Online Appendices

Endogenous Separations, Wage Rigidities and Unemployment Volatility

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A Appendix: Robustness of Empirical Results

This appendix addresses the robustness of the micro-econometric evidence. In Table 5 we present the first-stage results corresponding to the baseline results presented in column (1) of Table 1. As can be seen in both columns, the (excluded) instruments are strongly relevant with F statistics of 712 and 544, respectively. Also, a formal under-identification test confirms that the baseline IV specification is well identified (Kleibergen and Paap (2006) rk LM statistic: $\chi^2(1) = 427.44$, p-val $= 0.00$). Moreover, as expected under wage and price stickiness there is a positive relationship for each respective "own lag". The AR coefficient on $\ln mrp_{jt}$, given by the first-stage estimate presented in Table 5, is 0.23. This, in turn, provides an upper bound of the autocorrelation in marginal revenue product driven by the autocorrelation in idiosyncratic shocks under the assumption of no pricing frictions in the data. Moreover, looking at the relationship between $\ln w_{jt}$ and $\ln mrp_{jt-1}$, where only the arguably lower cross autocorrelation of prices and wages drives a wedge between the estimate and the true autocorrelation of shocks, the estimate falls to 0.07. Thus, we conclude that conditional on firm fixed effects the true autocorrelation in the idiosyncratic shock is very low.

Table 5: First-Stage Results for Baseline Specification

	(1)	(2)
Dependent Variable	$\ln mrp_{jt}$	$\ln w_{jt}$
$\ln mrp_{jt-1}$	0.229	0.070
	$(0.007)^{**}$	$(0.005)^{**}$
$\ln \hat{w}^{cc}_{jt-1}$	0.124	0.225
	$(0.013)^{**}$	$(0.010)^{**}$
Dummies:		
Firm	YES	YES
Sector by Time	YES	YES
F Stat($\ln mrp_{jt-1} = \ln \hat{w}_{jt-1} = 0$)	712^{**}	544^{**}
Observations	$306,205$	$306,205$
Firms	$42,656$	$42,656$

* (**) Denotes significance on the 5 (1) percent level from zero. Standard errors clustered on the firm level reported inside parenthesis. Sector denotes two-digit NACE codes. All regressions inlude lagged log value added to control for time variation in the initial firm size. Sample sizes are adjusted for singletons dropped in the estimation.

In Table 6 we perform various robustness exercises on our baseline results replicated in column (1) for convenience. In column (2) of Table 6 we first focus only on the manufacturing sector. As can be seen in the table, this does not change the results qualitatively. In column (3) we increase

Table 6: Robustness

	(1)	(2)	(3)	(4)	(5)
β_{mrp}	−3.769	−4.930	−3.608	−2.988	−3.002
	(0.139)**	(0.370)**	(0.166)**	(0.105)**	(0.244)**
β_w	1.638	3.647	2.003	1.675	2.033
	(0.261)**	(0.752)**	(0.432)**	(0.207)**	(0.316)**
Dummies:					
Firm	YES	YES	YES	YES	YES
Sector by Time	YES	YES	YES	YES	YES
Manufacturing Only	NO	YES	NO	NO	NO
$\geq$ # Full Time Employees	10	10	20	10	10
Separations Definition	BASELINE	BASELINE	BASELINE	LOOSE	BASELINE
Instrument lag order	1	1	1	1	2
Observations	306,205	78,730	149,988	307,132	266,471
Firms	42,656	10,039	20,867	43,034	38,400

* (**) Denotes significance on the 5 (1) percent level from zero. Standard errors clustered on the firm level reported inside parenthesis. Sector denotes two-digit NACE codes. All regressions inlude lagged log value added to control for time variation in the initial firm size. Sample sizes are adjusted for singletons dropped in the estimation.

the employment requirement to 20 full-time employees and find qualitatively the same results as in the base-line specification in column (2). In column (4) of Table 6, we use a much looser definition of employment when computing separations, using all employment spells of all workers regardless of their degree of firm attachment. This means that a worker is counted as employed regardless of the (monthly) wage or the timing or length of the spell within a year. Again, the results are qualitatively unchanged. In the final column of Table 6 we lag the instrument one additional time period. As can be seen in column (5) this increase the parameter estimate on the wage slightly, as expected from the discussion in the main text, but does not change the results qualitatively.

