ONLINE APPENDIX

The Young, the Old, and the Government:

Demographics and Fiscal Multipliers

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A Local Fiscal Multipliers: Further Evidence

A.1 The Role of the Age Structure

In the baseline regression, we consider as young individuals the white males between 20 and 29 years old. In the literature, there is no consensus on the definition of the young. For instance, Jaimovich and Siu (2009) and Jaimovich et al. (2013) consider the young as individuals between 15 and 29 years old, Wong (2019) looks at people between 25 and 35 years old, Ferraro and Fiori (2020) look at people between 16 and 34 years old, whereas Leahy and Thapar (2019) consider people between 20 and 35 years old. Our young group decision is aimed at striking a balance among all these different contributions, and at the same time to define a group which is as close as Jaimovich et al. (2013), since we borrow from this paper the capital-experience complementarity. We decide to abstract from the 15-19 years old, as our model emphasizes the differences across age groups in both labor supply and labor demand, and in 2015 only 22%

of these individuals were actually working, according to CPS data (whereas this share for the individuals between 20 and 29 years old is 76%).

In this section, we look at the robustness of our measure by estimating the age sensitivity of local multipliers in a set of alternative cases in which the share of young people is computed by considering the young as (i) those between 15 and 29 years old, (i) those between 15 and 34 years old, and (iii) those between 20 and 34 years old. The results of Table A.1 indicate that in all these cases the age sensitivity is around 0.05 and is highly statistically significant.

Then, we change the definition of the young group by replacing the share of white males between 20 and 29 years old in total population with (i) the share of white male workers between 20 and 29 years old in total employment, and (ii) the share of white male individuals between 20 and 29 years old in the labor force. We compare the results of the baseline case with these two alternatives in Table A.2, and find that the age sensitivity of local multipliers does not change substantially across these different definitions.

Finally, we emphasize that although in our empirical evidence the role of demographics is computed over the share of young people in total population, this measure is just a parsimonious way to capture the effect of the entire age structure of the population on the propagation of local government spending. To make this point, we replace the share of young people in total population with (i) the share of mature individuals (i.e., between 30 and 64 years old) in total population, and (ii) the share of old individuals (i.e., above 65 years old) in total population. We report the estimates of these regressions in Table A.3. The results indicate that local multipliers decrease with either the share of mature people or the share of old people, exactly the opposite of the implications of the baseline regression based on the share of young people. Hence, this evidence indicates that fiscal multipliers do not depend on the share of young people in isolation, but rather on the entire age structure of the population. As the population shifts towards older ages, the response of output to government spending shrinks down.

Table A.1: Different Definitions of the Young Group - Ages

	(1)	(2)	(3)	(4)
	Baseline	15-29 Years Old	15-34 Years Old	20-34 Years Old
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	1.510 (0.406)	1.261 (0.397)	1.364 (0.376)	1.514 (0.372)
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.047 (0.017)	0.047 (0.025)	0.051 (0.022)	0.040 (0.016)
$D_{i,t}$	0.003 (0.001)	0.003 (0.001)	0.003 (0.001)	$0.001 \\ (0.001)$
R^2	0.374	0.384	0.376	0.374
N. Observations	2374	2374	2374	2374

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, the independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. The only difference is that now the dependent variable is the state-level change in the employment-to-population ratio. Column (1) reports the results of the baseline regression and computes the share of young people as the ratio of 20-29 years old white males over the total population of white males, Column (2) computes the share of young people as the ratio of 15-29 years old white males over the total population of white males, Column (3) computes the share of young people as the ratio of 15-34 years old white males over the total population of white males, and Column (4) computes the share of young people as the ratio of 20-34 years old white males over the total population of white males. We include time and state fixed effects in all regressions. Standard errors are reported in brackets.

Table A.2: Different Definitions of the Young Group - Employment and Labor Force

	(1)	(2)	(3)
	Baseline	Share of Young Employment	Share of Young Labor Force
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	1.511 (0.406)	1.209 (0.397)	1.271 (0.375)
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.047 (0.017)	0.031 (0.020)	0.029 (0.022)
$D_{i,t}$	0.002 (0.001)	0.001 (0.001)	0.001 (0.001)
R^2	0.374	0.318	0.313
N. Observations	2374	1982	1982

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, the independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. The only difference is that now the dependent variable is the state-level change in the employment-to-population ratio. Column (1) reports the results of the baseline regression and computes the share of young people as the ratio of 20-29 years old white males over the total population of white males, Column (2) computes the share of young people as as the fraction of white male workers in total white male employment, and Column (3) computes the share of young people as the fraction of young white male individuals in the labor force. We include time and state fixed effects in all regressions. Standard errors are reported in brackets.

Table A.3: The Role of the Share of Mature and Old People

	(1)	(2)	(3)
	Baseline	Share of Mature People	Share of Old People
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	1.511 (0.406)	1.003 (0.326)	1.015 (0.313)
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.047 (0.017)	-0.018 (0.013)	-0.039 (0.017)
$D_{i,t}$	$0.002 \\ (0.001)$	-0.002 (0.001)	-0.001 (0.001)
R^2	0.374	0.373	0.351
N. Observations	2374	2374	2374

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, the independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. The only difference is that now the dependent variable is the state-level change in the employment-to-population ratio. Column (1) reports the results of the baseline regression and computes the share of young people as the ratio of 20-29 years old white males over the total population of white males, Column (2) replaces the share of young people with that of mature people, that is, the fraction of individuals between 35 and 64 years old in total population, and Column (3) replaces the share of young people with that of old people, that is, the fraction of individuals above 65 years old in total population. We include time and state fixed effects in all regressions. Standard errors are reported in brackets.

A.2 Additional State-Level Controls

The estimates of the age sensitivity of local fiscal multipliers in the baseline regressions could be biased if the exclusion restrictions of our IV approach are violated, which would happen in case there exist potential confounding factors which are highly correlated with both changes (across states and over time) in the current age structure of the population, in 20-30 year lagged birth rates, and in current government spending.

This section addresses this issue by reporting a comprehensive battery of robustness checks, in which we explicitly control for both the level and the interaction with changes in government spending of the lagged values of a set of key potential confounding factors. In this way, we can evaluate whether the age sensitivity of the local output multiplier keeps holding above and beyond the interaction that government spending may have with other state-level characteristics.

The first set regards heterogeneity across states in taxation. As we discuss in Section 2.4, in our empirical settings the time fixed effects absorb any variation in the financing side of government spending. In doing so, we implicitly assume that the financing of government spending affects symmetrically all states. This condition could be violated even if the statutory federal tax rate is common across states as long as state-level differences in the demographic composition of the population across states translate into state-level differences in the effective average and average marginal tax rates. Hence, we consider five variables that control for state heterogeneity in the financing of government spending: (i) state personal taxes from the U.S. Bureau of Economic Analysis (1969-2015a), (ii) local personal taxes from the U.S. Bureau of Economic Analysis (1969-2015b), (ii) federal personal taxes from the U.S. Bureau of Economic Analysis (1969-2015c), and (iv) the incidence of taxes at the top 10\% of the income distribution, and (v) the incidence of taxes at the bottom 90% of the income distribution, which both are from Zidar (2019). All these variables are normalized by total personal income from the U.S. Bureau of Economic Analysis (1969-2015d). The results of Table A.4 show that the age sensitivity of local multipliers holds above and beyond state-level heterogeneity in the amount and incidence of the financing of government spending.

Then, we consider state-level heterogeneity in transfers, since Oh and Reis (2012) show that transfers play a crucial role in the propagation of fiscal policy. To address this concern, we focus on five variables that control for the variation in transfers across states: (i) unemployment benefits from the U.S. Bureau of Economic Analysis (1969-2015e), (ii) total transfers benefits from the U.S. Bureau of Economic Analysis (1969-2015f), (iii) transfers from the government from the U.S. Bureau of Economic Analysis (1969-2015g), (iv) transfers from non-profit institutions from the U.S. Bureau of Economic Analysis (1969-2015h), and (v) transfers from businesses from the U.S. Bureau of Economic Analysis (1969-2015i). All these variables are normalized by total personal income from the U.S. Bureau of Economic Analysis (1969-2015d). The results of Table A.5 show that the age sensitivity of local multipliers holds above and beyond state-level heterogeneity in the transfers.

The third set of potential confounding factors relates to the heterogeneity in the sectoral composition of value added across states. This variation could affect the fiscal multiplier, since Bouakez et al. (2019) highlight how the propagation of government spending shocks depend on both the heterogeneity and the linkages across sectors. Also in this case we consider five variables: (i) the value added share of manufacturing, (ii) the value added share of construction, (iii) the value added share of services, (i) the value added share of personal services, and (v) the value added share of health care services. The information on total private GDP, manufacturing GDP, construction GDP, services GDP, personal services GDP, and health care services GDP come, respectively, from from the U.S. Bureau of Economic Analysis (1969-2015j), the U.S. Bureau of Economic Analysis (1969-2015h), the U.S. Bureau of Economic Analysis (1969-2015h), the U.S. Bureau of Economic Analysis (1969-2015o). The results of Table A.6 show that the age sensitivity of local multipliers holds above and beyond state-level heterogeneity in the sectoral composition of value added.

Finally, we consider one last set of potential confounding factors: (i) the change in real house prices, which come from the U.S. Federal Housing Finance Agency (1975-2015), (ii) the

unemployment rate rom the U.S. Bureau of Labor Statistics (1976-2015), (iii) the Gini index of labor earnings, (iv) female labor participation, and (v) the amount of skilled workers in total employment. The latter three variables are computed from data of the IPUMS Current Population Survey (CPS) (1977-2015). The decision of including these variables is based on the previous findings of the literature, which highlights how each of these potential confounding factors can influence either the size of the fiscal multiplier or the cyclicality of business cycle fluctuations (Auerbach and Gorodnichenko, 2012; Jaimovich et al., 2013; Nakamura and Steinsson, 2014; Brinca et al., 2016; Khan and Reza, 2017; Albanesi, 2019; Fukui et al., 2019; Hagedorn et al., 2019). Once again, the results of Table A.7 corroborate that the age sensitivity of local multipliers is not driven by heterogeneity across states in other key characteristics.

Table A.4: Additional State-Level Variables - Taxes

	(1)	(2)	(3)	(4)	(5)
	State Personal Taxes	Local Personal Taxes	Federal Personal Taxes	Incidence of Taxes on Top 10% Income Distribution	Incidence of Taxes on Bottom 90% Income Distribution
	IV	IV	IV	IV	IV
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	1.115 (0.312)	1.072 (0.243)	1.134 (0.321)	0.320 (0.451)	0.278 (0.466)
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.054 (0.014)	0.056 (0.013)	0.042 (0.017)	0.078 (0.027)	0.080 (0.028)
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times (VAR_{i,t} - \bar{V}AR)$	-0.144 (0.163)	-0.104 (0.973)	-0.306 (0.419)	-0.028 (0.018)	-0.020 (0.025)
$D_{i,t}$	0.001 (0.001)	0.001 (0.001)	$0.001 \\ (0.001)$	$0.001 \\ (0.001)$	$0.001 \\ (0.001)$
R^2	0.382	0.386	0.387	0.313	0.327
N. Obs.	2325	2325	2325	1400	1400

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, if not stated otherwise, the dependent and independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. All regressions add each time a new (lagged) state-level control, and its interaction with the change in per capita government spending. Column (1) introduces the ratio of state personal income taxes over total households' income, Column (2) introduces the ratio of local personal income taxes over total households' income, Column (3) introduces the ratio of federal personal income taxes over total households' income, and Columns (4) and (5) introduce respectively the measures of the incidence of taxes on top 10% and bottom 90% of the income distribution, using the variables defined in Zidar (2019). All regressions include time and state fixed effects. Robust standard errors clustered at the state level are reported in brackets.

Table A.5: Additional State-Level Variables - Transfers

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Benefits Personal Transfers Transfers Personal Transfers Transfers Transfers Personal Transfers Transfers Personal Transfers		(1)	(2)	(3)	(4)	(5)
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			Personal	Personal	Institutions Personal	Business Personal Transfers
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		IV	IV	IV	IV	IV
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$					1.296 (0.406)
$(VAR_{i,t} - \bar{V}AR)$ (5.136) (0.390) (0.408) (0.172) (0.12) $D_{i,t}$ (0.002 (0.001) (0.001) (0.001) (0.001) (0.001) R^2 (0.384 (0.369 (0.370) (0.380) (0.370)	-,					0.051 (0.016)
(0.001) (0.001) (0.001) (0.001) (0.001) (0.001) R^2 0.384 0.369 0.370 0.380 0.370	- 1, t - 2					-0.096 (0.120)
	$D_{i,t}$					0.002 (0.001)
N. Obs. 2325 2325 2325 2325 232	R^2	0.384	0.369	0.370	0.380	0.377
	N. Obs.	2325	2325	2325	2325	2325

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, if not stated otherwise, the dependent and independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. All regressions add each time a new (lagged) state-level control, and its interaction with the change in per capita government spending. Column (1) introduces the ratio of unemployment benefits over total households' income, Column (2) introduces the ratio of total personal transfers over total households' income, Column (3) introduces the ratio of government personal transfers over total households' income, Column (4) introduces the ratio of non-profit institutions personal transfers over total households' income, and Column (5) introduces the ratio of business personal transfers over total households' income. All regressions include time and state fixed effects. Robust standard errors clustered at the state level are reported in brackets.

Table A.6: Additional State-Level Variables - The Sectoral Composition of Value Added

	(1)	(2)	(3)	(4)	(5)
	Value Added Share of Manufacturing	Value Added Share of Construction	Value Added Share of Services	Value Added Share of Personal Services	Value Added Share of Health Care Services
	IV	IV	IV	IV	IV
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	1.301 (0.481)	1.302 (0.334)	1.064 (0.321)	1.316 (0.408)	1.298 (0.438)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times$	0.045	0.035	0.049	0.057	0.057
$ (D_{i,t} - \bar{D}) $ $ \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times $	(0.020) -0.006	(0.017) -0.552	(0.014)	(0.019) -0.072	(0.016) 0.011
$(VAR_{i,t} - \bar{V}AR)$ $D_{i,t}$	(0.040) 0.002	(0.168) 0.002	(0.011) 0.001	(0.202) 0.001	(0.146) 0.002
R^2	(0.001) 0.382	(0.001) 0.394	(0.001) 0.387	(0.001) 0.373	(0.001) 0.378
N. Obs.	2325	2325	2325	2325	2325

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, if not stated otherwise, the dependent and independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. All regressions add each time a new (lagged) state-level control, and its interaction with the change in per capita government spending. Column (1) introduces the value added share of manufacturing, Column (2) introduces the value added share of construction, Column (3) introduces the value added share of services, Column (4) introduces the value added share of personal services, and Column (5) introduces the value added share of health care services. All regressions include time and state fixed effects. Robust standard errors clustered at the state level are reported in brackets.

