

# Online Appendix for Pigouvian Cycles\*

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## A List of log-linearized equations

In this Online Appendix we list the log-linearized equations of the model introduced in Section I. Let barred variables denote steady-state values, and the hat over a lower case variable denote log-deviations from the steady-state value, i.e., let  $\hat{n}_t = \ln N_t - \ln \bar{N}$  denote log-deviations of employment from the steady-state. For variables that grow along the balanced growth path, such as consumption  $C_t$ , we denote by  $\tilde{C}_t = \frac{C_t}{A_t}$  the stationarized variable and by  $\tilde{C}$  the value it takes along the balanced growth path. In such a case  $\hat{c}_t = \ln \tilde{C}_t - \ln \tilde{C}$ .

1. Labor force

$$\hat{l}f_t = \frac{\bar{N}}{\bar{N} + \bar{U}} \hat{n}_t + \frac{\bar{U}}{\bar{N} + \bar{U}} \hat{u}_t.$$

2. Consumption Euler equation

$$\begin{aligned} -\hat{R}_t &= \left[ \frac{1}{\mu - \vartheta} + \frac{\vartheta}{(\mu - \vartheta)\mu} \right] \mu \hat{c}_t - \frac{\vartheta}{\mu - \vartheta} \hat{c}_{t-1} - \frac{\mu}{\mu - \vartheta} E_t \hat{c}_{t+1} \\ &\quad - \eta_t^p + E_t \eta_{t+1}^p + \frac{\vartheta}{\mu - \vartheta} \eta_t^A - \frac{\mu}{\mu - \vartheta} E_t \eta_{t+1}^A - E_t \pi_{t+1}. \end{aligned}$$

3. Marginal utility of consumption

$$\hat{\lambda}_t = -\frac{1}{1 - \frac{\vartheta}{\mu}} \hat{c}_t + \frac{\frac{\vartheta}{\mu}}{1 - \frac{\vartheta}{\mu}} (\hat{c}_{t-1} - \eta_t^A) + \hat{\eta}_t^p.$$

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4. Law of motion for employment

$$\hat{n}_t = (1 - \delta_N) \hat{n}_{t-1} + \delta_N \hat{h}_t.$$

5. Hiring

$$\hat{h}_t = \hat{u}_t + \frac{1}{1 - \bar{x}} \hat{x}_t.$$

6. Labor participation decision

$$\begin{aligned} \hat{v}_t^N + (1 - \bar{x})^{-1} \hat{x}_t &= \left( \eta_t^l + \varphi \hat{l}_t - \eta_t^p \right) \\ &+ \left[ \frac{\mu}{\mu - \vartheta} \hat{c}_t - \frac{\vartheta}{\mu - \vartheta} (\hat{c}_{t-1} - \eta_t^A) \right]. \end{aligned}$$

7. Value of employment to households

$$\begin{aligned} &\frac{\varpi (1 - \bar{x}) + \bar{x}}{\varpi (1 - \bar{x})} \left[ \hat{v}_t^N + \frac{\bar{x} [\varpi (1 - \bar{x}) + \bar{x}]}{1 - \bar{x}} \hat{x}_t \right] \\ &= \left\{ \frac{\varpi (1 - \bar{x}) + \bar{x}}{\varpi (1 - \bar{x})} - (1 - \delta_N) \beta \right\} \hat{w}_t^r + (1 - \delta_N) \beta \left( \hat{\pi}_{t+1} - \hat{R}_t + \hat{v}_{t+1}^N + \eta_{t+1}^A \right). \end{aligned}$$

8. Production function

$$\hat{f}_t = \hat{a}_t + \alpha \hat{n}_t + (1 - \alpha) (\hat{k}_{t-1} - \hat{\eta}_t^A).$$

9. Output function

$$\hat{y}_t = \frac{\tilde{f}}{\tilde{f} - \tilde{g}} \hat{f}_t - \frac{\tilde{g}}{\tilde{f} - \tilde{g}} \hat{g}_t.$$

10. Adjustment cost function

$$\hat{g}_t = 2 \left( \hat{h}_t - \hat{n}_t \right) - \eta^q \hat{q}_t + \hat{a}_t + \alpha \hat{n}_t + (1 - \alpha) \left( \hat{k}_{t-1} - \hat{\eta}_t^A \right).$$

11. Derivative of adjustment cost function ( $\partial H_t$ ):

$$\hat{g}_{H,t} = -\eta^q \hat{q}_t + \hat{h}_t - 2\hat{n}_t + \hat{f}_t.$$

12. Derivative of adjustment cost function ( $\partial K_t$ ):

$$\hat{g}_{K,t} = \hat{g}_t - \hat{k}_{t-1} + \hat{\eta}_t^A.$$

13. Derivative of adjustment cost function ( $\partial N_t$ ):

$$\tilde{g}_{N,t}\hat{g}_{N,t} = -e_2q^{-\eta^q}\delta_N^2\frac{\tilde{f}}{N}\left(-\eta^q\hat{q}_t + \hat{f}_t - 3\hat{n}_t + 2\hat{h}_t\right) + \frac{\alpha\tilde{g}}{N}(\hat{g}_t - \hat{n}_t).$$

14. Vacancy filling rate:

$$\hat{q}_t = -\frac{l}{1-l}\hat{x}_t.$$

15. Law of motion for capital

$$\hat{k}_t = (1 - \delta_K)\frac{1}{\mu}\left(\hat{k}_{t-1} - \hat{\eta}_t^A\right) + \frac{\tilde{I}}{\tilde{K}}(\hat{i}_t + \hat{\eta}_t^I).$$

16. FOC capital

$$\begin{aligned}\hat{q}_t^K &= E_t\hat{\pi}_{t+1} - \hat{R}_t + \frac{\bar{\Pi}}{\bar{R}}\left[\frac{\bar{\xi}(\tilde{f}_K - \tilde{g}_K)}{\bar{Q}_K}\right]E_t m\hat{c}_{t+1} \\ &+ \frac{\bar{\Pi}\bar{\xi}\tilde{f}_K}{\bar{Q}_K}E_t\hat{f}_{K,t+1} - \frac{\bar{\Pi}}{\bar{R}\bar{Q}_K}\bar{\xi}\tilde{g}_K E_t\hat{g}_{K,t+1} + \frac{\bar{\Pi}}{\bar{R}}[(1 - \delta_K)]E_t\hat{q}_{t+1}^K.\end{aligned}$$

17. FOC employment

$$\begin{aligned}\bar{\xi} &\left(\tilde{g}_K - \tilde{f}_N + \tilde{g}_N\right)\hat{\xi}_t + \bar{\xi}\tilde{g}_H \cdot \hat{g}_{H,t} = \\ &\bar{\xi}\tilde{f}_N \cdot \hat{f}_{N,t} - \bar{\xi}\tilde{g}_N \cdot \hat{g}_{N,t} - \tilde{W}^r \hat{w}_t^r \\ &+ (1 - \delta_N)\frac{\bar{\Pi}}{\bar{R}}\bar{\xi}\tilde{g}_H\mu\left[E_t\hat{\pi}_{t+1} - R_t + E_t\hat{\xi}_{t+1} + E_t\hat{g}_{H,t+1} + E_t\hat{\eta}_{t+1}^A\right].\end{aligned}$$

