Asset Price Booms and Macroeconomic Policy: a Risk-Shifting Approach

Franklin Allen Gadi Barlevy Douglas Gale

Online Appendix

Appendix A: Proofs of Propositions

Proof of Proposition 1: In the text, we showed there is a unique deterministic equilibrium. Here we allow for stochastic equilibrium paths for $\{p_t, R_t\}_{t=0}^{\infty}$ and confirm that the equilibrium is in fact deterministic.

First, note that for any date t, in equilibrium it must be the case that $0 < p_t \leq e$. If the price $p_t \leq 0$ there would be infinite demand for the asset given its dividend $d > 0$ and there is free disposal. But the supply of assets is finite, so this cannot be an equilibrium. At the same time, the most any cohort can spend to buy the assets is e. Let z_t denote the return to buying the asset, i.e., $z_t \equiv \frac{d+p_{t+1}}{p_t}$. This can be random if p_{t+1} is random. Let $G_t(z)$ denote the (possibly degenerate) distribution of the return z_t. Since $0 < p_t \leq e$ for all t, the maximum return $z_t^{\max}$ is finite, since $z_t^{\max} = \frac{d+p_{t+1}^{\max}}{p_t} \leq \frac{d+e}{p_t} < \infty$, where $p_{t+1}^{\max}$ is the maximum possible realization of the price at date $t+1$.

The equilibrium satisfies two conditions. First, as in equation (3) in the text, all resources will be used either to buy assets or to initiate production:

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e \tag{A1}$$

The implies $R_t = \rho(p_t)$ where $\rho'(\cdot) > 0$. Second, the interest rate on loans R_t must satisfy

$$(1 + R_t)\, p_t = d + p_{t+1}^{\max} \tag{A2}$$

If the interest rate on loans $1 + R_t$ exceeded $\frac{d+p_{t+1}^{\max}}{p_t}$, no agent would want to buy assets, which cannot be an equilibrium. If interest rate on loans $1 + R_t$ exceeded $\frac{d+p_{t+1}^{\max}}{p_t}$, agents could earn positive profits from borrowing, so demand for credit would be infinite. Substituting $R_t = \rho(p_t)$ into (A2) implies

$$p_{t+1}^{\max} = (1 + \rho(p_t))\, p_t - d$$

Suppose $p_t > p^d$. Consider the sequence $\{\widetilde{p}_\tau\}_{\tau=t}^{\infty}$ that comprises the upper support of prices at each date given the history of previous prices, starting from p_t. Formally, set $\widetilde{p}_t = p_t$ and define

$$\widetilde{p}_{\tau+1} = (1 + \rho(\widetilde{p}_\tau))\, \widetilde{p}_\tau - d$$

Since $p_t > p^d$, the sequence $\widetilde{p}_t$ would shoot off to infinity and would exceed e in finite time. This means there is a state of the world in which the price exceeds e, which cannot be an equilibrium. So $p_t \leq p^d$.

Next, suppose $p_t < p^d$. Again, we can construct the sequence $\{\widetilde{p}_\tau\}_{\tau=t}^{\infty}$ that comprises the upper support of prices at each date given the history of previous prices, starting from p_t. That is, we set $\widetilde{p}_t = p_t$ and then

$$\widetilde{p}_{\tau+1} = (1 + \rho(\widetilde{p}_\tau))\, \widetilde{p}_\tau - d$$

Since $p_t < p^d$, the sequence $\widetilde{p}_t$ would turn negative. Hence, there is a state of the world in which the price is negative, which cannot be an equilibrium. The distribution of the price at date t is degenerate with full support at p^d. From (A1), $R_t = \rho(p_t)$ is uniquely determined as well. ■

Proof of Proposition 2: Below we fill in some of the missing steps from the discussion in the text.

First, we need to show that at any date t in which $d_t = D$, the return on the asset will be higher if $d_{t+1} = D$. That is, we need to show that

$$p_{t+1}^D + D > p^d + d$$

Suppose $p_{t+1}^D + D \leq p^d + d$. Since $D > d$, this requires $p_{t+1}^D < p^d$. From equation (3) in the text, we know the equilibrium interest rate on loans R_{t+1}^D must equal $\rho\left(p_{t+1}^D\right)$. If $p_{t+1}^D < p^d$, then since $\rho'(\cdot) > 0$, we have

$$R_{t+1}^D = \rho\left(p_{t+1}^D\right) < \rho\left(p^d\right) = R^d$$

But then we would have

$$\left(1 + R_{t+1}^D\right) p_{t+1}^D < \left(1 + R^d\right) p^d = p^d + d.$$

This means that if $d_{t+1} = D$, an agent who borrows to buy assets at date $t+1$ can make positive profits if $d_{t+2} = d$. But then there would be infinite demand for borrowing to buy assets, which cannot be an equilibrium given supply of credit is finite. Since this is inconsistent with equilibrium, it follows that $p_{t+1}^D + D > p^d + d$.

The text establishes that the equilibrium interest rate on loans must equal the maximal return on the asset, and so $p_t^D = p^D$ and $R_t^D = \rho\left(p^D\right)$. The step that remains is to solve for $\left\{\alpha_t^D\right\}_{t=0}^{\infty}$. For this, we use the expected return to the asset, denoted $\bar{r}_t^D$, and the expected return to lending, denoted $\overline{R}_t^D$. The former is given by

$$1 + \bar{r}_t^D = (1 - \pi)\left(1 + \frac{D}{p^D}\right) + \pi\left(\frac{d + p^d}{p^D}\right) \equiv 1 + \bar{r}^D \tag{A3}$$

As for the expected return to lending, a fraction α_t^D of lending is used to buy assets and the rest finances production. Since all of the proceeds from asset purchases accrue to the lender, the expected return to these loans is just the expected return to buying an asset net of default costs, $1 + \bar{r}^D - \pi\Phi$. The remaining loans that finance production will be repaid in full, so the return on those loans is $1 + R^D$. This implies

$$\begin{aligned} 1 + \bar{R}_t^D &= \left(1 - \alpha_t^D\right)\left(1 + R^D\right) + \alpha_t^D\left(1 + \bar{r}^D - \pi\Phi\right) \\ &= \left(1 - \alpha_t^D\right)\left(1 + \frac{D}{p^D}\right) + \alpha_t^D\left(1 + \bar{r}^D - \pi\Phi\right) \end{aligned} \tag{A4}$$

If $\overline{R}_t^D > \bar{r}^D$, savers would prefer lending over buying assets. The only agents who would buy assets would be those who borrow to do so, and so $\alpha_t^D = \frac{p^D}{e}$. If $\overline{R}_t^D = \bar{r}^D$, savers would be indifferent between buying assets and lending. This means α_t^D can assume any value between 0 and $\frac{p^D}{e}$. Finally, if $\overline{R}_t^D < \bar{r}^D$, savers would prefer buying assets over lending. No agent would borrow to buy assets, implying $\alpha_t^D = 0$. Hence, the expected return to lending $\overline{R}_t^D$ and the share of lending used to buy assets α_t^D are jointly determined.

To solve for $\overline{R}_t^D$ and α_t^D, consider first the case where $\alpha^D = \frac{p^D}{e}$. This can only be an equilibrium if $\overline{R}_t^D \geq \bar{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, i.e., only if

$$\left(1 - \frac{p^D}{e}\right)\frac{D}{p^D} + \frac{p^D}{e}\left(\bar{r}^D - \pi\Phi\right) \geq \bar{r}^D$$

Rearranging this equation and substituting in for $\bar{r}^D$ implies $\alpha_t^D = \frac{p^D}{e}$ is an equilibrium only if

$$\Phi \leq \left(\tfrac{e}{p^D} - 1\right)\left(\tfrac{D+p^D-d-p^d}{p^D}\right) \equiv \Phi^* \tag{A5}$$

Next, consider the case where $\alpha_t^D \in \left(0, \frac{p^D}{e}\right)$. This can only be an equilibrium if $\overline{R}_t^D = r^D$ when we evaluate $\overline{R}_t^D$ at the relevant α_t^D. Since $\overline{R}_t^D$ is decreasing in α_t^D, this requires that $\overline{R}_t^D < \bar{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, or

$$\Phi > \Phi^* \tag{A6}$$

In this case, the equilibrium value of α_t^D is the one that equates $\overline{R}_t^D$ and $\bar{r}^D$, which implies

$$\alpha_t^D = \tfrac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} \tag{A7}$$

Finally, there cannot be an equilibrium in which $\alpha_t^D = 0$. This would require $\overline{R}_t^D \leq \bar{r}^D$ when $\alpha_t^D = 0$. But $\alpha_t^D = 0$ implies $\overline{R}_t^D = \frac{D}{p^D} > \bar{r}^D$. Hence, the value of α_t^D is unique and is either equal to $\frac{p^D}{e}$ or some value between 0 and $\frac{p^D}{e}$, depending on the cost of default Φ. ■

Proof of Proposition 4: Here we fill the missing steps in deriving the equilibrium at date 0 when there is a quota. In the text, we argued that $1 + R_0^D \geq \frac{p^D+D}{p_0^D}$. Suppose $1 + R_0^D$ strictly exceeded $\frac{p^D+D}{p_0^D}$. Then no agent would borrow to buy assets knowing they would default. With agents only borrowing to produce, lending would be safe and would yield a higher return than the asset. Savers would prefer to lend, but under the quota can lend at most $e - p^D$ and must use the remaining p^D to buy assets. Since only they buy the asset, $p_0^D = p^D$. From the market clearing condition (3) in the text, we have $R_0^D = \rho\left(p^D\right) = R^D$. But we know $1 + R^D = 1 + \frac{D}{p^D}$, which contradicts our supposition that $1 + R_0^D > \frac{p^D+D}{p_0^D}$. It follows that $1 + R_0^D = \frac{p^D+D}{p_0^D}$. Combining this with equation (3) in the text implies $R_0^D = R^D$ and $p_0^D = p^D$. Hence, imposing a lending cap of $e - p^D$ at date 0 will not change the price of the asset or the interest rate on loans relative to the equilibrium without a quota. Since savers spend at least p^D to buy assets under the quota and the value of assets is p^D, there can be no borrowers who buy the asset, so $\alpha_0^D = 0$.