In Table 7 we evaluate the use of the approximation $sep_{jt}/\overline{sep}_j$, where $\overline{sep}_j$ denotes the firm average of separations, instead of $\ln sep_{jt}$ in the regressions above. To this end we estimate the baseline specification on an overlapping sample where all zero separation observations have been removed. As can be seen in the table, the approximation works well with only a mild downward bias on β_w from using the approximation for separations we employ in the regressions as compared to using the log of separations.

Table 7: Comparisson between Normalized and Log Separations

	(1)	(2)
β_{mrp}	-3.572	-4.133
	$(0.143)^{**}$	$(0.163)^{**}$
β_w	2.022	2.510
	$(0.264)^{**}$	$(0.301)^{**}$
Dummies:		
Firm	YES	YES
Sector by Time	YES	YES
Dependent Variable	Normalized	Log
Observations	$270,266$	$270,266$
Firms	$40,388$	$40,388$

* (**) Denotes significance on the 5 (1) percent level from zero. Standard errors clustered on the firm level reported inside parenthesis. Sector denotes two-digit NACE codes. All regressions inlude lagged log value added to control for time variation in the initial firm size. Sample sizes are adjusted for singletons dropped in the estimation.

B Appendix: Derivations

B.1 Value Functions

Let H^s and H^{ns} denote worker surplus when the worker searches and does not search on the job, respectively. We assume that workers face a cost σ of searching on the job. With probability λ, workers' idiosyncratic productivity changes and is again drawn from the distribution G and with probability $(1-\lambda)$ that the probability is unchanged. Note that the wage will depend on idiosyncratic productivity a_t. Let $w^s(a_t)$ $(w^{ns}(a_t))$ denote the worker wages when searching (not searching). The expected net surplus for an employed worker in a firm that resets the wage this period is

$$\begin{aligned} H_t^i(a_t) &= w_t^i(a_t) - b - \mathbb{I}_t\sigma + \beta E_t \alpha \rho^i \left(\lambda \int H_{t+1}(r)\, dG(r) + (1-\lambda) H_{t+1}(a_t)\right) \\ &\quad + \beta E_t (1-\alpha)\rho^i \left(\lambda \int \hat{H}_{t+1}\left(r, w_t^i(a_t)\right) dG(r) + (1-\lambda)\hat{H}_{t+1}\left(a_t, w_t^i(a_t)\right)\right) \\ &\quad + \beta E_t \left(g^i - f(\theta_t)\right) H_{t+1}(a_{ub}), \end{aligned} \tag{B.1}$$

where $\mathbb{I}_t$ is an indicator function that is equal to one of the worker searches on the job and zero otherwise, again surpressing the aggregate state variable z_t. Moreover, b is the flow payoff of the worker when unemployed, $g^{ns}=0$ and $g^s=f(\theta_t)$, $\rho^{ns}=(1-s)$ and $\rho^s=(1-f(\theta_t))(1-s)$,

$$H_t(a_t)=\begin{cases}\max\left(H_t^{ns}(a_t),H_t^s(a_t)\right) & \text{if } a_t>R_t^{ns} \text{ and } a_t>R_t^s\\ H_t^{ns}(a_t) & \text{if } a_t>R_t^{ns} \text{ and } a_t\le R_t^s\\ H_t^s(a_t) & \text{if } a_t\le R_t^{ns} \text{ and } a_t>R_t^s\\ 0 & \text{otherwise,}\end{cases} \tag{B.2}$$

