Online Appendix for Sticky Wage Models and Labor Supply Constraints

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1 Details of the Estimation of Altig et al. (2011)

The system of the log-linearized equations for estimation and simulation is the following:

$$\mathbb{E}\left[\lambda_{z^*,t+1} - \frac{1}{1-\alpha}\mu_{\Upsilon t+1} - \mu_{zt+1} + \frac{\rho\rho_{t+1} + (1-\delta)\mu_{t+1}}{1-\delta+\rho}\middle|\Omega_t^p\right] = 0$$

$$\mathbb{E}\Bigg\{S''(\mu_\Upsilon\mu_{z^*})^2\left[i_t - i_{t-1} + \mu_{\Upsilon t} + \frac{\alpha}{1-\alpha}\mu_{\Upsilon t} + \mu_{zt}\right] -$$

$$\beta S''(\mu_\Upsilon\mu_{z^*})^2\left[i_{t+1} - i_t + \mu_{\Upsilon t+1} + \frac{\alpha}{1-\alpha}\mu_{\Upsilon t+1} + \mu_{zt+1}\right] - \mu_t\Big|\Omega_t^p\Bigg\} = 0$$

$$\frac{\nu R}{\nu R + 1 - \nu}R_t + w_t + \frac{1}{1-\alpha}\left(\frac{y}{y+\phi}y_t - k_t + \frac{\alpha}{1-\alpha}\mu_{\Upsilon t} + \mu_{zt} + \mu\Upsilon t\right) - \rho_t - \frac{1}{1-\alpha}u_t = 0$$

$$[\mu_\Upsilon\mu_{z^*} - (1-\delta)]i_t - \left\{\mu_\Upsilon\mu_{z^*}k_{t+1} - (1-\delta)\left[k_t - \frac{1}{1-\alpha}\mu_{\Upsilon t} - \mu_{zt}\right]\right\} = 0$$

$$\mathbb{E}[\beta(\pi_{t+1} - \pi_t) - \gamma s_t - (\pi_t - \pi_{t-1})|\Omega_t^p] = 0$$

$$c_t - \frac{R}{(R-1)(2+\sigma_\eta)}R_t - q_t = 0$$

$$\mathbb{E}\Bigg\{-\left(\frac{1}{c(1-b\mu_{z^*}^{-1})}\right)^2\left[cc_t - \frac{bc}{\mu_{z^*}}c_{t-1} + \frac{bc}{\mu_{z^*}}\left(\frac{\alpha}{1-\alpha}\mu_{\Upsilon t} + \mu_{zt}\right)\right] +$$

$$\beta\left(\frac{1}{c(1-b\mu_{z^*}^{-1})}\right)^2\left[cc_{t+1} - \frac{bc}{\mu_{z^*}}c_t + \frac{bc}{\mu_{z^*}}\left(\frac{\alpha}{1-\alpha}\mu_{\Upsilon t+1} + \mu_{zt+1}\right)\right] -$$

$$\lambda_{z^*}[(1+\eta(V)) + \eta'(V)V]\lambda_{z^*t} - \lambda_{z^*}\left[2 + \frac{\eta''(V)V}{\eta'(V)}\right]\eta'(V)V(c_t - q_t)\Big|\Omega_t^p\Bigg\} = 0$$

$$\mathbb{E}\left[-\lambda_{z^*t} + \lambda_{z^*t+1} + R_{t+1} - \pi_{t+1} - \frac{\alpha}{1-\alpha}\mu_{\Upsilon t+1} - \mu_{zt+1}\middle|\Omega_t^p\right] = 0$$

$$\frac{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}{(1-\xi_w)(1-\beta\xi_w)}\Bigg\{w_{t-1} + \left[-\frac{1+\beta\xi_w^2}{\xi_w} + \sigma_L\lambda_w\frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}\right]w_t + \beta w_{t+1} + \pi_{t-1} + \pi_t + \beta\pi_{t+1} +$$

$$\frac{(1-\xi_w)(1-\beta\xi_w)(1-\lambda_w)}{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}(-\sigma_L h_t + \lambda_{z^*t}) - (1-\vartheta)\mu_{zt} + \beta(1-\vartheta)\frac{\alpha}{1-\alpha}\mu_{\Upsilon t+1} + \beta(1-\vartheta)\mu_{zt+1}\Bigg\} = 0$$

$$(1+\eta)cc_t + \eta'\frac{c^2}{q}(c_t - q_t) + ii_t - (y+\phi)\left[\alpha\left(u_t k_t - \frac{1}{1-\alpha}\mu_{\Upsilon t} - \mu_{zt}\right) + (1-\alpha)h_t\right] + \rho\frac{k}{\mu_{z^*}\mu_\Upsilon}u_t = 0$$