A similar logic can be applied to a quota of $e - p^D$ at all dates as long as $d_t = D$. The market clearing condition (3) in the text remains unchanged at all dates. First, the argument that $p_{t+1}^D > p^d$ for all t only relies on the market clearing condition (16) in the text, and is true even if we introduce a quota. Next, to ensure demand for borrowing is finite, we need $1 + R_t^D \geq \frac{p_{t+1}^D+D}{p_t^D}$. Suppose $1 + R_t^D > \frac{p_{t+1}^D+D}{p_t^D}$. In that case, no agent would borrow to buy the asset for any $\phi > 0$, and savers would strictly prefer lending to buying assets. Because of the quota, they would have to spend p^D on the asset. Hence, $p_t^D = p^D$, and from the market clearing condition, $R_t^D = \rho\left(p^D\right) = R^D$. This would imply $1 + R^D > \frac{p_{t+1}^D+D}{p^D}$. It follows that $p_{t+1}^D < p^D$. But this is impossible, since the quota would require savers spend at least p^D on the asset at date $t+1$ if $d_{t+1} = D$. The contradiction implies $1 + R_t^D = \frac{p_{t+1}^D+D}{p_t^D}$. The equilibrium conditions are therefore the same as in Proposition 2. The unique equilibrium is given by $\left(p_t^D, R_t^D\right) = \left(p^D, R^D\right)$ for all t. The same argument as above implies $\alpha_t^D = 0$ for all t. ■

Appendix B: Monetary Policy

This appendix introduces within-period production, a monetary authority, and nominal price rigidity into our setup as in our discussion in Section 4. We set up the model and derive the results that underlie Propositions 5 and 6 in the text.

B.1 Agent Types and Endowments

Our approach largely follows Galí (2014) in how we incorporate production, nominal price rigidity, and monetary policy into an overlapping generations economy with assets. As in our benchmark model, agents live two periods and care only about consumption when old. Each cohort still consists of two types – savers who are endowed with resources but cannot produce intertemporally and entrepreneurs endowed with no resources who can convert goods at date t into goods at date $t + 1$. We continue to model entrepreneurs as in the benchmark model, but we now assume savers are endowed with the inputs to produce goods rather than with the goods themselves. This allows for an endogenous quantity of goods that can potentially vary with the stance of monetary policy.

More precisely, we assume two types of savers, each of mass 1. Half are workers, endowed with 1 unit of labor each who must choose how to allocate it. The other half are producers, endowed with the knowledge of how to convert labor into output but not with labor itself.[14] Producers set the price of the goods they produce and then hire the labor needed to satisfy their demand. Although producers and entrepreneurs both produce output, they differ in when and how they produce it. Producers born at date t convert labor into goods at date t. Entrepreneurs then convert the goods producers created at date t into goods at date $t + 1$. Producers operate within the period; entrepreneurs operate across periods.

B.2 Production, Pricing, and Labor Supply

Workers allocate their one unit of labor to home and market production. Home production yields the same good as the market, but using a technology $h(\ell)$ that is concave in the amount of labor ℓ allocated to home production. We assume $h'(0) = 1$ and $h'(1) = 0$ for reasons that will become clear below.

Workers who sell their labor on the market earn a wage W_t per unit labor. Their labor services are hired by producers, whom we index by $i \in [0, 1]$. Each producer can produce a distinct intermediate good according to a linear technology. In particular, if producer i hires n_{it} units of labor, she will produce $x_{it} = n_{it}$ units of intermediate good i. The different intermediate goods can then be combined to form final

[14] This setup borrows from Adam (2003) rather than Galí (2014). The latter assumes agents are homogeneous, selling labor when young and hiring labor when old. We want income to only accrue to the young as in our benchmark model.

consumption goods according to a constant elasticity of substitution (CES) production function available to all agents. That is, given x_{it} of each $i \in [0,1]$, the amount of final goods X_t that can be produced is

$$X_t = \left(\int_0^1 x_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \tag{B1}$$

with $\sigma > 1$. Let P_t denote the price of the final good and P_{it} denote the price of intermediate good i. At these prices, the x_{it} that maximize the profits of a final goods producer solve

$$\max_{x_{it}} P_t \left(\int_0^1 x_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} x_{it} di$$

The first-order condition with respect to x_{it} yields

$$x_{it} = X_t \left(\frac{P_{it}}{P_t}\right)^{-\frac{1}{\sigma}} \tag{B2}$$

If we set $X_t = 1$, we can compute the price of the cost of the optimal bundle of intermediate goods $x_{it} = \left(\frac{P_{it}}{P_t}\right)^{-1/\sigma}$ needed to produce one unit of the final good:

$$\int_0^1 P_{it} x_{it} di = \int_0^1 P_{it}^{1-\frac{1}{\sigma}} P_t^{\frac{1}{\sigma}} di$$

Since any agent can produce final goods, the price P_t must equal the per unit cost of producing a good in equilibrium. Equating the two yields the familiar CES price aggregator:

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{B3}$$

Each intermediate goods producer chooses their price P_{it} to maximize expected profits given demand (B2) and wage W_t. To allow producers to move either before or after the monetary authority, we condition producer i's choice on their information Ω_{it} when choosing their price. Each producer will set P_{it} to solve

$$\max_{P_{it}} E\left[\left(P_{it} - W_t\right) X_t \left(\frac{P_{it}}{P_t}\right)^{-1/\sigma} \middle| \Omega_{it}\right]$$

The optimal price is then

$$P_{it} = \frac{E\left[W_t X_t | \Omega_t\right]}{(1-\sigma) E\left[X_t | \Omega_t\right]} \tag{B4}$$

By symmetry, all producers will charge the same price, produce the same amount, and hire the same amount of labor, i.e., $n_{it} = n_t$ for all $i \in [0,1]$. The output of consumption goods is thus

$$X_t = \left(\int_0^1 n_t^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} = n_t$$

Workers receive $(W_t/P_t)\, n_t$ of these goods and producers get the remaining $(1 - W_t/P_t)\, n_t$. Workers also produce goods at home. Their income is thus $(W_t/P_t)\, n_t + h(1-n_t)$, which is maximized at

$$h'(1-n_t) = W_t/P_t \tag{B5}$$

By contrast, the total resources available to young agents is $e_t = n_t + h(1-n_t)$, which is maximized at

$$h'(1-n_t) = 1$$

Our assumption that $h'(0) = 1$ implies total resources are maximized when $n_t = 1$ and all goods are produced in the market, and $e_t = n_t + h(1-n_t)$ is increasing in n_t for all $n_t \in [0,1]$.

B.3 Assets, Credit, and Money

Since agents want to consume when old, they will wish to save their earnings $e_t = n_t + h(1 - n_t)$. As in the benchmark model, they can buy assets and make loans. Without money, this specification would be equivalent to our benchmark model, the only difference being that the income of savers e_t which before was exogenous and fixed is now endogenous and potentially time-varying. Equilibrium in the asset and credit markets involves the same conditions as in the benchmark model. First, regardless of the income they earn, the young will spend all of their resources either funding entrepreneurs or buying assets, and so we still have

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e_t$$

where p_t is the real price of the asset and R_t is the real interest rate on loans. The interest rate R_t must still ensure agents cannot earn profits by borrowing and buying assets. When $d_t = d$, this requires

$$\left(1 + R_t^d\right) p_t^d = d + p_{t+1}^d$$

and when $d_t = D$, this requires

$$\left(1 + R_t^D\right) p_t^D = D + p_{t+1}^D$$

We can then use R_t and p_t to solve for the expected return on loans:

$$\overline{R}_t = \begin{cases} R_t^d & \text{if } d_t = d \\ \max\left\{\overline{r}_t^D, \left(1 - \frac{p_t^D}{e_t}\right) R_t^D + \frac{p_t^D}{e_t}\left(\overline{r}_t^D - \pi\Phi\right)\right\} & \text{if } d_t = D \end{cases} \tag{B6}$$

where $\overline{r}_t^D$ is the expected real return to buying the asset. Below, we show that when prices are flexible or money is absent altogether, the equilibrium real wage W_t/P_t will be constant over time. Employment n_t and total earnings of all savers $e_t = n_t + h(1 - n_t)$ will then also be constant. The reduced form of our model in the absence of money thus coincides with our benchmark model.

To introduce money, we follow Galí (2014) in assuming money does not circulate in equilibrium. That is, money is the numeraire, and P_t and W_t denote the price of goods and labor relative to money. However, no agent actually holds money in equilibrium. The monetary authority announces a nominal interest rate i_t at each date t. The monetary authority commits to pay this rate at date $t+1$ to those who lend to it (with money it can always issue), and will charge i_t to those who borrow from it with full collateral. This is roughly in line with what central banks do in practice, paying interest on reserves and lending at the discount window against collateral. To ensure money doesn't circulate, the real return on lending to the monetary authority must equal the expected return on savings. Let $\Pi_t = P_{t+1}/P_t$ denote the gross inflation rate between dates t and $t+1$. Since agents always lend to entrepreneurs, the expected return on savings will equal $\overline{R}_t$, the expected return on loans. This implies

$$1 + i_t = \left(1 + \overline{R}_t\right) \Pi_t \tag{B7}$$

When the monetary authority changes the nominal interest rate i_t, either inflation Π_t or the expected return $1 + \overline{R}_t$ or both will have to adjust to ensure agents will neither borrow nor lend to the monetary authority.