and

$$\hat{H}_t(a_t,\hat{w}_t)=\begin{cases}\max\left(\hat{H}_t^{ns}(a_t,\hat{w}_t),\hat{H}_t^s(a_t,\hat{w}_t)\right) & \text{if } a_t>\hat{R}_t^{ns}(\hat{w}_t) \text{ and } a_t>\hat{R}_t^s(\hat{w}_t)\\ \hat{H}_t^{ns}(a_t,\hat{w}_t) & \text{if } a_t>\hat{R}_t^{ns}(\hat{w}_t) \text{ and } a_t\le\hat{R}_t^s(\hat{w}_t)\\ \hat{H}_t^s(a_t,\hat{w}_t) & \text{if } a_t\le\hat{R}_t^{ns}(\hat{w}_t) \text{ and } a_t>\hat{R}_t^s(\hat{w}_t)\\ 0 & \text{otherwise.}\end{cases} \tag{B.3}$$

In case wages are not reset but remain at the level $\hat{w}_t$ from the previous period, the wage $\hat{w}_t$ is a state variable and the surplus is

$$\begin{aligned}\hat{H}_t^i(a_t,\hat{w}_t) &= \hat{w}_t-b-\mathbb{I}_t\sigma+\beta E_t\alpha\rho^i\left(\lambda\int H_{t+1}(r)\,dG(r)+(1-\lambda)H_{t+1}(a_t)\right)\\ &\quad+\beta E_t(1-\alpha)\rho^i\left(\lambda\int\hat{H}_{t+1}(r,\hat{w}_t)\,dG(r)+(1-\lambda)\hat{H}_{t+1}(a_t,\hat{w}_t)\right)\\ &\quad+E_t\left(g^i-f(\theta_t)\right)\beta H_{t+1}(a_{ub}).\end{aligned} \tag{B.4}$$

For firms that change wages, the surplus is, when there is no on-the-job search

$$\begin{aligned}J_t^i(a_t) &= z_ta_t-w_t^i(a_t)+\beta E_t\rho^i\alpha\left(\lambda\int J_{t+1}(r)\,dG(r)+(1-\lambda)J_{t+1}(a_t)\right)\\ &\quad+\beta E_t(1-\alpha)\rho^i\left(\lambda\int\hat{J}_{t+1}\left(r,w_t^i(a_t)\right)dG(r)+(1-\lambda)\hat{J}_{t+1}\left(a_t,w_t^i(a_t)\right)\right),\end{aligned} \tag{B.5}$$

where

$$J_t(a_t)=\begin{cases}\max\left(J_t^{ns}(a_t),0\right) & \text{if } a_t>R_t^S\\ \max\left(J_t^s(a_t),0\right) & \text{if } a_t\le R_t^S\end{cases} \tag{B.6}$$

and

$$\hat{J}_t(a_t,\hat{w}_t)=\begin{cases}\max\left(\hat{J}_t^{ns}(a_t,\hat{w}_t),0\right) & \text{if } a_t>\hat{R}_t^S(\hat{w}_t)\\ \max\left(\hat{J}_t^s(a_t,\hat{w}_t),0\right) & \text{if } a_t\le\hat{R}_t^S(\hat{w}_t)\end{cases}. \tag{B.7}$$

In case wages are not reset but remain at the level $\hat{w}_t$ from the previous period, the values are

$$\begin{aligned}\hat{J}_t^i(a_t,\hat{w}_t) &= z_t a_t - \hat{w}_t + \beta E_t \rho^i \alpha \left(\lambda \int J_{t+1}(r)\, dG(r) + (1-\lambda) J_{t+1}(a_t)\right) \\ &\quad + \beta E_t (1-\alpha)\rho^i \left(\lambda \int \hat{J}_{t+1}(r,\hat{w}_t)\, dG(r) + (1-\lambda)\hat{J}_{t+1}(a_t,\hat{w}_t)\right).\end{aligned} \tag{B.8}$$

