Online Appendix: Firm Wages in a Frictional Labor Market

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Appendixes A-G in this online appendix provide the corresponding planner problem and unconstrained firm problem, a discussion of the role of endogenous separations/quits, proofs, details on the parametrization and solution methods, additional figures, as well as a version of the model with firm-level shocks.

A Unconstrained Firms and Efficient Allocations

An unconstrained firm's problem for hiring in period t may be written as

$$\max_{W_{it},v_{it}} q(\theta_{it})v_{it}(Z_t - W_{it}) - E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k \kappa(v_{it+k}, n_{it+k}) \text{ s.t. } X_t = \mu(\theta_{it})(W_{it} - Y_t),$$

with hired workers continuing to influence vacancy costs in subsequent periods according to the law of motion for the firm's workforce, $n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it})$ for all t. The first order conditions for this unconstrained problem coincide with (14) and (15) in the text.

By contrast, a benevolent planner maximizes the present value of producer and home output net of the costs of vacancy creation:

$$\begin{aligned} \max_{\{\theta_{it},v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{i\in I} [(n_{it} + q(\theta_{it})v_{it})z_t - \kappa(v_{it}, n_{it})] + [1 - \sum_{i\in I}(n_{it} + q(\theta_{it})v_{it})]b \right\} \\ \text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall i \in I, t \geq 0, \\ \sum_{i\in I} v_{it}/\theta_{it} = 1 - \sum_{i\in I} n_{it}, \ \forall t \geq 0, \end{aligned} \tag{A.1}$$

taking into account the law of motion for the workforce of each producer and that the planner's choices of θ_{it}, v_{it} must be consistent with the measure of unmatched workers each period, as the job seekers allocated to each producer, v_{it}/θ_{it}, must add up to the latter.

To connect these two problems, note that the planner objective may be rewritten, reorganizing terms and including the constraints (A.1) with Lagrange multipliers λ_t, as

$$E_0 \sum_{i\in I}\sum_{t=0}^{\infty} \beta^t[(n_{it}+q(\theta_{it})v_{it})(z_t-b)-\kappa(v_{it},n_{it})-\lambda_t(\frac{v_{it}}{\theta_{it}}+n_{it})],$$

subject to $n_{it+1}=(1-\delta)(n_{it}+q(\theta_{it})v_{it})$, $\forall i\in I, t\geq 0$.

Meanwhile, for the firm, substituting wages out and adding up across cohorts of workers hired over time yields the objective:

$$E_t \sum_{t=0}^{\infty} \beta^t[(n_{it}+q(\theta_{it})v_{it})(z_t-b)-\kappa(v_{it},n_{it})-X_t(\frac{v_{it}}{\theta_{it}}+n_{it})],$$

subject to the same laws of motion for the workforce n_{it} as above.

The unconstrained firm's objective thus coincides with the planner's, with the Lagrange multipliers replaced by the market value of search, and identical constraints on the two problems otherwise. Thus, the unconstrained equilibrium is efficient.

B Endogenous Separations/Quits

This section first highlights that workers are free to quit to look for a new job at any point, and that this assumption limits the monopsony power firms have in the model. It then extends the model to feature endogenous separations in equilibrium, showing that doing so need not change equilibrium wage setting in a significant way.

Endogenous separations/quits in the model Recall that in setting wages, firms face the constraint

$$U_t=\mu(\theta_t)E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(w_{t+k}+\beta\delta U_{t+1+k})+(1-\mu(\theta_t))(b+\beta E_tU_{t+1}), \tag{B.1}$$

which tells them what tightness to expect in response to their offered present value wages. Here a worker's value of accepting a job, $E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(w_{t+k}+\beta\delta U_{t+1+k})$, includes the present value of wages together with the continuation value of returning to search upon separation. Meanwhile, the worker's value of remaining unmatched and continuing search is $b+\beta E_tU_{t+1}$. For workers to be willing to accept employment, the former must dominate the latter: $E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(w_{t+k}+\beta\delta U_{t+1+k})\geq b+\beta E_tU_{t+1}$.

Due to the within-firm constraints, existing workers' wages satisfy the same constraint, meaning that an existing worker's value of remaining on the job, $E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(w_{t+k}+$

$\beta\delta U_{t+1+k}$), exceeds the value of quitting to start search for a new job tomorrow, $b+\beta E_t U_{t+1}$. As long as the firm remains in the market for new hires each period, it thus must be paying wages that guarantee that none of its existing workers want to quit.

Whether remaining in the market for new hires is optimal for the firm hinges on what it can do to its existing workers if it does not hire (in which case it can ignore constraint (B.1)). The paper assumes that existing workers are always free to take the outside option of quitting to search for a new job next period (with value $b+\beta E_t U_{t+1}$), and this limits how low wages firms can pay them. In solving the model I check to make sure that firm value from remaining in the market (and setting wages according to the first order conditions as discussed) dominates the value of the firm withdrawing from the market for new hires and paying existing workers just enough to keep them from quitting.

The above checks confirm that the interior solutions considered are optimal when workers are free to quit. By contrast, if one assumed that firms can pay existing workers arbitrarily low wages with the workers forced to remain with the firm, then the firm would always prefer to do that (assuming wages could be unboundedly low) and the equilibrium look different from the paper. Thus, the assumption that workers are free to quit limits the monopsony power firms have in setting wages.

Endogenous separations/quits in equilibrium Does incorporating endogenous separations/quits in equilibrium alter wage setting in an important way? This section extends the model to consider this question explicitly.

To begin, there must be a reason giving rise to worker reallocation across firms. A natural reason driving such reallocation is that a worker may discover that a particular job is not a good fit for them for match-specific reasons, and seek a better fit.[1] I formalize such behavior in the extension laid out below.

Consider the version of the model with worker heterogeneity, assuming workers have permanent differences in productivity, with J types $\{z_j\}_{j=1}^{J}$. Suppose then that each worker faces a small probability p each period that their productivity with their current employer drops, with the worker starting the next period at a permanently lower productivity level. To be concrete, suppose productivity drops from z_j to z_{j-1} for $j = 2, ..., J$, and from z_1 to b for the lowest productivity type. The worker's inherent productivity type remains unaffected,

[1]Such shocks are the drivers of quits in the directed search model of Menzio and Shi (JPE 2011) (with single worker firms). Absent such shocks their model would not generate quits, with the equilibrium featuring a single wage across firms. (Recall that equilibrium allocations in that model are efficient, and without shocks leading to mismatch, there is no need for a planner to reallocate workers across firms.)

however, leaving their outside options unchanged.

The firm pays workers based on their prevailing productivity, meaning that type z_{j+1} workers whose productivity declines to z_j following a p shock are paid the same per-period wage as workers with inherent productivity z_j. This wage may or may not be high enough to keep these workers with the firm, however, because their outside options are better than those of type j workers. If not, there will be quits in equilibrium.

In what follows, I first characterize equilibrium outcomes when p shocks cause workers to quit, and then consider conditions ensuring that doing so is optimal.

What does an equilibrium where p shocks lead to quits look like? In such an equilibrium, worker search value satisfies:

$$\begin{aligned}U_{jt} =& \mu(\theta_{jt})E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(1-p)^k[w_{jt+k}+\beta\delta U_{jt+k+1}+\beta(1-\delta)p(b+\beta U_{jt+k+2})]\\ &+(1-\mu(\theta_{jt}))(b+\beta E_t U_{jt+1}),\end{aligned} \tag{B.2}$$

where the value of getting hired differs from the baseline model due to the p shocks. When a worker is hired, they receive the wages paid until either the δ or p shock hits, valuing them according to their present value. If the δ shock hits, the worker returns to search. If the δ shock does not hit but the p shock does, the worker quits, remaining at home for a period and returning to search the next.[2]

Reorganizing terms to express (B.2) as $X_{jt} = \mu(\theta_{jt})(W_{jt}-Y_{jt})$ implies $X_{jt} = U_{jt}-b-\beta E_t U_{jt+1}$ and $Y_{jt} = b+\beta E_t U_{jt+1} - E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(1-p)^k(\beta\delta U_{jt+k+1}+\beta(1-\delta)p(b+\beta U_{jt+k+2}))$.

