# Optimal Currency Areas with Labor Market Frictions

Rohan Kekre

Online Appendix

## A Optimization problems facing Foreign agents

In this appendix I detail the optimization problems facing Foreign agents which were excluded from the main text for brevity.

The representative household chooses a state-contingent sequence of $c_t^*$, $B_{t+1}^*$, and $u_t^*$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u^*(c_t^*, n_t^*)$$

subject to

$$u^*(c_t^*, n_t^*) = \frac{c_t^{*1-\sigma}}{1-\sigma} - \chi^* \frac{n_t^{*1+\varphi^*}}{1+\varphi^*},$$

$$c_t^* = \left[(\gamma)^\varsigma (c_{Ht}^*)^{1-\varsigma} + (1-\gamma)^\varsigma (c_{Ft}^*)^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}},$$

$$c_{Ht}^* = \left[\int_0^1 c_{Ht}^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad c_{Ft}^* = \left[\int_0^1 c_{Ft}^*(j^*)^{\frac{\varepsilon-1}{\varepsilon}} dj^*\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

$$n_t^* = (1-\delta^*) n_{t-1}^* + p^*(\theta_t^*) u_t^*,$$

$$\int_0^1 P_{Ht}(j) c_{Ht}^*(j) dj + \int_0^1 P_{Ft}(j^*) c_{Ft}^*(j^*) dj^* + \mathbb{E}_t Q_{t,t+1} B_{t+1}^* \leq W_t^* n_t^* + B_t^* - T_t^*,$$

as well as the no-Ponzi constraint $\lim_{s\to\infty} \mathbb{E}_t Q_{t,t+s} B_{t+s}^* \geq 0$ at all $t$.

The representative intermediate good producer chooses a state-contingent sequence of $\nu_t^*$ and $n_t^*$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} Q_{0,t} \Pi_t^*$$

subject to

$$n_t^* = (1-\delta^*) n_{t-1}^* + q^*(\theta_t^*) \nu_t^*,$$

$$\Pi_t^* = (1-\gamma)\left(P_t^{I*} a_t^* \left[(1-\delta^*) n_{t-1}^* + q^*(\theta_t^*)\nu_t^* - k^* \nu_t^*\right] - W_t^* \left[(1-\delta^*) n_{t-1}^* + q^*(\theta_t^*)\nu_t^*\right]\right).$$

If retailer $j^* \in [0,1]$ can update its price in period $t$, it chooses $\mathcal{P}_{Ft}$ and a state-contingent sequence of $x_s^*(j^*)$ and $y_s^*(j^*)$ (for $s \geq t$) to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \Pi_s^{r*}(\mathcal{P}_{Ft}; j^*)$$

subject to

$$\Pi_s^{r*}(\mathcal{P}_{Ft}; j^*) = \mathcal{P}_{Ft} y_s^*(j^*) - (1 + \tau^{r*}) P_s^{I*} x_s^*(j^*),$$
$$y_s^*(j^*) = x_s^*(j^*),$$
$$y_s^*(j^*) = \left( \frac{\mathcal{P}_{Ft}}{P_{Fs}} \right)^{-\varepsilon} \left( \gamma c_{Fs} + (1 - \gamma) c_{Fs}^* \right).$$

If retailer $j^*$ cannot update its price in period $t$ it accommodates consumption demand at its preset price provided it earns non-negative profits.

# B Equilibrium

In this appendix I formally define the equilibrium and derive the equilibrium conditions for the environment described in the main text.

## B.1 Definition of equilibrium

The definition of equilibrium is standard:

**Definition B.1.** *An* equilibrium *is a state-contingent sequence of consumption, labor force participation, vacancies, tightness, employment, intermediate goods, and final goods, as well as nominal wages, prices, and profits, such that given a set of initial prices and portfolios and a state-contingent path for nominal interest rates and lump-sum taxes:*

1. *households solve (1) subject to (3)-(6);*
2. *producers solve (7) subject to (8)-(9);*
3. *retailers solve (10) subject to (11)-(13) if they can update prices and accommodate demand if not;*
4. *analogous conditions to those above are all satisfied in Foreign;*
5. *tightness is consistent with aggregate vacancies and job-seekers according to (16);*
6. *wages are Nash bargained each period (as detailed in appendix B.5);*

7. *government budgets are balanced according to (17) and (18);*
8. *the intermediate good markets clear according to (19) and (20);*
9. *the final good markets clear according to (21) and (22);*
10. *global asset markets in state-contingent securities and firm shares clear according to (23).*

## B.2 Equilibrium conditional on arbitrary final goods prices

I now derive equilibrium conditions conditional on arbitrary final goods prices consistent with producer-currency pricing. In the subsequent sections I complete the description of the natural allocation and then the equilibrium with nominal rigidity.

First, by households' optimal intratemporal allocation of consumption

$$\frac{c_{Ht}}{c_{Ft}} = \frac{\gamma}{1-\gamma}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{-\frac{1}{\varsigma}},$$
$$\frac{c^*_{Ht}}{c^*_{Ft}} = \frac{\gamma}{1-\gamma}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{-\frac{1}{\varsigma}}.$$

Given the definition of the terms of trade

$$s_t \equiv \frac{P_{Ht}}{P_{Ft}},$$

it follows that

$$s_t = \left(\frac{\gamma}{1-\gamma}\frac{c_{Ft}}{c_{Ht}}\right)^{\varsigma} = \left(\frac{\gamma}{1-\gamma}\frac{c^*_{Ft}}{c^*_{Ht}}\right)^{\varsigma}.$$

Now consider households' optimal choice of portfolios, which imply

$$\beta\frac{P_t}{P_{t+1}}\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} = Q_{t,t+1} = \beta\frac{P_t}{P_{t+1}}\frac{c_{t+1}^{*-\sigma}}{c_t^{*-\sigma}}$$

at all dates and states. We thus obtain the standard international risk-sharing condition

$$c_t = \Xi c_t^*,$$

where $\Xi$ reflects endowments in period 0. Following Assumption 1, endowments are such that $\Xi = 1$. Since households have the same homothetic preferences and face the same prices, it follows that

$$c_{Ht} = c^*_{Ht},$$
$$c_{Ft} = c^*_{Ft},$$

so that the terms of trade further satisfy

$$s_t = \left( \frac{\gamma}{1-\gamma} \frac{c^*_{Ft}}{c_{Ht}} \right)^{\varsigma}.$$

Turn now to equilibrium in the labor market. Producers' first-order condition for vacancy posting requires

$$P^I_t a_t - W_t - P^I_t \frac{k a_t}{q(\theta_t)} + (1-\delta)\mathbb{E}_t Q_{t,t+1} P^I_{t+1} \frac{k a_{t+1}}{q(\theta_{t+1})} = 0,$$
$$P^{I*}_t a^*_t - W^*_t - P^{I*}_t \frac{k^* a^*_t}{q^*(\theta^*_t)} + (1-\delta^*)\mathbb{E}_t Q_{t,t+1} P^{I*}_{t+1} \frac{k^* a^*_{t+1}}{q^*(\theta^*_{t+1})} = 0$$

for an interior optimum in vacancy posting to exist. As shown in appendix B.5, the Nash bargained wage in each country is

$$W_t = -P_t \frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} + \frac{\zeta}{1-\zeta}\left[ P^I_t \frac{k a_t}{q(\theta_t)} - (1-\delta)\mathbb{E}_t Q_{t,t+1}(1-p(\theta_{t+1}))P^I_{t+1}\frac{k a_{t+1}}{q(\theta_{t+1})} \right],$$
$$W^*_t = -P_t \frac{u^*_n(c^*_t,n^*_t)}{u^*_c(c^*_t,n^*_t)} + \frac{\zeta^*}{1-\zeta^*}\left[ P^{I*}_t \frac{k^* a^*_t}{q^*(\theta^*_t)} - (1-\delta^*)\mathbb{E}_t Q_{t,t+1}(1-p^*(\theta^*_{t+1}))P^{I*}_{t+1}\frac{k^* a^*_{t+1}}{q^*(\theta^*_{t+1})} \right].$$

Given the assumed preferences, the marginal rate of substitution between labor and consumption in each country is:

$$-P_t \frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} = P_{Ht}\chi\gamma^{-\varsigma} c_t^{\sigma-\varsigma} c_{Ht}^{\varsigma} n_t^{\varphi},$$
$$-P_t \frac{u^*_n(c^*_t,n^*_t)}{u^*_c(c^*_t,n^*_t)} = P_{Ft}\chi^*(1-\gamma)^{-\varsigma} c_t^{*\sigma-\varsigma} c_{Ft}^{*\varsigma} n_t^{*\varphi^*}.$$

Combining the Nash bargained wages with optimal vacancy posting, labor market equilibrium can be summarized by

$$P^I_t a_t = P_{Ht}\chi\gamma^{-\varsigma} c_t^{\sigma-\varsigma} c_{Ht}^{\varsigma} n_t^{\varphi} + \frac{1}{1-\zeta} P^I_t \frac{k a_t}{q(\theta_t)}$$
$$- (1-\delta)\mathbb{E}_t Q_{t,t+1}\left(1 + \frac{\zeta}{1-\zeta}(1-p(\theta_{t+1}))\right) P^I_{t+1}\frac{k a_{t+1}}{q(\theta_{t+1})},$$

$$P^{I*}_t a^*_t = P_{Ft}\chi^*(1-\gamma)^{-\varsigma} c_t^{*\sigma-\varsigma} c_{Ft}^{*\varsigma} n_t^{*\varphi^*} + \frac{1}{1-\zeta^*} P^{I*}_t \frac{k^* a^*_t}{q^*(\theta^*_t)}$$
$$- (1-\delta^*)\mathbb{E}_t Q_{t,t+1}\left(1 + \frac{\zeta^*}{1-\zeta^*}(1-p^*(\theta^*_{t+1}))\right) P^{I*}_{t+1}\frac{k^* a^*_{t+1}}{q^*(\theta^*_{t+1})}.$$

In the usual way, labor market frictions generate a wedge between the marginal rate of transformation and marginal rate of substitution between consumption and labor in each country. Moreover, provided workers have some bargaining power ($\zeta, \zeta^* > 0$), the equilibrium wage will be high enough that households will optimally ensure all initially unemployed members participate in the labor market:

$$u_t = 1 - (1-\delta)n_{t-1},$$
$$u^*_t = 1 - (1-\delta^*)n^*_{t-1}.$$

The employment rate in each market is thus

$$n_t = (1-\delta)n_{t-1} + p(\theta_t)(1-(1-\delta)n_{t-1}),$$
$$n^*_t = (1-\delta^*)n^*_{t-1} + p^*(\theta^*_t)(1-(1-\delta^*)n^*_{t-1}).$$

Lastly, consider final good market clearing for each variety produced by each country. Integrating over varieties, we obtain

$$\int_0^1 \gamma c_{Ht}(j)dj + \int_0^1 (1-\gamma)c^*_{Ht}(j)dj = \int_0^1 y_t(j)dj,$$
$$\int_0^1 \gamma c_{Ft}(j^*)dj^* + \int_0^1 (1-\gamma)c^*_{Ft}(j^*)dj = \int_0^1 y^*_t(j^*)dj^*.$$

Substituting in households' optimal consumption of each variety

$$c_{Ht}(j) = \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} c_{Ht}, \quad c_{Ft}(j^*) = \left(\frac{P_{Ft}(j^*)}{P_{Ft}}\right)^{-\varepsilon} c_{Ft},$$
$$c^*_{Ht}(j) = \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} c^*_{Ht}, \quad c^*_{Ft}(j^*) = \left(\frac{P_{Ft}(j^*)}{P_{Ft}}\right)^{-\varepsilon} c^*_{Ft},$$

and making use of retailers' production technologies $y_t(j) = x_t(j)$ and $y^*_t(j^*) = x^*_t(j^*)$ and intermediate good market clearing, we obtain

$$\left(\int_0^1 \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} dj\right)\left[\gamma c_{Ht} + (1-\gamma)c^*_{Ht}\right] = \gamma a_t\left(n_t - k\nu_t\right),$$

$$\left(\int_0^1 \left(\frac{P_{Ft}(j^*)}{P_{Ft}}\right)^{-\varepsilon} dj^*\right)[\gamma c_{Ft} + (1-\gamma)c^*_{Ft}] = (1-\gamma)a^*_t\left(n^*_t - k^*\nu^*_t\right).$$

Defining indices of price dispersion

$$D_{Ht} \equiv \int_0^1 \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} dj, \tag{B1}$$

$$D_{Ft} \equiv \int_0^1 \left(\frac{P_{Ft}(j^*)}{P_{Ft}}\right)^{-\varepsilon} dj^*, \tag{B2}$$

we have

$$\gamma c_{Ht} + (1-\gamma)c^*_{Ht} = \frac{1}{D_{Ht}}\gamma a_t\left(n_t - k\nu_t\right),$$
$$\gamma c_{Ft} + (1-\gamma)c^*_{Ft} = \frac{1}{D_{Ft}}(1-\gamma)a^*_t\left(n^*_t - k^*\nu^*_t\right).$$

Since $c^*_{Ht} = c_{Ht}$ and $c_{Ft} = c^*_{Ft}$, and making use of the definition of market tightness in each country, it follows that

$$c_{Ht} = \frac{1}{D_{Ht}}\gamma a_t\left(n_t - k(1-(1-\delta)n_{t-1})\theta_t\right),$$
$$c^*_{Ft} = \frac{1}{D_{Ft}}(1-\gamma)a^*_t\left(n^*_t - k^*(1-(1-\delta^*)n^*_{t-1})\theta^*_t\right).$$

