + \zeta v^c(S) \right) \right] \\ P^*(S) &= \arg \max_{\tilde{p}} \{ \Phi(\tilde{p}, S) + \beta(1+g)^{1-\sigma} v(\tilde{p}, S) \} \\ v^c(S) &= \Phi(P^*(S), S) + \beta(1+g)^{1-\sigma} v(P^*(S), S) \\ v^{nc}(\tilde{p}, S) &= \Phi(\tilde{p}, S) + \beta(1+g)^{1-\sigma} v(\tilde{p}, S) \\ \Phi(\tilde{p}, S) &= mu(S)Y(S)\tilde{p}^{-\gamma} (\tilde{p} - mc(S)) \\ \tilde{p}'(\tilde{p}) &= \begin{cases} \frac{\tilde{p}e^{\Delta_{a_l}}}{\Pi(S')} & \text{with prob. } p \\ \frac{\tilde{p}e^{\Delta_{a_h}}}{\Pi(S')} & \text{with prob. } 1-p \end{cases} \end{aligned}$$

with $\bar{\theta}(S) = \theta w(S')mu(S')$ and $\Delta_{a_k} \sim N(0, \sigma_{a,k})$ with $k \in \{l, h\}$.

- **Monetary policy and aggregate feasibility:**

$$\begin{aligned}\Pi(S) &= \left(\frac{1 - \Omega(S)}{1 - \Omega(S)(P^*(S))^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi(S) \\ mc(S) &= \iota(1 - \tau_{mc})w(S)^{1-\alpha} \\ R^*(S) &= \tilde{R}_-(S)\mathcal{R}^{1/(1-\phi_r)} \left(\left(\frac{\Pi(S)}{(1 + \bar{\pi})} \right)^{\phi_\pi} \left(\frac{mc(S)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc(S)}{mc_-(S)} \right)^{\phi_{dy}} \\ \tilde{R}(S') &= \left(\mathcal{R}^{-1/(1-\phi_r)} R^*(S) \right)^{\phi_r} \eta'_r \\ R(S) &= \max\{1, R^*(S)\} \\ 1 &= \frac{Y(S)}{(L(S) - \theta(\Omega(S) - \zeta))} \left(\frac{(1 - \alpha)}{\alpha w(S)} \right)^\alpha \Delta(S) \\ C(S) &= Y(S) \left(1 - \left(\frac{w(S)\alpha}{1 - \alpha} \right)^{1-\alpha} \Delta(S) \right) \frac{1}{\eta_{g-}(S)}\end{aligned}$$

– The original variables can be obtained as:

$$\tilde{m}c_t = mc(S_t) \quad ; \quad \tilde{Y}_t = Y(S_t)\eta_{zt}$$

- **Krusell-Smith cross-equation approximation:** Let $P^2(S)$ be a 2nd-order polynomial for the aggregate state

$$\log(\Delta(S)) = P^2(\log(S)) \quad ; \quad \log(\Omega(S)) = P^2(\log(S)) \quad ; \quad \log(\varphi(S)) = P^2(\log(S))$$

- **Exogenous shocks:**

$$\begin{aligned}\log(\eta_g(S')) &= (1 - \rho_g)\log(\eta_g^*) + \rho_g \log(\eta_g(S)) + \sigma_g \epsilon'_g \\ \log(d\eta_z(S')) &= (1 - \rho_z)\log(1 + g) + \rho_z \log(d\eta_z(S)) + \sigma_z \epsilon'_z \\ \log(\eta_r(S')) &= \sigma_r \epsilon'_r \\ \log(\eta_q(S')) &= (1 - \rho_q)\log(\eta_q^*) + \rho_q \log(\eta_q(S)) + \sigma_q \epsilon_q \\ \log(\eta_h(S')) &= (1 - \rho_h)\log(\eta_h^*) + \rho_h \log(\eta_h(S)) + \sigma_h \epsilon_h\end{aligned}$$

I.2. Menu Cost Model: Steady State

The equilibrium conditions in the steady state of the menu cost are given by

- Household optimality conditions

$$\begin{aligned}mu_{ss} &= \bar{u} \left(C_{ss} - \kappa \frac{L_{ss}^{1+\chi}}{1 + \chi} \right)^{-\sigma} \\ \kappa L_{ss}^\chi &= w_{ss} \\ u_{ss} &= \kappa \frac{\left(C_{ss} - \kappa \frac{L_{ss}^{1+\chi}}{1 + \chi} \right)^{1-\sigma}}{1 - \sigma} \\ U_{ss} &= (1 - \beta)u_{ss} + \beta(1 + g)^{1-\sigma} U_{ss}\end{aligned}$$

- Idiosyncratic equilibrium conditions

$$v(\tilde{p}) = \mathbb{E}_{\tilde{p}'} \left[\left((1-\zeta) \max_{c,nc} \left\{ \max_{\tilde{p}} V(\tilde{p}) - \theta w_{ss} m u_{ss}, V(\tilde{p}') \right\} + \zeta \max_{\tilde{p}} V(\tilde{p}) \right) \right]$$

$$V(\tilde{p}) = \Phi(\tilde{p}) + \beta g^{1-\sigma} v(\tilde{p})$$

$$\Phi(\tilde{p}) = m u_{ss} Y_{ss} \tilde{p}^{-\gamma} (\tilde{p} - m c_{ss})$$

$$\tilde{p}'(\tilde{p}) = \begin{cases} \frac{\tilde{p} e^{\Delta a_h}}{1+\bar{\pi}} & \text{with prob. } p \\ \frac{\tilde{p} e^{\Delta a_l}}{1+\bar{\pi}} & \text{with prob. } 1-p \end{cases}$$

$$\tilde{p}^* = \arg \max_{\tilde{p}} V(\tilde{p})$$

$$\Psi = \{x : V(\tilde{p}^*) - \theta w_{ss} m u_{ss} \leq V(\tilde{p})\}$$

$$C = \left\{ (\tilde{p}_-, \Delta a) : \frac{\tilde{p}_- e^{\Delta a}}{1+\bar{\pi}} \in \Psi \right\}$$

- Aggregate feasibility and monetary policy

$$m c_{ss} = (1 - \tau_{mc}) \iota w_{ss}^{1-\alpha}$$

$$\mathcal{M}_{ss} = \frac{(1 - \tau_{mc})}{m c_{ss}}$$

$$R_{ss}^* = \left(\frac{1 + \bar{\pi}}{\beta (1 + g)^{-\sigma}} \right) \quad R_{ss} = \left(\left(\frac{\beta g^{-\sigma}}{1 + \bar{\pi}} \right) R_{ss}^* \right)^{\phi_r}$$

$$1 + \bar{\pi} = \left(\frac{1 - \Omega_{ss}}{1 - \Omega_{ss} (\tilde{p}_{ss}^*)^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_{ss}$$

$$\Omega_{ss} = \zeta + (1 - \zeta) \int_{(\tilde{p}_-, \Delta a) \notin C} f_-(d\tilde{p}_-) g(d\Delta a)$$

$$\varphi_{ss} = \left(\int_{(\tilde{p}_-, \Delta a) \in C} \frac{(\tilde{p}_- e^{\Delta a})^{1-\gamma}}{1 - \Omega_{ss}} (1 - \zeta) f_-(d\tilde{p}_-) g(d\Delta a) \right)^{\frac{1}{1-\gamma}}$$

$$\Delta_{ss} = \int_{\tilde{p}} \tilde{p}^{-\gamma} f(\tilde{p}) \quad ; \quad f(\tilde{p}) := \text{distribution of posted prices}$$

$$L_{ss} = Y_{ss} \left(\frac{(1 - \alpha)}{\alpha w_{ss}} \right)^{\alpha} \Delta_{ss}$$

$$C_{ss} = Y_{ss} \left(1 - \left(\frac{w_{ss} \alpha}{1 - \alpha} \right)^{1-\alpha} \Delta_{ss} \right) / \eta_{g,ss}$$

NUMERICAL ALGORITHM FOR THE COMPUTATION OF THE STEADY STATE EQUILIBRIUM. — The algorithm to compute the equilibrium consists in finding the solution to the following system of equations, $F(X_{ss}) = 0$,

$$(I.1) \quad \Pi(X_{ss}) - \Pi = 0$$

$$(I.2) \quad \Omega_{ss} - \Omega(X_{ss}) = 0$$

$$(I.3) \quad \Delta_{ss} - \Delta(X_{ss}) = 0,$$

where $X_{ss} = [m c_{ss}, \Omega_{ss}, \Delta_{ss}]$ (i.e., real marginal cost, frequency of price changes and price dispersion) are arguments in the functions. $\Pi(X_{ss})$, $\Omega(X_{ss})$, and $\Delta(X_{ss})$ are equilibrium implied inflation, frequency of price changes, and price dispersion, respectively. Next, I describe the operations inside the $F(z)$ function.

1) **Step 1:** Given $(mc_{ss}, \Omega_{ss}, \Delta_{ss})$ solve

$$\begin{aligned}
w(X_{ss}) &= \left(\frac{mc_{ss}}{(1-\tau_{mc})\iota} \right)^{\frac{1}{1-\alpha}} \\
L(X_{ss}) &= \left(\frac{w(X_{ss})}{\kappa} \right)^{\frac{1}{\chi}} \\
Y(X_{ss}) &= \frac{L(X_{ss}) - (\Omega_{ss} - \zeta)\theta}{\Delta_{ss}} \left(\frac{\alpha w(X_{ss})}{1-\alpha} \right)^{\alpha} \\
C(X_{ss}) &= Y(X_{ss}) \left(1 - \left(\frac{w(X_{ss})\alpha}{1-\alpha} \right)^{1-\alpha} \Delta_{ss} \right) \frac{1}{\eta_{g,ss}} \\
u(X_{ss}) &= \bar{u} \frac{(C(X_{ss}) - \kappa \frac{L(X_{ss})^{1+\chi}}{1+\chi})^{1-\sigma}}{1-\sigma} \\
mu(X_{ss}) &= \bar{u} \left(C(X_{ss}) - \kappa \frac{L(X_{ss})^{1+\chi}}{1+\chi} \right)^{-\sigma} \\
U(X_{ss}) &= \frac{u(X_{ss})(1-\beta)}{1-\beta g^{1-\sigma}} \\
\Pi(X_{ss}) &= 1 + \bar{\pi} \\
R(X_{ss}) &= \frac{1 + \bar{\pi}}{\beta g^{-\sigma}} \\
\Sigma(X_{ss}) &= 1 \\
\tilde{R}(X_{ss}) &= 1
\end{aligned}$$

