ONLINE APPENDIX OF:
Temporary Price Changes, Inflation Regimes and the Propagation of Monetary Shocks

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This document contains detailed proofs and extensions of the material in the main body of the paper. *Keywords: sticky prices, menu cost models, temporary price changes, reference prices, price plans, price flexibility*

E. Details of proof of Proposition 5

PROOF:

Let $T(g)$ be the expected time until the next change of price plan, i.e. until $|g_n|$ reaches $\bar{g}$. We can index the state by $i = 0, \pm 1, ..., \pm \bar{n}$. We have the discrete time version of the Kolmogorov backward equation (KBE):

$$T_i \;=\; \Delta \;+\; \frac{1}{2}\,[T_{i+1} + T_{i-1}] \;\; \text{for all } i = 0, \pm 1, \pm 2, ..., \pm(\bar{n}-1)$$

and at the boundaries we have $T_{\bar{n}} = T_{-\bar{n}} = 0$. We use a guess a verify strategy, guessing a solution of the form:

$$T_i = a_0 + a_2\, i^2 \;\; \text{for all } i = 0, \pm 1, \pm 2, ..., \pm \bar{n}$$

for some constants a_0, a_2. Inserting this into the KBE we obtain

$$a_0 + a_2\, i^2 \;=\; \Delta + \frac{1}{2}\,\left[a_0 + a_2\,(i+1)^2 + a_0 + a_2\,(i-1)^2\right] \;\; \text{for all } i = 0, \pm 1, \pm 2, ..., \pm(\bar{n}-1)$$

so that $a_2 = -\Delta$. Using this value, into the equation for the boundary condition, we get:

$$a_0 - \Delta\,(\bar{n})^2 \;=0\,, \implies a_0 = \Delta\,(\bar{n})^2 \;,$$

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and since $\bar{n}\sqrt{\Delta}\,\sigma = \bar{g}$ and $T_0 = a_0$ we have:

$$T_0 = a_0 = \Delta\,(\bar{n})^2 = \Delta\left(\frac{\bar{g}}{\sqrt{\Delta}\sigma}\right)^2 = \left(\frac{\bar{g}}{\sigma}\right)^2 = \frac{1}{N_p}$$

□

PROOF:

(of Proposition 6) We now derive formally the expression that give the inequalities described in equation (15). The proof focus on the case $\bar{n} \geq 2$ (see the discussion following equation (14)), which is equivalent to $N_p\Delta \leq 1/4$. We first obtain an upper bound on the number of price changes within a price plan. We first state the 2 parts of the inequality in two lemmas, and then prove each of them.

LEMMA 3. Let $\Delta > 0$ be the length of the time period, and $\bar{g}$ be the width of the inaction band. Let n_w be the expected number of price changes during a price plan. We have:

$$n_w \leq \frac{2}{\sqrt{\Delta}}\,\frac{1}{\sqrt{N_p}} - \frac{1}{2} \tag{E1}$$

Hence the total number of price changes per unit of time within price plans, denoted by N_w, and equal to $n_w N_p$, satisfies:

$$N_w \leq 2\sqrt{\frac{N_p}{\Delta}} - \frac{N_p}{2}$$

where $N_p = \sigma^2/\bar{g}^2$ and $\bar{g}/(\sigma\sqrt{\Delta}) = 1/\sqrt{N_p\Delta}$ is an integer larger than 2.

It is straightforward to obtain an upper bound on expected number of all price changes $N = N_p + N_w$. We obtain:

$$N = N_w + N_p \leq 2\sqrt{\frac{N_p}{\Delta}} + \frac{N_p}{2}$$

□

PROOF:

(of Lemma 3) We first start with a lemma that relates the expected number of up-crossings within a plan to the expected number of plans.

LEMMA 4. In the discrete-time discrete-state model we have: $n_w = 2\,E\left[U(\tau)\right] - \frac{1}{2}$.

PROOF:

(of Lemma 4). We relate the price changes within a price plan to the number of up-crossings, $U(\tau)$, and number of down-crossings, $D(\tau)$, between $g = 0$ and $g = \sqrt{\Delta}\,\sigma$. We assume that the optimal policy within a plan that has just started at $g(0) = 0$ has a price $\hbar > 0$ if $g \geq \sqrt{\Delta}\,\sigma$ and price $-\hbar < 0$ if $g \leq 0$. We focus on up-crossings where g goes from $g(t) = 0$ to $g(t+\Delta) = \sqrt{\Delta}\,\sigma$, so there is a price increase. For a down-crossing, $g(t)$ goes from $g(t) = \sqrt{\Delta}\,\sigma$ to $g(t+\Delta) = 0$, so there is price decrease. We will denote by $U(\tau)$ the number of up-crossings, and $D(\tau)$ the number of down-crossings at the time when the price plan ends. Notice that in any path from $g(0) = 0$ to $g(\tau) = +\bar{g}$ there are $U(\tau) = D(\tau) + 1$ up-crossings, while in any path where $g(\tau) = -\bar{g}$ there are $U(\tau) = D(\tau)$ up-crossings. Since the number of price changes is the sum of up-crossings plus down-crossings, and since the price plan is as likely to end with $g(\tau) = \bar{g}$ as well as with $g(\tau) = -\bar{g}$, thus

$$\Pr\{U(\tau) - D(\tau) = 1\} = \Pr\{U(\tau) - D(\tau) = 0\} = \frac{1}{2}\ .$$

and hence: $n_w = 2\,E\left[U(\tau)\right] - \frac{1}{2}$. This finishes the proof of the lemma.

We now return to the proof of Lemma 3 and use Doob's inequality for the expected number of up-crossings obtaining:

$$(b-a)E\left[U(\tau)\right] \leq \sup_{t=0,\Delta,2\Delta,\ldots} \left(a + E\left[\,|g(t)|\,\right]\right)$$

so that using the values $a = 0$, $b = \sqrt{\Delta}\,\sigma$ and that $E\left[\,|g(t)|\,\right] \leq \bar{g}$ we have

$$E\left[U(\tau)\right] \leq \frac{\bar{g}}{\sqrt{\Delta}\,\sigma}$$

Hence:

$$n_w = 2\,E\left[U(\tau)\right] - \frac{1}{2} \leq 2\,\frac{\bar{g}}{\sqrt{\Delta}\,\sigma} - \frac{1}{2} = \frac{2}{\sqrt{\Delta}}\,\frac{1}{\sqrt{N_p}} - \frac{1}{2}\ .\ \square$$

Next we obtain a lower bound on the number of price changes within a plan.

LEMMA 5. *The expected number of price changes per unit of time within a plan N_w has the following lower bound:*

$$N_w \geq \frac{1}{\sqrt{\frac{\Delta}{N_p} + \frac{\Delta}{2}\left[\frac{1+\sqrt{\Delta N_p}}{1-\sqrt{\Delta N_p}}\right]}}\ ,$$

where $N_p = \sigma^2/\bar{g}^2$ and $\bar{g}/(\sigma\sqrt{\Delta}) = 1/\sqrt{N_p\Delta}$ is an integer larger than 2.

PROOF:

(of Lemma 5) The proof proceeds in several steps. First we define a stopping time that counts consecutive price changes, the first an increase of size $2\hbar$ and the second a decrease of $2\hbar$, starting from a normalized desired price $g = 0$ and ending in the same value $g = 0$. Call this event a cycle. Because of the Markovian nature of g and because it starts and ends at the same value then consecutive cycles are independent so that the expected number of cycles is, by the fundamental law of renewal theory, the inverse of the expected duration of such a cycle. We know that by construction each cycle has 2 price changes of the same absolute value, $2\hbar$. Second we decompose this into two events, whose expected values we compute separately. Third we use the fundamental theorem of renewal theory to compute the expected number of price changes per unit of time which do not involve a change in price plan. We use the following normalization for price changes within a plan:

$$p(t) = \begin{cases} p^*(t) + \hbar & \text{if } g(t) > 0 \\ p^*(t) - \hbar & \text{if } g(t) \leq 0 \end{cases} \tag{E2}$$

Note that the normalization consists on charging $p(t) = p^*(t) - \hbar$ when $g(t) = 0$. The normalization affects the definition below, but not the final result.

1) Define the stopping times τ^u and τ^d as:

(E3)
$$\tau^u = \min \left\{ t : p(t) - p(t - \Delta) = +2\,\hbar\,,\; g(t) = \sqrt{\Delta}\sigma\,,\; g(0) = 0\,,\; t = \Delta, 2\Delta, ... \right\}$$
(E4)
$$\tau^d = \min \left\{ t : p(t) - p(t - \Delta) = -2\,\hbar\,, g(t) = 0\,,\; g(0) = \sqrt{\Delta}\sigma\,,\; t = \Delta, 2\Delta, ... \right\}$$

In words τ^u is the time elapsed until the first price increase starting from a state where $g = 0$, i.e. at the beginning of a price plan. Instead τ^d is the time elapsed until the first price decrease starting from the state where $g = \sigma\sqrt{\Delta}$, i.e. after a price increase has just occurred. Note that at τ^d the state is the same as in the beginning of a price plan. The expected value of $\tau^u + \tau^d$ gives the expected value of a cycle of at least one price increase followed by a price decrease, within a price plan. In this cycle the initial state is equal to the final one, namely $g = 0$. Notice that in each cycle there are at least two price changes, one (or more) increases and one (or more) decreases. There could be more than two price changes because in each τ^u there could be price decreases and during each τ^d there can be price increases caused by changes of the plan.