$$w_t + h_t - \frac{xm(x_t + m_t) - qq_t}{xm - q} = 0$$

$$x_{zt} + x_{\Upsilon t} + x_{Mt} - x_t = 0$$

$$x_{t-1} - \pi_t - \frac{\alpha}{1-\alpha}\mu_{\Upsilon t} - \mu_{zt} + m_{t-1} - m_t = 0$$

$$yy_t - (y+\phi)\left[\alpha\left(u_t + k_t - \frac{1}{1-\alpha}\mu_{\Upsilon t} - \mu_{zt}\right) + (1-\alpha)h_t\right] + \rho\frac{k}{\mu_{z^*}\mu_\Upsilon}u_t = 0$$

$$\mathbb{E}\left[u_t - \frac{1}{\sigma_a}\rho_t\middle|\Omega_t^p\right] = 0$$

In this system, there are three shocks, $\{\epsilon_{Mt}, \epsilon_{\mu_z t}, \epsilon_{\mu_\Upsilon t}\}$, which are shocks to monetary policy, neutral technology, and embodied investment technology. The processes for various shocks are

$$\begin{aligned}
\mu_{zt} &= \rho_{\mu z}\mu_{zt-1} + \epsilon_{\mu_z t} \\
\mu_{\Upsilon t} &= \rho_{\mu\Upsilon}\mu_{\Upsilon t-1} + \epsilon_{\mu_\Upsilon t} \\
x_{Mt} &= \rho_M x_{Mt-1} + \epsilon_{Mt} \\
x_{zt} &= \rho_{xz}x_{zt-1} + c_z^p \epsilon_{\mu_z t-1} + c_z \epsilon_{\mu_z t} \\
x_{\Upsilon t} &= \rho_{x\Upsilon}x_{\Upsilon t-1} + c_\Upsilon^p \epsilon_{\mu_\Upsilon t-1} + c_\Upsilon \epsilon_{\mu_\Upsilon t}
\end{aligned}$$

In the estimation, we choose the parameters to minimize the distance between model-implied impulse responses and their data counterparts, as in Altig et al. (2011).

1.1 Additional Tables for the Estimation of the Altig et al. (2011) Economies

In this subsection, we included additional tables which show results discussed in Section **??**. Table 1 contains the estimated parameters under alternative wage markups and different sizes of the innovations to the impulse response functions. Table 2 and Table 3 display the results for the statistical properties of labor with the demand-determined solution and the approximated Drèze solution under different parameterization.

Table 1: Estimated Parameter Values for the Drèze Equilibria under Various Markups and Shock Sizes