Turning to the firm problem, suppose a firm has n_{jt} productivity z_j workers (not including the type $j+1$ workers hit with a p shock that quit). This measure of workers follows the law of motion $n_{jt+1} = (1-p)(1-\delta)(n_{jt}+q(\theta_{jt})v_{jt})$. With this, the firm problem involving productivity z_j workers becomes:

$$\begin{aligned}&\max_{W_j,v_j}(n_j+q(\theta_j)v_j)(Z_j(S)-W_j)-\kappa(v_j,n_j)+\beta E_S V_j(n_j',S')\\ &\text{s.t. } n_j' = (1-p)(1-\delta)(n_j+q(\theta_j)v_j),\\ &\qquad X_j(S) = \mu(\theta_j)(W_j-Y_j(S)),\end{aligned}$$

where $V_j(n_j,S) = q(\theta_j)v_j(Z_j(S)-W_j)-\kappa(v_j,n_j)+\beta E_S V_j(n_j',S')$. The present values now satisfy $W_{jt} = E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(1-p)^k w_{jt+k}$ and $Z_j = E_t\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(1-p)^k z_{jt+k}$.

[2] The responses to the δ and p shocks have slightly asymmetric timing. This timing for p shocks is desirable for internal consistency, because a worker's decision to quit is based on the same calculation as a searching worker's decision to accept employment, in terms of comparing the value of the job to the value of search. Meanwhile, I have maintained the timing of the δ shocks unchanged from the body of paper, to minimize changes to the model.

Scaling by n_j, the problem becomes:

$$\max_{W_j, x_j} (1 + q(\theta_j)x_j)(Z_j(S) - W_j) - \hat{\kappa}(x_j) + \beta(1-\delta)(1-p)(1 + q(\theta_j)x_j)E_S\hat{V}_j(S')$$
$$\text{s.t. } X_j(S) = \mu(\theta_j)(W_j - Y_j(S)),$$

where $\hat{V}_j(S) = q(\theta_j)x_j(Z_j(S) - W_j) - \hat{\kappa}(x_j) + \beta(1-\delta)(1-p)(1 + q(\theta_j)x_j)E_S\hat{V}_j(S')$.

The first order conditions for this problem remain very similar to those in the paper:

$$\kappa_v(x_j) = q(\theta_j)(Z_j(S) - W_j + \beta(1-\delta)(1-p)E_S\hat{V}_j(S')),$$
$$1 + q(\theta_j)x_j = q'(\theta_j)g^j_W(W_j)x_j(Z_j - W_j + \beta(1-\delta)(1-p)E_S\hat{V}_j(S')).$$

Turning to the adding up constraint for searching workers, suppose the sum of the n_{jt} workers across firms is denoted by N_{jt}. Searching type j workers include all type j workers not remaining with a firm (of measure $1 - N_{jt}$), except those hit with a p shock in the preceding period (of measure $\frac{p}{1-p}N_{jt}$) who will start search only in the subsequent period.[3] This brings the total measure of searching type j workers to $1 - N_{jt}/(1-p)$. The adding up constraint for these workers may then be written as $x_{jt}N_{jt} = \theta_{jt}(1 - N_{jt}/(1-p))$, with the firm-level laws of motion for workers implying: $N_{jt+1} = (1-p)(1-\delta)(N_{jt} + \mu(\theta_{jt})(1 - N_{jt}/(1-p)))$.

For the above equations to characterize an equilibrium, workers must prefer to quit when a p shock hits, and firms prefer to let them go instead of raising wages to retain them.

What ensures that workers prefer to quit when a p shock hits? For a type $j+1$ worker, the value of quitting in period t following a p shock is $b + \beta E_t U_{j+1t+1}$. The value of waiting for a period and then quitting is $w_{jt} + \beta\delta E_t U_{j+1t+1} + \beta(1-\delta)E_t(b + \beta U_{j+1t+2}) = w_{jt} + \beta E_t U_{j+1t+1} - \beta(1-\delta)E_t X_{j+1t+1}$ (using that $U_{j+1t+1} = b + X_{j+1t+1} + \beta E_{t+1}U_{j+1t+2}$). The worker prefers to quit in period t if $w_{jt} < b + \beta(1-\delta)E_t X_{j+1t+1}$. This condition ensures that p shock workers prefer to quit immediately following a p shock.

What ensures that firms prefer the above wage setting behavior over retaining quitting workers? If the firm raised this period's per-period wage above the equilibrium wage, the firm value from type j workers would fall (because the equilibrium wage maximizes this value), but at some point the wage would become high enough to retain the type $j+1$ workers hit with a p shock, leading to a jump up in firm value (as long as the required wage is not too high). What would be the implied firm value from raising the wage to that point?

The calculation to determine the threshold wage retaining p shock workers is identical to the one above, determining the threshold wage as $\underline{w}_{j+1t} = b + \beta(1-\delta)E_t X_{j+1t+1}$. If the firm

[3] I assume the total measure of type j workers is one.

set this wage today, the present value of wages for the type j workers at the firm would be $\underline{W}_{j+1t} = \underline{w}_{j+1t} + \beta(1-\delta)(1-p)E_t W_{j+1t+1}$, assuming equilibrium wages prevail after the current period. This present value determines the tightness the firm faces in hiring this period via $X_{jt} = \mu(\theta_{jt})(\underline{W}_{j+1t} - Y_{jt})$. Firm value from productivity z_j workers would then be determined by the problem:

$$\max_{v_j} n_j^p(z_j(S) - \underline{w}_{j+1}(S)) + (n_j + q(\theta_j)v_j)(Z_j(S) - \underline{W}_{j+1}(S)) - \kappa(v_j, n_j + n_j^p) + \beta E_S V_j(n_j', S'),$$

where the firm begins with measure n_j^p type $j+1$ workers hit with a p shock in the preceding period. Here $n_j' = (1-p)(1-\delta)(n_j + q(\theta_j)v_j)$, as the p shock workers would quit after this period due to equilibrium wages.

Scaling by n_j (and denoting $s_j^p = n_j^p/n_j$, $x_j = v_j/n_j$), the problem becomes

$$\begin{aligned}\max_{x_j}\ & s_j^p(z_j - \underline{w}_{j+1}) + (1 + q(\theta_j)x_j)(Z_j(S) - \underline{W}_{j+1}) - (1 + s_j^p)\hat{\kappa}(x_j/(1+s_j^p)) \\ & + \beta(1-\delta)(1-p)(1 + q(\theta_j)x_j)E_S\hat{V}_j(S'),\end{aligned}$$

where θ_j is determined via $X_j(S) = \mu(\theta_j)(\underline{W}_{j+1}(S) - Y_j(S))$. For equilibrium wage setting to be optimal, equilibrium firm value must dominate the value attained here.

Numerical example: To consider the impact of endogenous separations on wages in concrete terms, I return to the parametrization in the paper. Suppose we augment the baseline parametrization by assuming that half of separations are exogenous and half endogenous. Maintaining the target that jobs last 2.5 years on average, this requires that $p = d = 0.0168$.[4] Suppose, further, that we assume worker productivity declines by 5% upon a p shock, with workers of inherent productivity $z_{j+1} = 1.05$ falling to $z_j = 1$.

According to the above conditions characterizing equilibrium outcomes with p shocks, the equilibrium specifies that workers of inherent productivity $z_{j+1} = 1.05$ begin jobs with wage 0.940 and workers of inherent productivity $z_j = 1$ with wage 0.911. When a type $j+1$ worker is hit with a p shock, they then face the lower equilibrium wage 0.911. Meanwhile, the threshold wage that would retain these workers with the firm is 0.937. The workers thus prefer to quit because of their better outside options.

Equilibrium firm value from workers of inherent productivity z_j is 6.038 (per existing type j worker). For every type j worker, each firm receives roughly 0.017 p-shock workers, consistent

[4] An expected duration of employment of $1/(1-(1-p)(1-d)) = 30$ months implies a monthly total separation rate of 3.3%. According to the Job Openings and Labor Turnover Survey (JOLTS) of the U.S. Bureau of Labor Statistics, the overall quit rate is roughly half of the total separation rate, at 1.9% and 3.6% per month, respectively. The qualitative behavior of the example remains similar if one assumes all separations to be endogenous, as well as if one raises the total separation rate to 3.6% per month.

with the probability of the p shock hitting. In equilibrium firms let these workers quit. If a firm instead paid 0.937 today, retaining these workers, firm value from workers with productivity z_j would be 6.028 (per existing type j worker). Firms thus prefer to set the equilibrium wage, letting the p-shock workers quit.