2) We compute the expected value of τ^u and τ^d separately.

a) We discuss how to compute $E[\tau^u]$. For this quantity we use the operator T^u, for which $T^u(0) = E[\tau^u]$. The operator T^u is the expected first time for which g goes from 0 to $\sqrt{\Delta}\,\sigma$, which coincides with a price increase, conditional on $g(0) = 0$. Note that there may be none or several plan changes before this event occurs. The function T_u solves:

$$T^u(i) = \Delta + \frac{1}{2}\left[T^u(i-1) + T^u(i+1)\right] \quad \text{for } i = -1, -2, ..., -\bar{n}+1$$

which is a version of the Kolmogorov backward equation, and the boundary conditions: $T^u(-\bar{n}) = T^u(0)$, because when the price plan ends it is restarted at $g = 0$, or index $i = 0$, and $T^u(0) = \Delta + (1/2)T^u(-1)$, because at $g = \sqrt{\Delta}\sigma$, which is index $i = 1$ there is a price increase, and we stop counting time. We show that $T^u(i) = a + b\,i + c\,i^2$. First, the Kolmogorov Backward equation implies that $c = -\Delta$. We use this into the two boundary conditions. The boundary condition $T^u(-\bar{n}) = T^u(0)$ gives $a = a + b\bar{n} - \Delta(\bar{n})^2 = 0$ or $b = -\Delta(\bar{n})$. The boundary condition at $i = 0$ gives $a = \Delta + (1/2)[a - b - \Delta]$, or $a + b = \Delta$. These equations imply that $T^u(0) = a = \Delta - b = \Delta(1 + \bar{n})$.

b) Now we discuss how to compute $E[\tau^d]$. For this quantity we use the operator T^d, for which $T^d(1) = E[\tau^d]$. The operator T^d is the expected time for which g goes from $\sqrt{\Delta}\,\sigma$ to 0, which coincides with a price decrease, conditional on $g(0) = \sqrt{\Delta}\,\sigma$. Note that there may be none or several price plan changes before this event occurs, as well as none, one, or more price increases. The function T^d solves:

$$T^d(i) = \Delta + \frac{1}{2}\left[T^d(i-1) + T^d(i+1)\right] \quad \text{for } i = 1, 2, ..., \bar{n}-1$$

which is a version of the Kolmogorov backward equation, and the boundary conditions. At the top we have $T^d(\bar{n}) = T^u(0) + T^d(1)$, since at this point there is a price plan change which returns the process to $g = 0$ and thus there must be an increase in prices within a plan before we can have a decrease. The other boundary condition is $T^d(1) = \Delta + (1/2)T^d(2)$ which uses the fact that a price decrease within a price plan must occur when $g = \sqrt{\Delta}\sigma$ which corresponds to the $i = 1$ index. In this event we stop counting time. We try a solution of the type $T^d(i) = \alpha + \beta\,i + \gamma\,i^2$. Using the Kolmogorov Backward equation we obtain that $\gamma = -\Delta$. Using the boundary condition at the top, as well as the solution for $T^u(0)$, we obtain:

$$\alpha + \beta\bar{n} - \Delta\bar{n}^2 = \Delta(1 + \bar{n}) + \alpha + \beta - \Delta$$

This implies that $\beta = \Delta(\bar{n}^2 + 1)/(\bar{n} - 1)$. The other boundary gives:

$$\alpha + \beta - \Delta = \Delta + (1/2)\left[\alpha + \beta 2 - \Delta 4\right]$$

or $\alpha = (1/2)\alpha$ which implies $\alpha = 0$. Hence we have

$$T^d(1) = \beta - \Delta = \Delta(\bar{n}^2 + 1 - \bar{n} + 1)/(\bar{n} - 1) = \Delta\bar{n} + \Delta\frac{2}{\bar{n} - 1}$$

3) Now we use the previous result to obtain the desired expression for N_w. First note that

$$T^u(0) + T^d(1) = E[\tau^u] + E[\tau^d] = 2\Delta\bar{n} + \Delta\frac{2 + \bar{n} - 1}{\bar{n} - 1} = 2\Delta\bar{n} + \Delta\frac{1 + \bar{n}}{\bar{n} - 1}$$

Because the cycles start and end at $g = 0$ and consecutive cycles are independent, we can use the Fundamental theorem of renewal theory. Hence the expected number of cycles per unit of time is $1/(E[\tau^u] + E[\tau^d])$. Also recall that in each cycle there are at least two price changes, hence the expected number of price changes N_w per unit of time is at least two times the (reciprocal of) expected duration of the cycle, i.e.:

$$N_w \geq \frac{2}{2\Delta\bar{n} + \Delta\frac{1+\bar{n}}{\bar{n}-1}} = \frac{1}{\Delta\bar{n} + \Delta\frac{1+\bar{n}}{2(\bar{n}-1)}} \ .$$

Using $\sqrt{\Delta}\,\sigma\,\bar{n} = \bar{g}$ and $\bar{n} = \sqrt{1/(\Delta N_p)}$ we can write

$$N_w \geq \left(\sqrt{\frac{\Delta}{N_p}} + \frac{\Delta}{2}\left[\frac{1 + \sqrt{1/(\Delta N_p)}}{\sqrt{1/(\Delta N_p)} - 1}\right]\right)^{-1} \ .$$

□

F. Hazard rate of price changes

In this section we study the hazard rates of price changes and show that they are decreasing. We do this for two models. The first version is a model with price plans that change when the absolute value of the normalized desired price $|g|$ reaches a critical value, the threshold that we denote by $\bar{g}$. We refer to this version as the menu-cost version, and we denote the hazard rate for a price with duration $t > 0$ as $h_{MC}(t)$. In Appendix G we also consider a version of the model where price plans are changed at (exogenous) exponentially distributed times, in which case we denote the hazard rate for price changes by $h_{exp}(t)$. In both cases we provide an analytical solution to the hazard rate of price changes. These analytical expressions depend on only one parameter, namely N_p: the expected number of plan changes per unit of time. In the benchmark price plan model, the expected number of price plan changes per unit of time has a simple expression $N_p = \bar{g}^2/\sigma^2$, an expression whose derivation and interpretation we return to in Section II. In the version where price plans are changed at exponentially distributed times, N_p is simply the expected number of price plan changes per unit of time. Both hazard rates are downward slopping, very much so for low durations, behaving approximately as $1/2t$ for low t, and they asymptote to different constants. The asymptote for h_{MC} is a multiple of the number of price plan changes per year, namely $\mu^2/2\, N_p \approx 5\, N_p$. While the asymptote for in the exponential case is simply N_p. Appendix F.F1 provides more information and the exact definition of the hazard rates, and on their analytical characterization. We summarize that analysis in the following proposition:

PROPOSITION 14. *The hazard rate h_{MC} for the baseline model with price plans is:*

$$h_{MC}(t) = \sum_{m=1,3,5,\ldots}^{\infty} m^2\, N_p\, \frac{\mu^2}{2}\, \theta(t,m;N_p) \quad \text{where} \tag{F1}$$

$$\theta(t,m;N_p) \equiv \frac{e^{-t\, m^2\, N_p\, \frac{\mu^2}{2}}}{\sum_{m'=1,3,5,\ldots}^{\infty} e^{-t\,(m')^2\, N_p\, \frac{\mu^2}{2}}}$$

where for each $t > 0$, the $\theta(t,\cdot;N_p)$ are non-negative and add up to one over $m = 1,3,5,\ldots$. The hazard h_{MC} has the following properties:

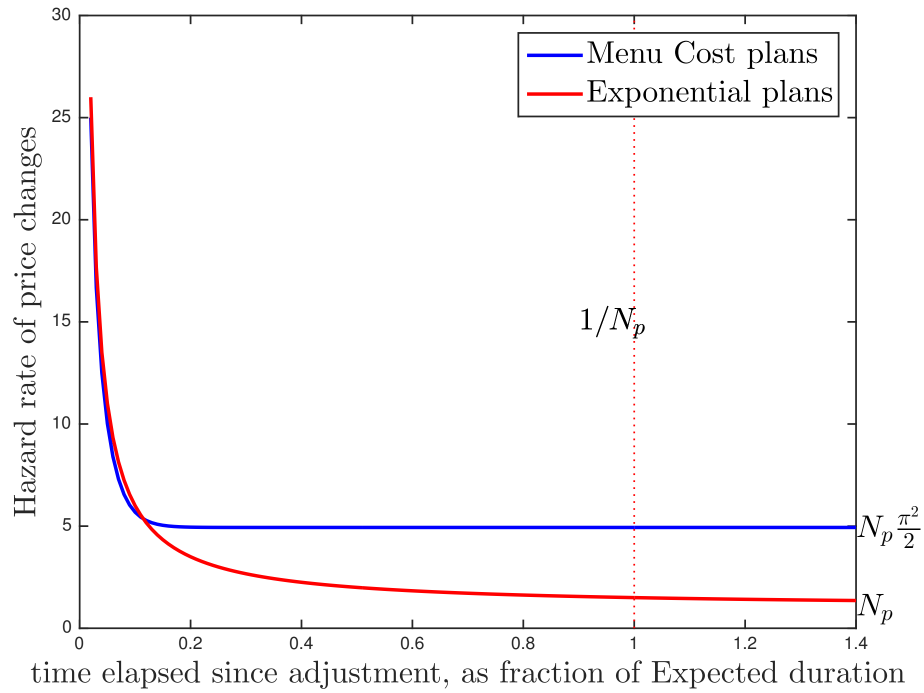
$$h'_{MC}(t) < 0 \text{ for } t > 0\,,\ \lim_{t\to 0} h_{MC}(t) = \infty\,,\ \lim_{t\to 0} h_{MC}(t)\, t = \frac{1}{2}\,,\ \text{and } \lim_{t\to\infty} h_{MC}(t) = \frac{\mu^2}{2} N_p\,.$$

For the case with exponentially distributed price plans times we have:

$$h_{exp}(t) = N_p + \frac{1}{2\,t} \text{ for all } t > 0\,. \tag{F2}$$

Figure F1 plots the two hazard rates. As explained in the proposition the hazard rate depends on one parameter, the expected number of plan changes, and hence $1/N_p$ is the expected time between price changes. In the figure we normalize N_p to one, so that duration, i.e. time, on the horizontal axis can be interpreted relative to the average duration of a plan. As it can be seen they are very similar for short durations, say for durations below 10 percent of the expected duration of a price plan, and very similar to the function $1/2t$. They differ in the level of asymptotic hazard rate, which is reached much sooner for the model with "state- dependent" plans and is reached later for the model with exponential plans.