Table A.7: Additional State-Level Variables - Other Potential Confounding Factors

	(1)	(2)	(3)	(4)	(5)
	House Price	Unemployment Rate	Gini Index Labor Earnings	Female Labor Participation	Skilled Labor
	IV	IV	IV	IV	IV
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	0.730	0.717	1.012	0.890	0.916
11,1-2	(0.479)	(0.452)	(0.400)	(0.389)	(0.372)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times$	0.080	0.062	0.072	0.076	0.081
$ig(D_{i,t}-ar{D}ig)$	(0.018)	(0.017)	(0.017)	(0.020)	(0.025)
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times$	-0.002	0.003	-0.158	0.058	0.046
$(VAR_{i,t} - \bar{V}AR)$	(0.001)	(0.002)	(0.106)	(0.153)	(0.064)
$D_{i,t}$	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)
D^{2}	,	, ,	` '	, ,	, ,
R^2	0.397	0.379	0.338	0.347	0.341
N. Obs.	1982	1982	1933	1933	1933

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, if not stated otherwise, the dependent and independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. All regressions add each time a new (lagged) state-level control, and its interaction with the change in per capita government spending. Column (1) introduces the change in real house prices, Column (2) introduces the unemployment rate, Column (3) introduces the Gini index of labor earnings, Column (4) introduces the share of female labor participation, and Column (5) introduces the share of skilled labor participation. All regressions include time and state fixed effects. Robust standard errors clustered at the state level are reported in brackets.

A.3 Additional National-Level Controls

In the baseline regressions we control for time fixed effects, which wash out the effects of national-level factor. Yet, if states differ in the responsiveness to national-level factors, and this heterogeneity correlates with the age structure of the population, then our estimates of the age sensitivity of local fiscal multipliers could be biased.

This section addresses this point by running a battery of regressions in which each time we control for the interaction between a key national-level variable and state fixed effects, so that the regressions control for states' heterogeneous responsiveness to these national-level variables. Namely, as national-level variables we consider the oil price (the annual average spot price of West Texas Intermediate from the U.S. Energy Information Administration, 1969-2015), households' debt to GDP (in which households' debt is the ratio of the credit market instruments - liability - of the households and nonprofit organizations from the U.S. Board of Governors of the Federal Reserve System, 1969-2015a), federal debt to GDP (which comes from the Federal Reserve Bank of St. Louis and U.S. Office of Management and Budget, 1969-2015), the military news variable of Ramey (2011) as computed by Ramey and Zubairy (2018), and the real interest rate (the difference between the effective federal funds rate from the U.S. Board of Governors of the Federal Reserve System, 1969-2015b, and the change in the Consumer Price Index for all urban consumers from the BLS, taken from the U.S. Bureau of Economic Analysis, 1969-2015p).

Table A.8 reports the results of all these regressions and highlights that the age sensitivity of local fiscal multipliers is always highly statistically significant, and again roughly constant across specifications.

Table A.8: Response of Output with Additional National-Level Variables

	(1)	(2)	(3)	(4)	(5)
	Oil Price	Households' Debt	Federal Debt	Real Interest Rate	Ramey News
	IV	IV	IV	IV	IV
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	0.929 (0.332)	1.661 (0.451)	1.511 (0.443)	1.499 (0.396)	1.401 (0.374)
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times (D_{i,t} - \bar{D})$	0.034 (0.016)	0.064 (0.023)	0.041 (0.017)	0.047 (0.017)	0.039 (0.015)
$D_{i,t}$	0.001 (0.001)	$0.002 \\ (0.001)$	0.002 (0.001)	$0.001 \\ (0.001)$	$0.001 \\ (0.001)$
R^2	0.559	0.371	0.398	0.405	0.420
N. Obs.	2276	2374	2374	2374	2374

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, if not stated otherwise, the dependent and independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. All regressions include one additional national-level control to the benchmark specification, which we interact with state-fixed effects. Column (1) includes the log-difference of the real oil price, Column (2) includes households' debt to GDP ratio, Column (3) includes federal debt to GDP ratio, Column (4) includes the level of the real interest rate, and Column (5) includes Ramey government spending news variable. Robust standard errors clustered at the state level are reported in brackets.

A.4 The Response of the Employment to Population Rate

The baseline regression estimates the local output multiplier, by computing the response of state-level real per capita output to a local government spending shocks. In this section, we also derive the local employment multiplier by estimating the effect of government spending on state employment rate with a similar regression as of Equation (1) of the main text, that is

$$\frac{E_{i,t} - E_{i,t-2}}{E_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \gamma \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \left(D_{i,t} - \bar{D}\right) + \zeta D_{i,t} + \epsilon_{i,t}.$$

in which the dependent variable is the growth rate of state employment rate $\frac{E_{i,t}-E_{i,t-2}}{E_{i,t-2}}$, in which the employment rate $E_{i,t}$ is computed as the ratio between state-level employment from the U.S. Bureau of Labor Statistics (1967-2015) and state-level total population.

We estimate the local employment multiplier in a set of cases that mirrors exactly the specifications considered in Table 1 of the main text. Namely, we estimate the response of the employment rate in the baseline regression (in which the share of young people is computed as the ratio of white male individuals between 20 and 29 years old over the total white male population), a regression which substitutes the standard errors clustered at the state level with the Driscoll-Kraay errors, a regression that abstracts from the observations associated with the District of Columbia, and finally a regression in which the share of young people is computed as the ratio of all individuals between 20 and 29 years old over the entire male and female population. We report the results of all these regressions in Table A.9.

Then, we perform further robustness checks to the local employment multiplier, again by following the cases considered in Table 2 of the main text for the local output multiplier. Table A.10 reports all these further checks, which consist in the baseline local employment multiplier regression, a regression which abstracts from the interaction of government spending with the share of young people, a regression estimated with OLS methods, the two "partial IV" regressions, in which we just instrument first the share of young people and then the state-level government spending, and finally a regression in which we change the normalization on the

Table A.9: Employment Response to a Government Expenditure Shock across U.S. States

	(1)	(2)	(3)	(4)
	Baseline	Driscoll-Kraay Errors	No DC	All Men and Women
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	1.095	1.095	1.069	1.077
	(0.215)	(0.191)	(0.195)	(0.220)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t}-\bar{D}\right)$	0.034	0.034	0.030	0.038
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	(0.011)	(0.009)	(0.011)	(0.017)
$D_{i,t}$	0.001	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.621	0.621	0.642	0.624
N. Observations	2374	2374	2327	2374

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, the independent variables – as well the instrumental variables – are those defined in the Note to Table 1 of the main text. The only difference is that now the dependent variable is the state-level change in the employment-to-population ratio. Column (1) computes the share of young people as the ratio of 20-29 years old white males over the total population of white males, and uses robust standard errors clustered at the state level, Column (2) considers Driscoll-Kraay standard errors, Column (3) abstracts from the observations of the District of Columbia, and Column (4) computes the share of young people as the ratio of all young men and women over total population. We include time and state fixed effects in all regressions. Standard errors are reported in brackets.

share of young people, to evaluate whether the variation in the age structure of the population comes from changes across states.

Table A.9 shows that for a state with an average share of young people, the local employment fiscal multiplier equals 1.10. Demographics affect also the local employment fiscal multiplier: increasing the share of young people by 1% in absolute terms above the average raises employment fiscal multipliers by 3.1%, from 1.10 up to 1.13. The size of the age sensitivity of the local employment multiplier is remarkably similar to the one of the local output multiplier. Importantly, in all the robustness checks of both Table A.9 and Table A.10 the age sensitivity of the local employment multiplier keeps being statistically significant, but for the case in which the share of young people is instrumented whereas government spending is not.

Table A.10: Response of Employment to Government Shocks - Robustness Checks

	(1) Baseline	(2) No Age Interaction	(3) Baseline	(4) No IV Govt. Spending	(5) No IV Birth Rates	(6) Age Interaction $(D_{i,t} - \bar{D}_t)$
	IV	IV	OLS	"Partial" IV	"Partial" IV	IV
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	1.095 (0.215)	1.220 (0.303)	0.180 (0.076)	0.217 (0.083)	1.046 (0.236)	1.070 (0.189)
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.034 (0.011)		0.001 (0.005)	-0.003 (0.001)	0.025 (0.010)	0.037 (0.009)
$D_{i,t}$	$0.001 \\ (0.001)$	$0.001 \\ (0.001)$	$0.001 \\ (0.001)$	$0.001 \\ (0.001)$	$0.001 \\ (0.001)$	$0.001 \\ (0.001)$
R^2	0.621	0.587	0.635	0.664	0.590	0.640
N. Observations	2374	2397	2397	2374	2397	2374

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1967 to 2015. In all regressions, the independent variables – as well the instrumental variables – are those defined in the Note to Table 2 of the main text. The only difference is that now the dependent variable is the state-level change in the employment-to-population ratio. Column (1) reports the baseline regression. Column (2) shows the results of the regression which abstracts from the interaction of government spending with demographics. Column (3) shows the results of the regression estimated by OLS. In Column (4) we instrument the share of young people but we do not instrument state-level government spending. In Column (5) we instrument state government spending but we do not instrument the share of young people. Column (6) considers the normalization $D_{i,t} - \bar{D}_i \equiv D_{i,t} - \sum_i \frac{D_{i,t}}{n_i} \text{ instead of } D_{i,t} - \bar{D} \equiv D_{i,t} - \sum_i \sum_t \frac{D_{i,t}}{n_i n_t}.$ We include time and state fixed effects in all the regressions. Robust standard errors clustered at the state level are reported in brackets.

A.5 The Dynamics of Local Multipliers

This section shows that the results on the age sensitivity of local fiscal multipliers do not change in case we explicitly take into account the dynamics of output, employment, and government spending. To do so, we extend the baseline regressions by introducing either the lagged two-year change in the dependent variable of interest (i.e., either output per capita or the employment rate), or the lagged two-year change in government spending, or both. We also consider a regression in which we control for state-specific time trends.

Table A.11 shows that although again the level of the local multipliers may change substantially, the variation in the age sensitivity is much more limited, especially in the case of the estimation of the local output multiplier. Moreover, in all cases the age sensitivity keeps being statistically significant, which only one case in which the significance is just at the 10%, and not at either the 1% or 5% level.

Table A.12 computes the 1-year, 2-year (baseline), 3-year, 4-year, and 5-year impact local output multipliers. In all these cases, the age sensitivity is rather constant and keeps always being statistically significant at either the 1% or 5% level.

Table A.11: The Role of the Dynamics of Output and Government Spending

	(1)	(2)	(3)	(4)
	IV	IV	IV	IV
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}}$	0.803 (0.316)	1.488 (0.597)	0.806 (0.441)	1.565 (0.404)
$\frac{G_{i,t}-G_{i,t-2}}{Y_{i,t-2}} \times (D_{i,t}-\bar{D})$	0.032 (0.016)	0.046 (0.017)	0.032 (0.016)	0.045 (0.017)
$D_{i,t}$	$0.001 \\ (0.001)$	0.002 (0.001)	$0.001 \\ (0.001)$	0.002 (0.001)
$\frac{Y_{i,t-1} - Y_{i,t-3}}{Y_{i,t-3}}$	0.626 (0.012)		0.627 (0.012)	
$\frac{G_{i,t-1} - G_{i,t-3}}{Y_{i,t-3}}$		-0.110 (0.301)	-0.060 (0.253)	
State-Specific Time Trend	NO	NO	NO	YES
R^2	0.625	0.372	0.625	0.387
N. Obs.	2325	2325	2325	2374

Note: The table reports the estimates of panel regressions across U.S. states using data from 1967 to 2015 at an annual frequency, following the same specifications of the regressions studied in Table 1 of the main text. Column (1) adds to the baseline specification the lagged value of the two-year change in real per-capita output, $Y_{i,t-1} - Y_{i,t-3}/Y_{i,t-3}$. Column (2) adds to the baseline specification the lagged value of the two-year change in real per-capita government spending, $G_{i,t-1} - G_{i,t-3}/Y_{i,t-3}$. Column (3) adds to the baseline specification the lagged value of both the two-year change in real per-capita output, $Y_{i,t-1} - Y_{i,t-3}/Y_{i,t-3}$, and the two-year change in real per-capita government spending, $G_{i,t-1} - G_{i,t-3}/Y_{i,t-3}$. Column (4) adds to the baseline specification state-specific time-trends. Robust standard errors clustered at the state level are reported in brackets.

Table A.12: Local Output Multipliers at Different Time Horizons

	(1)	(2)	(3)	(4)	(5)
	1 Year Changes	2 Year Changes	3 Year Changes	4 Year Changes	5 Year Changes
	IV	IV	IV	IV	IV
$\frac{G_{i,t} - G_{i,t-j}}{Y_{i,t-j}}$	0.964 (0.409)	1.511 (0.406)	1.977 (0.478)	1.960 (0.530)	2.090 (0.362)
$\frac{G_{i,t} - G_{i,t-j}}{Y_{i,t-j}} \times (D_{i,t} - \bar{D})$	0.024 (0.018)	0.047 (0.017)	0.052 (0.017)	0.046 (0.020)	0.038 (0.018)
$D_{i,t}$	$0.001 \\ (0.001)$	$0.002 \\ (0.001)$	0.003 (0.001)	0.004 (0.002)	$0.006 \\ (0.001)$
R^2	0.353	0.374	0.335	0.347	0.362
N. Obs.	2374	2374	2325	2276	2227

Note: The table reports the estimates of panel regressions across U.S. states using data from 1967 to 2015, following the some variants of the specifications of the regressions studied in Table 1 of the main text. Column (1) derives the changes in both real per-capita output and real per-capita military spending with a one-year lag, Column (2) derives the changes with a two-year lag, Column (3) derives the changes with a three-year lag, Column (4) derives the changes with a four-year lag, and Column (5) derives the changes with a five-year lag. In all cases, the dependent variable is the change in real per-capita output. Robust standard errors clustered at the state level are reported in brackets.

A.5.1 Cumulative Fiscal Multipliers

The econometric specification of the regression (1) in Section 2 of the main text computes a two-year impact output fiscal multiplier. Ramey and Zubairy (2018) argue that cumulative multipliers describe better the effectiveness of fiscal policy than impact multipliers.