Does having endogenous separations/quits occurring in equilibrium alter the level of wages significantly? The equilibrium wage of workers with inherent productivity $z_j = 1$ above is 0.911, while the steady-state wage of productivity $z = 1$ workers in the paper is 0.911. Wage setting is characterized by essentially the same optimality conditions across these two environments, with both shocks leading to separation, and the wage level little changed.

This example illustrates that wage setting need not be altered substantially by endogenous separations/quits taking place in equilibrium. Note, moreover, that it is also not clear that making the reallocation of mismatched workers easier (while maintaining the target for the duration of jobs) would alter this conclusion.

C Proofs and Details

<u>**The Static Model**</u>

Proof of Proposition 1 Equation (1) yields the derivative $g_w(w;U) = -\mu(\theta)/(\mu'(\theta)(w-b))$, equation (4) the wage $w = z - \kappa_v(x)/q(\theta)$ and the equilibrium condition the vacancy rate $x = \theta(1-N)/N$. Using these in (5) yields an equation determining equilibrium θ:

$$\frac{1}{\theta} + q(\theta)\frac{1-N}{N} = \frac{1-\varepsilon(\theta)}{\varepsilon(\theta)}\frac{1-N}{N}\frac{\kappa_v(\theta\frac{1-N}{N})}{z-b-\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)}},$$

where I denote the matching function elasticity by $\varepsilon(\theta) := \mu'(\theta)\theta/\mu(\theta)$. The left hand side is strictly decreasing and the right strictly increasing in θ, given the assumptions on the vacancy cost and matching function. The equation thus determines a unique equilibrium θ.

For the unconstrained model one simply leaves out the $1/\theta$ term on the left hand side, which implies that the tightness is strictly greater in the constrained case than in the unconstrained case, as is employment, $N + \mu(\theta)(1-N)$. From $x = \theta(1-N)/N$, the hiring rate in the constrained case is also strictly greater, as is total vacancy creation xN. The wage, from $w = z - \kappa_v(x)/q(\theta)$, is strictly lower in the constrained case.

Firm profits from hiring may be written (using the first order condition for vacancies) as $q(\theta)x(z-w) - \hat{\kappa}(x) = -\kappa_n(x)$, which is increasing in x, implying these profits are greater in the constrained case. Worker value from employment, w, is lower in the constrained case.

Using the first order condition for vacancy creation to substitute out the wage, we have $U = b + \mu(\theta)(w - b) = b + \mu(\theta)(z - b) - \theta\kappa_v(\theta(1-N)/N)$. This expression is non-monotonic, with derivative $\mu'(\theta)(z-b) - \kappa_v(\theta(1-N)/N) - \kappa_{vv}(\theta(1-N)/N)\theta(1-N)/N$, which is strictly decreasing: positive at $\theta = 0$ but negative at the unconstrained θ (where $\mu'(\theta)(z-b) = \kappa_v(x)$) and beyond. It follows that the worker value of search U is strictly lower in the constrained case.

Wages and Hiring Note that optimal wage setting implies

$$\frac{1+q(\theta)x}{q(\theta)x} = -\frac{q'(\theta)\mu(\theta)}{\mu'(\theta)q(\theta)}\frac{z-w}{w-b}$$

in the constrained model and

$$1 = -\frac{q'(\theta)\mu(\theta)}{\mu'(\theta)q(\theta)}\frac{z-w}{w-b}$$

in the unconstrained model. These equations follow from conditions (5) and (6), where (1) yields $g_w = -\mu(\theta)/(\mu'(\theta)(w-b))$. These equations imply that the wage can be written as the weighted average

$$w = (1-\gamma)b + \gamma z,$$

where $\gamma_c = [\frac{\frac{1}{qx}+1}{\frac{1}{\varepsilon}-1} + 1]^{-1}$ in the constrained case and $\gamma_u = [\frac{1}{\frac{1}{\varepsilon}-1} + 1]^{-1}$ in the unconstrained case. Here I denote the matching function elasticity as $\varepsilon := \mu'(\theta)\theta/\mu(\theta)$. For a simple illustration of wage outcomes, I treat ε as a constant.[5]

From the expressions above, it is easy to see that the wage is generally lower in the constrained case. To illustrate, note first that in a dynamic setting the steady-state hiring rate is related to the separation rate δ via $(1+qx)(1-\delta) = 1$. Adopting the values $\delta = 0.03$ and $\varepsilon = 0.5$ yields the weights $\gamma_c = 0.03$, $\gamma_u = 0.5$. The wage is clearly lower in the constrained case.

To consider how the wage responds to changes in productivity, hold γ fixed for a moment. From the expression for the wage it follows that the wage also responds less to changes in z in the constrained case, as $\Delta w = \gamma\Delta z$ with $\gamma_c < \gamma_u$.

In practice γ_c does respond to changes in market productivity, however (generally counteracting the above effect as an increase in z leads firms to place more weight on offering an attractive/high hiring wage instead of making profit on existing workers). Taking the change in γ_c into account, we have $\Delta w = \gamma_c\Delta z + (z-b)[\frac{\frac{1}{qx}+1}{\frac{1}{\varepsilon}-1} + 1]^{-2}[\frac{1}{\varepsilon} - 1]^{-1}(qx)^{-1}\frac{\Delta qx}{qx}$ in the constrained case, and $\Delta w = \gamma_u\Delta z$ in the unconstrained case.

Letting $z = 1$ and $\Delta z = 0.02$, we have $\Delta w = \gamma_u\Delta z = 0.01$ in the unconstrained case, implying a wage increase of half the increase in productivity. In the constrained case, we

[5]This would hold exactly with a Cobb-Douglas matching function, where the elasticity is constant.

arrive at the approximate upper bound wage response $\Delta w \approx 0.0006 + 0.0027 = 0.003$, where I have used $\frac{\Delta qx}{qx} \approx 0.1$ and $z - b \approx 1$. Despite the increase in γ_c, the wage response in the constrained case thus remains a fraction of that in the unconstrained case.[6]

For hiring, note that the equilibrium condition $1 - N = xN/\theta$ implies that an increase in x is associated with an increase in $\theta = xN/(1-N)$. With this, the left hand side of the optimality condition for vacancy creation $\kappa'(x)/q(\theta) = z - w$ is increasing in x. An increase in the right hand side thus implies an increase in x, as well as θ (and hence $\mu(\theta)$).

The Dynamic Model

Withdrawing from Hiring Consider the dynamic firm problem (13). The paper focuses on circumstances where the firm prefers to hire each period, with wages having to satisfy the job seeker constraint characterizing how the tightness responds to the offered present value wage each period. But, because the firm begins with a stock of existing workers, it could in some circumstances find it optimal to withdraw from hiring instead.

If a firm did not hire in the initial period, it would optimally set the present value wage so low as to make its existing workers indifferent between remaining with the firm and quitting into unemployment. Doing so would mean: $v_0 = 0$ and $W_0 - Y_0 = 0$. These initial period choices would leave the rest of the firm problem as: $n_{i0}[Z_0 - Y_0] + E_t \sum_{t=1}^{\infty} \beta^t [q(\theta_{it}) v_{it} (Z_t - W_{it}) - \kappa(v_{it}, n_{it})]$. With commitment to future wages, subsequent choices would then be consistent with those of unconstrained firms, and hence characterized by the first order conditions under standard conditions. It remains relevant to check that the value from the firm hiring throughout dominates the firm withdrawing from hiring in the initial period, however.

In the case of no commitment to future wages, problem (19), if a firm were to withdraw from hiring for a period, its (scaled) firm value would be $Z(S) - Y + \beta(1-\delta) E_S \hat{V}(S')$, where the continuation value $\hat{V}(S)$ follows (20). It remains relevant to verify that the firm values arising from solving the model based on the first order conditions described dominate this value from withdrawing from hiring for a period. In practice this requires keeping track of this alternative firm value and checking that equilibrium firm values dominate it. This can restrict parameter values, as well as the range of N, as hiring becomes less profitable when N is high.

