Online Appendix to "Collateral Shocks"

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This appendix is divided into five sections. Section A1 derives the full model. Section A2 lists all equilibrium equations. Section A3 describes the data and observation equations. Section A4 discusses the estimation of the parameters. Finally, Section A5 analyzes the role played by the other structural shocks of the model.

A1 Derivation of the Baseline Model

A1.1 Patient Households

The representative patient household maximizes utility subject to its budget constraint. The first-order conditions (FOCs) with respect to consumption C_t^p, housing $H_t^p = \bar{H}_t^p$, and deposits D_t are respectively

$$
\begin{aligned}
0 &= \Lambda_{z,t}^p(1+\tau^c)P_t - \zeta_{c,t}/(C_t^p - b_c^p C_{t-1}^p) + b_c^p \beta^p E_t \zeta_{c,t+1}/(C_{t+1}^p - b_c^p C_t^p),\\
0 &= 1/H_t^p - \Lambda_{z,t}^p Q_t^h + \beta^p E_t \Lambda_{z,t+1}^p Q_{t+1}^h,\\
0 &= \Lambda_{z,t}^p P_t - \beta^p P_t E_t \Lambda_{z,t+1}^p R_{t+1}.
\end{aligned}
$$

A1.2 Impatient Households

Workers.—The representative impatient household maximizes the utility of its workers subject to their budget constraint. The FOCs with respect to consumption C_t^i and housing services H_t^i are respectively

$$
\begin{aligned}
0 &= \Lambda_{z,t}^i(1+\tau^c)P_t - \zeta_{c,t}/(C_t^i - b_c^i C_{t-1}^i) + b_c^i \beta^i E_t \zeta_{c,t+1}/(C_{t+1}^i - b_c^i C_t^i),\\
0 &= 1/H_t^i - \Lambda_{z,t}^i P_t r_t^h.
\end{aligned}
$$

Real Estate Broker.—The real estate broker chooses a quantity of housing to maximize profit subject to the housing adjustment costs. The FOC with respect to $\bar{H}_t^i$ is

$$
\begin{aligned}
0 = \Lambda_{z,t}^i Q_t^h - \Lambda_{z,t}^i Q_t^h &\left[1 + S^h\left(\zeta_{h,t}\frac{\bar{H}_t^i}{\bar{H}_{t-1}^i}\right) + \zeta_{h,t}\frac{\bar{H}_t^i}{\bar{H}_{t-1}^i} S^{h\prime}\left(\zeta_{h,t}\frac{\bar{H}_t^i}{\bar{H}_{t-1}^i}\right)\right]\\
&+ \beta^i E_t \Lambda_{z,t+1}^i Q_{t+1}^h \zeta_{h,t+1}\left(\frac{\bar{H}_{t+1}^i}{\bar{H}_t^i}\right)^2 S^{h\prime}\left(\zeta_{h,t+1}\frac{\bar{H}_{t+1}^i}{\bar{H}_t^i}\right).
\end{aligned}
$$

Homeowners.—A non-defaulting homeowner j maximizes the present discounted value of dividends

$$V^i_{j,t} = \max_{\bar{H}^i_{j,t}, B^i_{j,t}} \left\{ \Delta^i_{j,t} + \beta^i E_t \Lambda^i_{z,t+1} / \Lambda^i_{z,t} \max\{0, V^i_{j,t+1}\} \right\},$$

$$\text{subject to} \quad N^i_{j,t} = R^h_t \omega^i_{j,t} Q^h_{t-1} \bar{H}^i_{j,t-1} - R^i_{j,t} B^i_{j,t-1},$$

$$Q^h_t \bar{H}^i_{j,t} + \Delta^i_{j,t} = N^i_{j,t} + P_t r^h_t \bar{H}^i_{j,t} + B^i_{j,t},$$

and the bank participation constraint. Substitute the two constraints into the value function

$$V^i_{j,t} = \max_{\bar{H}^i_{j,t}, B^i_{j,t}} \Big\{ R^h_t \omega^i_{j,t} Q^h_{t-1} \bar{H}^i_{j,t-1} - R^i_{j,t} B^i_{j,t-1} + P_t r^h_t \bar{H}^i_{j,t} + B^i_{j,t} - Q^h_t \bar{H}^i_{j,t}$$
$$+ \beta^i E_t \Lambda^i_{z,t+1} / \Lambda^i_{z,t} \max\{0, V^i_{j,t+1}\} \Big\}.$$

Following Ferrante (2019), we define $\eta^i_{j,t} \equiv B^i_{j,t} / \bar{H}^i_{j,t}$ and $g^i_{j,t} \equiv \bar{H}^i_{j,t} / \bar{H}^i_{j,t-1}$. Since $V^i_{j,t}$ is linearly homogeneous in $\bar{H}^i_{t-1}$, we rewrite the scaled value function $v^i_{j,t} \equiv V^i_{j,t} / \bar{H}^i_{t-1}$

$$v^i_{j,t} = \max_{g^i_{j,t}, \eta^i_{j,t}} \Bigg\{ R^h_t \omega^i_{j,t} Q^h_{t-1} - R^i_{j,t} \eta^i_{j,t-1} + P_t r^h_t g^i_{j,t} + \eta^i_{j,t} g^i_{j,t} - Q^h_t g^i_{j,t}$$
$$+ g^i_{j,t} \beta^i E_t \frac{\Lambda^i_{z,t+1}}{\Lambda^i_{z,t}} \left[\int_{\bar{\omega}^i_{j,t+1}}^{\infty} v^i_{j,t+1} dF^i(\omega^i_{j,t+1}) + (1 - \phi^i_t) \int_0^{\bar{\omega}^i_{j,t+1}} R^h_{t+1} \omega^i_{j,t+1} Q^h_t dF^i(\omega^i_{j,t+1}) \right] \Bigg\}.$$

The FOCs with respect to $g^i_{j,t}$ and $\eta^i_{j,t}$ are respectively

$$0 = P_t r^h_t + \eta^i_{j,t} - Q^h_t + \beta^i E_t \frac{\Lambda^i_{z,t+1}}{\Lambda^i_{z,t}} \int_0^{\infty} v^i_{j,t+1} dF^i(\omega^i_{j,t+1}),$$

$$1 = \beta^i E_t \frac{\Lambda^i_{z,t+1}}{\Lambda^i_{z,t}} [1 - F^i(\bar{\omega}^i_{j,t+1})] \left(\frac{\partial R^i_{j,t+1}}{\partial \eta^i_{j,t}} \eta^i_{j,t} + R^i_{j,t+1} \right).$$

Substitute the FOC for $g^i_{j,t}$ into the value function and multiply both sides by $\bar{H}^i_{j,t-1}$ to obtain the non-scaled value function

$$V^i_{j,t} = \{ R^h_t \omega^i_{j,t} Q^h_{t-1} \bar{H}^i_{j,t-1} - R^i_{j,t} B^i_{j,t-1} \} = N^i_{j,t}.$$

A default threshold $\bar{\omega}^i_{j,t}$ is such that the value of assets homeowner j pledged as collateral is lower than the cost of servicing debt. That is, homeowner j defaults when $\phi^i_{t-1} R^h_t \omega^i_{j,t} Q^h_{t-1} \bar{H}^i_{j,t-1} - R^i_{j,t} B^i_{j,t-1} = 0$, that is when $V^i_{j,t}(\bar{\omega}^i_{j,t}) = (1 - \phi^i_{t-1}) R^h_t \bar{\omega}^i_{j,t} Q^h_{t-1} \bar{H}^i_{j,t-1}$. This implies the following default threshold

$$\bar{\omega}^i_{j,t} = R^i_{j,t} B^i_{j,t-1} / (R^h_t \phi^i_{t-1} Q^h_{t-1} \bar{H}^i_{j,t-1}).$$

Finally, define $G^i(\bar{\omega}^i_{j,t+1}) \equiv \int_0^{\bar{\omega}^i_{j,t+1}} \omega^i_{j,t+1} dF^i(\omega^i_{j,t+1})$, use the bank participation constraint to compute the partial derivative $\partial R^i_{j,t+1}/\partial \eta^i_{j,t}$, and plug it into the FOC for $\eta^i_{j,t}$

$$\begin{aligned}1 = \beta^i E_t \Lambda^i_{z,t+1}/\Lambda^i_{z,t} \Big\{ & R_{t+1} - (1-\mu^i) R^i_{j,t+1} G^{i\prime}(\bar{\omega}^i_{j,t+1}) \\ & + F^{i\prime}(\bar{\omega}^i_{j,t+1})\bar{\omega}^i_{j,t+1}(1 - F^i(\bar{\omega}^i_{j,t+1})^{-1} \left[R_{t+1} - (1-\mu^i) G^i(\bar{\omega}^i_{j,t+1}) R^h_{t+1} \phi^i_t Q^h_t \bar{H}^i_{j,t}/B^i_{j,t} \right] \Big\}.\end{aligned}$$

A1.3 Entrepreneurs

Following Christiano, Motto, and Rostagno (2014, hereafter CMR), we define $\Gamma^e(\bar{\omega}^e_{j,t+1})$ as the expected gross share of entrepreneurial returns going to banks

$$\Gamma^e(\bar{\omega}^e_{j,t+1}) \equiv [1 - F^e(\bar{\omega}^e_{j,t+1})]\bar{\omega}^e_{j,t+1} + G^e(\bar{\omega}^e_{j,t+1}), \quad G^e(\bar{\omega}^e_{j,t+1}) \equiv \int_0^{\bar{\omega}^e_{j,t+1}} \omega^e_{j,t+1} dF^e(\omega^e_{j,t+1}).$$

Using these variables and the definitions of the default cutoff and leverage, rewrite expected net worth

$$E_t[1 - \phi^e_t \Gamma^e(\bar{\omega}^e_{j,t+1})] R^k_{t+1} Q^k_t \bar{K}_{j,t} = E_t[1 - \phi^e_t \Gamma^e(\bar{\omega}^e_{j,t+1})] R^k_{t+1} L^e_{j,t} N^e_{j,t},$$

Plug the definitions of default cutoff and leverage in the bank participation constraint

$$\phi^e_t \left[\Gamma^e(\bar{\omega}^e_{j,t+1}) - \mu^e G^e(\bar{\omega}^e_{j,t+1}) \right] = \frac{L^e_{j,t} - 1}{L^e_{j,t}} \frac{R_{t+1}}{R^k_{j,t+1}}.$$