## B.3 Natural allocation

When all retailers can set prices after the realization of productivity shocks ($\iota = \iota^* = 1$), their optimal price-setting conditions imply

$$P_{Ht}(j) = \frac{\varepsilon}{\varepsilon-1}(1+\tau^r)P^I_t, \quad P_{Ft}(j^*) = \frac{\varepsilon}{\varepsilon-1}(1+\tau^{r*})P^{I*}_t. \tag{B3}$$

Together with results of the previous subsection, we obtain the following system of equilibrium conditions defining the natural allocation (denoted with $n$ superscripts):

$$\begin{aligned}\frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^r)}a_t = {} & \chi\gamma^{-\varsigma}(c^n_t)^{\sigma-\varsigma}(c^n_{Ht})^{\varsigma}(n^n_t)^{\varphi} + \frac{1}{1-\zeta}\frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^r)}\frac{ka_t}{q(\theta^n_t)} \\ & -(1-\delta)\mathbb{E}_t\beta\frac{(c^n_t)^{\sigma-\varsigma}(c^n_{Ht})^{\varsigma}}{(c^n_{t+1})^{\sigma-\varsigma}(c^n_{Ht+1})^{\varsigma}}\left(1+\frac{\zeta}{1-\zeta}(1-p(\theta^n_{t+1}))\right)\frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^r)}\frac{ka_{t+1}}{q(\theta^n_{t+1})},\end{aligned} \tag{B4}$$

$$n^n_t = (1-\delta)n^n_{t-1} + p(\theta^n_t)(1-(1-\delta)n^n_{t-1}), \tag{B5}$$

$$c^n_{Ht} = \gamma a_t\left[n^n_t - k(1-(1-\delta)n^n_{t-1})\theta^n_t\right], \tag{B6}$$

$$\frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^{r*})}a_t^* = \chi^*(1-\gamma)^{-\varsigma}(c_t^{*n})^{\sigma-\varsigma}(c_{Ft}^{*n})^{\varsigma}(n_t^{*n})^{\varphi^*} + \frac{1}{1-\zeta^*}\frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^{r*})}\frac{k^* a_t^*}{q^*(\theta_t^{*n})}$$
$$-(1-\delta^*)\mathbb{E}_t\beta\frac{(c_t^{*n})^{\sigma-\varsigma}(c_{Ft}^{*n})^{\varsigma}}{(c_{t+1}^{*n})^{\sigma-\varsigma}(c_{Ft+1}^{*n})^{\varsigma}}\left(1+\frac{\zeta^*}{1-\zeta^*}(1-p^*(\theta_{t+1}^{*n}))\right)\frac{1}{\frac{\varepsilon}{\varepsilon-1}(1+\tau^{r*})}\frac{k^* a_{t+1}^*}{q^*(\theta_{t+1}^{*n})}, \tag{B7}$$

$$n_t^{*n} = (1-\delta^*)n_{t-1}^{*n} + p^*(\theta_t^{*n})(1-(1-\delta^*)n_{t-1}^{*n}), \tag{B8}$$

$$c_{Ft}^{*n} = (1-\gamma)a_t^*\left[n_t^{*n} - k^*(1-(1-\delta^*)n_{t-1}^{*n})\theta_t^{*n}\right], \tag{B9}$$

$$s_t^n = \left(\frac{\gamma}{1-\gamma}\frac{c_{Ft}^{*n}}{c_{Ht}^n}\right)^{\varsigma}, \tag{B10}$$

$$c_{Ft}^n = c_{Ft}^{*n}, \tag{B11}$$

$$c_{Ht}^{*n} = c_{Ht}^n, \tag{B12}$$

$$c_t^n = \left((\gamma)^{\varsigma}(c_{Ht}^n)^{1-\varsigma} + (1-\gamma)^{\varsigma}(c_{Ft}^n)^{1-\varsigma}\right)^{\frac{1}{1-\varsigma}}, \tag{B13}$$

$$c_t^{*n} = \left((\gamma)^{\varsigma}(c_{Ht}^{*n})^{1-\varsigma} + (1-\gamma)^{\varsigma}(c_{Ft}^{*n})^{1-\varsigma}\right)^{\frac{1}{1-\varsigma}}. \tag{B14}$$

As is evident, the natural allocation is fully determined without reference to nominal prices and wages in the global economy, reflecting a standard real/nominal dichotomy.

## B.4 Sticky price equilibrium

We now turn to the case with more general $\iota$ and $\iota^*$. Consider the problem facing a retailer which can update its price $\mathcal{P}_{Ht}$ in period $t$. Its optimal price satisfies

$$\mathbb{E}_t\sum_{s=t}^{\infty}(1-\iota)^{s-t}Q_{t,s}\mathscr{y}_s\left(\mathcal{P}_{Ht} - \frac{\varepsilon}{\varepsilon-1}(1+\tau^r)P_s^I\right) = 0$$

where

$$\mathscr{y}_s = \left(\frac{\mathcal{P}_{Ht}}{P_{Hs}}\right)^{-\varepsilon}(\gamma c_{Hs} + (1-\gamma)c_{Hs}^*).$$

Since only a randomly drawn fraction $\iota$ of retailers can update prices each period, the producer-price index in (14) evolves according to

$$(P_{Ht})^{1-\varepsilon} = (1-\iota)(P_{Ht-1})^{1-\varepsilon} + \iota(\mathcal{P}_{Ht})^{1-\varepsilon},$$

while the index of price dispersion in (B1) evolves according to

$$D_{Ht} = (1-\iota)D_{Ht-1}\left(\frac{P_{Ht-1}}{P_{Ht}}\right)^{-\varepsilon} + \iota\left(\frac{\mathcal{P}_{Ht}}{P_{Ht}}\right)^{-\varepsilon}.$$

Analogous conditions obtain in Foreign. Given the other conditions derived in appendix B.2, it is straightforward to summarize the equilibrium with a system in which the real and nominal sides of the global economy are now jointly determined, given a particular choice of union-wide monetary policy summarized by a state-contingent path of the riskless nominal interest rate

$$1 + i_t = \frac{1}{\mathbb{E}_t Q_{t,t+1}}.$$

Here, I proceed directly to characterize the set of implementable real allocations given possible paths of $\{i_t\}$, serving as constraints in the optimal policy problem studied in the main text:

$$\frac{P_t^I}{P_{Ht}} a_t = \chi \gamma^{-\varsigma} (c_t)^{\sigma-\varsigma} (c_{Ht})^{\varsigma} (n_t)^{\varphi} + \frac{1}{1-\zeta} \frac{P_t^I}{P_{Ht}} \frac{k a_t}{q(\theta_t)}$$

$$- (1-\delta) \mathbb{E}_t \beta \frac{(c_t)^{\sigma-\varsigma}(c_{Ht})^{\varsigma}}{(c_{t+1})^{\sigma-\varsigma}(c_{Ht+1})^{\varsigma}} \left(1 + \frac{\zeta}{1-\zeta}(1 - p(\theta_{t+1}))\right) \frac{P_{t+1}^I}{P_{Ht+1}} \frac{k a_{t+1}}{q(\theta_{t+1})}, \tag{B15}$$

$$n = (1-\delta) n_{t-1} + p(\theta_t)(1 - (1-\delta) n_{t-1}), \tag{B16}$$

$$c_{Ht} = \frac{1}{D_{Ht}} \gamma a_t \left[ n_t - k(1 - (1-\delta) n_{t-1}) \theta_t \right], \tag{B17}$$

$$\frac{P_t^{I*}}{P_{Ft}} a_t^* = \chi^* (1-\gamma)^{-\varsigma} (c_t^*)^{\sigma-\varsigma} (c_{Ft}^*)^{\varsigma} (n_t^*)^{\varphi^*} + \frac{1}{1-\zeta^*} \frac{P_t^{I*}}{P_{Ft}^*} \frac{k^* a_t^*}{q^*(\theta_t^*)}$$

$$- (1-\delta^*) \mathbb{E}_t \beta \frac{(c_t^*)^{\sigma-\varsigma}(c_{Ft}^*)^{\varsigma}}{(c_{t+1}^*)^{\sigma-\varsigma}(c_{Ft+1}^*)^{\varsigma}} \left(1 + \frac{\zeta^*}{1-\zeta^*}(1 - p^*(\theta_{t+1}^*))\right) \frac{P_{t+1}^{I*}}{P_{Ft+1}^*} \frac{k^* a_{t+1}^*}{q^*(\theta_{t+1}^*)}, \tag{B18}$$

$$n^* = (1-\delta^*) n_{t-1}^* + p^*(\theta_t^*)(1 - (1-\delta^*) n_{t-1}^*), \tag{B19}$$

$$c_{Ft}^* = \frac{1}{D_{Ft}} (1-\gamma) a_t^* \left[ n_t^* - k^*(1 - (1-\delta^*) n_{t-1}^*) \theta_t^* \right], \tag{B20}$$

$$s_t = \left( \frac{\gamma}{1-\gamma} \frac{c_{Ft}^*}{c_{Ht}} \right)^{\varsigma}, \tag{B21}$$

$$c_{Ft} = c_{Ft}^*, \tag{B22}$$

$$c_{Ht}^* = c_{Ht}, \tag{B23}$$

$$c_t = \left( (\gamma)^{\varsigma} (c_{Ht})^{1-\varsigma} + (1-\gamma)^{\varsigma} (c_{Ft})^{1-\varsigma} \right)^{\frac{1}{1-\varsigma}}, \tag{B24}$$

$$c_t^* = \left( (\gamma)^{\varsigma} (c_{Ht}^*)^{1-\varsigma} + (1-\gamma)^{\varsigma} (c_{Ft}^*)^{1-\varsigma} \right)^{\frac{1}{1-\varsigma}}, \tag{B25}$$

$$\mathbb{E}_t \sum_{s=t}^{\infty} (1-\iota)^{s-t} \beta^{s-t} \frac{(c_t)^{\sigma-\varsigma}(c_{Ht})^{\varsigma}}{(c_s)^{\sigma-\varsigma}(c_{Hs})^{\varsigma}} \left[ \left( \frac{\mathcal{P}_{Ht}}{P_{Hs}} \right)^{-\varepsilon} (\gamma c_{Hs} + (1-\gamma) c_{Hs}^*) \right] \times$$

$$\left( \frac{\mathcal{P}_{Ht}}{P_{Hs}} - \frac{\varepsilon}{\varepsilon - 1} (1 + \tau^r) \frac{P_s^I}{P_{Hs}} \right) = 0, \tag{B26}$$

$$(P_{Ht})^{1-\varepsilon} = (1-\iota)(P_{Ht-1})^{1-\varepsilon} + \iota (\mathcal{P}_{Ht})^{1-\varepsilon}, \tag{B27}$$

$$D_{Ht} = (1-\iota) D_{Ht-1} \left(\frac{P_{Ht-1}}{P_{Ht}}\right)^{-\varepsilon} + \iota \left(\frac{\mathcal{P}_{Ht}}{P_{Ht}}\right)^{-\varepsilon}, \tag{B28}$$

$$\mathbb{E}_t \sum_{s=t}^{\infty} (1-\iota^*)^{s-t} \beta^{s-t} \frac{(c_t^*)^{\sigma-\varsigma}(c_{Ft}^*)^{\varsigma}}{(c_s^*)^{\sigma-\varsigma}(c_{Fs}^*)^{\varsigma}} \left[\left(\frac{\mathcal{P}_{Ft}}{P_{Fs}}\right)^{-\varepsilon} (\gamma c_{Fs} + (1-\gamma) c_{Fs}^*)\right] \times$$

$$\left(\frac{\mathcal{P}_{Ft}}{P_{Fs}} - \frac{\varepsilon}{\varepsilon - 1}(1+\tau^{r*}) \frac{P_s^{I*}}{P_{Fs}}\right) = 0, \tag{B29}$$

$$(P_{Ft})^{1-\varepsilon} = (1-\iota^*)(P_{Ft-1})^{1-\varepsilon} + \iota^* (\mathcal{P}_{Ft})^{1-\varepsilon}, \tag{B30}$$

$$D_{Ft} = (1-\iota^*) D_{Ft-1} \left(\frac{P_{Ft-1}}{P_{Ft}}\right)^{-\varepsilon} + \iota^* \left(\frac{\mathcal{P}_{Ft}}{P_{Ft}}\right)^{-\varepsilon}, \tag{B31}$$

$$s_t = \frac{P_{Ht}}{P_{Ft}}. \tag{B32}$$

The monetary policy which implements such an allocation is simply given by

$$1 + i_t = \frac{1}{\mathbb{E}_t \beta \frac{P_t}{P_{t+1}} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}} = \frac{1}{\mathbb{E}_t \beta \frac{P_t}{P_{t+1}} \frac{c_{t+1}^{*-\sigma}}{c_t^{*-\sigma}}},$$

where it is straightforward to verify that second equality is satisfied at the given allocation using the equilibrium conditions above.

## B.5 Wage determination

Finally, I derive the Nash bargained wages used in the derivation of equilibrium. I focus on the problem at Home; the wage at Foreign can be derived analogously.