Proof of Proposition 2 From its definition, $Y = \frac{b+\beta(1-\delta)X}{1-\beta(1-\delta)}$. We thus have

$$X = \mu(\theta)(W - Y) = \mu(\theta)\left(W - \frac{b + \beta(1-\delta)X}{1-\beta(1-\delta)}\right) \Rightarrow \frac{X}{\mu(\theta)} = \frac{W - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

[6] This conclusion continues to hold when comparing percent changes in the wage due to the large difference in wage responses.

The first order condition for vacancy creation yields an expression for the present value wage

$$W = Z - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)}.$$

Using that $\kappa_n(x) = \hat{\kappa}(x) - x\kappa_v(x)$, we further arrive at

$$W = Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))}.$$

At the same time, combining the first order conditions for vacancy creation and wages yields

$$\frac{X}{\mu(\theta)} = \frac{\kappa_v(x)}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\frac{q(\theta)x}{1+q(\theta)x},$$

where the term $\frac{q(\theta)x}{1+q(\theta)x}$ equals $1/\delta$ in the constrained case, and reduces to 1 in the unconstrained case.

Equating the above two expressions for $\frac{X}{\mu(\theta)}$ yields the equation:

$$\frac{\kappa_v(x)}{q(\theta)}\frac{1-\varepsilon}{\varepsilon}\frac{q(\theta)x}{1+q(\theta)x} = \frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

Note that $q(\theta)x = \delta/(1-\delta)$ is a constant, which implies an increasing relationship between steady-state θ and x. With this, the left hand side of the above equation is strictly increasing in θ, rising from zero toward infinity as θ rises from zero to infinity, while the right hand side is strictly decreasing, falling from $Z - \frac{b}{1-\beta(1-\delta)}$ to negative values. Thus, there is a unique steady-state θ, and this value is strictly higher in the constrained case.

It follows that N and employment, $N/(1-\delta)$, are strictly greater in the constrained case. From $x = \delta/((1-\delta)q(\theta))$, x is higher in the constrained case. The expression for wages then implies that the present value hiring wage W is strictly lower in the constrained case.

Firm value from hiring may be written, using the first order condition for vacancies, as $q(\theta)x(Z-W) - \hat{\kappa}(x) + \beta(1-\delta)(1+q(\theta)x)\hat{V} = \hat{V} = -\beta(1-\delta)\kappa_n(x)/(1-\beta(1-\delta))$, which is increasing in x and thus greater in the constrained case where x is higher.

For worker values, the above expressions imply:

$$X = \mu(\theta)\frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

The right hand side expresses X as a product $\mu(\theta)f(\theta)$ where $f(\theta)$ is strictly decreasing from a positive value to zero, with both equilibrium θ in the range where it remains positive. The derivative of the right hand side $\mu'(\theta)f(\theta) + \mu(\theta)f'(\theta)$ is strictly decreasing in this range. To see this, note that for values of θ starting at zero onward: μ' is positive and strictly decreasing, f is positive and strictly decreasing toward zero, μ is positive and increasing and f' is negative and decreasing. It follows that the derivative is strictly decreasing. At the unconstrained equilibrium, moreover, the derivative is strictly negative.

It follows that X is lower in the constrained case than the unconstrained case. Thus, the value of searching for employment $U = (b + X)/(1 - \beta)$ and accepting employment $W + \beta\delta U/(1 - \beta(1 - \delta))$ are lower in the constrained case than the unconstrained case.

Wages and Hiring To arrive at equation (25), note that optimal wage setting implies

$$\frac{1 + q(\theta_t)x_t}{q(\theta_t)x_t} = -\frac{q'(\theta_t)\mu(\theta_t)}{\mu'(\theta_t)q(\theta_t)} \frac{Z_t - W_t - E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \kappa_n(x_{t+k})}{W_t - Y_t}$$

in the constrained case and

$$1 = -\frac{q'(\theta_t)\mu(\theta_t)}{\mu'(\theta_t)q(\theta_t)} \frac{Z_t - W_t - E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \kappa_n(x_{t+k})}{W_t - Y_t}$$

in the unconstrained case. These equations follow from the first order conditions for the present value wage, where $X_t = \mu(\theta_t)(W_t - Y_t)$ yields $g^t_{W_t} = -\mu(\theta_t)/(\mu'(\theta_t)(W_t - Y_t))$. As in the static model, these equations imply that the wage can be written as the weighted average

$$W_t = (1 - \gamma_t)Y_t + \gamma_t(Z_t - E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \kappa_n(x_{t+k})),$$

with the same weights $\gamma_{ct} = [\frac{\frac{1}{q_t x_t}+1}{\frac{1}{\varepsilon_t}-1} + 1]^{-1}$ in the constrained case and $\gamma_{ut} = [\frac{1}{\frac{1}{\varepsilon_t}-1} + 1]^{-1}$ in the unconstrained case.

To shed light on the implications of changes in productivity for hiring, consider steady-state comparative statics. Note that the steady-state relationship $(1 + q(\theta)x)(1 - \delta) = 1$ implies that an increase in x is associated with an increase in θ. With this, the left hand side of the optimality condition for vacancy creation $\kappa'(x)/q(\theta) = Z - W - E \sum_k \beta^k (1 - \delta)^k \kappa_n(x)$ is increasing in x. An increase in the right hand side thus implies an increase in x as well as θ (and hence $\mu(\theta)$). (Overall, functional forms and parameter values play a role in determining outcomes in the model, but in drawing the equilibrium wage toward the workers' opportunity cost, the constraints work to make wages less responsive to changes in productivity.)

Derivation of Equation (26) Note that the expression for the wage implies

$$W_t - Y_t = \gamma_t(Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t) \text{ and}$$
$$Z_t - W_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) = (1-\gamma_t)(Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t).$$

Note also that, from $Y_t = b + \sum_{k=1}\beta^k(1-\delta)^k(b+X_{t+k})$, $X_t = \mu(\theta_t)(W_t - Y_t)$ and the above, we have

$$\begin{aligned}
&Y_t - \beta(1-\delta)E_tY_{t+1} = b + \beta(1-\delta)E_tX_{t+1} = b + \beta(1-\delta)E_t\mu(\theta_{t+1})(W_{t+1} - Y_{t+1})\\
&= b + \beta(1-\delta)E_t\mu(\theta_{t+1})\gamma_{t+1}(Z_{t+1} - \sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+1+k}) - Y_{t+1})\\
&= b + \beta(1-\delta)E_t\mu(\theta_{t+1})\gamma_{t+1}\frac{\kappa_v(x_{t+1})}{q(\theta_{t+1})(1-\gamma_{t+1})} = b + \beta(1-\delta)E_t\frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&\frac{\kappa_v(x_t)}{q(\theta_t)(1-\gamma_t)} = Z_t - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+k}) - Y_t\\
&= z_t - b + \beta(1-\delta)E_t[Z_{t+1} - \sum_{k=1}^{\infty}\beta^k(1-\delta)^k\kappa_n(x_{t+1+k}) - Y_{t+1} - \kappa_n(x_{t+1}) - \frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}]\\
&= z_t - b + \beta(1-\delta)E_t[\frac{\kappa_v(x_{t+1})}{q(\theta_{t+1})(1-\gamma_{t+1})} - \kappa_n(x_{t+1}) - \frac{\gamma_{t+1}\theta_{t+1}\kappa_v(x_{t+1})}{(1-\gamma_{t+1})}].
\end{aligned}$$

Infrequent Adjustment

Proof of Proposition 3 The equilibrium with infrequent adjustment is characterized by the first order conditions

$$\begin{aligned}
\kappa_v(x) &= q(\theta)(Z - W + \beta(1-\delta)E_S(\alpha\hat{V}^r(S') + (1-\alpha)\hat{V}^f(w,S')))\\
(1+q(\theta)x)h_w &= q'(\theta)g_W(W;S)h_wx[Z - W + \beta(1-\delta)E_S[\alpha\hat{V}^r(S') + (1-\alpha)\hat{V}^f(w,S')]],\\
&+ \beta(1-\delta)(1+q(\theta)x)(1-\alpha)E_S\hat{V}^f_w(w,S').
\end{aligned}$$

In steady state, $\hat{V}^f_w(w) = h_w(w)$ and $\hat{V}^r = \hat{V}^f(w) = -\beta(1-\delta)\kappa_n(x)/(1-\beta(1-\delta))$. The remaining equilibrium conditions are, as before, $X = \mu(\theta)(W - Y)$, $(1+q(\theta)x)(1-\delta) = 1$ and $x = \theta(1-N)/N$. The latter two imply that $N = \mu(\theta)(1-\delta)/(\delta + \mu(\theta)(1-\delta))$.