B.4 Defining an Equilibrium

Given a path of nominal interest rates $\{1+i_t\}_{t=0}^{\infty}$, an equilibrium consists of a path of prices $\{P_t, W_t, p_t, R_t\}_{t=0}^{\infty}$ and a path of employment $\{n_t\}_{t=0}^{\infty}$ such that agents behave optimally and markets clear. Collecting the relevant conditions from above yields the following five equations for these five variables:

$$
\begin{array}{ll}
\text{(i) Optimal pricing:} & P_t = \dfrac{E\left[W_t X_t | \Omega_t\right]}{(1-\sigma)\, E\left[X_t|\Omega_t\right]} \\
\text{(ii) Optimal labor supply:} & h'\left(1-n_t\right) = W_t/P_t \\
\text{(iii) Optimal savings:} & \int_{R_t}^{\infty} n\left(y\right) dy + p_t = e_t \\
\text{(iv) Credit market clearing:} & 1+R_t = \begin{cases} \frac{D+p_{t+1}^D}{p_t^D} & \text{if } d_t = D \\ \frac{d+p_{t+1}^d}{p_t^d} & \text{if } d_t = d \end{cases} \\
\text{(v) Money market clearing:} & \Pi_t = \dfrac{1+i_t}{1+\overline{R}_t}
\end{array}
$$

where the expected return on loans $\overline{R}_t$ in the last condition is given by (B6).

B.5 Equilibrium with Flexible Prices

We begin with the case where producers set their prices P_{it} after observing the wage W_t. This corresponds to the case where prices are fully flexible, or alternatively where there is no money and so no sense in which nominal prices are set in advance. Producers can deduce what other producers will do and the labor workers will supply, they can perfectly anticipate total output X_t. Hence, their information set $\Omega_t = \{W_t, X_t\}$. It follows that $E\left[W_t X_t|\Omega_t\right] = W_t X_t$ and $E\left[X_t|\Omega_t\right] = X_t$. The optimal pricing rule (i) then implies

$$P_t = \frac{W_t}{1-\sigma}$$

The real wage is thus constant and equal to $1-\sigma$. Substituting this into (ii) yields

$$h'\left(1-n_t\right) = 1-\sigma \tag{B8}$$

Since $h\left(\cdot\right)$ is concave, n_t is equal to some constant n^* for all t. It follows that $e_t = n^* + h\left(1-n^*\right)$ is also constant for all t. We can then use (iii) and (iv) to solve for p_t and R_t as in the benchmark model, and then use (B6) to compute $\overline{R}_t$. Finally, given $\overline{R}_t$ we can use the implied Π_t from (v) to derive $\{P_t\}_{t=1}^{\infty}$ for any initial value for P_0. The initial price level P_0 is indeterminate, in line with the Sargent and Wallace (1975) result on the price level indeterminacy of pure interest rate rules. The nominal wage $W_t = (1-\sigma)\, P_t$.

B.6 Equilibria with Rigid Prices

We now turn to the case where producers set the price of their intermediate good P_{it} before the monetary authority moves. That is, producers set prices, the monetary authority sets $1+i_t$, and then producers hire workers at a nominal wage W_t. This formulation implies prices are only rigid for one period.

If monetary policy is deterministic, producers can perfectly anticipate the nominal interest rate and the equilibrium nominal wage W_t, and so $\Omega_t = \{W_t, X_t\}$ and $W_t/P_t = 1-\sigma$ as before.

Next, suppose monetary policy is contingent on some random variable, i.e., $i_t = i(\xi_t)$ where $\{\xi_t\}_{t=0}^{\infty}$ is a sequence of random variables. For simplicity, consider the case where ξ_t is only random at $t=0$, i.e.,

$$\xi_0 = \begin{cases} H & \text{w/prob } \chi \\ L & \text{w/prob } 1-\chi \end{cases}$$
$$\xi_t \text{ is deterministic for } t = 1, 2, ...$$

From date $t=1$ on, we know from the optimal price-setting condition (i) that $W_t/P_t = 1-\sigma$. It then follows that $n_t = n^*$ and $e_t = e^* \equiv n^* + h(1-n^*)$ for all $t \geq 1$, and we can determine p_t, R_t, and $\overline{R}_t$ for $t \geq 1$ just as in the case where prices are flexible. All we need is to solve for the equilibrium at date 0.

We use a superscript $\xi \in \{H, L\}$ to denote the value of a variable as for a given realization of ξ_0. Assume wlog that $i_0^H > i_0^L$. The optimal price setting condition (i) is now

$$\frac{\chi n_0^H \frac{W_0^H}{P_0} + (1-\chi) n_0^L \frac{W_0^L}{P_0}}{\chi n_0^H + (1-\chi) n_0^L} = 1-\sigma \tag{B9}$$

That is, the output-weighted average real wage over the two values of ξ is equal to $1-\sigma$. Optimal labor supply (ii) then implies

$$\begin{aligned} h'\left(1-n_0^H\right) &= \min\left\{\frac{W_0^H}{P_0}, 1\right\} \\ h'\left(1-n_0^L\right) &= \min\left\{\frac{W_0^L}{P_0}, 1\right\} \end{aligned}$$

These are three equations for four unknowns, meaning the set of all equilibria can be parameterized by a single parameter. Wlog, we choose the real wage when $\xi = H$ to be this parameter. The three equations above yield values for W_0^L/P_0, n_0^H, and n_0^L given W_0^H/P_0. From these, we can deduce earnings $e_0^\xi = n_0^\xi + h\left(1-n_0^\xi\right)$ for each $\xi \in \{H, L\}$. We can then use (iii) and (iv) to derive p_0^ξ and R_0^ξ by solving

$$\int_{R_0^\xi}^{\infty} n(y)\, dy + p_0^\xi = e_0^\xi \tag{B10}$$

$$\left(1+R_0^\xi\right) p_0^\xi = D + p^D \tag{B11}$$

and then compute the expected return on loans $\overline{R}_0^\xi$ using (B6), and, via (v), the inflation rate Π_0^ξ for each $\xi \in \{H, L\}$. As before, the price level P_0 is indeterminate. Optimal pricing only restricts the average real wage across states but not the real wage for each realization of ξ_0, introducing an indeterminacy. The equilibrium real wage can exceed $1-\sigma$ for one realization of ξ_0 if it falls below $1-\sigma$ for the other realization.

The case where monetary policy has no effect on real variables at date 0 remains an equilibrium. In this case, $W_0^H/P_0 = W_0^L/P_0 = 1-\sigma$. But price rigidity expands the set of equilibria to include ones in which real variables vary with the nominal interest rate. Since the nominal interest rate only serves as a signal to coordinate real activity but does not directly affect it, there are equilibria in which $W_0^H > W_0^L$ as well as equilibria in which $W_0^H < W_0^L$.[15] Since higher nominal interest rates seem to be contractionary in practice, we focus on equilibria in which $W_0^H/P_0 < 1-\sigma < W_0^L/P_0$, i.e., real wages are lower when the monetary authority unexpectedly raises the nominal interest rate. In this case, from condition (ii) we know that a higher nominal interest rate will be associated with lower employment ($n_0^H < n^* < n_0^L$) and hence lower earnings ($e_0^H < e^* < e_0^L$). From (B10), we can infer that $R_0^\xi = \rho^\xi\left(p_0^\xi\right)$ where $\rho^H(x) > \rho^L(x)$ for the same value x. As is clear from Figure 1, this implies a higher nominal interest rate will be associated with a lower real asset price ($p_0^H < p^D < p_0^L$). This also implies a higher real interest rate on loans ($R_0^H > R^D > R_0^L$). The real expected return to buying assets will also be higher ($\bar{r}_0^H > \bar{r}^D > \bar{r}_0^L$). However, whether the real expected return to lending $\overline{R}_0^H$ will be higher is ambiguous. (B6) implies $\overline{R}_0^\xi$ is either equal to $\bar{r}_0^\xi$ or to a weighted average of R_0^ξ and $\bar{r}_0^\xi$. In the latter case, although both terms are higher when $\xi = H$ the weight on $\bar{r}_0^\xi$, which is p_0^ξ/e_0^ξ, can be higher or lower for $\xi = H$. These results are summarized in Proposition 5 in the paper.

B.7 Redistribution and Welfare

We now argue that it will be possible to use redistribution to ensure that a monetary contraction is Pareto improving. To do this, ignore monetary policy temporarily and think about the effects of a lump sum tax τ_0 on savers at date 0 that is given to the old at that date. The wealth of savers is $e - \tau_0$. Our analysis above implies $\frac{dR_0^D}{d\tau_0} < 0$, i.e., impoverishing savers leads to a higher interest rate. From the market clearing condition, it follows that $0 < \frac{dp_0^D}{d\tau_0} < 1$. Hence, taxing savers and giving it to the old will make the old strictly better off. Since the derivatives $\frac{dp_0^D}{d\tau_0}$ and $\frac{dR_0^D}{d\tau_0}$ are independent of Φ, so the effect of the tax will be the same regardless of Φ. But from equation (14) in the text, when Φ is sufficiently large, a higher τ_0 will increase expected total consumption of the young. Thus, a redistribution from savers to the old will increase welfare for sufficiently large Φ. Intuitively, it is better to have the young give resources to the old directly than to lend them to speculators who use them to buy assets from the old.

Since a lump sum tax τ_0 makes both the old and the young better off, it will also make both sides better off if we discourage the young from working and reduce total output, as long as the fall in output is small. But this is exactly what contractionary monetary policy does. Hence, a redistribution combined with contractionary monetary policy can be Pareto improving.

[15] One way to avoid such multiplicity is to assume dynamic monetary policy rules that are conditioned on past economic variables. This allows a central bank to take actions that are unsustainable if a high interest rate today leads to certain outcomes, eliminating equilibria with those outcomes. See Cochrane (2011) for a discussion of these issues.

