Online Appendix to:
The Government Spending Multiplier in a Multi-Sector Economy

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A More on the Model

A.1 Households

The infinitely lived representative household has preferences over aggregate consumption, C_t, aggregate government spending, G_t, and aggregate labor, N_t, so that its expected lifetime utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(\left[\zeta^{\frac{1}{\mu}} C_t^{\frac{\mu-1}{\mu}} + (1-\zeta)^{\frac{1}{\mu}} G_t^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \right)^{1-\sigma}}{1-\sigma} - \theta \frac{N_t^{1+\eta}}{1+\eta} \right\}, \tag{A.1}$$

where β denotes the subjective time discount factor, σ is the degree of risk aversion, θ is a preference shifter that determines the disutility of labor, and η is the inverse of the Frisch elasticity of labor supply. We allow preferences to be non-separable in consumption and government services, with ζ denoting the weight of private consumption in the effective consumption aggregator, $\tilde{C}_t$, and μ the elasticity of substitution between private consumption and government spending.

The household trades one-period nominal bonds, B_t, and owns the stock of physical capital, K_t. Every period it purchases consumption goods at price $P_{C,t}$ and investment goods, I_t, at price $P_{I,t}$. Investment is subject to convex adjustment costs defined by the parameter Ω, such that the law of motion of physical capital is

$$K_{t+1} = (1-\delta) K_t + I_t \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right], \tag{A.2}$$

where δ is the depreciation rate. The household receives total labor income, $W_t N_t$, where W_t is the nominal aggregate wage; total capital income, $R_{K,t} K_t$, where $R_{K,t}$ is the nominal aggregate rental rate

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of capital; and total bond income, $R_{t-1}B_t$, where R_{t-1} is the nominal risk-free interest rate. Finally, the household pays a nominal lump-sum tax, T_t, and earns firms' nominal profits, D_t. Its budget constraint is therefore given by

$$P_{C,t}C_t + P_{I,t}I_t + B_{t+1} + T_t = W_t N_t + R_{K,t}K_t + B_t R_{t-1} + D_t. \tag{A.3}$$

The household chooses C_t, N_t, I_t, K_{t+1}, and B_{t+1} to maximize life-time utility (A.1) subject to the budget constraint (A.3), the law of motion of capital (A.2), and a no-Ponzi game condition.

We posit that the total amount of labor provided by the household is a CES function of the labor supplied to each sector, that is

$$N_t = \left[\sum_{s=1}^{S} \omega_{N,s}^{-\frac{1}{\nu_N}} N_{s,t}^{\frac{1+\nu_N}{\nu_N}}\right]^{\frac{\nu_N}{1+\nu_N}}, \tag{A.4}$$

where $\omega_{N,s}$ is the weight attached to labor provided to sector s, and $\nu_N > 0$ is the (absolute value of the) elasticity of substitution of labor across sectors, which captures the degree of labor mobility. This aggregator implies that also the nominal aggregate wage is a function of sectoral wages, $W_{s,t}$, that is

$$W_t = \left[\sum_{s=1}^{S} \omega_{N,s} W_{s,t}^{1+\nu_N}\right]^{\frac{1}{1+\nu_N}}. \tag{A.5}$$

In equilibrium, the optimal allocation of labor across sectors follow the first-order condition

$$N_{s,t} = \omega_{N,s}\left(\frac{W_{s,t}}{W_t}\right)^{\nu_N} N_t, \qquad s = 1, \ldots, S. \tag{A.6}$$

Analogously, aggregate capital, K_t, bundles sectoral capital services, that is

$$K_t = \left[\sum_{s=1}^{S} \omega_{K,s}^{-\frac{1}{\nu_K}} K_{s,t}^{\frac{1+\nu_K}{\nu_K}}\right]^{\frac{\nu_K}{1+\nu_K}}, \tag{A.7}$$

where $\omega_{K,s}$ is the weight attached to capital provided to sector s, and $\nu_K > 0$ is the (absolute value of the) elasticity of substitution of capital across sectors. The nominal aggregate return on capital equals

$$R_{K,t} = \left[\sum_{s=1}^{S} \omega_{K,s} {R_{K,s,t}}^{1+\nu_K}\right]^{\frac{1}{1+\nu_K}}, \tag{A.8}$$

which implies the following first-order conditions on the allocation of capital across sectors

$$K_{s,t} = \omega_{K,s}\left(\frac{R_{K,s,t}}{R_{K,t}}\right)^{\nu_K} K_t, \qquad s = 1, \ldots, S. \tag{A.9}$$

A.2 Producers

In each sector, there is a continuum of monopolistically competitive producers indexed by $j \in [0, 1]$ that use labor, capital, and a bundle of intermediate inputs to assemble a differentiated variety using the Cobb-Douglas technology

$$Z_{s,t}^{j} = \left({N_{s,t}^{j}}^{\alpha_{N,s}} {K_{s,t}^{j}}^{1-\alpha_{N,s}} \right)^{1-\alpha_{H,s}} {H_{s,t}^{j}}^{\alpha_{H,s}}, \tag{A.10}$$

where $Z_{s,t}^{j}$ is the gross output of the variety of producer j, $N_{s,t}^{j}$, $K_{s,t}^{j}$, and $H_{s,t}^{j}$ denote labor, capital, and the bundle of intermediate inputs used by this producer, respectively. Factor intensities, $\alpha_{N,s}$ and $\alpha_{H,s}$, are sector-specific. In equilibrium, labor-market clearing implies that the labor supplied by the households to each sector equals the sum of labor hired by each producer within each sector, that is, $N_{s,t} = \int_0^1 N_{s,t}^{j} dj$. Similarly, $K_{s,t} = \int_0^1 K_{s,t}^{j} dj$, and $H_{s,t} = \int_0^1 H_{s,t}^{j} dj$.

Each producer sets its price subject to Calvo-type frictions: producers can reset prices only with a constant probability $1 - \phi_s$, which varies across sectors. The optimal reset price, $P_{s,t}^{\star,j}$, maximizes the expected discounted stream of real dividends:

$$\max_{P_{s,t}^{j}} \mathbb{E}_t \left[\sum_{z=t}^{\infty} \beta^{z-t} \phi_s^{z-t} \frac{C_z^{-\sigma}}{C_t^{-\sigma}} \frac{D_{s,z}^{j}\left(P_{s,t}^{j}\right)}{P_z} \right], \tag{A.11}$$

where nominal dividends, $D_{s,t}^{j}$, are defined as the nominal value of the produced variety minus the nominal productions costs,

$$D_{s,t}^{j}\left(P_{s,t}^{j}\right) = P_{s,t}^{j} Z_{s,t}^{j} - W_{s,t} N_{s,t}^{j} - R_{K,s,t} K_{s,t}^{j} - P_{H,s,t} H_{s,t}^{j}. \tag{A.12}$$

Aggregate nominal profits equal the sum of profits across varieties and across sectors, that is, $D_t = \sum_{s=1}^{S} \int_0^1 D_{s,t}^{j} dj$.