FIGURE F1. THE HAZARD RATE OF PRICE CHANGES IN TWO MODELS



Next we provide an intuitive explanation of why the hazard rate of price plans are decreasing, while in the model without plans they are not. For instance, in the standard Calvo model of price setting without plans, hazard rates are constant by assumption. Likewise, hazard rates are increasing in the canonical menu-cost model, such as in Golosov and Lucas (2007), since right after a price change the firm charges the profit maximizing price, so that the probability to observe a new price change right after an adjustment is near zero. Instead, in the case of price plans with with two prices, the firm is indifferent between charging $p_i^* \pm \hbar$ right after a price change. Given that the upper threshold is preferred when $g > 0$ and the lower threshold is preferred when $g < 0$, the fact that $g = 0$ right after a price change makes it very likely that its sign with reverse many times, which triggers lots of price changes. We can also understand why $h(t) \approx 1/(2t)$ for small duration t. The reason is that a Brownian motion has, for a small enough time interval, approximately the same probability of an increase as a decrease, so if $g(t) > 0$, but $g(t)$ is small, then with probability roughly 1/2 it returns to zero, and thus the hazard rate is $1/(2t)$.

F1. Proofs for Hazard Rates

We compute the instantaneous hazard rate of price changes in two versions of our model. The first version has price plans that change when the (absolute value of the) normalized desired price $|g|$ reaches a critical value, a threshold that we denote by $\bar{g}$. We refer to this version as the menu cost version. We also consider another version where price plans are changed at exponentially distributed plans. In both cases we provide an analytical solution to the hazard rate of price changes. These analytical expressions depend on only one parameter, namely N_p: the expected number of price plans changes per unit of time. Both hazard rates are downward slopping, very much so for low durations (behaving approximately at $1/2t$), and they asymptote to different constants.

Hazard rate when price plans changes subject to menu cost. — To describe the hazard rate in this case we discuss the mathematical objects we use to define them and compute them. These results comes from the analysis in Alvarez, Shimer and Tourre (2015), which borrow some results from Kolkiewicz (2002). In our model a price change occurs when either a new price plan is in place or when within the same price plan prices are changed. In either case, at the instant right before price change takes place, the value of the desired normalized price satisfies $g = 0$. Thus, we compute the hazard rate for the following objects. We take $g(0) = \epsilon$, with $0 < \epsilon < \bar{g}$ and consider the following three stopping times:

$$\tilde{\tau}(\epsilon) = \inf_t \{\sigma\, W(t) \leq 0 \,|\, \sigma\, W(0) = \epsilon\} \tag{F3}$$

$$\bar{\tau}(\epsilon) = \inf_t \{\sigma\, W(t) \geq \bar{g} \,|\, \sigma\, W(0) = \epsilon\} \tag{F4}$$

$$\tau(\epsilon) = \min\{\bar{\tau}(\epsilon)\,,\, \tilde{\tau}(\epsilon)\} \tag{F5}$$

where W is a standard Brownian notion, so that we can use the desired normalized price until a price plan as $g(t) = \sigma W(t)$. The stopping time $\tilde{\tau}$ gives the first time that the desired normalized price g reaches back to 0, and hence the price changes, in logs, by $2\,\hbar$. Instead $\bar{\tau}$ gives the firms that the desired normalized price g reaches the upper barrier $\bar{g}$, and hence there is a new price plan, which new price. Thus, a price changes occurs, the first time that either event takes place, which is denoted by the stopping time τ. Note that in all cases we started with a normalized desired price equal to ϵ. Since right after price change $g = 0$, we will compute the limit of these stopping times as $\epsilon \to 0$. We require $g(0) = \epsilon$ to be small but strictly positive, because if we set $g(0)$ exactly equal to zero, then the distribution of τ is degenerate, i.e. $\tilde{\tau} = 0$ with probability one.[1] Convenient

[1]Give the symmetry of the problem we could have defined $\epsilon < 0$ and concentrate on the fist time that it comes back to zero, or it reaches $-\bar{g}$. Clearly we obtain the same stopping times.

expressions for the distribution of $\bar{\tau}(\epsilon)$ and $\tilde{\tau}(\epsilon)$ can be found in Kolkiewicz (2002) expressions (15) and (16). Alvarez, Shimer and Tourre (2015) derive the hazard rates, and compute the limit as $\epsilon \to 0$. Letting $h(t)$ the hazard rate of price changes, and adapting the expression in Alvarez, Shimer and Tourre (2015) we obtain equation (F1) in Proposition 14.

Hazard rate when price plans have exponentially distributed durations. — Again a price change occurs when either a new price plan is in place or within the same price plan. In this version we simply assume that price plans are changed at durations that are exponentially distributed, and independent of the normalized desired price g. This exponential distribution is assumed to have expected duration denoted by $1/N_p$, so N_p is the expected number of price plans per unit of time. Price changes within a plan are given by the stopping time as $\hat{\tau}$, define in equation (F4). The price changes that occur within a price plan are described by the same (limit of) the stopping time $\tilde{\tau}$ define above. Thus the stopping time for price changes is given by:

$$\tau(\epsilon) = \min\{\hat{\tau}\,,\, \tilde{\tau}(\epsilon)\} \tag{F6}$$

where W is $g(t)$ are defined as above. Since $\hat{\tau}$ and $\tilde{\tau}(\epsilon)$ are independent, then the hazard rate is simple the sum of the two hazard rates. The hazard rate corresponding to $\hat{\tau}$ is simply N_p. The hazard rate corresponding to $\tilde{\tau}$ can be computed as the hazard rate corresponding to the first time that a BM (with zero and volatility σ) and that starts at $\epsilon > 0$ and reaches 0. This stopping time is distributed according to the stable Levy law with density and CDF equal to[2]:

$$f(t;\epsilon) = \frac{\epsilon}{\sigma\sqrt{2\,\pi\,t^3}}\, e^{-\frac{\epsilon^2}{2\,t\,\sigma^2}}$$

$$F(t;\epsilon) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{\epsilon^2}{2\,t\,\sigma^2}}} e^{-z^2}\, dz\,.$$

Defining the hazard rate in terms of f and F, and taking ϵ to zero we obtain:

$$\tilde{h}(t) \equiv \lim_{\epsilon\to 0} \frac{f(t;\epsilon)}{1 - F(t;\epsilon)} = \lim_{\epsilon \to 0} \frac{\frac{\epsilon}{\sigma\sqrt{2\,\pi\,t^3}}\, e^{-\frac{\epsilon^2}{2\,t\,\sigma^2}}}{\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{\frac{\epsilon^2}{2\,t\,\sigma^2}}} e^{-z^2}\, dz} = \frac{\frac{1}{\sigma\sqrt{2\,\pi\,t^3}}}{\frac{2}{\sqrt{\pi}}\sqrt{\frac{1}{2\,t\,\sigma^2}}} = \frac{1}{2\,t}\,,$$

where we use L'Hopital rule to evaluate the limit. Thus we have equation (F2) in Proposition 14.

We briefly comment on the nature the limit hazard rates displayed in equation (F1)

[2]See Alvarez, Borovičková and Shimer (2015) for a derivation for the case of a BM with drift.

and equation (F2). We note that in continuous time both cases h_{MC} and h_{Exp} are not hazard rates that corresponds to a proper survivor function. The survivor function that correspond to $\epsilon = 0$ has $S(0) = 1$ and $S(t) = 0$ for all $t > 0$. The hazard rates in equation (F1) and equation (F2) are the limits of the approximation as $\epsilon \to 0$, so they should be regarded as approximations that are accurate for very small ϵ, or alternatively, as the hazard rates conditional on surviving a very small duration.[3]

G. Plans with exponentially distributed duration

In this section we consider an alternative model to the menu-cost model. Specifically, we assume that the duration of the price plan is exogenous and has a constant hazard rate λ, so that the duration of a plan is exponentially distributed. This version model corresponds to the well known Calvo (1983) pricing, if the price plan is a singleton. Thus this section can also be viewed as introducing price plans, or menu of prices, into the Calvo price setting. The reason for exploring this case is the pervasive use of the Calvo pricing in the sticky price literature. First we discuss the optimal value for $\hbar$. Then we characterize output's cumulative IRF to a monetary shock.

Optimal threshold $\hbar$. — The determination of the optimal threshold $\hbar$ follows exactly the same logic as in the case where the firm must pay a fixed cost, and thus price plans has duration given by the first time a top or bottom thresholds $\bar{g}$ or $-\bar{g}$ is hit. Instead in this case the stopping time is given by an exponentially distributed random variable, independent of g. Using the same first order condition as in Section I.B.

Proposition 15. The optimal threshold for the exponentially distributed price plan is:

$$\hbar = \frac{\sigma}{\sqrt{2\left(r+\lambda\right)}} \tag{G1}$$

The result in equation (G1) is intuitive: the threshold is increasing in σ since for higher values of it the deviations will be larger to each side. It is decreasing in $r + \lambda$ because this decreases the duration of the price plan, hence it is more likely that gaps will be smaller. Note also that it is the same as the limit obtained in Proposition 2 as $\bar{g} \to \infty$.

