Online Appendix for Trade Policies and Fiscal Devaluations

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Appendix

A1. Model Equations

Households. — Household $i \in H = [0,1]$ chooses $\{\bar{w}_t(i), w_t(i), n_t(i), c_t(i), a_{t,t+1}(i), B_{Ht}(i), B_{Ft}(i)\}$ to maximize

$$\max E_0 \Sigma_{t=0}^{\infty} \beta^t \left[\frac{[c_t(i)]^{1-\sigma}}{1-\sigma} - \frac{[n_t(i)]^{1+\eta}}{1+\eta} \right] \tag{A.1}$$

$$s.t$$

(A.2)

$$P_t c_t(i) + \Sigma_{t+1} q_{t,t+1} a_{t,t+1}(i) + B_{Ht}(i) + \varepsilon_t \left[B_{Ft}(i) + \frac{\chi}{2} \left(B_{Ft}(i) - \bar{B}_F \right)^2 \right] = R_{t-1} B_{Ht-1}(i) + \varepsilon_t R^*_{t-1} B_{Ft-1}(i) + P_t a_{t-1,t}(i) + w_t(i) n_t(i) + \widetilde{\Pi}_t + T_t$$

$$w_t(i) = \begin{cases} w_{t-1}(i) & w.p.\ \zeta_W \\ \bar{w}_t(i) & w.p.\ 1-\zeta_W \end{cases} \tag{A.3}$$

$$n_t(i) = \left[\frac{w_t(i)}{W_t} \right]^{-\gamma_n} N_t \tag{A.4}$$

where W_t is a wage index (described below) and $q_{t,t+1}$ is the price of a state contingent Arrow security paying one unit of consumption in a specific state at time $t+1$. We assume that a complete set of Arrow securities is traded domestically so that perfect risk sharing within each country allows for simple aggregation. Equation $(A.3)$ states that households can only adjust their wage with probability ζ_W. Equation $(A.4)$ is the firms' demand schedule for labor variety i, derived below.

Optimality conditions are

$$1=\beta E_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+1}}R_t\right] \tag{A.5}$$

$$1+\chi\left(B_{Ft}\left(i\right)-\bar{B}_F\right)=\beta E_t\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+1}}\frac{\varepsilon_{t+1}}{\varepsilon_t}R_t^*\right] \tag{A.6}$$

$$E_t\zeta_W^{s-t}\sum C_s^{-\sigma}\left\{\frac{\left[n_s\left(i\right)\right]^{\eta}}{C_s^{-\sigma}}\frac{\gamma_n}{\left(\gamma_n-1\right)}-\frac{\bar{W}_t}{P_s}\right\}n_s\left(i\right)=0 \tag{A.7}$$

Retailers. — The problem of retailers is as described in the main text.

Producers. — PCP pricing

Producer $i\in F=[0,1]$ chooses an optimal reset price $P_{Pt}\left(i\right)$, export prices $\{P_{Hs}^*(i)\}_{s\geq t}$ quantities $\{Y_{Hs}(i),Y_{Hs}^*(i)\}_{s\geq t}$ and employment $\left\{N_s\left(i\right),\{n_s\left(j;i\right)\}_j\right\}_{s\geq t}$ to maximize

(A.8)

$$\max E_t\sum_{s\geq t}\zeta_P^{s-t}\Lambda_{t,s}\left(1-\tau_s^{\pi}\right)\left\{\frac{\bar{P}_{Pt}(i)\left[Y_{Hs}(i)+\frac{s^*}{s}Y_{Hs}^*(i)\right]-\left(1-\varsigma_s^p\right)\int w_s\left(j\right)n_s\left(j;i\right)dj}{P_s}\right\}$$

s.t.