2) **Step 2:** Given $(Y(X_{ss}), mu(X_{ss}), w(X_{ss}), L(X_{ss}))$, solve

$$\begin{aligned}
v_1^{X_{ss}}(\tilde{p}) &= (1-\zeta) \max_z \left\{ \max_z \Phi^{X_{ss}}(z) + \beta g^{1-\sigma} v_1^{X_{ss}}(z) - mu(X_{ss})w(X_{ss})\theta, \Phi^{X_{ss}}(\tilde{p}) + \beta g^{1-\sigma} v_1^{X_{ss}}(\tilde{p}) \right\} + \dots \\
&\quad \dots + \zeta \max_z \left(\Phi(z) + \beta g^{1-\sigma} v_1^{X_{ss}}(z) \right) \\
v_2^{X_{ss}}(\tilde{p}) &= \mathbb{E}_{\tilde{p}'} \left[v_1^{X_{ss}}(\tilde{p}') \right] \\
\Phi^{X_{ss}}(\tilde{p}) &= mu(X_{ss})Y(X_{ss})\tilde{p}^{-\gamma} (\tilde{p} - mc_{ss}) \\
\tilde{p}'(\tilde{p}) &= \begin{cases} \frac{\tilde{p} e^{\sigma_h} e^a}{1+\bar{\pi}} & \text{with pr. } p \\ \frac{\tilde{p} e^{\sigma_l} e^a}{1+\bar{\pi}} & \text{with pr. } 1-p \end{cases}
\end{aligned}$$

and get $\tilde{p}^{*,X_{ss}}, \Psi^{X_{ss}}$ and $\mathcal{C}^{X_{ss}}$.

- a) *Technical remark 1:* I use 3rd order splines to approximate the value function in the firm's problem. It is important not to use 1st order splines, since this method generates jumps in the reset price in the iterations.
- b) *Technical remark 2:* I use value function iteration together with collocation to solve the firm's Bellman equation.
- c) *Technical remark 3:* I use the Brent optimization method to solve the firm's problem.

3) **Step 3:** Fix a grid between $[\tilde{p}_{min,s}, \tilde{p}_{max,s}]$ with n_s points close to the continuation region implied by the policy. Construct the three transition matrices $F_{\Delta_a}, F_{\Pi}, F_{p'}$ with $\mathcal{C}^{X_{ss}}$ and $\Psi^{X_{ss}}$ using linear splines over the grid $[\tilde{p}_{min,s}, \tilde{p}_{max,s}]$. The definitions of $F_{\Delta_a}, F_{\Pi}, F_{p'}$ are:

- a) F_{Δ_a} is given by the transition probability $\tilde{p}_1 = \tilde{p} \frac{A'}{A}$.
- b) F_{Π} is given by the transition probability $\tilde{p}_2 = \frac{\tilde{p}_1}{\Pi_{ss}}$.
- c) $F_{p'}^{X_{ss}}$ is given by the transition probability

$$\tilde{p}_3 = \begin{cases} \tilde{p}_2 I(\tilde{p} \in \mathcal{C}^{X_{ss}}) + I(\tilde{p} \notin \mathcal{C}^{X_{ss}}) p^{*,X_{ss}} & \text{with prob. } 1-\zeta \\ p^{*,X_{ss}} & \text{with prob. } \zeta \end{cases}$$

I compute the ergodic distribution as the eigenvector of the unit eigenvalue of $F_{\Delta a} F_{\Pi} F_{p'}$. The eigenvector associated to the unit eigenvalue gives the ergodic distribution $n^{X_{ss}}$. After obtaining the ergodic distribution, I compute the reset inflation, price dispersion, and frequency of price changes.

- $n_{a u \bar{x}}^{X_{ss}} = (n^{X_{ss}} F_{\Delta a} F_{\Pi}) I(p \in C^z)(1 - \zeta)$.
- $\Omega(X_{ss}) = 1 - \sum_i n_{a u \bar{x}}(i)$.
- $\varphi(X_{ss}) = (1 + \bar{\pi}) \left(\sum_i \tilde{p}(i)^{1-\gamma} \frac{n_{a u \bar{x}}^{X_{ss}}(i)}{\sum_i n_{a u \bar{x}}^{X_{ss}}(i)} \right)^{\frac{1}{1-\gamma}}$.
- $\Pi(X_{ss}) = \left(\frac{1 - \Omega(X_{ss})}{1 - \Omega(X_{ss})(p^*, X_{ss})^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi(X_{ss})$.
- $\Delta(X_{ss})^* = \sum_i \tilde{p}(i)^{-\gamma} n^{X_{ss}}(i)$

I.3. Menu Cost Model: Business Cycle Computation Algorithm

The algorithm to solve the model has three steps:

- Step 1: It uses a perturbation method to approximate the equilibrium dynamics without the ZLB with the reset price of the Calvo pricing model. It projects price dispersion, menu cost inflation and frequency of price changes onto the state.
- Step 2: It solves the equilibrium conditions with global methods ignoring the zero lower bound.
- Step 3: It solves the equilibrium conditions with global methods with the zero lower bound.

I.4. Step 1: Approximation of the Equilibrium

- 1) **Step 1:** Initiaze the Krusell-Smith projection

$$(I.4) \quad \Delta(S) = P_1(\log(S)) \quad ; \quad \Omega(S) = P_1(\log(S)) \quad ; \quad \varphi(S) = P_1(\log(S)) \quad ; \quad \tau_1^C$$

where P^1 denotes a linear projection.

- 2) **Step 2:** Given

$$(I.5) \quad \Delta(S) = P_i(\log(S)) \quad ; \quad \Omega(S) = P_i(\log(S)) \quad ; \quad \varphi(S) = P_i(\log(S)) \quad ; \quad \tau_i^C$$

solve the aggregate equilibrium equations using a first order perturbation method (with $\Omega^* =$

$p + \zeta$)

$$\begin{aligned}
mu_i(S) &= \beta \eta_{q-}(S) R_i(S) \mathbb{E}_{S'} \left[d\eta_z(S')^{-\sigma} \frac{mu_i(S)}{\Pi_i(S')} \middle| S \right] \\
\Sigma_i(S) &= \mathbb{E}_{S'} \left[\left(d\eta_z(S')^{1-\sigma} \frac{U_i(S')}{U_{ss}} \right) \middle| S \right] \\
mu_i(S) &= \bar{u} \left(C_i(S) - \kappa \frac{L_i(S)^{1+\chi}}{1+\chi} \right)^{-\sigma} \\
\kappa L_i(S)^{\chi} &= \eta_{h,-}(S) w_i(S) \\
u_i(S) &= \frac{\bar{u} \left(C_i(S) - \kappa \frac{L_i(S)^{1+\chi}}{1+\chi} \right)^{1-\sigma}}{1-\sigma} \\
U_i(S) &= (1-\beta)u_i(S) + \beta U_{ss} \Sigma_i(S) \\
\Pi_i(S) &= \left(\frac{1 - \Omega_i(S)}{1 - \Omega_i(S)(P_i^*(S))^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_i(S) \\
mc_i(S) &= \iota((1-\tau_{MC})w_i(S))^{1-\alpha} \\
R_i(S) &= \tilde{R}_-(S) \left(\frac{1+\bar{\pi}}{\beta g^{-\sigma}} \right) \left(\left(\frac{\Pi_i(S)}{1+\bar{\pi}} \right)^{\phi_\pi} \left(\frac{mc_i(S)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc_i(S)}{mc_-(S)} \right)^{\phi_{dy}} \\
\tilde{R}_i(S') &= \left(\left(\frac{\beta g^{-\sigma}}{1+\bar{\pi}} \right) R_i(S) \right)^{\phi_r} \eta'_r \\
0 &= -(L_i(S) - \theta(\Omega_i(S) - \zeta)) + Y_i(S) \left(\frac{1-\alpha}{\alpha w_i(S)} \right)^\alpha \Delta_i(S) \\
C_i(S) &= Y_i(S) \left(1 - \left(\frac{w_i(S)\alpha}{1-\alpha} \right)^{1-\alpha} \Delta_i(S) \right) \frac{1}{\eta_{g-}(S)} \\
H_i(S) &= \frac{\gamma}{\gamma-1} (1-\beta) mu_i(S) Y_i(S) mc_i(S) \tau_i^C + \dots \\
&\quad \dots + \beta g^{1-\sigma} (1-\Omega^*) \mathbb{E}_{S'} [d\eta_z(S')^{1-\sigma n} \Pi_i(S')^\gamma H_i(S') | S] \\
F_i(S) &= mu_i(S) Y_i(S) (1-\beta) + \dots \\
&\quad \dots + \beta g^{1-\sigma} (1-\Omega^*) \mathbb{E}_{S'} [d\eta_z(S')^{1-\sigma n} \Pi_i(S')^{\gamma-1} F_i(S') | S] \\
P_i^*(S) &= \frac{H_i(S)}{F_i(S)}
\end{aligned}$$

- a) *Warning:* The order of the projection has to be equal to the order of the perturbation. I use first order in both, the perturbation and the Krusell-Smith approximation.

$$\begin{aligned}
\Delta_i(S) &= P_i(\log(S)) \\
\Omega_i(S) &= P_i(\log(S)) \\
\varphi_i(S) &= P_i(\log(S)) \\
\log(\eta_g(S')) &= (1-\rho_g) \log(\eta_g^*) + \rho_g \log(\eta_g(S)) + \sigma_g \epsilon'_g \\
\log(\eta_z(S')) &= (1-\rho_z) \log(\eta_z^*) + \rho_z \log(\eta_z(S)) + \sigma_z \epsilon'_z \\
\log(\eta_r(S')) &= \sigma_r \epsilon'_r \\
\log(\eta_q(S')) &= (1-\rho_q) \log(\eta_q^*) + \rho_q \log(\eta_q(S)) + \sigma_q \epsilon'_q
\end{aligned}$$