[3] The derivation in Alvarez, Borovičková and Shimer (2015) takes the second limit, i.e. the hazard rates conditional on a strictly positive duration.

THE FIRM'S CONTRIBUTION TO THE IRF. — The logic of the firm's contribution to the cumulative output response after a shock is the same as in the benchmark case discussed in Section IV, so that the price gap is $\hat{p}(t) \equiv p(t) - p^*(t) = \hbar \ \mathrm{sgn}\,(g(t)) - g(t)$ for $\tau_i \leq t < \tau_{i+1}$. The difference concerns the stopping time that determines the change of plan, so the definition of $\hat{m}$ is the same as in equation (21).

The invariant distribution of the normalized desired prices is described by the density $f(g)$ which is a Laplace distribution, i.e.:

$$f\,(g) = \frac{\sqrt{2\,\lambda}\,/\sigma}{2}\, e^{-\sqrt{2\,\lambda}/\sigma\,|g|} \ \text{ for all } g\,.^4 \tag{G2}$$

Notice that the definition of the cumulative real output effect in equation (19) is, again except for the specification of τ, the same. Likewise, equation (22) also holds. Simple computations then lead to

LEMMA 6. With exponentially distributed revisions of plan the cumulative output effect after a small monetary shock δ is $\mathcal{M}(\delta) = \delta\ \mathcal{M}'(0) + o(\delta)$ where

$$\mathcal{M}'(0) = \frac{1}{2\ \lambda} = \frac{1}{2N_p} \tag{G3}$$

For comparison with the well known Calvo pricing with $N_C = \lambda$ price adjustments per period we define $\mathcal{M}_C(\delta) = \int_{-\bar{g}}^{\bar{g}} m\,(g+\delta)\ f\,(g)\ dg$ as the cumulative impulse response where $f(g)$ is the same exponential density defined above.[5] Simple analysis along the lines followed above reveals that the cumulative real effect of a small monetary shock in the Calvo model are given by:

$$\mathcal{M}_C(\delta) \ = \ \delta\ \frac{1}{\lambda} + o(\delta) \ \approx \ \delta\ \frac{1}{N_C}$$

PROPOSITION 16. Assume plans are adjusted at the exogenous constant rate λ. Let $N_p = \lambda$ be the mean number of plan changes per period. Let N_C denote the mean number of price changes per period in a Calvo model without plans. The ratio of the cumulative output responses in the two models is:

$$\lim_{\delta \downarrow 0, r \downarrow 0} \frac{\mathcal{M}(\delta)}{\mathcal{M}_C(\delta)} = \frac{N_C}{2\ N_p} \tag{G4}$$

[4]This is easily seen by noticing that the invariant density solves the Kolmogorov forward equation: $\lambda\, f\,(g) = \frac{\sigma^2}{2}\, f''\,(g)$ and also that $\int_0^\infty f(g)dg = 1/2$.

[5]As noted above, the price gap in the Calvo model is $\hat{p} = -g$. Since the density f is symmetric around zero this is also the density of price gaps.

The proposition shows that, as was observed for the menu cost model, the introduction of the plans introduces a flexibility that reduces the real effects of monetary shocks *assuming the number of plan changes is the same across models*, i.e. $N_p = N_C$.

Table G1—Synopsis of theoretical effect of price-plans across models: $\mathscr{M}'(0)$

"Menu cost model"		"Calvo model"	
Without Price Plans	With Price Plans	Without Price Plans	With Price Plans
$\frac{1}{6\ N}$	$\frac{1}{18\ N_p}$	$\frac{1}{N}$	$\frac{1}{2\ N_p}$

Note: N denotes the total number of price changes, N_p denotes the total number of plan changes.

Table G1 provides a summary of the effects of introducing price plans in the various models where the notation there uses N_p the number of plans and N for the total number of price changes in the model without plans. The cumulative output response in a model with exponentially distributed plan's adjustments is 1/2 of the effect in the corresponding Calvo model, as it appears comparing the expressions in the third and fourth panels of the table with $N = N_p$. This result is to be compared with the one in Proposition 12 where, for small r the ratio was 1/3.[6]

Impact effect. — It is immediate to see that, as was the case for the menu-cost model, the introduction of the plans leads to a non-negligible mass of adjustments on impact when the shock occurs. This happens because the monetary shock δ shifts the distribution of the normalized desired prices $f(g)$ given in equation (G2) and a mass of agents $\int_0^\delta f(g)\, dg$ switches from negative to positive values of g, therefore switching from the low to the high price within the price plan, i.e. each firm increases its price by $2\hslash$. The next proposition summarizes this result

Proposition 17. The impact effect of a monetary shock δ on the aggregate price level is:

$$\lim_{r\to 0} \tilde{\Theta}(\delta) = \lim_{r\to 0} \hslash \int_0^\delta f(g)\, dg = \delta \lim_{r\to 0} \sqrt{\frac{\lambda}{\lambda + r}} = \delta \quad .$$

[6] The table also shows that for models without price plans, the area under the output's IRF in the menu cost model is 1/6 of the area in a Calvo model, a result first proved by Alvarez, Le Bihan and Lippi (2016). For models with price plans, the table shows that ratio of the cumulated real effects is even smaller: the real effects of the menu cost model with plans is 1/9 of the real effect of a Calvo model with plans.

The proof follows immediately by using the density in equation (G2) and the expression for $\hbar$ in equation (G1). This result shows that the impact effect that results from the firm adjustments on impact yields an immediate jump of the price level of the same size of the monetary shock, so that output does not change at all on impact.

G1. *Proofs for the model with "Calvo" plans*

PROOF:

(of Proposition 15)

$$\begin{aligned}
\hbar &= \frac{\mathbb{E}\left[\int_0^\tau e^{-rt}\,|g(t)|\,dt \;\middle|\; g(0)=0\right]}{\mathbb{E}\left[\int_0^\tau e^{-rt}\,dt \middle|\; g(0)=0\right]} \\
&= \frac{\int_0^\infty \lambda\, e^{-(r+\lambda)t}\,\mathbb{E}\left[\,|g(t)|\,dt \;\middle|\; g(0)=0\right]\,dt}{\int_0^\infty \lambda\, e^{-(r+\lambda)t}\,dt} = \frac{\int_0^\infty e^{-(r+\lambda)t}\,\sigma\sqrt{t\,2/\pi}\,dt}{\int_0^\infty e^{-(r+\lambda)t}\,dt} \\
&= \int_0^\infty (r+\lambda)\,e^{-(r+\lambda)t}\,\sigma\sqrt{2\,t/\pi}\,dt = \int_0^\infty (r+\lambda)\,e^{-(r+\lambda)t}\,\sigma\sqrt{2\,t\,/\pi}\,dt \\
&= \frac{\sigma}{\sqrt{2\,(r+\lambda)}}
\end{aligned}$$

where we use that $g(t)$ is, conditional on $g(0)=0$, normally distributed with mean 0 and variance $\sigma^2\,t$, and hence $\mathbb{E}\left[\,|g(t)|\,dt \;\middle|\; g(0)=0\right] = \sigma\sqrt{2\,t/\pi}$. The last line follows by performing the integration. □

H. Costly adjustments within plan

This appendix generalizes the model of the paper by assuming that prices changes within the plan, i.e. changes back and forth between the low and the high price within the plan, are also costly. In particular we assume the firm must pay a menu cost ν to change the price within the plan, and a larger menu cost ψ to change the plan.

Let $\nu > 0$ be the cost for a price change within the plan $-\hbar \rightleftarrows +\hbar$. Our baseline model assumes that $\nu = 0$. This modified problem gives rise to 2 value functions $v_h(\cdot), v_l(\cdot)$; symmetric : $v_h(g) = v_l(-g)$, since the value of a given normalized price g depends on the price currently charged, i.e. $\pm\hbar$.

In such a setting the optimal policy is given by 3 thresholds: $-\underline{g} \leq 0 \leq \hbar < \bar{g}$, such that the profit maximizing firm sets the price $\hbar$ as long as $g \in (-\underline{g}, \bar{g})$ and $-\hbar$ for $g \in (-\bar{g}, \underline{g})$. We have that $\hbar, \bar{g}$ and value functions $v_h(\cdot), v_l(\cdot)$ solve for all

g:

$$r\,v_h(g) \le B\,(g-\hbar)^2 + \frac{\sigma^2}{2}\,v_h''(g)$$
$$r\,v_l(g) \le B\,(g+\hbar)^2 + \frac{\sigma^2}{2}\,v_l''(g)$$

with equality if inaction is optimal, and

$$v_h(g) \le \nu + v_l(g) \;\text{ and }\; v_l(g) \le \nu + v_h(g;\hbar)$$
$$v_h(g) \le \psi + v_h(0) \;\text{ and }\; v_l(g) \le \psi + v_l(0;\hbar)$$

if either changing from high to low price (or vice-versa) or if changing the plan is optimal. Thus at least one of this inequality must hold with equality at each g.

Solving the problem requires solving a system of 5 equations in 5 unknowns: $\underline{g}, \hbar, \bar{g}$ and the 2 parameters of the second order ODE for the Bellman equation. The five equations are given by:

$$\text{2 value matching at } \underline{g} \text{ and } \bar{g}: \quad v_h(-\underline{g}) = \nu + v_l(-\underline{g}) \quad , \quad v_h(\bar{g}) = \psi + v_h(0)$$

$$\text{smooth pasting at } \underline{g}: \quad v_h'(-\underline{g}) = v_l'(-\underline{g}) = -v_h'(\underline{g}) \quad \text{by symmetry}$$
$$\text{smooth pasting at } \bar{g}: \quad v_h'(\bar{g}) = 0 \quad \text{and the optimal return } \; v_h'(\tilde{g}) = 0\,.$$

This system can be solved numerically to deliver the three optimal thresholds $-\underline{g} \le 0 < \hbar < \bar{g}$. The classic menu-cost problem with one price is obtained when $\nu = \psi$ so that $\hbar = 0, \bar{g} > 0$ and $\underline{g} = \bar{g}$. The price plan model discussed in the paper has $\psi > 0$ and $\nu = 0$ so that $\hbar > 0, \bar{g} > 0$ and $\underline{g} = 0$.