The problem of entrepreneur j in period t is to choose leverage $L^e_{j,t}$ and cutoff $\bar{\omega}^e_{j,t+1}$ to maximize expected pre-dividend net worth in $t+1$ subject to the bank participation constraint. Current net worth $N^e_{j,t}$ does not appear in the constraint and is present in the objective only as a factor of proportionality. Therefore, all entrepreneurs select the same $L^e_t = L^e_{j,t}$ and $\bar{\omega}^e_{t+1} = \bar{\omega}^e_{j,t+1}$ regardless of their net worth. The FOCs with respect to leverage and the default cutoff are respectively

$$\begin{aligned} 0 &= E_t \left\{ \left[1 - \phi^e_t \Gamma^e(\bar{\omega}^e_{t+1}) \right] R^k_{j,t+1} N^e_{j,t} - \frac{\lambda^e_{j,t}}{(L^e_t)^2} \frac{R_{t+1}}{R^k_{j,t+1}} \right\}, \\ 0 &= E_t \left\{ -\phi^e_t \Gamma^{e\prime}(\bar{\omega}^e_{t+1}) R^k_{j,t+1} L^e_t N^e_{j,t} + \lambda^e_{j,t} \phi^e_t \Gamma^{e\prime}(\bar{\omega}^e_{t+1}) - \lambda^e_{j,t} \phi^e_t \mu^e G^{e\prime}(\bar{\omega}^e_{t+1}) \right\}, \end{aligned}$$

where $\lambda^e_{j,t}$ is the multiplier on the constraint. Substituting out for $\lambda^e_{j,t}$ we obtain

$$0 = E_t \left\{ \left[1 - \phi^e_t \Gamma^e(\bar{\omega}^e_{t+1})\right] \frac{R^k_{j,t+1}}{R_{t+1}} \right.$$
$$\left. + \frac{\Gamma^{e\prime}(\bar{\omega}^e_{t+1})}{\Gamma^{e\prime}(\bar{\omega}^e_{t+1}) - \mu^e G^{e\prime}(\bar{\omega}^e_{t+1})} \left(\frac{R^k_{j,t+1}}{R_{t+1}} \phi^e_t \left[\Gamma^e(\bar{\omega}^e_{t+1}) - \mu^e G^e(\bar{\omega}^e_{t+1})\right] - 1 \right) \right\}.$$

Utilization Rate.—Entrepreneur j also determines the utilization rate of capital $u_{j,t}$. Since the market for capital services is competitive, the user cost function must equal the return on renting out capital services

$$P_t \Upsilon^{-t} a(u_{j,t}) \omega^e_{j,t} \bar{K}_{j,t-1} = P_t \tilde{r}^k_t u_{j,t} \omega^e_{j,t} \bar{K}_{j,t-1}.$$

The FOC with respect to $u_{j,t}$ is

$$a'(u_t) = \Upsilon^t \tilde{r}^k_t,$$

where optimal utilization $u_t = u_{j,t}$ depends only on aggregate variables and is therefore common to all entrepreneurs. The derivative of the utilization adjustment cost function is $a'(u_t) = r^k \exp(\sigma_a[u_t - 1])$, and the FOC can be rewritten as $\Upsilon^t \tilde{r}^k_t = r^k \exp(\sigma_a[u_t - 1])$.

A1.4 Banks

The balance sheet of banks is

$$D_t = \int_0^1 B^i_{j,t} dj + \int_0^1 B^e_{j,t} dj.$$

A1.5 Productive Sector

Final Good Producers.—The representative final good firm chooses the quantity of inputs $Y_{j,t}$ to maximize output Y_t subject to the following budget constraint

$$\int_0^1 P_{j,t} Y_{j,t} dj = P_t Y_t.$$

The FOC with respect to intermediate good $Y_{j,t}$ is

$$\left[\int_0^1 Y_{j,t}^{\frac{1}{\lambda_{p,t}}} dj\right]^{\lambda_{p,t}-1} Y_{j,t}^{\frac{1-\lambda_{p,t}}{\lambda_{p,t}}} = x P_{j,t},$$

where x is the multiplier on the budget constraint. Integrate over all goods, solve for x, rearrange, and obtain the demand function for a generic intermediate good

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} Y_t.$$

Plug the demand function into the aggregator and obtain the aggregate price index

$$P_t = \left[\int_0^1 P_{j,t}^{\frac{1}{1-\lambda_{p,t}}} dj\right]^{1-\lambda_{p,t}}.$$

Intermediate Good Producers: Production.—Intermediate good producer j makes the following profit

$$P_{j,t}Y_{j,t} - W_t^p l_{j,t}^p - W_t^i l_{j,t}^i - P_t \tilde{r}_t^k u_t \bar{K}_{j,t-1},$$

where $P_t\tilde{r}_t^k$ represents the nominal rental rate of capital. The firm minimizes cost subject to the production function. The FOCs with respect to capital services $u_t\bar{K}_{j,t-1}$, patient labor $l_{j,t}^p$, and impatient labor $l_{j,t}^i$ are respectively

$$\begin{aligned}
P_t\tilde{r}_t^k &= S_{j,t}\alpha\varepsilon_t(u_t\bar{K}_{j,t-1})^{\alpha-1}(z_t l_{j,t}^{p,\kappa} l_{j,t}^{i,1-\kappa})^{1-\alpha}, \\
W_t^p l_{j,t}^p &= S_{j,t}(1-\alpha)\kappa\varepsilon_t(u_t\bar{K}_{j,t-1})^{\alpha}(z_t l_{j,t}^{p,\kappa} l_{j,t}^{i,1-\kappa})^{1-\alpha}, \\
W_t^i l_{j,t}^i &= S_{j,t}(1-\alpha)(1-\kappa)\varepsilon_t(u_t\bar{K}_{j,t-1})^{\alpha}(z_t l_{j,t}^{p,\kappa} l_{j,t}^{i,1-\kappa})^{1-\alpha},
\end{aligned}$$

where $S_{j,t}$ is the multiplier on the production function and is interpreted as the marginal cost. Combine each of the two FOCs for labor with the FOC for capital services

$$\frac{u_t\bar{K}_{j,t-1}}{l_{j,t}^p} = \frac{\alpha}{(1-\alpha)\kappa}\frac{W_t^p}{P_t\tilde{r}_t^k}; \qquad \frac{u_t\bar{K}_{j,t-1}}{l_{j,t}^i} = \frac{\alpha}{(1-\alpha)(1-\kappa)}\frac{W_t^i}{P_t\tilde{r}_t^k}.$$

These two capital-to-labor ratios depend only on aggregate quantities and are therefore common to all intermediate producers. If firms pay the same factor prices, receive the same aggregate shocks, and choose the same proportion of inputs, then they have the same marginal cost $S_t = S_{j,t}$

$$S_t = \frac{1}{\varepsilon_t}\left(\frac{P_t\tilde{r}_t^k}{\alpha}\right)^{\alpha}\left(\frac{W_t^{p,\kappa}W_t^{i,1-\kappa}}{(1-\alpha)\kappa^\kappa(1-\kappa)^{1-\kappa}z_t}\right)^{1-\alpha}.$$

Intermediate Good Producers: Prices.—Intermediate good producer j chooses a price $P_{j,t}$ to maximize the sum of future discounted profits from period t to $t+s$

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^{p,s} \Lambda_{z,t+s}^p \left[P_{j,t} \tilde{\Pi}_{t,t+s} Y_{j,t+s} - W_{t+s}^p l_{j,t+s}^p - W_{t+s}^i l_{j,t+s}^i - P_{t+s} \tilde{r}_{t+s}^k u_{t+s} \bar{K}_{j,t-1+s} \right],$$

subject to a demand function. Here, $\tilde{\Pi}_{t,t+s} \equiv \prod_{k=1}^{s} \tilde{\pi}_{t+k}$ and $\tilde{\pi}_t = \pi^{\iota_p} \pi_{t-1}^{1-\iota_p}$. Let $\Pi_{t,t+s} \equiv \prod_{k=1}^{s} \pi_{t+k}$. The firm discounts the future in the same way as the patient household it belongs to. Since the marginal cost equals the average variable cost we rewrite the problem as

$$\max_{P_{j,t}} E_t \sum_{s=0}^{\infty} \xi_p^s \beta^{p,s} \Lambda_{z,t+s}^p Y_{j,t+s} (P_{j,t} \tilde{\Pi}_{t,t+s} - S_{t+s}),$$

subject to the demand function. The FOC with respect to price $P_{j,t}$ is

$$0 = E_t \sum_{s=0}^{\infty} \xi_p^s \beta^{p,s} \Lambda_{z,t+s}^p Y_{t+s} \left(\frac{\tilde{P}_t \tilde{\Pi}_{t,t+s}}{P_t \Pi_{t,t+s}} \right)^{\frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}}} \frac{1}{1-\lambda_{p,t+s}} \left[\tilde{\Pi}_{t,t+s} - \lambda_{p,t+s} \frac{S_{t+s}}{\tilde{P}_t} \right],$$

where the optimal price $\tilde{P}_t \equiv P_{j,t}$ depends only on aggregate variables and is therefore common to all producers. Rearrange and anticipate that exponent terms $\lambda_{p,t+s}$ disappear in the log-linearized equilibrium

$$\frac{\tilde{P}_t}{P_t} = \frac{E_t \sum_{s=0}^{\infty} \xi_p^s \beta^{p,s} P_{t+s} \Lambda_{z,t+s}^p Y_{t+s} \left(\frac{\tilde{\Pi}_{t,t+s}}{\Pi_{t,t+s}} \right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} \frac{\lambda_{p,t+s}}{1-\lambda_{p,t+s}} \frac{S_{t+s}}{P_{t+s}}}{E_t \sum_{s=0}^{\infty} \xi_p^s \beta^{p,s} P_{t+s} \Lambda_{z,t+s}^p Y_{t+s} \left(\frac{\tilde{\Pi}_{t,t+s}}{\Pi_{t,t+s}} \right)^{\frac{1}{1-\lambda_p}} \frac{1}{1-\lambda_{p,t+s}}} \equiv \frac{K_{p,t}^p}{F_{p,t}^p}.$$

Express the infinite sums $K_{p,t}^p$ and $F_{p,t}^p$ in recursive form

$$K_{p,t}^p = P_t \Lambda_{z,t}^p Y_t \frac{\lambda_{p,t}}{1-\lambda_{p,t}} \frac{S_t}{P_t} + \xi_p \beta^p E_t \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_{p,t+1}}{1-\lambda_{p,t+1}}} K_{p,t+1}^p,$$

$$F_{p,t}^p = P_t \Lambda_{z,t}^p Y_t \frac{1}{1-\lambda_{p,t}} + \xi_p \beta^p E_t \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_{p,t+1}}} F_{p,t+1}^p.$$

Labor Contractors.—The representative labor contractor chooses the quantity of labor input $l_{k,t}^o$, $o \in \{p, i\}$ to maximize output l_t^o subject to the following budget constraint

$$\int_0^1 W_{k,t}^o l_{k,t}^o dk = W_t^o l_t^o, \quad o \in \{p, i\}.$$

The FOC with respect to differentiated labor $l_{k,t}^{o}$ is

$$\left[\int_0^1 l_{k,t}^{o,\frac{1}{\lambda_w}}dk\right]^{\lambda_w-1} l_{k,t}^{o,\frac{1-\lambda_w}{\lambda_w}} = xW_{k,t}^{o}, \quad o\in\{p,i\},$$

where x is the multiplier on the budget constraint. Integrate over all inputs, solve for x, rearrange, and obtain the demand function for a generic labor input

$$l_{k,t}^{o} = \left(\frac{W_{k,t}^{o}}{W_t^{o}}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_t^{o}, \quad o\in\{p,i\}.$$