18. Resource constraint

$$\frac{\tilde{Y}}{\eta^G}(\hat{y}_t - \hat{\eta}_t^G) = \tilde{C}\hat{c}_t + \tilde{I}(\hat{\eta}_t^q + \hat{i}_t).$$

19. Phillips curve

$$\left[1 + \frac{\bar{\Pi}\mu}{\bar{R}}\psi\right]\hat{\pi}_t = \psi\hat{\pi}_{t-1} + \frac{\epsilon - 1}{\zeta} \cdot \hat{\xi}_t + \frac{\bar{\Pi}\mu}{\bar{R}}E_t\hat{\pi}_{t+1} + \hat{\eta}_t^{mkp}.$$

20. Real wage equation

$$\begin{aligned}\tilde{W}^{r,NASH}\hat{w}_t^{r,NASH} &= \gamma\bar{\xi}\left[(\tilde{f}_N - \tilde{g}_N)\hat{\xi}_t + \tilde{f}_N\hat{f}_{N,t} - \tilde{g}_N\hat{g}_{N,t}\right] \\ &+ (1 - \gamma)\frac{\chi\bar{L}^\varphi}{\tilde{\lambda}_*}(\eta_t^l + \varphi\hat{l}_t - \hat{\lambda}_t).\end{aligned}$$

21. Inertial wage

$$\hat{W}_t^r = \omega \hat{W}_{t-1}^r + (1 - \omega) \hat{W}_t^{r,NASH}.$$

22. Taylor Rule

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) r_\pi \hat{\pi}_t + (1 - \rho_R) r_y \hat{y}_t + \hat{\eta}_{r,t}.$$

23. Marginal productivity of labor

$$\hat{f}_{N,t} = \hat{f}_t - \hat{n}_t.$$

24. Marginal productivity of capital

$$\hat{f}_{K,t} = \hat{f}_t - \hat{k}_{t-1} + \hat{\eta}_t^A.$$

25. Tobin's Q for capital

$$\hat{q}_t^K + \hat{\eta}_t^I = \hat{\eta}_t^q + S'' (1 + \beta) \hat{i}_t - S'' \hat{i}_{t-1} - \beta S'' \hat{i}_{t+1}. \quad (1)$$

26. Tobin's Q for employment

$$\hat{Q}_t^N = \hat{\xi}_t + \hat{g}_{H,t}.$$

## B The Data Set

Nominal consumption includes personal consumption expenditures: nondurable goods (PCND) and personal consumption expenditures in services (PCESV), which are computed by the U.S. Bureau of Economic Analysis (BEA) (NIPA tables). Nominal investments include personal consumption expenditures in durable goods (PCDG) and gross private domestic investment (GPDI), which are computed by the BEA (NIPA tables). We deflate GDP, consumption, and investment by using the implicit price deflator index (GDPDEF), computed by the BEA (NIPA tables) and then we divide the resulting variable by the civilian non-institutional population (CNP16OV), measured by the U.S. Bureau of Labor Statistics (BLS).

The employment rate and the participation rate are the quarterly averages of the civilian employment-to-population ratio (EMRATIO) and the civilian labor force participation rate (CIVPART), respectively. We measure wage growth by using the quarterly average of the wage and salary disbursements received by employees (A576RC1) divided by the civilian employment level (CE16OV). We divide the resulting series by the GDP deflator to obtain our measure of real wages. TFP growth rates are adjusted and unadjusted to capital utilization (Fernald 2012). We have three measures of inflation (GDP deflator, CPI, and PCE) in estimation. See Campbell et al. (2012) for a thorough description of this approach. We take the logs of these series. All

data used in estimation are quarterly and in percent.

The interest rate is the effective federal funds rate (DFF). For the second sample, which ranges from the fourth quarter of 2008 through the fourth quarter of 2016 we use the market-expected federal funds rates up to ten quarters forward to enforce the effective lower bound of the nominal interest rate. We construct this time series using the overnight index swap rates,  $OIS_{i,t}$ , where the underlying interest rate is the average Federal Funds rate over the next  $i$  quarters, starting from time  $t$ . Expected Federal Funds Rates  $i$ -quarters forward are computed as:  $EFFR_{i,t} = OIS_{i,t} * i - OIS_{i,t-1} * (i - 1)$ . We have experimented also using the data as in Campbell et al. (2017) and found very similar results.<sup>1</sup> As in that paper, we consider market expectations with forecasting horizons ranging from one quarter to ten quarters and introduce a two-factor model to parsimoniously capture the comovements of these expectations across horizons.<sup>2</sup>

## C Using Multiple TFP Growth Rates in Estimation

To ensure model consistency of the TFP series adjusted and unadjusted for variable capital utilization computed by Fernald (2014), we compute TFP growth using the number of employed workers instead of total hours. We do not adjust the TFP series for variations in the quality of workers over time because this time series is not available. Changes in the quality of employment is picked up by the labor-augmenting technology process,  $\hat{\eta}_t^A$ . Furthermore, we set the elasticity of output to employment,  $\alpha$ , to 0.66, which is consistent with how this parameter is calibrated in our analysis.

Note that we do not have to adjust Fernald's estimate of TFP for aggregate hiring costs  $g$  because these costs are modeled as forgone output. Hence, the measure of GDP in the data should be interpreted as already net of these costs.

The observation equations for the two TFP growth rates read as follows:

$$\Delta \ln TFP_t^N = c_{TFP,unadj}^m + \lambda_{TFP,unadj}^m [\hat{a}_t - \hat{a}_{t-1} + \alpha \hat{\eta}_t^A + 100\alpha \ln \mu] + \eta_{TFP,t}^N, \quad (2)$$

$$\Delta \ln TFP_t^A = c_{TFP,adj}^m + \lambda_{TFP,adj}^m [\hat{a}_t - \hat{a}_{t-1} + \alpha \hat{\eta}_t^A + 100\alpha \ln \mu] + \eta_{TFP,t}^A, \quad (3)$$

where  $\Delta \ln TFP_t^N$  and  $\Delta \ln TFP_t^A$  denote the observed series of unadjusted and adjusted TFP

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<sup>1</sup>The funds rate paths implied by these contracts include a 1 basis point- per-month adjustment for term premiums through 2011:Q2. We do not apply any adjustments after this date, when it appears that term premiums disappeared or perhaps turned negative. The unadjusted data yield very similar results.

<sup>2</sup>The forward guidance shocks in the Taylor rule are an array of i.i.d. shocks from the perspective of agents in the model. The factor model is part of the measurement equations and is introduced to capture the strong correlation of interest rates across their maturity horizons. We run a principal component analysis so as to verify that two factors are enough to explain most of the comovement among the expected interest rates in the period 2008:Q4-2016:Q4. This two-factor structure was introduced by Gürkaynak, Sack, and Swanson (2005).

growth expressed in percent quarterly rates;  $\lambda_{TFP,unadj}^m$  (normalized to unity) and  $\lambda_{TFP,adj}^m$  denote the loadings associated with the unadjusted and the adjusted series; and  $\eta_{TFP,t}^N$  and  $\eta_{TFP,t}^A$  are i.i.d. Gaussian measurement errors with mean zero and standard deviation  $\sigma_{TFP,unadj}^m$  and  $\sigma_{TFP,adj}^m$ , respectively. The parameters  $c_{TFP,unadj}^m$  and  $c_{TFP,adj}^m$  denote constant parameters. Furthermore,  $\hat{a}$  denotes log of TFP ( $\ln a_t$ ) and  $\hat{\eta}_t^a$  denotes log deviations of the growth rate of the labor-augmenting technology from its trend  $\mu$ .