B.2 Employment Flows and values

Let $e_{t-1}(a,\hat{w})$ denote employment for workers with idiosyncratic productivity at most a with wage $\hat{w}$ in period $t-1$. Total employment for workers with idiosyncratic productivity at most a in period $t-1$ is then

$$e_{t-1}^{agg}(a) = e_{t-1}^{c}(a) + \sum_{\hat{w}} e_{t-1}^{nc}(a,\hat{w}) \tag{B.9}$$

where e_{t-1}^{c} and e_{t-1}^{nc} are defined below. Employment evolution for workers with idiosyncratic productivity at most a that change wages is, when $a \in [R_{t-1}, R_{t-1}^S]$

$$\begin{aligned}e_t^c(a) &= \alpha\rho\left[\lambda\left(G(a)-G(R_t)\right)\left(e_{t-1}^{agg}(a_{ub}) - e_{t-1}^{agg}\left(R_{t-1}^S\right) + (1-f(\theta_t))\, e_{t-1}^{agg}\left(R_{t-1}^S\right)\right)\right. \\ &\quad \left. + (1-\lambda)(1-f(\theta_t))\left(e_{t-1}^{agg}(a) - e_{t-1}^{agg}(R_t)\right)\right],\end{aligned} \tag{B.10}$$

when $a \in [R_{t-1}^S, a_{ub})$

$$\begin{aligned}e_t^c(a) &= \alpha\rho\left[\lambda\left(G(a)-G(R_t)\right)\left(e_{t-1}^{agg}(a_{ub}) - e_{t-1}^{agg}\left(R_{t-1}^S\right) + (1-f(\theta_t))\, e_{t-1}^{agg}\left(R_{t-1}^S\right)\right)\right. \\ &\quad \left. + (1-\lambda)\left(e_{t-1}^{agg}(a) - e_{t-1}^{agg}\left(R_{t-1}^S\right) + (1-f(\theta_t))\left(e_{t-1}^{agg}\left(R_{t-1}^S\right) - e_{t-1}^{agg}(R_t)\right)\right)\right]\end{aligned} \tag{B.11}$$

and when $a = a_{ub}$

$$\begin{aligned}e_t^c(a) &= \alpha\rho\left[\lambda\left(G(a)-G(R_t)\right)\left(e_{t-1}^{agg}(a_{ub}) - e_{t-1}^{agg}\left(R_{t-1}^s\right) + (1-f(\theta_t))\, e_{t-1}^{agg}\left(R_{t-1}^S\right)\right)\right. \\ &\quad \left. + (1-\lambda)\left(e_{t-1}^{agg}(a) - e_{t-1}^{agg}\left(R_{t-1}^S\right) + (1-f(\theta_t))\left(e_{t-1}^{agg}\left(R_{t-1}^S\right) - e_{t-1}^{agg}(R_t)\right)\right)\right] \\ &\quad + f(\theta_t)\left(u_{t-1} + \phi_{t-1}\right),\end{aligned} \tag{B.12}$$

where ϕ_{t-1} are workers searching on the job. When $R_{t-1} > R_{t-1}^S$ we have, for $a \in [R_{t-1}, a_{ub})$

$$e_t^c(a) = \alpha\rho\left[\lambda\left(G(a)-G(R_t)\right) e_{t-1}^{agg}(a_{ub}) + (1-\lambda)\left(e_{t-1}^{agg}(a) - e_{t-1}^{agg}(R_t)\right)\right] \tag{B.13}$$

and for $a = a_{ub}$

$$\begin{aligned} e_t^c(a) &= \alpha\rho\left[\lambda\left(G(a)-G(R_t)\right)e_{t-1}^{agg}(a_{ub})+(1-\lambda)\left(e_{t-1}^{agg}(a)-e_{t-1}^{agg}(R_t)\right)\right] \\ &\quad +f(\theta_t)\left(u_{t-1}+\phi_{t-1}\right). \end{aligned} \tag{B.14}$$

Then we have, slightly abusing notation by letting $e_t^c(a-1)$ denote employment at the grid point below a,

$$n_t(a) = e_t^c(a) - e_t^c(a-1) + \sum_{\hat{w}}\left[e_t^{nc}(a,\hat{w}) - e_t^{nc}(a-1,\hat{w})\right].$$