To derive the cumulative local fiscal multipliers, we follow Dupor and Guerrero (2017). Namely, we estimate the following IV regression

$$\frac{\left(\sum_{j=1}^{2} Y_{i,t+1-j} - 2Y_{i,t-2}\right)}{Y_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{\left(\sum_{j=1}^{2} G_{i,t+1-j} - 2G_{i,t-2}\right)}{Y_{i,t-2}} + \dots$$

$$\dots + \gamma \frac{\left(\sum_{j=1}^{2} G_{i,t+1-j} - 2G_{i,t-2}\right)}{Y_{i,t-2}} \left(D_{i,t} - \bar{D}\right) + \zeta D_{i,t} + \epsilon_{i,t}$$

where the dependent variable is the two-year cumulative change in per capita output of state i, and the independent variables are state fixed effects α_i , time fixed effects δ_t , the two-year cumulative change in per capita state government spending $\frac{\left(\sum_{j=1}^2 G_{i,t+1-j} - 2G_{i,t-2}\right)}{Y_{i,t-2}}$, the interaction between the two-year cumulative in per capita state government spending and the demeaned log-share of young people in total population $D_{i,t} - \bar{D}$, where $\bar{D} = \sum_i \sum_t D_{i,t}$, and the log-share of young people in total population $D_{i,t}$ multiplied by 100. In this regression, β defines the two-year cumulative output local fiscal multiplier for a state with an average share of young people in total population and γ defines how two-year cumulative fiscal multipliers vary with the age structure of a state relative to the average. Analogously, we estimate two-year cumulative employment fiscal multipliers as

$$\frac{\left(\sum_{j=1}^{2} E_{i,t+1-j} - 2E_{i,t-2}\right)}{E_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{\left(\sum_{j=1}^{2} G_{i,t+1-j} - 2G_{i,t-2}\right)}{Y_{i,t-2}} + \dots$$

$$\dots + \gamma \frac{\left(\sum_{j=1}^{2} G_{i,t+1-j} - 2G_{i,t-2}\right)}{Y_{i,t-2}} \left(D_{i,t} - \bar{D}\right) + \zeta D_{i,t} + \epsilon_{i,t}.$$

In this case, the instrumenting strategy hinges on the following first-stage regression, which

leverage the cumulative change in per capita national government expenditures (as a fraction of per capita national GDP), that is

$$\frac{\left(\sum_{j=1}^{2} G_{i,t+1-j} - 2G_{i,t-2}\right)}{Y_{i,t-2}} = \alpha_i + \delta_t + \eta_i \frac{\left(\sum_{j=1}^{2} G_{t+1-j} - 2G_{t-2}\right)}{Y_{t-2}} + \zeta X_{i,t} + \epsilon_{i,t}$$

where $X_{i,t}$ includes the instruments for both the share of young people, and its interaction with two-year cumulative changes in government spending.

Table A.13 shows that the estimates of neither β nor γ change substantially when we estimate two-year cumulative multiplier rather than two-year impact multiplier.

Table A.13: Cumulative Local Fiscal Multipliers

	(1)	(2)
	Output per Capita	Employment Rate
$\frac{\left(\sum_{j=1}^{2} G_{i,t+1-j} - 2G_{i,t-2}\right)}{Y_{i,t-2}}$	1.453	1.019
- 0,0 - 2	(0.405)	(0.212)
$\frac{\left(\sum_{j=1}^{2} G_{i,t+1-j} - 2G_{i,t-2}\right)}{Y_{i,t-2}} \times \left(D_{i,t} - \bar{D}\right)$	0.046	0.033
-,	(0.016)	(0.011)
$D_{i,t}$	0.003	0.001
	(0.001)	(0.001)
R^2	0.369	0.618
N. Observations	2374	2374

Note: The table reports the estimates of a panel IV regression across U.S. states from 1967 to 2015, at an annual frequency. In regression (1) the dependent variable is the two-year cumulative change in output per capita. In regressions (2) the dependent variable is the two-year cumulative change in employment rate. The independent variables are the two-year cumulative change in per capita state government spending (as a fraction of per capita state GDP), $(G_{i,t} - G_{i,t-2})/Y_{i,t-2}$, the log-share of young people (aged 20-29) in total population times 100, $D_{i,t}$, and the interaction between the two-year cumulative change in per capita state government spending (as a fraction of per capita state GDP) and the log-share of young people, $[(G_{i,t} - G_{i,t-2})/Y_{i,t-2}] \times (D_{i,t} - \bar{D})$. In both regressions, two-year cumulative state-specific changes in per capita state government spending (as a fraction of per capita state GDP) are instrumented with the product of state fixed effects and the two-year cumulative change in per capita national government spending (as a fraction of per capita national GDP). The share of young people is instrumented with 20-30 year lagged birth rates. We include time and state fixed effects in all the regressions. Robust standard errors clustered at the state level are reported in brackets.

A.6 Population and Local Government Spending

Table A.14 studies the response of state population to a state-level government spending shock. In this case, we estimate a simplified regression in which we consider as independent variable just the change in state real government spending:

$$\frac{Pop_{i,t} - Pop_{i,t-2}}{Pop_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \epsilon_{i,t}$$

where $Pop_{i,t}$ denotes the population of state i at time t. In particular, we consider four different definitions of population: (i) overall population, (ii) young population (i.e., people between 20 and 29 years old), (iii) mature population (i.e., people between 30 and 64 years old), and (iv) old population (i.e., people above 65 years old). Given data availability on the disaggregation of total population across age groups, this set of regressions uses annual data from 1969 until 2015.

Column (1) of Table A.14 shows that the overall population does not change following a government spending shock. Yet, this aggregate result compounds different dynamics of the populations by age group. On the one hand, Column (2) shows that the young population does rise following a fiscal shock. On the other hand, Columns (3) and (4) show that mature and old population shrink following a government spending shock, even though this effect is not statistically significant.

These results are consistent with the findings of the literature on the sensitivity of state population to shocks. On the one hand, Blanchard and Katz (1992) show that state migration flows are important transmission mechanisms of changes in state unemployment rates over time. On the other hand, Nakamura and Steinsson (2014) find that overall state population does not react to government spending shocks at short horizon. Our results emphasize that although overall population may not change following a fiscal shock, this aggregate pattern masks heterogenous reactions in the population of different age groups.

This evidence validates our approach in instrumenting the share of young people with lagged

birth rates. Indeed, as the young population does react to fiscal shocks, using raw log-shares of the young people in total population would also capture the endogenous reaction of states' age structure to government spending shocks. Hence, instrumenting the log-share of young people with lagged birth rates is key to identify the causal effect of demographics on the size of fiscal multipliers.

Table A.14: Response of Population to a Government Spending Shock Across U.S. States

	(1)	(2)	(3)	(4)
	Overall Population	Young Population	Mature Population	Old Population
	IV	IV	IV	IV
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	-0.179 (0.303)	1.139 (0.408)	-0.393 (0.400)	-0.070 (0.211)
R^2	0.611	0.660	0.587	0.788
N. Observations	2295	2295	2295	2295

Note: The table reports the estimates of panel regressions across U.S. states on annual data from 1969 to 2015. In Column (1) the dependent variable is the state overall white male population. In Column (2) the dependent variable is the state white male young population (aged 20-29). In Column (3) the dependent variable is the state white male mature population (aged 30-64). In Column (4) the dependent variable is the state white male old population (aged 65+). The independent variable is the change in state-level per capita real government spending (as a fraction of state-level per capita real GDP), which is instrumented with the product of state fixed effects and the change in per capita national real government spending (as a fraction of per capita national real GDP). We include time and state fixed effects in all the regressions. Robust standard errors clustered at the state level are reported in brackets.

A.7 Labor Market and Local Government Spending

In the model, 40% of the age sensitivity of the output local fiscal multiplier hinges on the presence of age-specific differences in both labor supply and demand. On the one hand, we assume that the labor supply elasticity varies exogenously across age groups, such as the elasticity of young and old individuals is larger than the one of mature individuals. On the other hand, the production function is characterized capital-experience complementarity, such as the demand of experienced labor is relatively more persistent over the cycle as it is tied to the stock of capital. These two features makes both the hours worked and the hourly wage of young workers to relatively more elastic.

In this section, we validate in the data the model mechanism through which the age structure of the population affects the labor response to government spending shock, as the labor of young workers is relatively more responsive. To do so, we build a measure by state of total labor earnings, hours worked, and the hourly wage of both young workers (i.e., workers between 20 and 29 years old) and older (i.e., workers above 30 years old). Consistently with the definition of the share of young people in total population used in the baseline local multiplier regressions, we focus on white male workers employed in the private sector. We also exclude non full-time and self-employed workers.

Our choice of splitting the demographics in young vis-á-vis older workers is twofold. First, CPS data do not allow to define narrow groups at the state level which consider young, mature, and old individuals, as we do in the model. Since the data include only very few old workers, it would not be feasible to build state-level measures of old labor earnings, hours, and wages. Hence, we merge the old workers all together with any worker above 30 years old. Second, in the model the production function features capital-experience complementarity, in which young workers are considered as inexperienced, whereas mature and old workers are considered as experienced. From this point of view, grouping mature and old workers all together is consistent with the implications of the model on the demand of labor which is tied to the capital-experience complementarity.

Then, we estimate the following regression

$$\frac{X_{i,t} - X_{i,t-2}}{X_{i,t-2}} = \alpha_i + \delta_t + \beta \frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}} + \epsilon_{i,t}$$
(1)

where we consider each time a different dependent variable $X_{i,t}$, consisting of the per-capita labor earnings, per-capita hours worked, and per-capita hourly wage of young workers, and per-capita labor earnings, per-capita hours worked, and per-capita hourly wage of older workers.

Again, we instrument state military spending with a first-stage regression in which the independent variable is the product of a state fixed effect and the change in national military spending. Since the data start in 1977, we are left with 1887 observations, which is a substantial reduction in the sample size with respect our benchmark analysis, that spans from 1967 to 2015.

Table A.15 reports the estimates of the coefficient β , which defines the local multiplier for each of the dependent variables of interest. The results indicate that the labor market outcomes of young workers are more responsive than those of older workers. In all cases, the size of the multiplier of young workers is twice as large as the one of older workers, and is statistically different from zero in the case of labor earnings and the hourly wage, whereas the response of hours is surrounded by a high degree of uncertainty mainly because of the known measurement issues that characterize the accounting of hours worked.

This evidence is in line with the findings of Jaimovich and Siu (2009) and Jaimovich et al. (2013), which document that both the hours worked and the hourly wage of young workers are highly volatile over the business cycle. Moreover, the fact that both the hours worked and the hourly wage response of young individuals is larger than those of older workers further corroborates our modeling choices of the age-specific differences in labor demand and supply.

Table A.15: Labor Market Response to a Government Spending Shock Across U.S. States

	Labor Earnings		Hours Worked		Hourly Wage	
	(1)	(2)	(3)	(4)	(5)	(6)
	Young Workers	Older Workers	Young Workers	Older Workers	Young Workers	Older Workers
	IV	IV	IV	IV	IV	IV
$\frac{G_{i,t} - G_{i,t-2}}{Y_{i,t-2}}$	1.327 (0.678)	0.878 (0.648)	1.074 (0.769)	0.443 (0.661)	0.746 (0.450)	0.291 (0.670)
R^2	0.114	0.306	0.322	0.162	0.206	0.392
N. Observations	1887	1887	1887	1887	1887	1887

Note: The table reports the estimates of panel regressions across U.S. states from 1969 to 2015 at an annual frequency. In Column (1) the dependent variable is the two-year change in per-capita labor earnings of young workers (i.e., workers between 20 and 29 years old). In Column (2) the dependent variable is the two-year change in per-capita labor earnings of older workers (i.e., workers above 30 years old). In Column (3) the dependent variable is the two-year change in per-capita hours worked of young workers (i.e., workers between 20 and 29 years old). In Column (4) the dependent variable is the two-year change in per-capita hours worked of older workers (i.e., workers above 30 years old). In Column (5) the dependent variable is the two-year change in the per-capita hourly wage of young workers (i.e., workers between 20 and 29 years old). In Column (6) the dependent variable is the two-year change in the per-capita hourly wage of older workers (i.e., workers above 30 years old). In all regressions, the independent variable is the change in per capita state government spending (as a fraction of per capita national government spending (as a fraction of per capita national government spending (as a fraction of per capita national GDP). We include time and state fixed effects in all regressions. Robust standard errors clustered at the state level are reported in brackets.

A.8 Relevance of Birth Rates

In the baseline regression we instrument the share of young people in total population with lagged birth rates. This approach aims at avoiding any endogeneity of states' age structure with respect to the local government spending shocks. In particular, states' age structure would not be exogenous to local government spending shocks if they trigger migration flows. Blanchard and Katz (1992) provide empirical evidence on the fact that state migration responds to shocks. In addition, the results of Appendix A.6 show that the population of young individuals does react to local government spending shocks. Then, the use of lagged birth rates as an instrument imposes an identifying exclusion restriction which posits that, conditional on state and time fixed effects, whatever determines the cross-sectional variation in birth rates has no other long lasting effect on the size of fiscal multipliers 20-30 years later.

In this section we study the relevance of lagged birth rates as an instrument for the share of young people in total population, by reporting the results of the first-stage regression of the share of young people on lagged birth rates. We consider four different cases for the share of young white males, the share of young males, and the share of overall young people: (i) we regress the raw share of young people on the raw series of lagged birth rates and both time and state fixed effects; (ii) we regress the residual series of the raw share of young people on the residual series of the raw series of lagged birth rates. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is either the share of young people or the lagged birth rates and the independent variables are state and time fixed effects; (iii) we regress the log-share of young people on the series of lagged birth rates in logarithm and both time and state fixed effects; (iv) we regress the residual series of the log-share of young people on the residual series of the series of lagged birth rates in logarithm. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is either the log-share of young people or the logged lagged birth rates and the independent variables are state and time fixed effects.

Table A.16 reports the results on the first-stage regressions for the share of young white

Table A.16: First Stage Regression Share of Young White Males on Lagged Birth Rates

	(1)	(2)	(3)	(4)
	Share Young People	Share Young People Residuals	Share Young People Log	Share Young People Log, Residuals
Lagged Birth Rates	0.317 (0.062)			
Lagged Birth Rates (Residuals)		0.317 (0.014)		
$\begin{array}{c} {\rm Lagged~Birth~Rates} \\ {\rm (Log)} \end{array}$			0.509 (0.064)	
Lagged Birth Rates (Log, Residuals)				$0.509 \\ (0.018)$
State FE	YES	NO	YES	NO
Year FE	YES	NO	YES	NO
R^2	0.938	0.176	0.934	0.259
N. Observations	2374	2374	2374	2374

Note: The table reports the results of the first-stage regression in which the share of young white males (aged 20-29) in the total white male population is regressed on 20-30 year lagged birth rates. In Column (1) the raw share of young people is regressed on the raw series of lagged birth rates, in addition to state and year fixed effects. In Column (2) the residual series of the raw share of young people is regressed on the residual series of the raw series of lagged birth rates. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the raw series and the independent variables are state and time fixed effects. In Column (3) the log-share of young people is regressed on the series of lagged birth rates in logarithm, in addition to state and year fixed effects. In Column (4) the residual series of the log-share of young people is regressed on the residual series of the raw series of lagged birth rates in logarithm. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the log series and the independent variables are state and time fixed effects. Robust standard errors clustered at the state level are reported in brackets.