B.8 Promises of Future Intervention

Our last point concerns the effects of a promise at date 0 to be contractionary at date 1 if the boom continues into that date. In this case, ξ_0 and ξ_t for $t \geq 2$ are deterministic, while $\xi_1 = d_1 \in \{d, D\}$. That is, we assume producers set prices each period before d_t is revealed. Solving for equilibrium at date 1 is identical to how we solved for the equilibrium at date 0 when we assumed ξ_0 was random. Consider equilibria in which the real wage is lower if the boom continues, so

$$W_1^D/P_1 < 1 - \sigma < W_1^d/P_1.$$

This implies $n_1^D < n^* < n_1^d$ and so $e_1^D < e < e_1^d$. In other words, if dividends fall and the boom ends, monetary policy must be expansionary. By the same logic as above, such a policy would imply $p_1^D < p^D$ and $p_1^d > p^d$, as well as $R_1^D > R^D$ and $R_1^d < R^d$. Turning back to date 0, conditions (iii) and (iv) imply

$$\begin{aligned} \int_{R_0^D}^{\infty} n(y)\, dy + p_0^D &= e \\ \left(1 + R_0^D\right) p_0^D &= D + p_1^D \end{aligned}$$

Comparative statics of this system with respect to p_1^D reveals that $p_0^D < p^D$ and $R_0^D < R^D$. That is, while contractionary monetary policy at date 0 dampens p_0^D but raises R_0^D at date 0, a threat to enact contractionary monetary policy at date 1 if dividends remain high will dampen both p_0^D and R_0^D at date 0. These results are summarized in Proposition 6 in the paper.

Appendix C: Macroprudential Regulation

In this appendix, we define an equilibrium for an economy with multiple markets as in Section 5. We then show that for an equilibrium in which all markets are active, various aspects of the equilibrium are uniquely determined. We then discuss some comparative static results with respect to the set of active markets.

C.1 Defining an Equilibrium

We begin with some notation. Let p_t denote the price of the asset at date t. Given asset prices, we can define the return to buying the asset at date t as

$$z_t \equiv \frac{d_{t+1}+p_{t+1}}{p_t}$$

The return z_t can be random both because d_{t+1} might be uncertain (if $d_t = D$) and because p_{t+1} might in principle be stochastic. Let $G_t(z)$ denote the (possibly degenerate) cumulative distribution of the return z_t, i.e., $G(z) \equiv \Pr(z_t \leq z)$. Let $1 + r_t^{\max}$ denote the maximum possible return on the asset. As discussed in the text, $1 + r_t^{\max}$ is finite, since $r_t^{\max} \leq \frac{D+2\varphi e}{(1-\varphi)e}$. We will use $\overline{r}_t$ to denote the expected return to buying the asset at date t, i.e.,

$$1 + \overline{r}_t \equiv \int_0^{1+r_t^{\max}} z_t dG_t(z)$$

We now define variables for the different markets $\lambda \in [0, 1)$ agents can borrow in. Let $R_t(\lambda)$ denote the interest rate on loans in market λ, so an agent who agrees to pay a share λ of the project she undertakes will promise to pay back $1 + R_t(\lambda)$ for each unit she borrows. Since agents may default, let $\overline{R}_t(\lambda)$ denote what lenders expect to earn from lending in market λ given the possibility of default. Finally, we represent borrowing in markets with density functions $f_t^a(\lambda)$ and $f_t^p(\lambda)$ for all $\lambda \in [0, 1)$ such that the total amount of resources borrowed to buy assets and to produce are given by $\int_A f_t^a(\lambda)\, d\lambda$ and $\int_A f_t^p(\lambda)\, d\lambda$, respectively. Let $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$ denote the density of borrowing for any purpose in market λ.

Representing the quantities agents borrow in each market as a density function ignores the possibility that there may be equilibria in which agents borrow a positive mass of resources in certain markets. More generally, we can allow for a set $\Delta \subset [0, 1)$ with countably many elements such that each market $\lambda \in \Delta$ is associated with a positive mass of borrowing $m_t^x(\lambda) > 0$. The amount borrowed in any market $\lambda \in [0, 1)\backslash\Delta$ can still be represented with a density function. Heuristically, we can appeal to the Dirac-delta construction and represent the amount borrowed in any market as if it were a density. That is, for any $\lambda \in \Delta$, we set the density $f_t^x(\lambda) = m_t^x(\lambda)\,\delta_\lambda(\lambda)$, where $\delta_\lambda(q)$ is the Dirac-delta function defined so that $\delta_\lambda(q) = 0$ for $q \neq 0$ and $\int_0^1 \delta_\lambda(q)\, dq = 1$. This convention treats markets $\lambda \in \Delta$ as essentially having an infinite density. We will refer to a market λ as *inactive* if $f_t(\lambda) = 0$ and *active* if $f_t(\lambda) > 0$ or if $\lambda \in \Delta$.

Given these preliminaries, we define an equilibrium as a path $\left\{p_t, f_t^p(\lambda), f_t^a(\lambda), R_t(\lambda), \overline{R}_t(\lambda)\right\}_{t=0}^{\infty}$ that satisfies a series of conditions, (C1)-(C6), that ensure all markets clear when agents optimize.

The first few conditions we describe stipulate that all agents act optimally. We begin with lenders. Optimality requires that agents will only invest their wealth where the expected return is highest. Let $\overline{R}_t$ denote the maximal expected return to lending in any market λ, i.e.,

$$\overline{R}_t \equiv \sup_{\lambda \in [0,1)} \overline{R}_t(\lambda)$$

Optimal lending requires that agents lend in market λ' only if it they expect to earn $\overline{R}_t$ and if this rate exceeds the expected return to buying the asset, i.e.,

$$f_t(\lambda') > 0 \text{ only if } \overline{R}_t(\lambda') = \overline{R}_t \text{ and } \overline{R}_t \geq \overline{r}_t \tag{C1}$$

Entrepreneurs must also act optimally. We first argue this means they should use their endowment to produce. Recall entrepreneurs have productivity y^* where $y^* > r_t^{\max} \geq \overline{r}_t$ from condition (15) in the text, so producing is better than buying assets. But y^* must also exceed the expected return to lending $\overline{R}_t$. For suppose $\overline{R}_t$ were higher than y^*. Since $y^* > r_t^{\max}$, then $\overline{R}_t$ must also exceed $r_t^{\max}$. In that case, no agent would use their endowment to buy assets, nor would any agent borrow to buy assets given the interest rate on loans in any active market must be at least $\overline{R}_t$. Yet assets must trade in equilibrium: Owners sell their assets whenever the asset price is positive, while demand for the asset would be infinite if its price were nonnegative. Since production offers the highest return, entrepreneurs should use their endowment w to produce.

Since entrepreneurs can leverage their endowment to produce at a larger capacity, we also need to characterize their borrowing. If they borrow in market λ where $\lambda < w$, they can borrow enough to reach full capacity. Optimality requires that there will be borrowing to produce in market λ' only if some entrepreneur finds it optimal to borrow in that market from all $\lambda \in [0,1]$, including $\lambda = 1$ for no borrowing. This implies

$$f_t^p(\lambda') > 0 \text{ only if } \lambda' \in \arg\max_{\lambda \in [0,1]} \left\{ \begin{array}{ll} [1 + y - (1-w)(1 + R_t(\lambda))] & \text{if } \lambda \leq w \\ \dfrac{w}{\lambda}[1 + y - (1-\lambda)(1 + R_t(\lambda))] & \text{if } \lambda > w \end{array} \right\} \text{ for some } w \tag{C2}$$

Finally, agents who borrow to buy assets must act optimally. They will agree to borrow in market $\lambda \in [0,1)$ to buy assets only if doing so yields a higher expected return than lending out the same resources. Define

$$x_t(\lambda) \equiv (1 + R_t(\lambda))(1-\lambda)$$

The expected profits from borrowing in market λ to buy one consumption unit's worth of assets is

$$\int_{x_t(\lambda)}^{\infty} (z_t - x_t(\lambda))\, dG(z_t) \tag{C3}$$

Agents will borrow in market λ to buy assets only if (C3) equals $(1 + \overline{R}_t)\lambda$, the return they could have earned on any wealth that they use to buy assets. If (C3) were lower than $(1 + \overline{R}_t)\lambda$, no agent would borrow to buy assets. If (C3) were higher than $(1 + \overline{R}_t)\lambda$, then no one would ever lend given they can borrow in market λ', and so $f_t(\lambda') = 0$. But this contradicts the fact that $f_t^a(\lambda) > 0$. Optimality implies

$$f_t^a(\lambda') > 0 \text{ only if } \int_{x_t(\lambda')}^{\infty} (z_t - x_t(\lambda'))\, dG(z_t) = (1 + \overline{R}_t)\lambda' \tag{C4}$$

Next, we require that savers use their entire endowment e to ensure consumption when old rather than go to waste. Since entrepreneurs prefer to use their endowment w for production, all the resources used to buy the asset must come from savers. This implies that e must be either lent to entrepreneurs to produce or be spent on assets:

$$\int_0^1 f_t^p(\lambda)\, d\lambda + p_t = e \tag{C5}$$

Finally, we turn to equilibrium beliefs. In any active market λ', lenders must expect the return on lending $\overline{R}_t(\lambda')$ to conform with the actual fraction of borrowers who borrow in market λ' with the intent to produce and to buy assets, respectively. That is,

$$\overline{R}_t(\lambda') = \frac{f_t^p(\lambda')}{f_t(\lambda')} R_t(\lambda') + \frac{f_t^a(\lambda')}{f_t(\lambda')} E_t \min\left\{R_t(\lambda'), \frac{d_{t+1}+p_{t+1}}{p_t} - 1\right\} \text{ if } f_t(\lambda') > 0 \tag{C6}$$

In a market $\lambda \in \Delta$ with a positive mass of borrowing, the expression $\frac{f_t^x(\lambda')}{f_t(\lambda')}$ will be replaced by $\frac{m_t^x(\lambda)}{m_t(\lambda)}$. Condition (C6) does not impose any restrictions on expectations in inactive markets where $f_t(\lambda') = 0$.

C.2 Solving for Equilibrium

We now proceed to solve for an equilibrium. As in the text, we restrict attention to equilibria in which all markets $\lambda \in [0,1)$ are active. Such equilibria are natural given we focus on the effects of interventions to shut down markets. Our first result characterizes the schedule of interest rates in such an equilibrium.