$$Y_{Hs}(i)+\frac{s^*}{s}Y_{Hs}^*(i)=A_sN_s^{\alpha}\left(i\right) \tag{A.9}$$

$$N_s\left(i\right)=\left\{\int\left[n_s\left(j;i\right)\right]^{\frac{\gamma_n-1}{\gamma_n}}dj\right\}^{\frac{\gamma_n}{\gamma_n-1}} \tag{A.10}$$

$$Y_{Hs}(i)=\left[\frac{\bar{P}_{Pt}(i)}{P_{Ps}}\right]^{-\gamma}Y_{Hs} \tag{A.11}$$

$$Y_{Ht}^*(i)=\left[\frac{P_{Hs}^*(i)}{P_{Hs}^*}\right]^{-\gamma}Y_{Ht}^* \tag{A.12}$$

$$P_{Hs}^{*}(i) = \frac{(1+\tau_s^{m*})}{(1+\varsigma_t^{x})}\frac{\bar{P}_{Pt}(i)}{\varepsilon_s} \tag{A.13}$$

where s^* and s are the size of the foreign and home country respectively.

The optimality conditions for this problem are constraints $(A.9)-(A.13)$ as well as an optimal pricing condition as in the text:

$$E_t\Sigma_{s=t}^{\infty}\zeta_P^{s-t}\Lambda_{t,s}\left[Y_{Hs}(i)+\frac{s^*}{s}Y_{Hs}^{*}(i)\right](1-\tau_s^{\pi})\frac{1}{P_s}\left[\overline{P}_{Pt}(i)-\frac{\gamma}{\gamma-1}\frac{(1-\varsigma_s^{p})W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right]=0 \tag{A.14}$$

where W_s is the wage index

$$W_s = \left[\int [w_s(j)]^{1-\gamma_n}\, dj\right]^{\frac{1}{1-\gamma_n}} \tag{A.15}$$

LCP pricing

Producer i chooses optimal reset prices $\bar{P}_{Pt}(i)$ and $\bar{P}_{Xt}^{*}(i)$, where $\bar{P}_{Xt}^{*}(i)$ is the foreign currency price of domestic export net of tariffs, export prices $\{P_{Hs}^{*}(i)\}_{s\geq t}$, quantities $\{Y_{Hs}(i), Y_{Hs}^{*}(i)\}_{s\geq t}$ and employment $\left\{N_s(i), \{n_s(j;i)\}_j\right\}_{s\geq t}$ to maximize

$$\max E_t\sum_{s\geq t}\zeta_P^{s-t}\Lambda_{t,s}\left\{\frac{\bar{P}_{Pt}(i)Y_{Hs}(i)+\varepsilon_s P_{Xt}^{*}(i)(1+\varsigma_s^{x})\frac{s^*}{s}Y_{Hs}^{*}(i)-(1-\varsigma_s^{p})\int w_s(j)\,n_s(j;i)\,dj}{P_s}\right\} \tag{A.16}$$

$$s.t.$$

$$Y_{Hs}(i)+\frac{s^*}{s}Y_{Hs}^{*}(i) = A_s N_s^{\alpha}(i) \tag{A.17}$$

$$N_s(i) = \left\{\int [n_s(j;i)]^{\frac{\gamma_n-1}{\gamma_n}}\, dj\right\}^{\frac{\gamma_n}{\gamma_n-1}} \tag{A.18}$$

$$Y_{Hs}(i) = \left[\frac{\bar{P}_{Pt}(i)}{P_{Ps}}\right]^{-\gamma} Y_{Hs} \tag{A.19}$$

$$Y_{Ht}^{*}(i) = \left[\frac{P_{Hs}^{*}(i)}{P_{Hs}^{*}}\right]^{-\gamma} Y_{Ht}^{*} \tag{A.20}$$

$$P^*_{Hs}(i) = \left(1+\tau^{m*}_s\right) P^*_{Xt}(i) \tag{A.21}$$

The optimality conditions for this problem are constraints $(A.17)-(A.21)$ and optimal pricing conditions for domestic and foreign markets:

$$E_t \Sigma^{\infty}_{s=t} \zeta_P^{s-t} \Lambda_{t,s} \left(1-\tau^{\pi}_s\right) \frac{Y_{Hs}(i)}{P_s} \left[\bar{P}_{Pt}(i) - \frac{\gamma}{\gamma-1} \frac{\left(1-\varsigma^p_s\right) W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right] = 0 \tag{A.22}$$