Following Shimer (2010), at Home let $\tilde{v}_{nt}(\hat{W})$ denote the marginal value to the representative household of employing an additional worker at wage $\hat{W}$ and the equilibrium wage $\{W_s\}_{s=t+1}^{\infty}$ thereafter, and let $\tilde{J}_{nt}(\hat{W})$ denote the marginal value to the representative producer of employing an additional worker at wage $\hat{W}$ and the equilibrium wage $\{W_s\}_{s=t+1}^{\infty}$ thereafter. Then Nash bargained wages solve

$$W_t = \arg\max_{\hat{W}} \left[\tilde{v}_{nt}(\hat{W})\right]^{\zeta} \left[\tilde{J}_{nt}(\hat{W})\right]^{1-\zeta}. \tag{B33}$$

To compute $\tilde{v}_{nt}(\hat{W})$, let $\hat{v}_t(B, n, W; \epsilon, \hat{W})$ be the value to the representative household of $n$ workers employed at wage $W$ and $\epsilon$ workers employed at wage $\hat{W}$ in period $t$. Then

$$\hat{v}_t(B, n, W; \epsilon, \hat{W}) = \max_{\{c_H(j)\}_j, \{c_F(j^*)\}_{j^*}, B_{+1}} u(c, n+\epsilon) + \beta \mathbb{E}_t v_{t+1}(B_{+1}, n+\epsilon) \text{ s.t.}$$

$$\int_0^1 P_{Ht}(j) c_H(j) dj + \int_0^1 P_{Ft}(j^*) c_F(j^*) dj^* + \mathbb{E}_t Q_{t,t+1} B_{+1} \leq W n + \hat{W}\epsilon + B - T_t,$$

so we have

$$\tilde{v}_{nt}(\hat{W}) \equiv \frac{\partial}{\partial \epsilon}\hat{v}_t(B_t, n_t, W_t; \epsilon, \hat{W})|_{\epsilon=0}. \tag{B34}$$

To compute $\tilde{J}_{nt}(\hat{W})$ at Home, let $\hat{J}_t(n, W; \epsilon, \hat{W})$ be the profit for the representative producer of $n$ workers employed at $W$ and $\epsilon$ workers employed at $\hat{W}$ after vacancy posting costs have been sunk. Then

$$\hat{J}_t(n, W; \epsilon, \hat{W}) = (P_t^I a_t - W)n + (P_t^I a_t - \hat{W})\epsilon + \mathbb{E}_t Q_{t,t+1} J_{t+1}(n+\epsilon),$$

so we have

$$\tilde{J}_{nt}(\hat{W}) \equiv \frac{\partial}{\partial \epsilon}\hat{J}_t(n_t, W_t; \epsilon, \hat{W})|_{\epsilon=0}. \tag{B35}$$

Now, evaluating the right-hand side of (B34) using the Envelope Theorem, the marginal value to the representative household of an additional worker employed at $\hat{W}$ is

$$\tilde{v}_{nt}(\hat{W}) = \frac{\hat{W}}{P_t}u_{ct} + u_{nt} + \mathbb{E}_t v_{nt+1}(B_{t+1}, n_t).$$

Evaluating the right-hand side of (B35), the marginal value to the representative producer of an additional worker employed at $\hat{W}$ is

$$\tilde{J}_{nt}(\hat{W}) = P_t^I a_t - \hat{W} + \mathbb{E}_t Q_{t,t+1} J_{nt+1}(n_t).$$

It follows that the maximization problem in (B33) yields

$$\hat{W} + P_t\frac{u_{nt}}{u_{ct}} + P_t\frac{1}{u_{ct}}\beta \mathbb{E}_t v_{nt+1}(B_{t+1}, n_t) = \frac{\zeta}{1-\zeta}\left(P_t^I a_t - \hat{W} + \mathbb{E}_t Q_{t,t+1} J_{nt+1}(n_t)\right).$$

Evaluating this at the equilibrium wage $\hat{W} = W_t$ yields

$$W_t + P_t\frac{u_{nt}}{u_{ct}} + P_t\frac{1}{u_{ct}}\beta \mathbb{E}_t v_{nt+1}(B_{t+1}, n_t) = \frac{\zeta}{1-\zeta}P_t^I\frac{ka_t}{q(\theta_t)} \tag{B36}$$

where I have used the representative producer's first-order condition with respect to vacancies $\nu_t$ on the right hand side. Then, since

$$\begin{aligned} v_{nt+1}(B_{t+1}, n_t) &= (1-\delta)(1-p(\theta_{t+1}))\frac{1}{P_{t+1}}u_{ct+1}\times \\ &\qquad \left[W_{t+1} + P_{t+1}\frac{u_{nt+1}}{u_{ct+1}} + P_{t+1}\frac{1}{u_{ct+1}}\beta \mathbb{E}_{t+1} v_{nt+2}(B_{t+2}, n_{t+1})\right], \\ &= (1-\delta)(1-p(\theta_{t+1}))\frac{1}{P_{t+1}}u_{ct+1}\frac{\zeta}{1-\zeta}P_{t+1}^I\frac{ka_{t+1}}{q(\theta_{t+1})} \end{aligned}$$

where I have used the Envelope Theorem in the first equality and (B36) at $t+1$ in the second, we can write (B36) as

$$W_t + P_t \frac{u_{nt}}{u_{ct}} + \mathbb{E}_t Q_{t,t+1}(1-\delta)(1-p(\theta_{t+1}))\frac{\zeta}{1-\zeta} P^I_{t+1}\frac{ka_{t+1}}{q(\theta_{t+1})} = \frac{\zeta}{1-\zeta} P^I_t \frac{ka_t}{q(\theta_t)}$$

where I have used $Q_{t,t+1} = \beta \frac{P_t u_{ct+1}}{P_{t+1} u_{ct}}$ by households' intertemporal optimization. Re-arranging yields the Nash bargained wage described in appendix B.2.

# C First-order conditions characterizing optimal policy

In this appendix I characterize the first-order conditions of maximizing the quadratic objective in Lemma 4 subject to the linear implementability constraints in Lemmas 1-3, implicitly defining the Ramsey optimal allocation. Letting $\psi_{\pi_H t}$, $\psi_{\mu t}$, $\psi_{\pi_F t}$, $\psi_{\mu^* t}$, and $\psi_{st}$ denote the multipliers on the constraints, we obtain:

$$\omega \frac{\epsilon}{\lambda} \pi_{Ht} + \psi_{\pi_H t} - \psi_{\pi_H t-1} - \psi_{st} = 0, \tag{C1}$$

$$-\lambda \psi_{\pi_H t} + \psi_{\mu t} + \frac{\epsilon^f_{n_{-1}}}{\epsilon^f_n} \psi_{\mu t-1} = 0, \tag{C2}$$

$$\begin{aligned}
&\omega \left[\phi \left(\epsilon^f_n\right)^2 \tilde{n}_t + \beta \phi_{-1} \epsilon^f_n \epsilon^f_{n_{-1}} \mathbb{E}_t \tilde{n}_{t+1} + \phi_{-1} \epsilon^f_n \epsilon^f_{n_{-1}} \tilde{n}_{t-1}\right] \\
&\quad + \omega(1-\omega)(\sigma - \varsigma)\left[\phi' \epsilon^f_n \epsilon^{f^*}_{n^*} \tilde{n}^*_t + \epsilon^f_n \epsilon^{f^*}_{n^*_{-1}} \tilde{n}^*_{t-1} + \beta \epsilon^f_{n_{-1}} \epsilon^{f^*}_{n^*} \mathbb{E}_t \tilde{n}^*_{t+1}\right] \\
&\quad - \phi \epsilon^f_n \psi_{\mu t} - \beta \phi_{-1} \epsilon^f_{n_{-1}} \mathbb{E}_t \psi_{\mu t+1} - \phi_{-1} \epsilon^f_{n_{-1}} \psi_{\mu t-1} \\
&\quad - \omega(\sigma - \varsigma)\left[\phi' \epsilon^f_n \psi_{\mu^* t} + \frac{\epsilon^{f^*}_{n^*_{-1}}}{\epsilon^{f^*}_{n^*}} \epsilon^f_n \psi_{\mu^* t-1} + \beta \epsilon^f_{n_{-1}} \mathbb{E}_t \psi_{\mu^* t+1}\right] \\
&\quad - \varsigma\left[\epsilon^f_n \psi_{st} + \beta\left(-\epsilon^f_n + \epsilon^f_{n_{-1}}\right) \mathbb{E}_t \psi_{st+1} - \beta^2 \epsilon^f_{n_{-1}} \mathbb{E}_t \psi_{st+2}\right] = 0,
\end{aligned} \tag{C3}$$

$$(1-\omega)\frac{\epsilon}{\lambda^*} \pi_{Ft} + \psi_{\pi_F t} - \psi_{\pi_F t-1} + \psi_{st} = 0, \tag{C4}$$

$$-\lambda^* \psi_{\pi_F t} + \psi_{\mu^* t} + \frac{\epsilon^{f^*}_{n^*_{-1}}}{\epsilon^{f^*}_{n^*}} \psi_{\mu^* t-1} = 0, \tag{C5}$$

$$\begin{aligned}
&(1-\omega)\left[\phi^* \left(\epsilon^{f^*}_{n^*}\right)^2 \tilde{n}^*_t + \beta \phi^*_{-1} \epsilon^{f^*}_{n^*} \epsilon^{f^*}_{n^*_{-1}} \mathbb{E}_t \tilde{n}^*_{t+1} + \phi^*_{-1} \epsilon^{f^*}_{n^*} \epsilon^{f^*}_{n^*_{-1}} \tilde{n}^*_{t-1}\right] \\
&\quad + \omega(1-\omega)(\sigma - \varsigma)\left[\phi' \epsilon^f_n \epsilon^{f^*}_{n^*} \tilde{n}_t + \epsilon^f_{n_{-1}} \epsilon^{f^*}_{n^*} \tilde{n}_{t-1} + \beta \epsilon^f_n \epsilon^{f^*}_{n^*_{-1}} \mathbb{E}_t \tilde{n}_{t+1}\right] \\
&\quad - \phi^* \epsilon^{f^*}_{n^*} \psi_{\mu^* t} - \beta \phi^*_{-1} \epsilon^{f^*}_{n^*_{-1}} \mathbb{E}_t \psi_{\mu^* t+1} - \phi^*_{-1} \epsilon^{f^*}_{n^*_{-1}} \psi_{\mu^* t-1} \\
&\quad - (1-\omega)(\sigma - \varsigma)\left[\phi' \epsilon^{f^*}_{n^*} \psi_{\mu t} + \frac{\epsilon^f_{n_{-1}}}{\epsilon^f_n} \epsilon^{f^*}_{n^*} \psi_{\mu t-1} + \beta \epsilon^{f^*}_{n^*_{-1}} \mathbb{E}_t \psi_{\mu t+1}\right]
\end{aligned}$$

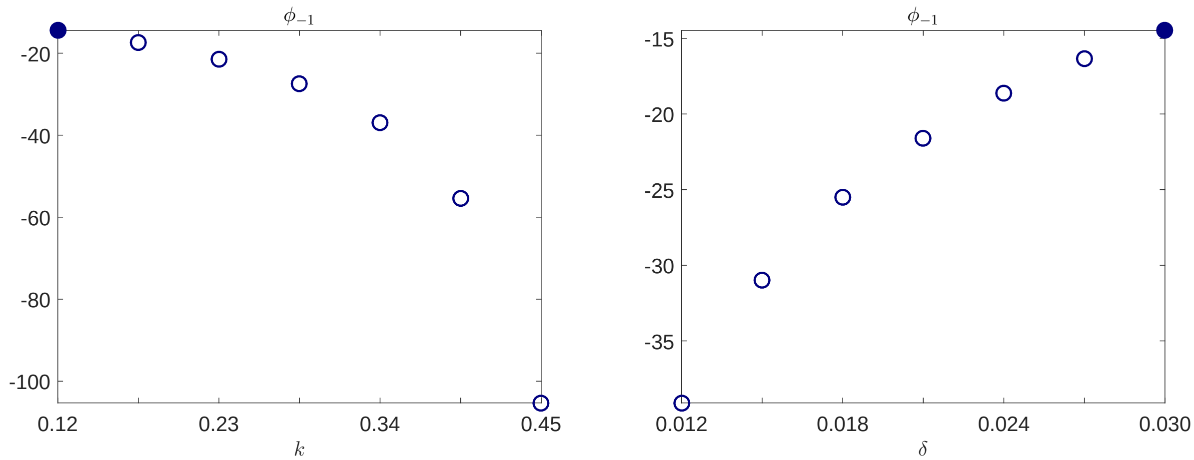

Figure 1: $\phi_{-1}$ as Home's labor market becomes more sclerotic

Note: $\phi_{-1}$ controls the welfare costs from and sensitivity of real marginal cost to lagged/future employment fluctuations in Home. Its formula is given in Lemma 2. A more sclerotic labor market is one with higher $k$ or lower $\delta$, and thus involves moving from the ends to the center of the figure.

$$+ \varsigma \left[ \epsilon^{f^*}_{n^*} \psi_{st} + \beta \left( -\epsilon^{f^*}_{n^*} + \epsilon^{f^*}_{n^*_{-1}} \right) \mathbb{E}_t \psi_{st+1} - \beta^2 \epsilon^{f^*}_{n^*_{-1}} \mathbb{E}_t \psi_{st+2} \right] = 0, \tag{C6}$$

where $\psi_{\pi_H -1} = \psi_{\mu -1} = \psi_{\pi_F -1} = \psi_{\mu^* -1} = 0$.