Proposition C1: *In an equilibrium where all markets are active, there exists a value $\Lambda_t \in [0,1]$ such that the equilibrium interest rate schedule will be given by*

$$1 + R_t(\lambda) = \begin{cases} \frac{\widetilde{x}_t(\lambda)}{1-\lambda} & \text{if } \lambda \in [0, \Lambda_t) \\ 1 + \overline{R}_t & \text{if } \lambda \in [\Lambda_t, 1) \end{cases} \tag{C7}$$

where $\widetilde{x}_t(\lambda)$ is the value of x that solves

$$\int_{z=x}^{1+r_t^{\max}} (z - x)\, dG_t(z) = \left(1 + \overline{R}_t\right)\lambda \tag{C8}$$

The schedule of interest rates $R_t(\lambda)$ is a decreasing and continuous function of λ for $\lambda \in [0, \Lambda_t]$.

Proof of Proposition C1: Our proof proceeds as two lemmas.

Lemma C1: In an equilibrium where all markets are active, $1 + R_t(\lambda) = \max\left\{\frac{\widetilde{x}_t(\lambda)}{1-\lambda}, 1 + \overline{R}_t\right\}$, where $\widetilde{x}_t(\lambda)$ equals the x that solves (C8) and $\overline{R}_t$ is the expected return to lending in any market λ.

Proof of Lemma C1: Equilibrium condition (C4) holds that agents are either indifferent between lending their wealth and using it as a down payment in some market λ to buy assets, or else they strictly prefer to lend their wealth. That is, for all $\lambda \in [0,1)$, we have

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) \leq \left(1+\overline{R}_t\right)\lambda \tag{C9}$$

In the latter case, since no agent borrows to buy the asset, we know that $R_t(\lambda) = \overline{R}_t$. This is one candidate for the interest rate in market λ. The other candidate is any value of $R_t(\lambda)$ which ensures (C9) holds with equality. We now argue that is exactly one such candidate.

Consider the expression $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$. It is strictly decreasing in x, it tends to $+\infty$ as $x \to -\infty$ and to 0 as $x \to 1+r_t^{\max}$. Hence, for any $\lambda \in [0,1)$ and any $\overline{R}_t \geq 0$, there exists a unique $x \in (-\infty, 1+r_t^{\max}]$ for which

$$\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z) = \left(1+\overline{R}_t\right)\lambda \tag{C10}$$

Denote $\widetilde{x}_t(\lambda)$ as the unique solution to equation (C10). If the LHS of (C10) represents the payoff to borrowing to buy an asset, the expression $\widetilde{x}_t(\lambda)$ would correspond to the debt obligation of an agent who borrows in market λ, i.e. $\widetilde{x}_t(\lambda)$ would equal $(1+R_t(\lambda))(1-\lambda)$. Hence, the unique interest rate that ensures (C9) holds with equality is given by

$$1 + R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$$

Thus, there are two candidate expressions for the equilibrium interest rate in any market λ, namely $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ and $\overline{R}_t$. To show that $1 + R_t(\lambda) = \max\left\{1+\overline{R}_t, \frac{\widetilde{x}_t(\lambda)}{1-\lambda}\right\}$, consider first a value of λ for which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} > 1+\overline{R}_t$. We want to argue that in this case, $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is the only possible equilibrium interest rate. Since $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$ is decreasing in x, it follows that

$$\int_{z=(1+\overline{R}_t)(1-\lambda)}^{1+r_t^{\max}} \left(z - (1+\overline{R}_t)(1-\lambda)\right) dG_t(z) > \int_{z=\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (z - \widetilde{x}_t(\lambda))\, dG_t(z) = \left(1+\overline{R}_t\right)\lambda$$

Since the equilibrium interest rate $R_t(\lambda)$ must satisfy (C9), we cannot $R_t(\lambda) = \overline{R}_t$. The only possible equilibrium for these values of λ is $1 + R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$. In other words, for any λ such that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} > 1+\overline{R}_t$, the equilibrium interest rate must leave agents just indifferent leveraging their wealth and borrowing to speculate in market λ and lending out the same wealth and earning an expected return of $\overline{R}_t$.

Next, consider a value of λ for which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} < 1+\overline{R}_t$. In this case, $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ cannot be an equilibrium interest rate for market λ, since it would mean the interest rate on loans in market is lower than the return lenders can earn elsewhere. That cannot be true in equilibrium. Hence, if $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} < 1+\overline{R}_t$, the only one of the two candidates that can be an equilibrium is $R_t(\lambda) = \overline{R}_t$. Given that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} < 1+\overline{R}_t$, we can conclude that the expected payoff from borrowing to buy the asset and defaulting if the return is low is worse than lending at the safe rate $\overline{R}_t$, so no agent will borrow to buy assets in market λ. This establishes the lemma. ■

Our second lemma establishes that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is a weakly decreasing and continuous function of λ. Combined with Lemma C1, this implies there exists a cutoff Λ_t such that $R_t(\lambda) = \overline{R}_t$ for $\lambda \geq \Lambda_t$.

Lemma C2: In any equilibrium where all markets are active, $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is nonincreasing and continuous in λ.

Proof of Lemma C2: The function $\widetilde{x}_t(\lambda)$ corresponds to the value of x which solves (C8). Note that even though the distribution $G_t(z)$ can contain mass points, the integral $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$ is continuous in x and so $\widetilde{x}_t(\lambda)$ is a continuous function of λ. However, $\widetilde{x}_t(\lambda)$ may exhibit kinks. To show that $\widetilde{x}_t(\lambda)$ is decreasing, it will suffice to show that its directional derivatives are nonpositive for all $\lambda \in [0,1)$. Totally differentiating (C8) with respect to λ implies

$$\frac{d\widetilde{x}_t(\lambda)}{d\lambda} = -\frac{1+\overline{R}_t}{\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}$$

For any λ where $\widetilde{x}_t(\lambda)$ is a mass point of $G_t(z)$, $\lim_{\lambda'\to\lambda^+} \int_{\widetilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z) \neq \lim_{\lambda'\to\lambda^-} \int_{\widetilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z)$. Nevertheless, both $\lim_{\lambda'\to\lambda^+} \frac{d\widetilde{x}_t(\lambda')}{d\lambda'}$ and $\lim_{\lambda'\to\lambda^-} \frac{d\widetilde{x}_t(\lambda')}{d\lambda'}$ are negative, so $\widetilde{x}_t(\lambda)$ is strictly decreasing in λ. But we want to show that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is decreasing in λ and not just $\widetilde{x}_t(\lambda)$.

Define $\widetilde{R}_t(\lambda) \equiv \frac{\widetilde{x}_t(\lambda)}{1-\lambda} - 1$. By construction, $\widetilde{R}_t(\lambda)$ is continuous in λ with possible kink-points. Differentiating the equation $\widetilde{x}_t(\lambda) = (1-\lambda)(1+\widetilde{R}_t(\lambda))$ implies

$$\frac{d\widetilde{x}_t(\lambda)}{d\lambda} = -(1+\widetilde{R}_t(\lambda)) + (1-\lambda)\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$$

Rearranging and using the expression for $\frac{d\widetilde{x}_t(\lambda)}{d\lambda}$ above yields

$$\begin{aligned}
\frac{d\widetilde{R}_t(\lambda)}{d\lambda} &= \frac{1}{1-\lambda}\left[1+\widetilde{R}_t(\lambda) + \frac{d\widetilde{x}_t(\lambda)}{d\lambda}\right] \\
&= \frac{1}{1-\lambda}\left[1+\widetilde{R}_t(\lambda) - \frac{1+\overline{R}_t}{\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}\right] \\
&= \frac{1}{(1-\lambda)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}\left[(1+\widetilde{R}_t(\lambda))\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) - \left(1+\overline{R}_t\right)\right] \qquad \text{(C11)}
\end{aligned}$$

We want to argue that the expression in brackets is negative. There are two possible cases. First, suppose $\widetilde{R}_t(\lambda) < \overline{R}_t$. Then

$$\begin{aligned}
(1+\widetilde{R}_t(\lambda))\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) &< \left(1+\overline{R}_t\right)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) \\
&\leq 1+\overline{R}_t
\end{aligned}$$

In that case, we have $\frac{d\widetilde{R}_t(\lambda)}{d\lambda} < 0$ from (C11) regardless of the direction we take the derivative.

Next, suppose $\widetilde{R}_t(\lambda) \geq \overline{R}_t$. Recall that $\widetilde{x}_t(\lambda)$ is the value of x that solves (C10). Substituting in $\widetilde{x}_t(\lambda) = (1+\widetilde{R}_t(\lambda))(1-\lambda)$, we can rewrite (C10) as

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[z - \left(1+\widetilde{R}_t(\lambda)\right)\right] dG_t(z) = \lambda\left[\left(1+\overline{R}_t\right) - \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left(1+\widetilde{R}_t(\lambda)\right) dG_t(z)\right]$$

The RHS of the equation above has the opposite sign as $\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$. Hence, we can establish that $\frac{d\widetilde{R}_t(\lambda)}{d\lambda} \leq 0$ if we can show that

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[z_t - \left(1 + \widetilde{R}_t(\lambda)\right)\right] dG_t(z) \geq 0$$

Here, we use the fact that $\widetilde{x}(\lambda) = (1 + \widetilde{R}(\lambda))(1 - \lambda)$ to rewrite the LHS of (C10) evaluated at $x = \widetilde{x}_t(\lambda)$ as

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (z - \widetilde{x}_t(\lambda))\, dG(z) = \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left[z_t - (1 + \widetilde{R}_t(\lambda))\right] dG(z) + \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG(z)$$

Note that when $\widetilde{R}(\lambda) > \overline{R}_t$, we must have $\widetilde{x}_t(\lambda) > 0$. When the equilibrium interest rate in market λ exceeds $\overline{R}_t$, some agents who borrow in market λ must default, since the only way the expected return to lending in market λ can equal $\overline{R}_t$ in this case is if some agents default. Hence, there must be some values of z for which an agent who borrows in market λ to buy assets defaults. But given the equilibrium price of the asset cannot be negative and the dividend $d > 0$, the lower support of z is bounded below by 0.