Employment for workers who do not change wages can be computed as follows. First, suppose $\hat{R}_{t-1}^S(\hat{w}) > \hat{R}_{t-1}(\hat{w})$. For wage state $\hat{w}$, when OJS is chosen, i.e., for $a \in [\hat{R}_{t-1}(\hat{w}), \hat{R}_{t-1}^S(\hat{w})]$, employment evolves according to

$$\begin{aligned} e_t^{nc}(a,\hat{w}) &= (1-\alpha)\rho\Big[\lambda\left(G(a)-G\left(\hat{R}_t(\hat{w})\right)\right) \\ &\quad \times\left(e_{t-1}^{nc}(a_{ub},\hat{w})-e_{t-1}^{nc}\left(\hat{R}_{t-1}^S(\hat{w}),\hat{w}\right)+(1-f(\theta_t))\,e_{t-1}^{nc}\left(\hat{R}_{t-1}^S(\hat{w}),\hat{w}\right)\right) \\ &\quad +(1-\lambda)(1-f(\theta_t))\left(e_{t-1}^{nc}(a,\hat{w})-e_{t-1}^{nc}\left(\hat{R}_t(\hat{w}),\hat{w}\right)\right)\Big] \end{aligned} \tag{B.15}$$

and, when OJS is not chosen, i.e., for $a \in [\hat{R}_{t-1}^S(\hat{w}), a_{ub}]$,

$$\begin{aligned} e_t^{nc}(a,\hat{w}) &= (1-\alpha)\rho\Big[\lambda\left(G(a)-G\left(\hat{R}_t(\hat{w})\right)\right)\Big\{e_{t-1}^{nc}(a_{ub},\hat{w})-e_{t-1}^{nc}\left(\hat{R}_{t-1}^S(\hat{w}),\hat{w}\right) \\ &\quad +(1-f(\theta_t))\,e_{t-1}^{nc}\left(\hat{R}_{t-1}^S(\hat{w}),\hat{w}\right)\Big\}+(1-\lambda)\Big\{e_{t-1}^{nc}(a,\hat{w})-e_{t-1}^{nc}\left(\hat{R}_{t-1}^S(\hat{w}),\hat{w}\right) \\ &\quad +(1-f(\theta_t))\left(e_{t-1}^{nc}\left(\hat{R}_{t-1}^S(\hat{w}),\hat{w}\right)-e_{t-1}^{nc}\left(\hat{R}_t,\hat{w}\right)\right)\Big\}\Big]. \end{aligned} \tag{B.16}$$

Now, suppose $\hat{R}_{t-1}^S(\hat{w}) \leq \hat{R}_{t-1}(\hat{w})$. Then, for $a \in [\hat{R}_{t-1}^S(\hat{w}), \hat{R}_{t-1}(\hat{w})]$ we have $e_t^{nc}(a,\hat{w}) = 0$ and for $a \in [\hat{R}_t(\hat{w}), a_{ub}]$ we have, modifying the expression above,[23]

$$\begin{aligned} e_t^{nc}(a,\hat{w}) &= (1-\alpha)\rho\Big[\lambda\left(G(a)-G\left(\hat{R}_t(\hat{w})\right)\right)e_{t-1}^{nc}(a_{ub},\hat{w}) \\ &\quad +(1-\lambda)\left(e_{t-1}^{nc}(a,\hat{w})-e_{t-1}^{nc}\left(\hat{R}_t(\hat{w}),\hat{w}\right)\right)\Big]. \end{aligned} \tag{B.17}$$

Finally, the unemployment to employment transitions are

$$UE_t = A\theta_{t-1}^{1-\alpha}u_{t-1} \tag{B.18}$$

[23] Note that workers with idiosyncratic productivity realization at or below $\hat{R}_{t-1}^S(\hat{w})$ search on the job and lose their job only in the current period.