Next, for given thresholds, we compute the density of the price gaps $f(g)$ as well as the density of high prices $\tilde{p}$, which we denote by $f_f(g)$, and the density of low prices $-\tilde{p}$, which we denote by $f_l(g)$. The density $f(g)$ is the Kolmogorov forward equation $0 = f''(g)\sigma^2/2$, which implies a linear density function, and the boundary conditions $f(\bar{g}) = f(-\bar{g}) = 0$ (due to the fact that these are exit points and no mass can be accumulated here) imply that the density $f(g)$ is

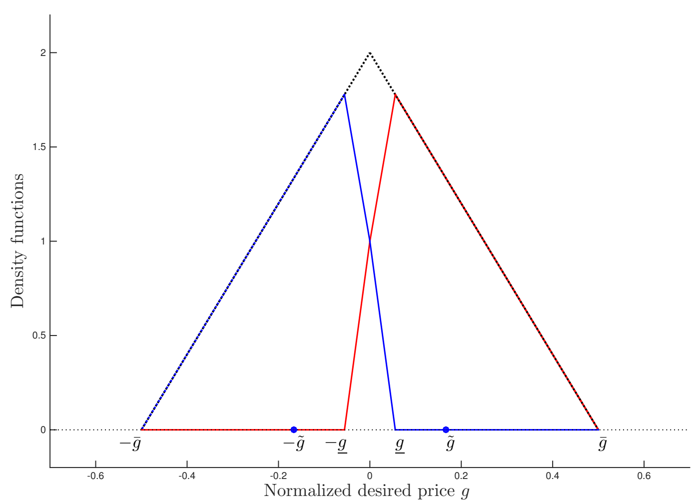
$$f(g) = \begin{cases} \frac{\bar{g}+g}{\bar{g}^2} & \text{for } g \in [-\bar{g}, 0] \\ \frac{\bar{g}-g}{\bar{g}^2} & \text{for } g \in [0, \bar{g}] \end{cases} \tag{H1}$$

Notice that the densities $f_h(g), f_l(g)$ follow the same Kolmogorov equation, hence they are linear, but they have different boundaries. In particular we have that the density $f_h(g)$ is continuous in $(-\bar{g}, \bar{g})$, the density is zero between $[-\bar{g}, -\underline{g})$, it is upward sloping between $-(\underline{g}, 0)$, it is upward sloping between $(0, \underline{g})$, and it coincides with $f(g)$ between $(\underline{g}, \bar{g})$. Using linearity and the boundary conditions $f_h(-\underline{g}) = 0$, $f_h(0) = f(0)/2$ and $f_h(\underline{g}) = f(\underline{g})$ yields the following density for high prices

$$f_h(g) = \begin{cases} 0 & \text{for } g \in [-\bar{g}, -\underline{g}) \\ \frac{1}{2\bar{g}} + \frac{1}{2\bar{g}\underline{g}}\, g & \text{for } g \in [-\underline{g}, 0) \\ \frac{1}{2\bar{g}} + \left(\frac{1}{2\bar{g}\underline{g}} - \frac{1}{\bar{g}^2}\right) g & \text{for } g \in [0, \underline{g}) \\ \frac{1}{\bar{g}} - \frac{1}{\bar{g}^2} g & \text{for } g \in [\underline{g}, \bar{g}] \end{cases} \tag{H2}$$

The density for low prices $f_l(g)$ is the symmetric counterpart of $f_h(g)$, in particular we have that $f_l(-g) = f_h(g)$. Figure H1 plots the two densities as an illustration of for the case in which $\nu > 0$ so the price gaps in the interval $(-\underline{g}, \underline{g})$ are associated with both high and low prices.

Figure H1. Density function for high and low prices when $\nu > 0$



Next we use the solution in equation (H2) to discuss the impact effect of a monetary shock. The next proposition shows that the impact effect is still approximately the same than the impact effect in a model where $\nu = 0$, i.e. that the shock δ has a first order effect on the aggregate price level, provided the fixed cost ν is small enough. More formally, the proposition states that the impact effect is continuous in ν, so that for small values of ν, which are necessary to get many temporary price changes as in the data, the impact effect is close to the impact of a model where $\nu = 0$:

Proposition 18. Continuity of the impact effect on $\underline{g}$. Fix a $0 < \delta < \bar{g}$ and

$\epsilon > 0$. Then there exist a $\underline{G}(\epsilon, \delta)$ such that for all $0 < \underline{g} < \underline{G}(\epsilon, \delta)$ the impact effect $|\tilde{\Theta}(\delta; \underline{g}) - \tilde{\Theta}(\delta; 0)| < \epsilon$.

Note that the optimal threshold $\underline{g} \to 0$ as the fixed cost $\nu \to 0$, so that the impact effect can be made arbitrarily close to the impact discussed in Proposition 11 in the main text.

I. Description of the Argentine CPI data for 1989-1997

Our dataset contains 8,618,345 price observations underlying the Argentinean CPI from December 1988 through September 1997. Each quote represents an item in a specific outlet for a specific time period. Goods and outlets are chosen to be representative of consumer expenditure in the 1986 consumer expenditure survey.[7] Goods are divided into two groups: homogeneous and differentiated goods. Homogeneous goods represent 49.5 percent of our sample and cover goods sold in super-market chains. Price quotes for differentiated goods are collected every month and cover mainly services.[8] We focus on homogeneous goods, and exclude price quotes for baskets of goods, rents, and fuel prices. We focus on these goods for two reasons: first because their prices are sampled every two weeks – versus heterogeneous goods are sampled every month–, and second because homogeneous goods are closer to the goods for which there are scanner price data, which increases the comparability of our study with ours. [9]

Next we discuss the sample that we use to compute different statistics. The main restrictions come from requiring that in each period of four months we can compute the reference prices, which are defined as the modal price for a store $\times$ good combination for a period of time. We discuss the definition of reference prices in detail below.

The data set has some missing observations and flags for stock-outs. We treat stock-outs and price quotes with no recorded information as missing observations. As a preliminary step we conduct two types of imputations. First, we impute missing observations when the price quotes before and after the missing value are the same, i.e. we "iron-out" the prices. Second, if in a given month a good $\times$ outlet has exactly one missing observation, we impute its price as the non-missing price of the same good $\times$ outlet for that month. The data set also contains a flag

[7] For a more detailed description see Alvarez et. al (2017).

[8] This is similar to the BLS, except that the frequency to which prices are gathered is twice as high in Argentina during this period, mostly because Argentina has a history of sustained inflation since the 1950's. Incidentally, during this period the agency in charge of measuring inflation, INDEC, was very prestigious and well regarded by other agencies. The intervention of the INDEC agency and the manipulation of the CPI started in the mid 2000's.

[9] Baskets correspond to around 9.91 percent of total expenditure and are excluded because their prices are gathered for any good in a basket, i.e., if one good is not available, it is substituted by any another in the basket. Examples are medicines and cigarettes. Rents are sampled monthly for a fixed set of representative properties. Reported prices represent the average of the sampled properties and include what is paid on that month, as opposed to what is paid for a new contract. Rents represent 2.33 percent of household expenditure. Fuel prices account for 4 percent of total expenditure and we exclude them because they were gathered in a separate database that we do not have access to.

for price substitutions. The statistical agency substitutes the price quote of an item for a similar item when the good is either discontinued by the producer or not sold any longer by an outlet. We define the relevant sample of four-month periods for a given good $\times$ outlet as those that have at most one substitution, at most one month where we impute its price, and they have no other missing prices for any other reasons –such as the outlet dropping from the sample, etc. Our final sample contains 4,759,584 price quotes from 198 different items and a total of 2877 unique stores. Around 5 percent of items have a sale flag, 1 percent have a substitution flag, and 0.1 percent are imputed prices. Overall, we have 594,948 four-month period $\times$ item $\times$ outlet combinations, i.e. it has 594,948 reference prices. We have 36 non overlapping four month periods, so in average we have about 132,000 price quotes and about 26,000 reference prices in each of the non overlapping four month period.

J. Additional moments: Argentine CPI and BPP data

This appendix provides additional detailed quantitative information on several price setting moments using two data sources: the Argentine CPI data as well as from the Billion Prices Project (BPP henceforth) by Cavallo and Rigobon (2016).