Table A.17: First Stage Regression Share of Young Males on Lagged Birth Rates

	(1)	(2)	(3)	(4)
	Share Young People	Share Young People Residuals	Share Young People Log	Share Young People Log, Residuals
Lagged Birth Rates	0.283 (0.061)			
Lagged Birth Rates (Residuals)		0.283 (0.013)		
$\begin{array}{c} {\rm Lagged~Birth~Rates} \\ {\rm (Log)} \end{array}$			0.447 (0.059)	
Lagged Birth Rates (Log, Residuals)				0.447 (0.017)
State FE	YES	NO	YES	NO
Year FE	YES	NO	YES	NO
R^2	0.915	0.164	0.916	0.230
N. Observations	2374	2374	2374	2374

Note: The table reports the results of the first-stage regression in which the share of young males (aged 20-29) in the total male population is regressed on 20-30 year lagged birth rates. In Column (1) the raw share of young people is regressed on the raw series of lagged birth rates, in addition to state and year fixed effects. In Column (2) the residual series of the raw share of young people is regressed on the residual series of the raw series of lagged birth rates. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the raw series and the independent variables are state and time fixed effects. In Column (3) the log-share of young people is regressed on the series of lagged birth rates in logarithm, in addition to state and year fixed effects. In Column (4) the residual series of the log-share of young people is regressed on the residual series of the raw series of lagged birth rates in logarithm. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the log series and the independent variables are state and time fixed effects. Robust standard errors clustered at the state level are reported in brackets.

Table A.18: First Stage Regression Share of Young People on Lagged Birth Rates

	(1)	(2)	(3)	(4)
	Share Young People	Share Young People Residuals	Share Young People Log	Share Young People Log, Residuals
Lagged Birth Rates	0.264 (0.056)			
Lagged Birth Rates (Residuals)		0.264 (0.012)		
$\begin{array}{c} {\rm Lagged~Birth~Rates} \\ {\rm (Log)} \end{array}$			0.428 (0.057)	
Lagged Birth Rates (Log, Residuals)				0.428 (0.016)
State FE	YES	NO	YES	NO
Year FE	YES	NO	YES	NO
R^2	0.922	0.162	0.923	0.227
N. Observations	2374	2374	2374	2374

Note: The table reports the results of the first-stage regression in which the share of young people (aged 20-29) in the total population is regressed on 20-30 year lagged birth rates. In Column (1) the raw share of young people is regressed on the raw series of lagged birth rates, in addition to state and year fixed effects. In Column (2) the residual series of the raw share of young people is regressed on the residual series of the raw series of lagged birth rates. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the raw series and the independent variables are state and time fixed effects. In Column (3) the log-share of young people is regressed on the series of lagged birth rates in logarithm, in addition to state and year fixed effects. In Column (4) the residual series of the log-share of young people is regressed on the residual series of the raw series of lagged birth rates in logarithm. Each residual variable is derived by taking the residuals of a regression in which the dependent variable is the log series and the independent variables are state and time fixed effects. Robust standard errors clustered at the state level are reported in brackets.

males, Table A.17 reports the results on the first-stage regressions for the share of young males, and Table A.18 reports the results on the first-stage regressions for the share of overall young people. The results indicate that in all cases the lagged birth rates are a relevant instrument for the current share of young people in total population, as the relative coefficient on the instrument is always highly statistically significant at the 1% level. Moreover, when we use state and time fixed effects, the R^2 of the regressions ranges between 91% and 94%. Even in the case we use the residual series and we abstract from the state and time fixed effects, the R^2 still ranges between 22% and 24%. Hence, birthrates in a state do have a predictive power for the future age composition in that state.

Furthermore, comparing the results of Tables A.16-A.18, we find that lagged birth rates are a more relevant instrument for the share of young white males than for the share of young males or the share of all young people. Indeed, the regressions with the share of young white males feature the highest values for the R^2 .

B National Fiscal Multipliers

The fact that at the state level demographics have an effect on fiscal multipliers which is statistically and economically significant does not necessarily imply that the same applies also at the national level. In this section we provide some suggestive evidence showing that also national fiscal multipliers depend on demographics. To do so, we run a SVAR à la Blanchard and Perotti (2002) on both a panel of developed countries and a panel of developing countries. In either case, we show that the long-run national output fiscal multiplier is larger in countries with higher shares of young people in total population.

B.1 Data

We take the data from Ilzetzki et al. (2013b). These authors compiled an unbalanced panel on government spending, GDP, current account, real effective exchange rate, and interest rates at quarterly frequency from 1960Q1 until 2009Q4 for 19 developed countries and 25 developing countries.¹ Then, we take the data on the demographic structure of each country at the annual frequency from the World Population Prospects prepared by the United Nations Department of Economic and Social Affairs (1970-2015).

B.2 Econometric Specification

We estimate fiscal multiplier using a SVAR system as in Blanchard and Perotti (2002), such that

$$AX_{i,t} = \sum_{k=1}^{K} C_k X_{i,t-k} + BU_{i,t}$$

¹The developed countries are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom and the United States. The developing countries are Argentina, Botswana, Brazil, Bulgaria, Chile, Colombia, Croatia, Czech Republic, Ecuador, El Salvador, Estonia, Hungary, Latvia, Lithuania, Malaysia, Mexico, Peru, Poland, Romania, Slovakia, Slovenia, South Africa, Thailand, Turkey, and Uruguay.

where $X_{i,t}$ is a vector that consists of the logarithm of real government expenditure, the logarithm of real GDP, the ratio of the real current account balance over GDP, and the log difference of the real effective exchange rate of country i. To identify government spending shocks, we follow the identification assumption of Blanchard and Perotti (2002): we assume that government spending reacts to changes in the other macroeconomic variables with the delay of a quarter. This assumption defines a Cholesky decomposition in which government spending is ordered first. For the selection of the lag structure of the panel SVAR we follow Ilzetzki et al. (2013a) by choosing K = 4 lags. The results do not change if we choose a number of lags between 1 and 8.

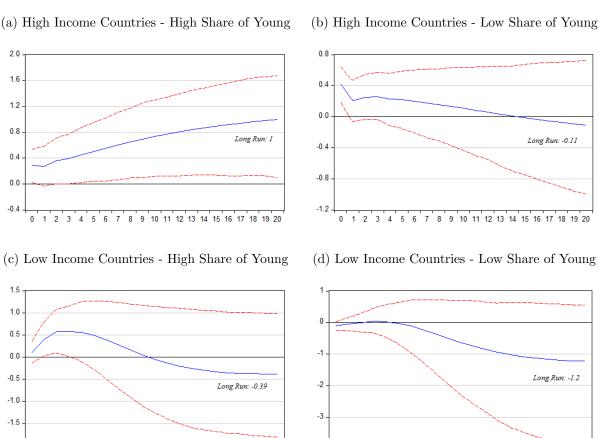
To identify the role of demographics on fiscal multipliers, we do the following. First, we take all the developed countries and split them in two sets: 9 countries with high shares of young people in total population, and 10 countries with low share of young people in total population. Second, we estimate the SVAR system on the two different panels and compare the results. Then, we repeat the same exercise for the developing countries. In this case, we find 11 countries with high shares of young people and 14 countries with low shares.^{2,3}

Finally, we follow Ilzetzki et al. (2013a) and define the long-run output fiscal multiplier as $\frac{\sum_{t=0}^{\infty} (1+r_i)^{-t} \Delta Y_{i,t}}{\sum_{t=0}^{\infty} (1+r_i)^{-t} \Delta G_{i,t}}$, where t=0 denotes the date in which the government expenditure shock occurs, and r_i is the median of the country specific nominal interest rate.

²We consider developed and developing countries separately because Ilzetzki et *al.* (2013a) show that national fiscal multipliers in developed countries are large and positive, while in developing countries are large and negative. The results of Ilzetzki et *al.* (2013a) suggest that other factors (e.g., the exchange rate policy rule, the degree of trade openness, and the level of public debt) could be explaining the differences in fiscal multipliers across our sets of countries.

³In the case of developed countries, the nine countries with high shares of young people have shares in the range of 15%-15.6%. Instead, the ten countries with low shares of young people have shares in the range of 13.5%-14.7%. In the case of developing countries, the eleven countries with high shares of young people have shares in the range of 16.4%-17.2%. Instead, the fourteen countries with low shares of young people have shares in the range of 14.7%-15.9%.

Figure B.1: National Fiscal Multipliers and Demographics.



Note: Panel (a) plots the cumulative national fiscal multipliers over twenty quarters following a government expenditure shock in a panel of nine high income countries with high shares of young people (i.e., age 20-29) in total population. Panel (b) plots the cumulative national fiscal multipliers in a panel of eleven high income countries with low shares of young people in total population. Panel (c) plots the cumulative national fiscal multipliers in a panel of eleven low income countries with high shares of young people in total population. Panel (d) plots the cumulative national fiscal multipliers in a panel of fourteen low income countries with low shares of young people in total population. In each Panel, the dotted lines display 90% confidence bands. The data on government expenditures and real GDP at quarterly frequency from 1960 until 2009 across 19 high income countries and 25 low income countries is from Ilzetzki et al. (2013b).

B.2.1 Results

Figure B.1 reports the response of national output to an increase in government spending in both developed countries and developing countries. We also report the estimates of the long-run fiscal multiplier. Panel (a) shows the response in developed countries with high shares of young people in total population whereas Panel (b) plots the response in developed countries with low shares of young people in total population.

Although the impact response is similar across groups, in countries with low shares of young people the fiscal multipliers becomes statistically insignificant from zero from the first quarter on, leading to a long-run multiplier of -0.11. Instead, in countries with high shares of young people the fiscal multiplier is always statistically significant and the long-run multiplier equals 1.

Panel (c) and Panel (d) report the same set of results for developing countries. As already pointed out in Ilzetzki et al. (2013a), fiscal multipliers in developing countries tend to be negative. Nevertheless, we find again that fiscal multipliers vary with the demographic structure of the countries. In the developing countries with high shares of young workers the impact responses are positive for the first ten periods, and interestingly the point estimate of the cumulative fiscal multiplier after two quarters is around 0.5, and is statistically different from zero. Then, the responses turn into negative values and as a result the long-run multiplier is -0.39. Instead, in the panel of developing countries with low shares of young people fiscal multipliers are much smaller. The impact responses are always negative and in the long-run the multiplier drops down to -1.2

C More on the Household Sector

In this Section we provide the maximization problems and the optimal conditions for each age group separately. We show that the optimal decisions of each individual are linear in wealth, so we can linearly aggregate the optimal choices of individuals within each age group to form a representative agent for each of the three age groups. For the sake of exposition, we derive the aggregation results only for the home economy. Nevertheless, the aggregation of the optimal choices of households within each age group in the foreign economy follows the same procedure. We derive all the problems and first-order conditions in real terms. We denote $\tilde{b}_{z,t}^j = \frac{b_{z,t}^j}{P_t}$ as the real bond-holdings of an individual i in the age group z at time t, $\tilde{a}_{z,t}^j = \frac{a_{z,t}^j}{P_t}$ is the real total return on assets of an individual i in the age group z at time t, $r_{k,t} = \frac{R_{k,t}}{P_t}$ is the real return on capital, and $w_{in,t} = \frac{W_{in,t}}{P_t}$ and $w_t = \frac{W_{ext,t}}{P_t}$ are the real wages. Finally, as in our calibration, we set $\psi_c = \psi_I$ such that $P_t = P_{I,t}$.

C.1 Old Agents

Assuming interior solutions for capital and bond holdings, the decision problem of an old agent i is

$$\max_{\substack{c_{o,t}^{i}, l_{o,t}^{i}, k_{o,t+1}^{i}, \tilde{b}_{o,t+1}^{i}}} v_{o,t}^{i} = \left\{ \left(c_{o,t}^{i} - \chi_{o} \frac{l_{o,t}^{i}}{1 + \frac{1}{\nu_{o}}} \right)^{\eta} + \beta \omega_{o} \mathbb{E}_{t} [v_{o,t+1}^{i}]^{\eta} \right\}^{1/\eta}$$
(C.1)

subject to

$$c_{o,t}^{i} + k_{o,t+1}^{i} + \tilde{b}_{o,t+1} + \frac{\varphi}{2} \left(\frac{k_{o,t+1}^{i}}{k_{o,t}^{i}} - \vartheta_{o} \right)^{2} \frac{k_{o,t}^{i}}{\omega_{o}} = \tilde{a}_{o,t}^{i} + w_{t} \xi_{o} l_{o,t}^{i} - \tau_{o,t}^{i}$$

$$\tilde{a}_{o,t}^{i} = \left\{ k_{o,t}^{i} \left[(1 - \delta) + r_{k,t} \right] + \tilde{b}_{o,t}^{i} \frac{R_{n,t}}{1 + \pi_{t}} \right\} \left(\frac{1}{\omega_{o}} \right).$$

In order to solve the stochastic version of the problem we follow Farmer (1990) closely.

