Firm Entry and Exit and Aggregate Growth

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Online Appendix

Online Appendix 1: Fixed Costs Denominated in Terms of Labor

In this section, we change our baseline model so that the fixed costs are denominated in terms of labor. We then calibrate the new model and redo the quantitative exercise in Section VI.

1.1. Model

All the equations that characterize the model in Section II remain the same except for the equations that we now characterize. Potential entrants pay an entry cost, κ, which is denominated in units of labor, to draw a marginal efficiency, x. The condition in (9) becomes

$$d_t(x) = \max_{\ell} x\ell_t(x)^{\alpha} - w_t\ell_t(x) - w_t f, \tag{1}$$

where f is the fixed continuation cost paid in units of labor. The mass of potential entrants, μ_t, characterized by (16), becomes

$$E_x\left[V_t(x)\right] = w_t\kappa. \tag{2}$$

The labor market condition, characterized by (20), becomes

$$1 = \sum_{j=1}^{\infty}\left[\mu_{t+j-1}\left(1-\delta\right)^{j-1}\int_{\hat{x}_{jt}}^{\infty}\ell_t(x)dF_{t+j-1}\left(x/\tilde{g}_{jt}\right)\right] + \eta_t f + \mu_t\kappa. \tag{3}$$

The goods market clearing condition, characterized by (21), becomes

$$C_t = Y_t = \sum_{j=1}^{\infty}\left[\mu_{t+j-1}\left(1-\delta\right)^{j-1}\int_{\hat{x}_{jt}}^{\infty}x\ell_t\left(x\right)^{\alpha}dF_{t-j+1}\left(x/\tilde{g}_{jt}\right)\right]. \tag{4}$$

Aggregate dividends, characterized by (22), become

$$D_t = \sum_{j=1}^{\infty}\left[\mu_{t+j-1}\left(1-\delta\right)^{j-1}\int_{\hat{x}_{jt}}^{\infty}d_t(x)dF_{t-j+1}\left(x/\tilde{g}_{jt}\right)\right] - \mu_t w_t\kappa. \tag{5}$$

1.2. Balanced Growth Path

Under the new formulation, we can show a proposition similar to Proposition 1 in which (23) is satisfied, along with $q_{t+1} = \beta / g_e$, $\mu_t = \mu$, $\eta_t = \eta$. The equations that characterize the balanced growth path become

$$\hat{x}_t = \frac{g_e^t}{\varphi}\left(\frac{\omega\mu}{\eta}\right)^{\frac{1}{\gamma}}, \tag{6}$$

$$w_t = \alpha\left(\frac{1-\alpha}{\alpha f}\right)^{1-\alpha} \hat{x}_t, \tag{7}$$

$$Y_t = \psi\alpha\left(\frac{1-\alpha}{\alpha f}\right)^{1-\alpha} \hat{x}_t, \tag{8}$$

$$\mu = \frac{\xi}{\gamma\kappa\omega}\psi, \tag{9}$$

$$\eta = \frac{\gamma(1-\alpha)-1}{\gamma f}\psi, \tag{10}$$

$$\psi = \frac{\gamma\omega}{(\gamma-1)\omega+\xi}. \tag{11}$$

As before, the efficiency cutoffs, real wages, and output grow at rate $g_e - 1$. The mass of potential entrants, μ, and the mass of operating firms, η, are also constant on the balanced growth path.

1.3. Measurement

As in our baseline, we treat fixed costs as investment. When a firm enters at time t, its nominal investment is $w_t(f+\kappa)$. The firm's real investment is $I_t = w_0 g_e^t (f+\kappa)$, where w_0 is the base year wages, and g_e^t reflects improvements in the quality of capital. After a firm enters, nominal investment each period is the fixed continuation cost $w_t f$, and its real investment is $I_t = w_0 g_e^t f$. Real capital, k_t, evolves as follows:

$$k_{t+1} = (1-\tilde{\delta}_{kt})k_t + I_{t+1}, \tag{12}$$

where $\tilde{\delta}_{kt}$ is the depreciation rate and I_{t+1} is real investment at time $t+1$. When a firm enters at time t, its real capital stock is $k_t = w_0 g_e^t (f+\kappa)$. If the depreciation rate is

$$\tilde{\delta}_{kt} = 1 - \frac{g_e \kappa}{f+\kappa}, \tag{13}$$

then the subsequent capital stock is

$$k_t = w_0 g_e^t (f+\kappa), \tag{14}$$

for all t after entry. As in the baseline model, the fact that all firms have the same fixed capital stock generates a clean relationship between efficiency and productivity.