Armed with this observation, we can rewrite (C10) as

$$\begin{aligned} \left(1 + \overline{R}_t\right)\lambda &= \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left[z_t - (1 + \widetilde{R}_t(\lambda))\right] dG(z) + \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG(z) \\ &\leq \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left[z_t - (1 + \widetilde{R}_t(\lambda))\right] dG(z) + \int_{0}^{1+r_t^{\max}} \lambda z_t dG(z) \\ &= \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left[z_t - (1 + R_t)\right] dG(z) + (1 + \overline{r}_t)\lambda \end{aligned} \tag{C12}$$

The inequality in the second row uses the fact that $\widetilde{x}_t(\lambda) \geq 0$ whenever $\widetilde{R}_t(\lambda) > \overline{R}_t$. But in an equilibrium where all markets are active, we must have $\overline{R}_t^D \geq \overline{r}_t^D$, i.e. since any saver can buy an asset, the return on savings $\overline{R}_t$ is at least as large as the return to buying an asset $\overline{r}_t$. This implies

$$0 \leq \left(\overline{R}_t - \overline{r}_t\right)\lambda \leq (1-\lambda)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (z_t - (1 + R_t))\, dG(z)$$

We have therefore confirmed that $\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (z_t - (1 + R_t))\, dG(z) \geq 0$. It follows that all directional derivatives $\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$ are nonnegative as claimed. ■

From Lemmas C1 and C2, define Λ_t as either 1 or the minimum value in $[0, 1]$ for which $R_t(\lambda) = \overline{R}_t$. It follows that $R_t(\lambda) > \overline{R}_t$ for $\lambda < \Lambda_t$ and $R_t(\lambda) = \overline{R}_t$ for all $\lambda \geq \Lambda_t$. This establishes the proposition. ■

We can use the schedule of interest rates in Proposition C1 to determine how much entrepreneurs should produce and in which markets to borrow if they do.

Proposition C2: *In an equilibrium where all markets are active, entrepreneurs with wealth w will borrow $1 - w$ units to produce, in a market with an interest rate equal to $R_t(w)$.*

Proof of Proposition C2: Consider an entrepreneur with wealth w. If she borrows in a market λ where $\lambda \leq w$, she can produce at full capacity and would only need to put down $\lambda\left(\frac{1-w}{1-\lambda}\right)$ resources to borrow $1-w$ to reach full capacity. This would earn her an expected profit of

$$1 + y^* - (1 + R_t(\lambda))(1 - w)$$

This value is maximized by choosing λ to minimize $R_t(\lambda)$. From Proposition C1, we know $R_t(\lambda)$ is weakly decreasing in λ and is therefore maximized at $\lambda = w$.

Next, suppose she borrows in a market λ where $\lambda > w$. In that case, she could not produce at full capacity. Since $y^* > r_t^{\max} = R_t(0) \geq R_t(\lambda)$ for all $\lambda \in [0,1)$, it will be optimal to borrow enough to produce at the maximal capacity possible. For $\lambda > w$, this maximum is $\frac{w}{\lambda}$. Her profits would thus equal

$$\frac{w}{\lambda}(1 + y^* - x_t(\lambda)) \tag{C13}$$

where recall $x_t(\lambda) = (1-\lambda)(1 + R_t(\lambda))$ is the amount a borrower is required to repay per each unit of resource she borrows. Since $R_t(\lambda) = \overline{R}_t$ for all $\lambda \in (\Lambda_t, 1)$, there would be no benefit to going to market $\lambda > \Lambda_t$: She would have to produce less at the same interest rate as in market Λ_t. The only case that remains is the interval of markets $\lambda \in [w, \Lambda_t]$. In that case, we can differentiate profits in (C13) to get

$$\begin{aligned}
\frac{d}{d\lambda}\left(\frac{w}{\lambda}(1 + y^* - x_t(\lambda))\right) &= -\frac{w}{\lambda^2}\left[(1 + y^* - x_t(\lambda)) + \lambda \frac{dx_t(\lambda)}{d\lambda}\right] \\
&= -\frac{w}{\lambda^2}\left[(1 + y^* - x(\lambda)) - \frac{\lambda\left(1 + \overline{R}_t\right)}{\int_x^{1+r_t^{\max}} dG_t(z)}\right] \\
&= -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)}\left[\int_x^{1+r_t^{\max}} (1 + y^* - x_t(\lambda))\, dG_t(z) - \lambda\left(1 + \overline{R}_t\right)\right]
\end{aligned}$$

Since $y^* > \frac{D+2\varphi e}{(1-\varphi)e} > r_t^{\max}$, we have

$$\frac{d}{d\lambda}\left(\frac{w}{\lambda}(1 + y^* - x_t(\lambda))\right) < -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)}\left[\int_x^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) - \lambda\left(1 + \overline{R}_t\right)\right]$$

But for $\lambda \leq \Lambda_t$, the expression in brackets is equal to 0. Hence, borrowing in a market with $\lambda > w$ will be strictly dominated by borrowing in the market with $\lambda = w$. At the optimum, each entrepreneur borrow $1-w$ at a rate of $R_t(w)$. ■

Proposition C3: *In an equilibrium where all markets are active, the equilibrium price of the asset will be given by* $p_t = (1-\varphi)e$

Proof of Proposition C3: Condition (C5) implies that all the resources of the young in cohort t will be used to either produce or to buy assets. From Proposition C2, we know that all entrepreneurs will produce at capacity, so the total amount used to produce is given by

$$\int_0^1 (2\varphi e)\, dw = 2\varphi e$$

This implies

$$p_t + 2\varphi e = (1+\varphi)\, e$$

and so $p_t = (1-\varphi)\, e$ as claimed. ■

Propositions C1-C3 did not involve any restrictions on the distribution of the return $z_t = \frac{p_{t+1}+d_{t+1}}{p_t}$. But given Proposition 3 and the process for dividends, we can determine the distribution of this return to obtain a sharper characterization. When $d_t = d$, the return on the asset $1+r_t$ will have a degenerate distribution with full mass at $\frac{d}{(1-\varphi)e}$. Substituting this into (C8) reveals that $\widetilde{x}_t(\lambda) = (1-\lambda)\left(1+\frac{d}{(1-\varphi)e}\right)$ for all λ, that $\frac{d\widetilde{R}(\lambda)}{d\lambda} = 0$ for all λ, and the cutoff $\Lambda_t = 0$. Hence, when all markets are active, $R_t(\lambda) = \overline{R}_t = \frac{d}{(1-\varphi)e}$ for all $\lambda \in [0,1)$ as described in the text. One equilibrium in which all markets are active if it entrepreneurs with wealth w borrow in market $\lambda = w$. But other equilibria in which all markets are active also exist.

Next, when dividends d_t follow the regime-switching process between d and D and at date t we have $d_t = D$, the return z_t would have a two-point distribution:

$$z_t = \begin{cases} 1+\frac{D}{(1-\varphi)e} & \text{w/prob } 1-\pi \\ 1+\frac{d}{(1-\varphi)e} & \text{w/prob } \pi \end{cases}$$

In this case, equation (C10) which defines $\widetilde{x}_t(\lambda)$ reduces to

$$(1-\pi)\left(1+\frac{D}{(1-\varphi)\,e} - \widetilde{x}_t(\lambda)\right) = \left(1+\overline{R}_t^D\right)\lambda$$

or

$$\widetilde{x}_t(\lambda) = 1 + \frac{D}{(1-\varphi)\,e} - \frac{1+\overline{R}_t^D}{1-\pi}\lambda \tag{C14}$$

From this we can derive the implied interest rate $1+\widetilde{R}_t^D(\lambda)$ in market λ while $d_t = D$:

$$\begin{aligned} 1+\widetilde{R}_t^D(\lambda) &= \frac{1}{1-\lambda}\left[1+\frac{D}{(1-\varphi)\,e} - \frac{1+\overline{R}_t^D}{1-\pi}\lambda\right] \\ &\equiv \frac{1-\kappa\lambda}{1-\lambda} \end{aligned}$$

where $\kappa \equiv \frac{\left(1+\overline{R}_t^D\right)/(1-\pi)}{1+D/((1-\varphi)e)}$. Since the return on savings is at least as large as the return to buying the asset,

$$\begin{aligned} 1+\overline{R}_t^D &\geq 1+\overline{r}_t^D \\ &= 1+\frac{(1-\pi)\,D+\pi d}{(1-\varphi)\,e} \\ &> (1-\pi)\left(1+\frac{D}{(1-\varphi)\,e}\right) \end{aligned}$$

This means $\kappa > 1$, which in turns implies the interest rate on loans $\widetilde{R}_t^D(\lambda)$ is strictly decreasing in λ for $\lambda > 0$, in line with what we discuss and depict in Figure 3 in the text.

Recall that, by definition, Λ_t^D is the minimum value of λ at which $\widetilde{R}_t^D(\lambda) = 1 + \overline{R}_t^D$. We can therefore solve for Λ_t^D by setting $\lambda = \Lambda_t^D$ in (C14) and equating $\widetilde{x}_t\left(\Lambda_t^D\right)$ with $1 + \overline{R}_t^D$, i.e. by setting

$$\tfrac{1}{1-\Lambda_t^D}\left[1 + \tfrac{D}{(1-\varphi)e} - \left(1 + \overline{R}_t^D\right)\tfrac{\Lambda_t^D}{1-\pi}\right] = 1 + \overline{R}_t^D$$

Rearranging, we have

$$\Lambda_t^D = \tfrac{1-\pi}{\pi\left(1+\overline{R}_t^D\right)}\left(\tfrac{D}{(1-\varphi)e} - \overline{R}_t^D\right) \tag{C15}$$

Since $R_t^D(\lambda)$ is decreasing in λ for $\lambda \in [0, \Lambda_t^D)$, Proposition C2 implies only borrowers with wealth w borrow in market $\lambda = w$ for $w \in [0, \Lambda_t^D)$. Hence, $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda_t^D)$. By contrast, $f_t^p(\lambda)$ is indeterminate for $\lambda \in [\Lambda_t^D, 1)$. However, we know that $f_t^p\left(\Lambda_t^D\right) > 0$, since borrowers with wealth $w = \Lambda_t^D$ will have to borrow in this market to borrow $1 - w$. As for the amount borrowed to buy assets, $f_t^a(\lambda)$, we can deduce $f_t^a(\lambda)$ for $\lambda \in [0, \Lambda_t^D]$ from $R_t^D(\lambda)$, $\overline{R}_t^D$, and $f_t^p(\lambda)$ using (C6). For $\lambda > \Lambda_t^D$, the fact that $\frac{dR_t^D(\lambda)}{d\lambda} < 0$ at $\lambda = \Lambda_t^D$, combined with the fact that $\frac{dR_t^D(\lambda)}{d\lambda} < 0$ for $\lambda > \Lambda_t^D$ from Lemma C2, implies that no agent would want to borrow to buy assets. So $f_t^a(\lambda) = 0$ for all $\lambda \geq \Lambda_t^D$.