3) **Step 3:** Given

$$(1.6) \quad mu_i(S), mc_i(S), U_i(S), Y_i(S), w_i(S), \Pi_i(S)$$

solve the firm's problem

$$\begin{aligned}
v_i(p, S) &= (1 - \zeta) \max \{v_i^c(S) - mu_i(S)w_i(S)\theta, v_i^{nc}(\tilde{p}, S)\} + \zeta v_i^c(S) \\
v_i^c(S) &= \max_x \Phi(x, S) + \beta v_2^i(x, S) \\
v_i^{nc}(\tilde{p}, S) &= \Phi\left(\frac{\tilde{p}}{\Pi_i(S)}, S\right) + \beta v_2^i\left(\frac{\tilde{p}}{\Pi_i(S)}, S\right) \\
Ev_i(p, S) &= \mathbb{E}_{\tilde{p}'} [d\eta_z(S')^{1-\sigma_n} v_1^i(\tilde{p}', S')] \\
\Phi(\tilde{p}, S) &= mu_i(S)Y_i(S)\tilde{p}^{-\gamma} (\tilde{p} - (1 - \tau_L)mc_i(S)) \\
\tilde{p}'(\tilde{p}, S') &= \begin{cases} \tilde{p}e^{\sigma_a \epsilon_h^a} & \text{with prob. } p \\ \tilde{p}e^{\sigma_a \epsilon_t^a} & \text{with prob. } 1 - p \end{cases}
\end{aligned}$$

and get $P_i^{mc}(S), \Psi_i(S), \mathcal{C}_i(S)$

$$\begin{aligned}
P_i^{mc}(S) &= \arg \max_x \Phi(x, S) + \beta g^{1-\sigma} v_1^i(x, S) \\
\Psi_i(S) &= \{\tilde{p} : v_i^{nc}(\tilde{p}, S) \geq v_i^c(S) - mu_i(S)w_i(S)\theta\} \\
\mathcal{C}_i(S) &= \{(\tilde{p}_-, \Delta a) : \tilde{p}_- e^{\Delta a} \in \Psi(S)\}
\end{aligned}$$

- a) *Technical remark 1:* I use splines to approximate the firm's relative price state and Smolyak polynomials for the aggregate state (see section J).
- b) *Technical remark 2:* I use value function iteration together with collocation to solve the firm's Bellman equation.
- c) *Technical remark 3:* I use the Brent optimization method to solve the firm's problem.
- d) *Technical remark 4:* To solve this problem and avoid the Kronecker product in the expectation, I preallocate the base in the optimization before solving the firm's problem

$$\begin{aligned}
\tilde{\Phi}(s, S) &= \sum_{\epsilon_s, \epsilon_S} w(\epsilon_s)w(\epsilon_S)d\eta_z(S')^{1-\sigma_n} \Psi(s'(s, \epsilon_s), S'(S, \epsilon_S)) \\
&= \sum_{\epsilon_S} \sum_{\epsilon_s} w(\epsilon_s)w(\epsilon_S)d\eta_z(S')^{1-\sigma_n} (\Psi_s(s'(s, \epsilon_s)) \otimes \Psi_S(S'(S, \epsilon_S))) \\
&= \left(\sum_{\epsilon_s} w(\epsilon_s)\Psi_s(s'(s, \epsilon_s)) \right) \otimes \left(\sum_{\epsilon_S} w(\epsilon_S)d\eta_z(S')^{1-\sigma_n} \Psi_S(S'(S, \epsilon_S)) \right).
\end{aligned}$$

Here $w(\epsilon_s)$ and $w(\epsilon_S)$ are the weights in the quadrature and Ψ denotes the base for the functional approximation.

Step 4: Given the policy

$$(I.7) \quad \Pi_i(S), mc_i(S), \tilde{R}_i(S), P_i^{mc}(S), \mathcal{C}_i(S)$$

and some initial conditions s_{idio} and S_{agge} , simulate the model.

- a) Compute the distribution after repricing decision

$$(I.8) \quad n_{aux,t}^1 = (n_{t-1} F_{\Delta a}) I(\tilde{p}e^{\Delta a} \in \mathcal{C}_i(S_{t-1}))(1 - \zeta)$$

- b) Compute frequency of price changes $\Omega_t = 1 - \sum_i n_{aux,t}(i)$.
- c) Compute menu cost inflation

$$(I.9) \quad \varphi_t = \left(\sum_i \tilde{p}(i)^{1-\gamma} \frac{n_{aux,t}(i)}{\sum_i n_{aux,t}(i)} \right)^{\frac{1}{1-\gamma}}$$

- d) Compute inflation

$$(I.10) \quad \Pi_t = \left(\frac{1 - \Omega_t}{1 - \Omega_t P_t^{1-\gamma}} \right)^{1/(1-\gamma)} \varphi_t$$

- e) Update the distribution $n_t = n_{t-1} F_{\Delta} F_{\Pi} F_p$
 f) Compute price dispersion $\varphi_t = \sum_i \tilde{p}^{-\gamma} n_t$
 g) Update state S_t from S_{t-1}

4) **Step 5:** Check convergence of the policy.

$$(I.11) \quad \max_{S \in S^{erg}} \left(\text{abs} \left(\frac{C_i(S) - C_{i-1}(S)}{C_{i-1}(S)}, \frac{L_i(S) - L_{i-1}(S)}{L_{i-1}(S)}, \frac{mc_i(S) - mc_{i-1}(S)}{mc_{i-1}(S)}, \dots \right. \right.$$

$$(I.12) \quad \left. \dots, R_i(S) - R_{i-1}(S), \frac{P_i^{mc}(S) - P_{i-1}^{mc}(S)}{P_{i-1}^{mc}(S)}, \frac{\Pi_i(S) - \Pi_{i-1}(S)}{\Pi_{i-1}(S)} \right) < tol_{conv}$$

where $S = \{S_1, S_2, S_3, \dots\}$ is obtained from the simulation in the model's ergodic set. Stop if convergence. Go to step 6 if no convergence.

5) **Step 6:** Update the coefficients in the projection

$$(I.13) \quad \Delta_t = P_{i+1}(\log(S_t)) \quad ; \quad \Omega_t = P_{i+1}(\log(S_t)) \quad ; \quad \varphi_t = P_{i+1}(\log(S_t)).$$

Update τ_{i+1}^C in such a way the inflation in the simulation is equal to the inflation target. Go to step 2.

I.5. Step 2: Global Solution Ignoring the Zero Lower Bound.

1) **Step 1:** Initiate the Krusell-Smith projection

$$\Delta(S) = P_1^2(\log(S)) \quad ; \quad \Omega(S) = P_1^2(\log(S)) \quad ; \quad \varphi(S) = P_1^2(\log(S))$$

where P^2 denotes a quadratic projection.

2) **Step 2:** Fix ξ as the limit in the iteration in the global solutions. i denote the i -th iteration in the Krusell-Smith projection and j denoted the j -th iteration in the global solution. Given

$$\Delta(S) = P_i^2(\log(S)) \quad ; \quad \Omega(S) = P_i^2(\log(S)) \quad ; \quad \varphi(S) = P_i^2(\log(S))$$

and

$$\Pi_{1,i}(S) \quad ; \quad U_{1,i}(S) \quad ; \quad \Sigma_{1,i}(S) \quad ; \quad mu_{1,i}(S)$$

- **Step 2.1:** Given $mu_{j,i}(S), U_{j,i}(S), \Sigma_{j,i}(S), mu_{j,i}(S)$, use the Euler equation to get

$$\Sigma_{j+1,i}(S) = \mathbb{E}_{S'} \left[\left(d\eta_z(S')^{1-\sigma_n} \frac{U_{j+1,i}(S')}{U_{ss}} \right) \middle| S \right]$$

$$mu_{j+1,i}(S) = \beta \eta_q(S) R_{j,i}(S) \mathbb{E}_{S'} \left[d\eta_z(S')^{-\sigma} \frac{mu_{j,i}(S')}{\Pi_{j,i}(S')} \middle| S \right]$$

- **Step 2.2:** With $mu_{j+1,i}(S)$ solve the following system

$$\kappa L_{j+1,i}(S)^\chi = \eta_{h,-}(S) w_{j+1,i}(S)$$

$$mu_{j+1,i}(S) = \bar{u} \left(C_{j+1,i}(S) - \kappa \frac{L_{j+1,i}(S)^{1+\chi}}{1+\chi} \right)^{-\sigma}$$

$$L_{j+1,i}(S) = Y^{i+1}(S) \left(\frac{(1-\alpha)}{\alpha w_{j+1,i}(S)} \right)^\alpha \Delta_i(S)$$

$$C_{j+1,i}(S) = Y_{j+1,i}(S) \left(1 - \left(\frac{w_{j+1,i}(S)\alpha}{1-\alpha} \right)^{1-\alpha} \Delta_i(S) \right) \frac{1}{\eta_g(S)}$$

Compute the marginal cost and period utility given by

$$mc_{j+1,i}(S) = \iota(1 - \tau_{mc}) w_{j+1,i}(S)^{1-\alpha} \quad u_{j+1,i}(S) = \bar{u} \frac{(C_{j+1,i}(S) - \kappa \frac{L_{j+1,i}(S)^{1+\chi}}{1+\chi})^{1-\sigma}}{1-\sigma}$$

- **Step 2.3:** Update the forward looking variable $U(S)$ and the Taylor rule

$$\begin{aligned} U_{j+1,i}(S) &= (1-\beta)u_{j+1,i}(S) + \beta U_{ss} \Sigma_{j+1,i}(S) \\ R_{j+1,i}^*(S) &= \tilde{R}_-(S) \left(\frac{1+\bar{\pi}}{\beta g^{-\sigma}} \right) \left(\left(\frac{\Pi_{j+1,i}(S)}{1+\bar{\pi}} \right)^{\phi_\pi} \left(\frac{\tilde{m}c_{j+1,i}(S)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc_{j+1,i}(S)}{\tilde{m}c(S)} \right)^{\phi_{dy}} \\ \tilde{R}_{j+1,i}(S) &= \left(\left(\frac{\beta g^{-\sigma}}{1+\bar{\pi}} \right) (R^*)_{j+1,i}(S) \right)^{\phi_r} \quad R_{j+1,i}(S) = (R^*)_{j+1,i}(S); \end{aligned}$$

- **Step 2.4:** If $j+1 = \xi$ go to step 2.5. If

$$\begin{aligned} error &= \text{mean}_S (|mu^i(S) - mu^{i+1}(S)| + |\Pi^i(S) - \Pi^{i+1}(S)| + |U^i(S) - U^{i+1}(S)|) < \epsilon \\ &\text{go to step 2.5. Otherwise, go to step 2.1.} \end{aligned}$$