1.4. Quantitative Exercises

To calibrate the model, we follow a strategy similar to the one we used before. The calibrated parameters are reported in Table A13. One thing to note is that firm size is measured in units of variable labor, as in the baseline calibration.

Table A1: Calibrated parameters

Parameter		Value	Target
Operating cost (technological)	f^T	0.69×5	Average U.S. plant size: 14.0
Entry cost (technological)	κ^T	0.57	Entry cost/continuation cost: 0.82
Tail parameter	γ	6.10	S.D. of U.S. plant size: 89.0
Entrant efficiency growth	g_e	1.02^5	BGP growth rate of U.S.: 2 percent
Returns to scale	α	0.67	BGP labor share of U.S.: 0.67
Death rate	δ	$1-0.960^5$	Exiting plant employment share of U.S.: 19.3 percent
Discount factor	β	0.98^5	Real interest rate of U.S.: 4 percent
Firm growth	g_c	1.019^5	Effect of entry and exit on U.S. manufacturing productivity growth: 25 percent
Spillover term	ε	0.64	Authors' estimate

As before, we create distorted economies to study the transition dynamics when we remove distortions. We report only the results for the quantitative exercise that involve lowering entry costs, since reducing barriers to technology adoption yields similar results. In the distorted economy, we raise the entry cost to 1.53 so that income is 15 percent lower on the balanced growth path. We report the results of the reforms to entry costs in Table A14. We find that as was the case before, GDP and productivity growth rates increase immediately after the reform. We also see an increase in the contribution of net entry. The results are quantitatively similar to our baseline.

Table A2: Productivity decompositions (entry cost reform)

Model periods (five years)	Entry cost	Real GDP growth (percent, annualized)	Aggregate productivity growth (percent, annualized)	Contribution of net entry to aggregate productivity (percent)
0–3	1.53	2.0	1.3	25.0
4 (reform)	0.57	4.7	4.3	56.5
5	0. 57	2.4	1.3	34.0
6	0. 57	2.2	1.3	28.0
7+	0. 57	2.0	1.3	24.9

Online Appendix 2: Model with a Composite of Variable Labor and Capital

In this section, we recalibrate the model presented in Online Appendix 1, in which fixed costs are denominated in terms of labor. We reinterpret labor in the model to be equipped labor. Equipped labor is created by a bundler that uses a Cobb-Douglas technology to combine variable labor and variable capital, both of which are owned by the household. The bundler operates under perfect competition.

The new interpretation of α is the span-of-control parameter in Lucas's (1978) model. We use a value of $\alpha = 0.85$, which is consistent with Gomes and Kuehn (2017) and Atkeson and Kehoe (2005). Given the new value of α, we recalibrate all other parameters as described in Table A15. Most targets are similar to the baseline calibration. One point to make is that after normalizing the labor and capital endowments, we can calculate the targets that use employment statistics in the model, such as the average plant size. We also set the coefficient of variable labor in the bundler's Cobb-Douglas technology so that the model matches a labor share of 0.67.

Table A3: Calibration of model with a composite of variable labor and capital

Parameter		Value	Target
Operating cost (technological)	f^T	0.25×5	Average U.S. plant size: 14.0
Entry cost (technological)	κ^T	0.20	Entry cost/continuation cost: 0.82
Tail parameter	γ	13.42	S.D. of U.S. plant size: 89.0
Entrant efficiency growth	g_e	1.02^5	BGP growth rate of U.S.: 2 percent
Span-of-control	α	0.85	Atkeson and Kehoe (2005)
Death rate	δ	$1-0.963^5$	Exiting plant employment share of U.S.: 19.3 percent
Discount factor	β	0.98^5	Real interest rate of U.S.: 4 percent
Firm growth	g_c	1.019^5	Effect of entry and exit on U.S. manufacturing productivity growth: 25 percent
Spillover term	ε	0.64	Authors' estimates

We create a distorted economy by raising the entry cost so that income on the balanced growth path is 15 percent lower. We then conduct a reform by reducing entry costs and study the subsequent transition to the new balanced growth path. Table A16 summarizes the new results. We find that the results are qualitatively the same as before. The main difference is that the contribution of net entry in the period of reform is higher than in the baseline case (74.0 vs. 59.6). Notice, however, that the model can still account for the positive correlation between GDP growth and the contribution of net entry.