Define

$$f_o(Q_{o,t}) \equiv \left(1 + (\beta \omega_o)^{\frac{1}{1-\eta}} Q_{o,t}^{\frac{\eta}{1-\eta}}\right)^{\frac{1-\eta}{\eta}} \tag{C.2}$$

$$g_o(Q_{o,t}) \equiv \left(1 + (\beta \omega_o)^{\frac{1}{1-\eta}} Q_{o,t}^{\frac{\eta}{1-\eta}}\right)^{-1} \tag{C.3}$$

$$Q_{o,t} \equiv \mathbb{E}_t \left(\frac{f_o(Q_{o,t+1})R_{n,t+1}}{\omega_o(1+\pi_{t+1})} \right) \tag{C.4}$$

We conjecture that the old consume a fraction of a measure of wealth $(W_{o,t})$, define as the sum of financial assets $(a_{o,t}^i)$ and the present value of human capital gains $(HC_{o,t}^i)$, net of the present value of taxes $(T_{o,t}^i)$ and the present value of adjustment costs $(ADJ_{o,t}^i)$. Moreover, the value function is given by

$$c_{o,t}^{i} = \varepsilon_{t} \varsigma_{t} \left[\tilde{a}_{o,t}^{i} + H C_{o,t}^{i} - T_{o,t}^{i} - A D J_{o,t}^{i} \right] = \varepsilon_{t} \varsigma_{t} W_{o,t}$$
 (C.5)

$$v_{o,t}^{i} = (\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}} \left(c_{o,t}^{i} - \chi_{o} \frac{(l_{o,t}^{i})^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right)$$
 (C.6)

Finally we set $(\varepsilon_t \varsigma_t)^{\frac{\eta-1}{\eta}} \equiv f_o(Q_{o,t})$, and thus $g_o(Q_{o,t}) = \varepsilon_t \varsigma_t$

Using the conjecture for the value and policy functions

$$v_{o,t}^{i} = (\varepsilon_{t}\varsigma_{t})^{\frac{\eta-1}{\eta}}W_{o,t} - (\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}}\chi_{o}\frac{(l_{o,t}^{i})^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}$$

$$v_{o,t}^{i} = f_{o}(Q_{o,t})\left[\tilde{a}_{o,t}^{i} + HC_{o,t}^{i} - T_{o,t}^{i} - ADJ_{o,t}^{i}\right] - (\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}}\chi_{o}\frac{(l_{o,t}^{i})^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}$$

Rearranging the budget constraint we have that

$$\tilde{a}_{o,t+1}^{i} = \frac{R_{n,t+1}}{(1+\pi_{t+1})\,\omega_o} \Big(\tilde{a}_{o,t}^{i} + w_t \xi_o l_{o,t}^{i} - \tau_{o,t}^{i} - adj_{o,t}^{i} - c_{o,t}^{i} \Big).$$

where
$$adj_{o,t}^{i} = \left(1 - \frac{(1-\delta+r_{k,t+1})(1+\pi_{t+1})}{R_{n,t+1}}\right)k_{o,t+1}^{i} + \frac{\varphi}{2}\left(\frac{k_{o,t+1}^{i}}{k_{o,t}^{i}} - \vartheta_{o}\right)^{2}\frac{k_{o,t}^{i}}{\omega_{o}}$$
. Thus,
$$v_{o,t}^{i} = f_{o}(Q_{o,t})\left[\frac{R_{n,t}}{(1+\pi_{t})\omega_{o}}\left(\tilde{a}_{o,t-1}^{i} + w_{t-1}\xi_{o}l_{o,t-1}^{i} - \tau_{o,t-1}^{i} - adj_{o,t-1}^{i} - c_{o,t-1}^{i}\right) + HC_{o,t}^{i} - T_{o,t}^{i} - ADJ_{o,t}^{i}\right] - (\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}}\chi_{o}\frac{\left(l_{o,t}^{i}\right)^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}$$

$$v_{o,t}^{i} = \frac{f_{o}(Q_{o,t})R_{n,t}}{(1+\pi_{t})\omega_{o}}\left(\tilde{a}_{o,t-1}^{i} + w_{t-1}\xi_{o}l_{o,t-1}^{i} - \tau_{o,t-1}^{i} - adj_{o,t-1}^{i} - c_{o,t-1}^{i}\right) + f_{o}(Q_{o,t})HC_{o,t}^{i} - f_{o}(Q_{o,t})T_{o,t}^{i} - f_{o}(Q_{o,t})ADJ_{o,t}^{i} - (\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}}\chi_{o}\frac{\left(l_{o,t}^{i}\right)^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}$$

Taking expectations \mathbb{E}_{t-1} , and using (C.4)⁴

$$\begin{split} \mathbb{E}_{t-1}(v_{o,t}^{i}) &= Q_{o,t-1}\Big(\tilde{a}_{o,t-1}^{i} + w_{t-1}\xi_{o}l_{o,t-1}^{i} - \tau_{o,t-1}^{i} - \mathbb{E}_{t-1}adj_{o,t-1}^{i} - c_{o,t-1}^{i}\Big) + \\ &+ \mathbb{E}_{t-1}f_{o}(Q_{o,t})HC_{o,t}^{i} - \mathbb{E}_{t-1}f_{o}(Q_{o,t})T_{o,t}^{i} - \mathbb{E}_{t-1}f_{o}(Q_{o,t})ADJ_{o,t}^{i} - \\ &- \mathbb{E}_{t-1}\left[\left(\varepsilon_{t}\varsigma_{t}\right)^{\frac{-1}{\eta}}\chi_{o}\frac{\left(l_{o,t}^{i}\right)^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}\right] \end{split}$$

Then, define

$$\begin{split} HC_{o,t}^{i} &\equiv w_{t}\xi_{o}l_{o,t}^{i} + \mathbb{E}_{t}\left[\frac{f(Q_{o,t+1})HC_{o,t+1}^{i}}{Q_{o,t}}\right] + \\ &\quad + \frac{(Q_{o,t}\beta\omega_{o})^{\frac{1}{1-\eta}}}{Q_{o,t}}\left[\chi_{o}\frac{(l_{o,t}^{i})^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}\right] - \mathbb{E}_{t}\left[\left(\varepsilon_{t+1}\varsigma_{t+1}\right)^{\frac{-1}{\eta}}\chi_{o}\frac{(l_{o,t+1}^{i})^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}\right] \\ T_{o,t}^{i} &\equiv \tau_{o,t}^{i} + \mathbb{E}_{t}\left[\frac{f(Q_{o,t+1})T_{o,t+1}^{i}}{Q_{o,t}}\right] \\ ADJ_{o,t}^{i} &\equiv \mathbb{E}_{t}adj_{o,t}^{i} + \mathbb{E}_{t}\left[\frac{f(Q_{o,t+1})ADJ_{o,t+1}^{i}}{Q_{o,t}}\right] \end{split}$$

Using these results and adding and subtracting $(Q_{o,t-1}\beta\omega_o)^{\frac{1}{1-\eta}}\left[\chi_o\frac{(l_{o,t-1}^i)^{1+\frac{1}{\nu_o}}}{1+\frac{1}{\nu_o}}\right]$, the expected

⁴We assume $\mathbb{E}_{t-1} \frac{f_o(Q_{o,t})R_{n,t}}{(1+\pi_t)\omega_o} adj_{o,t-1}^i \approx Q_{o,t-1}\mathbb{E}_{t-1}adj_{o,t-1}^i$, essentially ignoring the effect of uncertainty on the portfolio allocation between bonds and capital. To a first order approximation the agent is indifferent between holding bonds or capital.

value function simplifies to

$$\mathbb{E}_{t-1}(v_{o,t}^{i}) = Q_{o,t-1}\left(\tilde{a}_{o,t-1}^{i} + HC_{o,t-1}^{i} - T_{o,t-1}^{i} - ADJ_{o,t-1}^{i} - c_{o,t-1}^{i} - \frac{(Q_{o,t-1}\beta\omega_{o})^{\frac{1}{1-\eta}}}{Q_{o,t-1}}\left[\chi_{o}\frac{(l_{o,t-1}^{i})^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}\right]\right)$$

$$= Q_{o,t-1}\left(W_{o,t-1} - c_{o,t-1}^{i} - \frac{(Q_{o,t-1}\beta\omega_{o})^{\frac{1}{1-\eta}}}{Q_{o,t-1}}\left[\chi_{o}\frac{(l_{o,t-1}^{i})^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}\right]\right)$$

We can now use this result into the objective function to obtain

$$\max v_{o,t}^{i} = \left\{ \left(c_{o,t}^{i} - \chi_{o} \frac{l_{o,t}^{i}^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right)^{\eta} + \beta \omega_{o} \left[Q_{o,t} \left(W_{o,t} - c_{o,t}^{i} - \frac{(Q_{o,t}\beta\omega_{o})^{\frac{1}{1-\eta}}}{Q_{o,t}} \chi_{o} \frac{(l_{o,t}^{i})^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right) \right]^{\eta} \right\}^{1/\eta}$$

The first order condition with respect to consumption gives,

$$v_{o,t}^{i}^{1-\eta} \left(\left(c_{o,t}^{i} - \chi_{o} \frac{l_{o,t}^{i}^{1+\frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right)^{\eta-1} + \beta \omega_{o} \left[Q_{o,t} \left(W_{o,t} - c_{o,t}^{i} - \frac{(Q_{o,t}\beta\omega_{o})^{\frac{1}{1-\eta}}}{Q_{o,t}} \chi_{o} \frac{(l_{o,t}^{i})^{1+\frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right) \right]^{\eta-1} (-Q_{o,t}) \right) = 0$$

$$\left(c_{o,t}^{i} - \chi_{o} \frac{l_{o,t}^{i}^{1+\frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right) = (\beta\omega_{o}Q_{o,t})^{\frac{1}{\eta-1}} \left[Q_{o,t} \left(W_{o,t} - c_{o,t}^{i} - \frac{(Q_{o,t}\beta\omega_{o})^{\frac{1}{1-\eta}}}{Q_{o,t}} \chi_{o} \frac{(l_{o,t}^{i})^{1+\frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right) \right]$$

$$c_{o,t}^{i} = \left(1 + (\beta\omega_{o})^{\frac{1}{1-\eta}} Q_{o,t}^{\frac{\eta}{1-\eta}} \right)^{-1} W_{o,t} = \varepsilon_{t} \varsigma_{t} W_{o,t}$$

We can now replace the solution for $c_{o,t}^i$, which matches our conjecture, into the value function to obtain

$$v_{o,t}^{i} = \left[\left(c_{o,t}^{i} - \chi_{o} \frac{l_{o,t}^{i}^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right)^{\eta} + \beta \omega_{o} \left[Q_{o,t} \left(\left[1 + (\beta \omega_{o})^{\frac{1}{1 - \eta}} Q_{o,t}^{\frac{\eta}{1 - \eta}} \right] c_{o,t}^{i} - c_{o,t}^{i} - \frac{(Q_{o,t} \beta \omega_{o})^{\frac{1}{1 - \eta}}}{Q_{o,t}} \chi_{o} \frac{(l_{o,t}^{i})^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right) \right]^{\eta} \right]^{1/\eta}$$

$$v_{o,t}^{i} = \left\{ \left(1 + (\beta \omega_{o})^{\frac{1}{1 - \eta}} Q_{o,t}^{\frac{\eta}{1 - \eta}} \right) \left(c_{o,t}^{i} - \chi_{o} \frac{l_{o,t}^{i}^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right)^{\eta} \right\}^{1/\eta}$$

Using the definition for $f_o(Q_{o,t})$ we confirm our guess, obtaining

$$v_{o,t}^{i} = (\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}} \left(c_{o,t}^{i} - \chi_{o} \frac{(l_{o,t}^{i})^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}} \right)$$

Combining (C.2) and (C.4) we obtain the condition that determines the marginal propensity to consume of the old.

$$Q_{o,t} = ((\varepsilon_t \varsigma_t)^{-1} - 1)^{\frac{1-\eta}{\eta}} (\beta \omega_o)^{\frac{-1}{\eta}}$$
$$((\varepsilon_t \varsigma_t)^{-1} - 1)^{\frac{1-\eta}{\eta}} = \mathbb{E}_t \left(\frac{R_{n,t+1} (\beta \omega_o)^{\frac{1}{\eta}} (\varepsilon_{t+1} \varsigma_{t+1})^{\frac{\eta-1}{\eta}}}{\omega_o (1 + \pi_{t+1})} \right)$$

Finally, from the first order conditions of (C.1) labor is set such that

$$l_{o,t}^i = \left(\frac{\xi_o w_t}{\chi_o}\right)^{\nu_o},$$

and, to a first order approximation the individual is indifferent between holding bonds or capital. The no-arbitrage condition on investment posits that the expected return on capital should equalize the expected return on bonds, that is,

$$\left(\frac{R_{n,t+1}}{1+\pi_{t+1}}\right) = \left(\frac{(1-\delta) + r_{k,t+1} - \frac{\varphi}{2} \left(\frac{k_{o,t+2}^{i}}{k_{o,t+1}^{i}} - \vartheta_{o}\right)^{2} + \varphi\left(\frac{k_{o,t+2}^{i}}{k_{o,t+1}^{i}} - \vartheta_{o}\right)\frac{k_{o,t+2}^{i}}{k_{o,t+1}^{i}}}{\left[1 + \frac{\varphi}{\omega_{o}} \left(\frac{k_{o,t+1}^{i}}{k_{o,t}^{i}} - \vartheta_{o}\right)\right]}\right)$$

If the constraint binds, we no longer have an interior solution. In this case the consumption policy function can be easily obtained from the budget constraint of the agent. The labor optimality condition remains the same.

C.2 Mature Agents

The decision problem of a mature agent i, assuming interior solutions for capital and bond holdings, is

$$\max_{c_{m,t}^{i}, l_{m,t}^{i}, k_{m,t+1}^{i}, b_{m,t+1}^{i}} v_{m,t}^{i} = \left\{ \left(c_{m,t}^{i} - \chi_{m} \frac{l_{m,t}^{i}}{1 + \frac{1}{\nu_{m}}} \right)^{\eta} + \beta \mathbb{E}_{t} \left[\omega_{m} v_{m,t+1}^{i} + (1 - \omega_{m}) v_{o,t+1}^{i} \right]^{\eta} \right\}^{1/\eta}$$
(C.7)

subject to

$$k_{m,t+1}^{i} + \tilde{b}_{m,t+1}^{i} + c_{m,t}^{i} + \frac{\varphi}{2} \left(\frac{k_{m,t+1}^{i}}{k_{m,t}^{i}} - \vartheta_{m} \right)^{2} k_{m,t}^{i} = \tilde{a}_{w,t}^{i} + w_{t} l_{m,t}^{i} + (1 - \tau_{d}) d_{m,t}^{i} - \tau_{m,t}^{i}$$

$$\tilde{a}_{m,t}^{i} = k_{m,t}^{i} ((1 - \delta) + r_{k,t}) + \tilde{b}_{m,t}^{i} \frac{R_{nt}}{1 + \pi_{t}}.$$

Define

$$f_m(Q_{m,t}) \equiv \left(1 + (\beta)^{\frac{1}{1-\eta}} Q_{m,t}^{\frac{\eta}{1-\eta}}\right)^{\frac{1-\eta}{\eta}}$$
 (C.8)

$$g_m(Q_{m,t}) \equiv \left(1 + (\beta)^{\frac{1}{1-\eta}} Q_{m,t}^{\frac{\eta}{1-\eta}}\right)^{-1}$$
 (C.9)

$$Q_{m,t} \equiv \mathbb{E}_t \left(\frac{\mathfrak{Z}_{t+1} R_{n,t+1} f_m(Q_{m,t+1})}{(1+\pi_{t+1})} \right)$$
 (C.10)

where $\mathfrak{Z}_{t+1} = (\omega_m + (1 - \omega_m)\varepsilon_{t+1}^{\frac{\eta-1}{\eta}}).$

We conjecture that the mature consume a fraction of a measure of wealth $(W_{m,t})$, define as the sum of financial assets $(a_{m,t}^i)$, the present value of human capital gains $(HC_{m,t}^i)$ and dividends $(D_{m,t}^i)$, net of the present value of taxes $(T_{m,t}^i)$ and the present value of adjustment costs $(ADJ_{m,t}^i)$. Moreover, the value function is given by

$$c_{m,t}^{i} = \varsigma_{t} \left[\tilde{a}_{m,t}^{i} + H C_{m,t}^{i} + D_{m,t}^{i} - T_{m,t}^{i} - A D J_{m,t}^{i} \right] = \varsigma_{t} W_{m,t}$$
 (C.11)

$$v_{m,t}^{i} = \left(\varsigma_{t}\right)^{\frac{-1}{\eta}} \left(c_{m,t}^{i} - \chi_{o} \frac{\left(l_{m,t}^{i}\right)^{1 + \frac{1}{\nu_{m}}}}{1 + \frac{1}{\nu_{m}}}\right) \tag{C.12}$$

Finally we set $(\varsigma_t)^{\frac{\eta-1}{\eta}} \equiv f_m(Q_{m,t})$, and thus $g_m(Q_{m,t}) = \varsigma_t$.