## D Measurement Equations

1. Real GDP growth

$$100\Delta \ln RGDP_t = \hat{y}_t - \hat{y}_{t-1} + \hat{\eta}_t^A + 100 \ln \mu.$$

2. Real Consumption

$$100\Delta \ln RConsump_t = \hat{c}_t - \hat{c}_{t-1} + \hat{\eta}_t^A + 100 \ln \mu.$$

3. Real Investment

$$100\Delta RINV_t = \hat{i}_t - \hat{i}_{t-1} + \hat{\eta}_t^A + 100 \ln \mu.$$

4. Inflation rate (multiple indicator)

$$100 \cdot GDPDEF L_t = c_{\pi,1}^m + \lambda_{\pi,1} \hat{\pi}_t + 100 \ln \Pi_* + \sigma_{\pi,1}^m \eta_{1,t}^\pi,$$

$$100\Delta PCE_t = c_{\pi,2}^m + \hat{\pi}_t + 100 \ln \Pi_* + \sigma_{\pi,2}^m \eta_{2,t}^\pi,$$

$$100\Delta CPI_t = c_{\pi,3}^m + \lambda_{\pi,3} \hat{\pi}_t + 100 \ln \Pi_* + \sigma_{\pi,3}^m \eta_{3,t}^\pi.$$

5. Real wage growth

$$100\Delta \ln RW_t = c_w^m + \hat{w}_t^r - \hat{w}_{t-1}^r + \hat{\eta}_t^A + 100 \ln \mu + \sigma_w^m \eta_{w,t}.$$

where the constant  $c_w^m$  accounts for the difference in sample means with the growth rate of GDP, consumption, and investment.

6. Unemployment rate ( $u_* = 0.056$ )<sup>3</sup>

$$100 \ln UR_t = \hat{u}_t - \hat{l}f_t + 100 \ln u_*.$$

7. Unemployment rate ( $u_* = 0.056$ )<sup>4</sup>

$$100 \ln E_t^{spf} UR_{t+h} = E_t \hat{u}_{t+h} - E_t \hat{l}f_{t+h} + 100 \ln u_* + \sigma_{u,h}^m \eta_{h,t}^u, \quad h \in \{1, 2, 3, 4\}.$$

8. Participation rate ( $lf_* = 0.65$ )

$$\begin{aligned} 100 \ln PartR_t &= 100 \ln \frac{LF_t}{Pop_t} \\ &= \hat{l}f_t + 100 \ln lf_*. \end{aligned}$$

9. Employment rate ( $n_*$  is implied by  $u_*$  and  $lf_*$ )

$$100 \ln ER_t = \hat{n}_t + 100 \ln n_* + \sigma_E^m \eta_{e,t}.$$

10. Federal funds rate (quarterly and in percent)

$$FFR_t = \ln R_t + 100 \ln R_*.$$

11. Multiple indicator for TFP growth adjusted for capital utilization  $\Delta TFP_t^A$  and non-

<sup>3</sup>To get this, observe that

$$\begin{aligned} 100 \ln \frac{UR_t\%}{100} &= 100 \ln \frac{U_t}{LF_t} \\ &= 100 \ln \frac{U_t}{\bar{U}} - 100 \ln \frac{LF_t}{\bar{LF}} + 100 \ln \frac{\bar{U}}{\bar{LF}} \\ &= \hat{u}_t - \hat{l}f_t + 100 \ln \bar{U}^r, \end{aligned}$$

where  $\bar{U}^r \equiv \frac{\bar{U}}{\bar{LF}}$  denotes the steady-state unemployment rate.

<sup>4</sup>To get this, observe that

$$\begin{aligned} 100 \ln \frac{UR_t\%}{100} &= 100 \ln \frac{U_t}{LF_t} \\ &= 100 \ln \frac{U_t}{\bar{U}} - 100 \ln \frac{LF_t}{\bar{LF}} + 100 \ln \frac{\bar{U}}{\bar{LF}} \\ &= \hat{u}_t - \hat{l}f_t + 100 \ln \bar{U}^r, \end{aligned}$$

where  $\bar{U}^r \equiv \frac{\bar{U}}{\bar{LF}}$  denotes the steady-state unemployment rate.

adjusted for capital utilization  $\Delta TFP_t^N$

$$100\Delta \ln TFP_t^A = c_{TFP,adj}^m + \lambda_{TFP,adj}^m [\hat{a}_t - \hat{a}_{t-1} + \alpha \hat{\eta}_t^A + 100\alpha \ln \mu] + \eta_{TFP,t}^A,$$

$$100\Delta \ln TFP_t^N = c_{TFP,unadj}^m + \lambda_{TFP,unadj}^m [\hat{a}_t - \hat{a}_{t-1} + \alpha \hat{\eta}_t^A + 100\alpha \ln \mu] + \eta_{TFP,t}^N.$$

12. Expected future federal funds rate (only in the second sample): The forward guidance shocks in the Taylor rule,  $\xi_{r,t}^l$  with  $l \in \{0, \dots, 10\}$  are disciplined by the following two-factor model

$$\xi_{r,t}^l = \Lambda_T f_T + \Lambda_P f_P + \eta_{l,t}^{FG}, \text{ with } l \in \{0, \dots, 10\}$$

where  $f_T$  and  $f_P$  are two i.i.d. Gaussian factors with standard deviations  $\sigma_{f,T}$  and  $\sigma_{f,P}$ ,  $\Lambda_T$  and  $\Lambda_P$  are their respective loadings, and  $\eta_{l,t}^{FG}$  are eleven i.i.d. measurement error shocks. We impose restrictions on the two vectors of loadings allowing us to identify the two factors: a target factor that moves the current policy rate and a path factor that moves the slope of the term structure of future interest rates (i.e., it moves only expected future rates). The crucial restrictions to interpret factors this way are that  $\Lambda_T(0) = 1$  and  $\Lambda_P(0) = 0$ .

## E Model's Impulse Response Functions to TFP Shocks

Figures 1-3 show the posterior median and the 68-percent credible set of the impulse response functions of unemployment rate, employment rate, real wages, GDP, consumption, and investment to a one-standard deviation surprise TFP shock, a one-standard deviation four-quarter-ahead news shock to TFP, a one-standard deviation eight-quarter-ahead news shock to TFP, respectively.