Table A4: Productivity decompositions (entry cost reform)

Model periods (five years)	Entry cost	Real GDP growth (percent, annualized)	Aggregate productivity growth (percent, annualized)	Contribution of net entry to aggregate productivity (percent)
0–3	1.80	2.0	1.7	25.0
4 (reform)	0.20	5.0	4.8	74.0
5	0.20	2.3	1.6	33.5
6	0.20	2.1	1.7	26.6
7+	0.20	2.0	1.7	25.0

Online Appendix 3: Imperfect Competition

In this section, we change our baseline model so that monopolistically competitive firms produce different varieties of goods. The fixed costs are denominated in terms of labor, as in Online Appendix 1. We then calibrate the new model and redo the quantitative exercise in Section VI.

3.1. Model

All the equations that characterize the model in Online Appendix 1 remain the same except for the equations that we now characterize. Perfectly competitive final good firms purchase intermediate goods and assemble them to produce the final good. The representative final good firm solves

$$\begin{aligned}&\min_{\{y_t(\nu)\}} \int_0^{\eta_t} p_t(\nu) y_t(\nu) d\nu \\ &\text{s.t. } \left(\int_0^{\eta_t} y_t(\nu)^{\rho} d\nu\right)^{\frac{1}{\rho}} = Y_t,\end{aligned} \tag{15}$$

where $p_t(\nu)$ and $y_t(\nu)$ are the price and quantity of intermediate good ν and η_t is the measure of intermediate goods firms. The elasticity of substitution between intermediate goods is $1/(1-\rho) > 1$, and Y_t is real aggregate output. Solving the final good firm's problem yields the demand function for the intermediate good ν,

$$y_t(\nu) = Y_t \left(\frac{p_t(\nu)}{P_t}\right)^{\frac{-1}{1-\rho}}, \tag{16}$$

where the price aggregator is given by

$$P_t = \left(\int_0^{\eta_t} p_t(\nu)^{\frac{-\rho}{1-\rho}} d\nu\right)^{\frac{1-\rho}{-\rho}}, \tag{17}$$

which is normalized to 1.

Potential entrants pay an entry cost, κ, which is denominated in units of labor to draw a marginal efficiency, x. Firm ν in the intermediate goods sector has the production function

$$y(\nu) = x(\nu)\ell(\nu). \tag{18}$$

Conditional on existing, firm ν solves

$$d_t(\nu) = \max_{p(\nu)} \left[p(\nu) - \frac{w_t}{x(\nu)} \right] Y_t p_t(\nu)^{\frac{-1}{1-\rho}} - w_t f, \tag{19}$$

where f is the fixed continuation cost paid in units of labor. The solution is the usual markup over marginal cost,

$$p(\nu) = \frac{w_t}{\rho x(\nu)}. \tag{20}$$

As every firm with productivity x chooses the same price, we no longer characterize a good by its name ν but by the productivity x of the firm that produces it.

The goods market clearing condition, characterized by (4), becomes

$$C_t = Y_t = \left[\sum_{j=1}^{\infty} \left[\mu_{t+j-1} (1-\delta)^{j-1} \int_{\hat{x}_{jt}}^{\infty} y_t(x)^{\rho} \, dF_{t-j+1}(x / \tilde{g}_{jt}) \right] \right]^{\frac{1}{\rho}}. \tag{21}$$