(A.23)
$$E_t \Sigma^{\infty}_{s=t} \zeta_P^{s-t} \Lambda_{t,s} \left(1-\tau^{\pi}_s\right) \frac{Y^*_{Hs}(i)}{P_s} \left[\varepsilon_s \left(1+\varsigma^x_s\right) P^*_{Xt}(i) - \frac{\gamma}{\gamma-1} \frac{\left(1-\varsigma^p_s\right) W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right] = 0$$

where W_s is the wage index

$$W_s = \left\{ \int \left[w_s\left(j\right)\right]^{1-\gamma_n} dj \right\}^{\frac{1}{1-\gamma_n}} \tag{A.24}$$

An analogous problem for the foreign producers yield

$$E_t \Sigma^{\infty}_{s=t} \zeta_P^{s-t} \Lambda_{t,s} \frac{Y^*_{Fs}(i)}{P_s} \left[\bar{P}^*_{Pt}(i) - \frac{\gamma}{\gamma-1} \frac{W^*_s}{\alpha A^*_s N^*_s(i)^{\alpha-1}}\right] = 0 \tag{A.25}$$

$$E_t \Sigma^{\infty}_{s=t} \zeta_P^{s-t} \Lambda_{t,s} \frac{Y_{Fs}(i)}{P_s} \left[\frac{1}{\varepsilon_s} \left(1+\varsigma^{x*}_s\right) \bar{P}_{X^*t}(i) - \frac{\gamma}{\gamma-1} \frac{W^*_s}{\alpha A^*_s N^*_s(i)^{\alpha-1}}\right] = 0 \tag{A.26}$$

where

$$P_{Fs}(i) = \frac{\left(1+\tau^m_s\right) \bar{P}_{X^*t}(i)}{\left(1-\tau^v_s\right)}$$

A2. Equilibrium equations

Equations $(A.27)-(A.58)$ below determine the equilibrium process $\left\{\Psi\left(s^t\right)\right\}_{s^t\in(S)^t, t\geq 0}$ for any initial value $(\mathcal{M}_{-1}, s_0)$ where s_0 is the policy regime at time 0 and $\mathcal{M}_{-1}$ collects bond holdings and the distribution of prices and wages:

$$\mathcal{M}_{-1} = \{\mathcal{A}_{-1}, \mathcal{P}_{-1}\}$$

$$\mathcal{A}_{-1} = \left\{B_{H,-1}R_{-1}, B_{F,-1}R^*_{-1}, B^*_{F,-1}R^*_{-1}, B^*_{H,-1}R_{-1}\right\}.$$

$$\mathcal{P}_{-1} = \left\{ \left\{P_{P,-1}\left(j\right), P^*_{X,-1}\left(j\right)\right\}_{j\in J}, \left\{W\left(i\right)\right\}_{i\in I}, \left\{P^*_{P,-1}\left(j\right), P_{X^*,-1}\left(j\right)\right\}_{j\in J^*}, \left\{W^*\left(i\right)\right\}_{i\in I^*} \right\}$$

For ease of exposition we group elements of Ψ into variables that we associate with households optimality conditions,Ψ_{HH} and Ψ^*_{HH} abroad, retailers optimality conditions, Ψ_{RE} and Ψ^*_{RE}, firms optimality conditions, Ψ_{FI} and Ψ^*_{FI}, price indexes, Ψ_{PI} and Ψ^*_{PI}, and market clearing conditions, Ψ_{MC}. We have that $\Psi = \{\Psi_{HH}, \Psi^*_{HH}, \Psi_{RE}, \Psi^*_{RE}, \Psi_{FI}, \Psi^*_{FI}, \Psi_P, \Psi^*_P, \Psi_{MC}\}$