As for the baseline dynamic model (see proof above), we have that:

$$\frac{X}{\mu(\theta)} = \frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

Combining the first order conditions for vacancy creation and wages yields:

$$\frac{X}{\mu(\theta)} = \frac{\kappa_v}{q(\theta)} \frac{1-\varepsilon}{\varepsilon} \frac{q(\theta)x}{1+q(\theta)x} \frac{1}{1-\beta(1-\delta)(1-\alpha)}.$$

Combining the two yields the equation (using $(1+q(\theta)x)(q(\theta)x) = \delta$):

$$\frac{\kappa_v(x)}{q(\theta)} \frac{1-\varepsilon}{\varepsilon} \frac{\delta}{1-\beta(1-\delta)(1-\alpha)} = \frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1+\frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

Note that $q(\theta)x = \delta/(1-\delta)$ is a constant, which implies an increasing relationship between steady-state θ and x. As before, the left hand side of the above equation is strictly increasing in θ, rising from zero toward infinity as θ rises from zero to infinity, while the right hand side is strictly decreasing, falling from $Z - \frac{b}{1-\beta(1-\delta)}$ to negative values. Thus, there is a unique steady-state θ, and this value is strictly higher than efficient and increasing in α. From $x = \delta/((1-\delta)q(\theta))$, x which is higher than efficient and increasing in α. It follows that N and employment, $N/(1-\delta)$, are strictly greater than efficient and increasing in α. The expression for wages

$$W = Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))},$$

then implies that the present value wage W is strictly below efficient and decreasing in α. Even with α approaching zero, the term $\delta/(1-\beta(1-\delta)(1-\alpha))$ remains strictly below one if $\beta < 1$, implying allocations fall short of efficient.

Firm value $Z - W - \beta(1-\delta)\kappa_n(x)/(1-\beta(1-\delta))$ is strictly greater than efficient and falls as α falls from one toward zero. For worker values, we have

$$X = \mu(\theta)\frac{Z - \frac{\beta(1-\delta)\hat{\kappa}(x)}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)x\kappa_v(x)}{\delta(1-\beta(1-\delta))} - \frac{b}{1-\beta(1-\delta)}}{1+\frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}.$$

The right hand side is again decreasing from the efficient θ toward higher values, implying that X is below efficient and falls in α. Thus, the value of searching for and accepting employment are below efficient and fall in α. Firm and worker values also fall short of efficient even as α approaches zero.

D Calibration Details

The law of motion for matches implies that the steady-state level of unemployment satisfies:

$$u = 1 - N - \mu(\theta)(1-N) = \frac{\delta(1-\mu(\theta))}{\delta(1-\mu(\theta))+\mu(\theta)}.$$

Given a value for δ, a target for steady-state u then determines $\mu(\theta)$.

Given a target for the tightness θ, the matching function parameter ℓ is then pinned down (uniquely) from $\mu(\theta) = \theta/(1+\theta^{\ell})^{1/\ell}$. This further pins down the steady-state value of $x = \theta(1-N)/N = \delta\theta/((1-\delta)\mu(\theta))$.

The above values must be consistent with the model equation (26) with the appropriate weight γ. Note that the text compares the constrained model where firms reoptimize each period—and the constrained γ thus applies to both sides of (26)—to the unconstrained model where the unconstrained γ applies to both sides of the equation. Given the above, equation (26) pins down a unique value of $(z-b)/\kappa_0$ for each of the two models. This still allows alternative combinations of b, κ_0 that are consistent with any such value.

The scaled steady-state firm value may be written, using the first order condition for vacancy creation, as

$$(1+q(\theta)x)(Z - W - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)}) - \hat{\kappa}(x) = (1+q(\theta)x)\frac{\kappa_v(x)}{q(\theta)} - \hat{\kappa}(x) = \frac{\kappa_v(x)}{q(\theta)} - \kappa_n(x).$$

Flow profits per employed worker thus equal $(1-\beta)(\frac{\kappa_v(x)}{q(\theta)} - \kappa_n(x))/(1+q(\theta)x)$.

From the same first order condition, the wage may be written as:

$$w = W(1-\beta(1-\delta)) = (Z - \frac{\beta(1-\delta)\kappa_n(x)}{1-\beta(1-\delta)} - \frac{\kappa_v(x)}{q(\theta)})(1-\beta(1-\delta)).$$

For the share of profit and wage to remain unchanged across the two cases, given the above, κ_0 must remain unchanged across cases. Thus, only b adjusts across the two cases, essentially rising in the constrained model to keep the wage from falling.

If b is held fixed across cases, κ_0 must increase in the constrained case to keep hiring from rising while firm value rises and the wage falls.

E Solving: Firm Wages with Aggregate Shocks

The dynamic system for the firm wage equilibrium with aggregate shocks is given below. The last five equations define some variables of interest based on the solution (employment, unemployment, the vacancy-unemployment ratio, firm value, and firm value if the firm did not hire in the current period at all).

$$
\begin{aligned}
&\kappa_v(x_t) = q(\theta_t)(Z_t - W_t + \beta(1-\delta)E_t\hat{V}_{t+1}) \\
&1 + q(\theta_t)x_t = q'(\theta_t)g_{Wt}x_t(Z_t - W_t + \beta(1-\delta)E_t\hat{V}_{t+1}) \\
&g_{Wt} = -\mu(\theta_t)/(\mu'(\theta_t)(W_t - Y_t)) \\
&\hat{V}_t = -\kappa_n(x_t) + \beta(1-\delta)E_t\hat{V}_{t+1} \\
&N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t)) \\
&x_t = v_t/N_t \\
&v_t = \theta_t(1-N_t) \\
&X_t = \mu(\theta_t)(W_t - Y_t) \\
&W_t = w_t + \beta(1-\delta)E_tW_{t+1} \\
&Y_t = b + \beta(1-\delta)E_t(X_{t+1} + Y_{t+1}) \\
&Z_t = z_t + \beta(1-\delta)E_tZ_{t+1} \\
&E_tz_{t+1} - 1 = \rho_z(z_t - 1) \\
&e_t = N_t + \mu(\theta_t)(1-N_t) \\
&u_t = 1 - e_t \\
&vuratio_t = v_t/u_t \\
&\hat{V}_{obj,t} = Z_t - W_t + \hat{V}_t \\
&\hat{V}_{objnh,t} = Z_t - Y_t + \beta(1-\delta)E_t\hat{V}_{t+1}
\end{aligned}
$$

F Additional Figures

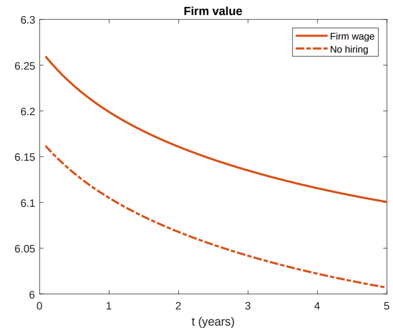

Figure F.1: Impulse Responses in Firm Wage Model: Optimality of Hiring

Notes: The figure refers to the impulse response in Figure 1, showing that the firm value attained by following the first order conditions dominates withdrawing from hiring for a period, throughout the response.