# D Supplemental numerical results

In this appendix I provide supplemental numerical results for the exploration of the optimal monetary policy in section III.C of the main text.

## D.1 Additional comparative statics

I first provide additional comparative statics of interest as Home's labor market grows more sclerotic, accompanying Figure 2 in the main text.

Figure 1 provides the comparative statics for $\phi_{-1}$. We see that a more sclerotic labor market features a more negative $\phi_{-1}$. Provided that employment is persistent, this offsets the amplified $\phi$ in Figure 2 in determining the welfare losses and effects on real marginal cost from employment fluctuations.

Figure 2 provides the comparative statics for the employment rates in each country $n$ and $n^*$, as well as the global expenditure share on Home-produced goods $\omega$. As is standard, higher $k$ and lower $\delta$ feature offsetting effects on $n$, so that a more sclerotic labor market need not have higher steady-state unemployment.[1] Because $\sigma > \varsigma$ in this parameterization, the

[1] Quantitatively, I also note that the variation in employment is well within the variation observed in

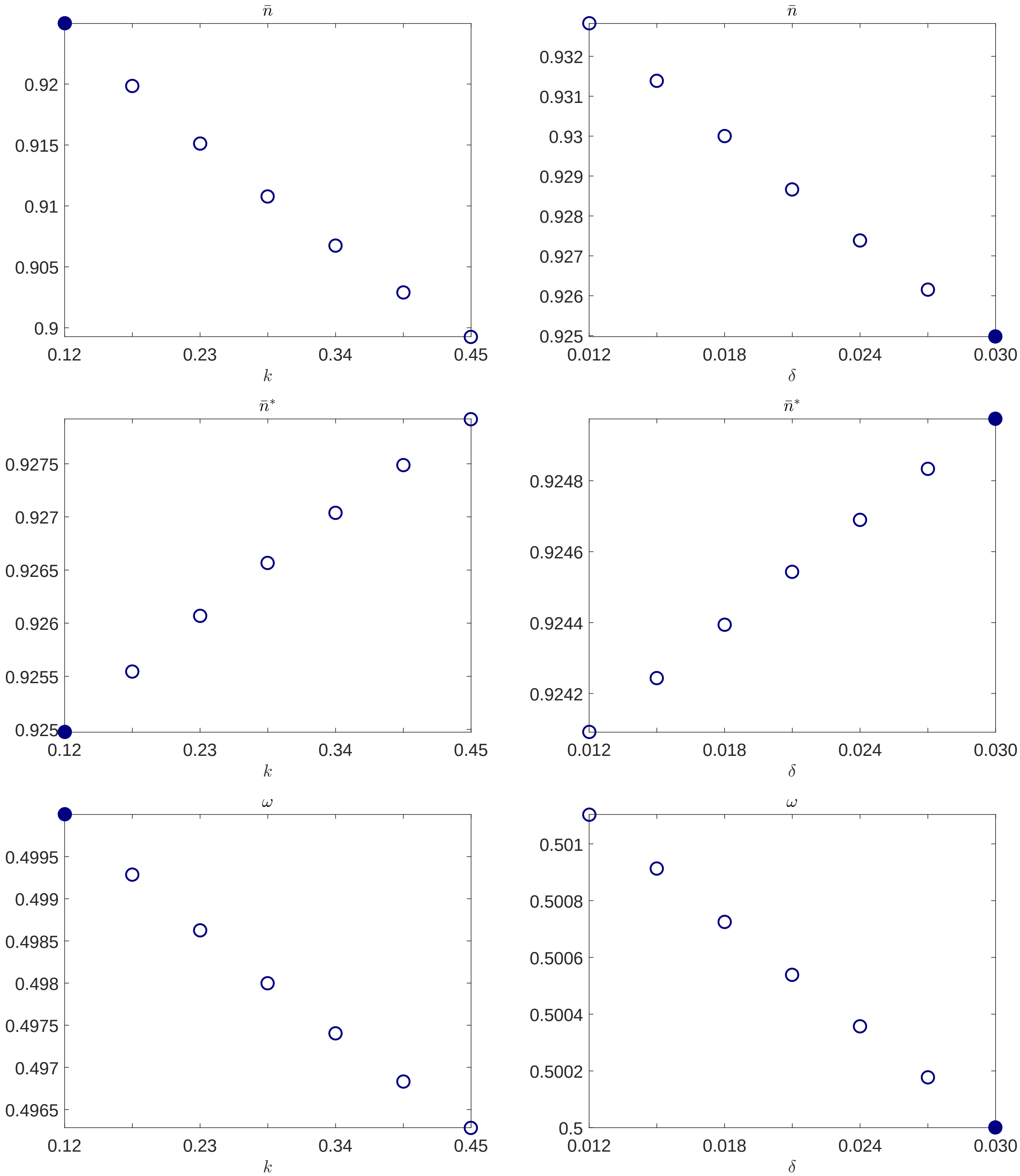


Figure 2: $\bar{n}$, $\bar{n}^*$, and $\omega$ as Home's labor market becomes more sclerotic

Note: $\omega$ is the global expenditure share on Home-produced goods. A more sclerotic labor market is one with higher $k$ or lower $\delta$, and thus involves moving from the ends to the center of the figure.

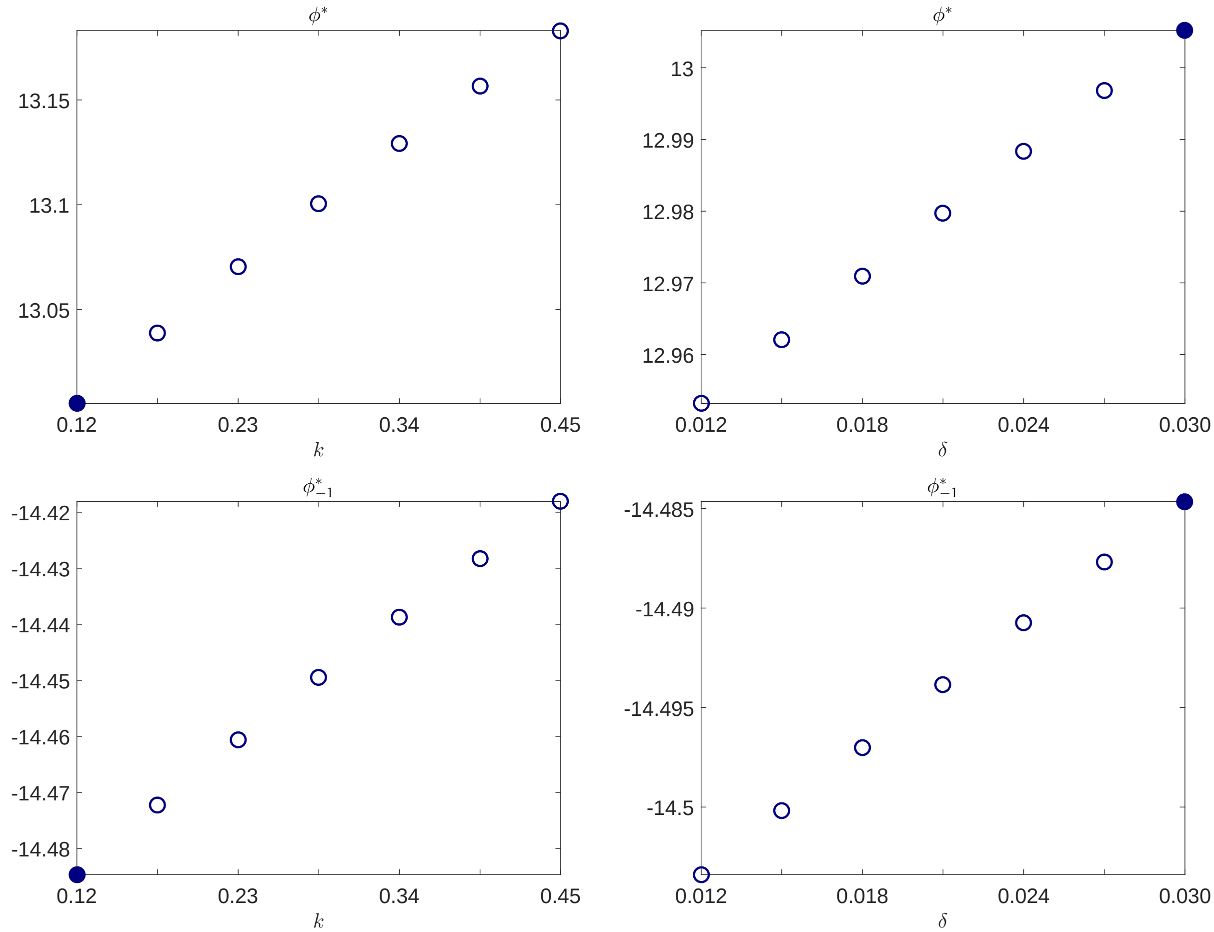


Figure 3: $\phi^*$ and $\phi^*_{-1}$ as Home's labor market becomes more sclerotic

Note: $\phi^*$ and $\phi^*_{-1}$ control the welfare costs from and sensitivity of real marginal cost to employment fluctuations in Foreign. Their formulas are given in the proof of Lemma 2. A more sclerotic labor market is one with higher $k$ or lower $\delta$, and thus involves moving from the ends to the center of the figure.

risk-sharing effects of changes in foreign employment dominate their terms of trade effects, so that greater employment at Home raises marginal cost and thus lowers employment in Foreign. It follows that greater employment at Home is associated with greater relative production of Home-produced goods. Since $\varsigma < 1$ in this parameterization, this raises the global expenditure share on Home-produced goods $\omega$.

Figure 3 then provides the comparative statics for $\phi^*$ and $\phi^*_{-1}$. Higher employment in Home is associated with lower $\phi^*$ and $\phi^*_{-1}$ because $\sigma > \varsigma$ and $\omega$ is rising in Home employment. However, we see that these spillovers are modest in size.

Eurozone economies summarized in Table 1.

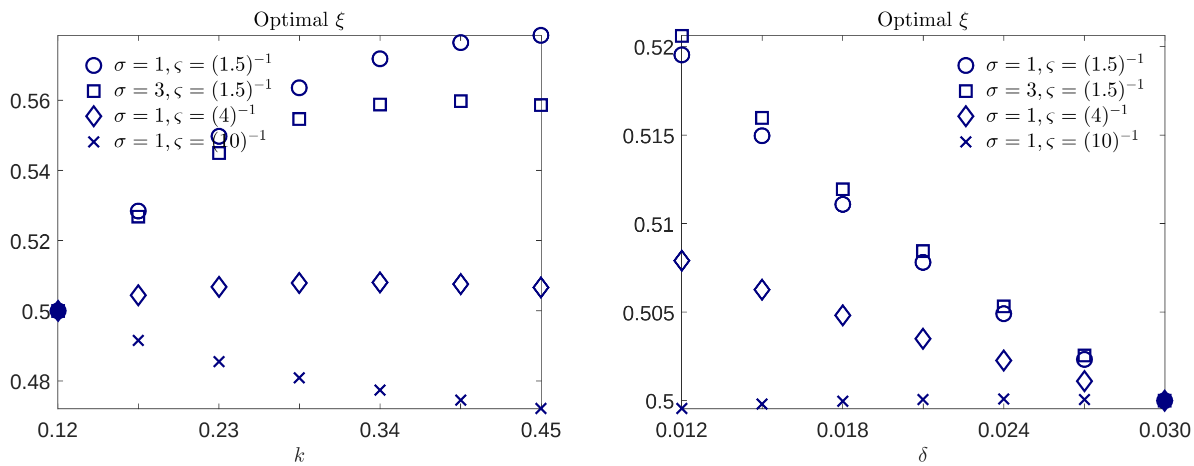


Figure 4: optimal $\xi$ as Home's labor market becomes more sclerotic

Note: the optimal $\xi$ is computed by minimizing the average welfare loss across many histories of shocks. Shaded markers depict the symmetric benchmark in which $k = k^* = 0.12$ and $\delta = \delta^* = 0.030$. A more sclerotic labor market is one with higher $k$ or lower $\delta$, thus moving from the ends to the center of the figure.

## D.2 Alternative values of $\sigma$ or $\varsigma$

I now characterize how the relationship between labor market frictions and the optimal policy varies with the coefficient of relative risk aversion $\sigma$ and inverse trade elasticity $\varsigma$. I continue to focus on the optimal weight on Home producer-price inflation $\xi$ in the rule (33).

Recall that in the parameterization in the main text, I set $\sigma = 1$ and $\varsigma = (1.5)^{-1}$. The former is consistent with balanced growth. The latter is consistent with Backus, Kehoe, and Kydland (1994) and used extensively in the international macro literature.

For alternative parameterizations still within the typical range of the literature, I continue to find that the optimal $\xi$ rises as Home's labor market grows more sclerotic. Figure 4 reproduces this result for the baseline parameterization and demonstrates that the qualitative patterns remain the same for $\sigma = 3$ or $\varsigma = (4)^{-1}$. The former is consistent with some of the lowest estimates of the intertemporal elasticity of substitution in the literature (see, for instance, the survey in Hall (2009)). The latter is consistent with trade elasticity estimates in the international trade literature (see, for instance, the survey in Costinot and Rodriguez-Clare (2014)), which typically estimate a trade elasticity higher than that estimated in the international macro literature.