Households optimality

$\Psi_{HH} = \left\{w_t\left(i\right), \bar{W}_t, n_t\left(i\right), C_t, B_{Ht}\right\}$ (leaving out budget constraint and B_{Ft})

$$w_t\left(i\right) = \begin{cases} w_{t-1}\left(i\right) & w.p.\ \zeta_W \\ \bar{W}_t & w.p.\ 1-\zeta_W \end{cases} \tag{A.27}$$

$$E_t \zeta_W^{s-t} \sum C_s^{-\sigma} \left[\frac{\left[n_s\left(i\right)\right]^\eta}{C_s^{-\sigma}} \frac{\gamma_n}{(\gamma_n - 1)} - \frac{\bar{W}_t}{P_s} \right] n_s\left(i\right) = 0 \tag{A.28}$$

$$n_t\left(i\right) = \left(\frac{w_t\left(i\right)}{W_t}\right)^{-\gamma_n} N_t \tag{A.29}$$

$$1 = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} R_t \right] \tag{A.30}$$

$$1 + \chi\left(B_{Ft}\left(i\right) - \bar{B}_F\right) = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}}{\varepsilon_t} R_t^* \right] \tag{A.31}$$

and symmetric conditions for $\Psi^*_{HH} = \left\{w^*_t\left(i\right), \bar{W}^*_t, n^*_t\left(i\right), C^*_t, B^*_{Ft}\right\}$ abroad

Retailers optimality

$\Psi_{RE} = \{Y_{Ht}, Y_{Ft}, Y_{Ht}(i), Y_{Ft}(i)\}$

$$Y_{Ht} = \omega \left[\frac{P_{Ht}}{P_t}\right]^{-\theta} C_t \tag{A.32}$$

$$Y_{Ft} = (1-\omega) \left[\frac{P_{Ft}}{P_t}\right]^{-\theta} C_t \tag{A.33}$$

$$Y_{Ht}(i) = \left(\frac{P_{Pt}(i)}{P_{Pt}}\right)^{-\gamma} Y_{Hs} \tag{A.34}$$

$$Y_F(i) = \left(\frac{P_{Ft}(i)}{P_{Ft}}\right)^{-\gamma} Y_{Ft} \tag{A.35}$$

and symmetric conditions for $\Psi_{RE} = \{Y^*_{Ft}, Y^*_{Ht}, Y^*_{Ft}(i), Y^*_{Ht}(i)\}$

Firms optimality

$$\Psi_{FI} = \left\{P^*_{Hs}(i), P_{Ft}(i), P_{Pt}(i), P^*_{Pt}(i), \overline{P}_{Pt}(i), \overline{P}^*_{Pt}(i), \bar{P}^*_{Xt}(i), \bar{P}^*_{X^*t}(i), P^*_{Xt}(i), P_{X^*t}(i)\right\}$$

$$P^*_{Ht}(i) = (1 + \tau^{m*}_t)\, P^*_{Xt}(i) \tag{A.36}$$

$$P_{Ft}(i) = \frac{1 + \tau^m_t}{1 - \tau^v_t} P_{X^*t}(i) \tag{A.37}$$

$$P_{Pt}(i) = \begin{cases} P_{Pt-1}(i) & w.p.\ \zeta_p \\ \bar{P}_{Pt}(i) & w.p.\ 1 - \zeta_p \end{cases} \tag{A.38}$$

$$P^*_{Pt}(i) = \begin{cases} P^*_{Pt-1}(i) & w.p.\ \zeta_p \\ \bar{P}^*_{Pt}(i) & w.p.\ 1 - \zeta_p \end{cases} \tag{A.39}$$

$$E_t \Sigma^{\infty}_{s=t} \zeta_P^{s-t} \Lambda_{t,s} \left[Y_{Hs}(i) + \tfrac{s^*}{s} Y^*_{Hs}(i)\right] \tfrac{1}{P_s} \left[\overline{P}_{Pt}(i) - \tfrac{\gamma}{\gamma-1} \tfrac{\left(1-\varsigma^p_s\right) W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right] = 0 \quad PCP \tag{A.40a}$$