Plug the demand function into the Dixit-Stiglitz aggregator and obtain the aggregate wage index of patient and impatient workers

$$W_t^{o} = \left[\int_0^1 W_{k,t}^{o,\frac{1}{1-\lambda_w}}dk\right]^{1-\lambda_w}, \quad o\in\{p,i\}.$$

Monopoly Unions.—Worker union k discounts the future in the same way as the household it represents. It chooses a wage $W_{k,t}^{o}$, $o\in\{p,i\}$, to maximize the sum of future utilities from period t to $t+s$

$$E_t\sum_{s=0}^{\infty}\xi_w^s\beta^{o,s}\left[-\psi_l\int_0^1\frac{l_{k,t+s}^{o,1+\sigma_l}}{1+\sigma_l}dk + \Lambda_{z,t+s}^{o}(1-\tau^l)W_{k,t}^{o}\tilde{\Pi}_{t,t+s}^{w}l_{k,t+s}^{o}\right], \quad o\in\{p,i\},$$

$$\text{subject to} \quad l_{k,t+s}^{o} = \left(\frac{W_{k,t}^{o}\tilde{\Pi}_{t,t+s}^{w}}{W_{t+s}^{o}}\right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+s}^{o},$$

where $\tilde{\Pi}_{t,t+s}^{w} = \prod_{k=1}^{s}\mu_{z^*}\tilde{\pi}_{w,t+k}$ and $\tilde{\pi}_{w,t} = \pi^{\iota_w}\pi_{t-1}^{1-\iota_w}$. Let $\Pi_{t,t+s}^{w} = \prod_{k=1}^{s}\pi_{w,t+k}$. The FOC with respect to wage $W_{k,t}^{o}$, $o\in\{p,i\}$, is

$$0 = E_t\sum_{s=0}^{\infty}\xi_w^s\beta^{o,s}l_{t+s}^{o}\left(\frac{\tilde{W}_t^{o}\tilde{\Pi}_{t,t+s}^{w}}{W_t^{o}\Pi_{t,t+s}^{w}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\left[\Lambda_{z,t+s}^{o}(1-\tau^l)\tilde{\Pi}_{t,t+s}^{w} - \frac{\psi_l\lambda_w}{\tilde{W}_t^{o}}\left(\frac{\tilde{W}_t^{o}\tilde{\Pi}_{t,t+s}^{w}}{W_{t+s}^{o}}\right)^{\frac{\lambda_w\sigma_l}{1-\lambda_w}} l_{t+s}^{o,\sigma_l}\right].$$

The optimal wage $\tilde{W}_t^{o}\equiv W_{k,t}^{o}$ depends only on aggregate variables and is therefore common to all worker unions. That is, there is one optimal wage $\tilde{W}_t^{p}$ for patient workers and another one $\tilde{W}_t^{i}$ for impatient workers. Divide by $W_t^{o} = W_{t+s}^{o}/(\pi_{w,t+s}\ldots\pi_{w,t+1})$ and rearrange

$$\left(\frac{\tilde{W}_t^{o}}{W_t^{o}}\right)^{\frac{1-\lambda_w(1+\sigma_l)}{1-\lambda_w}}\frac{W_t^{o}}{P_t}\frac{1}{\psi_l} = \frac{E_t\sum_{s=0}^{\infty}\xi_w^s\beta^{o,s}\left(\frac{\tilde{\Pi}_{t,t+s}^{w}}{\Pi_{t,t+s}^{w}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_l)} l_{t+s}^{o,1+\sigma_l}}{E_t\sum_{s=0}^{\infty}\xi_w^s\beta^{o,s}\frac{1-\tau^l}{\lambda_w}l_{t+s}^{o}\left(\frac{\tilde{\Pi}_{t,t+s}^{w}}{\Pi_{t,t+s}^{w}}\right)^{\frac{1}{1-\lambda_w}}\left(\frac{\Pi_{t,t+s}^{w}}{\Pi_{t,t+s}}\right)\Lambda_{z,t+s}^{o}P_{t+s}} \equiv \frac{K_{w,t}^{o}}{F_{W,t}^{o}}.$$

Express the infinite sums $K^o_{w,t}$ and $F^o_{W,t}$, $o \in \{p, i\}$, in recursive form

$$K^o_{w,t} = l_t^{o,1+\sigma_l} + \xi_w \beta^o E_t (\tilde{\pi}_{w,t+1} \pi^{-1}_{w,t+1} \mu_{z^*})^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_l)} K^o_{w,t+1},$$
$$F^o_{W,t} = (1-\tau^l)\lambda_w^{-1} l^o_t P_t \Lambda^o_{z,t} + \xi_w \beta^o E_t \left(\tilde{\pi}_{w,t+1}\mu_{z^*}\right)^{\frac{1}{1-\lambda_w}} \pi_{w,t+1}^{\frac{\lambda_w}{\lambda_w - 1}} \pi^{-1}_{t+1} F^o_{W,t+1}.$$

Therefore, the optimal wage writes

$$\frac{\tilde{W}^o_t}{W^o_t} = \left[\frac{\psi_l}{W^o_t / P_t} \frac{K^o_{w,t}}{F^o_{W,t}}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_l)}}, \quad o \in \{p, i\}.$$

Capital Producers.—The representative capital producer discounts the future in the same way as the patient household it belongs to. It chooses investment to maximize profit subject to its capital production technology. The FOC with respect to investment I_t is

$$\begin{aligned} 0 = \Lambda^p_{z,t} Q^k_t &\left[1 - S^k\left(\zeta_{i,t}\frac{I_t}{I_{t-1}}\right) - \zeta_{i,t}\frac{I_t}{I_{t-1}} S^{k\prime}\left(\zeta_{i,t}\frac{I_t}{I_{t-1}}\right)\right] \\ &- \frac{\Lambda^p_{z,t} P_t}{\Upsilon^t \mu_{\Upsilon,t}} + \beta^p E_t \Lambda^p_{z,t+1} Q^k_{t+1} \zeta_{i,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S^{k\prime}\left(\zeta_{i,t+1}\frac{I_{t+1}}{I_t}\right). \end{aligned}$$

A1.6 Aggregation and Market Clearing

Productive Sector.—All intermediate goods producers have the same capital to labor ratio and the same marginal cost. Therefore, aggregate output writes

$$Y_t = \varepsilon_t (u_t \bar{K}_{t-1})^{\alpha} (z_t l_t)^{1-\alpha} - \theta z^*_t.$$

Households.—Aggregate impatient homeowner debt is given by $B^i_t = \int_0^1 B^i_{j,t} dj$. Since the mean of $\omega^i_{i,t}$ is unity, aggregate homeowner housing stock writes

$$\bar{H}^i_t = \int_0^1 \int_0^\infty \omega^i_{j,t} \bar{H}^i_{j,t} dF^i(\omega^i_{j,t}) dj.$$

The value function of homeowners is linear in housing net worth. This implies that all homeowners select the same leverage L^i_t and default cutoff $\bar{\omega}^i_{t+1}$ regardless of their housing net worth. Perfect insurance within the impatient household ensures all homeowners begin the next period with the same level of net worth. Aggregate net worth is given by

$$N^i_t = \int_0^1 [1 - \phi^i_{t-1}\Gamma^i(\bar{\omega}^i_t)] R^h_t Q^h_{t-1} \bar{H}^i_{j,t-1} dj = [1 - \phi^i_{t-1}\Gamma^i(\bar{\omega}^i_t)] R^h_t Q^h_{t-1} \bar{H}^i_{t-1},$$

where $\Gamma^i(\bar{\omega}^i_t) \equiv [1 - F^i(\bar{\omega}^i_t)]\bar{\omega}^i_t + G^i(\bar{\omega}^i_t)$. We assume government transfers are weighted according to households' respective share in total labor income

$$T^p_t = \kappa T_t; \qquad T^i_t = (1-\kappa)T_t.$$

Entrepreneurs.—Market clearing requires that the quantity of physical capital produced by capital producers equal the quantity purchased by entrepreneurs, $\bar{K}_t = \int_0^1 \bar{K}_{j,t}dj$. As explained above, all entrepreneurs select the same utilization regardless of their idiosyncratic shock. Therefore, the return on capital $R^k_t = R^k_{j,t}$ is common to all entrepreneurs. Entrepreneurs also choose the same leverage L^e_t and default cutoff $\bar{\omega}^e_{t+1}$. Since the mean of $\omega^e_{j,t}$ is unity, we determine the aggregate supply of capital services by entrepreneurs as

$$K_{t-1} = \int_0^1 \int_0^\infty u_t \omega^e_{j,t} \bar{K}_{j,t-1} dF^e(\omega^e_{j,t}) = u_t \bar{K}_{t-1}.$$

Market clearing in capital services requires that the supply of capital services by entrepreneurs equal the demand by intermediate good producers, $K_t = \int_0^1 K_{j,t}dj$.

We make the following assumption to facilitate aggregation. All entrepreneurs insure each other through transfers, so that they start off the next period with the same level of net worth. Transfers, however, take place only after the default decision is made. No entrepreneur goes out of business because even defaulting ones have nonzero net worth, as they are left with the fraction $1 - \phi^e_t$ of assets that was not pledged to the bank. Also, to avoid that entrepreneurs accumulate net worth to the point where they are completely self-financed, we impose that they pay a fixed dividend δ^e each period to patient households. As mentioned in the main text, we include an equity shock γ^e_t that modifies the net worth of entrepreneurs.

Our assumption on perfect insurance ensures entrepreneurs finish period t with the same level of net worth. Aggregate net worth after dividend payments is

$$N^e_t = \gamma^e_t[1 - \phi^e_{t-1}\Gamma^e(\bar{\omega}^e_t)]R^k_t Q^k_{t-1}\bar{K}_{t-1} - \delta^e N^e_t.$$

The aggregate balance sheet of entrepreneurs writes $Q^k_t \bar{K}_t = N^e_t + B^e_t$, where $B^e_t = \int_0^1 B^e_{j,t}dj$ is aggregate entrepreneurial debt.

Banks.—Aggregate resources used by banks to monitor defaulting borrowers are

$$D^b_t = \phi^i_{t-1}\mu^i G^i(\bar{\omega}^i_t) R^h_t Q^h_{t-1}\bar{H}^i_{t-1}/P_t + \phi^e_{t-1}\mu^e G^e(\bar{\omega}^e_t) R^k_t Q^k_{t-1}\bar{K}_{t-1}/P_t.$$

A2 Summary of Equilibrium Conditions

In this section we list all the stationary equilibrium conditions of our baseline model. We also describe the alternative model specification mentioned in the main text.