Table J1— Pricing Statistics - Argentina CPI (largest mode)

Date	Inflation	Freq. - Regular	Freq. - Reference	Freq.- Reference Adj.	Distinct	Fraction to New	Novelty
1989-1	228.6	0.613	0.832	0.882	94.6		
1989-2	792.9	0.720	0.831	0.831	98.7		
1989-3	193.7	0.454	0.788	0.819	91.9	0.941	0.660
1990-1	488.6	0.688	0.879	0.966	96.8	0.960	0.668
1990-2	153.3	0.533	0.859	0.959	94.1	0.904	0.482
1990-3	70.6	0.398	0.774	0.961	84.8	0.841	0.340
1991-1	125.2	0.463	0.763	0.825	91.3	0.848	0.393
1991-2	44.4	0.293	0.710	0.795	81.1	0.774	0.229
1991-3	10.6	0.255	0.628	0.795	77.1	0.701	0.167
1992-1	32.3	0.293	0.600	0.639	79.1	0.707	0.189
1992-2	14.0	0.195	0.556	0.621	68.2	0.659	0.112
1992-3	2.0	0.175	0.511	0.619	62.4	0.623	0.090
1993-1	15.4	0.188	0.442	0.472	63.8	0.594	0.094
1993-2	5.8	0.166	0.423	0.467	63.4	0.614	0.087
1993-3	3.0	0.144	0.383	0.457	55.2	0.556	0.066
1994-1	-2.7	0.157	0.361	0.389	54.1	0.587	0.075
1994-2	9.4	0.135	0.342	0.387	55.9	0.569	0.061
1994-3	3.0	0.138	0.325	0.388	60.6	0.559	0.064
1995-1	1.9	0.158	0.381	0.404	59.9	0.586	0.081
1995-2	-1.1	0.135	0.360	0.400	60.3	0.615	0.070
1995-3	2.2	0.139	0.340	0.395	54.9	0.550	0.063
1996-1	-4.5	0.145	0.341	0.365	59.4	0.549	0.065
1996-2	6.8	0.131	0.331	0.372	59.0	0.579	0.062
1996-3	-6.5	0.133	0.307	0.372	59.5	0.603	0.060
1997-1	-2.5	0.129	0.328	0.350	52.5	0.569	0.056
1997-2	6.1	0.147	0.294	0.349	62.6	0.610	0.074

Note: The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. Regular price changes are defined as any price change in a two-week period without a substitution flag. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the largest mode. The statistic reported in the table is fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed. The Distinct Index is the number of distinct prices minus two divided by the number of price spells minus two. The index is computed only on four-month periods with 3 price spells. Fraction to New indicates the fraction of price changes where the last price is a new price. The Novelty Index is the fraction of prices that are new, prices that do not appear in the last 12 months for the same item.

Table J2— Pricing Statistics - Argentina CPI (smallest mode)

Date	Inflation	Freq. - Regular	Freq. - Reference	Freq.- Reference Adj.	Distinct	Fraction to New	Novelty
1989-1	228.6	0.613	0.864	0.916	94.6		
1989-2	792.9	0.720	0.880	0.880	98.7		
1989-3	193.7	0.454	0.840	0.871	91.9	0.941	0.660
1990-1	488.6	0.688	0.882	0.969	96.8	0.960	0.668
1990-2	153.3	0.533	0.863	0.964	94.1	0.904	0.482
1990-3	70.6	0.398	0.777	0.965	84.8	0.841	0.340
1991-1	125.2	0.463	0.755	0.816	91.3	0.848	0.393
1991-2	44.4	0.293	0.707	0.792	81.1	0.774	0.229
1991-3	10.6	0.255	0.624	0.790	77.1	0.701	0.167
1992-1	32.3	0.293	0.594	0.632	79.1	0.707	0.189
1992-2	14.0	0.195	0.555	0.621	68.2	0.659	0.112
1992-3	2.0	0.175	0.510	0.618	62.4	0.623	0.090
1993-1	15.4	0.188	0.443	0.473	63.8	0.594	0.094
1993-2	5.8	0.166	0.426	0.471	63.4	0.614	0.087
1993-3	3.0	0.144	0.386	0.461	55.2	0.556	0.066
1994-1	-2.7	0.157	0.358	0.386	54.1	0.587	0.075
1994-2	9.4	0.135	0.339	0.383	55.9	0.569	0.061
1994-3	3.0	0.138	0.323	0.384	60.6	0.559	0.064
1995-1	1.9	0.158	0.376	0.398	59.9	0.586	0.081
1995-2	-1.1	0.135	0.356	0.396	60.3	0.615	0.070
1995-3	2.2	0.139	0.337	0.392	54.9	0.550	0.063
1996-1	-4.5	0.145	0.344	0.368	59.4	0.549	0.065
1996-2	6.8	0.131	0.333	0.374	59.0	0.579	0.062
1996-3	-6.5	0.133	0.309	0.375	59.5	0.603	0.060
1997-1	-2.5	0.129	0.319	0.339	52.5	0.569	0.056
1997-2	6.1	0.147	0.286	0.338	62.6	0.610	0.074

Note: The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. Regular price changes are defined as any price change in a two-week period without a substitution flag. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the smallest mode. The statistic reported in the table is fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed. The Distinct Index is the number of distinct prices minus two divided by the number of price spells minus two. The index is computed only on four-month periods with 3 price spells. Fraction to New indicates the fraction of price changes where the last price is a new price. The Novelty Index is the fraction of prices that are new, prices that do not appear in the last 12 months for the same item.

Table J3— Frequency of Price Adjustment by Types - Argentina CPI (largest mode)

		Frequency - Regular			Frequency - Reference			Frequency - Reference Adj.		
Date	Inflation (%)	All	Positive	Negative	All	Positive	Negative	All	Positive	Negative
1989-1	228.6	0.613	0.477	0.052	0.832	0.749	0.082	0.882	0.795	0.087
1989-2	792.9	0.720	0.598	0.072	0.831	0.701	0.130	0.831	0.701	0.130
1989-3	193.7	0.454	0.345	0.091	0.788	0.653	0.135	0.819	0.678	0.141
1990-1	488.6	0.688	0.501	0.165	0.879	0.841	0.038	0.966	0.924	0.042
1990-2	153.3	0.533	0.452	0.065	0.859	0.817	0.042	0.959	0.910	0.049
1990-3	70.6	0.398	0.295	0.091	0.774	0.738	0.036	0.961	0.914	0.047
1991-1	125.2	0.463	0.352	0.097	0.763	0.668	0.095	0.825	0.719	0.106
1991-2	44.4	0.293	0.214	0.070	0.710	0.609	0.101	0.795	0.673	0.121
1991-3	10.6	0.255	0.156	0.091	0.628	0.540	0.087	0.795	0.676	0.119
1992-1	32.3	0.293	0.203	0.081	0.600	0.470	0.130	0.639	0.496	0.142
1992-2	14.0	0.195	0.119	0.070	0.556	0.434	0.122	0.621	0.481	0.140
1992-3	2.0	0.175	0.098	0.072	0.511	0.399	0.112	0.619	0.480	0.139
1993-1	15.4	0.188	0.113	0.069	0.442	0.293	0.149	0.472	0.313	0.159
1993-2	5.8	0.166	0.094	0.067	0.423	0.283	0.140	0.467	0.313	0.154
1993-3	3.0	0.144	0.075	0.065	0.383	0.256	0.127	0.457	0.307	0.151
1994-1	-2.7	0.157	0.082	0.068	0.361	0.214	0.147	0.389	0.229	0.160
1994-2	9.4	0.135	0.077	0.053	0.342	0.204	0.138	0.387	0.231	0.156
1994-3	3.0	0.138	0.072	0.061	0.325	0.194	0.131	0.388	0.232	0.156
1995-1	1.9	0.158	0.084	0.069	0.381	0.226	0.155	0.404	0.238	0.166
1995-2	-1.1	0.135	0.067	0.064	0.360	0.215	0.146	0.400	0.238	0.163
1995-3	2.2	0.139	0.074	0.061	0.340	0.204	0.136	0.395	0.237	0.157
1996-1	-4.5	0.145	0.068	0.071	0.341	0.178	0.164	0.365	0.190	0.175
1996-2	6.8	0.131	0.071	0.055	0.331	0.173	0.157	0.372	0.196	0.176
1996-3	-6.5	0.133	0.061	0.068	0.307	0.160	0.146	0.372	0.197	0.175
1997-1	-2.5	0.129	0.061	0.064	0.328	0.166	0.163	0.350	0.178	0.172
1997-2	6.1	0.147	0.085	0.058	0.294	0.151	0.144	0.349	0.179	0.169

Note: The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. It reports the statistics for all price changes, positive price changes, and negative price changes. Regular price changes are defined as any price change in a two-week period without a substitution flag. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the largest mode. The statistic reported in the table is fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed.

Table J4— Frequency of Price Adjustment by Types - Argentina CPI (smallest mode)

		Frequency - Regular			Frequency - Reference			Frequency - Reference Adj.		
Date	Inflation (%)	All	Positive	Negative	All	Positive	Negative	All	Positive	Negative
1989-1	228.6	0.613	0.477	0.052	0.864	0.819	0.045	0.916	0.868	0.048
1989-2	792.9	0.720	0.598	0.072	0.880	0.814	0.067	0.880	0.814	0.067
1989-3	193.7	0.454	0.345	0.091	0.840	0.770	0.069	0.871	0.799	0.073
1990-1	488.6	0.688	0.501	0.165	0.882	0.862	0.020	0.969	0.947	0.023
1990-2	153.3	0.533	0.452	0.065	0.863	0.840	0.022	0.964	0.937	0.027
1990-3	70.6	0.398	0.295	0.091	0.777	0.759	0.019	0.965	0.939	0.025
1991-1	125.2	0.463	0.352	0.097	0.755	0.677	0.077	0.816	0.729	0.087
1991-2	44.4	0.293	0.214	0.070	0.707	0.624	0.084	0.792	0.690	0.103
1991-3	10.6	0.255	0.156	0.091	0.624	0.551	0.073	0.790	0.688	0.102
1992-1	32.3	0.293	0.203	0.081	0.594	0.468	0.126	0.632	0.495	0.137
1992-2	14.0	0.195	0.119	0.070	0.555	0.438	0.117	0.621	0.486	0.135
1992-3	2.0	0.175	0.098	0.072	0.510	0.403	0.107	0.618	0.484	0.134
1993-1	15.4	0.188	0.113	0.069	0.443	0.295	0.148	0.473	0.314	0.159
1993-2	5.8	0.166	0.094	0.067	0.426	0.287	0.139	0.471	0.317	0.154
1993-3	3.0	0.144	0.075	0.065	0.386	0.259	0.127	0.461	0.309	0.152
1994-1	-2.7	0.157	0.082	0.068	0.358	0.211	0.147	0.386	0.226	0.159
1994-2	9.4	0.135	0.077	0.053	0.339	0.201	0.138	0.383	0.227	0.156
1994-3	3.0	0.138	0.072	0.061	0.323	0.192	0.131	0.384	0.228	0.156
1995-1	1.9	0.158	0.084	0.069	0.376	0.220	0.156	0.398	0.231	0.166
1995-2	-1.1	0.135	0.067	0.064	0.356	0.211	0.145	0.396	0.234	0.162
1995-3	2.2	0.139	0.074	0.061	0.337	0.202	0.135	0.392	0.235	0.157
1996-1	-4.5	0.145	0.068	0.071	0.344	0.176	0.168	0.368	0.188	0.180
1996-2	6.8	0.131	0.071	0.055	0.333	0.172	0.161	0.374	0.194	0.180
1996-3	-6.5	0.133	0.061	0.068	0.309	0.159	0.150	0.375	0.194	0.181
1997-1	-2.5	0.129	0.061	0.064	0.319	0.161	0.158	0.339	0.172	0.167
1997-2	6.1	0.147	0.085	0.058	0.286	0.146	0.140	0.338	0.173	0.165