A.3 Wholesalers

Producers' different varieties are aggregated into a single sectoral final good by perfectly competitive wholesalers using the following CES production technology:

$$Z_{s,t} = \left[\int_0^1 {Z_{s,t}^{j}}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{A.13}$$

where $Z_{s,t}^{j}$ is the output of sector s, and ϵ is the elasticity of substitution across varieties within sectors. This technology implies that the price of the sectoral good of sector s is

$$P_{s,t} = \left[\int_0^1 {P_{s,t}^{j}}^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}. \tag{A.14}$$

Thus, the representative wholesaler in sector s purchases each single variety $Z^j_{s,t}$ by solving the problem

$$\max_{Z^j_{s,t}} P_{s,t}Z_{s,t} - \int_0^1 P^j_{s,t}Z^j_{s,t}dj$$
$$s.t. \qquad Z_{s,t} = \left[\int_0^1 {Z^j_{s,t}}^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}},$$

which implies the optimal demand of each variety j

$$Z^j_{s,t} = \left(\frac{P^j_{s,t}}{P_{s,t}}\right)^{-\epsilon} Z_{s,t}, \qquad j \in [0,1]\,, \;\; s = 1,\ldots,S. \tag{A.15}$$

Finally, the wholesaler sells the final sector good to consumption, investment, government and intermediate input retailers, such that

$$Z_{s,t} = C_{s,t} + I_{s,t} + G_{s,t} + \sum_{x=1}^{S} H_{x,s,t}. \tag{A.16}$$

A.4 Consumption-good retailers

The final consumption good is assembled by perfectly competitive consumption-good retailers according to the following CES technology:

$$C_t = \left[\sum_{s=1}^{S} \omega_{C,s}^{\frac{1}{\nu_C}} C_{s,t}^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}}, \tag{A.17}$$

where $C_{s,t}$ is the purchase of consumption goods from sector s, $\omega_{C,s}$ denotes the weight of good s in the consumption bundle, such that $\sum_{s=1}^{S} \omega_{C,s} = 1$, and ν_C is the elasticity of substitution of consumption across sectors. This aggregator implies that the price of the consumption bundle is

$$P_{C,t} = \left[\sum_{s=1}^{S} \omega_{C,s} P_{s,t}^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}}. \tag{A.18}$$

The optimal amount of consumption goods to be purchased from the wholesalers of each sectors is

$$C_{s,t} = \omega_{C,s}\left(\frac{P_{s,t}}{P_{C,t}}\right)^{-\nu_C} C_t, \qquad s = 1,\ldots,S, \tag{A.19}$$

which is derived as the first-order condition of the problem of the consumption-good retailer:

$$\max_{C_{s,t}} P_{C,t}C_t - \sum_{s=1}^{S} P_{s,t}C_{s,t}$$
$$s.t. \qquad C_t = \left[\sum_{s=1}^{S} \omega_{C,s}^{\frac{1}{\nu_C}} C_{s,t}^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}}.$$

A.5 Investment-good retailers

The final investment good is assembled by perfectly competitive investment-good retailers according to the following CES technology:

$$I_t = \left[\sum_{s=1}^{S} \omega_{I,s}^{\frac{1}{\nu_I}} I_{s,t}^{\frac{\nu_I - 1}{\nu_I}} \right]^{\frac{\nu_I}{\nu_I - 1}}, \tag{A.20}$$

where $I_{s,t}$ is the purchase of investment goods from sector s, $\omega_{I,s}$ denotes the weight of good s in the investment bundle, such that $\sum_{s=1}^{S} \omega_{I,s} = 1$, and ν_I is the elasticity of substitution of investment across sectors. This aggregator implies that the price of the investment bundle is

$$P_{I,t} = \left[\sum_{s=1}^{S} \omega_{I,s} P_{s,t}^{1-\nu_I} \right]^{\frac{1}{1-\nu_I}}. \tag{A.21}$$

The optimal amount of investment goods to be purchased from the wholesalers of each sectors is

$$I_{s,t} = \omega_{I,s} \left(\frac{P_{s,t}}{P_{I,t}} \right)^{-\nu_I} I_t, \qquad s = 1, \dots, S, \tag{A.22}$$

which is derived as the first-order condition of the problem of the investment-good retailer:

$$\max_{I_{s,t}} P_{I,t} I_t - \sum_{s=1}^{S} P_{s,t} I_{s,t}$$

$$s.t. \qquad I_t = \left[\sum_{s=1}^{S} \omega_{I,s}^{\frac{1}{\nu_I}} I_{s,t}^{\frac{\nu_I - 1}{\nu_I}} \right]^{\frac{\nu_I}{\nu_I - 1}}.$$

A.6 Government-consumption-good retailers

The final government-consumption good is assembled by perfectly competitive government-consumption-good retailers according to the following Cobb-Douglas technology:

$$G_t = \prod_{s=1}^{S} G_{s,t}^{\omega_{G,s}}, \tag{A.23}$$

where $G_{s,t}$ is the purchase of government-consumption goods from sector s, and $\omega_{G,s}$ denotes the weight of good s in the government-consumption bundle, such that $\sum_{s=1}^{S} \omega_{G,s} = 1$. This aggregator implies that the price of the government bundle is

$$P_{G,t} = \prod_{s=1}^{S} \frac{P_{s,t}^{\omega_{G,s}}}{\omega_{G,s}^{\omega_{G,s}}}. \tag{A.24}$$

The optimal amount of government-consumption goods to be purchased from the wholesalers of each sectors is

$$G_{s,t} = \omega_{G,s} \frac{P_{G,t} G_t}{P_{s,t}}, \qquad s = 1, \dots, S, \tag{A.25}$$

which is derived as the first-order condition of the problem of the government-consumption-good retailer:

$$\max_{G_{s,t}} P_{G,t}G_t - \sum_{s=1}^{S} P_{s,t}G_{s,t}$$
$$s.t. \qquad G_t = \prod_{s=1}^{S} G_{s,t}^{\omega_{G,s}}.$$

A.7 Intermediate-input retailers

The final intermediate inputs used by producers of sector s are assembled by perfectly competitive intermediate-inputs retailers according to the following CES technology:

$$H_{s,t} = \left[\sum_{x=1}^{S} \omega_{H,s,x}^{\frac{1}{\nu_H}} H_{s,x,t}^{\frac{\nu_H-1}{\nu_H}}\right]^{\frac{\nu_H}{\nu_H-1}}, \tag{A.26}$$

where $H_{s,x,t}$ is the purchase of intermediate goods from sector x, $\omega_{H,s,x}$ denotes the weight of good x in the bundle of intermediate inputs used by producers in sector s, such that $\sum_{x=1}^{S} \omega_{H,s,x} = 1$, and ν_H is the elasticity of substitution of intermediate inputs across sectors. This aggregator implies that the price of the intermediate-input bundle is

$$P_{H,s,t} = \left[\sum_{x=1}^{S} \omega_{H,s,x} P_{x,t}^{1-\nu_H}\right]^{\frac{1}{1-\nu_H}}. \tag{A.27}$$

The optimal amount of goods to be purchased from the wholesalers of each sector x for the production of intermediate inputs used by sector s is

$$H_{s,x,t} = \omega_{H,s,x} \left(\frac{P_{x,t}}{P_{H,s,t}}\right)^{-\nu_H} H_{s,t}, \qquad s, x = 1, \ldots, S, \tag{A.28}$$

which is derived as the first-order condition of the problem of the intermediate-input retailer of sector s:

$$\max_{H_{s,x,t}} P_{H,s,t}H_{s,t} - \sum_{x=1}^{S} P_{x,t}H_{s,x,t}$$
$$s.t. \qquad H_{s,t} = \left[\sum_{x=1}^{S} \omega_{H,s,x}^{\frac{1}{\nu_H}} H_{s,x,t}^{\frac{\nu_H-1}{\nu_H}}\right]^{\frac{\nu_H}{\nu_H-1}}.$$