A2.1 Stationary Equilibrium in the Baseline Model

In order to solve our model, we need to stationarize it. Scaled variables are as follows

$$
\begin{aligned}
b_t &= B_t/(z_t^* P_t), & F_{w,t}^p &= F_{W,t}^p z_t^*, & n_t^e &= N_t^e/(z_t^* P_t), & t_t^p &= T_t^p/(z_t^* P_t),\\
b_t^e &= B_t^e/(z_t^* P_t), & g_t &= G_t/z_t^*, & n_t^i &= N_t^i/(z_t^* P_t), & w_t &= W_t/(z_t^* P_t),\\
b_t^i &= B_t^i/(z_t^* P_t), & h_t &= \bar{H}_t/z_t^*, & \tilde{p}_t &= \tilde{P}_t/P_t, & w_t^i &= W_t^i/(z_t^* P_t),\\
c_t &= C_t/z_t^*, & h_t^i &= \bar{H}_t^i/z_t^*, & q_t^k &= Q_t^k \Upsilon^t/P_t, & w_t^p &= W_t^p/(z_t^* P_t),\\
c_t^i &= C_t^i/z_t^*, & h_t^p &= \bar{H}_t^p/z_t^*, & q_t^h &= Q_t^h/P_t, & y_{z,t} &= Y_t/z_t^*,\\
c_t^p &= C_t^p/z_t^*, & i_t &= I_t/(z_t^* \Upsilon^t), & r_t^k &= \Upsilon^t \tilde{r}_t^k, & y_t &= Y_t^{gdp}/z_t^*,\\
d_t &= D_t/z_t^*, & k_t &= \bar{K}_t/(z_t^* \Upsilon^t), & s_t &= S_t/P_t, & \mu_{z^*,t} &= z_t^*/z_{t-1}^*,\\
d_t^b &= D_t^b/z_t^*, & \lambda_{z,t}^i &= \Lambda_{z,t}^i P_t z_t^*, & t_t &= T_t/(z_t^* P_t), & z_t^* &= z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}.\\
F_{w,t}^i &= F_{W,t}^i z_t^*, & \lambda_{z,t}^p &= \Lambda_{z,t}^p P_t z_t^*, & t_t^i &= T_t^i/(z_t^* P_t), & &
\end{aligned}
$$

Prices.—Optimal price equations

$$F_{p,t}^p = \lambda_{z,t}^p y_{z,t} + \xi_p \beta^p E_t (\tilde{\pi}_{t+1} \pi_{t+1}^{-1})^{\frac{1}{1-\lambda_{p,t+1}}} F_{p,t+1}^p. \tag{1}$$

$$K_{p,t}^p = \lambda_{z,t}^p y_{z,t} \lambda_{p,t} s_t + \xi_p \beta^p E_t (\tilde{\pi}_{t+1} \pi_{t+1}^{-1})^{\frac{\lambda_{p,t+1}}{1-\lambda_{p,t+1}}} K_{p,t+1}^p. \tag{2}$$

$$K_{p,t}^p = \left(\left[1 - \xi_p (\tilde{\pi}_t \pi_t^{-1})^{\frac{1}{1-\lambda_{p,t}}} \right] (1 - \xi_p)^{-1} \right]^{1-\lambda_{p,t}} F_{p,t}^p. \tag{3}$$

Wages.—Optimal patient and impatient household wage equations and aggregate wage

$$F_{w,t}^p = (1-\tau^l) \lambda_w^{-1} \lambda_{z,t}^p l_t^p + \xi_w \beta^p \mu_{z^*}^{\frac{1}{1-\lambda_w}} E_t \mu_{z^*,t+1}^{-1} \pi_{w,t+1}^{\frac{\lambda_w}{\lambda_w - 1}} \tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}} \pi_{t+1}^{-1} F_{w,t+1}^p. \tag{4}$$

$$K_{w,t}^p = l_t^{p,1+\sigma_l} + \xi_w \beta^p E_t (\tilde{\pi}_{w,t+1} \pi_{w,t+1}^{-1} \mu_{z^*})^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_l)} K_{w,t+1}^p. \tag{5}$$

$$K_{w,t}^p = \psi_l^{-1} \left[\left(1 - \xi_w (\tilde{\pi}_{w,t} \pi_{w,t}^{-1} \mu_{z^*})^{\frac{1}{1-\lambda_w}} \right) (1-\xi_w)^{-1} \right]^{1-\lambda_w(1+\sigma_l)} w_t^p F_{w,t}^p. \tag{6}$$

$$F_{w,t}^i = (1-\tau^l) \lambda_w^{-1} \lambda_{z,t}^i l_t^i + \xi_w \beta^i \mu_{z^*}^{\frac{1}{1-\lambda_w}} E_t \mu_{z^*,t+1}^{-1} \pi_{w,t+1}^{\frac{\lambda_w}{\lambda_w - 1}} \tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}} \pi_{t+1}^{-1} F_{w,t+1}^i. \tag{7}$$

$$K_{w,t}^i = l_t^{i,1+\sigma_l} + \xi_w \beta^i E_t (\tilde{\pi}_{w,t+1} \pi_{w,t+1}^{-1} \mu_{z^*})^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_l)} K_{w,t+1}^i. \tag{8}$$

$$K_{w,t}^i = \psi_l^{-1} \left[\left(1 - \xi_w (\tilde{\pi}_{w,t} \pi_{w,t}^{-1} \mu_{z^*})^{\frac{1}{1-\lambda_w}} \right) (1-\xi_w)^{-1} \right]^{1-\lambda_w(1+\sigma_l)} w_t^i F_{w,t}^i. \tag{9}$$

$$w_t = (1-\kappa) w_t^p + \kappa w_t^i. \tag{10}$$

Production.—Capital utilization and rental rate, patient and impatient labor demand, capital accumulation, return on capital and housing, and aggregate production function

$$r_t^k = r^k \exp(\sigma_a[u_t - 1]). \tag{11}$$
$$r_t^k = \alpha \varepsilon_t (\Upsilon \mu_{z^*,t} l_t)^{1-\alpha} (u_t k_{t-1})^{\alpha-1} s_t. \tag{12}$$
$$w_t^p l_t^p = (1-\alpha)\kappa s_t \varepsilon_t \Upsilon^{-\alpha} (\mu_{z^*,t}^{-1} u_t k_{t-1})^{\alpha} l_t^{1-\alpha}. \tag{13}$$
$$w_t^i l_t^i = (1-\alpha)(1-\kappa) s_t \varepsilon_t \Upsilon^{-\alpha} (\mu_{z^*,t}^{-1} u_t k_{t-1})^{\alpha} l_t^{1-\alpha}. \tag{14}$$
$$k_t = (1-\delta)\Upsilon^{-1}\mu_{z^*,t}^{-1} k_{t-1} + [1 - S^k\left(\Upsilon \mu_{z^*,t}\zeta_{i,t} i_t / i_{t-1}\right)] i_t. \tag{15}$$
$$R_t^k = \left[(1-\tau^k)[u_t r_t^k - a(u_t)] + (1-\delta) q_t^k\right] \Upsilon^{-1} \pi_t q_{t-1}^{k,-1} + \tau^k \delta. \tag{16}$$
$$R_t^h = \pi_t q_t^h / q_{t-1}^h. \tag{17}$$
$$y_{z,t} = \varepsilon_t (\Upsilon^{-1} \mu_{z^*,t}^{-1} u_t k_{t-1})^{\alpha} l_t^{1-\alpha} - \theta. \tag{18}$$

Resource Constraints.—Aggregate output, consumption, hours, and housing; and GDP

$$y_{z,t} = g_t + c_t + \mu_{\Upsilon,t}^{-1} i_t + a(u_t)\Upsilon^{-1}\mu_{z^*,t}^{-1} k_{t-1} + d_t^b. \tag{19}$$
$$c_t = c_t^p + c_t^i. \tag{20}$$
$$l_t = l_t^{p,\kappa} l_t^{i,1-\kappa}. \tag{21}$$
$$h = h_t^p + h_t^i. \tag{22}$$
$$y_t = g_t + c_t + \mu_{\Upsilon,t}^{-1} i_t. \tag{23}$$

Government.—Monetary policy rule and government budget constraint

$$R_t - R = \rho_p(R_{t-1} - R) + (1-\rho_p)\left[\alpha_\pi(E_t \pi_{t+1} - \pi) + \alpha_{\Delta y}(g_{y,t} - \mu_{z^*})\right] + \varepsilon_t^p, \tag{24}$$
$$g_t + t_t = \tau^k \left([u_t r_t^k - a(u_t)]\Upsilon^{-1} - \pi_t^{-1}\delta q_{t-1}^k\right)\mu_{z^*,t}^{-1} k_{t-1} + \tau^l (w_t^i l_t^i + w_t^p l_t^p) + \tau^c c_t. \tag{25}$$

Capital Producers.—Optimal capital investment

$$\begin{aligned} 0 = {} & \lambda_{z,t}^p q_t^k \left[1 - S^k\left(\Upsilon\mu_{z^*,t}\zeta_{i,t} i_t / i_{t-1}\right) - \Upsilon \mu_{z^*,t}\zeta_{i,t} i_t i_{t-1}^{-1} S^{k\prime}\left(\Upsilon\mu_{z^*,t}\zeta_{i,t} i_t / i_{t-1}\right)\right] \\ & - \mu_{\Upsilon,t}^{-1}\lambda_{z,t}^p + \beta^p E_t (\Upsilon\mu_{z^*,t+1})^{-1}\lambda_{z,t+1}^p q_{t+1}^k \zeta_{i,t+1}\left(\Upsilon\mu_{z^*,t+1} i_{t+1}/i_t\right)^2 S^{k\prime}\left(\Upsilon\mu_{z^*,t+1}\zeta_{i,t+1} i_{t+1}/i_t\right). \end{aligned} \tag{26}$$

Patient Households.—Optimal consumption, housing, and deposits

$$0 = (1+\tau^c)\lambda_{z,t}^p - \mu_{z^*,t}\zeta_{c,t}/(\mu_{z^*,t} c_t^p - b_c^p c_{t-1}^p) + b_c^p \beta^p E_t \zeta_{c,t+1}/(\mu_{z^*,t+1} c_{t+1}^p - b_c^p c_t^p). \tag{27}$$
$$0 = 1/h_t^p - \lambda_{z,t}^p q_t^h + \beta^p E_t \mu_{z^*,t+1}^{-1} \lambda_{z,t+1}^p q_{t+1}^h. \tag{28}$$
$$0 = \lambda_{z,t}^p - \beta^p E_t (\pi_{t+1}\mu_{z^*,t+1})^{-1}\lambda_{z,t+1}^p R_{t+1}. \tag{29}$$