3.2. Balanced Growth Path

Under the new formulation, we can show a proposition similar to Proposition 1, in which (23) is satisfied, along with $q_{t+1} = \beta / g_e$, $\mu_t = \mu$, $\eta_t = \eta$. The equations that characterize the balanced growth path become

$$\hat{x}_t = \frac{g_e^t}{\varphi} \left(\frac{\omega\mu}{\eta} \right)^{\frac{1}{\gamma}}, \tag{22}$$

$$w_t = \rho \left(\psi \frac{1-\rho}{f} \right)^{\frac{1-\rho}{\rho}} \hat{x}_t, \tag{23}$$

$$Y_t = \psi\rho \left(\psi \frac{1-\rho}{f} \right)^{\frac{1-\rho}{\rho}} \hat{x}_t, \tag{24}$$

$$\mu = \frac{\xi\rho}{\gamma\kappa\omega} \psi, \tag{25}$$

$$\eta = \frac{\gamma(1-\rho)-\rho}{\gamma f} \psi, \tag{26}$$

$$\psi = \frac{\gamma\omega}{(\gamma-\rho)\omega+\rho\xi}. \tag{27}$$

As before, the efficiency cutoffs, real wages, and output grow at rate $g_e - 1$. The mass of potential entrants, μ, and the mass of operating firms, η, are also constant on the balanced growth path.

3.3. Measurement

While the model with imperfect competition has equilibrium conditions similar to those in our baseline model with perfect competition and decreasing returns to scale, there is a difference in how the model efficiencies map into measured productivities. Measured productivity in the model with imperfect competition, complicated by the fact that higher efficiency firms charge lower prices, is given by

$$\log\left(z_t(x)\right) = \log\left(p_t(x)y_t(x)\right) - \alpha_{\ell t}\log\left(\ell_t(x)\right) - \alpha_{kt}\log\left(k_t\right), \tag{28}$$

where α_ℓ and α_k are calculated as explained in Section IV and k_t is defined as in Appendix F.3. Substituting optimality conditions, we obtain

$$\log\left(z_t(x)\right) = \rho\log x + (1-\rho)\log\left(Y_t\right) - \alpha_{kt}\log\left(\kappa_t + f_t\right), \tag{29}$$

which still has the characteristic that log productivity is an affine transformation of log efficiency.

3.4. Quantitative Exercises

To calibrate the model, we follow a strategy similar to the one we used before, except that we set the elasticity of substitution to 3, as in Asturias et al. (2016). The calibrated parameters are reported in Table A17. Firm size is measured in units of variable labor, as in the baseline calibration and in Online Appendix 1.

Table A5: Calibrated parameters

Parameter		Value	Target
Operating cost (technological)	f^T	0.69×5	Average U.S. plant size: 14.0
Entry cost (technological)	κ^T	0.57	Entry cost/continuation cost: 0.82
Tail parameter	γ	4.09	S.D. of U.S. plant size: 89.0
Entrant efficiency growth	g_e	1.02^5	BGP growth rate of U.S.: 2 percent
Elasticity of substitution	$1/(1-\rho)$	3.00	Asturias et al. (2016)
Death rate	δ	$1-0.960^5$	Exiting plant employment share of U.S.: 19.3 percent
Discount factor	β	0.98^5	Real interest rate of U.S.: 4 percent
Firm growth	g_c	1.019^5	Effect of entry and exit on U.S. manufacturing productivity growth: 25 percent
Spillover term	ε	0.64	Authors' estimate

As we did before, we create distorted economies to study the transition dynamics when we remove distortions. We report only the results for the quantitative exercise that involves lowering entry costs, since reducing barriers to technology adoption yields similar results. In the distorted economy, we raise the entry cost to 1.11 so that income is 15 percent lower on the balanced growth path. In Table A18, we report the results of the reforms to entry costs. We find that as they did before, GDP and productivity growth rates increase immediately after the reform. We also see an increase in the contribution of net entry. The results are quantitatively similar to our baseline.

Table A6: Productivity decompositions (entry cost reform)

Model periods (five years)	Entry cost	Real GDP growth (percent, annualized)	Aggregate productivity growth (percent, annualized)	Contribution of net entry to aggregate productivity (percent)
0–3	1.11	2.0	1.3	25.0
4 (reform)	0.57	4.2	3.7	48.1
5	0. 57	2.8	1.7	29.5
6	0. 57	2.3	1.5	26.6
7+	0. 57	2.0	1.3	25.0

Online Appendix 4: Model with Idiosyncratic Growth Shocks

In this section, we change our baseline model so that incumbent efficiency growth is subject to idiosyncratic shocks. We then calibrate the new model and redo the quantitative exercise in Section VI. Our findings indicate that the two models—the idiosyncratic growth shock model and the baseline model—behave similarly, as long as the average growth rate of continuing firms is calibrated so that entry and exit account for the same proportion of productivity growth.