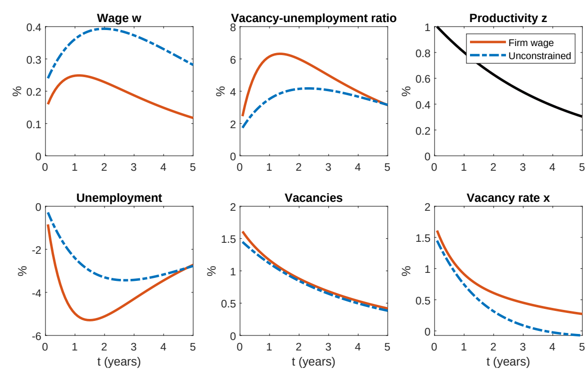

Figure F.2: Impulse Responses in Firm Wage Model: Identical Parameters

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the unconstrained model. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The two models have the same parameter values, with $b = 0.89$. Steady state unemployment is three times higher in the unconstrained model than in the firm wage model, with market tightness less than half of that in the constrained model.

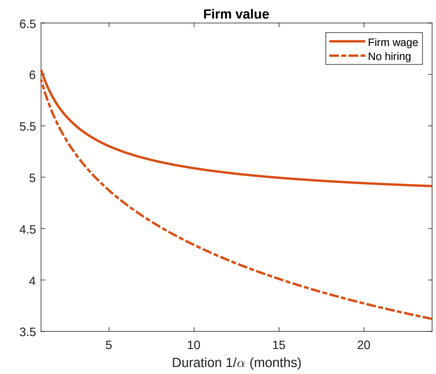

Figure F.3: Equilibrium with Infrequent Wage Adjustment: Optimality of Hiring

Notes: The figure refers to Figure 3, showing that the firm value attained by following the first order conditions dominates withdrawing from hiring for the duration of the wage, across equilibrium wage durations.

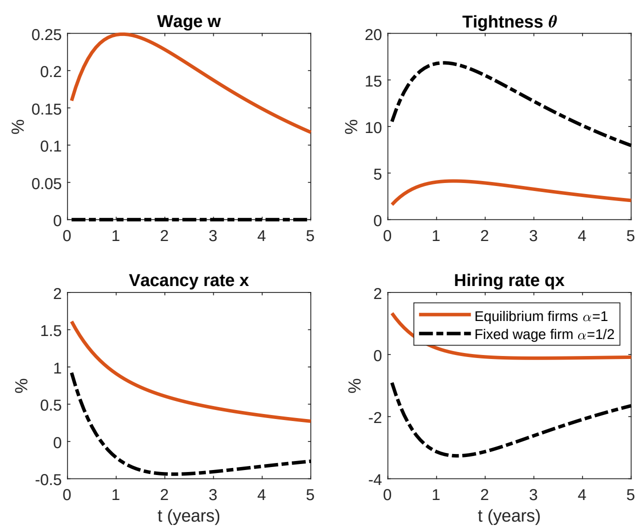

Figure F.4: Impulse Responses in Firm Wage Model: Impact of Fixed Wage

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and for a single firm deviating to a longer wage commitment. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

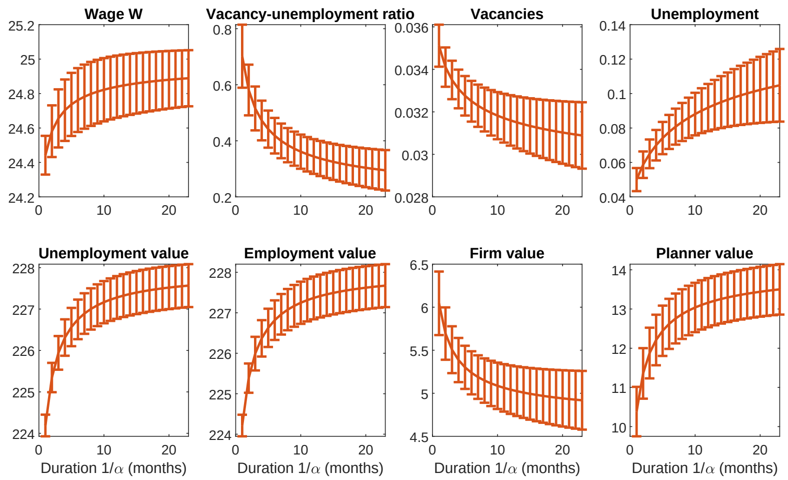

Figure F.5: Equilibrium with Infrequent Wage Adjustment and Aggregate Shocks

Notes: The figure plots simulation means together with standard deviation bounds for the equilibrium with infrequent adjustment and aggregate shocks, as a function of the duration of wages. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The firm and planner values plotted are the scaled values, but the unscaled values remain monotonic in wage duration.

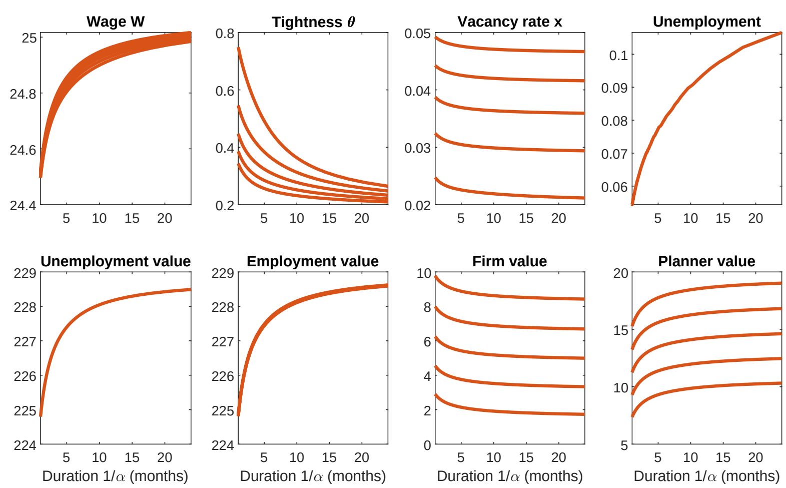

Figure F.6: Equilibrium with Infrequent Wage Adjustment and Firm Shocks

Notes: The figure plots the equilibrium with infrequent wage adjustment with firm-level shocks, as a function of the equilibrium duration of wages. The model is solved on a 5-state grid for productivity, approximating an $AR(1)$ with autocorrelation $\rho_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm and planner values plotted are the scaled values, but the unscaled values remain monotonic in wage duration.