A.8 Government

The government consists of a fiscal authority and a monetary authority. The fiscal authority purchases government goods, G_t, at price $P_{G,t}$ from the government-consumption-good retailers. Government spending is determined by the process

$$\log G_t = (1-\rho)\log G + \rho \log G_{t-1} + \epsilon_t, \tag{A.29}$$

where G defines the steady-state amount of government spending,[1] ρ measures the persistence of the process, and the government spending shock, u_t, is a zero-mean normally distributed innovation. Government purchases are financed through lump-sum taxes paid by households, which implies the following budget constraint for the government:

$$P_{G,t}G_t = T_t. \tag{A.30}$$

The monetary authority sets the nominal interest rate according to the Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\varphi_R}\left[\left(\frac{Y_t}{Y_t^{\text{flex}}}\right)^{\varphi_Y}(1+\pi_t)^{\varphi_\Pi}\right]^{1-\varphi_R}, \tag{A.31}$$

where R is the steady-state nominal interest rate, φ_R captures the amount of interest-rate smoothing, Y_t is the real aggregate value added, Y_t^{flex} is the real aggregate value added of a counterfactual flexible-price economy, φ_Y and φ_Π denote the degree to which the nominal interest rate responds to changes in the output gap, $\frac{Y_t}{Y_t^{\text{flex}}}$, and aggregate inflation, $\pi_t = \frac{P_t}{P_{t-1}} - 1$, where P_t is the GDP deflator.

A.9 Aggregation

We denote the nominal value added of producer j in sector s as $Y_{s,t}^j$, which is defined as the nominal value of gross output net of the nominal value of intermediate inputs,

$$\mathcal{Y}_{s,t}^j = P_{s,t}^j Z_{s,t}^j - P_{H,s,t}H_{s,t}^j. \tag{A.32}$$

The nominal value added of sector s sums the nominal value added of all producers, that is

$$\mathcal{Y}_{s,t} = \int_0^1 \mathcal{Y}_{s,t}^j dj = P_{s,t}Z_{s,t} - P_{H,s,t}H_{s,t}. \tag{A.33}$$

Summing nominal dividends across producers within sectors and then across sectors, to then substitute dividends into households' budget constraint, yields the definition of nominal aggregate value added:

$$\mathcal{Y}_t = \sum_{s=1}^{S}\mathcal{Y}_{s,t} = P_{C,t}C_t + P_{I,t}I_t + P_{G,t}G_t. \tag{A.34}$$

We define real aggregate value added as the ratio between nominal aggregate value added and the GDP deflator:

$$Y_t = \frac{\mathcal{Y}_t}{P_t}. \tag{A.35}$$

[1] Throughout the text, variables without a time subscript denote steady-state values.

Finally, we define analogously real sectoral value added:

$$Y_{s,t} = \frac{\mathcal{Y}_{s,t}}{P_t}, \quad s = 1, \ldots, S. \tag{A.36}$$

B More on the Calibration of the Model

This section presents further information on the calibration of the model. Tables B.1–B.3 report the list of the 57 production sectors we consider. This level of disaggregation roughly corresponds to the three-digit level of the NAICS codes. Notice that we have excluded all the financial sectors. Table B.4 shows the values of the parameters that are common to all sectors. We also report the target or the source that disciplines our calibration choice. The tables reporting the parameters that vary across sectors are available upon request.

Table B.1: Sectors 1-20.

1	Farms
2	Forestry, fishing, and related activities
3	Mining
4	Utilities
5	Construction
6	Wood products
7	Nonmetallic mineral products
8	Primary metals
9	Fabricated metal products
10	Machinery
11	Computer and electronic products
12	Electrical equipment, appliances, and components
13	Motor vehicles, bodies and trailers, and parts
14	Other transportation equipment
15	Furniture and related products
16	Miscellaneous manufacturing
17	Food and beverage and tobacco products
18	Textile mills and textile product mills
19	Apparel and leather and allied products
20	Paper products

Table B.2: Sectors 21-40.

21	Printing and related support activities
22	Petroleum and coal products
23	Chemical products
24	Plastics and rubber products
25	Wholesale trade
26	Motor vehicle and parts dealers
27	Food and beverage stores
28	General merchandise stores
29	Other retail
30	Air transportation
31	Rail transportation
32	Water transportation
33	Truck transportation
34	Transit and ground passenger transportation
35	Pipeline transportation
36	Other transportation and support activities
37	Warehousing and storage
38	Publishing industries, except internet (includes software)
39	Motion picture and sound recording industries
40	Broadcasting and telecommunications

Table B.3: Sectors 41-57.

41	Data processing, internet publishing, and other information services
42	Legal services
43	Computer systems design and related services
44	Miscellaneous professional, scientific, and technical services
45	Management of companies and enterprises
46	Administrative and support services
47	Waste management and remediation services
48	Educational services
49	Ambulatory health care services
50	Hospitals
51	Nursing and residential care facilities
52	Social assistance
53	Performing arts, spectator sports, museums, and related activities
54	Amusements, gambling, and recreation industries
55	Accommodation
56	Food services and drinking places
57	Other services, except government

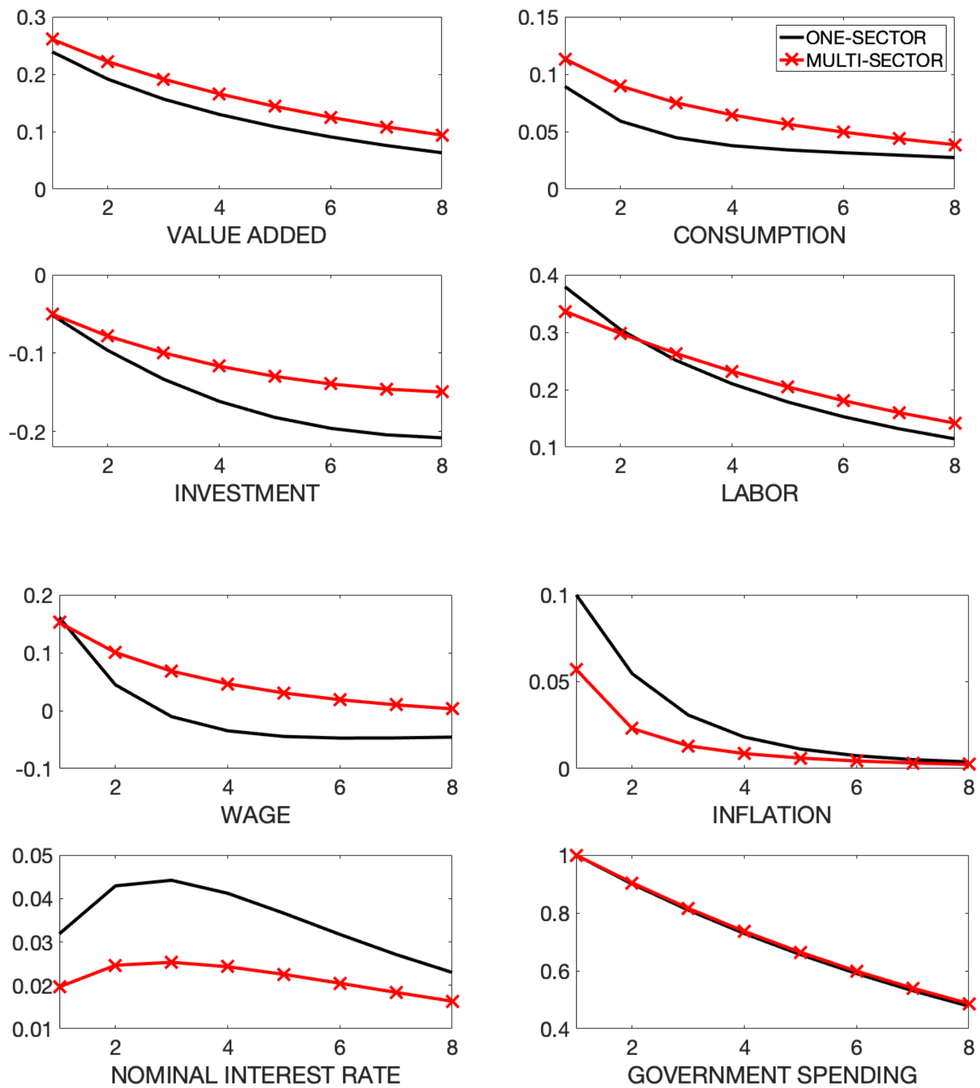
Table B.4: Calibration of Economy-Wide Parameters.