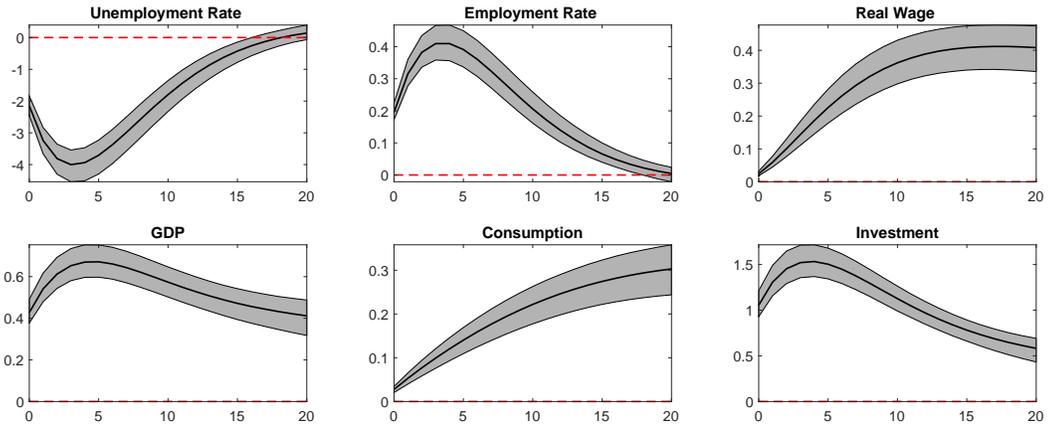


Figure 1: Posterior median of the response of unemployment rate, employment rate, real wage, GDP, consumption, and investment to a surprise shock to TFP. The gray areas denote the sixty-eight-percent posterior credible sets. The responses of unemployment and employment rates are expressed in percentage points deviations from the steady-state rate. All other responses are in percentage deviations from their steady-state value. The size of the initial shocks is one percentage point.

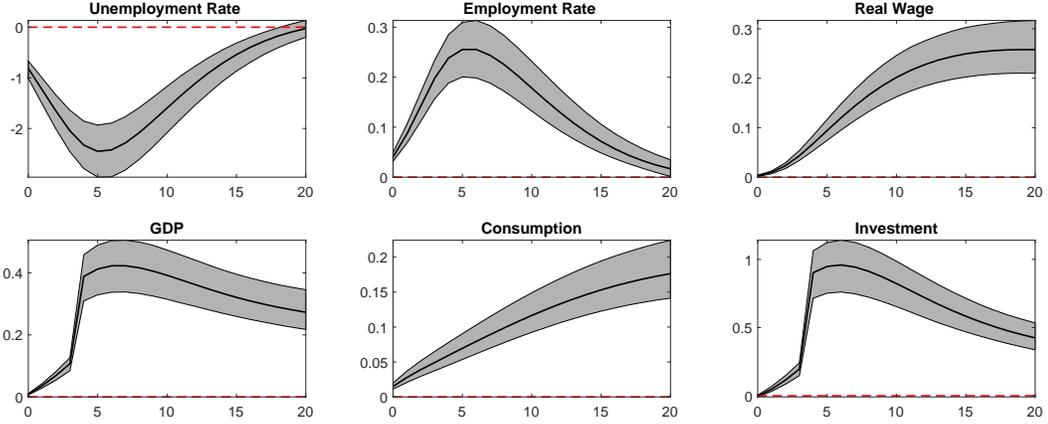


Figure 2: Posterior median of the response of unemployment rate, employment rate, real wage, GDP, consumption, and investment to a four-quarter-ahead shock to TFP. The gray areas denote the sixty-eight-percent posterior credible sets. The responses of unemployment and employment rates are expressed in percentage points deviations from the steady-state rate. All other responses are in percentage deviations from their steady-state value. The size of the initial shocks is one percentage point.

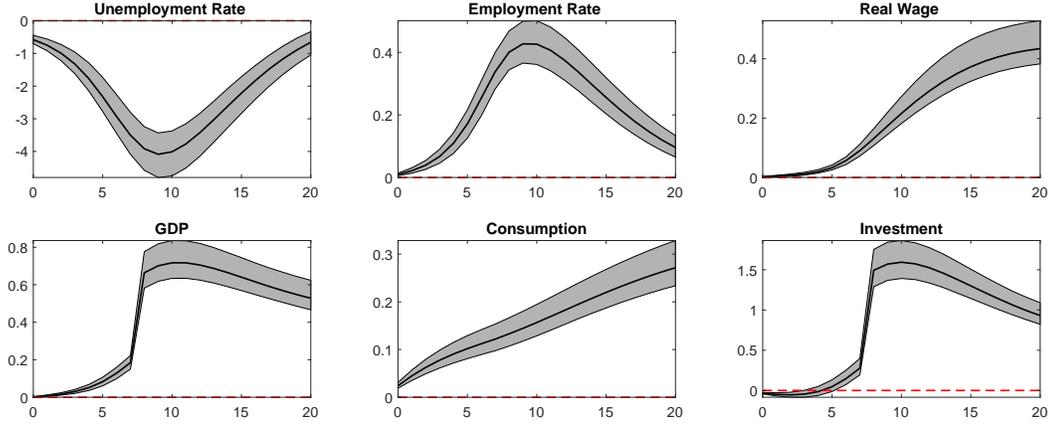


Figure 3: Posterior median of the response of unemployment rate, employment rate, real wage, GDP, consumption, and investment to an eight-quarter-ahead shock to TFP. The gray areas denote the sixty-eight-percent posterior credible sets. The responses of unemployment and employment rates are expressed in percentage points deviations from the steady-state rate. All other responses are in percentage deviations from their steady-state value. The size of the initial shocks is one percentage point.

## F Recovering Noise from the Estimated Models with News Shocks

The goal of this Online Appendix is to show how the estimated news representation can be used to tease out the historical series of noise shocks and assess their historical contribution to the U.S. business cycle. We will proceed toward this goal in three steps. We first apply the representation theorem introduced by Chahrour and Jurado (2018) to obtain the implied parameter of the model ( $\sigma_\theta$ ,  $\sigma_{4,\nu}$ , and  $\sigma_{8,\nu}$ ) from the estimated parameters ( $\sigma_{0,a}$ ,  $\sigma_{4,a}$ , and  $\sigma_{8,a}$ ) defined in the news representation. Second, with the parameter values of our model with signals at hand, we use the two-sided filtered series of TFP news and surprise shocks (obtained using the estimated

news representation of our model) to tease out the implied series of noise shocks. Third, we construct the historical dynamics of the business cycle variables implied by the estimated in-sample realizations of noise shocks alone.

**Step 1: Fetching the Parameters of the Model from the Estimated News Representation (Chahrour and Jurado 2018)**

The news representation of the model shares all the parameters of our model except for the standard deviations of TFP fundamentals and noise; that is,  $\sigma_\theta$ ,  $\sigma_{4,\nu}$ , and  $\sigma_{8,\nu}$ . As shown by Chahrour and Jurado (2018), for given parameter values of the estimated news representation, the parameter values of the observationally equivalent model with noisy signals are given by:

$$\sigma_{8,\nu}^2 = (\sigma_{0,a}^2 + \sigma_{4,a}^2 + \sigma_{8,a}^2) \left( \frac{\sigma_{0,a}^2 + \sigma_{4,a}^2}{\sigma_{8,a}^2} \right), \quad (4)$$

$$\sigma_{4,\nu}^2 = (\sigma_{0,a}^2 + \sigma_{4,a}^2) \frac{\sigma_{0,a}^2}{\sigma_{4,a}^2}, \quad (5)$$

and

$$\sigma_\theta^2 = \sigma_{a,0}^2 + \sigma_{4,a}^2 + \sigma_{8,a}^2. \quad (6)$$

We can use the estimated variance of TFP shocks ( $\sigma_{a,0}^2$ ,  $\sigma_{a,4}^2$ , and  $\sigma_{a,8}^2$ ) in the news representation to pin down the estimated variances for noise and fundamental shocks  $\sigma_{4,\nu}^2$ ,  $\sigma_{8,\nu}^2$ , and  $\sigma_\theta^2$ .