and separations evolve according to, letting $\mathbb{I}_t = 1$ if $\hat{R}^S_{t-1}(\hat{w}) > \hat{R}_{t-1}(\hat{w})$ and $\mathbb{I}_t = 0$ otherwise,

$$\begin{aligned}
EU_t &= (1-\rho)\left(e^c_{t-1}(a_{ub}) + \sum_{\hat{w}} e_{t-1}(a_{ub}, \hat{w})\right) + \\
&+\alpha\rho\left[\lambda G(R_t)\left(e^{agg}_{t-1}(a_{ub}) - e^{agg}_t\left(R^S_{t-1}\right) + (1-f(\theta_t))\, e^{agg}_{t-1}\left(R^S_{t-1}\right)\right)\right. \\
&\left.+(1-\lambda)(1-f(\theta_t))\, e^{agg}_{t-1}(R_t)\right] \qquad \text{(B.19)} \\
&+(1-\alpha)\sum_{\hat{w}} \rho\left[\lambda G\left(\hat{R}_t(\hat{w})\right)\left(e_{t-1}(a_{ub}, \hat{w}) - \mathbb{I}_t f(\theta_t)\, e_{t-1}\left(\hat{R}^S_{t-1}(\hat{w}), \hat{w}\right)\right)\right. \\
&\left.+(1-\lambda)\left(\mathbb{I}_t(1-f(\theta_t)) + (1-\mathbb{I}_t)\right) e_{t-1}\left(\hat{R}_t(\hat{w}), \hat{w}\right)\right].
\end{aligned}$$

B.3 The Algorithm

Since the system (16), (19), (B.1), (B.4), (B.5) and (B.8) does not depend directly on unemployment, we can solve without using unemployment as a state variable. Now, for clarity, we do not surpress the dependence of wages, surpluses and labor market tightness on aggregate productivity. Then, since the values of newly created firms and newly hired workers depend on current and future productivities only (through future surpluses, tightness and H^e), the current wage depends only on the current productivities and tightness depends only on aggregate productivity. Hence, w^i_t is a function of z_t and a_t only. Then, for firm-worker pairs that did not reset their wage today, the wage depends on the productivity when the wage was last reset, say $\hat{z}$ and $\hat{a}$. We then write $\hat{w}(\hat{z}, \hat{a})$. Then worker surpluses are

$$\begin{aligned}
H^i(z_t, a_t) &= w^i(z_t, a_t) - b - \mathbb{I}_t\sigma + \beta E_t \alpha\rho^i\left(\lambda\int H(z_{t+1}, r)\, dG(r) + (1-\lambda) H(z_{t+1}, a_t)\right) \\
&+\beta E_t(1-\alpha)\rho^i\left(\lambda\int \hat{H}\left(z_{t+1}, r, w^i_t(z_t, a_t)\right) dG(r)\right. \qquad \text{(B.20)} \\
&\left.+(1-\lambda)\hat{H}\left(z_{t+1}, a_t, w^i_t(z_t, a_t)\right)\right) + \beta E_t\left(g^i - f(\theta(z_t))\right) H(z_{t+1}, a_{ub}).
\end{aligned}$$

In case wages are not reset but remain at the level $\hat{w}_t$ from the previous period, the wage $\hat{w}_t$ is a state variable and the values are

$$\begin{aligned}
\hat{H}^i(z_t, a_t, \hat{w}(\hat{z}, \hat{a})) &= \hat{w}(\hat{z}, \hat{a}) - b - \mathbb{I}_t\sigma + \beta E_t \alpha\rho^i\left(\lambda\int H(z_{t+1}, r)\, dG(r) + (1-\lambda) H(z_{t+1}, a_t)\right) \\
&+\beta E_t(1-\alpha)\rho^i\left(\lambda\int \hat{H}(z_{t+1}, r, \hat{w}(\hat{z}, \hat{a}))\, dG(r)\right. \qquad \text{(B.21)} \\
&\left.+(1-\lambda)\hat{H}(z_{t+1}, a_t, \hat{w}(\hat{z}, \hat{a}))\right) + E_t\left(g^i - f(\theta(z_t))\right) H(z_{t+1}, a_{ub}).
\end{aligned}$$

We can proceed similarly for the remaining value equations so that surpluses when wages are reset depend on current productivity only and surpluses when wages are not rebargained depend on productivity at the last rebargain together with the current productivity.