However, for sufficiently low $\varsigma$ I find that the optimal $\xi$ falls as Home's labor market grows more sclerotic, consistent with Proposition 8. For instance, for $\varsigma = (10)^{-1}$, Figure 4 demonstrates that the optimal $\xi$ falls as Home's labor market grows more sclerotic. Figure 5 depicts the impulse responses under this optimal rule when $\varsigma = (10)^{-1}$. As is evident, when Home's labor market is more sclerotic the optimal policy now accommodates larger relative inflation/deflation at Home. Nonetheless, even in this case the optimal policy continues to target smaller output and employment distortions at Home than in Foreign, consistent with the discussion of this result in the main text.

# E Real wage rigidity

In this appendix I demonstrate that the paper's result on relative accommodation of the more sclerotic union member is robust to real wage rigidity.

## E.1 Environment and equilibrium revisited

The environment is exactly as in section I, except product wages are given by

$$\frac{W_t}{P_{Ht}} = (1-\alpha)w + \alpha \frac{W_t^{nb}}{P_{Ht}},$$
$$\frac{W_t^*}{P_{Ft}} = (1-\alpha^*)w^* + \alpha^* \frac{W_t^{nb*}}{P_{Ft}},$$

where $W_t^{nb}$ and $W_t^{nb*}$ are the wages which would be obtained under Nash bargaining and $w$ and $w^*$ are the product wages in the deterministic steady-state.

The Nash bargained nominal wage $W_t^{nb}$ derived in appendix B.5 is

$$\begin{aligned} W_t^{nb} &= P_{Ht}\chi\gamma^{-\varsigma}c_t^{\sigma-\varsigma}c_{Ht}^{\varsigma}n_t^{\varphi} \\ &\quad + \frac{\zeta}{1-\zeta}\left[P_t^I \frac{ka_t}{q(\theta_t)} - \mathbb{E}_t Q_{t,t+1}(1-\delta)(1-p(\theta_{t+1}))P_{t+1}^I \frac{ka_{t+1}}{q(\theta_{t+1})}\right] \end{aligned}$$

Combined with firms' optimal vacancy posting, labor market equilibrium is thus characterized by

$$\begin{aligned} P_t^I a_t &= P_{Ht}\chi\gamma^{-\varsigma}c_t^{\sigma-\varsigma}c_{Ht}^{\varsigma}n_t^{\varphi} + \frac{1}{1-\zeta}P_t^I \frac{ka_t}{q(\theta_t)} \\ &\quad - (1-\delta)\mathbb{E}_t Q_{t,t+1}\left(1 + \frac{\zeta}{1-\zeta}(1-p(\theta_{t+1}))\right)P_{t+1}^I \frac{ka_{t+1}}{q(\theta_{t+1})} + (W_t - W_t^{nb}). \end{aligned}$$

Given the form of the equilibrium wage $W_t$, it follows that

$$\begin{aligned} P_t^I a_t &= P_{Ht}\chi\gamma^{-\varsigma}c_t^{\sigma-\varsigma}c_{Ht}^{\varsigma}n_t^{\varphi} + \frac{1}{1-\zeta}P_t^I \frac{ka_t}{q(\theta_t)} \\ &\quad - (1-\delta)\mathbb{E}_t Q_{t,t+1}\left(1 + \frac{\zeta}{1-\zeta}(1-p(\theta_{t+1}))\right)P_{t+1}^I \frac{ka_{t+1}}{q(\theta_{t+1})} - (1-\alpha)\left(W_t^{nb} - wP_{Ht}\right). \end{aligned}$$

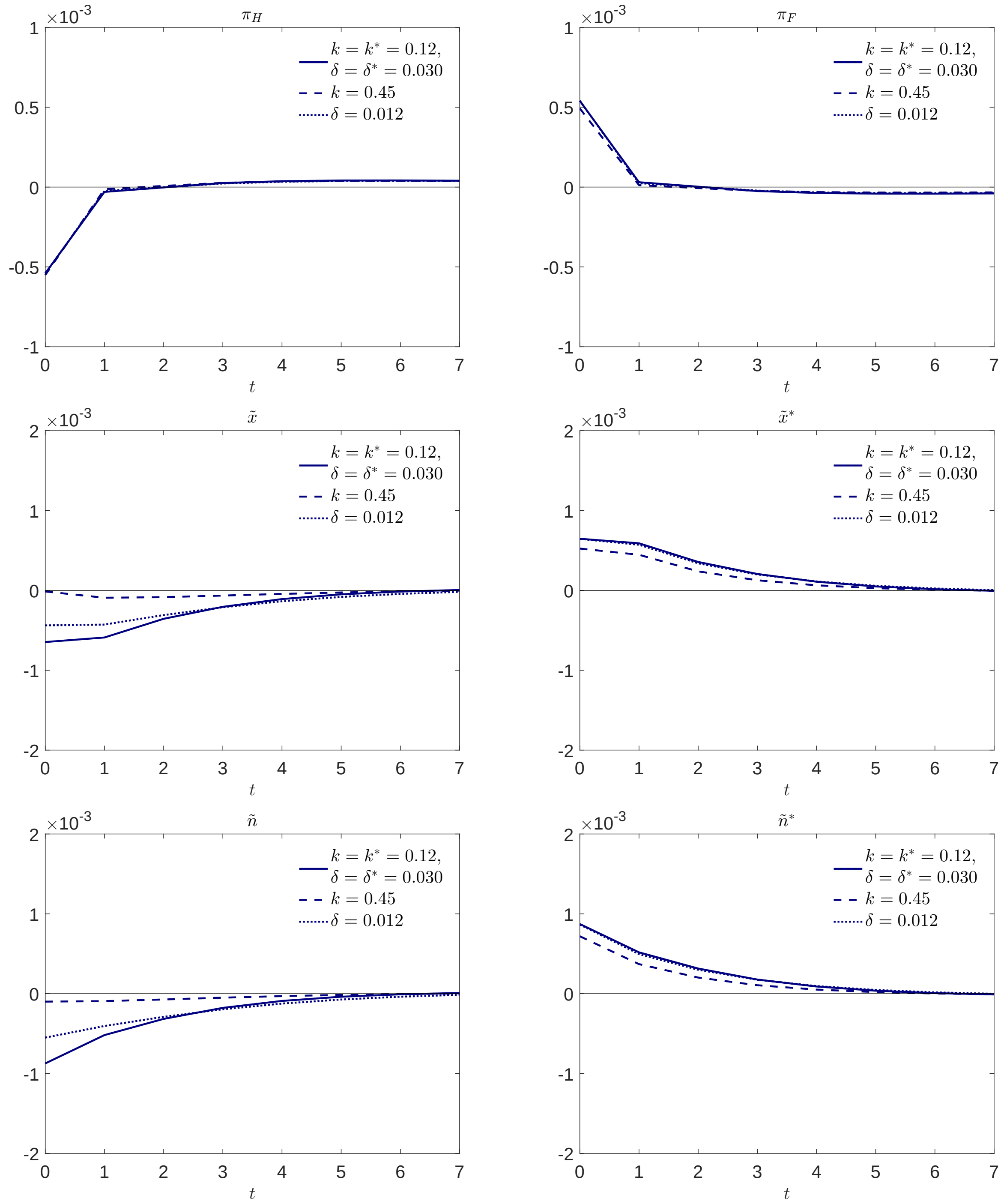


Figure 5: response to depreciation in Home's natural terms of trade (optimal $\xi$, $\varsigma = (10)^{-1}$)

Note: depreciation in Home's natural terms of trade induced by 1% positive innovation to $a_t$ at $t = 0$.

Analogously, in Foreign, we have

$$
\begin{aligned}
W_t^{*nb} &= P_{Ft}^* \chi^* (1-\gamma)^{-\varsigma} c_t^{*\sigma-\varsigma} c_{Ft}^{*\varsigma} n_t^{*\varphi^*} \\
&\quad + \frac{\zeta^*}{1-\zeta^*} \left[ P_t^{I*} \frac{k^* a_t^*}{q^*(\theta_t^*)} - \mathbb{E}_t Q_{t,t+1} (1-\delta^*)(1-p^*(\theta_{t+1}^*)) P_{t+1}^{I*} \frac{k^* a_{t+1}^*}{q^*(\theta_{t+1}^*)} \right], \\
P_t^{I*} a_t^* &= P_{Ft}^* \chi^* (1-\gamma)^{-\varsigma} c_t^{*\sigma-\varsigma} c_{Ft}^{*\varsigma} n_t^{*\varphi^*} + \frac{1}{1-\zeta^*} P_t^{I*} \frac{k^* a_t^*}{q^*(\theta_t^*)} \\
&\quad - (1-\delta^*) \mathbb{E}_t Q_{t,t+1} \left( 1 + \frac{\zeta^*}{1-\zeta^*} (1-p^*(\theta_{t+1}^*)) \right) P_{t+1}^{I*} \frac{k^* a_{t+1}^*}{q^*(\theta_{t+1}^*)} - (1-\alpha^*) \left( W_t^{*nb} - w^* P_{Ft} \right).
\end{aligned}
$$

The remainder of the equilibrium is as characterized in appendix B.

## E.2 Stabilization trade-offs and optimal policy

We study the stabilization problem in this environment again employing a linear-quadratic approximation.

We replace Assumption 2 with:

**Assumption E.1.** *A retailer subsidy offsets the distortions from monopolistic competition ($\tau^r = \tau^r = -\frac{1}{\epsilon}$) and the Hosios condition is satisfied in each country ($\zeta = 1 - \eta$ and $\zeta^* = 1 - \eta^*$).*

This ensures that in steady-state, the natural allocation (which has flexible prices but rigid real wages) remains constrained efficient as in the main text. However, this allocation will now be generically inefficient in response to shocks because of the rigidity in real wages. For that reason, deviations from the natural allocation will no longer be the appropriate measure of inefficiency in this economy. For any variable $z_t$, we thus redefine the tilde notation to be

$$\tilde{z}_t \equiv \log z_t - \log z_t^{ce}$$

and further define

$$\hat{z}_t^{ce} \equiv \log z_t^{ce} - \log z,$$

where the *ce* superscript denotes the value of that variable in the allocation with flexible prices, no rigidity in real wages, and assumption E.1, which is constrained efficient.

We continue to make Assumption 3 as in the main text. In the results which follow, it will be useful to refer to the product wages $w_t \equiv \frac{W_t}{P_{Ht}}$ and $w_t^* \equiv \frac{W_t^*}{P_{Ft}}$ and Nash bargained product wages $w_t^{nb} \equiv \frac{W_t^{nb}}{P_{Ht}}$ and $w_t^{*nb} \equiv \frac{W_t^{*nb}}{P_{Ft}}$.

### E.2.1 Implementability constraints

Lemma 1 characterizing the New Keynesian Phillips curves is unchanged.

Lemma 2 must now be generalized to account for real wage rigidity in the labor market:

**Lemma E.1.** *Up to first order around the deterministic steady-state,*

$$\begin{aligned}
\tilde{\mu}_t + \frac{\epsilon^f_{n_{-1}}}{\epsilon^f_n}\beta\mathbb{E}_t\tilde{\mu}_{t+1} &= \phi\epsilon^f_n\tilde{n}_t + \phi_{-1}\epsilon^f_{n_{-1}}\tilde{n}_{t-1} + \phi_{-1}\epsilon^f_{n_{-1}}\beta\mathbb{E}_t\tilde{n}_{t+1} \\
&\quad + (\sigma-\varsigma)(1-\omega)\left[\phi'\epsilon^{f^*}_{n^*}\tilde{n}^*_t + \epsilon^{f^*}_{n^*_{-1}}\tilde{n}^*_{t-1} + \frac{\epsilon^f_{n_{-1}}}{\epsilon^f_n}\epsilon^{f^*}_{n^*}\beta\mathbb{E}_t\tilde{n}^*_{t+1}\right] \\
&\quad - (1-\alpha)\frac{wn}{af(n,n)}\frac{1}{\epsilon^f_n}\hat{w}^{nb}_t, \\
\tilde{\mu}^*_t + \frac{\epsilon^{f^*}_{n^*_{-1}}}{\epsilon^{f^*}_{n^*}}\beta\mathbb{E}_t\tilde{\mu}^*_{t+1} &= \phi^*\epsilon^{f^*}_{n^*}\tilde{n}^*_t + \phi^*_{-1}\epsilon^{f^*}_{n^*_{-1}}\tilde{n}^*_{t-1} + \phi^*_{-1}\epsilon^{f^*}_{n^*_{-1}}\beta\mathbb{E}_t\tilde{n}^*_{t+1} \\
&\quad + (\sigma-\varsigma)\,\omega\left[\phi'\epsilon^f_n\tilde{n}_t + \epsilon^f_{n_{-1}}\tilde{n}_{t-1} + \frac{\epsilon^{f^*}_{n^*_{-1}}}{\epsilon^{f^*}_{n^*}}\epsilon^f_n\beta\mathbb{E}_t\tilde{n}_{t+1}\right] \\
&\quad - (1-\alpha^*)\frac{w^*n^*}{a^*f^*(n^*,n^*)}\frac{1}{\epsilon^{f^*}_{n^*}}\hat{w}^{*nb}_t,
\end{aligned}$$

*where $\phi$, $\phi_{-1}$, $\phi^*$, $\phi^*_{-1}$, and $\phi'$ are as defined in Lemma 2.*

There are two differences relative to Lemma 2. First, we express these dynamics in terms of employment distortions relative to the constrained efficient allocation, rather than the natural allocation. Second, real wage rigidity together with adjustment in the Nash bargained wages mean that generically there must be inefficiency in employment and/or distortions in real marginal cost, which in turn drives inflation according to Lemma 1.