Impatient Households.—Optimal consumption, housing services, physical housing, leverage; budget constraint, bank participation constraint, default cutoff, net worth, leverage, and spread

$$0 = (1+\tau^c)\lambda^i_{z,t} - \mu_{z^*,t}\zeta_{c,t}/(\mu_{z^*,t}c^i_t - b^i_c c^i_{t-1}) + b^i_c\beta^i E_t\zeta_{c,t+1}/(\mu_{z^*,t+1}c^i_{t+1} - b^i_c c^i_t). \tag{30}$$

$$0 = 1/h^i_t - \lambda^i_{z,t}r^h_t. \tag{31}$$

$$\begin{aligned} 0 = {} & \lambda^i_{z,t}(r^h_t h^i_t + b^i_t) - \lambda^i_{z,t}q^h_t h^i_t[1 + S^h(\mu_{z^*,t}\zeta_{h,t}h^i_t/h^i_{t-1}) + \mu_{z^*,t}\zeta_{h,t}h^i_t h^{i,-1}_{t-1}S^{h\prime}(\mu_{z^*,t}\zeta_{h,t}h^i_t/h^i_{t-1})] \\ & + \beta^i E_t\mu^{-1}_{z^*,t+1}\lambda^i_{z,t+1}q^h_{t+1}h^i_t\zeta_{h,t+1}(\mu_{z^*,t+1}h^i_{t+1}/h^i_t)^2 S^{h\prime}(\mu_{z^*,t+1}\zeta_{h,t}h^i_{t+1}/h^i_t) \\ & + \beta^i E_t(\pi_{t+1}\mu_{z^*,t+1})^{-1}\lambda^i_{z,t+1}[1 - \phi^i_t\Gamma^i(\bar{\omega}^i_{t+1})]R^h_{t+1}q^h_t h^i_t. \end{aligned} \tag{32}$$

$$\begin{aligned} 0 = {} & \lambda^i_{z,t} - \beta^i E_t(\pi_{t+1}\mu_{z^*,t+1})^{-1}\lambda^i_{z,t+1}\Big[R_{t+1} - (1-\mu^i)R^i_{t+1}G^{i\prime}(\bar{\omega}^i_{t+1}) \\ & + \bar{\omega}^i_{t+1}F^{i\prime}(\bar{\omega}^i_{t+1})[1 - F^i(\bar{\omega}^i_{t+1})]^{-1}\left(R_{t+1} - (1-\mu^i)G^i(\bar{\omega}^i_{t+1})R^h_{t+1}\phi^i_t q^h_t h^i_t/b^i_t\right)\Big]. \end{aligned} \tag{33}$$

$$0 = (1-\tau^l)w^i_t l^i_t + (\pi_t\mu_{z^*,t})^{-1}[1 - \phi^i_t\Gamma^i(\bar{\omega}^i_t)]R^h_t q^h_{t-1}h^i_{t-1} + b^i_t + t^i_t - (1+\tau^c)c^i_t - q^h_t h^i_t. \tag{34}$$

$$0 = R_t b^i_{t-1} - [1 - F^i(\bar{\omega}^i_t)]R^i_t b^i_{t-1} - (1-\mu^i)G^i(\bar{\omega}^i_t)R^h_t\phi^i_{t-1}q^h_{t-1}h^i_{t-1}. \tag{35}$$

$$\bar{\omega}^i_t = R^i_t b^i_{t-1}/(R^h_t\phi^i_{t-1}q^h_{t-1}h^i_{t-1}). \tag{36}$$

$$n^i_t = (\pi_t\mu_{z^*,t})^{-1}[1 - \phi^i_{t-1}\Gamma^i(\bar{\omega}^i_t)]R^h_t q^h_{t-1}h^i_{t-1}. \tag{37}$$

$$L^i_t = q^h_t h^i_t/n^i_t. \tag{38}$$

$$S^i_t = R^i_t/R_t. \tag{39}$$

Entrepreneurs.—FOC, bank participation constraint, default cutoff, net worth, leverage, debt, and spread

$$\begin{aligned} 0 = E_t\Big\{ & \left[1 - \phi^e_t\Gamma^e(\bar{\omega}^e_{t+1})\right]R^k_{t+1}/R_{t+1} + \Gamma^{e\prime}(\bar{\omega}^e_{t+1})[\Gamma^{e\prime}(\bar{\omega}^e_{t+1}) - \mu^e G^{e\prime}(\bar{\omega}^e_{t+1})]^{-1} \\ & \left(R^k_{t+1}R^{-1}_{t+1}\phi^e_t\left[\Gamma^e(\bar{\omega}^e_{t+1}) - \mu^e G^e(\bar{\omega}^e_{t+1})\right] - 1\right)\Big\}. \end{aligned} \tag{40}$$

$$0 = R_t b^e_{t-1} - [1 - F^e(\bar{\omega}^e_t)]R^e_t b^e_{t-1} - (1-\mu^e)G^e(\bar{\omega}^e_t)R^k_t\phi^e_{t-1}q^k_{t-1}k_{t-1}. \tag{41}$$

$$\bar{\omega}^e_t = R^e_t b^e_{t-1}/(R^k_t\phi^e_{t-1}q^k_{t-1}k_{t-1}). \tag{42}$$

$$n^e_t = \gamma^e_t(\pi_t\mu_{z^*,t})^{-1}[1 - \phi^e_{t-1}\Gamma^e(\bar{\omega}^e_t)]R^k_t q^k_{t-1}k_{t-1} - \delta^e n^e_t. \tag{43}$$

$$L^e_t = q^k_t k_t/n^e_t. \tag{44}$$

$$b^e_t = q^k_t k_t - n^e_t. \tag{45}$$

$$S^e_t = R^e_t/R_t. \tag{46}$$

Banks.—Optimal housing and capital requirements and total bank credit

$$\phi_t^i = \nu_t \eta_t^i. \tag{47}$$

$$\phi_t^e = \nu_t \eta_t^e. \tag{48}$$

$$b_t = b_t^i + b_t^e. \tag{49}$$

A2.2 Auxiliary Expressions

Prices and Wages.—Price and wage indexation and wage inflation

$$\tilde{\pi}_t = \pi^{\iota_p} \pi_{t-1}^{1-\iota_p}. \tag{A1}$$

$$\tilde{\pi}_{w,t} = \pi^{\iota_w} \pi_{t-1}^{1-\iota_w}. \tag{A2}$$

$$\pi_{w,t} = \pi_t \mu_{z^*,t} w_t / w_{t-1}. \tag{A3}$$

Adjustment Costs.—Utilization, investment, and housing adjustment costs

$$a(u_t) = r^k (\exp[\sigma_a(u_t - 1)] - 1)/\sigma_a. \tag{A4}$$

$$S^k(\zeta_{i,t} \mu_{z^*,t} \Upsilon i_t / i_{t-1}) = e^{\sqrt{S^{k\prime\prime}/2}\Upsilon\left(\zeta_{i,t}\mu_{z^*,t}\frac{i_t}{i_{t-1}} - \mu_{z^*}\right)} + e^{-\sqrt{S^{k\prime\prime}/2}\Upsilon\left(\zeta_{i,t}\mu_{z^*,t}\frac{i_t}{i_{t-1}} - \mu_{z^*}\right)} - 2. \tag{A5}$$

$$S^h(\zeta_{h,t} \mu_{z^*,t} h_t^i / h_{t-1}^i) = e^{\sqrt{S^{h\prime\prime}/2}\left(\zeta_{h,t}\mu_{z^*,t}\frac{h_t^i}{h_{t-1}^i} - \mu_{z^*}\right)} + e^{-\sqrt{S^{h\prime\prime}/2}\left(\zeta_{h,t}\mu_{z^*,t}\frac{h_t^i}{h_{t-1}^i} - \mu_{z^*}\right)} - 2. \tag{A6}$$

Distribution Functions.—Default probability, bank monitoring returns, and gross share of profits going to banks

$$F^i(\bar{\omega}_t^i, \sigma^i) = \Phi\left([\ln(\bar{\omega}_t^i) + (\sigma^i)^2/2]/\sigma^i\right), \tag{A7}$$

$$G^i(\bar{\omega}_t^i, \sigma^i) = \Phi\left([\ln(\bar{\omega}_t^i) + (\sigma^i)^2/2]/\sigma^i - \sigma^i\right), \tag{A8}$$

$$\Gamma^i(\bar{\omega}_t^i, \sigma^i) = \bar{\omega}_t^i[1 - F^i(\bar{\omega}_t^i, \sigma^i)] + G^i(\bar{\omega}_t^i, \sigma^i), \tag{A9}$$

$$F^e(\bar{\omega}_t^e, \sigma^e) = \Phi\left([\ln(\bar{\omega}_t^e) + (\sigma^e)^2/2]/\sigma^e\right), \tag{A10}$$

$$G^e(\bar{\omega}_t^e, \sigma^e) = \Phi\left([\ln(\bar{\omega}_t^e) + (\sigma^e)^2/2]/\sigma^e - \sigma^e\right), \tag{A11}$$

$$\Gamma^e(\bar{\omega}_t^e, \sigma^e) = \bar{\omega}_t^e[1 - F^e(\bar{\omega}_t^e, \sigma^e)] + G^e(\bar{\omega}_t^e, \sigma^e), \tag{A12}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Banks.—Aggregate resources for monitoring impatient households and entrepreneurs

$$d_t^b = (\pi_t \mu_{z^*,t})^{-1} \left[\mu^i G^i(\bar{\omega}_t^i) R_t^h \phi_{t-1}^i q_{t-1}^h h_{t-1}^i + \mu^e G^e(\bar{\omega}_t^e) R_t^k \phi_{t-1}^e q_{t-1}^k k_{t-1}\right]. \tag{A13}$$

A2.3 Alternative Model with No Impatient Households

In the paper, we compare our baseline model to an alternative model without impatient households, without household credit, and without housing. With respect to our baseline model, the differences are as follows. All equations related to impatient households and housing drop: (7), (8), (9), (14), (17), (22), (25), (28), (30), (31), (35), (32), (34), (36), (37), (38), (39), (40), (48), (A6), (A7), (A8), (A9), and (A13). In addition, the following equations change: aggregate wage (10), aggregate consumption (20), aggregate hours (21), and total bank credit (50)

$$w_t = w_t^p; \qquad c_t = c_t^p; \qquad l_t = l_t^{p,\kappa}; \qquad b_t = b_t^e.$$

Regarding parameters, the share κ of patient labor in total labor equals one, while β^i, b_c^i, $F^i(\bar{\omega}^i)$, μ^i, and $S^{h\prime\prime}$ drop. Parameters associated to housing-specific shocks, ρ_{η^i}, σ_{η^i}, ρ_{ζ_h}, and σ_{ζ_h}, also drop.

A3 Data and Observation Equations

A3.1 Data Sources

All macroeconomic and financial data are extracted from the Federal Reserve Economic Data (FRED) database. To compute effective tax rates, we use annual data from the Organization for Economic Co-operation and Development (OECD). Table A1 lists all the series with their associated mnemonic and source.

A3.2 Data Treatment

Table A2 shows how we build our observable variables. We construct additional variables to match steady-state ratios and provide out-of-sample evidence. For the tax rates we follow the methodology developed by Mendoza, Razin, and Tesar (1994). The formulas are theirs; we use OECD mnemonics reported in Table A1.