	Demand Determined	Drèze, 5 percent markup		Drèze, 10 percent markup		Drèze, 15 percent markup	
		1 std	1.5 std	2.5 std	3 std	3 std	4 std
ρ_{μ_z}	0.902 (0.102)	0.697 (0.240)	0.579 (0.002)	0.824 (0.119)	0.736 (0.183)	0.902 (0.112)	0.898 (0.063)
$\sigma_{\mu z}$	0.068 (0.046)	0.140 (0.089)	0.110 (0.000)	0.112 (0.055)	0.132 (0.071)	0.070 (0.035)	0.089 (0.035)
ρ_M	-0.037 (0.111)	-0.040 (0.130)	-0.078 (0.121)	-0.019 (0.148)	-0.018 (0.125)	-0.030 (0.123)	-0.021 (0.131)
σ_M	0.331 (0.084)	0.325 (0.078)	0.319 (0.074)	0.339 (0.077)	0.331 (0.077)	0.334 (0.075)	0.332 (0.077)
ρ_{μ_Υ}	0.241 (0.224)	0.318 (0.176)	0.344 (0.377)	0.841 (0.138)	0.839 (0.146)	0.833 (0.178)	0.840 (0.143)
$\sigma_{\mu\Upsilon}$	0.303 (0.042)	0.286 (0.046)	0.287 (0.046)	0.296 (0.053)	0.297 (0.053)	0.304 (0.051)	0.296 (0.053)
ρ_{xz}	0.343 (0.266)	0.195 (0.480)	0.130 (0.553)	0.315 (0.380)	0.251 (0.413)	0.347 (0.285)	0.356 (0.338)
c_z	2.997 (2.310)	1.027 (0.749)	1.008 (0.704)	1.435 (0.875)	1.108 (0.717)	2.840 (1.823)	2.023 (1.124)
c_z^p	1.327 (1.381)	0.665 (0.650)	0.715 (0.724)	0.697 (0.551)	0.598 (0.427)	1.373 (0.974)	0.946 (0.717)
c_Υ^p	0.135 (0.238)	0.107 (0.244)	0.110 (0.270)	0.149 (0.247)	0.133 (0.247)	0.125 (0.247)	0.150 (0.244)
$\rho_{x\Upsilon}$	0.824 (0.154)	0.832 (0.132)	0.882 (0.066)	0.841 (0.138)	0.839 (0.146)	0.833 (0.178)	0.840 (0.143)
c_Υ	0.246 (0.244)	0.305 (0.266)	0.318 (0.276)	0.226 (0.253)	0.240 (0.250)	0.224 (0.230)	0.221 (0.245)
ϵ	0.808 (0.208)	0.779 (0.193)	0.722 (0.170)	0.823 (0.208)	0.799 (0.208)	0.818 (0.193)	0.796 (0.199)
S''	3.281 (2.038)	4.275 (2.378)	3.246 (2.030)	4.539 (2.462)	4.257 (2.252)	3.216 (1.758)	4.252 (2.623)
ξ_w	0.722 (0.123)	0.825 (0.043)	0.801 (0.135)	0.850 (0.036)	0.854 (0.050)	0.806 (0.096)	0.872 (0.040)
b	0.706 (0.045)	0.698 (0.058)	0.719 (0.078)	0.717 (0.051)	0.711 (0.055)	0.717 (0.042)	0.721 (0.046)
σ_a	1.995 (2.222)	4.564 (7.070)	0.932 (0.834)	3.373 (3.627)	3.688 (4.297)	1.744 (2.414)	3.075 (3.410)
γ	0.040 (0.029)	0.054 (0.039)	0.103 (0.144)	0.060 (0.036)	0.057 (0.039)	0.041 (0.026)	0.059 (0.037)

Table 2: Effects on Labor of Shocks over Each Solution Concept: 10 percent Markup, 3 Std Shock

	Estimated with Demand-Determined				Estimated with Approximated Drèze: 3 std shock			
	mean	var	corr with output	labor violation	mean	var	corr with output	labor violation
Neutral Technology Shock								
Demand-Determined	—	0.17	0.86	3.50	—	0.23	0.97	5.57
Approximated Drèze	-0.44	0.25	0.85	—	-0.83	0.35	0.87	—
Investment Technology Shock								
Demand-Determined	—	0.62	0.99	0.90	—	0.47	0.99	1.04
Approximated Drèze	-0.03	0.55	0.99	—	-0.05	0.43	0.99	—
Monetary Shock								
Demand-Determined	—	0.50	0.99	0.05	—	0.36	1.00	0.00
Approximated Drèze	0.01	0.50	0.99	—	0.00	0.36	1.00	—
All Shocks								
Demand-Determined	—	1.35	0.96	5.58	—	1.12	0.95	7.59
Approximated Drèze	-0.74	0.99	0.94	—	-1.21	0.95	0.92	—

Note: All the variables except for the mean of labor are logged and HP filtered.

Table 3: Effects on Labor of Shocks over Each Solution Concept: 15 percent Markup, 4 Std Shock

	Estimated with Demand-Determined				Estimated with Approximated Drèze: 4 std shock			
	mean	var	corr with output	labor violation	mean	var	corr with output	labor violation
Neutral Technology Shock								
Demand-Determined	—	0.17	0.86	1.21	—	0.09	0.94	3.68
Approximated Drèze	-0.14	0.17	0.84	—	-0.71	0.23	0.83	—
Investment Technology Shock								
Demand-Determined	—	0.62	0.99	0.06	—	0.47	0.99	0.11
Approximated Drèze	0.01	0.62	0.99	—	0.00	0.46	0.99	—
Monetary Shock								
Demand-Determined	—	0.50	0.99	0.00	—	0.36	1.00	0.00
Approximated Drèze	0.01	0.50	0.99	—	0.00	0.36	1.00	—
All Shocks								
Demand-Determined	—	1.35	0.96	2.28	—	0.98	0.94	4.92
Approximated Drèze	-0.25	1.11	0.95	—	-0.97	0.90	0.93	—

Note: All the variables except for the mean of labor are logged and HP filtered.