**Step 2: Teasing Out the Historical Realizations of Noise Shocks**

In the estimated news representation, revisions of expectations about future TFP innovations  $\theta_{t+8}^a$  in period  $t$ ,  $t+4$ , and  $t+8$  are given by the realizations of news and surprise shocks  $\varepsilon_{a,t}^i$  with  $i \in \{0, 4, 8\}$ , respectively. In symbols, this would be as follows:

$$E_t \theta_{t+8}^a = \varepsilon_{a,t}^8, \quad (7)$$

$$E_{t+4} \theta_{t+8}^a - E_t \theta_{t+8}^a = \varepsilon_{a,t+4}^4, \quad (8)$$

$$\theta_{t+8}^a - E_{t+4} \theta_{t+8}^a = \varepsilon_{a,t+8}^0. \quad (9)$$

For the news representation to be observationally equivalent to our model with noisy signals, expectations about eight-quarter-ahead TFP changes in the model and in the estimated news representation must be identical. Therefore, we write the following condition:

$$\kappa_8 (\theta_{t+8}^a + \nu_{8,t}) = E_t \theta_{t+8}^a = \varepsilon_{a,t}^8, \quad (10)$$

where  $\kappa_8 \equiv (\sigma_{0,a}^2 + \sigma_{4,a}^2 + \sigma_{8,a}^2) / (\sigma_{0,a}^2 + \sigma_{4,a}^2 + \sigma_{8,a}^2 + \sigma_{8,\nu}^2)$  is the Kalman gain in terms of the estimated parameters of the news representation. The Kalman gain captures the precision of

signals and depends on the parameter mappings (4)-(6) from the estimated news representation to our model with signals. Equation (10) decomposes the expectations about the eight-quarter-ahead TFP innovations,  $E_t\theta_{t+8}^a$ , into a fundamental component  $\kappa_8\theta_{t+8}^a$ , which will affect TFP in eight quarters, and a noise component  $\kappa_8\nu_{8,t}$ , which will never affect TFP. Substituting the estimated TFP innovations  $\hat{\theta}_{t+8}^a = \hat{\varepsilon}_{a,t+8}^0 + \hat{\varepsilon}_{a,t+4}^4 + \hat{\varepsilon}_{a,t}^8$  in equation (10), we obtain the equation that can be used to tease out the noise component of the estimated eight-quarter-ahead TFP news shocks:

$$\kappa_8\hat{\nu}_{8,t} = (1 - \kappa_8)\hat{\varepsilon}_{a,t}^8 - \kappa_8(\hat{\varepsilon}_{a,t+8}^0 + \hat{\varepsilon}_{a,t+4}^4). \quad (11)$$

It should be noted that the noise component depends on the timing of information about  $\theta_{t+8}^a$ , which is distributed from period  $t$  through  $t + 8$ , and on the degree of imperfect information as captured by the Kalman gain  $(1 - \kappa_8)$ .

As far as the four-quarter-ahead expectation revisions,  $E_t\theta_{t+4}^a - E_{t-4}\theta_{t+4}^a$ , are concerned, we can analogously establish the following relation between the news representation and the model:

$$\begin{aligned} E_t\theta_{t+4}^a - E_{t-4}\theta_{t+4}^a &= \kappa_4(\theta_{t+4}^a + \nu_{4,t} - E_{t-4}\theta_{t+4}^a), \\ &= \kappa_4(\varepsilon_{a,t+4}^0 + \varepsilon_{a,t}^4 + \nu_{4,t}) = \varepsilon_{a,t}^4, \end{aligned} \quad (12)$$

where  $\kappa_4 \equiv (\sigma_{0,a}^2 + \sigma_{4,a}^2) / (\sigma_{0,a}^2 + \sigma_{4,a}^2 + \sigma_{4,\nu}^2)$  is the Kalman gain in terms of the estimated parameters of the model with news. In the last row we made use of the fact  $E_{t-4}\theta_{t+4}^a = \varepsilon_{a,t-4}^8$ . Substituting the estimated TFP innovations  $\hat{\theta}_{t+8}^a = \hat{\varepsilon}_{a,t+8}^0 + \hat{\varepsilon}_{a,t+4}^4 + \hat{\varepsilon}_{a,t}^8$  in equation (12), we obtain the equation that can be used to tease out the noise component of the estimated four-quarter-ahead TFP news shocks:

$$\kappa_4\hat{\nu}_{4,t} = (1 - \kappa_4)\hat{\varepsilon}_{a,t}^4 - \kappa_4\hat{\varepsilon}_{a,t+4}^0. \quad (13)$$

Equations (11) and (13) show that noise shocks are a particular linear combination of TFP news shocks and future surprise shocks. Specifically, they depend on the magnitude of the news shocks realized today relative to the magnitude of the future news and surprise shocks. As a result, noise shocks will arise even if both news and surprise shocks are i.i.d, as their existence does not require any correlation between the two.

**Step 3: Assessing the Historical Contribution of Noise Shocks** Equation (10) allows us to decompose eight-quarter-ahead news shocks into a fundamental component  $\kappa_8\theta_{t+8}^a$ , which will affect TFP in eight quarters, and a noise component  $\kappa_8\nu_{8,t}$ , which is orthogonal to future changes in TFP. Equation (12) allows for a similar decomposition of the four-quarter-ahead TFP news shocks. Equipped with the time series of noise shocks retrieved from equations (11) and

(13), we can compute the counterfactual series for TFP news and surprise shocks that generate revisions in expectations orthogonal to future fundamentals. Starting from the Kalman equation (10) and simply zeroing the fundamental component, we obtain

$$\tilde{\varepsilon}_{a,t}^8 = \kappa_8 \hat{\nu}_{8,t}. \quad (14)$$

Next, we substitute  $E_{t-4}\theta_{t+4}^a = \kappa_8 \left( \hat{\theta}_{t+4}^a + \hat{\nu}_{8,t-4} \right)$  from equation (10) into the first line of equation (12) and then zero the realization of fundamentals  $\hat{\theta}_{t+4}^a$  to obtain the counterfactual series of the four-quarter-ahead TFP news shocks:

$$\tilde{\varepsilon}_{a,t}^4 = \kappa_4 \hat{\nu}_{4,t} - k_4 k_8 \hat{\nu}_{8,t-4}. \quad (15)$$

Analogously, combining equations (8), (9), (10), and (12) and then zeroing the fundamental component  $\theta_{t+8}^a$ , we get

$$\tilde{\varepsilon}_{a,t}^0 = -\kappa_4 (\hat{\nu}_{4,t-4} - \kappa_8 \hat{\nu}_{8,t-8}) - \kappa_8 \hat{\nu}_{8,t-8}. \quad (16)$$

These counterfactual news and surprise shocks can be used to simulate the estimated news representation and obtain the sought contribution of noise shocks to business fluctuations.<sup>5</sup> Note that these counterfactual news and surprise shocks have no effect on time- $t$  innovation to TFP  $\theta_t^a$ , since  $\tilde{\varepsilon}_{a,t}^0 + \tilde{\varepsilon}_{a,t-4}^4 + \tilde{\varepsilon}_{a,t-8}^8 = 0$  for every  $t$  over our sample period. This is because these counterfactual shocks are orthogonal to fundamentals by construction.