We solve by fixing a solution for the wage, surpluses and tightness and then use value function iteration to find revised surpluses, wages and tightness. Given convergence of the value function iteration, we can then proceed to compute employment, unemployment, vacancies and separations.

B.4 Large firms

This appendix outlines how the simple model, described above, with only one worker firms can be recasted in a constant-returns framework into a model where firms may employ many workers, each subject to individual-specific shocks. For incumbent workers, some wages are renegotiated and some are not and separations are given by the firm's share of workers in each wage cohort multiplied by the share of workers in the cohort below the cutoffs in (8) and (9), respectively. For new hires, on the other hand, all wages are negotiated at entry and the separation decision is given by (8). Thus, again, rigid wages affect separations for incumbent workers.

The set of feasible productivities is finite and is denoted T. The value of the firm is

$$V\left(n_{jt}\right) = \max_{v_{jt}} \sum_{r \in T} p_{jt} z_t r n_{jt}\left(r\right) - \sum_{r \in T} w_{jt}\left(r\right) n_{jt}\left(r\right) dr - c v_{jt} + \beta E_t V\left(n_{jt+1}\right),$$

where

$$n_{jt+1}\left(r'\right) = \mathbb{I}\left(r' > R_{ijt+1}\right) g\left(r'\right) \left(q_t v_{jt} + \rho \sum_{r \in T} n_{jt}\left(r\right)\right)$$

The effect of an increase in $n_{jt}\left(r\right)$ is, noting that the envelope theorem ensures that the indirect effect of $n_{jt}\left(r\right)$ on v_{jt} is zero,

$$\frac{\partial V\left(n_{jt}\right)}{\partial n_{jt}\left(r\right)} = p_{jt} z_t r - w_{jt}\left(r\right) + \beta E_t \sum \frac{\partial V\left(n_{jt+1}\right)}{\partial n_{jt}\left(r\right)}$$

Noting that $\frac{\partial n_{jt+1}(r)}{\partial n_{jt}(r)} = \rho \mathbb{I}\left(r > R_{ijt+1}\right)$ and defining $J_{jt}\left(r\right) = \frac{\partial V(n_{jt})}{\partial n_{jt}(r)}$ gives

$$\begin{aligned} J_{jt}\left(r\right) &= p_{jt} z_t r - w_{jt}\left(r\right) + \beta \rho E_t \frac{\partial V\left(n_{jt+1}\right)}{\partial n_{jt+1}\left(r\right)} g\left(r'\right) \mathbb{I}\left(r' > R_{ijt+1}\right) \\ &= p_{jt} z_t r - w_{jt}\left(r\right) + \beta \rho E_t \sum_{r'} g\left(r'\right) \max\left\{J_{jt+1}\left(r'\right), 0\right\} \end{aligned} \tag{B.22}$$

This is similar to expression (2). Worker values are as in (3), replacing $dG\left(r'\right)$ with $g\left(r'\right)$. Noting that the wage is determined in bilateral bargaining over the marginal surplus, it follows that the cutoff is given by

$$R_{jt} = \frac{b - \left[\beta\rho\sum_r \max\{S_{jt+1}(r), 0\} g(r) - \beta s_t \varphi S^e_{t+1}\right]}{p_{jt} z_t} \tag{B.23}$$

which is similar to (5). When wages are sticky, a similar argument establishes that

$$\hat{R}_{jt}(\hat{w}_{jt}) = \frac{\hat{w}_{jt} - \alpha\beta\rho\sum_r \max\{J_{jt+1}(r), 0\} g(r) - (1-\alpha)\beta\rho\sum_r \max\{\hat{J}_{jt+1}(r, \hat{w}_{jt}), 0\} g(r)}{p_{jt} z_t}. \tag{B.24}$$

where $\hat{J}_{jt+1}(r, \hat{w}_{jt})$ is determined along the lines of (B.22), noting that $\hat{w}_{jt}$ is a state variable, as in (7).