$$E_t \Sigma^{\infty}_{s=t} \zeta_P^{s-t} \Lambda_{t,s} \tfrac{Y_{Hs}(i)}{P_s} \left[\bar{P}_{Pt}(i) - \tfrac{\gamma}{\gamma-1} \tfrac{\left(1-\varsigma^p_s\right) W_s}{\alpha A_s N_s(i)^{\alpha-1}}\right] = 0 \quad LCP \tag{A.40b}$$

$$E_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s}^* \left[\frac{s}{s^*} Y_{Fs}(i) + Y_{Fs}^*(i) \right] \frac{1}{P_s^*} \left[\overline{P}_{Pt}^*(i) - \frac{\gamma}{\gamma-1} \frac{W_s^*}{\alpha A_s N_s^*(i)^{\alpha-1}} \right] = 0 \quad PCP$$
(A.41a)

$$E_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s}^* \frac{Y_{Fs}^*(i)}{P_s^*} \left[\overline{P}_{Pt}^*(i) - \frac{\gamma}{\gamma-1} \frac{W_s^*}{\alpha A_s N_s^*(i)^{\alpha-1}} \right] = 0 \quad LCP$$
(A.41b)

$$P_{Ht}^*(i) = \frac{(1+\tau_t^{m*})}{(1+\varsigma_t^x)} \frac{P_{Pt}(i)}{\varepsilon_t} \quad PCP$$
(A.42a)

$$E_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s} \frac{Y_{Hs}^*(i)}{P_s} \left[\varepsilon_s (1+\varsigma_s^x) \bar{P}_{Xt}^*(i) - \frac{\gamma}{\gamma-1} \frac{(1-\varsigma_s^p) W_s}{\alpha A_s N_s(i)^{\alpha-1}} \right] = 0 \quad LCP$$
(A.42b)

$$P_{Ft}(i) = \frac{1+\tau_t^m}{1-\tau_t^v} \frac{P_{Pt}^*(i)\varepsilon_t}{(1+\varsigma_t^{x*})} \quad PCP$$
(A.43a)

$$E_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{t,s}^* \frac{Y_{Hs}^*(i)}{P_s} \left[\frac{(1+\varsigma_s^{x*})}{\varepsilon_s} \bar{P}_{X^*t}(i) - \frac{\gamma}{\gamma-1} \frac{W_s^*}{\alpha A_s N_s^*(i)^{\alpha-1}} \right] = 0 \quad LCP$$
(A.43b)

$$P_{X^*t}(i) = \bar{P}_{X^*t}(i) \quad PCP \tag{A.44a}$$

$$P_{X^*t+1}(i) = \begin{cases} P_{X^*t}(i) & w.p.\ \zeta_W \\ \bar{P}_{X^*t+1}(i) & w.p.\ 1-\zeta_W \end{cases} \quad LCP \tag{A.44b}$$

and symmetric conditions for $\Psi^*_{FI} = \left\{ N^*_t(i), P_{Ft}(i), P^*_{Pt}(i), \overline{P}^*_{Pt}(i), \bar{P}_{X^*t}(i), P_{X^*t}(i) \right\}$

Price indexes

$\Psi_{PI} = \{P_t, P_{Ht}, P_{Pt}, P_{Ft}, W_t\}$

$$P_t = \left[\omega P_{Ht}^{1-\theta} + (1-\omega) P_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{A.45}$$