Using the conjecture for the value and policy functions

$$v_{m,t}^{i} = (\varsigma_{t})^{\frac{\eta-1}{\eta}} W_{m,t} - (\varsigma_{t})^{\frac{-1}{\eta}} \chi_{m} \frac{(l_{m,t}^{i})^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}$$

$$v_{m,t}^{i} = f_{m}(Q_{m,t}) \left[\tilde{a}_{m,t}^{i} + HC_{m,t}^{i} + D_{m,t}^{i} - T_{m,t}^{i} - ADJ_{m,t}^{i} \right] - (\varsigma_{t})^{\frac{-1}{\eta}} \chi_{m} \frac{(l_{m,t}^{i})^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}$$

Rearranging the budget constraint, we have that

$$\tilde{a}_{m,t+1}^{i} = \frac{R_{n,t+1}}{(1+\pi_{t+1})} \left(\tilde{a}_{m,t}^{i} + w_{t} l_{m,t}^{i} + (1-\tau_{d}) d_{m,t}^{i} - \tau_{m,t}^{i} - a d j_{m,t}^{i} - c_{m,t}^{i} \right).$$

where
$$adj_{m,t}^i = \left(1 - \frac{(1-\delta+r_{k,t+1})(1+\pi_{t+1})}{R_{n,t+1}}\right)k_{m,t+1}^i + \frac{\varphi}{2}\left(\frac{k_{m,t+1}^i}{k_{m,t}^i} - \vartheta_m\right)^2k_{m,t}^i.$$

Thus,

$$v_{m,t}^{i} = f_{m}(Q_{m,t}) \left[\frac{R_{n,t}}{(1+\pi_{t})} \left(\tilde{a}_{m,t-1}^{i} + w_{t-1} l_{m,t-1}^{i} + (1-\tau_{d}) d_{m,t}^{i} - \tau_{m,t-1}^{i} - a d j_{m,t-1}^{i} - c_{m,t-1}^{i} \right) + HC_{m,t}^{i} + D_{m,t}^{i} - T_{m,t}^{i} - ADJ_{m,t}^{i} \right] - \left(\varsigma_{t}\right)^{\frac{-1}{\eta}} \chi_{m} \frac{\left(l_{m,t}^{i}\right)^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}$$

$$v_{m,t}^{i} = \frac{f_{m}(Q_{m,t})R_{n,t}}{(1+\pi_{t})} \left(\tilde{a}_{m,t-1}^{i} + w_{t-1} l_{m,t-1}^{i} + (1-\tau_{d}) d_{m,t}^{i} - \tau_{m,t-1}^{i} - a d j_{m,t-1}^{i} - c_{m,t-1}^{i} \right) + f_{m}(Q_{m,t}) \left(HC_{m,t}^{i} + D_{m,t} - T_{m,t}^{i} - ADJ_{m,t}^{i} \right) - \left(\varsigma_{t}\right)^{\frac{-1}{\eta}} \chi_{m} \frac{\left(l_{m,t}^{i}\right)^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}$$

The value function of the old at time t who was a mature individual at time t-1 can be written as 5

$$v_{o,t}^{i} \mid_{m,t-1} = f_{o}(Q_{o,t}) \left[\tilde{a}_{o,t}^{i} + HC_{o,t}^{i} - T_{o,t}^{i} - ADJ_{o,t}^{i} \right] - (\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}} \chi_{o} \frac{\left(l_{o,t}^{i} \right)^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}}$$

$$v_{o,t}^{i} \mid_{m,t-1} = \frac{f_{o}(Q_{o,t})R_{n,t}}{(1 + \pi_{t})} \left(\tilde{a}_{m,t-1}^{i} + w_{t-1}l_{m,t-1}^{i} + (1 - \tau_{d})d_{m,t}^{i} - \tau_{m,t-1}^{i} - adj_{m,t-1}^{i} - c_{m,t-1}^{i} \right) +$$

$$+ f_{o}(Q_{o,t})HC_{o,t}^{i} - f_{o}(Q_{o,t})T_{o,t}^{i} - f_{o}(Q_{o,t})ADJ_{o,t}^{i} - (\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}} \chi_{o} \frac{\left(l_{o,t}^{i} \right)^{1 + \frac{1}{\nu_{o}}}}{1 + \frac{1}{\nu_{o}}}$$

 $[\]overline{}^5$ We assume that $adj_{m,t-1}^i \approx adj_{m,t-1|o,t}^i$ for an individual transitioning from mature to the old age. The difference might only arise due to the second order effects when the covariance between the rates of return on bonds and capital is considered.

We can now obtain $[\omega_m v_{m,t}^i + (1 - \omega_m) v_{o,t}^i]$,

$$\begin{split} &\omega_{m}v_{m,t}^{i}+(1-\omega_{m})v_{o,t}^{i}=\\ &=\frac{(\omega_{m}f_{m}(Q_{m,t})+(1-\omega_{m})f_{o}(Q_{o,t}))R_{n,t}}{(1+\pi_{t})}\left(\tilde{a}_{m,t-1}^{i}+w_{t-1}l_{m,t-1}^{i}+\right.\\ &\left.+(1-\tau_{d})d_{m,t}^{i}-\tau_{m,t-1}^{i}-adj_{m,t-1}^{i}-c_{m,t-1}^{i}\right)-\omega_{m}(\varsigma_{t})^{\frac{-1}{\eta}}\chi_{m}\frac{\left(l_{m,t}^{i}\right)^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}+\\ &\left.+\omega_{m}\left(f_{m}(Q_{m,t})HC_{m,t}^{i}+f_{m}(Q_{m,t})D_{m,t}-f_{m}(Q_{m,t})T_{m,t}^{i}-f_{m}(Q_{m,t})ADJ_{m,t}^{i}\right)+\\ &\left.+(1-\omega_{m})\left(f_{o}(Q_{o,t})HC_{o,t}^{i}-f_{o}(Q_{o,t})T_{o,t}^{i}-f_{o}(Q_{o,t})ADJ_{o,t}^{i}\right)-(1-\omega_{m})(\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}}\chi_{o}\frac{\left(l_{o,t}^{i}\right)^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}\right) \end{split}$$

Note that

$$\frac{(\omega_{m}f_{m}(Q_{m,t}) + (1 - \omega_{m})f_{o}(Q_{o,t}))R_{n,t}}{(1 + \pi_{t})} = \frac{(\omega_{m}(\varsigma_{t})^{\frac{\eta-1}{\eta}} + (1 - \omega_{m})(\varepsilon_{t}\varsigma_{t})^{\frac{\eta-1}{\eta}})R_{n,t}}{(1 + \pi_{t})}$$

$$= \frac{3_{t}(\varsigma_{t})^{\frac{\eta-1}{\eta}}R_{n,t}}{(1 + \pi_{t})} = \frac{3_{t}f_{m}(Q_{m,t})R_{n,t}}{(1 + \pi_{t})}$$

Therefore, taking expectations \mathbb{E}_{t-1} , and using $(C.10)^6$

$$\begin{split} &\mathbb{E}_{t-1}[\omega_{m}v_{m,t}^{i} + (1-\omega_{m})v_{o,t}^{i}] = \\ &= Q_{m,t-1}\Big(\tilde{a}_{m,t-1}^{i} + w_{t-1}l_{m,t-1}^{i} + (1-\tau_{d})d_{m,t}^{i} - \tau_{m,t-1}^{i} - \mathbb{E}_{t-1}adj_{m,t-1}^{i} - c_{m,t-1}^{i}\Big) + \\ &+ \mathbb{E}_{t-1}\omega_{m}f_{m}(Q_{m,t})\left(HC_{m,t}^{i} + D_{m,t} - T_{m,t}^{i} - ADJ_{m,t}^{i}\right) - \mathbb{E}_{t-1}\omega_{m}(\varsigma_{t})^{\frac{-1}{\eta}}\chi_{m}\frac{\left(l_{m,t}^{i}\right)^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}} \\ &+ \mathbb{E}_{t-1}(1-\omega_{m})f_{o}(Q_{o,t})\left(HC_{o,t}^{i} - T_{o,t}^{i} - ADJ_{o,t}^{i}\right) - \mathbb{E}_{t-1}(1-\omega_{m})(\varepsilon_{t}\varsigma_{t})^{\frac{-1}{\eta}}\chi_{o}\frac{\left(l_{o,t}^{i}\right)^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}} \end{split}$$

⁶Once again, we assume $\mathbb{E}_{t-1} \frac{f_m(Q_{m,t})R_{n,t}}{(1+\pi_t)} adj_{m,t-1}^i \approx Q_{m,t-1}\mathbb{E}_{t-1}adj_{m,t-1}^i$, essentially ignoring the effect of uncertainty on the portfolio allocation between bonds and capital. To a first order approximation the agent is indifferent between holding bonds or capital.

Then, define

$$\begin{split} HC_{m,t}^{i} &\equiv w_{t}l_{m,t}^{i} + \mathbb{E}_{t}\omega_{m}\frac{f(Q_{m,t+1})HC_{m,t+1}^{i}}{Q_{m,t}} + \mathbb{E}_{t}(1-\omega_{m})\frac{f(Q_{o,t+1})HC_{o,t+1}^{i}}{Q_{m,t}} - \\ &- \mathbb{E}_{t}\frac{\left(\varsigma_{t}\right)^{\frac{-1}{\eta}}}{Q_{m,t}}\left[\omega_{m}\chi_{m}\frac{\left(l_{m,t+1}^{i}\right)^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}} + \left(\varepsilon_{t+1}\right)^{\frac{-1}{\eta}}(1-\omega_{m})\chi_{o}\frac{\left(l_{o,t+1}^{i}\right)^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}\right] + \\ &+ \frac{\left(Q_{m,t}\beta\right)^{\frac{1}{1-\eta}}}{Q_{m,t}}\left[\chi_{m}\frac{\left(l_{m,t}^{i}\right)^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}\right] \\ D_{m,t}^{i} &\equiv (1-\tau_{d})d_{m,t}^{i} + \mathbb{E}_{t}\frac{\omega_{m}f(Q_{m,t+1})D_{m,t+1}^{i}}{Q_{m,t}} \\ T_{m,t}^{i} &\equiv \tau_{m,t}^{i} + \mathbb{E}_{t}\frac{\omega_{m}f(Q_{m,t+1})T_{m,t+1}^{i}}{Q_{m,t}} + \mathbb{E}_{t}\frac{(1-\omega_{m})f(Q_{o,t+1})T_{o,t+1}^{i}}{Q_{m,t}} \\ ADJ_{m,t}^{i} &\equiv \mathbb{E}_{t}adj_{m,t}^{i} + \mathbb{E}_{t}\frac{\omega_{m}f(Q_{m,t+1})ADJ_{m,t+1}^{i}}{Q_{m,t}} + \mathbb{E}_{t}\frac{(1-\omega_{m})f(Q_{o,t+1})ADJ_{o,t+1}^{i}}{Q_{m,t}} \end{split}$$

Using these results and adding and subtracting $(Q_{m,t-1}\beta)^{\frac{1}{1-\eta}}\left[\chi_m\frac{(l_{m,t-1}^i)^{1+\frac{1}{\nu_m}}}{1+\frac{1}{\nu_m}}\right]$, we obtain

$$\mathbb{E}_{t-1}[\omega_{m}v_{m,t}^{i} + (1 - \omega_{m})v_{o,t}^{i}] =$$

$$= Q_{m,t-1}\left(\tilde{a}_{m,t-1}^{i} + HC_{m,t-1}^{i} + D_{m,t-1}^{i} - T_{m,t-1}^{i} - ADJ_{m,t-1}^{i} - c_{m,t-1}^{i} - \frac{(Q_{m,t-1}\beta)^{\frac{1}{1-\eta}}}{Q_{m,t-1}}\left[\chi_{m}\frac{(l_{m,t-1}^{i})^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}\right]\right)$$

$$= Q_{m,t-1}\left(W_{m,t-1} - c_{m,t-1}^{i} - \frac{(Q_{m,t-1}\beta)^{\frac{1}{1-\eta}}}{Q_{m,t-1}}\left[\chi_{m}\frac{(l_{m,t-1}^{i})^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}\right]\right)$$

We can now use this result into the Bellman equation to obtain

$$\max v_{m,t}^{i} = \left\{ \left(c_{m,t}^{i} - \chi_{m} \frac{l_{m,t}^{i}^{1 + \frac{1}{\nu_{m}}}}{1 + \frac{1}{\nu_{m}}} \right)^{\eta} + \beta \left[Q_{m,t} \left(W_{m,t} - c_{m,t}^{i} - \frac{(Q_{m,t}\beta)^{\frac{1}{1 - \eta}}}{Q_{m,t}} \chi_{m} \frac{(l_{m,t}^{i})^{1 + \frac{1}{\nu_{m}}}}{1 + \frac{1}{\nu_{m}}} \right) \right]^{\eta} \right\}^{1/\eta}$$

The first order condition with respect to consumption allow us to obtain,

$$c_{m,t}^{i} = \left(1 + (\beta)^{\frac{1}{1-\eta}} Q_{m,t}^{\frac{\eta}{1-\eta}}\right)^{-1} W_{m,t} = \varsigma_t W_{m,t}$$