Parameter	Target/Source
$\beta = .995$	2 percent steady-state annual interest rate R
$\sigma = 2$	Standard value
$\theta = 41.01$	0.33 Steady-state total hours N
$\eta = 1.25$	Frisch elasticity = 0.8
$\mu = 0.3$	Bouakez and Rebei (2007), Sims and Wolff (2018)
$\zeta = 0.7$	Ratio of nominal value of consumption expenditures over the sum of consumption and government expenditures
$\delta = 0.025$	10 percent annual depreciation rate
$\Omega = 20$	8 quarters peak response of investment
$\nu_C = 2$	Hobijn and Nechio (2019)
$\nu_I = 2$	$\nu_C = \nu_I$
$\nu_H = 0.1$	Barrot and Sauvagnat (2016), Atalay (2017), Boehm, Flaaen and Pandalai-Nayar (2019)
$\nu_N = 1$	Horvath (2000)
$\nu_K = 1$	$\nu_K = \nu_N$
$\epsilon = 4$	33 percent steady-state markup
$\varphi_R = 0.8$	Clarida, Gali and Gertler (2000)
$\varphi_\Pi = 1.5$	Clarida, Gali and Gertler (2000)
$\varphi_Y = 0.2$	Clarida, Gali and Gertler (2000)
$\rho = 0.9$	Standard value

C Impulse-Response Functions

Figure C.1 reports the impulse responses of key aggregate variables in the one-sector and multi-sector economies to an aggregate government spending shock.

Figure C.1: Impulse-Response Functions: One-Sector vs. Multi-Sector Economy.



Note: The graph reports the responses of aggregate value added, aggregate consumption, aggregate investment, aggregate employment, the aggregate wage, aggregate inflation, the nominal interest rate, and aggregate government spending to a 1 percent aggregate government spending shock. The continuous black line denotes the responses implied by the one-sector economy, whereas the red crossed line denotes the responses implied by the multi-sector economy.

D More on the Analytical Results

This appendix shows the derivation of the simplified model employed in Section 4.4. After assuming (i)-(ix), the following set of equations summarizes the key aggregators, preferences, technological and budget constraints:

$$U(C_t, N_t) = \ln C_t - \theta \frac{N_t^{1+\eta}}{1+\eta}, \tag{D.37}$$

$$P_{C,t}C_t + B_{t+1} + T_t = W_t N_t + B_t R_{t-1} + D_t, \tag{D.38}$$

$$C_t = \prod_{s=1}^{S} C_{s,t}^{\omega_{C,s}}, \tag{D.39}$$

$$P_{C,t} = \prod_{s=1}^{S} \frac{P_{s,t}^{\omega_{C,s}}}{\omega_{C,s}^{\omega_{C,s}}}, \tag{D.40}$$

$$G_t = \prod_{s=1}^{S} G_{s,t}^{\omega_{G,s}}, \tag{D.41}$$

$$P_{G,t} = \prod_{s=1}^{S} \frac{P_{s,t}^{\omega_{G,s}}}{\omega_{G,s}^{\omega_{G,s}}}, \tag{D.42}$$

$$H_{s,t} = \prod_{x=1}^{S} H_{s,x,t}^{\omega_{H,s,x}}, \tag{D.43}$$

$$P_{H,s,t} = \prod_{x=1}^{S} \frac{P_{x,t}^{\omega_{H,s,x}}}{\omega_{H,s,x}^{\omega_{H,s,x}}}, \tag{D.44}$$

$$N_t = \left[\sum_{s=1}^{S} \omega_{N,s}^{-\frac{1}{\nu_N}} N_{s,t}^{\frac{1+\nu_N}{\nu_N}}\right]^{\frac{\nu_N}{1+\nu_N}}, \tag{D.45}$$

$$W_t = \left[\sum_{s=1}^{S} \omega_{N,s} W_{s,t}^{1+\nu_N}\right]^{\frac{1}{1+\nu_N}}, \tag{D.46}$$

$$Z_{s,t}^j = N_{s,t}^{j\ 1-\alpha_H} H_{s,t}^{j\ \alpha_H}, \tag{D.47}$$

$$Z_{s,t} = \left[\int_0^1 Z_{s,t}^{j\ \frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{D.48}$$

$$Z_{s,t} = G_{s,t} + C_{s,t} + \sum_{x=1}^{S} H_{x,s,t}, \tag{D.49}$$

$$T_t = P_{G,t} G_t, \tag{D.50}$$

$$R_t = \left(\frac{1+\pi_t}{1+\pi}\right)^{\varphi_\Pi}, \tag{D.51}$$

$$\frac{G_t}{G} = \left(\frac{G_{t-1}}{G}\right)^{\rho} \exp(\epsilon_t). \tag{D.52}$$

In this environment, the consumption price, the price of the government spending bundle, and the numeraire of the economy (i.e., the GDP deflator) coincide, that is, $P_{C,t} = P_{G,t} = P_t = 1$. Throughout this section, we define the relative price of the final sectoral goods in terms of the numeraire as $Q_{s,t} = \frac{P_{s,t}}{P_{C,t}}$, and the relative price of sectoral intermediate inputs as $Q_{H,s,t} = \frac{P_{H,s,t}}{P_{C,t}}$. Our set of assumptions implies that $Q_{s,t} = Q_{H,s,t},\ \forall s$. Finally, the aggregate inflation rate can be defined as a weighted average of sectoral inflation rates, that is, $\pi_t = \prod_{s=1}^{S} \pi_{s,t}^{\omega_{C,s}} = \prod_{s=1}^{S} \pi_{s,t}^{1/S}$.

D.1 Log-linear Economy

We log-linearize the analytical framework by taking a first-order approximation of the equilibrium conditions around the steady state. This subsection deals with the derivation of the log-linear setting employed to analyze the amplification of an aggregate shock to fiscal spending. Throughout this analysis, we denote by v_t the log-deviation of a generic variable V_t from its steady-state value, V.

Log-linearizing the first-order condition for bonds yields

$$c_t = \mathbb{E}_t c_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1}) \,. \tag{D.53}$$

Using the log-linearized Taylor rule in Equation (A.31) to substitute for r_t yields Equation (13) in the main text.