Table A1: Data Sources

Mnemonic	Description	Unit	Source
A. Macroeconomic Series			
GDP	Gross domestic product	$bn	BEA
GDPDEF	Gross domestic product: implicit price deflator	idx	BEA
PCND	Personal consumption expenditures: nondurables	$bn	BEA
PCESV	Personal consumption expenditures: services	$bn	BEA
PCDG	Personal consumption expenditures: durables	$bn	BEA
GPDI	Gross private domestic investment	$bn	BEA
A006RD3Q086SBEA	Gross private domestic investment: price deflator	idx	BEA
HOANBS	Nonfarm business sector: hours of all persons	idx	BEA
FEDFUNDS	Effective federal funds rate	%	BOG
CNP16OV	Civilian noninstitutional population	ppl	BLS
LABSHPUSA156NR	Share of labor compensation in GDP, annual	%	UoG
B. Financial Series			
CMDEBT	Households & nonprofits: debt securities & loans	$bn	BOG
MORTGAGE30US	30-year fixed rate mortgage average in the US	%	FHLMC
TBSDODNS	Nonfinancial business: debt securities & loans	$bn	BOG
BAA	Moody's seasoned Baa corporate bond yield	%	Moody's
TABSNNCB	Nonfinancial corporate business: total assets	$bn	BOG
TNWMVBSNNCB	Nonfinancial corporate business: net worth	$bn	BOG
TABSNNB	Nonfinancial noncorporate business: total assets	$bn	BOG
TNWBSNNB	Nonfinancial noncorporate business: net worth	$bn	BOG
NCBREMV	Nonfinancial corporate business: real estate	$m	BOG
NNBREMV	Nonfinancial noncorporate business: real estate	$bn	BOG
H0SUBLPDHMSNQ	Net % of banks tightening standards for mortgages	%	BOG
DRTSPM	Net % of banks tightening standards for prime mtgs.	%	BOG
DSUBLPDHMSENQ	Net % of banks tightening standards for GSE mtgs.	%	BOG
DRTSCILM	Net % of banks tightening standards for C&I loans	%	BOG
NFCICREDIT	Chicago Fed national financial conditions credit	idx	FRBC
C. Tax Series			
1100	Taxes on income, profits, & capital gains of individuals	$bn	OECD
1200	Taxes on income, profits, & capital gains of corporates	$bn	OECD
2000	Social security contributions	$bn	OECD
2200	Employers social security contributions	$bn	OECD
3000	Tayes on payroll and workforce	$bn	OECD
4400	Taxes on financial and capital transactions	$bn	OECD
5110	General taxes on goods and services	$bn	OECD
5121	Excises	$m	OECD
P31NC	Final consumption expenditures of households	$m	OECD
P3CG	Final consumption expenditure, general government	$m	OECD
D1CG	Total compensation of government employees	$m	OECD
SB3G	Mixed income, gross	$m	OECD
NFD4R	Property income	$m	OECD
NFB4G	Entrepreneurial income, gross	$m	OECD
NFD11P	Wages and salaires	$m	OECD
SB2GB3G	Operating surplus and mixed income, gross	$m	OECD

Notes: BEA: Bureau of Economic Analysis; BLS: Bureau of Labor Statistics; BOG: Board of Governors; FHLMC: Freddie Mac; FRBC: Federal Reserve Bank of Chicago, UoG: University of Groningen.

Table A2: Data Treatment

Constructed Series	Formula	Remark
Population	= HPfilter(CNP16OV, λ = 10 000)	To remove breaks
A. Observable Variables		
GDP	= GDP/(GDPDEF × Population)	First diff, demean
Consumption	= (PCND + PCESV)/(GDPDEF × Population)	First diff, demean
Investment	= (PCDG + GPDI)/(GDPDEF × Population)	First diff, demean
Hours	= HOANBS/Population	Demean
Inflation	= ln(GDPDEF) − ln(GDPDEF)$_{-1}$	Demean
Price of investment	= ln(A006RD3Q086SBEA$_t$) − ln(GDPDEF)	Demean
Nominal interest rate	= FEDFUNDS/4	Demean
Household credit	= CMDEBT/(GDPDEF × Population)	First diff, demean
Household spread	= (MORTGAGE30US − FEDFUNDS)/4	Demean
Business credit	= TBSDODNS/(GDPDEF × Population)	First diff, demean
Business spread	= (BAA − FEDFUNDS)/4	Demean
B. Other Variables		
Exogenous spending	= GDP − Consumption − Investment	Avg 1985–2019
Capital share	= 100 − LABSHPUSA156NRUG	Avg 1985–2017
Total credit	= Household credit + Business credit	Avg 1985–2019
Productive capital	$= \frac{\text{TABSNNCB+TABSNNB−NCBREMV−NNBREMV}}{\text{GDPDEF×Population}}$	Avg 1985–2019
Business leverage	$= \frac{\text{TABSNNCB+TABSNNB}}{\text{TNWMVBSNNCB+TNWBSNNB}}$	Avg 1985–2019
Lending standards H	= H0SUBLPDHMSNQ; DRTSPM; DSUBLPDHMSENQ	Merge 3 series
Lending standards B	= DRTSCILM	
Consumption tax	= (5110+5121)/(P31NC+P3CG−D1CG−5110−5121)	Methodology
Household tax rate	= 1100/(SB3G + NFD4R − NFB4G + NFD11P) ≡ τ^h	developed by
Labor income tax	= (τ^hD1CG + 2000 + 3000)/(D1CG + 2200)	Mendoza, Razin,
Capital income tax	= [τ^h(SB3G+NFD4R+NFB4G)+1200+4400]/SB2GB3G	and Tesar (1994)

A3.3 Observation Equations

We specify the model observation equations that match our treatment of the data. The superscript *obs* denotes an observable variable.

Gross domestic product: $y_t^{obs} = 1 + \ln(y_t \mu_{z^*,t}/y_{t-1}) - \ln \mu_{z^*} = y_t \mu_{z^*,t}/(y_{t-1}\mu_{z^*})$.

Consumption: $c_t^{obs} = c_t \mu_{z^*,t}/(c_{t-1}\mu_{z^*})$.

Investment: $i_t^{obs} = i_t \mu_{z^*,t}/(i_{t-1}\mu_{z^*})$.

Hours: $l_t^{obs} = 1 + \ln l_t - \ln l = l_t/l$.

Inflation: $\pi_t^{obs} = 1 + \ln \pi_t - \ln \pi = \pi_t/\pi$.

Nominal interest rate: $R_t^{obs} = R_t - R$.

Price of investment: $\mu_{\Upsilon,t}^{obs} = \mu_{\Upsilon,t}/\mu_{\Upsilon,t-1}$.

Household credit: $b_t^{i,obs} = b_t^i \mu_{z^*,t}/(b_{t-1}^i \mu_{z^*})$.

Household spread: $S_t^{i,obs} = R_t^i - R_t - (R^i - R)$.

Business credit: $b_t^{e,obs} = b_t^e \mu_{z^*,t}/(b_{t-1}^e \mu_{z^*})$.

Business spread: $S_t^{e,obs} = R_t^e - R_t - (R^e - R)$.

A4 Estimation, Complement

This section complements Section III of the main text. We discuss the set of parameters we calibrate, the set of parameters we estimate, and measures of model fit.

A4.1 Calibrated Parameters

Table A3 reports the calibrated parameters. For those in Panel A, we use our data set directly. The share of capital in production α averages 0.39 between 1985 and 2019. The steady-state government spending-to-GDP ratio η_g equals 0.16, the mean in our sample. Annualized steady-state inflation π is set to 2.15%, the average over the period. The mean growth rate of real per capita GDP μ_z^* is fixed at 1.50% on an annual basis. We set the annualized rate of investment-specific technological change Υ to 0.93%, which corresponds to the average rate of decline in the relative price of investment goods over the period. The tax rates on consumption τ^c, capital income τ^k, and labor income τ^l are computed following the methodology developed by Mendoza, Razin, and Tesar (1994). Over the 1985-2018 period, we find $\tau^c = 0.048$, $\tau^k = 0.229$, and $\tau^l = 0.200$. The dividend paid by entrepreneurs δ^e is set to match the average business sector leverage of 1.75.

Table A3: Calibrated Parameters

Description	Par.	Value	Target / Source
A. Parameters Calibrated Using Data Set			
Capital share in production	α	0.3906	Sample mean
Steady-state gov. spending-GDP ratio	η_g	0.1648	Sample mean
Steady-state inflation, annual	π	2.1450	Sample mean
Growth rate of the economy, annual	μ_{z^*}	1.4987	Sample mean
Trend rate of IST change, annual	Υ	0.9268	Sample mean
Tax rate on consumption	τ^c	0.0476	Sample mean
Tax rate on capital income	τ^k	0.2290	Sample mean
Tax rate on labor income	τ^l	0.2005	Sample mean
Entrepreneurial dividend share	δ^e	0.0192	$L^e = 1.7479$
B. Other Parameters			
Depreciation rate of capital	δ	0.0250	10% annual
Labor supply elasticity	σ_l	1.0000	CMR
Patient discount factor	β^p	0.9993	R = 3.95% annual
Impatient discount factor	β^i	0.9750	Iacoviello (2005)
Steady state price markup	λ_p	1.2000	CMR
Steady state wage markup	λ_w	1.0250	CMR
Disutility weight on labor	ψ_l	0.8142	Hours $l = 1$

We calibrate the remaining parameters, in Panel B, as follows. We set the depreciation rate δ to 0.025 to match an annual rate of 10%. The labor supply elasticity σ_l equals 1. The patient household discount factor β^p is fixed at 0.9993, which pins down the annualized fed funds rate R to 3.95%. The impatient household discount factor β^i must be lower than

β^p. We put it at 0.975, which lies between the values used by Iacoviello (2005, 0.95) and Krusell and Smith (1998, 0.99). Following CMR, we calibrate the steady-state price markup λ_p at 1.20 and the steady-state wage markup λ_w at 1.025. The disutility weight on labor ψ_l is fixed so that total hours worked are normalized to one in steady state.

A4.2 Estimated Parameters

Table A4 reports the estimated parameters. In Panel A we collect the structural ones. Many of these parameters are standard in the DSGE literature, and we apply similarly standard priors.[1] These include the Taylor rule coefficients, $a_{\Delta y}$, a_π, and ρ_p, the Calvo price and wage stickiness parameters, ξ_p and ξ_w, the indexation coefficients, ι_p, and ι_w, and the curvature parameters for utilization and investment, σ_a and $S^{k\prime\prime}$. For most of these parameters, our posterior estimates are in line with the literature.[2] One exception is the higher posterior for the two Calvo parameters. This is probably due to subdued inflation during most of the sample, especially in the ten years since the 2009 recession—data not available in previous studies. Our estimate of the Calvo price stickiness, at 0.85, implies a Phillips curve with a slope coefficient of 0.013.[3] Note that this value does not mean that prices stay unchanged for $1/(1 - 0.85) = 6.7$ quarters, because at each period prices that are not re-optimized are indexed to past inflation.

We now discuss the less habitual parameters. The cost of adjusting housing $S^{h\prime\prime}$ is essential to smooth the dynamics of impatient household housing, and hence household debt, a variable we observe. Since it is costly to sell housing immediately, impatient households react more gradually to shocks. The posterior mode of $S^{h\prime\prime}$, at 63.2, is what it takes to discipline the dynamics of household debt. Next, we set the prior mean of the steady-state default probability of households and entrepreneurs, $F^i(\bar{\omega}^i)$ and $F^e(\bar{\omega}^e)$, to an annual percentage rate of 3.2 and 2.95, respectively.[4] We find a higher posterior value for both, implying our model overshoots the actual default rates of households and firms. The two monitoring costs have a prior mean of 0.3. It is difficult to measure precisely the cost of bankruptcy. Andrade and Kaplan (1998) estimate it at 20 percent for firms. A 2008 survey by the Joint Economic Committee of Congress finds lenders pay an average of \$50,000 for each foreclosure, or 24 percent of that year's median home price. Our posterior estimates of 11 and 38 percent for μ^e and μ^i, respectively, are not too far from

[1] We refer to Smets and Wouters (2007), Justiniano, Primiceri, and Tambalotti (2010), and CMR.