Note: The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. It reports the statistics for all price changes, positive price changes, and negative price changes. Regular price changes are defined as any price change in a two-week period without a substitution flag. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the smallest mode. The statistic reported in the table is fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed.

Table J5— Time at the Reference Price - Argentina CPI (largest mode)

Year	Inflation (%)	At Ref. Price	Below Ref. Price	Below/Above	Below/(1-At)	Sales
1989-1	228.6	0.362	0.318	0.993	0.498	0.023
1989-2	792.9	0.287	0.606	5.701	0.851	0.054
1989-3	193.7	0.528	0.202	0.749	0.428	0.064
1990-1	488.6	0.309	0.544	3.715	0.788	0.126
1990-2	153.3	0.406	0.380	1.771	0.639	0.048
1990-3	70.6	0.544	0.303	1.986	0.665	0.075
1991-1	125.2	0.428	0.471	4.650	0.823	0.071
1991-2	44.4	0.636	0.270	2.860	0.741	0.056
1991-3	10.6	0.690	0.215	2.262	0.693	0.076
1992-1	32.3	0.610	0.253	1.837	0.648	0.056
1992-2	14.0	0.726	0.178	1.856	0.650	0.055
1992-3	2.0	0.768	0.157	2.083	0.676	0.058
1993-1	15.4	0.753	0.155	1.667	0.625	0.049
1993-2	5.8	0.785	0.142	1.947	0.661	0.056
1993-3	3.0	0.821	0.118	1.924	0.658	0.052
1994-1	-2.7	0.795	0.128	1.675	0.626	0.048
1994-2	9.4	0.830	0.106	1.655	0.623	0.043
1994-3	3.0	0.824	0.110	1.699	0.629	0.048
1995-1	1.9	0.801	0.118	1.468	0.595	0.051
1995-2	-1.1	0.820	0.115	1.754	0.637	0.053
1995-3	2.2	0.823	0.118	1.984	0.665	0.049
1996-1	-4.5	0.816	0.116	1.702	0.630	0.055
1996-2	6.8	0.840	0.100	1.669	0.625	0.046
1996-3	-6.5	0.827	0.113	1.860	0.650	0.053
1997-1	-2.5	0.840	0.098	1.605	0.616	0.049
1997-2	6.1	0.813	0.107	1.345	0.573	0.047

Note: The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. For each non overlapping four-month interval and product $\times$ store combinations there are 8 two-weeks periods. In each of these two-week periods the price can be either at, above or below the reference price. The table reports the fraction of time the price is at the reference price, below the reference price, the ratio of time below and above the reference price, and the ratio of time below divided by one minus the time at the reference price. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. Reference prices are computed using the largest mode. The table also reports the fraction of sales which is the average number of product $\times$ store combinations with a sales flag in each two-weeks period. The reported statistic is the average in each non overlapping four-month period.

Table J6— Time at the Reference Price - Argentina CPI (smallest mode)

Year	Inflation (%)	At Ref. Price	Below Ref. Price	Below/Above	Below/(1-At)	Sales
1989-1	228.6	0.362	0.129	0.254	0.202	0.023
1989-2	792.9	0.287	0.310	0.771	0.435	0.054
1989-3	193.7	0.528	0.095	0.253	0.202	0.064
1990-1	488.6	0.309	0.264	0.620	0.383	0.126
1990-2	153.3	0.406	0.175	0.417	0.295	0.048
1990-3	70.6	0.544	0.151	0.493	0.330	0.075
1991-1	125.2	0.428	0.276	0.933	0.483	0.071
1991-2	44.4	0.636	0.158	0.767	0.434	0.056
1991-3	10.6	0.690	0.129	0.715	0.417	0.076
1992-1	32.3	0.610	0.124	0.467	0.318	0.056
1992-2	14.0	0.726	0.091	0.500	0.333	0.055
1992-3	2.0	0.768	0.091	0.645	0.392	0.058
1993-1	15.4	0.753	0.088	0.556	0.357	0.049
1993-2	5.8	0.785	0.084	0.635	0.388	0.056
1993-3	3.0	0.821	0.066	0.586	0.370	0.052
1994-1	-2.7	0.795	0.077	0.601	0.375	0.048
1994-2	9.4	0.830	0.066	0.642	0.391	0.043
1994-3	3.0	0.824	0.061	0.538	0.350	0.048
1995-1	1.9	0.801	0.065	0.490	0.329	0.051
1995-2	-1.1	0.820	0.064	0.553	0.356	0.053
1995-3	2.2	0.823	0.071	0.670	0.401	0.049
1996-1	-4.5	0.816	0.066	0.555	0.357	0.055
1996-2	6.8	0.840	0.059	0.589	0.371	0.046
1996-3	-6.5	0.827	0.062	0.562	0.360	0.053
1997-1	-2.5	0.840	0.059	0.587	0.370	0.049
1997-2	6.1	0.813	0.062	0.493	0.330	0.047

Note: The table shows several pricing statistics computed using the Argentina CPI data for each four-month period between 1989 and 1997. The inflation reported is the annualized continuously compounded inflation rate in percent. For each non overlapping four-month interval and product × store combinations there are 8 two-weeks periods. In each of these two-week period the price can be either at, above or below the reference price. The table reports the fraction of time the price is at the reference price, below the reference price, the ratio of time below and above the reference price, and the ratio of time below divided by one minus the time at the reference price. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. Reference prices are computed using the smallest mode. The table also reports the fraction of sales which is the average number of product × store combinations with a sales flag in each two-weeks period. The reported statistic is the average in each non overlapping four-month period.

Table J7— Pricing Statistics - BPP (largest mode)

Country	Date	Inflation	Freq. - Reg.	Freq. - Ref.	Freq. - Ref. Adj.	Distinct	Fract. to New	Novelty
	2008	5.700	0.171	0.358	0.602	73.900	0.678	0.125
Argentina	2009	4.168	0.190	0.562	0.632	73.660	0.719	0.150
	2010	8.905	0.168	0.596	0.635	74.062	0.730	0.137
	2008	1.452	0.253	0.369	0.624	79.505	0.687	0.179
Brazil	2009	1.419	0.267	0.725	0.813	88.121	0.781	0.251
	2010	0.234	0.320	0.886	0.978	90.183	0.748	0.250
	2008	3.039	0.169	0.285	0.485	59.718	0.616	0.116
Chile	2009	0.102	0.156	0.311	0.341	49.207	0.510	0.085
	2010	0.919	0.147	0.284	0.297	41.876	0.420	0.066
	2008	2.397	0.236	0.394	0.650	77.577	0.814	0.226
Colombia	2009	1.762	0.258	0.690	0.714	80.778	0.778	0.205
	2010	1.486	0.250	0.771	0.791	72.438	0.732	0.188
	2008	2.910	0.299	0.000		47.124		
USA	2009	-1.651	0.207	0.350	0.386	41.167	0.336	0.064
	2010	4.328	0.275	0.343	0.370	66.713	0.374	0.145

Note: The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. Regular price changes are defined as any price change in a two-week period without a substitution flag. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the largest mode. The statistic reported in the table is the fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed. The Distinct Index is the number of distinct prices minus two divided by the number of price spells minus two. The index is computed only on four-month periods with 3 price spells. Fraction to New indicates the fraction of price changes where the last price is a new price. The Novelty Index is the fraction of prices that are new, prices that do not appear in the last 12 months for the same item.