To derive Equation (14), we start combining the (log-linearized) first-order condition for the optimal price and the definition of the sectoral price index to obtained the following sectoral New Keynesian Phillips curve:

$$\pi_{s,t} = \beta \mathbb{E}_t \pi_{s,t+1} + \kappa_s (mc_{s,t} - q_{s,t}), \tag{D.54}$$

where $mc_{s,t}$ denotes the (log-linear) real marginal cost of production in sector s. The latter can be expressed as a linear combination of the sector's real wage (i.e., $w_{s,t} - p_t$) and relative price, $q_{s,t}$:

$$mc_{s,t} = (1 - \alpha_H)(w_{s,t} - p_t) + \alpha_H q_{s,t}. \tag{D.55}$$

Log-linearizing the sectoral resource constraint yields

$$z_{s,t} = \frac{C_s}{Z_s} c_{s,t} + \frac{G_s}{Z_s} g_{s,t} + \frac{H_s}{Z_s} h_{s,t}. \tag{D.56}$$

Using the linearized production function to substitute for $h_{s,t}$, we obtain

$$z_{s,t} = \frac{C_s}{Z_s} c_{s,t} + \frac{G_s}{Z_s} g_{s,t} + \frac{H_s}{Z_s} \left(\frac{1}{\alpha_H} z_{s,t} - \frac{1-\alpha_H}{\alpha_H} n_{s,t} \right). \tag{D.57}$$

By virtue of the production subsidy, the steady-state distortion due to mark-up pricing is neutralized, so that $H_s/Z_s = \alpha_H$. In the steady state, sectoral government spending is assumed to be a fraction $\gamma \in [0,1]$ of sectoral value added, $Y_{s,t}$, so that $G_s/Z_s = \gamma(1-\alpha_H)$ and $C_s/Z_s = (1-\gamma)(1-\alpha_H)$. Thus, Equation (D.57) becomes

$$n_{s,t} = (1-\gamma) c_{s,t} + \gamma g_{s,t}. \tag{D.58}$$

Imposing $g_{s,t} = g_t$, and substituting (in linearized form) the labor-supply equation for sector s (i.e., $n_{s,t} = \nu_N (w_{s,t} - w_t) + n_t$), the aggregate labor-supply equation (i.e., $\eta n_t + c_t = w_t - p_t$), and the demand for good s (i.e., $c_{s,t} = c_t - q_{s,t}$) into Equation (D.55) and, in turn, into the New Keynesian Phillips curve, we obtain

$$\pi_{s,t} = \beta \mathbb{E}_t \pi_{s,t+1} + \kappa_s (1 - \alpha_H) (\Theta q_{s,t} + \Xi c_t + \Psi g_t), \tag{D.59}$$

where

$$\begin{aligned} \Theta &= -\frac{1 - \gamma + \nu_N}{\nu_N} < 0, \\ \Xi &= 1 + \eta (1 - \gamma) > 0, \\ \Psi &= \gamma \eta > 0. \end{aligned}$$

E More on the Difference between Government Spending and Monetary Policy Shocks

As discussed in the main text, our baseline results indicate that input-output linkages play a larger role than heterogeneity in price rigidity in amplifying the spending multiplier. On the other hand, Pasten, Schoenle and Weber (2020) find the opposite result when it comes to the amplification of the aggregate effects of monetary policy shocks; an observation that also holds in an extended version of our model in which the Taylor rule is augmented with a monetary policy shock. Nonetheless, we argue that this outcome, albeit robust across a wide range of modelling assumptions and parameter values, is not general and may not hold in specific regions of the parameter space. To shed further light on this issue, this section clarifies the difference in the transmission of government spending and monetary policy shocks. We show that despite the fact that these shocks are both demand disturbances, they propagate differently and in a way that prevents one from drawing unambiguous conclusions about the relative importance of the two sources of amplification just discussed.

Consider the stylized model presented in Section 4.4 and assume that the monetary policy rule features a monetary policy shock, ϑ_t, that follows an identical process to that governing g_t. For ease of interpretation, assume that positive realizations of the shock correspond to a monetary easing. Abstracting from sectoral heterogeneity in price rigidity, system (16–17) therefore becomes:

$$c_t = \mathbb{E}_t c_{t+1} - \left(\varphi_\Pi \pi_t - \mathbb{E}_t \pi_{t+1} - \vartheta_t\right), \tag{E.60}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left(1 - \alpha_H\right) \left(\Xi c_t + \Psi g_t\right). \tag{E.61}$$

The absence of endogenous state variables implies that the equilibrium allocation satisfies

$$\pi_t = -\frac{1-\rho}{\varphi_\Pi - \rho} c_t + \frac{1}{\varphi_\Pi - \rho} \vartheta_t, \tag{E.62}$$

$$\pi_t = \frac{\kappa \left(1 - \alpha_H\right)}{1 - \beta\rho} \left(\Xi c_t + \Psi g_t\right). \tag{E.63}$$

This system shows that while the government spending shock move the economy along the IS curve, the monetary policy shock shifts this curve. This observation reflects the fact that changes in government purchases affect the real interest rate 'indirectly' via the inflation rate, whereas monetary policy shocks affect it 'directly' through their incidence on the policy rate.

From (E.62)–(E.63), it is easy to show that the consumption response to a monetary policy shock, $\varpi \equiv \frac{dc_t}{d\vartheta_t}$, is given by

$$\varpi = \frac{1 - \beta\rho}{\left(1-\rho\right)\left(1 - \beta\rho\right) + \left(\varphi_\Pi - \rho\right)\left(1 - \alpha_H\right)\kappa\Xi},$$

and that

$$\frac{\partial \varpi}{\partial \alpha_H} = \frac{\left(1 - \beta\rho\right)\kappa\Xi}{\left[\left(1-\rho\right)\left(1 - \beta\rho\right) + \left(\varphi_\Pi - \rho\right)\left(1 - \alpha_H\right)\kappa\Xi\right]^2}.$$

Now consider the special case of an infinite Frisch elasticity of labor supply ($\eta = 0$). We know from the analysis in Section 4.4 that, in this case, the consumption response to a spending shock, ξ, is nil and is independent of the intensity of intermediate inputs ($\frac{\partial \xi}{\partial \alpha_H} = 0$). On the other hand, as long as prices are not fully flexible, consumption still responds to a monetary policy shock and the presence of intermediate inputs still amplifies this response ($\frac{\partial \varpi}{\partial \alpha_H} > 0$) even if the Frisch elasticity is infinite. This in turn implies that, once we extend the model to allow for heterogeneity in price rigidity, this feature will tend to account for most of the amplification of the spending multiplier as η tends towards very low values. Whether input-output interactions explain relatively more or less of the amplification of the spending multiplier than heterogeneity in price rigidity is therefore ultimately a quantitative question.

F More on the Sensitivity Analysis

This section reports further quantitative results about the government spending multiplier in a variety of alternative specifications of the baseline economy.

Tables F.1–F.4 decompose the sources of amplification of the aggregate value multiplier in the multi-sector economy when: (i) we vary the values of the Taylor-rule parameters φ_Π and φ_R, and consider alternative monetary policy rules that correspond to strict inflation targeting, price-level targeting, and nominal-GDP targeting, as well as a variant in which the output gap is replaced with output growth; (ii) we consider i.i.d., moderately persistent, and highly persistent spending shocks.

Tables F.5–F.6 compare the spending multiplier in the one-sector and multi-sector economies in the following cases: (i) we abstract from the complementarity between private and public consumption; (ii) we assume that additional government spending (in excess of its steady-state level) is financed through distortionary labor-income taxes, instead of lump-sum taxes; (iii) we consider a model with sticky wages à la Erceg, Henderson and Levin (2000), in which differentiated labor-service varieties are supplied monopolistically by households to unions; (iv) we consider alternative values of the parameters ν_I and ν_k; (v) we alter the way in which we calibrate price stickiness in the one-sector model based on the duration of sectoral prices.

Table F.1: Sources of Amplification of the Aggregate Value-Added Multiplier - Sensitivity.