4.1. Model

The equations characterizing the model in Section II remain the same except for the equations described below. We modify the model so that continuing firms' productivity grows by g_{ht} with probability p and grows by a lower $g_{lt} \le g_{ht}$ with probability $1-p$. This implies that the value of a firm with efficiency x is

$$V_t(x) = \max\left\{d_t(x) + q_{t+1}(1-\delta)\left[pV_{t+1}\left(xg_{h,t+1}\right) + \left(1-p\right)V_{t+1}\left(xg_{l,t+1}\right)\right], 0\right\}. \tag{30}$$

We continue with the formulation in (12), in which firms benefit from spillovers from the rest of the economy. In particular, g_{ht} and g_{lt} are determined by the following equations:

$$g_{ht} = \overline{g}_h g_t^{\varepsilon}, \tag{31}$$

$$g_{lt} = \overline{g}_l g_t^{\varepsilon}, \tag{32}$$

where $\overline{g}_h$ and $\overline{g}_l$ are constants that satisfy $\overline{g}_h \ge \overline{g}_l$, g_t is the growth factor of the unweighted mean efficiency of all firms that operate, and ε measures the degree of spillovers from the aggregate economy as defined in the baseline model.

The mass of firms of age j in operation, η_{jt}, given in equation (17) becomes

$$\eta_{jt} = \mu_{t+1-j}\left(1-\delta\right)^{j-1}\left(1-G_{jt}\left(\hat{x}_{jt}\right)\right), \tag{33}$$

where $G_{jt}\left(x\right)$ characterizes the efficiency distribution of firms of age j at time t and is as follows:

$$1-G_{jt}\left(x\right) = \prod_{k=1}^{j-1}\left[pg_{h,t-k+1}^{\gamma} + \left(1-p\right)g_{l,t-k+1}^{\gamma}\right]\left[1-F_{t-j+1}\left(x\right)\right]. \tag{34}$$

This expression can be obtained by recursively applying

$$1-G_{jt}\left(x\right)=p\left[1-G_{j-1,t-1}\left(\frac{x}{g_{ht}}\right)\right]+\left(1-p\right)\left[1-G_{j-1,t-1}\left(\frac{x}{g_{lt}}\right)\right], \tag{35}$$

noting that $G_{1t}\left(x\right)=F_t\left(x\right)$.

4.2. Balanced Growth Path

The analytical characterizations of the FHK decompositions along the balanced growth path, given in equations (37)–(39), become

$$\frac{\Delta\log Z^{Entry}}{\Delta\log Z}=1-(1-\delta)\left[p\left(\frac{g_h}{g_e}\right)^{\gamma}+\left(1-p\right)\left(\frac{g_l}{g_e}\right)^{\gamma}\right], \tag{36}$$

$$-\frac{\Delta\log Z^{Exit}}{\Delta\log Z}=\left(1-\delta\right)\left[\begin{array}{l} p\left(\frac{g_h}{g_e}\right)^{\gamma-\frac{1}{1-\alpha}}\frac{\log\left(g_e\right)-\log\left(g_h\right)}{\left(1-\alpha_k\right)\log\left(g_e\right)} \\ +\left(1-p\right)\left(\frac{g_l}{g_e}\right)^{\gamma-\frac{1}{1-\alpha}}\frac{\log\left(g_e\right)-\log\left(g_l\right)}{\left(1-\alpha_k\right)\log\left(g_e\right)}\end{array}\right], \tag{37}$$

$$\frac{\Delta\log Z^{C}}{\Delta\log Z}=(1-\delta)\left\{\begin{array}{l} p\left(\frac{g_h}{g_e}\right)^{\gamma-\frac{1}{1-\alpha}}\left[\left(\frac{g_h}{g_e}\right)^{\frac{1}{1-\alpha}}-\frac{\log(g_e)-\log(g_h)}{(1-\alpha_k)\log(g_e)}\right] \\ +\left(1-p\right)\left(\frac{g_l}{g_e}\right)^{\gamma-\frac{1}{1-\alpha}}\left[\left(\frac{g_l}{g_e}\right)^{\frac{1}{1-\alpha}}-\frac{\log\left(g_e\right)-\log\left(g_l\right)}{\left(1-\alpha_k\right)\log\left(g_e\right)}\right]\end{array}\right\}. \tag{38}$$

Note that if we set $p=0$ and $g_l=g_c$, then (36)–(38) are identical to (37)–(39).