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\gamma}\, di\right]^{\frac{1}{1-\gamma}} \tag{A.46}$$

$$P_{Pt} = P_{Ht}(1-\tau^v_t) \tag{A.47}$$

$$P_{Ft} = \left[\int_0^1 P_{Ft}(i)^{1-\gamma}\, di\right]^{\frac{1}{1-\gamma}} \tag{A.48}$$

$$W_s = \left[\int [w_s(j)]^{1-\gamma_n}\, dj\right]^{\frac{1}{1-\gamma_n}} \tag{A.49}$$

and symmetric conditions for $\Psi^*_{PI} = \{P^*_t, P^*_{Ft}, P^*_{Pt}, P^*_{Ht}, W^*_t\}$

Market Clearing

$\Psi_{MC} = \{N_t(i), N^*_t(i), N_t, N^*_t, B_{Ft}, B^*_{Ht}, \varepsilon_t, R_t, R^*_t\}$

$$Y_{Ht}(i) + \frac{s^*}{s} Y^*_{Ht}(i) = A_t N^\alpha_t(i) \tag{A.50}$$

$$Y^*_{Ft}(i) + \frac{s}{s^*}Y_{Ft}(i) = A_t N_t^{*\alpha}(i) \tag{A.51}$$

$$N_t = \int_{j\in F} N_t(j)\,dj \tag{A.52}$$

$$N^*_t = \int_{j\in F} N^*_t(j)\,dj \tag{A.53}$$

$$B_{Ft} + B^*_{Ft} = 0 \tag{A.54}$$

$$B_{Ht} + B^*_{Ht} = 0 \tag{A.55}$$

(A.56)

$$B_{Ft} - \frac{B^*_{Ht}}{\varepsilon_t} = B_{Ft-1}R^*_{t-1} - \frac{B^*_{Ht-1}}{\varepsilon_t}R_{t-1} + \frac{P_{Pt}}{(1+\varsigma^x_t)\,\varepsilon_t}\left[Y^*_{Ht} - \frac{(1+\varsigma^x_t)}{(1+\varsigma^{x*}_t)}\varepsilon_t\frac{P^*_{Pt}}{P_{Pt}}Y_{Ft}\right]$$

$$R^*_t = \frac{1}{\beta}\left(\frac{P^*_{pt}}{P^*_{pt-1}}\right)^{\varphi_\pi}\left(\frac{Y_{Ft}+Y^*_{Ft}}{Y^{flex}_{Ft}+Y^{*flex}_{Ft}}\right)^{\varphi_y}\left(\frac{\varepsilon_t}{\bar{\varepsilon}_t}\right)^{\varphi^*_\varepsilon} \tag{A.57}$$

$$R_t = \frac{1}{\beta}\left(\frac{P_{pt}}{P_{pt-1}}\right)^{\varphi_\pi}\left(\frac{Y_{Ht}+Y^*_{Ht}}{Y^{flex}_{Ht}+Y^{*flex}_{Ht}}\right)^{\varphi_y}\left(\frac{\varepsilon_t}{\bar{\varepsilon}_t}\right)^{\varphi_\varepsilon} \tag{A.58}$$

A3. Proof of Proposition 1

We let the policy regime s_t be a vector collecting all policy variables at time t

$$s_t = (\tau^m_t, \varsigma^x_t, \tau^v_t, \varsigma^p_t, \bar{\epsilon}_t, \tau^{m*}_t, \varsigma^{x*}_t)$$

We start by giving defining what it means to implement a new policy in our Markov Switching regime framework.

DEFINITION 1: *Assume that s_t is governed by $\{S,\Omega\}$ from $t = 0, ..., t^*$. A new policy from t^* is defined by a new stochastic process $\left\{\tilde{S},\tilde{\Omega}\right\}$ and a function $\tilde{\sigma} : S \to \tilde{S}$ that determines how the policy configuration at t^* changes, $\tilde{s}_{t^*} = \tilde{\sigma}(s^*_t)$, upon introduction of the new policy.*

We next define neutrality of a policy and equivalence between policies.