- **Step 2.5:** Solve firm's Bellman equation

$$\begin{aligned} v_{j+1,i}(p, S) &= (1-\zeta) \max \{v_{j+1,i}^c(S) - mu_{j+1,i}(S)w_{j+1,i}(S)\theta, v_{j+1,i}^{nc}(\tilde{p}, S)\} + \zeta v_{j+1,i}^c(S) \\ v_{j+1,i}^c(S) &= \max_x \Phi(x, S) + \beta g^{1-\sigma} E v_{j+1,i}(x, S) \\ v_{j+1,i}^{nc}(\tilde{p}, S) &= \Phi \left(\frac{\tilde{p}}{\Pi_{j+1,i}(S)}, S \right) + \beta E v_{j+1,i} \left(\frac{\tilde{p}}{\Pi_i(S)}, S \right) \\ E v_{j+1,i}(p, S) &= \mathbb{E}_{\tilde{p}'} [d\eta_z(S')^{1-\sigma} v_{j+1,i}(\tilde{p}', S')] \\ \Phi(\tilde{p}, S) &= (1-\beta)mu_{j+1,i}(S)Y_{j+1,i}(S)\tilde{p}^{-\gamma} (\tilde{p} - mc_{j+1,i}(S)) \\ \tilde{p}'(\tilde{p}, S') &= \begin{cases} \tilde{p} & \text{with prob. } p \\ \tilde{p}e^{\sigma_a \epsilon^a - \frac{\sigma_a^2}{2}} & \text{with prob. } 1-p \end{cases} \end{aligned}$$

and get $P_{j+1,i}(S)$ from

$$\begin{aligned} P_{j+1,i}^{mc}(S) &= \arg \max_x \Phi(x, S) + \beta g^{1-\sigma} v_1^i(x, S) \\ \Pi_{j+1,i}(S) &= \left(\frac{1 - \Omega_i(S)}{1 - \Omega_i(S)(P_{j+1,i}^*(S))^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi_i(S) \end{aligned}$$

- **Step 2.6:** If

$$(I.14) \quad \max_{S \in S^{erg}} \left(\text{abs} \left(\frac{C_{j+1,i}(S) - C_{j,i}(S)}{C_{j,i}(S)}, \frac{L_{j+1,i}(S) - L_{j,i}(S)}{L_{j,i}(S)}, \frac{mc_{j+1,i}(S) - mc_{j,i}(S)}{mc_{j,i}(S)}, \dots \right) \right)$$

$$(I.15) \quad \dots, R_{j+1,i}(S) - R_{j,i}(S), \frac{P_{j+1,i}(S) - P_{j,i}(S)}{P_{j,i}(S)}, \frac{\Pi_{j+1,i}(S) - \Pi_{j,i}(S)}{\Pi_{j,i}(S)}) \geq tol_{conv}$$

go to step 2.1. If

$$\begin{aligned} \max_{S \in S^{erg}} \left(\text{abs} \left(\frac{C_{j+1,i}(S) - C_{j,i}(S)}{C_{j,i}(S)}, \frac{L_{j+1,i}(S) - L_{j,i}(S)}{L_{j,i}(S)}, \frac{mc_{j+1,i}(S) - mc_{j,i}(S)}{mc_{j,i}(S)}, \dots \right) \right) \\ \dots, R_{j+1,i}(S) - R_{j,i}(S), \frac{P_{j+1,i}(S) - P_{j,i}(S)}{P_{j,i}(S)}, \frac{\Pi_{j+1,i}(S) - \Pi_{j,i}(S)}{\Pi_{j,i}(S)}) < tol_{conv} \end{aligned}$$

Set

$$\begin{aligned} C_{1,i+1}(S) &= C_{j+1,i}(S) \quad ; \quad L_{1,i+1}(S) = L_{j+1,i}(S) \\ mc_{1,i+1}(S) &= mc_{j+1,i}(S) \quad ; \quad \Pi_{1,i+1}(S) = \Pi_{j+1,i}(S) \end{aligned}$$

and check if

$$\max_{S \in S^{erg}} \left(\text{abs} \left(\frac{C_{j,i+1}(S) - C_{1,i}(S)}{C_{1,i}(S)}, \frac{L_{1,i+1}(S) - L_{1,i}(S)}{L_{1,i}(S)}, \frac{mc_{1,i+1}(S) - mc_{1,i}(S)}{mc_{1,i}(S)}, \dots \right. \right. \\ \left. \left. \dots, R_{1,i+1}(S) - R_{1,i}(S), \frac{P_{1,i+1}(S) - P_{1,i}(S)}{P_{1,i}(S)}, \frac{\Pi_{1,i+1}(S) - \Pi_{1,i}(S)}{\Pi_{1,i}(S)} \right) \right) < tol_{conv}$$

If this is the case, we found the equilibrium. If previous inequality doesn't hold, go to step 3.

Step 3: Given the policy

$$(I.16) \quad \Pi_i(S), mc_i(S), \tilde{R}_i(S), P_i^{mc}(S), C_i(S)$$

and some initial conditions n_0 and S_0 simulate the equilibrium.

a) Compute the distribution after repricing decision

$$n_{aux,t} = (n_{t-1} F_{\Delta a}) I(\tilde{p} e^{\Delta} \in C_i(S_{t-1})) (1 - \zeta)$$

b) Compute frequency of price change $\Omega_t = 1 - \sum_i n_{aux,t}(i)$.

c) Compute menu cost inflation

$$\varphi_t = \left(\sum_i \tilde{p}(i)^{1-\gamma} \frac{n_{aux,t}(i)}{\sum_i n_{aux,t}(i)} \right)^{\frac{1}{1-\gamma}}$$

d) Compute inflation

$$\Pi_t = \left(\frac{1 - \Omega_t}{1 - \Omega_t P_t^{1-\gamma}} \right)^{1/(1-\gamma)} \varphi_t$$

e) Update the distribution $n_t = n_{t-1} F_{\Delta A} F_{\Pi_t} F_p$

f) Compute price dispersion $\varphi_t = \sum_i \tilde{p}^{-\gamma} n_t$

g) Update state S_t from S_{t-1}

3) **Step 4:** Update the coefficient in the projection

$$\Delta_t = P_{i+1}^2(\log(S_{t-1})) \quad ; \quad \Omega_t = P_{i+1}^2(\log(S_{t-1})) \quad ; \quad \varphi_t = P_{i+1}^2(\log(S_{t-1}))$$

Go to step 2.

I.6. Step 3: Global Solution with ZLB

I repeat the same algorithm as in step 2 using the global solution of the model without the zero lower bound from Step 2 to initialize the policy function. The only difference comes in step 2.4 where I solve the following system:

$$(R^*)^{i+1}(S) = \tilde{R}_-(S) \left(\frac{1 + \bar{\pi}}{\beta(1+g)^{-\sigma}} \right) \left(\left(\frac{\Pi^{i+1}(S)}{(1+\bar{\pi})} \right)^{\phi_\pi} \left(\frac{\tilde{m}c^{i+1}(S)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc^{i+1}(S)}{\tilde{m}c(S)} \right)^{\phi_{dy}}$$

$$\tilde{R}^{i+1}(S) = \left(\left(\frac{\beta(1+g)^{-\sigma}}{1+\bar{\pi}} \right) (R^*)^{i+1}(S) \right)^{\phi_r}$$

$$R^{i+1}(S) = \max \{1, (R^*)^{i+1}(S)\}$$

For the model with ZLB, I make sure the interest rate \tilde{R} stays always inside the hypercube. In order to achieve this goal, let \underline{R} and \underline{mc} be the lower bound of the hypercube for the interest rate and the

marginal cost. I add the following wedge τ^C into the Euler equation:

$$mu(S) = \beta(1 - \tau^C(S))(1 + g)^{-\sigma} \eta_{q-}(S) R(S) \mathbb{E}_{S'} \left[\frac{mu(S')}{\Pi(S')} \middle| S \right]$$

$$\tau^c(S) = \begin{cases} 0 & \text{if } R(S) \geq \underline{R} \text{ \& } mc(S) \geq \underline{mc} \\ 1 - \frac{1}{\beta(1+g)^{-\sigma} \eta_{q-}(S) R(S) \mathbb{E}_{S'} \left[\frac{mu(S')}{\Pi(S')} \middle| S \right]} & \text{otherwise} \end{cases} .$$

This wedge is never active at the optimal inflation target.

J. PROJECTION METHOD FOR THE FIRM'S RECURSIVE PROBLEM IN THE MC MODEL

This section discusses the projection method I use to approximate the firm's value function in the MC model. The approximation method is important since the optimal reset price and Ss bands come from the numerical approximation of the value function; hence, it is important to have a reliable method to approximate the firm's value function. The main idea of this method is to use 2nd-order splines for the firm's state variable dimension and a Smolyak sparse grid for the aggregate state variables dimension. First, I will describe the construction of the grid and then the approximation method.

- **Grid:** Let $s = [\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_{n_s}]' \in R^{n_s \times 1}$ be the grid on the firm's idiosyncratic state variable with $n_s - 1$ breakpoints and let $S = [S_1, S_2, \dots, S_7]' \in R^{n_s \times 7}$ be the grid on the aggregate state variables. I follow Judd et al. (2014) to construct a sparse grid in S . If we order the firm's state last, so that the state vector is (S, s) , then I construct the grid for the firm's state as

$$(J.1) \quad (S, s) := [\mathcal{I}_{n_s \times 1} \otimes S, s \otimes \mathcal{I}_{n_s \otimes 1}] \in R^{(n_s * n_s) \times 8}$$

- **Function bases:** let $Z \in R^{H \times 8}$ be H arbitrary point in the grid (S, s) . To generate the base for Z , I generate the base for the idiosyncratic state variable $\Phi_s(Z_s) \in R^{H \times n_s}$ using 2nd-order splines, and I generate the base for the aggregate state variable $\Phi_S(Z_S) \in R^{H \times n_S}$ using Smolyak polynomials as in Judd et al. (2014). Then I take the Kronecker product

$$(J.2) \quad \Phi(Z) = \Phi_s(Z_s) \otimes \Phi_S(Z_S) \in R^{H \times (n_s * n_S)}$$

Figure J.II describes the values of changing the price and not changing the price, the optimal price, and the expected continuation value in the steady state and in the model with business cycle fluctuations. In the steady state, I'm using 3rd-order splines. In the model with business cycles fluctuations, I'm using Smolyak sparse grid in the dimensions for the idiosyncratic and aggregate state variables. We can see three properties: (i) There is a large difference in the value of not changing the price in the steady state and in the model with business cycles, (ii) the expected continuation value with business cycles has oscillations that the steady state expected value function doesn't have, and (iii) the Ss bands in the model with business cycle are bigger than the Ss bands in the steady state since it cannot capture the concave-convex shape of the value function. These properties are errors coming from the inaccurate approximation in the interpolation method. The main reasons for this numerical errors are: (i) the sparsity in the grid of the idiosyncratic state, and (ii) the Chebyshev polynomial cannot capture the shape of the value function.