Finally, the constant terms, ω and ξ, in Proposition 1 become

$$\omega=\sum_{i=1}^{\infty}\left(1-\delta\right)^{i-1}g_e^{\gamma(1-i)}\left[pg_h^{\gamma}+\left(1-p\right)g_l^{\gamma}\right]^{i-1}, \tag{39}$$

$$\xi=\sum_{i=1}^{\infty}\left(1-\delta\right)^{i-1}\beta^{i-1}g_e^{\gamma(1-i)}\left[pg_h^{\gamma}+\left(1-p\right)g_l^{\gamma}\right]^{i-1}. \tag{40}$$

4.3. Quantitative Exercises

To calibrate the model, we first set $\bar{g}_h = g_e^{1-\varepsilon}$. This value of $\bar{g}_h$ implies that on the balanced growth path, high-growth firm efficiencies grow at the same rate as the aggregate economy, g_e. As a starting point, we set $p = 0.5$ and explore alternative values of p in the next subsection. The calibration strategy is similar to the one described in Table 4, with the exception that the g_l parameter is set so that the model matches the FHK contribution of entry and exit to U.S. manufacturing productivity growth of 25 percent. Table A19 reports the calibrated parameters.

Table A7: Calibration of model with idiosyncratic growth shocks for $p = 0.50$

Parameter		Value	Target
Operating cost (technological)	f^T	0.46×5	Average U.S. plant size: 14.0
Entry cost (technological)	κ^T	0.38	Entry cost/continuation cost: 0.82
Tail parameter	γ	6.10	S.D. of U.S. plant size: 89.0
Entrant efficiency growth	g_e	1.02^5	BGP growth rate of U.S.: 2 percent
Returns to scale	α	0.67	BGP labor share of U.S.: 0.67
Death rate	δ	$1-0.96^5$	Exiting plant employment share of U.S.: 19.3 percent
Discount factor	β	0.98^5	Real interest rate of U.S.: 4 percent
Low growth	g_l	1.018^5	FHK contribution of entry and exit to U.S. manufacturing productivity growth: 25 percent
Spillover term	ε	0.64	Authors' estimate

We study the properties of the transition path by creating a distorted economy with higher entry costs. Table A20 reports the results of the transition after conducting a reform in this distorted economy. We find that as was the case before, GDP and productivity growth rates increase immediately after the reform. We also see an increase in the contribution of net entry. The results are very similar to those of the baseline case reported Table 6 and are identical up to the first decimal digit. Reducing barriers to technology adoption yields results similar to those in Table A20.

Table A8: Productivity decompositions for $p = 0.50$ (entry cost reform)

Model periods (five years)	Entry cost	Real GDP growth (percent, annualized)	Aggregate productivity growth (percent, annualized)	Contribution of net entry to aggregate productivity (percent)
0–3	0.74	2.0	1.3	25.0
4 (reform)	0.38	4.6	2.9	59.6
5	0.38	2.5	1.5	36.5
6	0.38	2.1	1.3	28.1
7+	0.38	2.0	1.3	25.0

4.4. Further Exploring Quantitative Results

The quantitative results from the baseline model and the model with idiosyncratic growth shocks are remarkably similar. To better understand the implications of these idiosyncratic growth shocks, we calibrate the model using different values of p, ranging from 0 to 0.95. Table A21 reports the values of p and the corresponding value of g_l that emerge from the calibration. Note that higher values of p (that is, a larger fraction of high-growth firms) require lower g_l in order to match the FHK net entry contribution to productivity growth of 25 percent. Furthermore, the case of $p = 0$ and $g_l = 1.019^5$ corresponds to the baseline case, meaning that all continuing firms have the same productivity growth and this growth is the same as g_c in the baseline case.