2 Details of the Estimation of the Smets and Wouters (2007)

The system of the log-linearized equations for estimation and simulation are the following (the only changes from Smets and Wouters (2007) is the wage Philips curve):

$$
\begin{aligned}
y_t &= c_y c_t + i_y i_t + k_y \bar{r}^k u_t + \epsilon_t^g \\
c_t &= \frac{h/\zeta}{1+h/\zeta} c_{t-1} + \frac{1}{1+h/\zeta}\mathbb{E}_t c_{t+1} - \frac{1-h/\zeta}{1+h/\zeta}(r_t - E_t\pi_{t+1} + \epsilon_t^b) \\
i_t &= \frac{1}{1+\beta}\left(i_{t-1} + \beta\mathbb{E}_t i_{t+1} + \frac{1}{\zeta^2\varphi}q_t\right) + \epsilon_t^i \\
q_t &= \frac{\bar{r}^k}{1-\delta+\bar{r}^k} r_{t+1}^k + \frac{1-\delta}{1-\delta+\bar{r}^k}\mathbb{E}_t q_{t+1} - (r_t - \mathbb{E}_t\pi_{t+1} + \epsilon_t^b) \\
y_t &= \Phi\left(\alpha(k_{t-1}+u_t) + (1-\alpha)\ell_t + \epsilon_t^a\right) \\
r_t^k &= \frac{1-\psi}{\psi}u_t \\
k_t &= \left(1 - \frac{1-\delta}{\zeta}\right)k_{t-1} + \frac{1-\delta}{\zeta}i_t + \epsilon_t^i \\
\pi_t &= \frac{1}{1+\beta\iota_p}\left(\iota_p\pi_{t-1} + \beta\mathbb{E}_t\pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p((\Phi-1)\varepsilon_p+1)}(\alpha(k_{t-1}+u_t-\ell_t) - w_t + \epsilon_t^a)\right) + \epsilon_t^p \\
r_t^k &= w_t + \ell_t - k_{t-1} - u_t \\
w_t &= \frac{1}{1+\beta}\Bigg\{\beta\mathbb{E}_t[w_{t+1}] + w_{t-1} + \beta\mathbb{E}_t[\pi_{t+1}] - (1+\beta\iota_w)\pi_t + \iota_w\pi_{t-1} \\
&\quad + \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\gamma\epsilon_w)\xi_w}\left(\gamma\ell_t + \frac{1}{1-h/\zeta}c_t - \frac{h/\zeta}{(1-h/\zeta}c_{t-1} - w_t\right)\Bigg\} + \epsilon_t^w \\
r_t &= \rho r_{t-1} + (1-\rho)(r_\pi\pi_t + r_y(y_t - y_t^p)) + r_{\Delta y}(y_t - y_t^p - (y_{t-1} - y_{t-1}^p)) + \epsilon_t^m
\end{aligned}
$$

In this system, there are seven shocks, $\{\epsilon_t^a, \epsilon_t^i, \epsilon_t^b, \epsilon_t^p, \epsilon_t^w, \epsilon_t^g, \epsilon_t^m\}$, which are shocks to TFP, investment technology, risk premium, price markup, wage markup, government spending, and monetary policy. Some of the shocks are rescaled to facilitate estimation. The processes for various shocks are

$$
\begin{aligned}
\epsilon_t^g &= \rho_g\epsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a \\
\epsilon_t^b &= \rho_b\epsilon_{t-1}^b + \eta_t^b \\
\epsilon_t^i &= \rho_i\epsilon_{t-1}^i + \eta_t^i \\
\epsilon_t^a &= \rho_a\epsilon_{t-1}^a + \eta_t^a \\
\epsilon_t^m &= \rho_m\epsilon_{t-1}^m + \eta_t^m \\
\epsilon_t^p &= \rho_p\epsilon_{t-1}^p + \eta_t^p - \mu_p\eta_{t-1}^p \\
\epsilon_t^w &= \rho_w\epsilon_{t-1}^w + \eta_t^w - \mu_w\eta_{t-1}^w
\end{aligned}
$$

As in Smets and Wouters (2007), we fixed a few parameters, which are the depreciation rate $\delta = 0.025$, the average government spending to output ratio $g_y = 0.18$, the Kimball aggregator parameter

$\varepsilon_p = 10$, and we use log utility function for consumption.[1] We make two further modifications to Smets and Wouters (2007), in line with most of the recent literature: the utility function is additively separable in consumption and leisure, and the aggregator of labor inputs is the standard Dixit-Stiglitz aggregator.

References

Altig, D., Christiano, L. J., Eichenbaum, M., and Linde, J. (2011). Firm-specific capital, nominal rigidities and the business cycle. *Review of Economic Dynamics*, 14(2):225–247.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: a Bayesian DSGE approach. *American Economic Review*, 97(3):586–606.

[1] The parameter estimation results are available upon request.