Figure J.I describes the values of changing the price and not changing the price, the optimal price, and the expected value in the steady state and the model with business cycle fluctuations. In the steady state, I'm using 3rd-order splines. In the model with business cycle fluctuations, I'm using 2nd-order splines for the firm's state variable dimension and a Smolyak sparse grid for the aggregate state variables dimension. As we can see, all of the previous problems disappear.

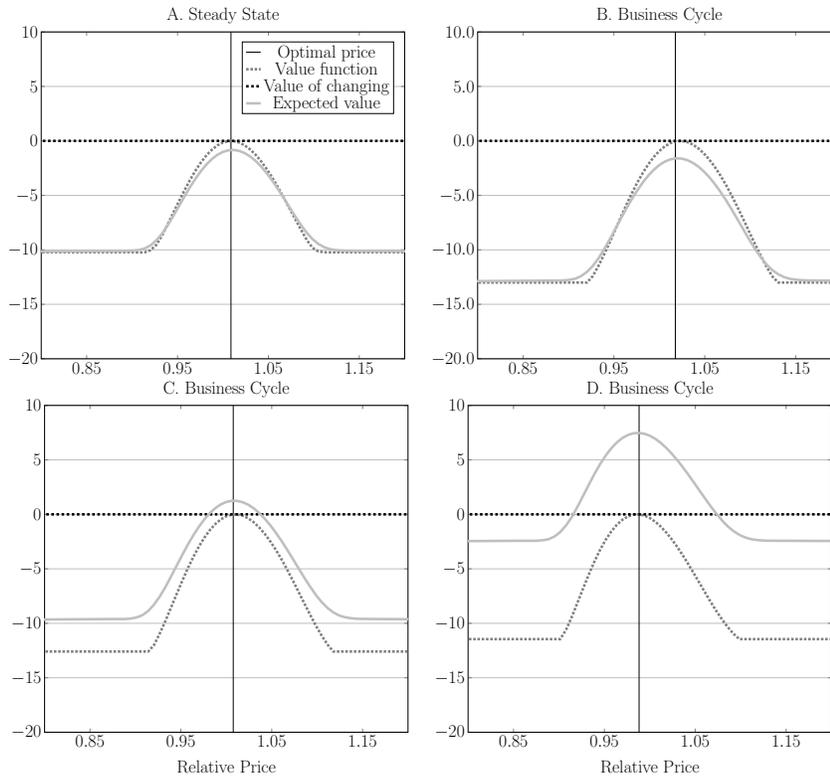


FIGURE J.I. VALUE FUNCTIONS WITH COMPLETED SMOLYAK SPARSE-GRID INTERPOLATION METHOD

Note: Panel A describes the value function, the optimal price, the value of changing the price—without the menu cost—and the expected continuation value in the steady state. Panel B to D describe the same variables with business cycle. All the value functions are normalized.
Source: Author's calculations

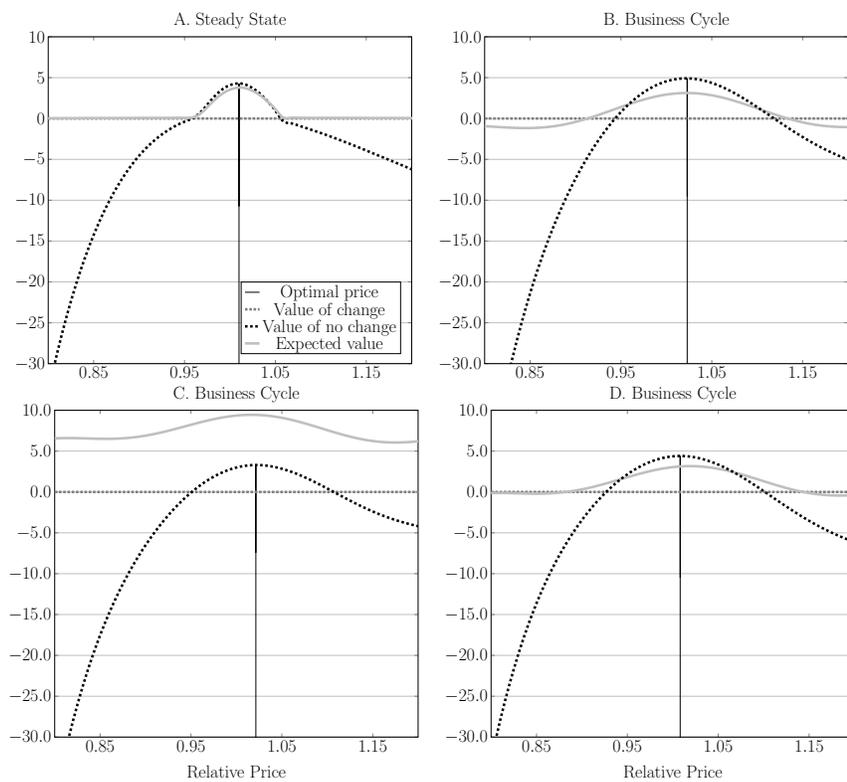


FIGURE J.II. VALUE FUNCTIONS WITH SMOLYAK SPARSE-GRID INTERPOLATION METHOD

Note: Panel A describes the value function of no changing the price, the optimal price, the value of changing the price—without the menu cost—and the expected continuation value in the steady state. Panel B to D describe the same variables with business cycle. All the value functions are normalized.
Source: Author's calculations

K. KRUSELL-SMITH IN STICKY PRICE MODELS WITH A TAYLOR RULE

This section of the Online Appendix explains the solution method to compute the equilibrium in a simplified environment, since the computation of the equilibrium poses a sizeable challenge: The combination of a Taylor rule for monetary policy and an infinite-dimensional aggregate state requires a nonstandard application and evaluation of accuracy of the Krusell-Smith algorithm. I explain the challenge and the solution, and provide an analytical example in which it is easy to see this problem.

For exposition, and for exposition only, I simplify the model in several dimensions: Preferences are given by period utility $u(C, L) = \log\left(C - \frac{L^{1+\chi}}{1+\chi}\right)$; the only input of production is labor $y_i = A_i l_i$ and menu costs are constant; the risk premium shock is the only structural shock in the economy; and the Taylor rule is given by $R_t = \max\left\{\frac{1+\bar{\pi}}{\beta} \left(\frac{\Pi_t}{1+\bar{\pi}}\right)^{\phi_\pi}, 1\right\}$. I abstract from productivity growth rate; thus $g = 0$.

K.1. Equilibrium Conditions

FIRM'S EQUILIBRIUM CONDITIONS. — The relevant idiosyncratic state variable for firm i at time t is $\tilde{p}_{ti} = \frac{p_{ti} A_{ti}}{P_t}$, the relative price multiplied by productivity. For simplicity, I refer to this object as the relative price. The assumption that productivity shocks also affect the demand of the intermediate input implies that the firm's static profits depend only on the relative price; thus, it is the only idiosyncratic state variable for the firm. Let $v(\tilde{p}_-, S)$ be the present discounted value of a firm with previous relative price \tilde{p}_- and current aggregate state S . Then $v(\tilde{p}_-, S)$ satisfies

$$\begin{aligned} v(\tilde{p}_-, S) &= \mathbb{E}_{\Delta a} \left[\max_{\text{change, no change}} \left\{ V^c(S), V^{nc} \left(\frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)}, S \right) \right\} \right], \\ V^{nc}(\tilde{p}, S) &= mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta \mathbb{E}_{S'} [v(\tilde{p}, S') | S], \\ \text{(K.1)} \quad V^c(S) &= -\theta w(S)mu(S) + \max_{\tilde{p}} \{ mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta \mathbb{E}_{S'} [v(\tilde{p}, S') | S] \}, \end{aligned}$$

where $mu(S), C(S), w(S)$ and $\Pi(S)$ denote the marginal utility, aggregate consumption, real wage, and inflation, respectively. The timing of the firm's optimization problem is as follows: First, aggregate and idiosyncratic uncertainty are realized; then the firm has the option of either changing the price or keeping it the same. If it changes the price, it has to pay the menu cost θ .

The policy of the firm is characterized by two objects: (1) a reset price and (2) a continuation region. Let $P^*(S)$ be the reset price, i.e., the firm's relative price with respect to the aggregate price level. Then

$$\text{(K.2)} \quad P^*(S) = \max_{\tilde{p}} \{ mu(S)C(S)\tilde{p}^{-\gamma}(\tilde{p} - w(S)) + \beta \mathbb{E}_{S'} [v(\tilde{p}, S') | S] \}.$$

The firm's relative price does not depend on the idiosyncratic shock; it only depends on the aggregate state of the economy, and therefore it is the same across resetting firms. The continuation region is given by all relative prices such that the value of changing the price is less than the value of not changing the price. Let $\Psi(S)$ be the continuation region. Then

$$\text{(K.3)} \quad \Psi(S) = \{ \tilde{p} : V^{nc}(\tilde{p}, S) \geq V^c(S) \}.$$

Since the firm makes the pricing decision after aggregate and idiosyncratic shocks are realized, the firm's policy is given by changing the price and setting a relative price equal to $P^*(S)$ if and only if $\frac{\tilde{p}_- e^{\Delta a}}{\Pi(S)} \notin \Psi(S)$.

As is typical in models with heterogeneity, the firm needs to forecast equilibrium prices and quantities and the aggregate state law of motion. If the firm knows these functions, then it has all the elements to make the optimal decision in (K.1).

AGGREGATE CONDITIONS. — The aggregate equilibrium conditions are given by the household optimality conditions, feasibility, and the monetary policy rule:

$$(K.4) \quad mu(S) = \beta R(S) \eta_q(S) \mathbb{E}_{S'} \left[\frac{mu(S')}{\Pi(S')} \middle| S \right],$$

$$(K.5) \quad \kappa L(S)^\chi = w(S) \quad ; \quad mu(S) = \bar{u}(C(S) - (1 + \chi)^{-1} L^{1+\chi})^{-1},$$

$$(K.6) \quad R(S) = \max \left\{ \frac{1 + \bar{\pi}}{\beta} \left(\frac{\Pi(S)}{1 + \bar{\pi}} \right)^{\phi_\pi}, 1 \right\} \quad ; \quad C(S) = \frac{L(S) - \Omega(S)\theta}{\Delta(S)},$$

where $\Delta(S)$ is labor productivity depending on inefficient price dispersion, given by

$$(K.7) \quad \Delta(S) = \int \bar{p}^\gamma f(d\bar{p});$$

here, $f(\bar{p})$ is the distribution of relative prices after repricing, and $\Omega(S)$ is the measure of firms changing the price.