Finally, we need to solve for $\overline{R}_t^D$. Consider the return on all forms of savings in this economy. First, savings are used to finance production by entrepreneurs, which yields savers a payoff of

$$\int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi e)\,dw$$

Second, savings are used to buy assets, directly or indirectly through loans. The expected earnings from these investments equal $\left(1 + \overline{r}_t^D\right)p_t^D$. From this, we must net out expected default costs. We use γ_t^D to denote the fraction of spending on assets that is financed with some debt. These purchases will result in default if returns are low. Since default is proportional to the size of the borrower's project, expected default costs equal $\pi\gamma_t^D\Phi p_t^D = \pi\gamma_t^D\Phi(1-\varphi)e$. These payoffs must add up to $(1 + \overline{R}_t^D)e$, i.e.,

$$(1 + \overline{R}_t^D)e = \left[1 + \overline{r}^D - \pi\gamma_t^D\Phi\right](1-\varphi)e + \int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi e)\,dw \tag{C16}$$

We also need an equation to characterize γ_t^D. When the expected return to lending $\overline{R}_t^D$ exceeds the expected return to buying the asset $\overline{r}^D$, only agents who borrow will buy the asset. In that case, $\gamma_t^D = 1$, and we can solve for $1 + \overline{R}_t^D$ by plugging in $\gamma_t^D = 1$ in (C16). When $\overline{R}_t^D = \overline{r}^D$, then γ_t^D would have to ensure that $\overline{R}_t^D$ is indeed equal to $\overline{r}^D$, where we know the latter is equal to $\frac{(1-\pi)D+\pi d}{(1-\varphi)e}$. We can combine these two conditions into a single equation:

$$1 + \overline{R}_t^D = \max\left\{1 + \overline{r}^D, \left[1 + \overline{r}^D - \pi\Phi\right](1-\varphi) + \int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi)\,dw\right\} \tag{C17}$$

Equation (C17) ensures that when $\overline{R}_t^D > \overline{r}^D$, we must have $\gamma_t^D = 1$, and when $\overline{R}_t^D = \overline{r}^D$, the value of γ_t^D must equate the two returns. Since $\overline{r}^D$ is time invariant, the solutions to these equations, $\overline{R}^D$ and γ^D, are also time invariant. Given a value for $\overline{R}^D$, we can solve for the time invariant cutoff Λ^D as the smallest value of λ for which $R^D(\lambda) = \overline{R}^D$. This completes the characterization of an equilibrium when all markets are active.

C.3 Comparative Statics with a floor

Finally, we consider equilibria where all markets above some floor $\underline{\lambda}$ are active. These results correspond to Propositions 9 and 10 in the text. The first result concerns how the equilibrium changes with $\underline{\lambda}$.

Proof of Proposition 9: In the text, we derive the equilibrium values p^D and $\overline{r}^D$ are show that they are increasing and decreasing in $\underline{\lambda}$, respectively. Here, we show that $\overline{R}^D$ is decreasing in $\underline{\lambda}$. Recall that in equilibrium, $\overline{R}^D \geq \overline{r}^D$, i.e. the return on savings must be at least as high as the return agents can earn from buying the asset. We need to show that $\overline{R}^D$ is decreasing in $\underline{\lambda}$ when $\overline{R}^D > \overline{r}^D$.

When $\overline{R}^D > \overline{r}^D$, we have $\gamma^D = 1$, and the equilibrium conditions for $\overline{R}^D$ and Λ^D are given by two equations. First, since Λ^D corresponds to the minimum value of λ for which $R^D(\lambda) = \overline{R}^D$, we know from (C14) that

$$1 + \overline{R}^D = \frac{1}{1-\Lambda^D}\left[1 + \tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \left(1+\overline{R}^D\right)\tfrac{\Lambda^D}{1-\pi}\right] \tag{C18}$$

Second, using the same approach to compute the return on savings as before, we have a similar equation for $\overline{R}^D$ as in (C16):

$$\begin{aligned} 1+\overline{R}^D &= (1-\varphi(1-\underline{\lambda}))\left[1+\overline{r}^D - \pi\Phi\right] + \\ & 2\varphi\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\} - w\right]\left[1+R^D(\max\{w,\underline{\lambda}\})\right]dw \end{aligned} \tag{C19}$$

If $\overline{R}^D > \overline{r}^D$, we argue that the floor $\underline{\lambda}$ must be below the cutoff Λ^D. For suppose $\underline{\lambda} \geq \Lambda^D$. Then all markets where agents might default will be shut down. But without default, the expected return on lending and the expected return on the asset must be equal to ensure both the credit market and asset market clear. Since $\underline{\lambda} < \Lambda^D$, we can expand the integral term in (C19) into the sum of three distinct terms:

$$\begin{aligned} \int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\} - w\right]\left[1+R^D(\max\{w,\underline{\lambda}\})\right] &= (1+R(\underline{\lambda}))\left(\frac{1}{\underline{\lambda}}-1\right)\int_0^{\underline{\lambda}} wdw + \\ & \int_{\underline{\lambda}}^{\Lambda^D}(1+R(w))(1-w)\,dw + \left(1+\overline{R}^D\right)\int_{\Lambda^D}^1 (1-w)\,dw \end{aligned}$$

We then use the fact that $1+R^D(\lambda) = \frac{1}{1-\lambda}\left[1 + \frac{D}{(1-(1-\underline{\lambda})\varphi)e} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right]$ to evaluate the three terms above:

$$(1+R(\underline{\lambda}))\left(\frac{1}{\underline{\lambda}}-1\right)\int_0^{\underline{\lambda}} wdw = \left[1+\tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \underline{\lambda}\left(\tfrac{1+\overline{R}^D}{1-\pi}-1\right)\right]\frac{\underline{\lambda}}{2} \tag{C20}$$

$$\int_{\underline{\lambda}}^{\Lambda^D}(1+R(w))(1-w)\,dw = \int_{\underline{\lambda}}^{\Lambda^D}\left[1+\tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \tfrac{w\left(1+\overline{R}_t^D\right)}{1-\pi}\right]dw \tag{C21}$$

$$\left(1+\overline{R}^D\right)\int_{\Lambda^D}^1 (1-w)\,dw = \frac{1}{2}\left(1+\overline{R}^D\right)\left(1-\Lambda^D\right)^2 \tag{C22}$$

We can write (C18) and (C19) more compactly as

$$\begin{aligned} h_1\left(\overline{R}^D, \Lambda^D\right) &= 0 \\ h_2\left(\overline{R}^D, \Lambda^D\right) &= 0 \end{aligned}$$

Totally differentiating this system of equations gives us the comparative statics of the equilibrium $\overline{R}^D$ and Λ^D with respect to any variable a as

$$\begin{bmatrix} \frac{\partial h_1}{\partial \overline{R}^D} & \frac{\partial h_1}{\partial \Lambda^D} \\ \frac{\partial h_2}{\partial \overline{R}^D} & \frac{\partial h_2}{\partial \Lambda^D} \end{bmatrix} \begin{bmatrix} d\overline{R}^D/da \\ d\Lambda^D/da \end{bmatrix} = \begin{bmatrix} -\frac{\partial h_1}{\partial a} \\ -\frac{\partial h_2}{\partial a} \end{bmatrix}$$

Differentiating (C18) and (C19) using expressions (C20)-(C22) yields

$$\begin{aligned} \frac{\partial h_1}{\partial \overline{R}^D} &= 1 - \Lambda^D + \frac{\Lambda^D}{1-\pi} & \frac{\partial h_1}{\partial \Lambda^D} &= \frac{\pi(1+\overline{R}^D)}{1-\pi} \\ \frac{\partial h_2}{\partial \overline{R}^D} &= 1 + \varphi\left[\frac{1}{1-\pi}\left(\Lambda^D\right)^2 - \left(1-\Lambda^D\right)^2\right] & \frac{\partial h_2}{\partial \Lambda^D} &= 0 \end{aligned}$$