Table J8— Pricing Statistics – BPP (smallest mode)

Country	Date	Inflation	Freq. - Reg.	Freq. - Ref.	Freq. - Ref. Adj.	Distinct	Fract. to New	Novelty
	2008	5.700	0.171	0.377	0.634	73.900	0.678	0.125
Argentina	2009	4.168	0.190	0.571	0.642	73.660	0.719	0.150
	2010	8.905	0.168	0.502	0.534	74.062	0.730	0.137
	2008	1.452	0.253	0.373	0.631	79.505	0.687	0.179
Brazil	2009	1.419	0.267	0.714	0.803	88.121	0.781	0.251
	2010	0.234	0.320	0.887	0.979	90.183	0.748	0.250
	2008	3.039	0.169	0.282	0.485	59.718	0.616	0.116
Chile	2009	0.102	0.156	0.339	0.371	49.207	0.510	0.085
	2010	0.919	0.147	0.294	0.307	41.876	0.420	0.066
	2008	2.397	0.236	0.392	0.648	77.577	0.814	0.226
Colombia	2009	1.762	0.258	0.695	0.721	80.778	0.778	0.205
	2010	1.486	0.250	0.791	0.811	72.438	0.732	0.188
	2008	2.910	0.299	0.000		47.124		
USA	2009	-1.651	0.207	0.382	0.416	41.167	0.336	0.064
	2010	4.328	0.275	0.304	0.332	66.713	0.374	0.145

Note: The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. Regular price changes are defined as any price change in a two-week period without a substitution flag. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the smallest mode. The statistic reported in the table is the fraction of product × store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed. The Distinct Index is the number of distinct prices minus two divided by the number of price spells minus two. The index is computed only on four-month periods with 3 price spells. Fraction to New indicates the fraction of price changes where the last price is a new price. The Novelty Index is the fraction of prices that are new, prices that do not appear in the last 12 months for the same item.

Table J9— Frequency of Price Adjustment by Types – BPP (largest mode)

			Frequency - Regular			Frequency - Reference			Frequency - Reference Adj.		
Country	Date	Inflation (%)	All	Positive	Negative	All	Positive	Negative	All	Positive	Negative
	2008	5.700	0.171	0.121	0.041	0.358	0.331	0.027	0.602	0.555	0.048
Argentina	2009	4.168	0.190	0.129	0.058	0.562	0.465	0.096	0.632	0.526	0.106
	2010	8.905	0.168	0.130	0.037	0.596	0.563	0.032	0.635	0.599	0.035
	2008	1.452	0.253	0.130	0.111	0.369	0.206	0.163	0.624	0.349	0.275
Brazil	2009	1.419	0.267	0.167	0.096	0.725	0.485	0.239	0.813	0.540	0.273
	2010	0.234	0.320	0.133	0.183	0.886	0.427	0.459	0.978	0.465	0.513
	2008	3.039	0.169	0.102	0.059	0.285	0.240	0.045	0.485	0.408	0.077
Chile	2009	0.102	0.156	0.079	0.075	0.311	0.196	0.115	0.341	0.214	0.127
	2010	0.919	0.147	0.079	0.068	0.284	0.136	0.149	0.297	0.142	0.155
	2008	2.397	0.236	0.129	0.094	0.394	0.248	0.145	0.650	0.410	0.240
Colombia	2009	1.762	0.258	0.150	0.106	0.690	0.457	0.233	0.714	0.473	0.241
	2010	1.486	0.250	0.142	0.107	0.771	0.546	0.225	0.791	0.558	0.233
	2008	2.910	0.299	0.143	0.119	0.000	0.000	0.000			
USA	2009	-1.651	0.207	0.098	0.107	0.350	0.163	0.187	0.386	0.183	0.203
	2010	4.328	0.275	0.177	0.096	0.343	0.123	0.220	0.370	0.131	0.239

Note: The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. It reports the statistics for all price changes, positive price changes, and negative price changes. Regular price changes are defined as any price change in a two-week period without a substitution flag. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the largest mode. The statistic reported in the table is the fraction of product × store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed.

Table J10— Frequency of Price Adjustment by Types – BPP (smallest mode)

			Frequency - Regular			Frequency - Reference			Frequency - Reference Adj.		
Country	Date	Inflation (%)	All	Positive	Negative	All	Positive	Negative	All	Positive	Negative
	2008	5.700	0.171	0.121	0.041	0.377	0.352	0.025	0.634	0.589	0.045
Argentina	2009	4.168	0.190	0.129	0.058	0.571	0.471	0.100	0.642	0.532	0.110
	2010	8.905	0.168	0.130	0.037	0.502	0.450	0.052	0.534	0.478	0.057
	2008	1.452	0.253	0.130	0.111	0.373	0.201	0.171	0.631	0.340	0.291
Brazil	2009	1.419	0.267	0.167	0.096	0.714	0.483	0.231	0.803	0.539	0.264
	2010	0.234	0.320	0.133	0.183	0.887	0.386	0.501	0.979	0.421	0.559
	2008	3.039	0.169	0.102	0.059	0.282	0.232	0.050	0.485	0.399	0.086
Chile	2009	0.102	0.156	0.079	0.075	0.339	0.215	0.124	0.371	0.233	0.137
	2010	0.919	0.147	0.079	0.068	0.294	0.154	0.140	0.307	0.161	0.146
	2008	2.397	0.236	0.129	0.094	0.392	0.247	0.145	0.648	0.408	0.239
Colombia	2009	1.762	0.258	0.150	0.106	0.695	0.457	0.238	0.721	0.474	0.247
	2010	1.486	0.250	0.142	0.107	0.791	0.556	0.235	0.811	0.569	0.242
	2008	2.910	0.299	0.143	0.119	0.000	0.000	0.000			
USA	2009	-1.651	0.207	0.098	0.107	0.382	0.174	0.209	0.416	0.191	0.225
	2010	4.328	0.275	0.177	0.096	0.304	0.124	0.180	0.332	0.135	0.197

Note: The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We convert the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. The tables reports the frequency of adjustment of regular price, reference prices, and adjusted reference prices. It reports the statistics for all price changes, positive price changes, and negative price changes. Regular price changes are defined as any price change in a two-week period without a substitution flag. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. If they are different, it constitutes a reference price change. Reference prices are computed using the smallest mode. The statistic reported in the table is the fraction of product $\times$ store combination with a change. The frequency of adjustment is a four-month frequency. The frequency of adjustment of adjusted reference prices is computed in the same way as the frequency of adjustment of reference price changes but counts as missing periods in which the reference price cannot be computed.

Table J11— Time at the Reference Price – BPP (largest mode)

Country	Year	Inflation (%)	At Ref. Price	Below Ref. Price	Below/Above	Below/(1-At)	Sales
	2008	5.7	0.769	0.144	1.655	0.623	0.029
Argentina	2009	4.2	0.759	0.131	1.197	0.545	0.031
	2010	8.9	0.722	0.201	2.599	0.722	0.023
	2008	1.5	0.713	0.189	1.940	0.660	0.031
Brazil	2009	1.4	0.671	0.215	1.894	0.654	0.026
	2010	0.2	0.590	0.201	0.961	0.490	0.044
	2008	3.0	0.798	0.137	2.113	0.679	0.039
Chile	2009	0.1	0.835	0.120	2.636	0.725	0.044
	2010	0.9	0.867	0.092	2.224	0.690	0.038
	2008	2.4	0.698	0.192	1.741	0.635	0.029
Colombia	2009	1.8	0.670	0.207	1.686	0.628	0.029
	2010	1.5	0.683	0.204	1.807	0.644	0.032
	2008	2.9	0.717	0.228	4.175	0.807	0.083
USA	2009	-1.7	0.799	0.151	3.025	0.752	0.069
	2010	4.3	0.730	0.142	1.107	0.526	0.061

Note: The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. For each non overlapping four-month interval and product $\times$ store combinations there are 8 two-weeks periods. In each of these two-week period the price can be either at, above or below the reference price. The table reports the fraction of time the price is at the reference price, below the reference price, the ratio of time below and above the reference price, and the ratio of time below divided by one minus the time at the reference price. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. Reference prices are computed using the largest mode. The table also reports the fraction of sales which is the average number of product $\times$ store combinations with a sales flag in each two-weeks period. The reported statistic is the average in each non overlapping four-month period.

Table J12— Time at the Reference Price – BPP (smallest mode)

Country	Year	Inflation (%)	At Ref. Price	Below Ref. Price	Below/Above	Below/(1-At)	Sales
	2008	5.7	0.769	0.094	0.690	0.408	0.029
Argentina	2009	4.2	0.759	0.095	0.648	0.393	0.031
	2010	8.9	0.722	0.100	0.557	0.358	0.023
	2008	1.5	0.713	0.129	0.820	0.451	0.031
Brazil	2009	1.4	0.671	0.187	1.313	0.568	0.026
	2010	0.2	0.590	0.137	0.500	0.333	0.044
	2008	3.0	0.798	0.099	0.964	0.491	0.039
Chile	2009	0.1	0.835	0.094	1.318	0.569	0.044
	2010	0.9	0.867	0.076	1.323	0.569	0.038
	2008	2.4	0.698	0.134	0.793	0.442	0.029
Colombia	2009	1.8	0.670	0.137	0.706	0.414	0.029
	2010	1.5	0.683	0.137	0.757	0.431	0.032
	2008	2.9	0.717	0.183	1.833	0.647	0.083
USA	2009	-1.7	0.799	0.110	1.212	0.548	0.069
	2010	4.3	0.730	0.103	0.616	0.381	0.061

Note: The table shows several pricing statistics computed using data collected by thee Billion Prices Project every day between October 2007 and August 2010 for over 250 thousand individual products in five countries: Argentina, Brazil, Chile, Colombia, and the United States. We converted the daily data to biweekly for comparison with the Argentinean CPI data. The inflation reported is the computed using geometric means reported in percent. For each non overlapping four-month interval and product $\times$ store combinations there are 8 two-weeks periods. In each of these two-week period the price can be either at, above or below the reference price. The table reports the fraction of time the price is at the reference price, below the reference price, the ratio of time below and above the reference price, and the ratio of time below divided by one minus the time at the reference price. Reference price changes are computed as follows: in each non overlapping four month interval, we compare the reference price of that four-month interval with the one in the previous four-month interval. Reference prices are computed using the smallest mode. The table also reports the fraction of sales which is the average number of product $\times$ store combinations with a sales flag in each two-weeks period. The reported statistic is the average in each non overlapping four-month period.

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