We now characterize the adjustment in Nash bargained product wages:

**Lemma E.2.** *Up to first order around the deterministic steady-state,*

$$\begin{aligned}
\hat{w}^{nb}_t &= \hat{w}^{ce}_t + \epsilon^w_\mu\tilde{\mu}_t + \epsilon^w_{\mu_{+1}}\beta\mathbb{E}_t\tilde{\mu}_{t+1} + \epsilon^w_n\epsilon^f_n\tilde{n}_t + \epsilon^w_{n_{-1}}\epsilon^f_{n_{-1}}\tilde{n}_{t-1} + \epsilon^w_{n_{+1}}\epsilon^f_n\beta\mathbb{E}_t\tilde{n}_{t+1} \\
&\quad + \epsilon^w_{n^*}\epsilon^{f^*}_{n^*}\tilde{n}^*_t + \epsilon^w_{n^*_{-1}}\epsilon^{f^*}_{n^*_{-1}}\tilde{n}^*_{t-1} + \epsilon^w_{n^*_{+1}}\epsilon^{f^*}_{n^*}\beta\mathbb{E}_t\tilde{n}^*_{t+1}, \\
\hat{w}^{*nb}_t &= \hat{w}^{*ce}_t + \epsilon^{w^*}_{\mu^*}\tilde{\mu}^*_t + \epsilon^{w^*}_{\mu^*_{+1}}\beta\mathbb{E}_t\tilde{\mu}^*_{t+1} + \epsilon^{w^*}_{n^*}\epsilon^{f^*}_{n^*}\tilde{n}^*_t + \epsilon^{w^*}_{n^*_{-1}}\epsilon^{f^*}_{n^*_{-1}}\tilde{n}^*_{t-1} + \epsilon^{w^*}_{n^*_{+1}}\epsilon^{f^*}_{n^*}\beta\mathbb{E}_t\tilde{n}^*_{t+1} \\
&\quad + \epsilon^{w^*}_n\epsilon^f_n\tilde{n}_t + \epsilon^{w^*}_{n_{-1}}\epsilon^f_{n_{-1}}\tilde{n}_{t-1} + \epsilon^{w^*}_{n_{+1}}\epsilon^f_n\beta\mathbb{E}_t\tilde{n}_{t+1},
\end{aligned}$$

*where*

$$\epsilon^w_\mu = (1-\Phi)\frac{1}{1-(1-\delta)\beta(1-p(\theta))},$$

$$
\begin{aligned}
\epsilon^w_{\mu_{+1}} &= -(1-\Phi)\frac{(1-\delta)(1-p(\theta))}{1-(1-\delta)\beta(1-p(\theta))},\\
\epsilon^w_n &= \Phi\left(\sigma + \frac{\varphi}{\epsilon^f_n} - (\sigma-\varsigma)(1-\omega)\right)\\
&\quad + (1-\Phi)\Bigg(-\frac{1}{1-(1-\delta)\beta(1-p(\theta))}\frac{f_n}{1-f_n}\frac{\epsilon^{f_n}_n}{\epsilon^f_n}\\
&\quad - \frac{(1-\delta)\beta(1-p(\theta))}{1-(1-\delta)\beta(1-p(\theta))}\Bigg((\sigma-(\sigma-\varsigma)(1-\omega))\left(1-\frac{\epsilon^f_{n_{-1}}}{\epsilon^f_n}\right)\\
&\quad + \frac{f_{n_{-1}}}{f_{n_{-1}}-(1-\delta)\eta(1-f_n)}\frac{\epsilon^{f_{n_{-1}}}_{n_{-1}}}{\epsilon^f_n} + \frac{(1-\delta)\eta f_n}{f_{n_{-1}}-(1-\delta)\eta(1-f_n)}\frac{\epsilon^{f_n}_{n_{-1}}}{\epsilon^f_n}\Bigg)\Bigg),\\
\epsilon^w_{n_{-1}} &= \Phi\left(\sigma-(\sigma-\varsigma)(1-\omega)\right)\\
&\quad + (1-\Phi)\Bigg(-\frac{1}{1-(1-\delta)\beta(1-p(\theta))}\frac{f_n}{1-f_n}\frac{\epsilon^{f_n}_{n_{-1}}}{\epsilon^f_{n_{-1}}}\\
&\quad - \frac{(1-\delta)\beta(1-p(\theta))}{1-(1-\delta)\beta(1-p(\theta))}(\sigma-(\sigma-\varsigma)(1-\omega))\Bigg),\\
\epsilon^w_{n_{+1}} &= (1-\Phi)\frac{(1-\delta)(1-p(\theta))}{1-(1-\delta)\beta(1-p(\theta))}\Bigg(\sigma-(\sigma-\varsigma)(1-\omega)\\
&\quad - \frac{f_{n_{-1}}}{f_{n_{-1}}-(1-\delta)\eta(1-f_n)}\frac{\epsilon^{f_{n_{-1}}}_n}{\epsilon^f_n} - \frac{(1-\delta)\eta f_n}{f_{n_{-1}}-(1-\delta)\eta(1-f_n)}\frac{\epsilon^{f_n}_n}{\epsilon^f_n}\Bigg),\\
\epsilon^w_{n^*} &= (\sigma-\varsigma)(1-\omega)\left(\Phi-(1-\Phi)\frac{(1-\delta)\beta(1-p(\theta))}{1-(1-\delta)\beta(1-p(\theta))}\left(1-\frac{\epsilon^{f^*}_{n^*_{-1}}}{\epsilon^{f^*}_{n^*}}\right)\right),\\
\epsilon^w_{n^*_{-1}} &= (\sigma-\varsigma)(1-\omega)\left(\Phi-(1-\Phi)\frac{(1-\delta)\beta(1-p(\theta))}{1-(1-\delta)\beta(1-p(\theta))}\right),\\
\epsilon^w_{n^*_{+1}} &= (\sigma-\varsigma)(1-\omega)(1-\Phi)\frac{(1-\delta)(1-p(\theta))}{1-(1-\delta)\beta(1-p(\theta))},
\end{aligned}
$$

*where* $\Phi \equiv \frac{\chi\gamma^{-\varsigma}c^{\sigma-\varsigma}c_H^{\varsigma}n^{\varphi}}{w}$ *is the steady-state ratio of the marginal rate of substitution between consumption and labor and the product wage, with analogous expressions for* $\epsilon^{w^*}_{\cdot}$.

Absent inefficiency in real marginal cost and employment, the Nash bargained product wage is given by the constrained efficient product wage. Inefficiency in real marginal cost and employment change the Nash bargained product wage based on their impact on the marginal rate of substitution between consumption and labor and on present and future hiring costs.

Finally, Lemma 3 is replaced by

**Lemma E.3.** *Up to first order around the deterministic steady-state,*

$$\hat{s}_t^{ce} - \hat{s}_{t-1}^{ce} = \left(\pi_{Ht} + \varsigma\left[\epsilon_n^f \tilde{n}_t + \epsilon_{n_{-1}}^f \tilde{n}_{t-1} - \epsilon_n^f \tilde{n}_{t-1} - \epsilon_{n_{-1}}^f \tilde{n}_{t-2}\right]\right) - \left(\pi_{Ft} + \varsigma\left[\epsilon_{n^*}^{f^*} \tilde{n}_t^* + \epsilon_{n_{-1}^*}^{f^*} \tilde{n}_{t-1}^* - \epsilon_{n^*}^{f^*} \tilde{n}_{t-1}^* - \epsilon_{n_{-1}^*}^{f^*} \tilde{n}_{t-2}^*\right]\right).$$

We now have on the left-hand side the constrained efficient terms of trade and on the right-hand side employment distortions relative to the constrained efficient allocation.

### E.2.2 Social welfare

Lemma 4 is replaced by

**Lemma E.4.** *Up to second order around the deterministic steady-state,*

$$\begin{aligned} U_0 - U = -\frac{C^{1-\sigma}}{2}\sum_{t=0}^{\infty}\mathbb{E}_0\beta^t\Bigg[&\omega\left(\frac{\varepsilon}{\lambda}(\pi_{Ht})^2 + \phi\left(\epsilon_n^f\right)^2(\tilde{n}_t)^2 + 2\beta\phi_{-1}\epsilon_n^f\epsilon_{n_{-1}}^f\tilde{n}_t\tilde{n}_{t+1}\right) \\ &+ (1-\omega)\left(\frac{\varepsilon}{\lambda^*}(\pi_{Ft})^2 + \phi^*\left(\epsilon_{n^*}^{f^*}\right)^2(\tilde{n}_t^*)^2 + 2\beta\phi_{-1}^*\epsilon_{n^*}^{f^*}\epsilon_{n_{-1}^*}^{f^*}\tilde{n}_t^*\tilde{n}_{t+1}^*\right) \\ &+ 2\omega(1-\omega)(\sigma-\varsigma)\left(\phi'\epsilon_n^f\epsilon_{n^*}^{f^*}\tilde{n}_t\tilde{n}_t^* + \epsilon_n^f\epsilon_{n_{-1}^*}^{f^*}\tilde{n}_t\tilde{n}_{t-1}^* + \beta\epsilon_{n_{-1}}^f\epsilon_{n^*}^{f^*}\tilde{n}_t\mathbb{E}_t\tilde{n}_{t+1}^*\right)\Bigg] + tips \end{aligned}$$

*where tips denotes terms independent of policy.*

This is the same as Lemma 4 except again inefficiency in employment is captured by its deviation from the constrained efficient rather than natural allocation.

### E.2.3 Optimal policy

We now consider the problem of maximizing the welfare objective in Lemma E.4 subject to the implementability constraints in Lemmas 1 and E.1-E.3. As is evident, even if the constrained efficient terms of trade are constant, there will be distortions in this environment if the constrained efficient product wages adjust. And because of the monetary union, it will generally be the case that the optimal policy will call for distortions in both countries even if only the constrained efficient product wage in one country needs to adjust.

The first-order conditions characterizing the optimal policy generalize (C1)-(C6) in appendix C. Here we numerically explore the optimal policy by introducing real wage rigidity into the analysis of section III.G. We assume the same parameters as in the numerical analysis of that section except for $\alpha = \alpha^* < 1$. We obtain two main insights.

First, the result on relative accommodation of the more sclerotic labor market remains robust to symmetric real wage rigidity across the union. We continue to focus on the optimal

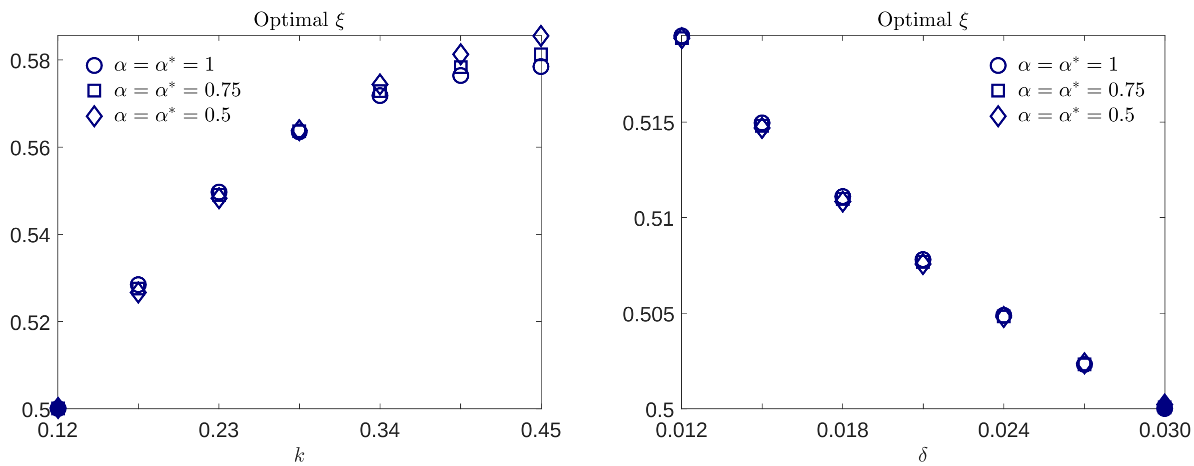


Figure 6: optimal $\xi$ as Home's labor market becomes more sclerotic

Note: the optimal $\xi$ is computed by minimizing the average welfare loss across many histories of shocks. Shaded markers depict the symmetric benchmark in which $k = k^* = 0.12$ and $\delta = \delta^* = 0.030$. A more sclerotic labor market is one with higher $k$ or lower $\delta$, thus moving from the ends to the center of the figure.

inflation targeting rule of the form (33). In Figure 6, we compare the optimal weight on Home producer-price inflation $\xi$ absent real wage rigidity ($\alpha = \alpha^* = 1$), reproduced from Figure 3, with that under higher degrees of real wage rigidity ($\alpha = \alpha^* = 0.75$ and $\alpha = \alpha^* = 0.5$). It remains the case that more weight should be placed on Home as its labor market grows more sclerotic. In the case of a higher $k$, this result is in fact amplified with more real wage rigidity.