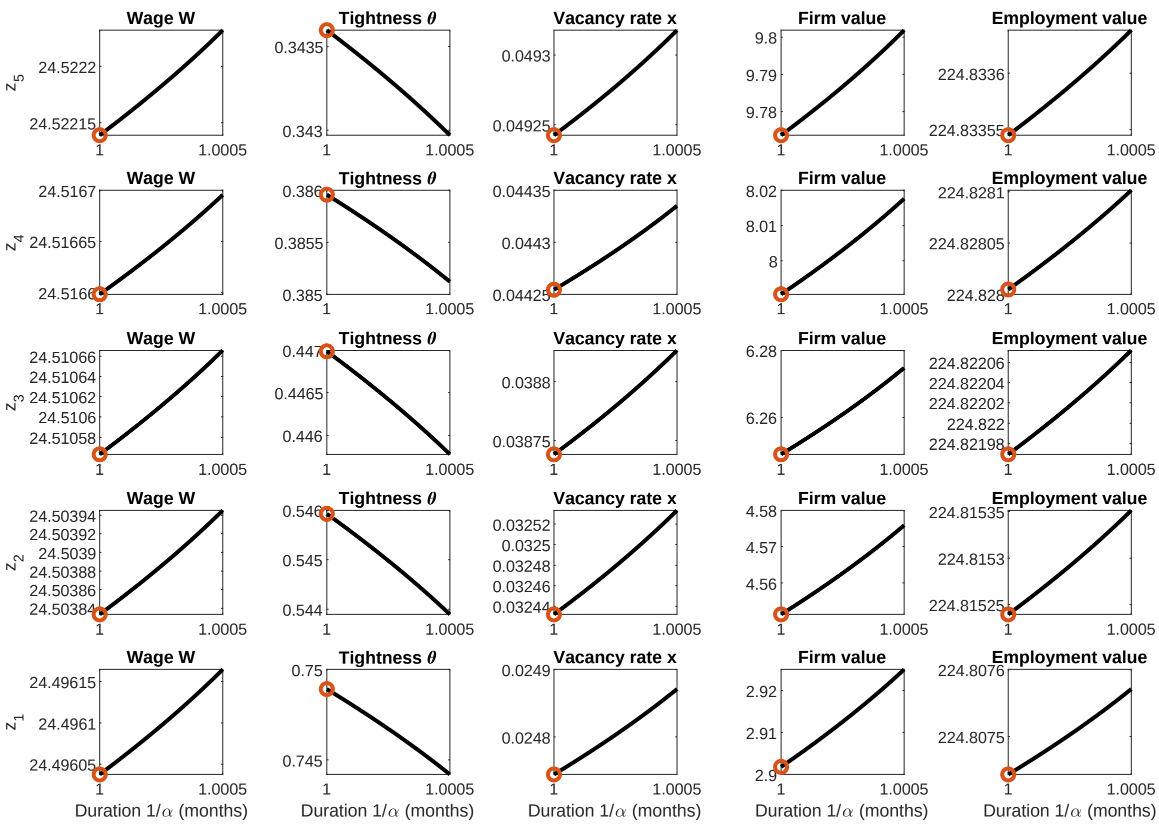

Figure F.7: Single Firm Deviation to Longer Wage Duration with Firm Shocks

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize monthly, and the figure plots corresponding values for a firm deviating to longer wage duration as a function of the expected duration of wages $1/\alpha$. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

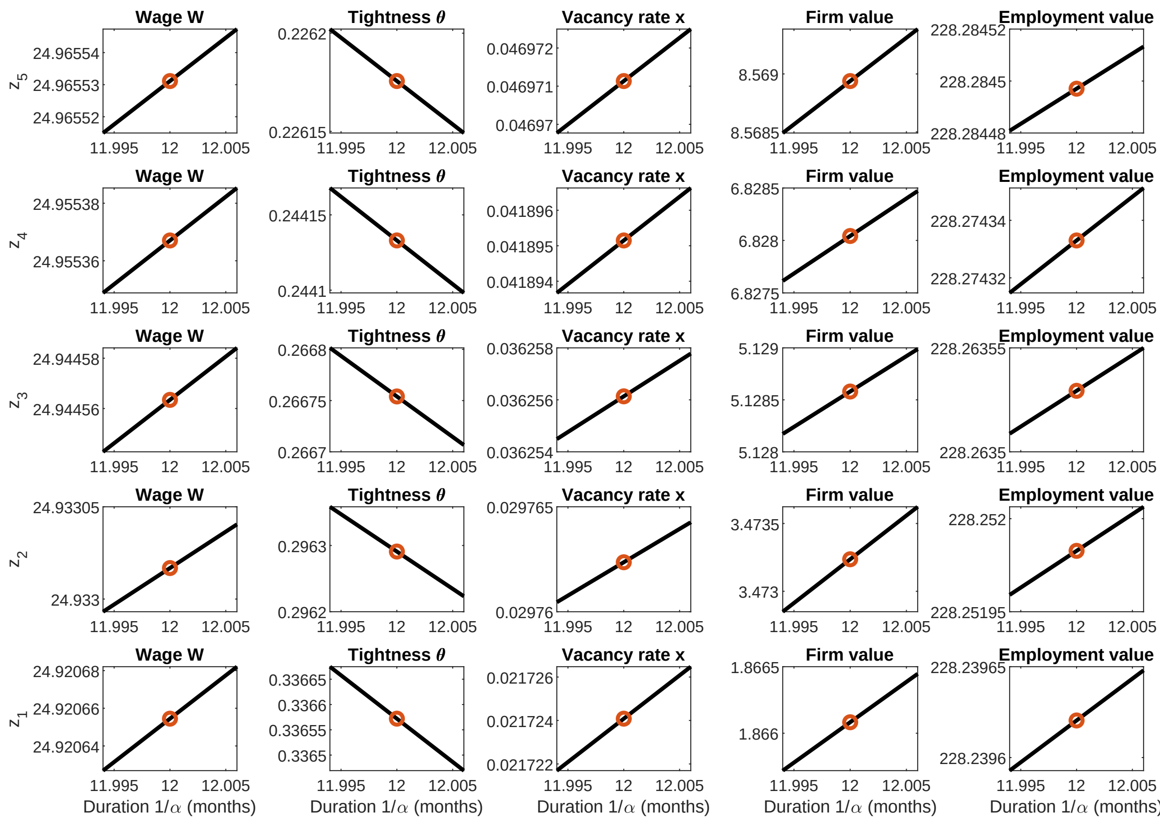

Figure F.8: Single Firm Deviation to Longer/Shorter Wage Duration with Firm Shocks

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize annually, and the figure plots corresponding values for a firm deviating to longer/shorter wage duration as a function of the expected duration of wages $1/\alpha$. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

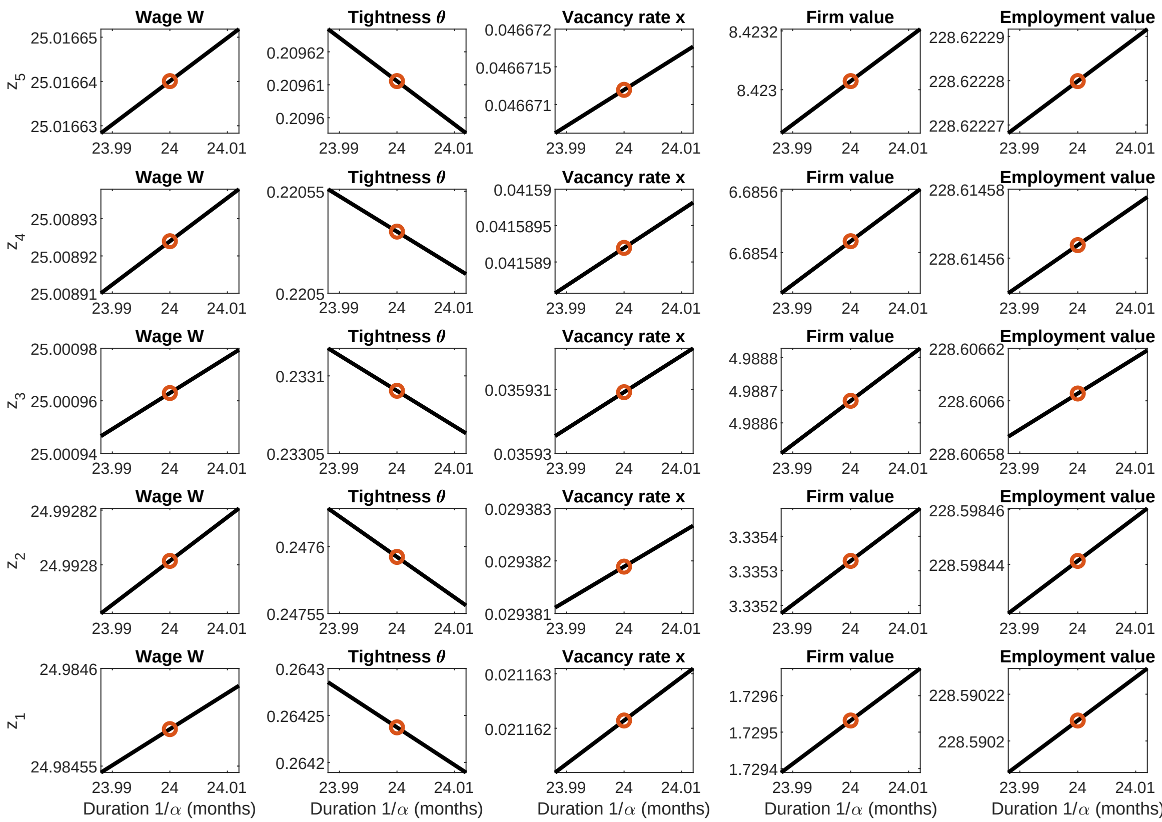

Figure F.9: Single Firm Deviation to Longer/Shorter Wage Duration with Firm Shocks

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize biennially, and the figure plots corresponding values for a firm deviating to longer/shorter wage duration as a function of the expected duration of wages $1/\alpha$. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