DEFINITION 2: *Assume that a new policy* $\left\{\tilde{S},\tilde{\Omega};\tilde{\sigma}\right\}$ *is implemented at time* t^* *replacing* $\{S,\Omega\}$. *The implementation of the policy has no allocative effects, i.e. it is neutral, if for any endogenous state* $\mathcal{M}_{t^*-1}$ *and any (continuation) equilibrium process* $\left\{\Psi\left(s^t\right)\right\}_{s^t\in(S)^{t+1-t^*},t\geq t^*}$ *under* $\{S,\Omega\}$, *there is an equilibrium process,* $\left\{\tilde{\Psi}\left(\tilde{s}^t\right)\right\}_{\tilde{s}^t\in\left(\tilde{S}\right)^{t+1-t^*},t\geq t^*}$ *under* $\left\{\tilde{S},\tilde{\Omega}\right\}$ *that induces the same probability distribution for the real allocation.*

That is, letting

$$\Xi=\left\{C\left(s^t\right),C^*\left(s^t\right),\left\{n\left(i,s^t\right),n^*\left(i,s^t\right),Y_H\left(i,s^t\right),Y_F\left(i,s^t\right),Y_H^*\left(i,s^t\right),Y_F^*\left(i,s^t\right)\right\}\right\}_{s^t\in(S)^{t+1-t^*},t\geq t^*}$$

$$\tilde{\Xi}=\left\{\tilde{C}\left(\tilde{s}^t\right),\tilde{C}^*\left(\tilde{s}^t\right),\left\{\tilde{n}\left(i,\tilde{s}^t\right),\tilde{n}^*\left(i,\tilde{s}^t\right),\tilde{Y}_H\left(i,\tilde{s}^t\right),\tilde{Y}_F\left(i,\tilde{s}^t\right),\tilde{Y}_H^*\left(i,\tilde{s}^t\right),\tilde{Y}_F^*\left(i,\tilde{s}^t\right)\right\}\right\}_{\tilde{s}^t\in\left(\tilde{S}\right)^{t+1-t^*},t\geq t^*}$$

denote the real allocation under $\left\{\Psi\left(s^t\right)\right\}_{s^t\in(S)^{t+1-t^*},t\geq t^*}$ *and* $\left\{\tilde{\Psi}\left(\tilde{s}^t\right)\right\}_{\tilde{s}^t\in\left(\tilde{S}\right)^{t+1-t^*},t\geq t^*}$ *respectively. For any* $\bar{s}_i\in S$

$$\Pr_{\left(\tilde{S},\tilde{\Omega}\right)}\left\{\tilde{\Xi}(\tilde{s}^{n+1})=\xi|\ \tilde{s}_{t^*}=\tilde{\sigma}(\bar{s}_i)\right\}=\Pr_{(S,\Omega)}\left\{\Xi(s^{n+1})=\xi|\ s_{t^*}=\bar{s}_i)\right\}$$

We also say that two policies described by $\left\{\hat{S},\hat{\Omega},\hat{\sigma}\right\}$ *and* $\left\{\tilde{S},\tilde{\Omega},\tilde{\sigma}\right\}$ *are equivalent if they induce the same probability distribution for the real allocation.*

Finally we give a definition of IX and VP policies.

DEFINITION 3: *Assume that* s_t *is governed by* $\{S,\Omega\}$ *from* $t=0,...,t^*$. *A unilateral implementation of IX of size* δ *is described by* $\left\{GP^{IX},\Omega^{IX},\sigma_\delta^{IX}\right\}$ *with* $GP^{IX}=S\cup S^{IX}$ *where the new set of states is*

$$S^{IX}=\left\{\tilde{s}=(\tilde{\tau}^m,\tilde{\varsigma}^x,\tau^v,\varsigma^p,\bar{\epsilon},\tau^{m*},\varsigma^{x*})\;\middle|\;\begin{array}{c}\frac{1+\tilde{\tau}^m}{1+\tau^m}=\frac{1+\tilde{\varsigma}^x}{1+\varsigma^x}=1+\delta\\ \exists s=(\tau^m,\varsigma^x,\tau^v,\varsigma^p,\bar{\epsilon},\tau^{m*},\varsigma^{x*})\in S\end{array}\right\},$$

the transition matrix

$$\Omega^{IX}=\left[\begin{array}{cc}\left(1-\pi^{IX}\right)\Omega & \pi^{IX}\Omega\\ (1-\rho)\,\Omega & \rho\Omega\end{array}\right]\tag{A.59}$$

allows for the possibility that the tax change is anticipated with probability π^{IX} *and then reversed with probability* ρ.