[2] For instance, the response of the interest rate to inflation is 2.28, close to 2.40 found by CMR. The investment adjustment cost curvature, at 5.07, is similar to 5.48 estimated by Smets and Wouters (2007).

[3] Mavroeidis, Plagborg-Møller, and Stock (2014) find that the slope coefficient of the New Keynesian Philips curve varies from 0.001 to 0.141 according to different model specifications and estimation methods. This is a fairly wide range and the authors warn of specification uncertainty and weak identification issues.

[4] These values correspond to the average delinquency rates on consumer loans and commercial and industrial loans, respectively, over the period 1987-2019.

Table A4: Estimated Parameters

Description	Param.	Prior			Posterior	
		Distrib.	Mean	SD	Mode	SD
A. Economic Parameters						
Taylor rule output	$a_{\Delta y}$	normal	0.2	0.05	0.3589	0.0418
Taylor rule inflation	a_{π}	normal	1.5	0.25	2.2768	0.1656
Taylor rule smoothing	ρ_p	beta	0.8	0.15	0.8506	0.0128
Calvo price stickiness	ξ_p	beta	0.6	0.15	0.8550	0.0308
Calvo wage stickiness	ξ_w	beta	0.6	0.15	0.8725	0.0172
Price indexation on inflation	ι_p	beta	0.5	0.2	0.9531	0.0343
Wage indexation on inflation	ι_w	beta	0.5	0.2	0.6773	0.1708
Capital utilization cost	σ_a	normal	1	0.5	1.4303	0.4184
Investment adjustment cost	$S^{k\prime\prime}$	normal	2	1	5.0979	0.6333
Housing adjustment cost	$S^{h\prime\prime}$	normal	15	20	63.390	11.834
Patient consumption habit	b_c^p	beta	0.6	0.1	0.8252	0.0401
Impatient consumption habit	b_c^i	beta	0.6	0.1	0.8746	0.0382
Impatient default probability	$F^i(\bar{\omega}^i)$	beta	0.008	0.003	0.0115	0.0026
Entrepreneur default probability	$F^e(\bar{\omega}^e)$	beta	0.007	0.003	0.0274	0.0031
Impatient monitoring cost	μ^i	beta	0.3	0.1	0.3839	0.0716
Entrepreneur monitoring cost	μ^e	beta	0.3	0.1	0.1123	0.0195
Share of patient in total labor	κ	beta	0.5	0.1	0.8438	0.0645
B. Shock Parameters						
Autocorr. stationary technology	ρ_{ε}	beta	0.5	0.2	0.9076	0.0139
Autocorr. government spending	ρ_g	beta	0.5	0.2	0.9433	0.0200
Autocorr. entrepreneur equity	ρ_{γ^e}	beta	0.5	0.2	0.4357	0.0711
Autocorr. price markup	ρ_{λ_p}	beta	0.5	0.2	0.8282	0.0595
Autocorr. investment technology	ρ_{μ_Υ}	beta	0.5	0.2	0.9494	0.0176
Autocorr. technology trend	$\rho_{\mu_{z^*}}$	beta	0.5	0.2	0.4437	0.1816
Autocorr. collateral	ρ_{ν}	beta	0.5	0.2	0.9649	0.0095
Autocorr. housing redeployment	ρ_{η^i}	beta	0.5	0.2	0.9729	0.0125
Autocorr. capital redeployment	ρ_{η^e}	beta	0.5	0.2	0.1717	0.1011
Autocorr. preference	ρ_{ζ_c}	beta	0.5	0.2	0.2797	0.1135
Autocorr. housing	ρ_{ζ_h}	beta	0.5	0.2	0.7534	0.0586
Autocorr. marginal eff. investment	ρ_{ζ_i}	beta	0.5	0.2	0.4273	0.0685
SD stationary technology	σ_{ε}	invg2	0.01	1	0.0045	0.0004
SD monetary policy	σ_{ε^p}	invg2	0.01	1	0.0011	0.0001
SD government spending	σ_g	invg2	0.01	1	0.0197	0.0015
SD entrepreneur equity	σ_{γ^e}	invg2	0.01	1	0.0048	0.0003
SD price markup	σ_{λ_p}	invg2	0.01	1	0.0229	0.0107
SD investment technology	σ_{μ_Υ}	invg2	0.01	1	0.0045	0.0003
SD technology trend	$\sigma_{\mu_{z^*}}$	invg2	0.01	1	0.0032	0.0013
SD collateral	σ_{ν}	invg2	0.01	1	0.0312	0.0034
SD housing redeployment	σ_{η^i}	invg2	0.01	1	0.0225	0.0020
SD capital redeployment	σ_{η^e}	invg2	0.01	1	0.0095	0.0018
SD preference	σ_{ζ_c}	invg2	0.01	1	0.0198	0.0047
SD housing	σ_{ζ_h}	invg2	0.01	1	0.0026	0.0005
SD marginal efficiency investment	σ_{ζ_i}	invg2	0.01	1	0.0216	0.0017

Note: SD stands for standard deviation, invg2 for the inverse gamma distribution of type 2.

these numbers. Another important coefficient is the share κ of patient labor in total labor. We set its prior to 0.5 based on the observation that at least half of households in the US

Table A5: Static and Dynamic Properties, Model Versus Data

A. Steady-State Variables						Model	Data
Consumption-to-GDP ratio	c/y					0.58	0.58
Investment-to-GDP ratio	i/y					0.25	0.26
Government-spending-to-GDP ratio	g/y					0.16	0.16
Productive-capital-to-GDP ratio	$k/(4y)$					2.03	2.09
Debt-to-GDP ratio	$b/(4y)$					1.07	1.36
Inflation, annual rate	π					2.15	2.15
Fed funds rate, annual rate	R					3.97	3.62
Entrepreneurial leverage	L^e					1.75	1.75
B. Dynamic Variables		Corr. with GDP		Standard Deviation		Autocorrelation	
		Model	Data	Model	Data	Model	Data
GDP		1	1	1	1	0.91	0.88
Consumption		0.82	0.81	0.62	0.71	0.91	0.84
Investment		0.94	0.91	2.81	4.39	0.91	0.91
Hours		0.90	0.87	1.21	1.64	0.87	0.95
Household Credit		0.22	0.51	1.39	1.31	0.94	0.92
Household Spread		−0.47	−0.62	0.09	0.21	0.78	0.87
Business Credit		0.31	0.31	1.15	2.60	0.93	0.96
Business Spread		−0.39	−0.70	0.11	0.27	0.80	0.89

Notes: The sample period is 1985Q1–2019Q1. In Panel A data values show the sample mean. Model values are computed at the posterior mode. In Panel B data variables are detrended with a HP filter to permit comparison with stationary model variables.

hold some form of collateralized debt.[5] We find a posterior mode of 0.84, implying 16 percent of households are debt-constrained. This is slightly lower than the 20-25 percent share of hand-to-mouth households estimated by Kaplan, Violante, and Weidner (2014) with micro data.

Finally, we turn to the exogenous processes. Panel B of Table A4 reports their values. We fix the autoregressive parameter of the monetary policy shock ρ_{ε^p} to zero. We find that several shocks are highly persistent, including the collateral shock, with an autocorrelation coefficient of 0.965. The estimated standard deviation of the collateral shock, at 0.031, is the largest of any shock, presaging a strong impact on endogenous variables.

A4.3 Model Fit

We present two measures of model fit. In the first one, we ask whether our estimated model at the steady state is a reliable representation of the US economy. Panel A of Table A5 reports selected model variables and ratios evaluated at the posterior mode along with their empirical counterpart. Overall, the model and data match well. This is the case by construction for the ratio of government spending to GDP, inflation, and nonfinancial firm

[5]According to The Pew Charitable Trusts (2015), eight in ten Americans hold some form of debt. The most frequently held forms are mortgage debt (44%), unpaid credit card balances (39%), car loans (37%), and student loans (21%). In our model debt is backed by collateral, so that corresponds to all mortgage debt as well as a large share of auto loans.

leverage. One exception to the good fit is the ratio of debt to GDP, which is too low in the model. At the cost of an additional parameter, one could easily put more weight on housing in the utility function of impatient households, thereby increasing the ratio of debt to output.[6]

As a second measure of model fit, we compute moments of selected variables when the model is at its posterior mode, and we compare them to the data. Since model variables are stationary, we need to stationarize the data as well, and we do that using a standard HP filter (we find similar results using a bandpass filter). Panel B of Table A5 shows the results. On the whole, the model captures well the dynamic properties of the data.

A5 Competing With The Collateral Shock

The literature on business cycles puts forward other shocks as potential driving forces. This section examines the dynamic properties of some of these shocks and explains why they are displaced by the collateral shock.

A5.1 Preference Shocks

The consumption preference shifter $\zeta_{c,t}$ is the quintessential demand shock. It plays a critical role in the business cycle literature—as the main driver of consumption. Previous studies repeatedly find that it accounts for 45 to 75 percent of the variance in consumption, but has very little impact on other variables. In stark contrast, our analysis puts its contribution at 18 percent. We interpret this as follows. The estimation procedure minimizes the residuals of the exogenous variables. Thus, it favors the shocks that best explain the entire set of observables rather than those that account separately for the movements in individual series. Put differently, we no longer "need" the preference shock now that the collateral shock does the job.

To illustrate this insight, Table A6 reports the contribution of the collateral and preference shocks to the covariances of output, consumption, and investment at business cycle frequency. Panel A shows estimates for the baseline model, while Panel B shows estimates for the model without impatient households (and thus without household credit). We emphasize two results. First, in the baseline model the collateral shock explains a large share of the three covariances. Second, in the model without impatient households the collateral shock is unable to account for the output-consumption and consumption-investment covariances. Instead, it is the preference shock that fills the gap.

[6]A previous version included this parameter but we decided to drop it for parsimony.

Table A6: Covariance Decomposition at Business Cycle Frequency

	Collateral Shock ν_t	Preference Shock $\zeta_{c,t}$
A. Baseline Model		
Output–Consumption	49	8
Output–Investment	46	0
Consumption–Investment	55	0
B. Model with No Impatient Households and No Household Credit		
Output–Consumption	0	48
Output–Investment	21	2
Consumption–Investment	4	−11

Notes: The covariance decomposition is computed at the posterior mode. Business cycle frequency encompasses periodic components with cycles of 6-32 quarters.

A5.2 Financial Shocks

Our model includes two sector-specific financial shocks on housing η_t^i and capital η_t^e. These disturbances alter the cost of redeploying assets in their respective market. As mentioned in the main text, they are observationally equivalent to risk shocks in the first-order approximation of the model. In particular, η_t^e is equivalent to CMR's well-known risk shock σ_t.