Letting n^{ns}_{jt} denote total employment before separations, total separations are

$$n^{ns}_{jt} \sum_{r \le R_{jt}} g(r)$$

when the wage is rebargained and

$$n^{ns}_{jt} \sum_{r \le \hat{R}_{jt}(\hat{w}_{jt})} g(r)$$

otherwise.

B.5 Appendix: Swedish Calibration

In the Swedish calibration the parameters β, φ and σ_a are set to the same values as in the U.S. calibration, see Table 8 in the main text. Following the estimates from Swedish data presented in Adlolfsson, Laseen, Linde, and Villani (2008), we set the Calvo probability of wage adjustment to 0.091 on a monthly basis. The parameters σ_μ, c, σ_G, σ, λ and s are set matching the same moments as for the US calibration; see Table 9 for details.

Table 8: Calibration of the Swedish Model.

Calibration Parameters		
β	*Time preference*	0.9966
φ	*Family bargaining power*	0.5
σ_a	*Matching function*	0.5
α	*Calvo prob. of wage adjustment*	0.091
b	*Payoff when unemployed*	0.78
Moment-Matched Parameters		
σ_μ	*Matching function productivity*	0.335
c	*Vacancy cost*	1.409
σ_G	*Idiosyncratic productivity distr. variance*	0.257
σ	*Search cost*	0.042
λ	*Prob. of new idiosyncratic draw*	0.576
s	*Exogenous separation rate*	0.005

Table 9 gives the moments in data and in the model that results when calibrating the parameters σ_μ, c, σ_G, σ, λ and s.

Table 9: Comparison of Matched Moments Sweden

Moments	Data	Model
Mean v_t/u_t	0.29	0.290
Mean Separation rate	0.005	0.005
Mean Job-to-job transition rate	0.010	0.010
Mean Job finding rate	0.101	0.105
Std. Separation rate	0.091	0.093
Persistence Separation rate	0.660	0.549

Note: The sample period for data is 2005:Q3-2016:Q4. The targets for separations, the job-to-job transition rate and the job finding rate are from the quarterly series provided by the Labor Force Survey (AKU), The series for v/u is computed using monthly data provided by the Labor Force Survey (AKU) and the Unemployment Board Statistics, respectively. To compute the persistence and standard deviation for separations, we take logs and HP filter the series for separations with a penalty parameter of 1,600.

Although the Swedish labor markets flows are clearly lower than in the U.S., the fit between the simulated moments and the data moments is high, and in fact slightly better than for the U.S. calibration. In Table 10, the moments for unemployment, the job finding rate, the separation rate, vacancies and the vacancy/unemployment rate are illustrated. Again the model performs well when it comes to the volatility of unemployment, vacancies and tightness. Although, as for the U.S. the

Table 10: Comparison of Moments, Sweden

	u_t	Job find. rate	Sep. rate	v_t	v_t/u_t
		Standard Deviation			
Data	0.085	0.077	0.091	0.175	0.247
Model	0.070	0.037	0.093	0.267	0.304
		Correlation with Labor Productivity			
Data	−0.153	0.462	−0.656	0.628	0.498
Model	−0.669	0.957	−0.460	0.825	0.882

Note: All variables are logged and HP-filtered with a penalty parameter equal to 1,600. The sample period for data is 2005:Q3-2016:Q4. Unemployment and vacancies is quarterly averages of monthly data from the Labor Force surveys and the Unemployment Board Statistics, respectively. The job finding and separation rates are constructed from quarterly series provided by the Labor Force Survey (AKU), SCB. Labor productivity is from the National Accounts (Nationalräkenskaperna) and on a quarterly frequency.

model volatility of the job-finding rate is substantially lower than in the data. With respect to the business-cycle correlations, the performance is slightly below that of the U.S. calibration, but the overall conclusions is unaffected.