The estimated time series of noise shocks is obtained from the estimated news shocks in combination with equations (11) and (13). The estimated series of noise shocks are the black bars in Figure 5 (after rescaling by the appropriate Kalman gain). The white bars are the remainder ( $\kappa_8 \theta_{t+8}^a$  and  $\kappa_4 \theta_{t+4}^a$ ) given that we know the estimated TFP news shocks  $\hat{\varepsilon}_{a,t}^8$  and  $\hat{\varepsilon}_{a,t}^4$ , which capture the expectations revisions about future fundamentals in our model. The historical role of noise in the U.S. postwar period can be worked out by simulating the model using the estimated noise shocks in combination with equations (14), (15), and (16). Specifically, those equations give us the counterfactual news shocks that allow us to evaluate the historical contribution of noise shocks to the model's variables. Figure 6 plots the historical contribution of noise shocks to the unemployment rate, GDP growth, consumption growth, and investment growth.

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<sup>5</sup>This is one way to assess the contribution of noise. Alternatively, one could simulate the model with noisy signals in Step 1, using the series of noise shocks obtained in Step 2. However, our approach can be implemented by using only the observationally equivalent news representation with no need to solve the model with noisy signals.

## G Historical Realizations of Shocks

Figure 4 shows the historical realizations (smoothed estimates) of four- and eight-quarter-ahead TFP news shocks along with their estimated distribution in the model. There are no realizations of these shocks lying in the tails of their distribution. When a large number of realizations lie in the tails of the distribution, it is often a symptom of misspecification and violation of rationality. We conclude that the historical realizations of TFP news shocks are not too big. Figure 5 shows that similar conclusions apply when considering actual TFP shocks: the large majority of the historical realizations of these shocks fall within the two-standard-deviation bands around their zero mean.

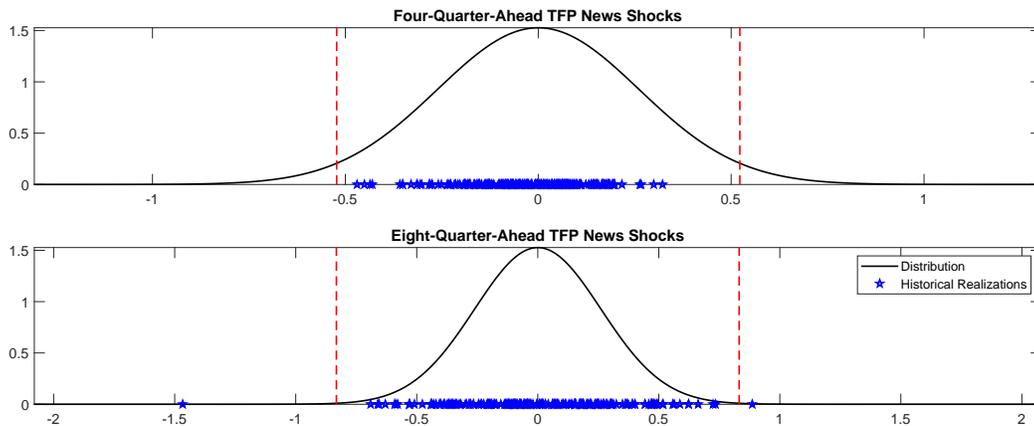


Figure 4: Distribution of the four- (top) and eight-quarter-ahead (bottom) TFP news shocks in the estimated model (black line). The blue stars mark the historical realizations of these shocks obtained from the Kalman smoother. The red dashed vertical lines denote the two-standard-deviation interval around the zero mean of these shocks.

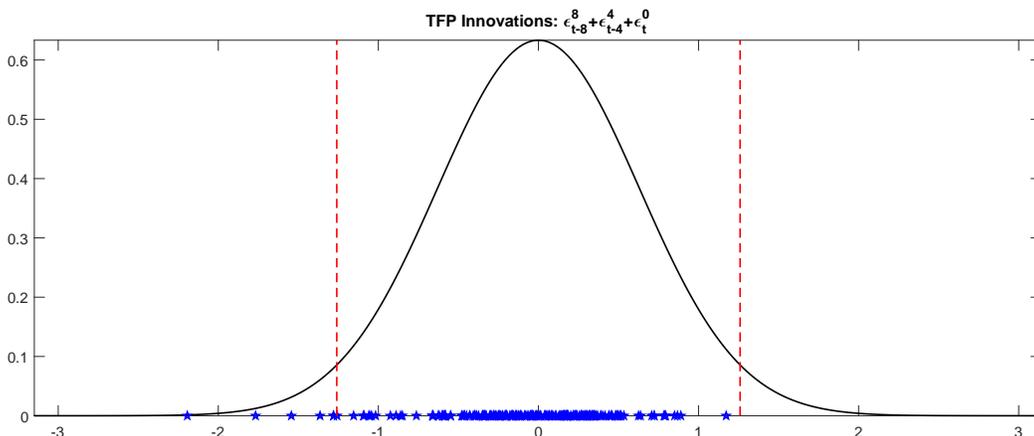


Figure 5: Distribution of the actual TFP innovations in the estimated model (black line). The blue stars mark the historical realizations of these shocks obtained from the Kalman smoother. The red dashed vertical lines denote the two-standard-deviation interval around the zero mean of these shocks.

Figure 6 compare the historical realizations of noise shocks to the estimated distribution of

these shocks in the model. The realized noise shocks are not in the tails of their distribution. This check ensures that the Kalman gains in the model, which depends on the standard deviation of the Gaussian distribution of noise shocks, are consistent with the in-sample standard deviations of the estimated noise shocks.

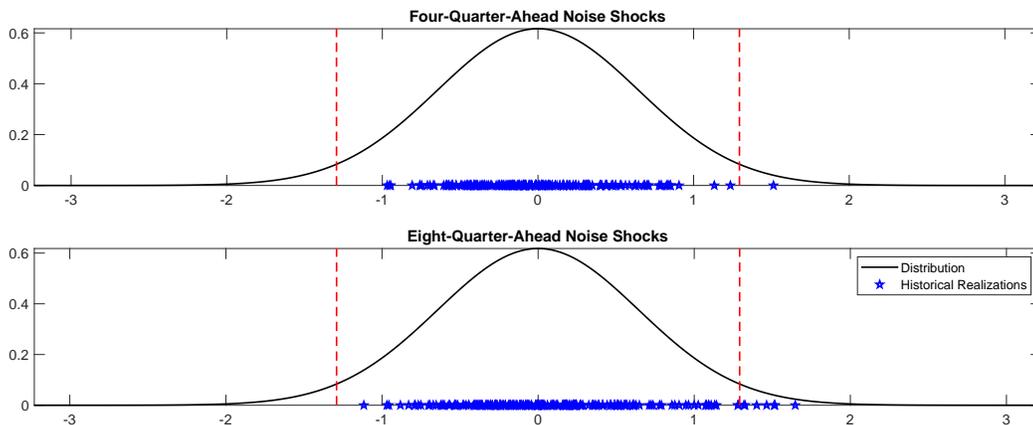


Figure 6: Distribution of the four- (top) and eight-quarter-ahead (bottom) noise shocks in the estimated model (black line). The blue stars mark the historical realizations of these shocks obtained from the Kalman smoother. The red dashed vertical lines denote the two-standard-deviation interval around the zero mean of these shocks.