Second, while the optimal policy departs from an inflation targeting rule in the presence of real wage rigidity (even with symmetric countries), it remains the case that putting more weight on the more sclerotic labor market eliminates incremental welfare losses from labor market heterogeneity. I again summarize the consumption-equivalent of welfare losses under an alternative policy to that under the optimal policy with $\psi$, given by (34). In Figure 7 I characterize $\psi$ under the HICP-targeting rule $\xi = \omega$ and the optimal inflation targeting rule when $\alpha = \alpha = 0.5$, to be compared to the case without real wage rigidity in Figure 5 in the main text. First, we see that in the symmetric benchmark, a positive value of $\psi$ indicates that the inflation targeting rules are suboptimal, consistent with the suboptimality of inflation targeting found by Blanchard and Gali (2010) in a closed economy with real wage rigidity (though I note that HICP-targeting remains optimal within the class of inflation targeting rules). Second, as $k$ rises, the welfare losses from the HICP-targeting rule grow, but optimizing over the weight on Home $\xi$ can eliminate nearly all of these incremental losses. Interestingly, as $\delta$ falls, the welfare losses from HICP-targeting in fact fall, though again by optimizing over $\xi$ we can further reduce welfare losses relative to the optimal policy.

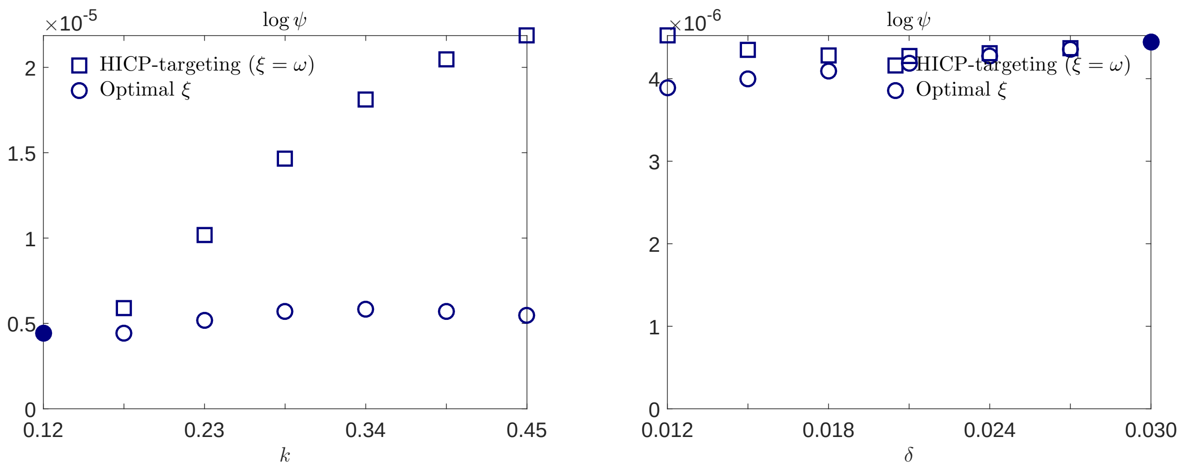


Figure 7: welfare losses as Home's labor market becomes more sclerotic ($\alpha = \alpha^* = 0.5$)

Note: average welfare loss from fluctuations under each policy is computed across many histories of shocks, and then compared to that under the optimal policy by computing $\psi$ as in (34). Shaded markers depict the symmetric benchmark in which $k = k^* = 0.12$ and $\delta = \delta^* = 0.030$. A more sclerotic labor market is one with higher $k$ or lower $\delta$, thus moving from the ends to the center of the figure.

While not the focus of the paper, we can also ask how the optimal policy would respond to asymmetric degrees of real wage rigidity across countries. Figure 8 depicts the optimal $\xi$ as Home real wage rigidity varies (with $\alpha$ between 0.1 and 1), assuming that Foreign has no real wage rigidity ($\alpha^* = 1$) and all labor market parameters are at their symmetric benchmark values. As is evident, $\xi$ should fall as Home real wage rigidity strengthens. This is consistent with the closed economy results of Blanchard and Gali (2010): inflation targeting generates welfare losses in the presence of real wage rigidity, so the optimal inflation targeting rule should place less weight on stabilizing inflation in the country with more real rigidity.

# F Heterogeneity in opportunity costs of employment

In this appendix I provide analytical and numerical results regarding the effects of heterogeneity in the disutilities of labor $\{\chi, \chi^*\}$ on optimal monetary policy in the union. As noted in the main text, these parameters can capture the effects of heterogeneity in the opportunity cost of employment across countries. I indeed use them in this way in the calibration to the Eurozone in section IV.

## F.1 Optimal policy in the $\beta \to 0$ limit

I begin with analytical results in the $\beta \to 0$ limit, as in section III.F.

We can first characterize the effect of lower $\chi$ on $(\varphi - \epsilon_n^{f_n})/\epsilon_n^f$:

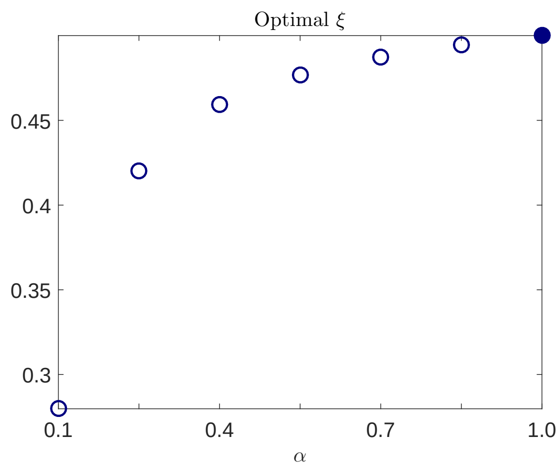


Figure 8: optimal $\xi$ as Home features more real wage rigidity

Note: the optimal $\xi$ is computed by minimizing the average welfare loss across many histories of shocks. The shaded marker depicts the symmetric benchmark in which $\alpha = \alpha^* = 1$. More real wage rigidity implies lower $\alpha$, thus moving from the right to the left of the figure.

**Proposition F.1.** *Suppose $\beta \to 0$. Around the symmetric benchmark and at least for small $\{k, k^*\}$, the lower is $\chi$, the higher is $(\varphi - \epsilon_n^{f_n})/\epsilon_n^f$.*

Intuitively, a lower opportunity cost of employment encourages vacancy creation, leading to higher hiring costs per hire. This is as in a more sclerotic labor market characterized in Proposition 5, implying that $\phi$ rises for the same reasons as described in that result.

Since a lower $\chi$ raises $\phi$ through this channel, it means that Home inflation is more sensitive to output fluctuations and Home output fluctuations themselves are more costly in welfare terms. For both reasons, we obtain an analogous result to Proposition 8:

**Proposition F.2.** *Suppose $\beta \to 0$ and $\iota = \iota^*$. Around the symmetric benchmark and at least for small $\{k, k^*\}$, the optimal weight on Home $\xi$ rises relative to $\omega$ as $\chi$ decreases, unless $\varsigma$ is sufficiently below 1.*

## F.2 Optimal policy in the general case

I now numerically demonstrate that these results hold in the more general case. Starting with the same symmetric parameterization in section III.G, I lower $\chi$ and keep all other parameters unchanged. The first panel of Figure 9 demonstrates that $\phi$ rises as $\chi$ falls, consistent with Proposition F.1. The second panel demonstrates that the optimal inflation targeting rule then places more weight on Home, consistent with Proposition F.2.

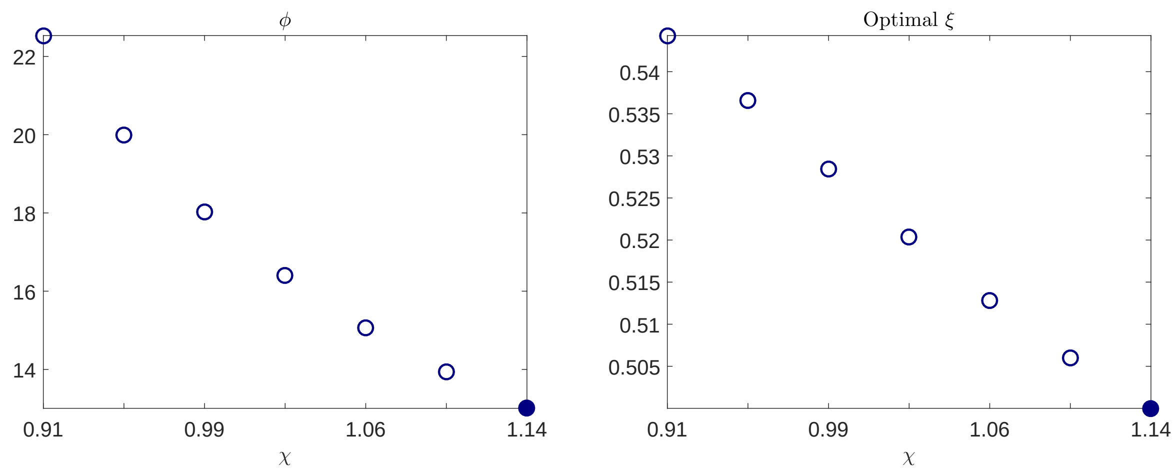


Figure 9: $\phi$ and optimal $\xi$ as Home's opportunity cost of employment falls

Note: $\phi$ controls the welfare costs from and sensitivity of real marginal costs to contemporaneous employment fluctuations. Its formula is given in Lemma 2. The optimal $\xi$ is computed by minimizing the average welfare loss across many histories of shocks. Shaded markers depict the symmetric benchmark in which $\chi = \chi^* = 1.14$. A lower opportunity cost of employment is one with lower $\chi$, moving from the right to the left of the figure.

# G Supplemental results for calibration to Eurozone

In this appendix I provide supplemental results accompanying the quantitative calibration to the Eurozone in section IV.

## G.1 Measurement of labor market flows

I first provide additional details on the measurement of labor market flows for the Eurozone countries summarized in the main text.

### G.1.1 Methodology

I exactly follow the approach in Elsby, Hobijn, and Sahin (2013) who in turn build on Shimer (2012), and so I only briefly the summarize the steps here.

The outflow hazard rate from unemployment $f$ is estimated using data on unemployment by duration as well as overall unemployment. For instance, if monthly data were available, the outflow probability during month $t$ could be recovered from the stock of unemployed workers at the end of month $t$, less those who report being unemployed for less than one month, divided by the stock of unemployed workers at the end of month $t-1$. Elsby et al. (2013) demonstrate how to apply this approach using data sampled less frequently, and using data on duration spells longer than one month to improve the precision of the estimates. The

| Country (code) | Sample size (persons) | $p$-value | Null rejected? |
|---|---|---|---|
| Austria (AT) | 57,500 | 0% | yes |
| Belgium (BE) | 37,500 | 2% | no |
| Finland (FI) | 36,000 | 0% | yes |
| France (FR) | 182,500 | 0% | yes |
| Germany (DE) | 170,750 | 0% | yes |
| Greece (GR) | 67,500 | 3% | no |
| Italy (IT) | 150,000 | 0% | yes |
| Netherlands (NL) | 109,375 | 0% | yes |
| Portugal (PT) | 56,430 | 7% | no |
| Spain (ES) | 160,000 | 1% | yes |

Table 1: hypothesis test for duration dependence in outflow rate from unemployment

Notes: as in Elsby et al. (2013), the null hypothesis is that the probabilities an unemployed worker completes their spell within $d$ months, for $d \in \{3, 6, 12\}$, are equal.

latter is particularly relevant for European economies for which short-term unemployment is quite noisily measured, but will not bias the estimated outflow rate only if there is no duration dependence in outflow rates. I test the hypothesis of no duration dependence for each country using the approach further described in their paper.

Given the outflow rate $f$, the inflow rate $s$ over period $t$ is implied by how the unemployment rate changes from the end of period $t-1$ to the end of $t$.

### G.1.2 Data

I use three sets of data on unemployment for each country from the OECD: annual data on unemployment, annual data on unemployment by duration, and quarterly data on unemployment rates. I use data over 1999-2018 period after the introduction of the euro.

To test the hypothesis of no duration dependence in outflow rates from unemployment, I also require information on the quarterly sample size used in each country's labor force surveys to estimate unemployment. I use the latest available survey size reported on the website of each country's appropriate statistical agency.[2]

### G.1.3 Additional details on estimated flows

In Table 1, I provide information regarding the hypothesis test that there is no duration dependence in outflow rates from unemployment. The first column provides the sample size assumed for that country's labor force surveys necessary to perform the test. The second column provides the $p$-value associated with the null that the probabilities an unemployed worker completes their spell within $d$ months, for $d \in \{3, 6, 12\}$, are equal. The third column summarizes whether or not we reject the null at a 1% significance level. Comparing this table to Table 2 in Elsby et al. (2013), it is notable that we can reject the null hypothesis for France, Germany, Italy, and Spain over the more recent sample period. For countries in which I reject the null hypothesis, I use only data on unemployment duration less than one month to estimate the outflow rate; for those in which I cannot, I use data for longer durations to estimate the outflow rate with greater precision, optimally weighting the various estimates as explained in Elsby et al. (2013).

In Figure 10, I depict the estimated outflow and inflow rates by year. For the countries and years which overlap with the analysis of Elsby et al. (2013), the estimates are broadly consistent with Figures 2 and 3 in that paper.

## G.2 Net replacement rates

I now provide evidence on net replacement rates in unemployment used to discipline the heterogeneity in opportunity costs of employment in the main text.