Aggregate equilibrium conditions depend on two outcomes of the firm problem: inflation and price dispersion. Price dispersion only depends on the distribution of relative prices. This is a direct consequence of the assumption of idiosyncratic quality shocks. To see this, notice that the technological assumptions over the idiosyncratic shocks imply that labor demand is given by

$$(K.8) \quad \int l_i(S) di = C(S) \left[\int \bar{p}^\gamma f(d\bar{p}) \right] + \theta \Omega(S),$$

where $l_i(S)$ are firms' demand functions.

The key cross-equation restriction in which the MC model deviates from the Calvo model is the cross-equation restriction with respect to inflation. The next proposition shows the equilibrium condition for inflation:

PROPOSITION 5: *Define*

$$C(S) = \left\{ (\bar{p}_-, \Delta a) : \frac{\bar{p}_- e^{\Delta a}}{\Pi(S)} \in \Psi(S) \right\}.$$

Inflation dynamic is given by

$$(K.9) \quad \begin{aligned} \Pi(S) &= \left(\frac{1 - \Omega(S)}{1 - \Omega(S) P^*(S)^{1-\gamma}} \right)^{\frac{1}{1-\gamma}} \varphi(S), \\ \Omega(S) &= \int_{(\bar{p}_-, \Delta a) \notin C(S)} f(d\bar{p}_-) g(d\Delta a), \\ \varphi(S) &= \left(\int_{(\bar{p}_-, \Delta a) \in C(S)} \frac{(\bar{p}_- e^{\Delta a})^{1-\gamma}}{1 - \Omega(S)} f(d\bar{p}_-) g(d\Delta a) \right)^{\frac{1}{1-\gamma}}, \end{aligned}$$

where $g(\Delta a)$ is the distribution of quality shock innovations and $f(\bar{p}_-)$ is the distribution of relative prices previous period.

Inflation is a function of three elements: the reset price, the firm's inaction set, and the distribution of relative prices from the previous period. Inflation depends on forward-looking variables like the reset price and the continuation set—they are forward-looking because they solve the firm's problem—and a backward-looking variable given by the distribution of relative prices, which is backward-looking because it depends on the history of firms' previous choices.

AGGREGATE STATE. — Given that inflation and price dispersion are the aggregation of the relative prices, the distribution of relative prices is a state in the economy. I denote with S the state of the economy with the law of motion $\Gamma(S'|S)$. Therefore, in this simplified economy, the state of the economy is $S = (f(\bar{p}_-), \eta^Q)$ with the law of motion $\Gamma(S'|S)$.

K.2. Solution Method

To solve this model numerically, I modify the KS algorithm. Next, I describe this development.

MODIFICATION OF THE KS ALGORITHM. — Given that the distribution of relative prices is part of the state, I use the Krusell-Smith algorithm to solve this problem. However, the standard way of implementing Krusell-Smith does not work for this problem. The main reason is the following: Whenever solving equilibrium conditions, the KS algorithm replaces a model's equilibrium conditions with an approximation of the equilibrium policies obtained in the simulation. In this model—as in many others—this could generate indeterminacy at the time of solving the aggregate equilibrium equations. Next, I explain the steps in Krusell-Smith and indicate where it fails. Then, I describe the modification of the Krusell-Smith algorithm and evaluation of its accuracy in this model.

The Krusell-Smith algorithm consists of projecting aggregate prices and quantities on a small set of moments of the distribution of relative prices—one of the states of this economy—and the exogenous state. Let us define S_{SK} as the set of finite moments of the distribution in Krusell-Smith together with the exogenous shock η^Q ; $\Gamma(S'_{SK}|S_{SK})$ the law of motion of the state; and $\Pi(S_{SK})$, $\Omega(S_{SK})$ and $\Delta(S_{SK})$ the projections of inflation and price dispersion onto the state. Formally, the algorithm is given by

- 1) Given $\Pi(S_{KS})$, $\Delta(S_{KS})$, $\Omega(S_{KS})$ and $\Gamma(S'_{KS}|S_{KS})$, solve aggregate conditions (K.4) to (K.6).
- 2) With the solution of (K.4) to (K.6), solve the firm's value function (K.1).
- 3) Simulate and update $\Pi(S_{KS})$, $\Delta(S_{KS})$, $\Omega(S_{SK})$ and $\Gamma(S'_{KS}|S_{KS})$. Check convergence. If $\Pi(S_{KS})$, $\Delta(S_{KS})$ and $\Gamma(S'_{KS}|S_{KS})$ haven't converge, go to step 1.

To my knowledge, all Krusell-Smith formulations use this approach. The next proposition shows how the standard method generates multiplicity of solutions of equilibrium equations at the step of solving aggregate conditions.

PROPOSITION 6: For any $\Pi(S_{KS})$, $\Delta(S_{KS})$, $\Gamma(S'_{KS}|S_{KS})$ and $\lambda > 0$, if

$$(K.10) \quad \{mu(S_{KS}), C(S_{KS}), L(S_{KS}), R(S_{KS}), w(S_{KS})\}$$

is a solution for (K.4) to (K.6), then $\{\lambda mu(S_{KS}), \tilde{C}(S_{KS}), \tilde{L}(S_{KS}), R(S), \tilde{w}(S)\}$ is a solution, where $\tilde{C}(S_{KS})$, $\tilde{L}(S_{KS})$, and $\tilde{w}(S)$ solve

$$(K.11) \quad \kappa \tilde{L}(S_{KS})^\chi = \tilde{w}(S_{KS}) \quad ; \quad \tilde{C}(S_{KS}) = \frac{\tilde{L}(S_{KS}) - \Omega(S_{KS})\theta}{\Delta(S_{KS})} \quad ; \quad \lambda mu(S_{KS}) = \bar{u} \left(\tilde{C}(S_{KS}) - \frac{\tilde{L}^{1+\chi}}{1+\chi} \right)^{-1}.$$

PROOF:

It is easy to see that $\{\lambda mu(S_{KS}), \tilde{C}(S_{KS}), \tilde{L}(S_{KS}), R(S), \tilde{w}(S)\}$ satisfies all the equilibrium conditions.

From Proposition 6, we can extract three observations. First, since we didn't make any assumption about S_{KS} , the main result in Proposition 6 does not depend on the selected moments in the Krusell-Smith approximation. Second, the result does not depend on using global or projection methods. Third and more importantly, this is not saying that the economy has multiplicity of equilibria, but rather saying that the method for computing the equilibrium generates multiplicity at the moment of solving aggregate equilibrium conditions.

To understand the implication of previous propositions, we need to understand how models with price rigidities work. Assume a deviation from the equilibrium with an increase in consumption. This increase in consumption raises the real marginal cost, and due to the Phillips curve, it also raises inflation. If the Taylor principle is satisfied, this change in inflation impacts the real rate, feeding back to consumption, and undoing the original increase. It is the general equilibrium effect between households, firms, and the central bank that generates uniqueness of equilibrium. The KS algorithm breaks this general equilibrium effect at the moment of solving equilibrium equations since the Phillips curve is replaced by the inflation policy obtained in the simulation.

The main problem until now is that in the aggregate equilibrium conditions, there is no information on the relationship between inflation and real marginal cost, i.e., the Phillips curve. The solution I propose is to apply the Krusell-Smith algorithm to the frequency of price changes, and menu cost inflation, and solve *jointly* the aggregate and idiosyncratic equilibrium conditions. Even if this method generates some numerical challenges—which I have solved—it provides the central bank with the cross-equation restriction of the intensive margin of the Phillips curve, breaking the multiplicity mentioned above in step 1. Through numerical computation, it seems a reliable method, since it provides a unique solution when solving equilibrium conditions.³³

³³Aggregate kinks eliminate from consideration methods like that of Reiter (2009), and implemented in menu cost models by Costain and Nakov (2011).

Before describing the solution, I need to find the state. In models with nominal rigidities, the important object for the repricing of the firm is the markup, the ratio between the relative price and real marginal cost. Therefore, I use real marginal cost in the previous period as the state in Krusell-Smith. This variable is significant for predicting menu cost inflation. Moreover, I use price dispersion in the previous period. This variable approximates the second moment of the relative prices distribution and predicts menu cost inflation and itself. Next, I describe the algorithm used to solve the model.

- 1) Guess $\Delta(w_-, \Delta_-, \eta^Q), \Omega(w_-, \Delta_-, \eta^Q), \varphi(w_-, \Delta_-, \eta^Q)$ as functions of the state.
- 2) Solve the equilibrium conditions: the joint system of (K.1), (K.4) to (K.6), and (K.9). Get the law of motion for inflation and real wage $(mu, C, w, \Pi)(w_-, \Delta_-, \eta^Q)$, and the continuation set and reset price $(\Psi(w_-, \Delta_-, \eta^Q), P^*(w_-, \Delta_-, \eta^Q))$.
- 3) Simulate a measure of firms and compute $\{\Omega, \varphi, S\}_t$.
- 4) Project $\Delta, \Omega,$ and φ on the state. Check convergence. If convergence has not been achieved, update and go to step 2.

Note that the only law of motion of the state obtained in the simulation is that of price dispersion. Since the business cycle fluctuations of price dispersion are small, the laws of motion for all relevant endogenous states comes from solving the aggregate and idiosyncratic equilibrium conditions in step 2—not from the simulation. Second, even without the ZLB, I need to solve the model globally, given the kinks in the idiosyncratic policies. Third, this method breaks the separability in the equilibrium solution. With the previous implementation of Krusell-Smith, it was possible to solve the *aggregate conditions separate from idiosyncratic conditions*, dividing the system into two subsystems. Now, the aggregate and idiosyncratic equilibrium conditions must be solved together.