Multi-Sector Economy	Counterfactual Multi-Sector Economies				
	Excluding Input-Output Matrix	Excluding Heterogeneity in Price Rigidity	Excluding Heterogeneity in Consumption & Investment Shares	Excluding Heterogeneity in Factor Intensities	Excluding off-Diagonal Elements of I-O Matrix
Taylor Rule Parameter $\varphi_{\Pi} = 15$					
Panel A: Aggregate Value-Added Multiplier					
0.4806	0.1300	0.4405	0.7112	0.5311	0.6363
Panel B: Marginal Contribution of the Excluded Feature					
-	73.0%	8.3%	-48.0%	-10.5%	-32.4%
Taylor Rule Parameter $\varphi_{R} = 0$					
Panel A: Aggregate Value-Added Multiplier					
0.8871	0.3352	0.6086	1.2416	0.9253	0.9757
Panel B: Marginal Contribution of the Excluded Feature					
-	62.2%	31.4%	-40.0%	-4.3%	-10.0%
Taylor Rule Parameter $\varphi_{R} = 0.4$					
Panel A: Aggregate Value-Added Multiplier					
0.8428	0.2999	0.5910	1.1759	0.8856	0.9426
Panel B: Marginal Contribution of the Excluded Feature					
-	64.4%	29.9%	-39.5%	-5.1%	-11.8%

Table F.2: Sources of Amplification of the Aggregate Value-Added Multiplier - Sensitivity.

Multi-Sector Economy	Counterfactual Multi-Sector Economies				
	Excluding Input-Output Matrix	Excluding Heterogeneity in Price Rigidity	Excluding Heterogeneity in Consumption & Investment Shares	Excluding Heterogeneity in Factor Intensities	Excluding off-Diagonal Elements of I-O Matrix
Strict Inflation Targeting ($\varphi_Y = 0$ and $\varphi_R = 0$)					
Panel A: Aggregate Value-Added Multiplier					
1.2020	0.3364	0.6357	1.5969	1.2339	1.1896
Panel B: Marginal Contribution of the Excluded Feature					
-	72.0%	47.1%	-32.9%	-2.7%	-1.0%
Price Level Targeting					
Panel A: Aggregate Value-Added Multiplier					
0.4635	0.1229	0.4437	0.5003	0.5166	0.6234
Panel B: Marginal Contribution of the Excluded Feature					
-	73.5%	4.3%	-7.9%	-11.4%	-34.5%

Table F.3: Sources of Amplification of the Aggregate Value-Added Multiplier - Sensitivity.

Multi-Sector Economy	Counterfactual Multi-Sector Economies				
	Excluding Input-Output Matrix	Excluding Heterogeneity in Price Rigidity	Excluding Heterogeneity in Consumption & Investment Shares	Excluding Heterogeneity in Factor Intensities	Excluding off-Diagonal Elements of I-O Matrix
Nominal GDP Targeting					
Panel A: Aggregate Value-Added Multiplier					
0.5618	0.1121	0.6687	0.5313	0.6409	0.7569
Panel B: Marginal Contribution of the Excluded Feature					
-	80.0%	-19.0%	5.4%	-14.1%	-34.7%
Output Growth in Taylor Rule					
Panel A: Aggregate Value-Added Multiplier					
0.9641	0.2500	0.5992	1.1951	1.0132	1.0264
Panel B: Marginal Contribution of the Excluded Feature					
-	74.1%	37.8%	-24.0%	-5.1%	-6.5%

Table F.4: Sources of Amplification of the Aggregate Value-Added Multiplier - Sensitivity.

Multi-Sector Economy	Counterfactual Multi-Sector Economies				
	Excluding Input-Output Matrix	Excluding Heterogeneity in Price Rigidity	Excluding Heterogeneity in Consumption & Investment Shares	Excluding Heterogeneity in Factor Intensities	Excluding off-Diagonal Elements of I-O Matrix
Autocorrelation of Government Spending $\rho = 0$					
Panel A: Aggregate Value-Added Multiplier					
1.3856	0.7286	1.3357	1.8712	1.4151	1.4509
Panel B: Marginal Contribution of the Excluded Feature					
-	47.4%	3.6%	-35.1%	-2.1%	-4.7%
Autocorrelation of Government Spending $\rho = 0.45$					
Panel A: Aggregate Value-Added Multiplier					
1.2644	0.6041	1.1497	1.7096	1.2996	1.3734
Panel B: Marginal Contribution of the Excluded Feature					
-	52.2%	9.1%	-35.2%	-2.8%	-8.6%
Autocorrelation of Government Spending $\rho = 0.95$					
Panel A: Aggregate Value-Added Multiplier					
0.9394	0.5000	0.8395	1.2972	0.9921	1.0198
Panel B: Marginal Contribution of the Excluded Feature					
-	46.8%	10.6%	-38.1%	-5.6%	-8.6%

Table F.5: Value-Added Spending Multipliers - Multi-Sector vs. One Sector - Additional Sensitivity.

Average One-Sector Economy	Multi-Sector Economy	Δ %	Δ \$
(1)	(2)	(3)	(4)
Panel A: No Complementarity between C_t and G_t			
0.2204	0.3518	+59.6%	0.1314
Panel B: Distortionary Labor-Income Taxation			
-0.2465	0.0804	+132.6%	0.3270
Panel C: Sticky Wages			
0.5367	0.8007	+49.2%	0.2640
Panel D: Flexible Prices			
0.3363	0.4057	+20.7%	0.0694
Panel E: Elasticity of Substitution of Capital across Sectors $\nu_K = 0.1$			
0.4247	0.7599	+78.9%	0.3352
Panel F: Elasticity of Substitution of Investment across Sectors $\nu_I = 4$			
0.4247	0.7396	+74.2%	0.3149
Panel G: $\nu_K = 0.1$ and $\nu_I = 4$			
0.4247	0.7550	+77.8%	0.3304

Note: The table reports the aggregate value-added multipliers implied by a version of the model without complementarity between consumption and government spending (Panel A), a version of the model in which additional government spending is financed with distortionary labor-income taxes (Panel B), a version of the model with sticky wages (Panel C), a version of the model with flexible prices (Panel D), a version of the model in which the elasticity of substitution of capital across sectors is set to $\nu_K = 0.1$ (Panel E), a version of the model in which the elasticity of substitution of investment across sectors is set to $\nu_I = 4$ (Panel F), and a version of the model in which the elasticity of substitution of capital across sectors is set to $\nu_K = 0.1$ and the elasticity of substitution of investment across sectors is set to $\nu_I = 4$ (Panel G). Column (1) reports the multipliers implied by one-sector models in each of these economies, Column (2) reports the multipliers implied by the multi-sector model, Columns (3) and (4) report the amplification of the multiplier in the multi-sector model vis-à-vis the one-sector model in percentage terms and in absolute values, respectively.

Table F.6: Value-Added Spending Multipliers - Multi-Sector vs. One Sector - Additional Sensitivity.