In order to study our quantitative results more closely, we further break down the continuing term described in (5), $\Delta \log Z_{it}^C$, in the same manner as FHK as follows:

$$\Delta \log Z_{it}^C = \underbrace{\sum_{e \in C_{it}} s_{ei,t-1} \Delta \log z_{et}}_{\text{within term}} + \underbrace{\sum_{e \in C_{it}} \Delta \log z_{et} \Delta s_{et}}_{\text{covariance term}} + \underbrace{\sum_{e \in C_{it}} \left(\log z_{e,t-1} - \log Z_{i,t-1} \right) \Delta s_{et}}_{\text{between term}} . \tag{41}$$

The within term remains the same as before. The reallocation term is broken down into the covariance and between terms. The covariance term measures productivity growth that is accounted for by a positive covariance of changes in productivity and market shares (for example, firms with increasing productivity also see an increase in market shares). The between term measures productivity growth that is due to changing shares across firms (for example, firms that are relatively productive at the beginning of the window see an increase in market share).

Table A9: FHK decompositions in the balanced growth path

FHK decomposition (BGP)	Probability of high growth					
	(Baseline) $p=0$ $g_l=1.019^5$	$p=0.25$ $g_l=1.019^5$	$p=0.5$ $g_l=1.018^5$	$p=0.75$ $g_l=1.017^5$	$p=0.90$ $g_l=1.011^5$	$p=0.95$ $g_l=0.997^5$
Net entry	25.0	25.0	25.0	25.0	25.0	25.0
Entry	20.3	20.3	20.3	20.3	20.2	20.2
Exit	–4.7	–4.7	–4.7	–4.7	–4.8	–4.8
Continuing	75.0	75.0	75.0	75.0	75.0	75.0
Within	75.9	75.9	75.9	75.9	75.9	75.8
Covariance	–0.9	–0.9	–0.8	–0.7	–0.3	0.6
Between	–0.1	–0.1	–0.1	–0.2	–0.6	–1.4

Table A21 reports the sub-components of the continuing term and the net entry term on the balanced growth path for the idiosyncratic growth-shocks model and the baseline model. In the first row, we see that the contribution of net entry, which is a calibrated target, is 25 percent in all cases. The breakdown of the contributions of net entry shows that the entry and exit contributions are stable across the various specifications. Thus, the model with idiosyncratic growth shocks does not significantly change the composition of the net entry term on the balanced growth path.

The fourth row of the table indicates the contribution of the continuing term. The breakdown of the continuing term shows that the within term is very stable, even with different values of p. The breakdown also shows that there are changes in the covariance term. In the baseline model, the covariance term is negative because the productivity of continuing firms increases, while their market shares decline. In order to understand why this is the case, it is useful to study the market share of a firm with efficiency x in the model, which is given by

$$s_t(x)=\left(\frac{\alpha x}{w_t}\right)^{\frac{1}{1-\alpha}}. \tag{42}$$

Thus, changes in the market share of a firm depend on the growth of the firm's efficiency compared with the growth in wages. On the balanced growth path, w_t grows by g_e, whereas continuing firms' efficiencies grow by $g_c < g_e$. Why is it that there is an increase in the covariance term with an increase in p? First, it is important to note that the efficiency of high-growth firms grows at

$g_h = g_e$, implying that the market share of the high-growth firms remains the same, or $\Delta s_{et} = 0$. Similarly, the market share of the low-growth firms declines because $g_l < g_h$, which implies that $\Delta s_{et} < 0$. Furthermore, for very large p, we get that $\Delta \log z_{et} < 0$ for the low-growth firms. For example, when $p = 0.95$, we find that $g_l = 0.997^5$, which implies that the low-growth firms experience productivity losses. These two results would yield a positive covariance term for large enough p.

We also see that the between term becomes increasingly negative as p increases. Note that in the baseline model, the between term is negative. The reason is that the market share of continuing firms declines, $\Delta s_{et} < 0$, because $g_c < g_e$. Furthermore, the selection effect implies that the set of continuing firms is on average more productive than the set of all firms, which includes exiting firms. Hence, $\sum_{e \in C_{it}} \left(\log z_{e,t-1} - \log Z_{i,t-1} \right) > 0$. Why is it that the within term declines with an increase in p? As before, the market share of the high-growth firms remains unchanged because $g_h = g_e$, and the market share of the low-growth firms declines because $g_l < g_h$. For the low-growth productivity firms, as p increases, Δs_{et} becomes increasingly negative. The reason is that a higher p implies a lower g_l in the calibrated model.

Finally, the transition dynamics when we calibrate the model to the different values of p are very similar to the transition dynamics when $p = 0.5$, as reported in Table A20. For example, when $p = 0.95$, the contribution of net entry in the transition rises to 59.5 percent.