We can now replace the solution for $c_{m,t}^i$, which matches our conjecture, into the value function to obtain

$$v_{m,t}^{i} = \left(\varsigma_{t}\right)^{\frac{-1}{\eta}} \left(c_{m,t}^{i} - \chi_{m} \frac{\left(l_{m,t}^{i}\right)^{1 + \frac{1}{\nu_{m}}}}{1 + \frac{1}{\nu_{m}}}\right)$$

Combining (C.8) and (C.10) we obtain the condition that determines the marginal propensity to consume of the mature agents.

$$Q_{m,t} = ((\varsigma_t)^{-1} - 1)^{\frac{1-\eta}{\eta}} (\beta)^{\frac{-1}{\eta}}$$
$$((\varsigma_t)^{-1} - 1)^{\frac{1-\eta}{\eta}} = \mathbb{E}_t \left(\frac{\mathfrak{Z}_{t+1} R_{n,t+1}(\beta)^{\frac{1}{\eta}} (\varsigma_t)^{\frac{\eta-1}{\eta}}}{(1 + \pi_{t+1})} \right)$$

Finally, from the first order conditions of (C.7) labor is set such that

$$l_{m,t}^i = \left(\frac{w_t}{\chi_m}\right)^{\nu_m},$$

and, to a first order approximation the individual is indifferent between holding bonds or capital. The no-arbitrage condition on investment posits that the expected return on capital should equalize the expected return on bonds, that is,

$$\left(\frac{R_{n,t+1}}{1+\pi_{t+1}}\right) = \left(\frac{(1-\delta) + r_{k,t+1} - \frac{\varphi}{2} \left(\frac{k_{m,t+2}^{i}}{k_{m,t+1}^{i}} - \vartheta_{m}\right)^{2} + \varphi\left(\frac{k_{m,t+2}^{i}}{k_{m,t+1}^{i}} - \vartheta_{m}\right)\frac{k_{m,t+2}^{i}}{k_{m,t+1}^{i}}}{\left(1 + \varphi\left(\frac{k_{m,t+1}^{i}}{k_{m,t}^{i}} - \vartheta_{m}\right)\right)}\right)$$
(C.13)

If the constraint binds, we no longer have an interior solution. In this case the consumption policy function can be easily obtained from the budget constraint of the agent. The labor optimality condition remains the same.

C.3 Young Agents

For the problem of the young we follow a similar procedure to obtain

$$f_y(Q_{y,t}) \equiv (\varepsilon_{y,t}\varsigma_t)^{\frac{\eta-1}{\eta}}$$
 (C.14)

$$Q_{y,t} = ((\varepsilon_{y,t}\varsigma_t)^{-1} - 1)^{\frac{1-\eta}{\eta}} (\beta)^{\frac{-1}{\eta}}$$
(C.15)

$$((\varepsilon_{y,t}\zeta_{t})^{-1} - 1)^{\frac{1-\eta}{\eta}} = \mathbb{E}_{t} \left(\frac{\mathfrak{Z}_{y,t+1}R_{n,t+1}(\beta)^{\frac{1}{\eta}}(\zeta_{t})^{\frac{\eta-1}{\eta}}}{(1+\pi_{t+1})} \right)$$
(C.16)

$$c_{y,t}^i = \varepsilon_{y,t} \varsigma_t \left[\tilde{a}_{y,t}^i + H C_{y,t}^i - T_{y,t}^i - A D J_{y,t}^i \right] = \varepsilon_{y,t} \varsigma_t W_{y,t}$$
 (C.17)

$$v_{y,t}^{i} = \left(\varepsilon_{y,t}\varsigma_{t}\right)^{\frac{-1}{\eta}} \left(c_{y,t}^{i} - \chi_{y} \frac{\left(l_{y,t}^{i}\right)^{1 + \frac{1}{\nu_{y}}}}{1 + \frac{1}{\nu_{y}}}\right) \tag{C.18}$$

where $\mathfrak{Z}_{y,t+1} = (\omega_y \varepsilon_{y,t+1}^{(\eta-1)/\eta} + (1 - \omega_y))$ and,

$$\begin{split} HC_{y,t}^{i} & \equiv w_{in,t}\xi_{y}l_{y,t}^{i} + \mathbb{E}_{t}\omega_{y}\frac{f(Q_{y,t+1})HC_{y,t+1}^{i}}{Q_{y,t}} + \mathbb{E}_{t}(1-\omega_{y})\frac{f(Q_{m,t+1})HC_{m,t+1}^{i}}{Q_{y,t}} - \\ & - \mathbb{E}_{t}\frac{\left(\varsigma_{t}\right)^{\frac{-1}{\eta}}}{Q_{y,t}}\left[\omega_{y}\varepsilon_{y,t}^{\frac{-1}{\eta}}\chi_{y}\frac{\left(l_{y,t+1}^{i}\right)^{1+\frac{1}{\nu_{y}}}}{1+\frac{1}{\nu_{y}}} + (1-\omega_{y})\chi_{m}\frac{\left(l_{m,t+1}^{i}\right)^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}\right] + \\ & + \frac{\left(Q_{y,t}\beta\right)^{\frac{1}{1-\eta}}}{Q_{y,t}}\left[\chi_{y}\frac{\left(l_{y,t}^{i}\right)^{1+\frac{1}{\nu_{y}}}}{1+\frac{1}{\nu_{y}}}\right] \\ T_{y,t}^{i} & \equiv \tau_{y,t}^{i} + \mathbb{E}_{t}\frac{\omega_{y}f(Q_{y,t+1})T_{y,t+1}^{i}}{Q_{y,t}} + \mathbb{E}_{t}\frac{(1-\omega_{y})f(Q_{m,t+1})(T_{m,t+1}^{i}-D_{m,t+1}^{i})}{Q_{y,t}} \\ ADJ_{y,t}^{i} & \equiv \mathbb{E}_{t}adj_{y,t}^{i} + \mathbb{E}_{t}\frac{\omega_{y}f(Q_{y,t+1})ADJ_{y,t+1}^{i}}{Q_{y,t}} + \mathbb{E}_{t}\frac{(1-\omega_{y})f(Q_{m,t+1})ADJ_{m,t+1}^{i}}{Q_{y,t}} \end{split}$$

Finally, labor is set such that

$$l_{y,t}^i = \left(\frac{\xi_y w_{in,t}}{\chi_y}\right)^{\nu_y},$$

and, the no-arbitrage condition on investment posits that the expected return on capital should

equalize the expected return on bonds, that is,

$$\left(\frac{R_{n,t+1}}{1+\pi_{t+1}}\right) = \left(\frac{(1-\delta) + r_{k,t+1} - \frac{\varphi}{2} \left(\frac{k_{y,t+2}^{i}}{k_{y,t+1}^{i}} - \vartheta_{m}\right)^{2} + \varphi\left(\frac{k_{y,t+2}^{i}}{k_{y,t+1}^{i}} - \vartheta_{m}\right)\frac{k_{y,t+2}^{i}}{k_{y,t+1}^{i}}}{\left(1 + \varphi\left(\frac{k_{y,t+1}^{i}}{k_{y,t}^{i}} - \vartheta_{m}\right)\right)}\right)$$
(C.19)

If the borrowing constraint for the young binds then equations (C.14) - (C.18) no longer describe the optimal conditions. The consumption policy function can be easily obtained from the budget constraint of the agent. In this case $c_{y,t}^i = w_t \xi_y l_{y,t}^i - \tau_{y,t}^i$.

C.4 Aggregation

In this Section we show that we can linearly aggregate the optimal choices of individuals across each age group, such that for a variable $x_{z,t}$ we have that $x_{z,t} = \int_0^{N_{z,t}} x_{y,t}^i di$.

Firstly we must ensure that at steady state adjustment costs are zero. Given the arbitrage conditions (C.1), (C.13), and its counterpart for the young problem, we have that the ratio of capital for any agent within a type is constant, which is to say that $\frac{k_{y,t+1}^i}{k_{y,t}^i} = \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}}$, $\frac{k_{m,t+1}^i}{k_{m,t}^i} = \frac{\hat{k}_{w,t+1}}{\hat{k}_{w,t}}$, and $\frac{\hat{k}_{o,t+1}^i}{\hat{k}_{o,t}} = \frac{\hat{k}_{o,t+1}}{\hat{k}_{o,t}}$, where $\frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}}$, $\frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}}$, and $\frac{\hat{k}_{o,t+1}}{\hat{k}_{o,t}}$ define given age-group specific values for the ratio of physical capital holdings over time. Then given that individuals are born with no capital at steady state we have that

$$k_{y,t+1} = \int_0^{N_{y,t}} k_{y,t+1}^i = \int_0^{N_{y,t+1}} k_{y,t+2}^i = k_{y,t+2} = k_{y,SS}$$

As the young individuals who become mature are selected randomly

$$k_{y,SS} = \int_0^{N_{y,t+1}} k_{y,t+2}^i = \int_0^{\omega_y N_{y,t+1}} \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} k_{y,t+1}^i = \omega_y \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} \int_0^{N_{y,t}} k_{y,t+1}^i = \omega_y \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} k_{y,SS}^i$$

Hence, $\frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}}\mid_{SS}=\frac{1}{\omega_y}.$ For mature agents we have that

$$k_{m,t+1} = \int_0^{N_{m,t}} k_{m,t+1}^i = \int_0^{N_{m,t+1}} k_{m,t+2}^i = k_{m,t+2} = k_{my,SS}$$

where

$$k_{m,SS} = \int_{0}^{N_{m,t+1}} k_{m,t+2}^{i} = \int_{0}^{\omega_{m}N_{m,t+1}} \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} k_{m,t+1}^{i} + \int_{0}^{(1-\omega_{y})N_{y,t+1}} \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} k_{y,t+1}^{i} = \dots$$

$$\dots = \omega_{m} \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} \int_{0}^{N_{m,t}} k_{m,t+1}^{i} + (1-\omega_{y}) \frac{\hat{k}_{y,t+1}}{\hat{k}_{y,t}} \int_{0}^{N_{y,t}} k_{y,t+1}^{i} = \dots$$

$$\dots = \omega_{m} \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} k_{m,SS} + \frac{(1-\omega_{y})}{\omega_{y}} k_{y,SS}.$$

As a result, we have that

$$\frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} \mid_{SS} = \frac{1}{\omega_m} \left(1 - \frac{(1 - \omega_y)}{\omega_y} \frac{k_{y,SS}}{k_{m,SS}} \right).$$

Analogously, we have that

$$\frac{\hat{k}_{o,t+1}}{\hat{k}_{o,t}} \mid_{SS} = \frac{1}{\omega_o} \left(1 - (1 - \omega_m) \frac{\hat{k}_{m,t+1}}{\hat{k}_{m,t}} \mid_{SS} \frac{k_{m,SS}}{k_{o,SS}} \right).$$

Thus, if we set

$$\vartheta_{y} = \frac{1}{\omega_{y}}$$

$$\vartheta_{m} = \frac{1}{\omega_{m}} \left(1 - \frac{(1 - \omega_{y})}{\omega_{y}} \frac{k_{y,SS}}{k_{m,SS}} \right)$$

$$\vartheta_{o} = \frac{1}{\omega_{o}} \left(1 - (1 - \omega_{m}) \vartheta_{m} \frac{k_{m,SS}}{k_{o,SS}} \right)$$

we ensure that at steady state capital adjustment costs are zero. At steady state agents accumulate or reduce capital at a constant rate while within a group $z \in \{y, m, o\}$. Nonetheless, as individuals transition across groups through their life cycle, the aggregate capital holdings of

each group remain constant and no adjust cost of capital is paid.

Ensuring that at steady state adjustment costs are zero is important for aggregation since the only non-linear term left in the consumption decision is the quadratic term in the adjustment cost condition. As we solve a linearized version of the model around the steady state this quadratic term disappears such that the choice variables across agents within a group can be easily aggregated to find a condition for each group. Consequently, for instance, the aggregate consumption of all old agents at time t is simply given by

$$c_{o,t} = \varepsilon_t \varsigma_t \left[\tilde{a}_{o,t} + HC_{o,t} - T_{o,t} - ADJ_{o,t} \right].$$

where we excluded the quadratic terms which are irrelevant in a first order approximated solution and thus, $ADJ_{o,t} = \tilde{adj}_{o,t} + \frac{(1+\pi_{t+1})\omega_o}{R_{n,t+1}}ADJ_{o,t+1}$ and $\tilde{adj}_{o,t}^i = \left(1 - \frac{(1-\delta+r_{k,t+1})(1+\pi_{t+1})}{R_{n,t+1}}\right)k_{o,t+1}^i$. Therefore, the equilibrium conditions can be defined without explicitly incorporating the heterogeneity within age groups.