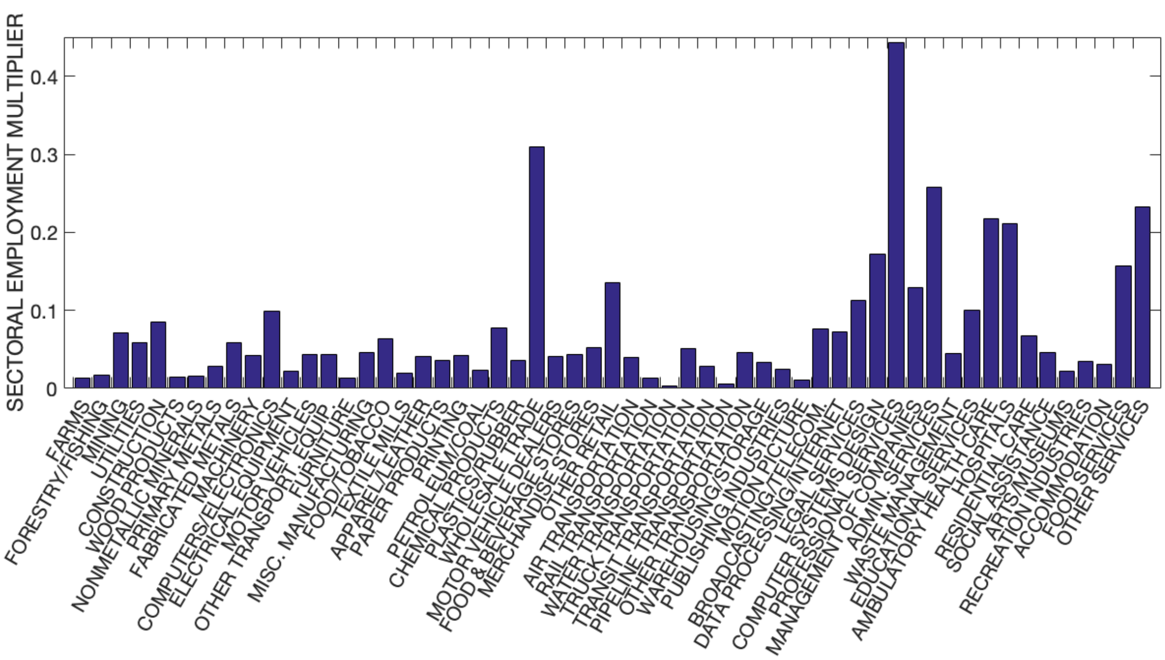
Average One-Sector Economy	Multi-Sector Economy	Δ %	Δ \$
(1)	(2)	(3)	(4)
Panel A: Median Price Duration			
0.4728	0.7444	+57.5%	0.2717
Panel B: Weighted Mean Price Duration			
0.4570	0.7444	+62.9%	0.2874
Panel C: Weighted Median Price Duration			
0.4889	0.7444	+52.3%	0.2555
Panel D: 12 Month Price Duration (in both economies)			
0.4638	0.7920	+70.8%	0.3283

Note: The table reports the aggregate value-added multipliers implied by versions of the one-sector economy in which the degree of price rigidity is calibrated to match the median price duration across sectors (Panel A), the weighted mean price duration across sectors (Panel B), the weighted median price duration across sectors (Panel C). Panel D reports the multipliers implied by versions of the one-sector and multi-sector economies in which the price duration is set to 12 months. Columns (3) and (4) report the amplification of the multiplier in the multi-sector model vis-à-vis the one-sector model in percentage terms and in absolute values, respectively.

G More on the Sectoral Implications

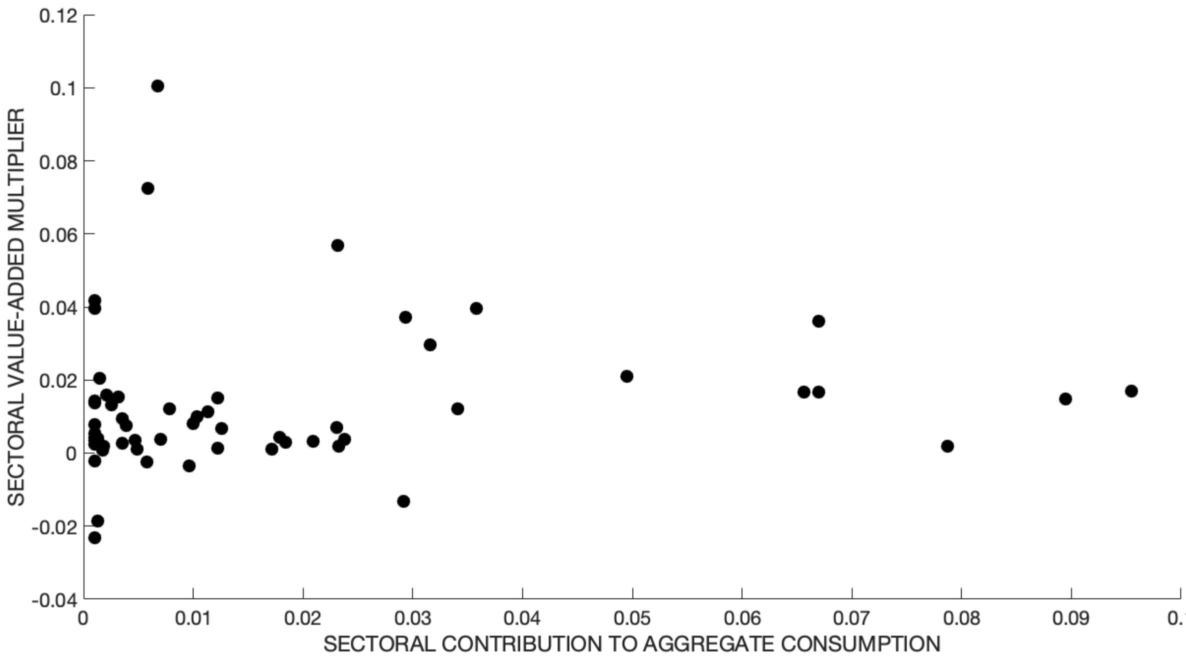
This section reports further details on the sectoral implications of a government spending shock. First, Figure G.1 reports the sectoral employment multipliers. Then, we show that the dispersion in the sectoral government spending multipliers does not correlate with heterogeneity across industries in their contribution to final consumption (i.e., variation in $\omega_{C,s}$ - see Figure G.2), contribution to final investment (i.e., variation in $\omega_{I,s}$ - see Figure G.3), heterogeneity in the value-added-based labor intensity (i.e., variation in $\alpha_{N,s}$ - see Figure G.4), and heterogeneity in the degree of price rigidity (i.e., variation in ϕ_s - see Figure G.5).

Figure G.1: The Response of Sectoral Employment.