L. EVALUATION OF ACCURACY OF THE MODIFIED KRUSELL-SMITH ALGORITHM

I evaluate the accuracy of the KS algorithm in the simulation. First, I construct the time series of simulated inflation, price dispersion, and frequency of price changes, together with the simulated marginal cost, interest rate, and structural shocks—i.e., the states in the economy. Let X_t be the simulated vector with these variables, given by

$$(L.1) \quad X_t = \left\{ mc_-(S_t^s), \Delta_{t-1}^s, \tilde{R}_-(S_t^s), d\eta_z(S_t^s), \eta_g(S_t^s), \eta_q(S_t^s), \eta_h(S_t^s), \Pi_t^s, \Omega_t^s \right\}_t.$$

Let Y_t be the model-implied equilibrium functions in the simulation using the projections,

$$(L.2) \quad Y_t = \{mc(S_t^s), mu(S_t^s), R(S_t^s), R^*(S_t^s), Y(S_t^s), C(S_t^s), L(S_t^s), P^*(S_t^s), w(S_t^s)\}_t,$$

and let Y_t^s be the solution of the equilibrium equations in the simulation using X_t ,

$$(L.3) \quad \hat{Y}_t^s = \left\{ \hat{m}c_t^s, \hat{m}u_t^s, \hat{R}_t^s, \hat{R}_t^{*,s}, \hat{Y}_t^s, \hat{C}_t^s, \hat{L}_t^s, \hat{P}_t^{*,s}, w_t^s \right\}_t.$$

Next, I describe the construction of each function. Table L.I shows the standardized errors $\sigma_y^{error} = \frac{Std[\log(|\hat{Y}_t^s/Y_t|)]}{Std(\log(Y_t))} \times 100$ for each variable and Figures L.I to L.VI show the time series for \hat{Y}_t^s and Y_t (with and without the ZLB).

- **Method for the construction of \hat{Y}_t^s using the static equations and $mu(S_t^s)$.** Given $mu(S_t^s)$ and X_t , I construct \hat{Y}_t^s solving the static system of equations

$$\begin{aligned} \hat{R}_t^{*,s} &= \tilde{R}_-(S_t^s) \left(\frac{1 + \bar{\pi}}{\beta(1+g)^{-\sigma_{np}}} \right) \left(\left(\frac{\Pi_t^s}{(1+\bar{\pi})} \right)^{\phi_{\pi}} \left(\frac{mc_-(S_{t+1}^s)}{mc_{ss}} \right)^{\phi_y} \right)^{1-\phi_r} \left(\frac{mc_t^s}{mc_-(S_t^s)} \right)^{\phi_{dy}} \\ \hat{R}_t^s &= \max\{1, \hat{R}_t^{*,s}\} \\ mu(S_t^s) &= \left(\hat{C}_t^s - \kappa \frac{(\hat{L}_t^s)^{1+\chi}}{1+\chi} \right)^{-\sigma} \\ \eta_h^s \kappa (\hat{L}_t^s)^\chi &= \hat{w}_t^s \eta_{h-}(S_t^s) \\ (\hat{L}_t^s - \theta(\Omega_t^s - hz)) &= \hat{Y}_t^s \left(\frac{(1-\alpha)}{\alpha \hat{w}_t^s} \right)^\alpha \Delta_t^s \\ \hat{C}_t^s &= \hat{Y}_t^s \left(1 - \left(\frac{\hat{w}_t^s \alpha}{1-\alpha} \right)^{1-\alpha} \Delta_t^s \right) \frac{1}{\eta_{g-}(S)} \\ \hat{m}c_t^s &= \iota(1 - \tau_{mc})(\hat{w}_t^s)^{1-\alpha} \end{aligned}$$

- **Method for the construction of the marginal utility $\hat{m}u_t^s$ using forward-looking equations.** I construct $\hat{m}u_t^s$ using the household Euler equation. First, I project realized inflation onto the state. Let $\tilde{\Pi}(S)$ be this function. Then, I construct the marginal utility in the simulation as

$$\hat{m}u_t^s = \beta \eta_{q-}(S_t^s) \hat{R}_t^s \mathbb{E}_{S'} \left[d\eta_z(S_t^s)^{-\sigma} \frac{mu(S')}{\tilde{\Pi}(S')} \middle| S_t^s \right]$$

where $S_t^s = (mc_-(S_t^s), \Delta_{t-1}^s, \tilde{R}_-(S_t^s), d\eta_z(S_t^s), \eta_g(S_t^s), \eta_q(S_t^s), \eta_h(S_t^s))$.

TABLE L.I—ACCURACY OF THE KRUSELL-SMITH ALGORITHM

Variables	Annual Inflation Target (σ_y^{error} no ZLB , σ_y^{error} ZLB)		
	1.3	3	5
Nominal rate	(0.038,0.624)	(0.053,0.329)	(0.086,0.133)
Gross output	(0.511,3.518)	(0.577,0.968)	(1.359,1.217)
Real wage	(0.001,0.005)	(0.001,0.001)	(0.001,0.001)
Consumption	(0.505,4.358)	(0.571,0.989)	(1.378,1.223)
Period utility	(0.055,1.924)	(0.062,0.070)	(0.170,0.141)

Note: The table describes the ratio between the variance of predicted errors to the total variance for nominal interest rate, gross output, real wages, consumption, and period utility.

Source: Author's calculations

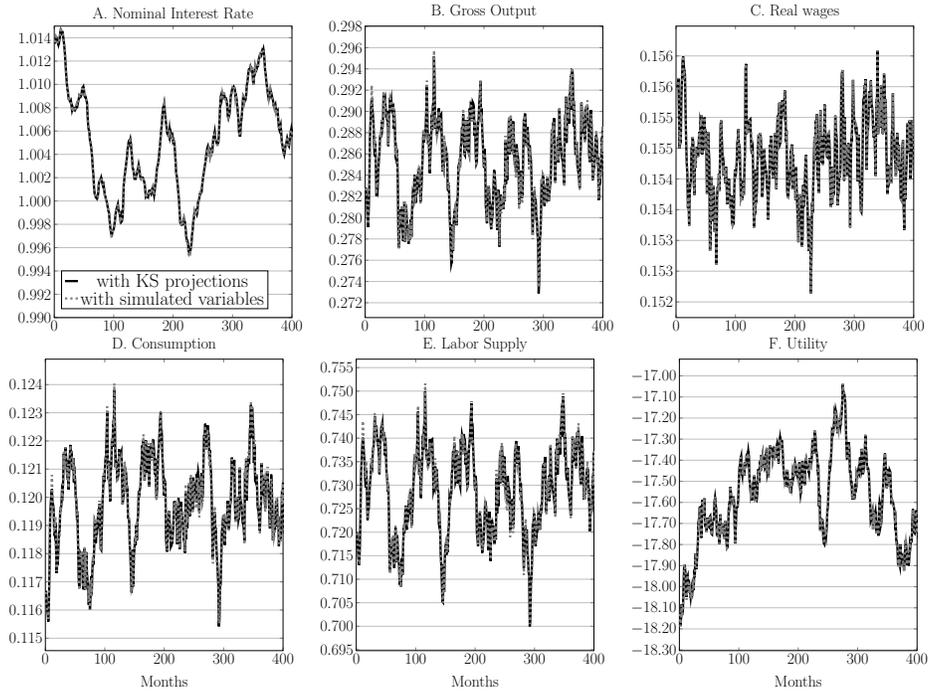


FIGURE L.I. PREDICTED AND SIMULATED AGGREGATE TIME SERIES WITH NO ZLB AT A 1.3% INFLATION TARGET

Note: The figure describes the macroeconomic time series for one simulation at a 1.3% inflation target with no ZLB constraint. Panels A to F plot (in the following order) nominal interest rate, gross output, real wages, consumption, labor supply, and period utility. Black solid lines describe the model-implied aggregate variables with Krusell-Smith projections, and gray dotted lines describe the implied macroeconomic time series with simulated inflation, frequency of price changes, and price dispersion.
Source: Author's calculations

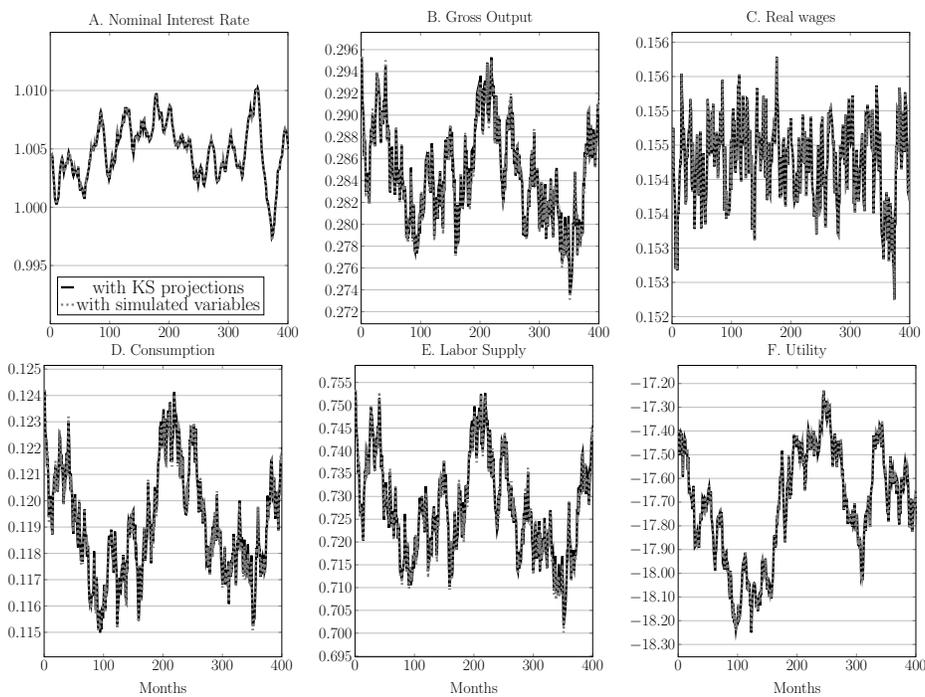


FIGURE L.II. PREDICTED AND SIMULATED AGGREGATE TIME SERIES WITH NO ZLB AT A 3% INFLATION TARGET

Note: The figure describes the macroeconomic time series for one simulation at a 3% inflation target with no ZLB constraint. Panels A to F plot (in the following order) nominal interest rate, gross output, real wages, consumption, labor supply, and period utility. Black solid lines describe the model-implied aggregate variables with Krusel-Smith projections, and gray dotted lines describe the implied macroeconomic time series with simulated inflation, frequency of price changes, and price dispersion.
Source: Author's calculations