As some young agents become mature and some mature agents become old every period, when we aggregate and discard the quadratic adjustment terms, the flow of assets are given by

$$k_{y,t+1} + \tilde{b}_{y,t+1} = \omega_y(\tilde{a}_{y,t} + l_{y,t}\xi_y w_{in,t} + \tau_{y,t} - c_{y,t})$$
(C.20)

$$\tilde{a}_{y,t} = k_{y,t} \left[(1 - \delta) + r_{k,t} \right] + \tilde{b}_{y,t} \frac{R_{n,t+1}}{1 + \pi_{t+1}}$$
(C.21)

$$k_{m,t+1} + \tilde{b}_{m,t+1} = \omega_m(\tilde{a}_{m,t} + l_{m,t}w_t + d_{m,t} + \tau_{m,t} - c_{m,t}) + \dots$$
 (C.22)

$$\cdots + (1 - \omega_y)(\tilde{a}_{y,t} + l_{y,t}\xi_y w_t + \tau_{y,t} - c_{y,t})$$
 (C.23)

$$\tilde{a}_{m,t} = k_{m,t} \left[(1 - \delta) + r_{k,t} \right] + \tilde{b}_{m,t} \frac{R_{n,t+1}}{1 + \pi_{t+1}}$$
(C.24)

$$k_{o,t+1} + \tilde{b}_{o,t+1} = \tilde{a}_{o,t} + \xi_o l_{o,t} w_t + \text{tr}_{o,t} - c_{o,t} + \dots$$
 (C.25)

$$\cdots + (1 - \omega_m)(\tilde{a}_{m,t} + l_{m,t}w_t + d_{m,t} + \tau_{m,t} - c_{m,t})$$
 (C.26)

$$\tilde{a}_{o,t} = k_{o,t} \left[(1 - \delta) + r_{k,t} \right] + \tilde{b}_{o,t} \frac{R_{n,t+1}}{1 + \pi_{t+1}}$$
(C.27)

We then define the stochastic discount factor for the mature group as

$$Q_{t}^{m} = \beta \mathfrak{Z}_{t+1} \frac{\left[\omega_{m} \left(c_{m,t+1} - \chi_{m} \frac{l_{m,t+1}^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}\right) + (1-\omega_{m})\varepsilon_{t+1}^{\frac{-1}{\eta}} \left(c_{o,t+1} - \chi_{o} \frac{l_{o,t+1}^{1+\frac{1}{\nu_{o}}}}{1+\frac{1}{\nu_{o}}}\right)\right]^{(1-\eta)}}{\left(c_{m,t} - \chi_{m} \frac{l_{m,t}^{1+\frac{1}{\nu_{m}}}}{1+\frac{1}{\nu_{m}}}\right)^{(1-\eta)}}$$

Finally, given that we are interested in a solution under a linear approximation,

$$\begin{pmatrix} k_{m,t+1}^{i} - \vartheta_{m} \end{pmatrix} = \begin{pmatrix} \hat{k}_{m,t+1} - \vartheta_{m} \end{pmatrix} \approx \vartheta_{m} \begin{pmatrix} \hat{k}_{m,t+1} - \hat{k}_{m,t+1} \mid_{SS} - \frac{\hat{k}_{m,t} - \hat{k}_{m,t} \mid_{SS}}{\hat{k}_{m,t} \mid_{SS}} \end{pmatrix} \\
= \vartheta_{m} \begin{pmatrix} \frac{1}{N_{m,t}} \frac{k_{m,t+1} - k_{m,t+1} \mid_{SS}}{k_{m,t+1} \mid_{SS}} - \frac{1}{N_{m,t-1}} \frac{k_{m,t} - k_{m,t} \mid_{SS}}{k_{m,t} \mid_{SS}} \end{pmatrix} \\
\approx \vartheta_{m} \begin{pmatrix} \frac{k_{m,t+1} N_{m,t-1}}{k_{m,t} N_{m,t}} - 1 \end{pmatrix}$$

then the aggregated arbitrage condition for mature agents becomes

$$\frac{R_{n,t+1}}{1+\pi_{t+1}} = \frac{(1-\delta) + r_{k,t+1} + \varphi \vartheta_m^2 \left(\frac{k_{m,t+1}}{k_{m,t}} \frac{N_{m,t}}{N_{m,t+1}} - 1\right) \frac{k_{m,t+1}}{k_{m,t}} \frac{N_{m,t}}{N_{m,t+1}}}{\left(1 + \vartheta_m \varphi \left(\frac{k_{m,t}}{k_{m,t-1}} \frac{N_{m,t-1}}{N_{m,t}} - 1\right)\right)}$$
(C.28)

Given the hump-shaped life-cycle earnings profile, the young wants to borrow, the mature wants to save for retirement and the old dissaves (see Constantinides et al. (2002) for an simple OLG model with the same features). Thus, $\tilde{a}_{y,t} = 0$ and from (C.20) we obtain the consumption of the young. (C.23) simplifies to $k_{m,t+1} + \tilde{b}_{m,t+1} = \omega_m(\tilde{a}_{m,t} + l_{m,t}w_t + d_{m,t} + \tau_{m,t} - c_{m,t})$.

⁷Given the probabilistic nature of the death transition, a very small share of old individuals might live for a very long time. In such cases assets would eventually be completely consumed and the borrowing constraint would bind. As the mass of old individuals in this situation is very small, for simplicity we assume the intermediary that offers the annuity provides consumption to the old individuals living for too long such that the condition (C.5) always holds within this age group.

D More on Calibration

This section reports the values of the entire set of parameters of the model. Table D.1 reports the calibration choices on the set of parameters that govern the demographics in the model, and Table D.2 reports the calibration of the parameters that define the life cycle of hours worked and wages. Finally, Table D.3 reports the calibration choices of the block of parameters that comes with the structure of a standard open-economy New Keynesian model.

Table D.1: Calibration - Demographics

Parameter	Value	Target
Birth Rate of New Young Agents	$\omega_n = 0.0024$	Share of Young in Population
Probability Transition from Young to Mature	$1 - \omega_y = 0.0250$	Avg. Number of Years as Young: 10y
Probability Transition from Mature to Old	$1 - \omega_m = 0.0071$	Avg. Number of Years as Mature: 30y
Death Probability of Old Agents	$1 - \omega_o = 0.0274$	Share of Old in Population
Relative Size Population Home Economy	$N/N^u = 0.02$	Average Size U.S. State

Table D.2: Calibration - Life-Cycle of Hours and Wages

Parameter	Value	Target
Complementarity Experience Labor and Capital	$\kappa = 0.2$	Jaimovich et $al.$ (2013)
Complementarity Inexperience Labor and Capital	$\sigma = 0.7$	Jaimovich et $al.$ (2013)
Weight Experience Labor	$\alpha = 0.27$	Share of Capital $= 0.33$
Weight Inexperience Labor	$\mu = 0.36$	Wage Young = 71% Wage Mature
Disutility Labor for Young Agents	$\chi_y = 2.4$	Fraction of Hours Worked = 0.324
Disutility Labor for Mature Agents	$\chi_m = 131.9$	Fraction of Hours Worked = 0.35
Disutility Labor for Old Agents	$\chi_o = 14.5$	Fraction of Hours Worked $= 0.08$
Efficiency Units of Hours for Young Agents	$\xi_y = 1$	Normalization
Efficiency Units of Hours for Mature Agents	$\xi_m = 1$	Normalization
Efficiency Units of Hours for Old Agents	$\xi_o = 0.72$	Wage Old = 72% Wage Mature
Labor Supply Elasticity for Young Agents	$\nu_y = 0.71$	Weighted Average Labor Supply Elasticity $= 0.4$
Labor Supply Elasticity for Mature Agents	$\nu_m = 0.2$	Chetty et <i>al.</i> (2013)
Labor Supply Elasticity for Old Agents	$\nu_o = 0.75$	Rogerson and Wallenius (2013)

Table D.3: Calibration - Standard Parameters

Parameter	Value	Target/Source
Time Discount Factor	$\beta=0.995$	Standard Value
Elasticity Intertemporal Substitution	$\eta = -9$	EIS = 0.1
Capital Depreciation Rate	$\delta = 0.025$	Standard Value
Capital Adjustment Cost	$\varphi = 200$	Peak Investment Response After 8 Quarters
Home Bias in Consumption & Investment	$\lambda = 0.69$	Nakamura and Steinsson (2014)
Elasticity Substitution Home & Foreign Consumption	$\psi_c = 2$	Nakamura and Steinsson (2014)
Elasticity Substitution Home & Foreign Investment	$\psi_i = 2$	$\psi_i = \psi_c$
Elasticity Substitution Across Varieties	$\epsilon = 9$	Standard Value
Calvo Parameter	$\zeta = 0.75$	Standard Value
Dividend Tax Rate	$\tau_d = 0.9394$	Mature Agents Receive 60% Total Dividends
Steady-State Government Spending to Output Ratio	$\frac{G_{H,SS} + G_{F,SS}}{Y_{SS}^u} = 0.204$	Data
Persistence Government Spending Shock	$\rho_G = 0.953$	Data
Inertia of Government Debt	$\rho_{bg} = 0.95$	Dynamic Response to Spending of Government Debt
Response to Spending of Government Debt	$\phi_G = 4.5$	Dynamic Response to Spending of Government Debt
Response to Spending of Taxation	$\phi_T = 0.01$	Dynamic Response to Spending of Taxation
Inertia of Taylor Rule	$\psi_R = 0.8$	Clarida et $al.$ (2000)
Taylor Rule Response to Inflation	$\psi_{\pi}=1.5$	Clarida et $al.$ (2000)
Taylor Rule Response to Output Gap	$\psi_Y = 0.2$	Clarida et $al.$ (2000)

E The Model Implications with Progressive Taxes

In this section we present the extension of the model with progressive income taxation. The household problem now changes such that at time t the agent i of the age group $z = \{y, m, o\}$ chooses consumption $c_{z,t}^i$, labor supply $l_{z,t}^i$, capital $k_{z,t+1}^i$, and nominal bonds $b_{z,t+1}^i$ to maximize

$$\max_{\substack{c_{z,t}^{i}, l_{z,t}^{i}, k_{z,t+1}^{i}, b_{z,t+1}^{i} \\ v_{z,t}^{i} = \left\{ \left(c_{z,t}^{i} - \chi_{z} \frac{l_{z,t}^{i}}{1 + \frac{1}{\nu_{z}}} \right)^{\eta} + \beta \mathbb{E}_{t} [v_{z',t+1}^{i} \mid z]^{\eta} \right\}^{1/\eta}} \\ \text{s.t.} \quad P_{t} c_{z,t}^{i} + P_{I,t} k_{z,t+1}^{i} + P_{I,t} \varphi_{z,t+1}^{i} + b_{z,t+1}^{i} + P_{t} \tau_{z,t}^{i} = \dots \\ \dots = a_{z,t}^{i} + \varpi(W_{z,t} \xi_{z} l_{z,t}^{i})^{1-\tau_{PI}} + (1-\tau_{d}) d_{z,t}^{i} \mathbb{I}_{\{z=m\}}} \\ \begin{cases} a_{z,t}^{i} = P_{I,t} (1-\delta) k_{z,t}^{i} + R_{k,t} k_{z,t}^{i} + R_{n,t} b_{z,t}^{i} & \text{if } z = \{y,m\} \\ a_{z,t}^{i} = \frac{1}{\omega_{z}} \left[P_{I,t} (1-\delta) k_{z,t}^{i} + R_{k,t} k_{z,t}^{i} + R_{n,t} b_{z,t}^{i} \right] & \text{if } z = \{o\} \end{cases} \\ k_{z,t+1}^{i} = (1-\delta) k_{z,t}^{i} + x_{z,t}^{i} - \varphi_{z,t+1}^{i} \\ k_{z,t+1}^{i} \geqslant 0, \ b_{z,t}^{i} \geqslant 0 \\ c_{z,t}^{i} = \left[\lambda^{1/\psi_{c}} c_{H,z,t}^{i} \frac{\psi_{c-1}}{\psi_{c}} + (1-\lambda)^{1/\psi_{c}} c_{F,z,t}^{i} \frac{\psi_{c-1}}{\psi_{c}} \right] \frac{\psi_{c}}{\psi_{c-1}} \\ x_{z,t}^{i} = \left[\lambda^{1/\psi_{I}} x_{H,z,t}^{i} \frac{\psi_{I}^{i-1}}{\psi_{I}} + (1-\lambda)^{1/\psi_{I}} x_{F,z,t}^{i} \frac{\psi_{I}^{i-1}}{\psi_{I}} \right] \frac{\psi_{I}}{\psi_{I}^{i-1}} . \end{cases}$$

With respect to the baseline model, we modify the net labour income earnings of households such that their after tax nominal labor income now equals $\varpi(W_{z,t}\xi_z l_{z,t}^i)^{1-\tau_{PI}}$, where $W_{z,t}$ denotes the wage of agents of the age group $z = \{y, m, o\}$ and ξ_z denotes the age-specific efficiency units of hours worked, ϖ controls the level of income tax and τ_{PI} determines the progressivity of the tax system. We calibrate τ_{PI} following Heathcote, Storesletten, and Violante (2017), which derive the degree of progressivity of the U.S. tax system using the NBERs TAXSIM program and by looking into the sum of pre-government income and public transfers minus federal and state income taxes.

The budget constraint of the government is also modified and now reads

$$P_{H,t}G_{H,t} + P_{F,t}G_{F,t} + B_{g,t+1} = B_{g,t}R_{n,t} + P_{t}T_{t} + P_{t}^{\star}T_{t}^{\star} + \Psi_{t} + \Psi_{t}^{\star} + \tau_{d}(D_{m,t} + D_{m,t}^{\star})$$

where $\Psi_t = \sum_{z=\{y,m,o\}} [W_{z,t}\xi_z l_{z,t} - \varpi(W_{z,t}\xi_z l_{z,t})^{1-\tau_{PI}}]$ is the revenue from labor income taxation in the home region and $\Psi_t^* \sum_{z=\{y,m,o\}} [W_{z,t}^*\xi_z l_{z,t}^* - \varpi(W_{z,t}^*\xi_z l_{z,t}^*)^{1-\tau_{PI}}]$ is the revenue from labor income taxation in the foreign region. Direct transfers/lump-sum taxes $(T_t \text{ and } T_t^*)$ are still used to re-distribute dividend taxation and ensure the steady state in the extension model is similar to the benchmark case.

We assume that the government follows a fiscal rule which determines the response of debt and tax to the exogenous changes in government spending. In this specification only labor income taxation changes to ensure the government budget constraint holds. Thus, ϖ and government borrowing adjust such that

$$\frac{\widehat{B}_{g,t+1}}{Y_{SS}^u} = \rho_{bg} \frac{\widehat{B}_{g,t}}{Y_{SS}^u} + \phi_G \frac{\widehat{P}_{H,t} \widehat{G}_{H,t}}{Y_{SS}^u} + \phi_G \frac{\widehat{P}_{F,t} \widehat{G}_{F,t}}{Y_{SS}^u} + \phi_\Psi \frac{\widehat{\Psi}}{Y_{SS}^u} + \phi_T \frac{\widehat{\Psi}_t^*}{Y_{SS}^u}$$

where Y_{SS}^u denotes the steady-state value of the output of the monetary union, and $\hat{Z}_t \equiv Z_t - Z_{SS}$ denotes the absolute deviation from steady-state. As in the benchmark case, the parameters ρ_{bg} , ϕ_G , and ϕ_{Ψ} control to what extent debt and labor income tax finance an increase in government spending and how long the government takes to raise taxes to bring government debt back to the steady state level. For instance, when $\phi_G = 0$, $\rho_{bg} = 0$, and $\phi_{\Psi} = 0$, spending is fully financed through taxes. As ϕ_G and ρ_{bg} increase, government spending becomes partially financed through debt. As ϕ_{Ψ} increases, debt levels above steady-state trigger tax adjustments through increases in ϖ , affecting both the home and the foreign countries. Although the degree of progressivity of the tax system is kept constant, when agents experience a rise in their labor income following a government spending shock, then they also face a relatively larger tax burden.

We find that in this version of the model the level of local fiscal multipliers are slightly

reduced, going down from 1.46 to 1.35, as the government spending is not anymore financed with a lump-sum tax, but rather with a distortionary labor income tax. For the same reason the introduction of a progressive labor income tax slightly reduces the age sensitivity of local multipliers, that drops from 2.7% to 2.5%. Overall the main takeaways from the model are unchanged when we consider progressive income taxation.

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