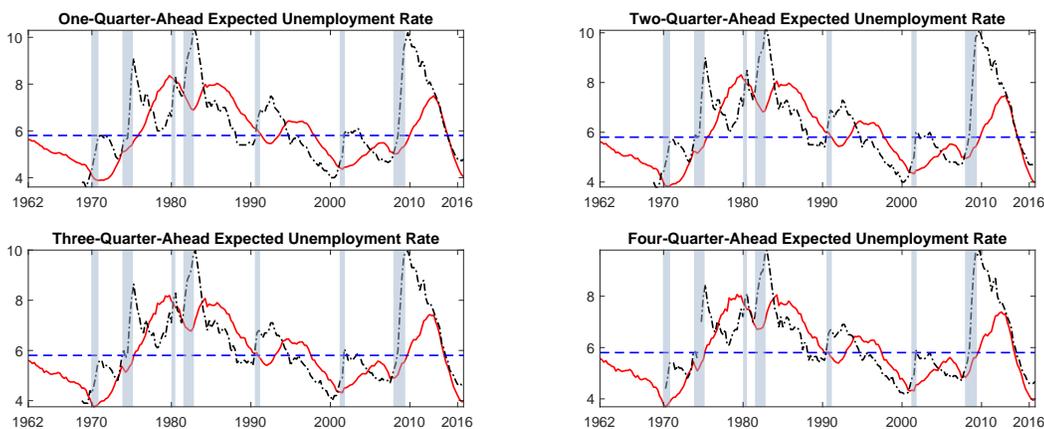


Figure 7: Expectations of U.S. unemployment rates (black dashed-dotted line), along with the counterfactual unemployment rate obtained by simulating the model using only the smoothed estimate of the four- and eight-quarter-ahead TFP news shocks (red solid lines). The counterfactual series are computed by setting the model parameters to their posterior modes, which are reported in Tables 1 and 2. Shaded areas denote NBER recessions.

## H The Role of Expected Unemployment Rates in Identifying TFP Shocks

To show how we achieve identification of the revisions of expectations about future TFP is useful to look at the historical analysis of TFP news shocks. The right plot in Figure 7 reports the

Statistic	$Y$	$C$	$I$	$FFR$	$EMPL$	$PART$	$E_t U_{t+1}$	$E_t U_{t+2}$
Data	0.68	0.49	2.92	0.81	2.28	0.79	22.01	21.10
Model	0.77	0.57	3.22	1.00	1.81	0.78	17.98	17.47
Statistic	$E_t U_{t+3}$	$E_t U_{t+4}$	$W/P$	$P^{def}$	$P^{pce}$	$P^{cpi}$	$TFP^{adj}$	$TFP^{unadj}$
Data	20.32	18.59	0.61	0.59	0.62	0.73	0.75	0.87
Model	16.80	15.98	0.46	0.90	0.90	0.90	0.66	0.66

Table 1: Unconditional standard deviations of the observable variables and their model counterparts. The model’s standard deviations are obtained under the assumption that measurement errors are shut down and loadings for the multiple indicators are one for every variable. The observable series for employment and labor force participation rates have been detrended by subtracting their respective trends implied by the labor disutility shock before computing their standard deviation. For the sake of consistency, the standard deviations of employment and participation in the model are obtained by shutting down the contribution of the labor disutility shocks. All standard deviations are expressed in logs and in percent.

expected unemployment rate (black dashed-dotted line) along with the counterfactual time series obtained by simulating the news representation of the model using only the smoothed estimates of the four- and eight-quarters-ahead TFP news shocks (red solid lines). These shocks appear to have been a key driver of the expected rates of unemployment at lower frequencies over the postwar period, in line with the insights of Figure 3.

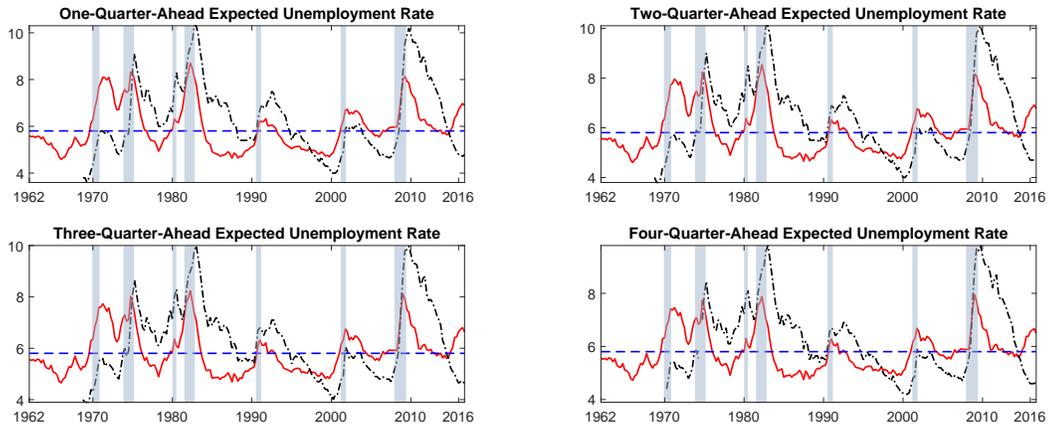


Figure 8: Expectations of U.S. unemployment rates (black dashed-dotted line), along with the counterfactual unemployment rate obtained by simulating the model using only the smoothed estimate of the surprise TFP shocks (red solid line). The counterfactual series are computed by setting the model parameters to their posterior modes, which are reported in Tables 1 and 2. Shaded areas denote NBER recessions.

Figure 8 shows the U.S. expected unemployment rate (black dashed-dotted line) along with the counterfactual time series obtained by simulating the estimated model using only the smoothed estimate of the TFP surprise shocks (red solid lines). Surprise TFP shocks seem to primarily affect the dynamics of unemployment rate at the business cycle frequencies, in line with Figure 3.