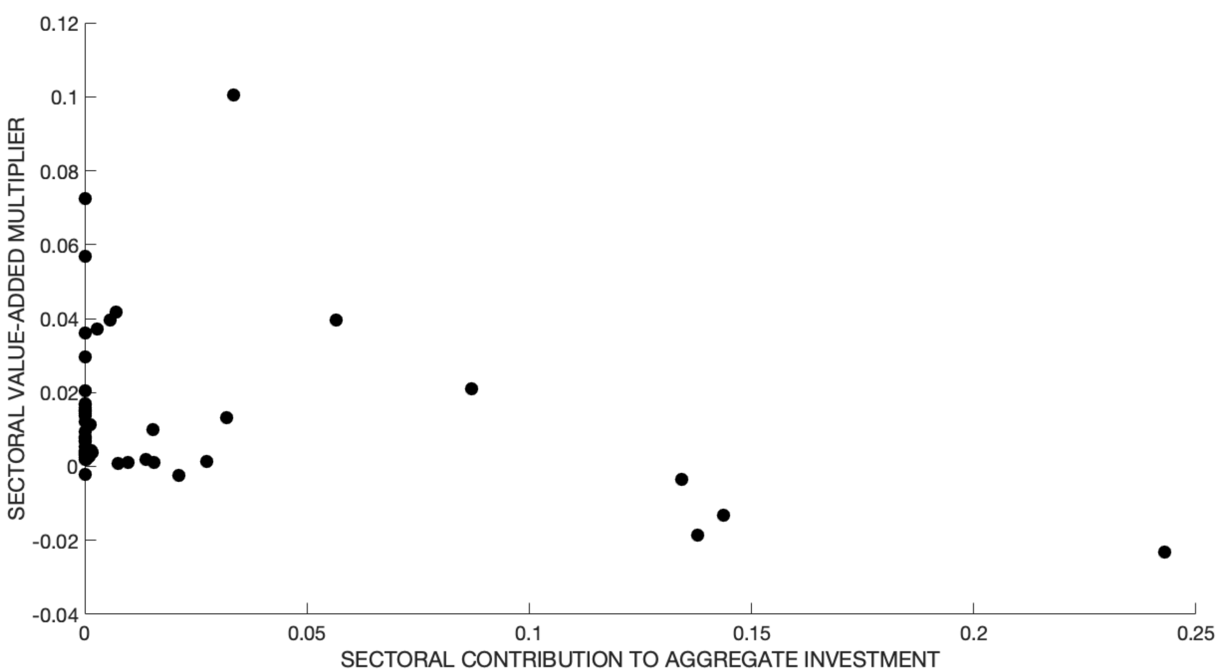
Note: The graph reports the employment multiplier for each of the 57 sectors.

Figure G.2: The Response of Sectoral Value Added and Sectoral Contribution to Consumption.



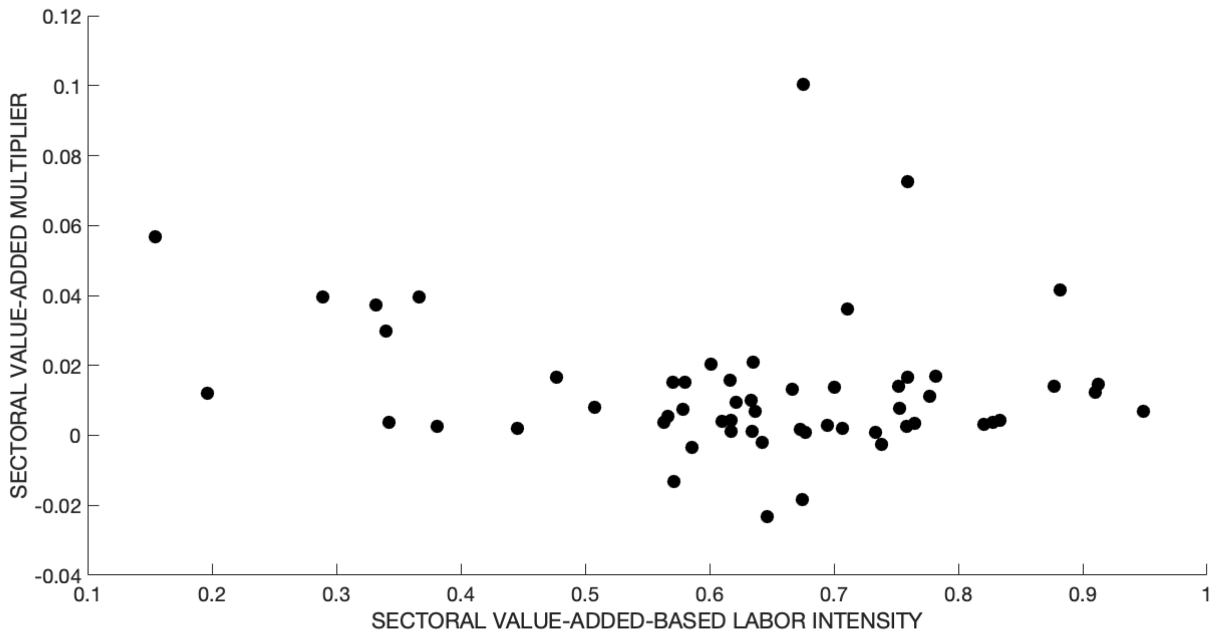
Note: The graph reports a scatter that links the sectoral value-added multiplier of each sector (measured on the y-axis) to its contribution to aggregate consumption $\omega_{C,s}$ (measured on the x-axis).

Figure G.3: The Response of Sectoral Value Added and Sectoral Contribution to Investment.



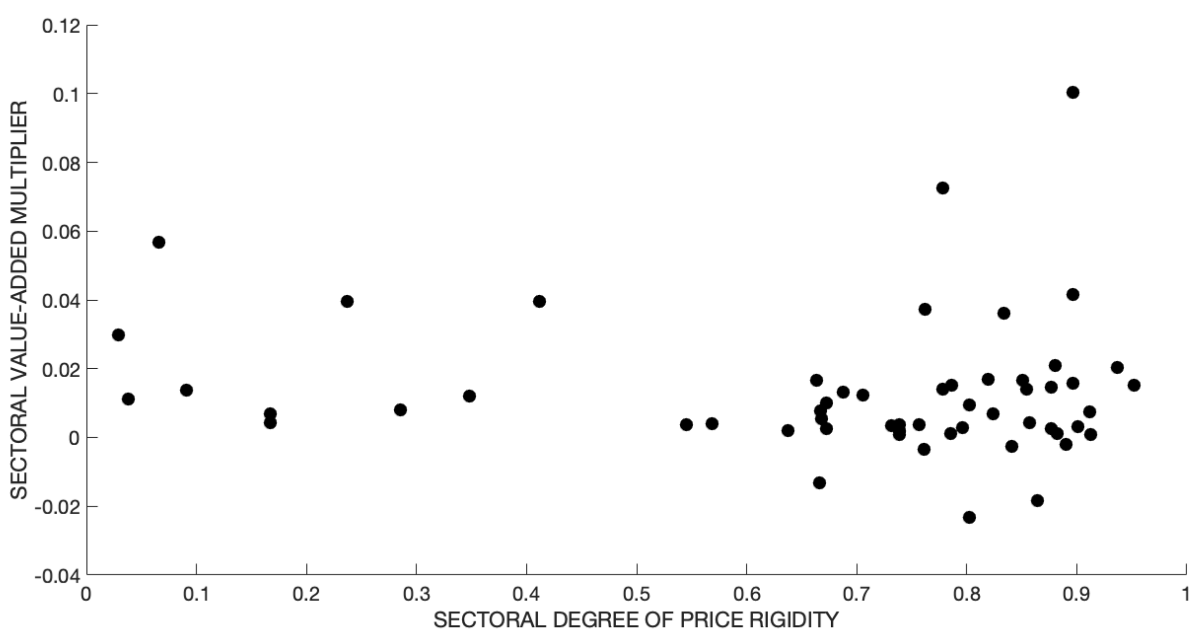
Note: The graph reports a scatter that links the sectoral value-added multiplier of each sector (measured on the y-axis) to its contribution to aggregate investment $\omega_{I,s}$ (measured on the x-axis).

Figure G.4: The Response of Sectoral Value Added and Sectoral Value-Added-Based Labor Intensity.



Note: The graph reports a scatter that links the sectoral value-added multiplier of each sector (measured on the y-axis) to its value-added-based labor intensity $\alpha_{N,s}$ (measured on the x-axis).

Figure G.5: The Response of Sectoral Value Added and Sectoral Price Rigidity.



Note: The graph reports a scatter that links the sectoral value-added multiplier of each sector (measured on the y-axis) to its degree of price rigidity ϕ_s (measured on the x-axis).

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