When we evaluate comparative statics with respect to $\underline{\lambda}$, we now have

$$\begin{aligned} \begin{bmatrix} d\overline{R}^D/d\underline{\lambda} \\ d\lambda^D/d\underline{\lambda} \end{bmatrix} &= \begin{bmatrix} \frac{\partial h_1}{\partial \overline{R}^D} & \frac{\partial h_1}{\partial \Lambda^D} \\ \frac{\partial h_2}{\partial \overline{R}^D} & \frac{\partial h_2}{\partial \Lambda^D} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dh_1}{d\underline{\lambda}} \\ \frac{dh_2}{d\underline{\lambda}} \end{bmatrix} \\ &= \frac{\varphi}{\kappa} \begin{bmatrix} 0 & \frac{\pi}{1-\pi}\left(1+\overline{R}^D\right) \\ 1+\varphi\frac{\left(\Lambda^D\right)^2}{1-\pi} - \varphi\left(1-\Lambda^D\right)^2 & -\left(1-\Lambda^D+\frac{\Lambda^D}{1-\pi}\right) \end{bmatrix} \begin{bmatrix} -\frac{D}{(1-(1-\underline{\lambda})\varphi)^2 e} \\ -\frac{2D\left(1+\Lambda^D\varphi\right)}{(1-(1-\underline{\lambda})\varphi)^2 e} - (1+\pi\Phi) \end{bmatrix} \end{aligned}$$

where $\kappa = \frac{\pi(1+\overline{R}^D)}{1-\pi}\left(1+\varphi\frac{\left(\Lambda^D\right)^2}{1-\pi} - \varphi\left(1-\Lambda^D\right)^2\right) > 0$. It follows that

$$\frac{d\overline{R}^D}{d\underline{\lambda}} = -\varphi\left(1+\varphi\left[\frac{1}{1-\pi}\left(\Lambda^D\right)^2 - \left(1-\Lambda^D\right)^2\right]\right)^{-1}\left[\frac{2D\left(1+\Lambda^D\varphi\right)}{\left(1-\left(1-\underline{\lambda}\right)\varphi\right)^2 e} + (1+\pi\Phi)\right] < 0$$

Since $\overline{R}^D$ is decreasing in $\underline{\lambda}$ whether $\overline{R}^D > \bar{r}^D$ or $\overline{R}^D = \bar{r}^D$, the claim follows. ■

Proposition 10 concerns how changing $\underline{\lambda}$ affects the expected costs of default $\gamma^D\Phi p^D$. Since we already know p^D is increasing in $\underline{\lambda}$, any changes in expected default costs occur entirely through γ^D. Our next result argues that there exists cutoffs Λ_0 and Λ_1 such that $d\gamma^D/d\underline{\lambda} = 0$ when $\underline{\lambda} < \Lambda_0$ or $\underline{\lambda} > \Lambda_1$. When $\Lambda_0 < \underline{\lambda} < \Lambda_1$, we only claim it must be decreasing for some $\underline{\lambda}$ in this interval.

Proof of Proposition 10: Define

$$\rho\left(\underline{\lambda}\right) = \frac{\overline{R}^D}{\left(1-\left(1-\underline{\lambda}\right)\varphi\right)}$$

Using the fact that $\frac{d\overline{R}^D}{d\underline{\lambda}} < 0$, we have

$$\frac{d\rho\left(\underline{\lambda}\right)}{d\underline{\lambda}} = \frac{d\overline{R}^D/d\underline{\lambda} - \varphi\rho\left(\underline{\lambda}\right)}{1-\left(1-\underline{\lambda}\right)\varphi} < 0$$

Since

$$\overline{R}^D/\bar{r}^D = \left[\left(1-\pi\right)D + \pi d\right]\rho\left(\underline{\lambda}\right)$$

it follows that the ratio $\overline{R}^D/\bar{r}^D$ is decreasing in $\underline{\lambda}$. Hence, there exists a value $\Lambda_0 \geq 0$ such that $\overline{R}^D > \bar{r}^D$ for $\underline{\lambda} < \Lambda_0$ and $\overline{R}^D = \bar{r}^D$ for $\underline{\lambda} \geq \Lambda_0$. Since $\overline{R}^D > \bar{r}^D$ when $\underline{\lambda} < \Lambda_0$, then $\gamma^D = 1$ for $\underline{\lambda} < \Lambda_0$. It follows that expected default costs $\pi\gamma^D\Phi p^D = \pi\Phi p^D$ are increasing in $\underline{\lambda}$ in this region. A higher $\underline{\lambda}$ for $\lambda < \Lambda_0$ reduces the amount entrepreneurs produce and increases the foregone output when dividends fall. Each cohort will therefore be left with fewer goods to consume.

We next turn to the case where $\underline{\lambda} \geq \Lambda_0$. Here, we know $\overline{R}^D = \overline{r}^D$. Substituting this into (C18) yields

$$\left(1-\Lambda^D\right)\left(1+\overline{r}^D\right) = \left[1+\tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \tfrac{\Lambda^D}{1-\pi}\left(1+\overline{r}^D\right)\right]$$

which, upon rearranging,

$$\Lambda^D = \tfrac{(1-\pi)(D-d)}{(1-(1-\underline{\lambda})\varphi)e+(1-\pi)D+\pi d}$$

From this, we can conclude that $\Lambda^D \geq \underline{\lambda}$ if

$$\tfrac{(1-\pi)(D-d)}{(1-(1-\underline{\lambda})\varphi)e+(1-\pi)D+\pi d} \geq \underline{\lambda}$$

or, upon rearranging, if

$$(1-\pi)(D-d) \geq \underline{\lambda}\left[(1-(1-\underline{\lambda})\varphi)e+(1-\pi)D+\pi d\right] \tag{C23}$$

The RHS of (C23) is a quadratic in $\underline{\lambda}$ with a positive coefficient on the quadratic term. The inequality is satisfied when $\underline{\lambda} = 0$ and violated when $\underline{\lambda} = 1$. Hence, there exists a cutoff $\Lambda_1 \in (0,1)$ such that $\Lambda^D > \underline{\lambda}$ if $\underline{\lambda} \in [0, \Lambda_1)$ and $\Lambda^D < \underline{\lambda}$ if $\underline{\lambda} \in (\Lambda_1, 1)$. By definition, Λ_0 is the smallest value of $\underline{\lambda} \geq 0$ for which setting $\underline{\lambda} \geq \Lambda_0$ ensures $\overline{R}^D = \overline{r}^D$. By contrast, Λ_1 is the smallest value of $\underline{\lambda} \geq 0$ for which setting $\underline{\lambda} \geq \Lambda_1$ ensures that no agent borrows to speculate in any market above $\underline{\lambda}$. But in that case, all lending is riskless, and we know that the equilibrium interest rate on loans will equal the return on the asset. Hence, $\Lambda_1 \geq \Lambda_0$. To show that the inequality is strict, recall that when $\underline{\lambda} = 0$, we know that $\gamma^D > 0$ since some agents borrow to buy the asset. But γ^D is continuous in $\underline{\lambda}$, and we know that $\gamma^D = 0$ when $\underline{\lambda} \geq \Lambda_1$. Hence, there must be some value of $\underline{\lambda} \in [0, \Lambda_1)$ for which $\gamma^D < 1$. But $\gamma^D < 1$ iff $\overline{R}^D = \overline{r}^D$. It follows that $\Lambda_1 > \Lambda_0$.

When $\underline{\lambda} > \Lambda_1$ no agent will borrow to buy the asset, so $\gamma^D = 0$. Expected default costs are 0, and so the only effect of increasing $\underline{\lambda}$ is to reduce production. This will leave fewer goods for each cohort to consume.

Finally, we turn to the case where $\Lambda_0 < \underline{\lambda} < \Lambda_1$. We do not analyze this case in general. However, when $\Lambda^D = \underline{\lambda}$, the interest rate in all active markets would equal $\overline{R}^D$, since the only active markets are those with $\lambda \geq \underline{\lambda} = \Lambda^D$. Since $\underline{\lambda} \geq \Lambda_0$, we know that $\overline{R}^D = \overline{r}^D$ and so the interest rate in all active markets is $\overline{r}^D$. The equilibrium condition that determines γ^D is given by

$$\begin{aligned}
\left(1+\overline{r}^D\right) &= (1-(1-\underline{\lambda})\varphi)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right] + 2\varphi\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\}-w\right]\left[1+R^D\left(\max\left\{w,\underline{\lambda}\right\}\right)\right]dw \\
&= (1-(1-\underline{\lambda})\varphi)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right] + 2\varphi\left(1+\overline{r}^D\right)\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\}-w\right]dw \\
&= (1-(1-\underline{\lambda})\varphi)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right] + 2\varphi\left(1+\overline{r}^D\right)\left[\underline{\lambda}/2+(1-\underline{\lambda})-1/2\right] \\
&= 1+\overline{r}^D-\gamma^D(1-(1-\underline{\lambda})\varphi)\pi\Phi
\end{aligned}$$

Hence, when $\underline{\lambda} = \Lambda_1$, we have $\gamma^D = 0$. For $\underline{\lambda} < \Lambda_1$, however, $\gamma^D > 0$, since

$$\int_0^1 \left[1 + R^D\left(\max\left\{w, \underline{\lambda}\right\}\right)\right]\left[\min\left\{\frac{w}{\underline{\lambda}}, 1\right\} - w\right] dw$$

will be strictly greater than $\frac{1}{2}\left(1 + \bar{r}^D\right)\left(1 - \underline{\lambda}\right)$. Hence, in the limit as $\underline{\lambda} \uparrow \Lambda_1$, we have $d\gamma^D / d\underline{\lambda} < 0$ expected default costs $\pi\gamma^D \Phi p^D$ must be decreasing in $\underline{\lambda}$ since this expression goes from a positive value to 0.

To show that this can generate a Pareto improvement, observe that increasing $\underline{\lambda}$ while dividends are high will make the initial old at date 0 better off given p_0^D increases. Cohorts born after dividends have fallen will be unaffected if $\underline{\lambda}$ is only increased while dividends are high. Cohorts who are born while dividends are high expect to consume the dividends from the asset net of default costs $E\left[d_{t+1}\right] - \Phi\pi\gamma^D p_t^D$ as well as the output produced by entrepreneurs. If Φ is sufficiently large and φ is small, we can promise these agents a higher expected consumption. ■

References

[1] Adam, Klaus, 2003. "Learning and Equilibrium Selection in a Monetary Overlapping Generations Model with Sticky Prices" *Review of Economic Studies*, 70(4), October, p887-907.

[2] Cochrane, John, 2011. "Determinacy and Identification with Taylor Rules" *Journal of Political Economy*, 119(3), June, p565-615.

[3] Galí, Jordi, 2014. "Monetary Policy and Rational Asset Price Bubbles" *American Economic Review*, 104(3), March, p721-52.

[4] Sargent, Thomas and Neil Wallace, 1975. "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule" *Journal of Political Economy*, 83, p241–254.