I use the annual estimates provided by the OECD over 2001-2018.[3] Averaging over this period, Table 2 reports the net replacement rate in unemployment for a worker at two months of unemployment duration, earning the average wage prior to job loss, and including housing-related benefits. The OECD reports these replacement rates for workers in various family situations. The last column is a simple arithmetic average across these. It is based on the last column that, in the calibration provided in the main text, I calibrate an opportunity cost of employment in France which is 75%/77% = 97% of that in Germany.

## G.3 Alternative calibration: Germany and Italy

I now provide an alternative calibration to Germany and Italy to complement that for Germany and France in the main text. By Table 1, the Italian labor market features a lower

[2] Only the number of households in the survey are reported for Austria, Belgium, France, Greece, Netherlands, and Portugal. For these countries I assume 2.5 persons per household as in Elsby et al. (2013).

[3] These data only become available starting in 2001.

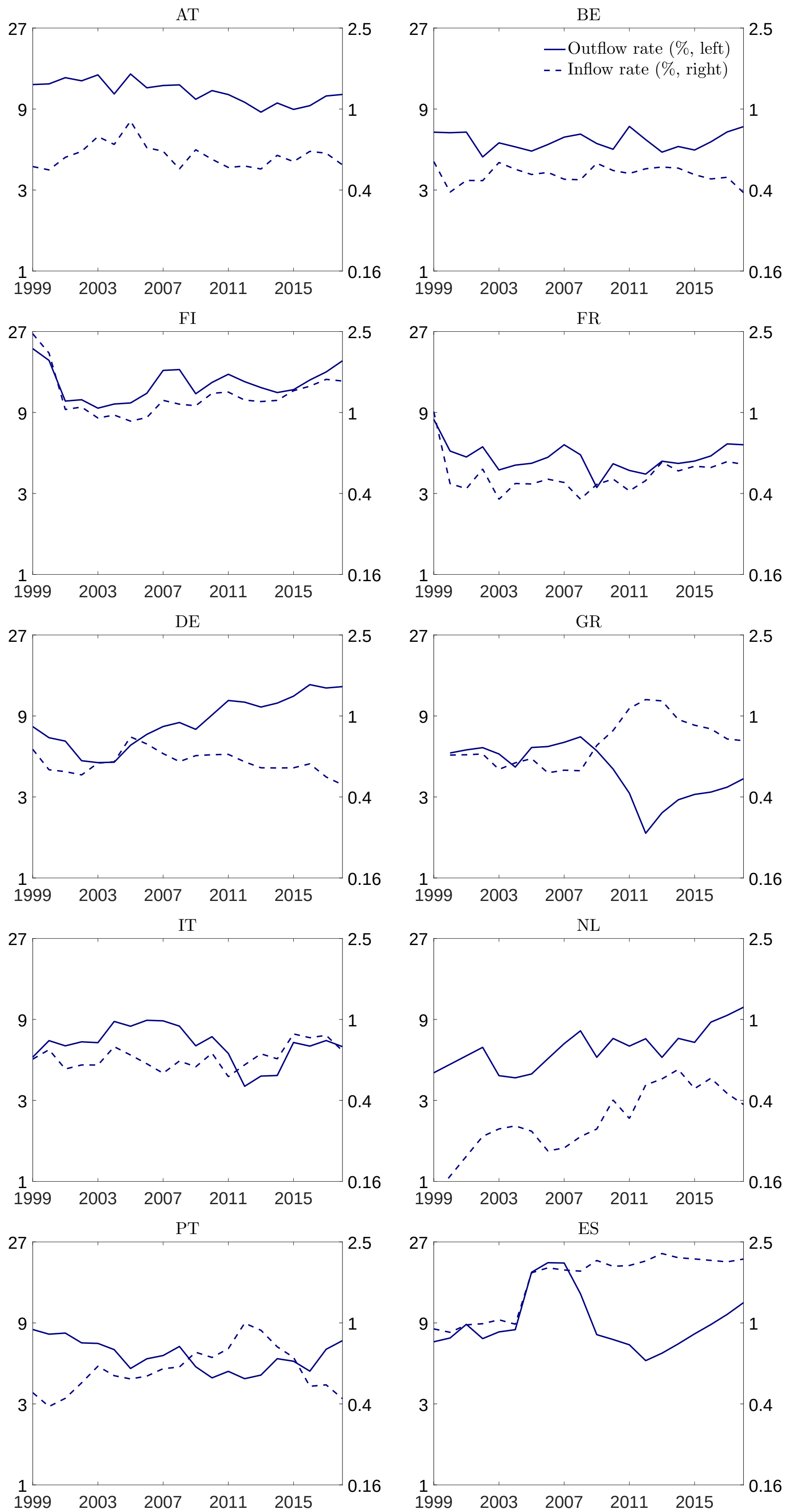


Figure 10: estimates of monthly outflow and inflow rates by year

| Size: | Single | | Couple | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Children: | 0 | 2 | 0 | 0 | 0 | 2 | 2 | 2 | |
| Partner wage / avg wage: | n/a | n/a | 0% | 67% | 100% | 0% | 67% | 100% | Avg |
| Austria (AT) | 58% | 70% | 63% | 75% | 78% | 75% | 79% | 82% | 72% |
| Belgium (BE) | 62% | 67% | 56% | 71% | 78% | 59% | 73% | 80% | 68% |
| Finland (FI) | 57% | 78% | 66% | 75% | 78% | 78% | 79% | 82% | 74% |
| France (FR) | 67% | 71% | 66% | 80% | 84% | 68% | 82% | 84% | 75% |
| Germany (DE) | 60% | 73% | 60% | 85% | 87% | 74% | 89% | 91% | 77% |
| Greece (GR) | 32% | 44% | 34% | 57% | 63% | 44% | 63% | 67% | 51% |
| Italy (IT) | 61% | 70% | 64% | 76% | 79% | 70% | 79% | 80% | 72% |
| Netherlands (NL) | 72% | 71% | 73% | 82% | 85% | 80% | 79% | 82% | 78% |
| Portugal (PT) | 90% | 90% | 88% | 97% | 99% | 87% | 99% | 100% | 94% |
| Spain (ES) | 59% | 74% | 59% | 76% | 80% | 73% | 84% | 86% | 74% |

Table 2: net replacement rates in unemployment in the largest 10 Eurozone economies

Notes: each column reports for a given family situation, the average net replacement rate over 2001-2018 for an unemployed worker with duration 2 months, earning 100% of the average wage prior to job loss, and including housing-related benefits.

outflow rate from unemployment but slightly higher inflow rate to unemployment than the German labor market. I model Italy as Home and Germany as Foreign.

I parameterize the model as follows. The externally set parameters are identical to the calibration in the main text. The calibrated parameters are summarized in Table 3. I directly set $\delta = 0.021$ and $\delta^* = 0.018$ to match the quarterly separation probabilities implied by the monthly inflow rates to unemployment for Italy and Germany in Table 1. I again set $k^*$ and $\chi^*$ to target recruiting costs which are 23% of the quarterly wage in Germany and a quarterly job-finding probability of 23.2% as implied by the monthly outflow rate from unemployment for Germany in Table 1. I set $\chi$ such that the opportunity cost of employment in Italy is 94% of that in Germany, consistent with the fact that the net replacement rate during unemployment has been 94% of that in Germany over this period, per Table 2. I then set $k$ to target a quarterly job-finding probability of 18.0% as implied by the monthly outflow rate from unemployment for Italy reported in Table 1. I use $\gamma$ to target $\omega = 0.37$, the average ratio of Italy's nominal GDP relative to the sum of Italy and Germany over 1999-2018. I set the persistence, volatility, and correlation of productivity shocks to be consistent with the corresponding moments estimated on labor productivity for Italy and Germany over the 1999-2018 period. Finally, given these driving forces, I calibrate the degree of real

| | Value | Moment | Target | Source |
|---|---|---|---|---|
| *Heterogeneity in labor markets* | | | | |
| $\delta$ | 0.021 | qtr. inflow prob., IT | 2.1% | Table 1 |
| $k$ | 0.59 | qtr. outflow prob., IT | 18.0% | Table 1 |
| $\chi$ | 1.10 | opp cost employment IT / DE | 94% | Table 2 |
| $\delta^*$ | 0.018 | qtr. inflow prob., DE | 1.8% | Table 1 |
| $k^*$ | 0.16 | recruiting costs / qtr. wage, DE | 23% | MP (2016) |
| $\chi^*$ | 1.10 | qtr. outflow prob., DE | 23.2% | Table 1 |
| *Other features of economy* | | | | |
| $\gamma$ | 0.37 | nominal GDP IT/(IT+DE) | 37% | OECD |
| $\rho$ | 0.85 | persistence log labor prod., IT | 0.85 | OECD |
| $\rho^*$ | 0.87 | persistence log labor prod., DE | 0.87 | OECD |
| $\zeta_H$ | 0.0063 | std. dev. log labor prod., IT | 0.0063 | OECD |
| $\zeta_F$ | 0.0082 | std. dev. log labor prod., DE | 0.0082 | OECD |
| $\rho_{HF}$ | 0.55 | corr log labor prod. IT, DE | 0.55 | OECD |
| $\alpha = \alpha^*$ | 0.25 | std. dev. real wages / unemp. | 1.00 | OECD |

Table 3: calibrated parameters for Italy (Home) and Germany (Foreign)

Note: all model-implied moments exactly hit the targets, so I do not separately report these. See text for values of other externally set parameters.

wage rigidity across the union $\alpha = \alpha^*$ to match the weighted average of log product wage volatilities over 1999-2018 relative to the weighted average of unemployment volatilities over the same period (with weights $\omega = 0.37$ on Italy and $1 - \omega = 0.63$ on Germany). I generate the same moment in simulated data in the model assuming a HICP-targeting policy rule with $\xi = \omega = 0.37$.

Table 4 compares the quantitative fit of the model using a HICP-targeting rule to untargeted second moments over the 1999-2019 period in the data. In general we see that the model underpredicts volatilities of quantities for Italy. This suggests that labor productivity shocks alone are incapable of generating the observed fluctuations in Italian aggregates over this period, consistent with the focus of the recent literature on financial conditions, fiscal policy, and other driving forces in Southern Europe (as emphasized by Gourinchas, Philippon, and Vayanos (2017), Martin and Philippon (2017), and Chodorow-Reich, Karabarbounis, and Kekre (2019), among others). As in the data, however, the HICP-targeting rule does capture the fact that inflation volatility is higher in Italy than Germany.

Relative to the HICP-targeting rule, the optimal inflation targeting rule again features

| | $\hat{x}_t$ | $\hat{x}_t^*$ | $\hat{n}_t$ | $\hat{n}_t^*$ | $\hat{w}_t$ | $\hat{w}_t^*$ | $\pi_{Ht}$ | $\pi_{Ft}$ |
|---|---|---|---|---|---|---|---|---|
| SD data | 3.05% | 1.75% | 1.73% | 1.23% | 1.27% | 1.50% | 0.43% | 0.34% |
| SD model | 1.60% | 2.65% | 0.55% | 1.17% | 0.77% | 1.04% | 0.30% | 0.18% |

Table 4: untargeted second moments

Notes: empirical second moments estimated on Q1/99-Q4/18 OECD data, where $\hat{x}_t$ and $\hat{x}_t^*$ are linearly detrended log GDP per capita, $\hat{n}_t$ and $\hat{n}_t^*$ are one minus linearly detrended harmonised unemployment rate, $\hat{w}_t$ and $\hat{w}_t^*$ are linearly detrended log labor compensation per employed person less log GDP deflator, and $\pi_{Ht}$ and $\pi_{Ft}$ are linearly detrended difference in log GDP deflator. Corresponding model moments are estimated after simulating many histories of shocks and assuming an HICP-targeting rule.

| Model | $\xi$ | $\psi$ ($\times 10^{-6}$) |
|---|---|---|
| HICP | 0.37 | 21.6 |
| Optimal $\xi$ | 0.48 | 10.9 |
| HICP, $\alpha = \alpha^* = 1$ | 0.37 | 2.1 |
| Optimal $\xi$, $\alpha = \alpha^* = 1$ | 0.42 | 0.4 |
| Optimal $\xi$, $\alpha = \alpha^* = 1$, $k = k^*$ | 0.37 | 0.0 |

Table 5: HICP-targeting versus optimal inflation targeting rule

Notes: $\xi$ denotes weight on Home producer-price inflation in (33) and $\psi$ denotes the proportional change in consumption at all dates and states to render welfare the same as under the optimal policy, following (34).

11*pp* more weight on Italy. As summarized in the first two rows of Table 5, while the HICP-targeting rule calls for $\xi = \omega = 0.37$, the optimal inflation targeting rule features $\xi = 0.48$. Re-weighting the inflation targeting rule in this way again eliminates nearly half of the welfare losses from fluctuations relative to the optimal policy. The third and fourth rows of the table eliminate real wage rigidity in both Italy and Germany and keep all other parameters the same. It remains the case that the optimal inflation targeting rule features a higher weight on Italy. Finally, the fifth row of this table sets $k$ to the value of $k^*$ and leaves all other parameters unchanged. We again see that the optimal inflation targeting rule is consistent with HICP-targeting, so that the difference in hiring costs which rationalize the difference in outflow rates from unemployment between these economies quantitatively drives the departure from HICP-targeting in the optimal inflation targeting rule.

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