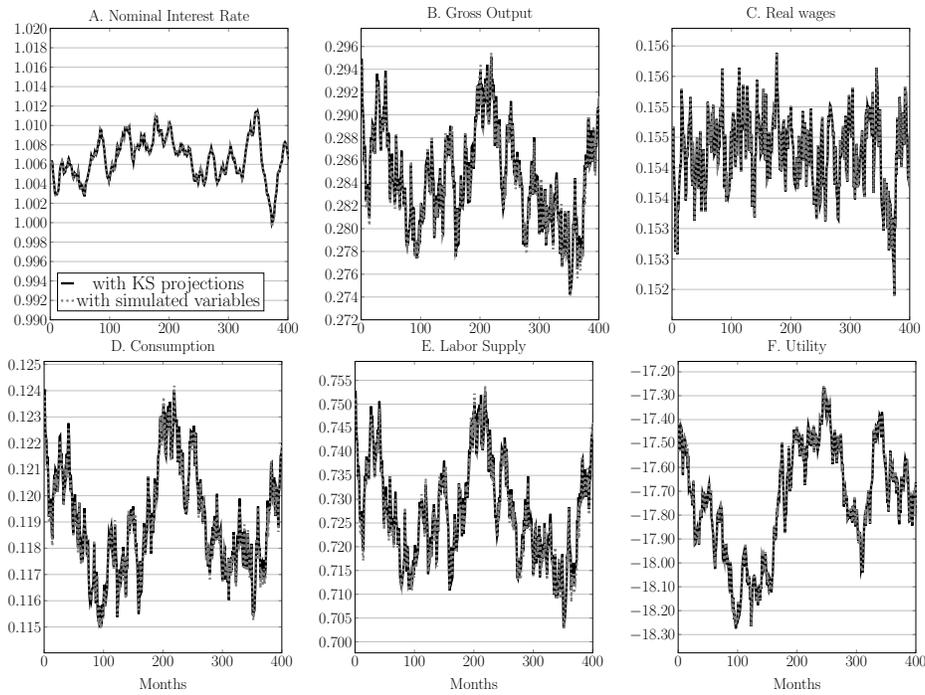


FIGURE L.III. PREDICTED AND SIMULATED AGGREGATE TIME SERIES WITH NO ZLB AT A 5% INFLATION TARGET

Note: The figure describes the macroeconomic time series for one simulation at a 5% inflation target with no ZLB constraint. Panels A to F plot (in the following order) nominal interest rate, gross output, real wages, consumption, labor supply, and period utility. Black solid lines describe the model-implied aggregate variables with Krusell-Smith projections, and gray dotted lines describe the implied macroeconomic time series with simulated inflation, frequency of price changes, and price dispersion.
Source: Author's calculations

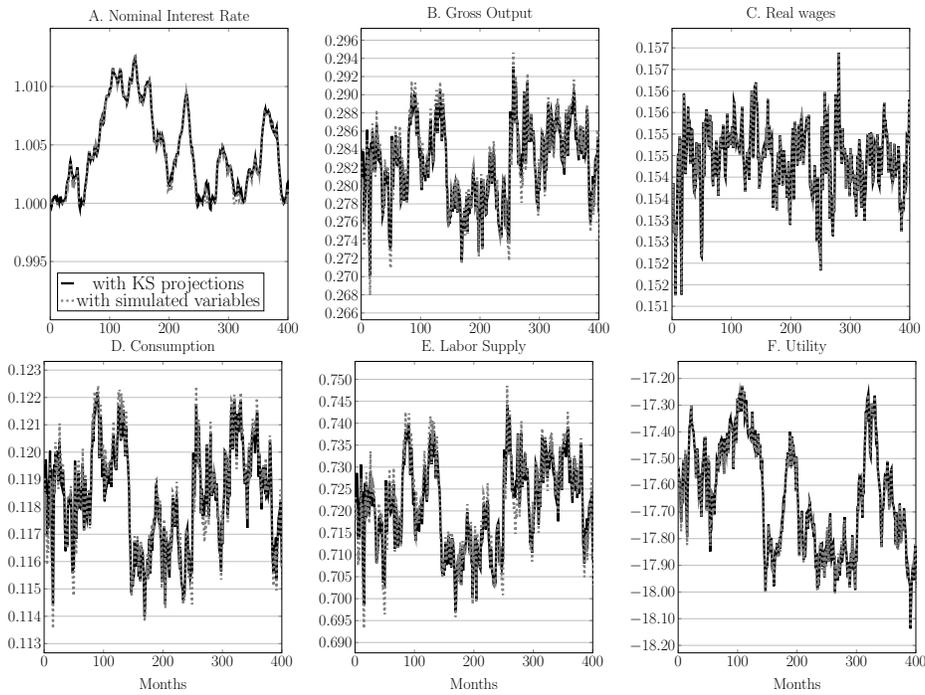


FIGURE L.IV. PREDICTED AND SIMULATED AGGREGATE TIME SERIES WITH ZLB AT A 1.3% INFLATION TARGET

Note: The figure describes the macroeconomic time series for one simulation at a 1.3% inflation target with the ZLB constraint. Panels A to F plot (in the following order) nominal interest rate, gross output, real wages, consumption, labor supply, and period utility. Black solid lines describe the model-implied aggregate variables with Krusell-Smith projections, and gray dotted lines describe the implied macroeconomic time series with simulated inflation, frequency of price changes, and price dispersion.
Source: Author's calculations

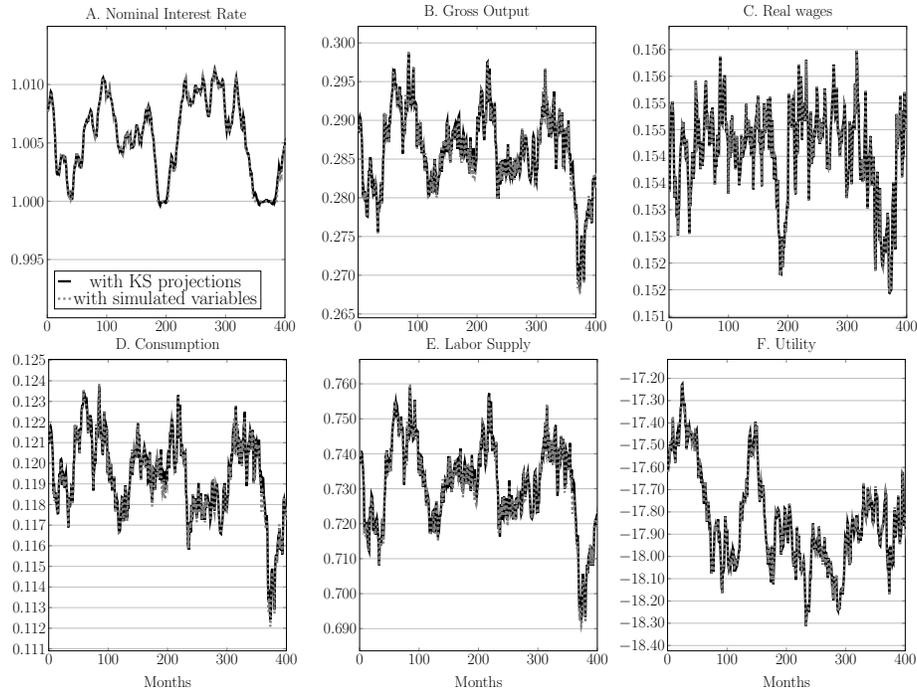


FIGURE L.V. PREDICTED AND SIMULATED AGGREGATE TIME SERIES WITH ZLB AT A 3% INFLATION TARGET

Note: The figure describes the macroeconomic time series for one simulation at a 3% inflation target with the ZLB constraint. Panels A to F plot (in the following order) nominal interest rate, gross output, real wages, consumption, labor supply, and period utility. Black solid lines describe the model-implied aggregate variables with Krusell-Smith projections, and gray dotted lines describe the implied macroeconomic time series with simulated inflation, frequency of price changes, and price dispersion.
Source: Author's calculations

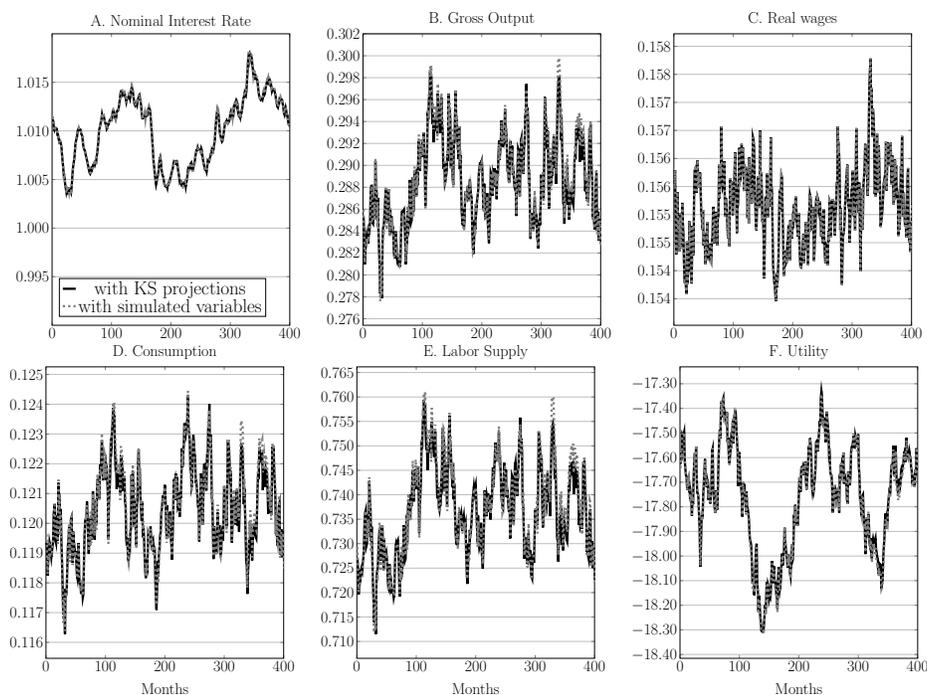


FIGURE L.VI. PREDICTED AND SIMULATED AGGREGATE TIME SERIES WITH ZLB AT A 5% INFLATION TARGET

Note: The figure describes the macroeconomic time series for one simulation at a 5% inflation target with the ZLB constraint. Panels A to F plot (in the following order) nominal interest rate, gross output, real wages, consumption, labor supply, and period utility. Solid black lines describe the model-implied aggregate variables with Krusell-Smith projections, and dotted gray lines describe the implied macroeconomic time series with simulated inflation, frequency of price changes, and price dispersion.

Source: Author's calculations