The collateral shock affects the two sectors at once. We interpret it as coming from banks, their common interlocutor. As one would expect, some key variables behave differently whether they respond to collateral or sector-specific shocks. Credit spreads on household and business loans are the best example. These variables, which we use as observables in the estimation, move hand in hand in the data, with a correlation of 0.85. We compute the response of credit spreads to a collateral shock in our baseline model. Then, we switch off the collateral shock, we re-estimate the model, and we compute the equivalent response to the two sector-specific shocks. Figure A1 displays the results.

An adverse collateral shock raises the two credit spreads simultaneously, as discussed in the paper. The two sectoral shocks, by contrast, increase their respective market's spread but have virtually no effect on the other one's.[7] Not surprisingly, in order to match the data, the two estimated sectoral shocks in the alternative model turn out to be highly correlated, at 0.72.[8] This contradicts the premise that shocks are structural, independent and identically distributed random variables.

Why is a common collateral shock more plausible than sector-specific forces? In our opinion, the financial sector is the center of the modern market economy. It is

[7]The household spread does increase a tiny bit a few periods after a capital redeployment cost shock hits. As firms cut back on employment, impatient households suffer from a lower income and become slightly more prone to default.

[8]In our baseline estimation, the correlation is −0.02 and non-significant.

Figure A1: Response of Credit Spreads to Three Shocks

Notes: The solid line plots impulse responses from the baseline model. The dashed and dotted lines come from the model without collateral shocks. Spreads are in annual percentage deviation from steady state.

the place where all economic actors meet: savers, borrowers, individuals, firms, banks, governments. Our collateral redeployment cost function is a crude way to microfound how shocks that originate in financial markets propagate to different classes of borrowers. Better understanding how weaknesses build up in the financial system and make it more prone to large reversals of confidence is a promising avenue of research.

A5.3 Credit Demand Shocks

Besides the financial shocks, two disturbances in our model directly affect the demand for loans—through the demand for housing and capital. These are the housing $\zeta_{h,t}$ and entrepreneurial equity γ_t^e (or capital quality) shocks. In the estimation, they are driven out by the collateral shock for the following reasons. The housing shock induces countercyclical consumption. As impatient households find housing services less desirable, they consume more. The equity shock implies countercyclical business credit. Entrepreneurs with lower net worth are less able to self-finance their projects and compensate by taking on more debt.[9]

Nonetheless, these forces play a nontrivial role in accounting for household and business bankruptcy. Figure A2 decomposes historical default rates into the contribution of the model's different shocks. The first takeaway is that the collateral shock—a disturbance to the credit supply—accounts for the bulk of the evolution. The second takeaway is that shocks directly affecting the demand for loans are responsible for most of the remaining share. These findings are consistent with a literature that emphasizes credit demand shocks.[10] They are also in line with anecdotal evidence of bold homeowners and firms buying larger property in the hope that real estate prices would continue to rise indefinitely.

[9]See CMR for an extensive discussion of the equity shock.

[10]Recent examples include Kahle and Stulz (2013) and Jiménez et al. (2017).

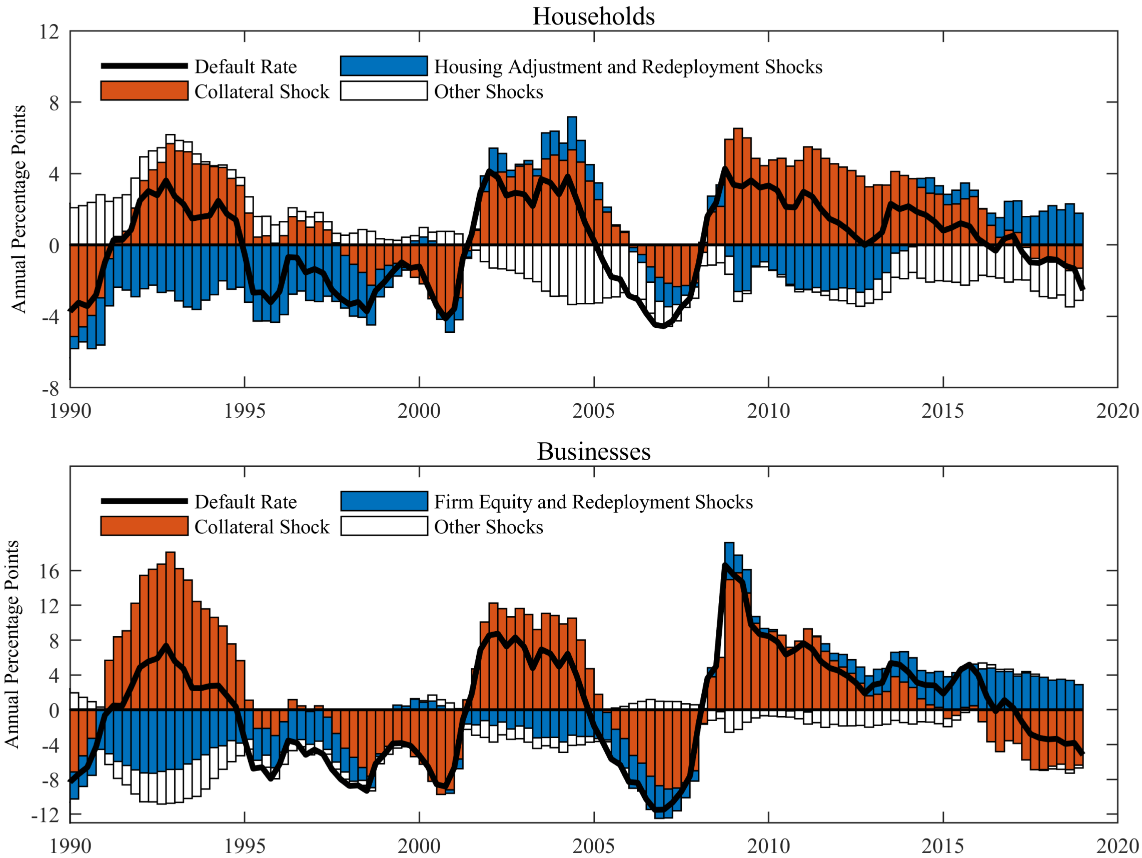

Figure A2: Shock Decomposition of Default Rates

Notes: The solid lines correspond to the default rates of impatient households (top) and entrepreneurs (bottom). The vertical bars show the contributions of the different shocks to the evolution of default rates.

A5.4 Remaining Shocks

Figure A3 plots the response of key variables to all the shocks of the model (except the collateral shock). Overall, the responses are in step with those of the literature. None of these impulses is able to generate the comovements observed in the data. For example, the monetary policy shock implies countercyclical nominal interest rates and a muted initial response of credit. The government-spending shock generates countercyclical consumption, a well-known puzzle in the fiscal policy literature. The TFP shock implies countercyclical hours, as New Keynesians contend. The two investment shocks $\zeta_{i,t}$ and $\mu_{\Upsilon,t}$ predict countercyclical asset prices and business credit. And so on.

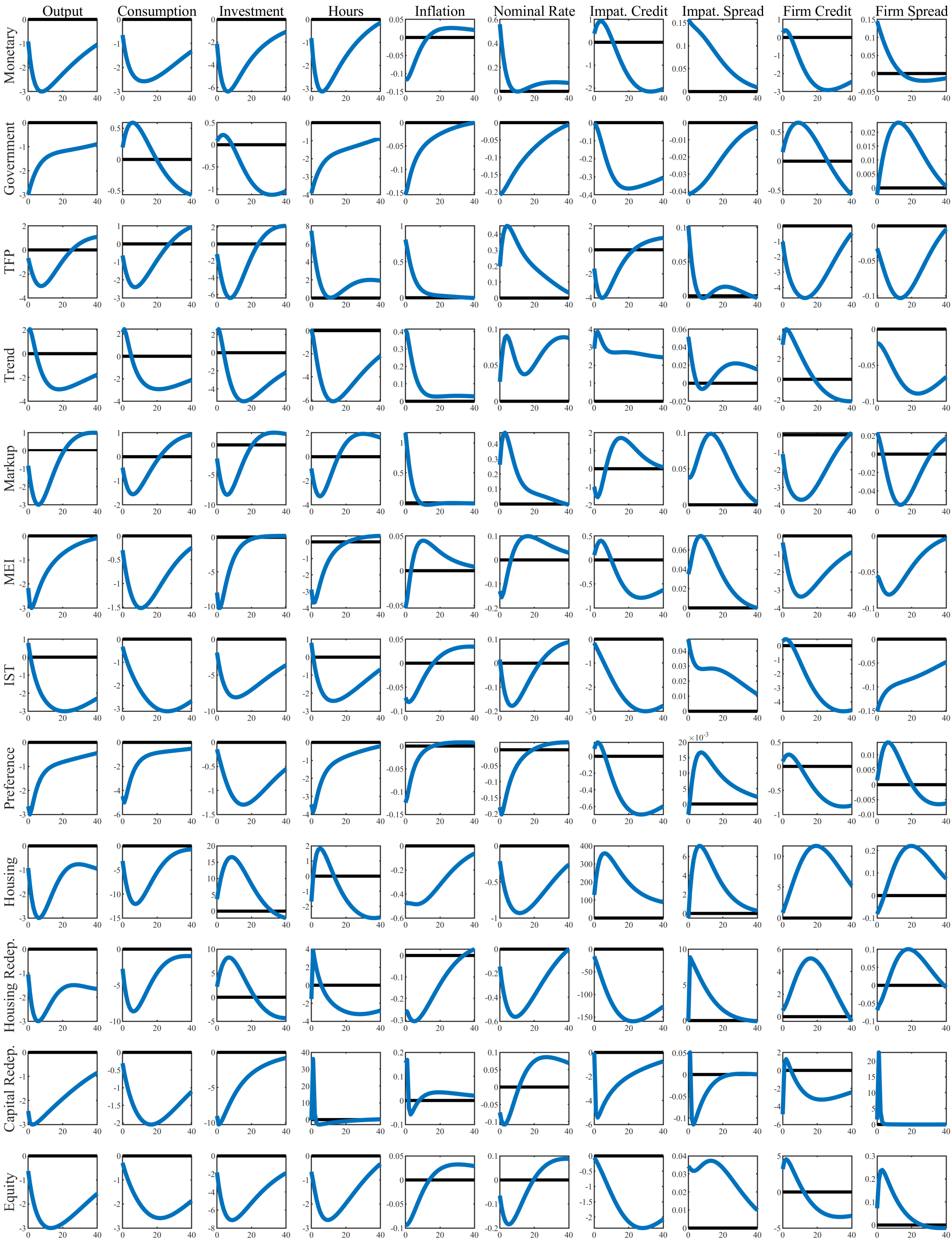

Figure A3: Response to All Other Shocks

Notes: Impulse responses are normalized so that the maximum fall in output is three percent. All variables are expressed in percentage deviation from their steady state. The horizontal axis is time, one period is a quarter.

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