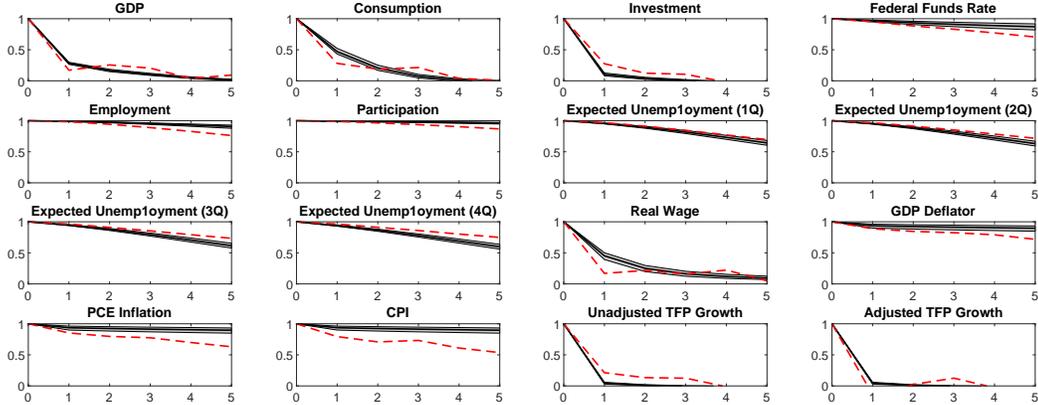


Figure 9: Posterior autocorrelation functions computed for every 100 posterior draws. The red dashed line denotes the empirical autocorrelation function and the solid black line denotes the posterior median for the autocorrelation implied by the model after shutting down its measurement errors. The gray areas denote the 90-percent posterior credible set. Sample period: 1962:Q1-2008:Q3)

## I Autocorrelation Functions

As far as the empirical fit of the model is concerned, we report in Table 1 the standard deviations of the observable variables predicted by the estimated model and compare them with the data. Overall, the estimated model matches well the empirical second moments. The volatility of investment is slightly overestimated, which implies that the volatility of output is also somewhat above its empirical counterpart. The volatility of adjusted TFP implied by the model is very close to the one measured in the data. As we shall explain in the next section, the countercyclicity of the shadow value of output and marginal hiring costs conditional on technology shocks allows the model to generate volatility in unemployment rates that comes close to the data. To provide further evidence on the ability of the model to fit the data, in Appendix Section I we show that the model does well at matching the empirical autocorrelation functions, overestimating only slightly the persistence of the rates of inflation and participation.

To provide further evidence on the ability of the model to fit the data, we show in Figure 9 the autocorrelation functions for the endogenous variables. Overall, the model does well at matching these moments, overestimating only slightly the persistence of the rates of inflation and participation.

## J Internal Detrending of Employment and Participation Rates

A key challenge of using unfiltered labor market data to estimate a structural model is to account for the trends in the rates of employment and labor force participation in the postwar period. Recall that we set a dogmatic prior that restricts the value for the autocorrelation parameter of

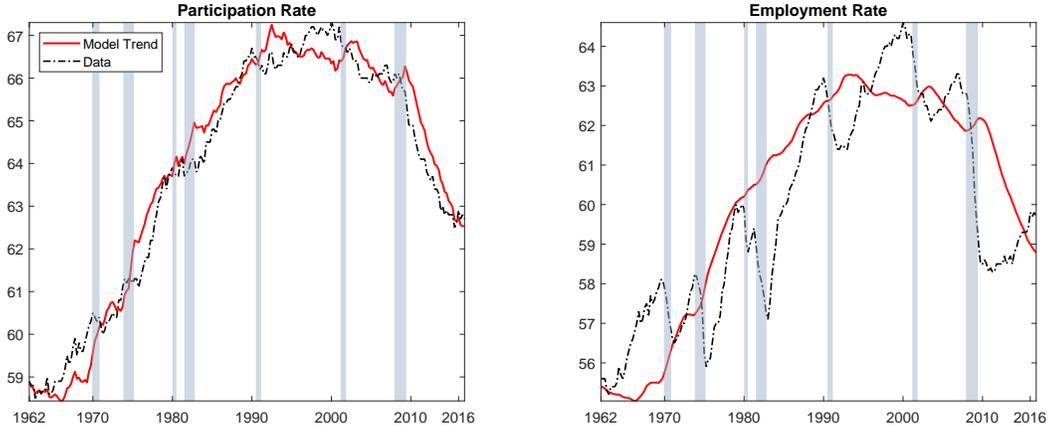


Figure 10: Detrending the rate of participation and the rate of employment. The black dashed-dotted lines denote the data and the red solid lines denote the two rates simulated from the estimated model by using only the filtered (one-sided) estimates for the labor disutility shocks. The estimated model’s parameters are set to their values at the posterior mode, which are reported in Tables 1 and 2. Shaded areas denote NBER recessions.

labor disutility shocks to be close to unity. The idea is to introduce an almost-unit-root process so as to endow the model with a persistent exogenous process that can account for these labor market trends. Figure 10 shows the U.S. rates of participation and employment (black dashed-dotted lines) along with their counterfactuals simulated from the estimated model using only the one-sided filtered labor disutility shocks (solid red lines).<sup>6</sup> This picture suggests that labor disutility shocks effectively detrend the employment and participation rates in estimation.

## K How Accurately are TFP Surprise and News Shocks Identified?

Now we formally evaluate how accurate our estimates of TFP news shocks are. To do so, we compute the reduction in the econometrician’s uncertainty (measured by the variance) about the in-sample estimates of the two news shocks due to observing our entire data set relative to their unconditional variance (i.e., if no data were observed).<sup>7</sup> If shocks were observed or implied by the data, the uncertainty conditional on the data would be zero and this ratio would be equal to unity. If the data conveyed no information whatsoever about the shocks, then the

<sup>6</sup>Simulating the model using the two-sided estimates of the shocks would not materially change the solid red line in Figure 10. We work with the one-sided estimates because they are obtained from the filter that we use to evaluate the likelihood of the model and to estimate the model parameters.

<sup>7</sup>This analysis is conditional on the posterior mode of the model parameters, which is shown in Table 1 and Table 2, and abstracts from parameter uncertainty, which is very small. The unconditional variance of the shocks depends on the estimated values of the model parameters. The conditional variance of the shocks is computed by running the Kalman smoother. Since the smoother is a two-sided filter, it returns the uncertainty of the shocks in every period conditional on the entire data set described in Section 3.1. To correct for the relatively larger uncertainty at the beginning and at the end of the sample period, we take the smallest value of the variances in the sample. Results would not change if we used the median of the variances instead.

conditional uncertainty would be equal to the unconditional uncertainty and the ratio would be equal to zero. The information content of our data set is 79%, 38%, and 61% for the TFP surprise shocks, the four-quarters-ahead TFP news shocks, and the eight-quarters-ahead TFP news shocks, respectively. These numbers are one order of magnitude larger than those found in leading studies with the same news structure, in which the information content about TFP news shocks is only 2% (Iskrev 2018).

## L The role of other expectation data

In this Section we elaborate on the results reported in footnote 29 of the paper. The Table below is an extension to Table 3 of the paper, where we now add one more column to consider the case where we include SPF expectations on both GDP growth and inflation to the set of observables, retaining employment growth as the only labor-market-quantity variable, following Miyamoto and Nguyen (2020). Specifically, the estimation is based on observing SPF expectations data 1 to 4 quarters ahead on both real GDP growth and the inflation rate of the GDP deflator. The results reported in the last column of the Table show that adding these SPF expectation data on top of the observables used to produce the results in column (4) is not sufficient to retrieve Pigouvian Cycles in estimation.

Table 2: Noise and Business Cycles

	Baseline	No expectations	No expectations, no participation no employment	No expectations, no participation employment growth	No expectations, no participation employment growth, SPF GDP Growth, SPF Inflation
GDP	0.48	0.27	0.00	0.03	0.18
Consumption	0.36	0.10	0.00	0.00	0.04
Investment	0.38	0.26	0.00	0.05	0.09
Unemployment Rate	0.48	0.35	0.22	0.01	0.13
Real Wages	0.60	0.36	0.02	0.06	0.17

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