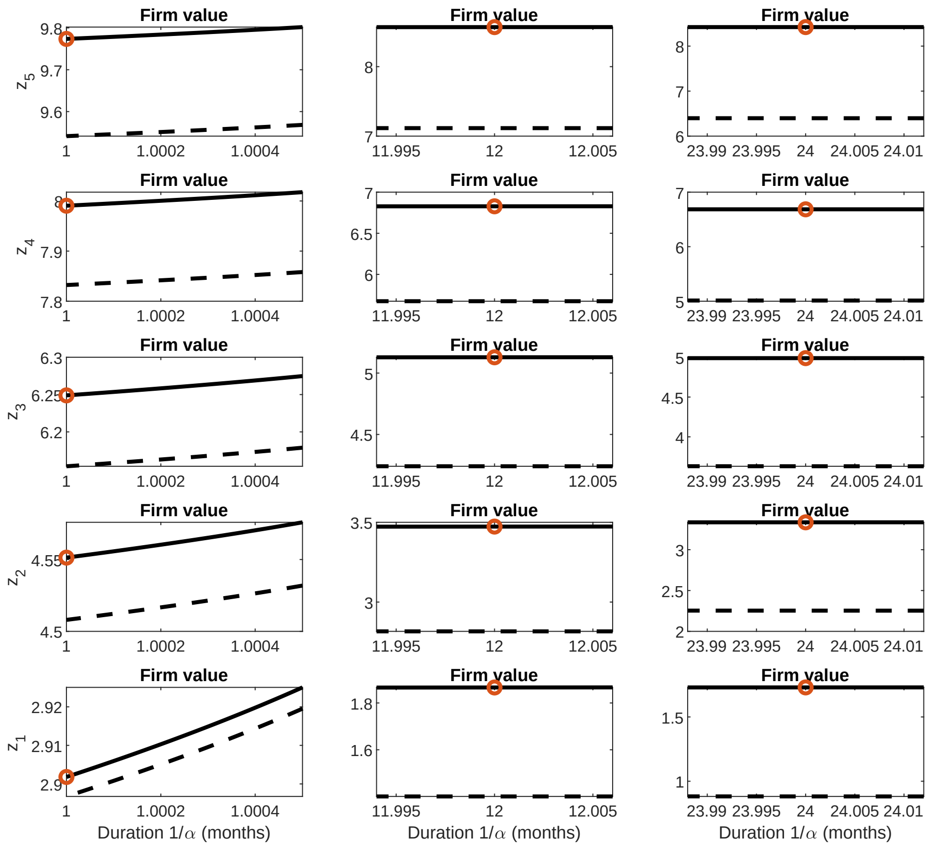

Figure F.10: Firm Deviation with Firm Shocks: Optimality of Hiring

Notes: The circles denote the equilibrium with infrequent wage adjustment where firms reoptimize monthly, annually and biennially, and the figure plots corresponding values for a firm deviating to longer/shorter wage duration as a function of the expected duration of wages $1/\alpha$. It shows that the firm value attained by following the first order conditions dominates withdrawing from hiring for the duration of the wage. The model is solved on a 5-state grid for productivity, approximating an AR(1) with autocorrelation $\sigma_z = 0.88$ and standard deviation $\sigma_z = 0.2$ based on the Rouwenhorst method. The firm value plotted is the scaled value.

G Model with Firm-Level Shocks

In a stationary equilibrium with idiosyncratic firm-specific shocks to productivity, the aggregate measure of matches N and value of search U (and hence also X, Y) remain constant, while shocks lead to reallocation of labor across firms over time.[7]

The firm problem may be written:

$$\begin{aligned}
&\max_{W,v}(n+q(\theta)v)(Z-W)-\kappa(v,n)+\beta E_z V(n',z') \\
&\text{s.t. } n'=(1-\delta)(n+q(\theta)v), \\
&\qquad X=\mu(\theta)(W-Y),
\end{aligned}$$

where the continuation value satisfies $V(n,z)=q(\theta)v(Z-W)-\kappa(v,n)+\beta E_z V(n',z')$.

Scaling by size yields the size-independent problem:

$$\begin{aligned}
&\max_{W,x}(1+q(\theta)x)(Z-W)-\hat{\kappa}(x)+\beta(1-\delta)(1+q(\theta)x)E_z\hat{V}(z') \\
&\text{s.t. } X=\mu(\theta)(W-Y),
\end{aligned}$$

where $\hat{V}(z)=q(\theta)x(Z-W)-\hat{\kappa}(x)+\beta(1-\delta)(1+q(\theta)x)E_z\hat{V}(z')$.

Infrequent Wage Adjustment Consider a firm setting a fixed wage for a probabilistic period of time. The firm's beliefs regarding the market tightness continue to be determined by the constraint $X=\mu(\theta)(h(w,z)-Y)$ each period, where $h(w,z)$ represents the present value of wages.[8]

The firm problem may be written

$$\begin{aligned}
&\max_{w,v}(n+q(\theta)v)(Z-h(w,z))-\kappa(v,n)+\beta E_z(\alpha V^r(n',z')+(1-\alpha)V^f(n',w,z')) \\
&\text{s.t. } n'=(1-\delta)(n+q(\theta)v), \\
&\qquad X=\mu(\theta)(h(w,z)-Y).
\end{aligned}$$

Here the value of reoptimizing satisfies

$$V^r(n,z)=q(\theta)v(Z-h(w,z))-\kappa(v,n)+\beta E_z(\alpha V^r(n',z')+(1-\alpha)V^f(n',w,z'))$$

[7]I abstract from entry and exit but one could easily incorporate exit shocks into the firm problem, with exiting firms replaced by new ones. The behavior of new and existing firms is identical if new firms enter with at least one worker.

[8]Suppose z lives on a grid and the transition probability matrix is denoted Π. Denote the vector of equilibrium present values of wages for a reoptimizing firm across z as $\mathbf{W}^r$ and the present value of wages of a firm holding its wage w fixed as $\mathbf{W}^f(w)$. We have that $\mathbf{W}^f(w)=w\mathbf{i}+\beta(1-\delta)[\alpha\Pi\mathbf{W}^r+(1-\alpha)\Pi\mathbf{W}^f(w)]$, where $\mathbf{i}$ a vector of ones. This gives the present value wage for a firm with wage w as $\mathbf{W}^f(w)=(I-\beta(1-\delta)(1-\alpha)\Pi)^{-1}(w\mathbf{i}+\beta(1-\delta)\alpha\Pi\mathbf{W}^r)$. I denote the components of this vector in the text by $h(w,z)$.

with the above firm choices. The value of holding the wage fixed satisfies

$$
\begin{aligned}
V^f(n', w, z') = \max_v & \, q(\theta)v(Z - h(w,z)) - \kappa(v,n) + \beta E_z(\alpha V^r(n', z') + (1-\alpha)V^f(n', w, z')) \\
\text{s.t. } & \, n' = (1-\delta)(n + q(\theta)v), \\
& \, X = \mu(\theta)(h(w,z) - Y).
\end{aligned}
$$

Scaling these problems, firms reoptimizing wages solve

$$
\begin{aligned}
& \max_{w,x}(1 + q(\theta)x)(Z - h(w,z)) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_z(\alpha \hat{V}^r(z') + (1-\alpha)\hat{V}^f(w, z')) \\
& \text{s.t. } X = \mu(\theta)(h(w,z) - Y).
\end{aligned}
$$

Here the value of reoptimizing satisfies

$$
\hat{V}^r(z) = q(\theta)x(Z - h(w,z)) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_z(\alpha \hat{V}^r(z') + (1-\alpha)\hat{V}^f(w, z'))
$$

with the above choices. The value of holding the wage fixed satisfies

$$
\begin{aligned}
\hat{V}^f(w, z') = \max_x & \, q(\theta)x(Z - h(w,z)) - \hat{\kappa}(x) + \beta(1-\delta)(1 + q(\theta)x)E_z(\alpha \hat{V}^r(z') + (1-\alpha)\hat{V}^f(w, z')) \\
\text{s.t. } & \, X = \mu(\theta)(h(w,z) - Y).
\end{aligned}
$$