The implementation of IX is ***anticipated*** *if* $\pi^{IX}>0$ *and* σ_δ^{IX} *is the identity function, i.e.* $\sigma_\delta^{IX}(s)=s$ *for any* $s\in S$.

The implementation of IX is ***unanticipated*** *if* $\pi^{IX} = 0$ *and* σ_{δ}^{IX} *maps each element of* S *to its associated element in* S^{IX}. *That is for any* $s = (\tau^{m}, \varsigma^{x}, \tau^{v}, \varsigma^{p}, \bar{\epsilon}, \tau^{m*}, \varsigma^{x*}) \in S$

$$\sigma_{\delta}^{IX}(s) = (\tilde{\tau}^{m}, \tilde{\varsigma}^{x}, \tau^{v}, \varsigma^{p}, \bar{\epsilon}, \tau^{m*}, \varsigma^{x*}) \quad s.t. \quad \frac{1+\tilde{\tau}^{m}}{1+\tau^{m}} = \frac{1+\tilde{\varsigma}^{x}}{1+\varsigma^{x}} = 1+\delta \tag{A.60}$$

We define an anticipated and unanticipated VP policy analogously. The policy is described by $\left\{GP^{VP}, \Omega^{VP}, \sigma_{\delta}^{VP}\right\}$ *with* $GP^{VP} = S \cup S^{VP}$ *where the new set of states is*

$$S^{VP} = \left\{ \tilde{s} = (\tau^{m}, \varsigma^{x}, \tilde{\tau}^{v}, \tilde{\varsigma}^{p}, \bar{\epsilon}, \tau^{m*}, \varsigma^{x*}) \left| \begin{array}{c} \frac{1-\tilde{\tau}^{v}}{1-\tau^{v}} = \frac{1-\tilde{\varsigma}^{p}}{1-\varsigma^{p}} = \frac{1}{1+\delta} \\ \exists s = (\tau^{m}, \varsigma^{x}, \tau^{v}, \varsigma^{p}, \bar{\epsilon}, \tau^{m*}, \varsigma^{x*}) \in S \end{array} \right. \right\},$$

the transition matrix is

$$\Omega^{VP} = \begin{bmatrix} \left(1-\pi^{VP}\right)\Omega & \pi^{VP}\Omega \\ (1-\rho)\Omega & \rho\Omega \end{bmatrix}, \tag{A.61}$$

and the function describing the unanticipated transition to VP is given by

$$\sigma_{\delta}^{VP}(s) = (\tau^{m}, \varsigma^{x}, \tilde{\tau}^{v}, \tilde{\varsigma}^{p}, \bar{\epsilon}, \tau^{m*}, \varsigma^{x*}) \quad s.t. \quad \frac{1-\tilde{\tau}^{v}}{1-\tau^{v}} = \frac{1-\tilde{\varsigma}^{p}}{1-\varsigma^{p}} = \frac{1}{1+\delta}. \tag{A.62}$$

Notice that the process $\left\{GP^{IX}, \Omega^{IX}\right\}$ does not encompass the possibility of retaliation which we will introduce below.

PROPOSITION 1: *In an economy with flexible exchange rates ($\varphi_{\varepsilon} = 0$) a unilateral IX policy of size δ and a unilateral VP policy of size $\frac{\delta}{1+\delta}$ are both neutral and cause a $\delta-$ percent appreciation of the real exchange rate if*

1) *The policy is permanent and unanticipated;*
2) *Foreign holdings of home-currency-denominated bonds are always zero ($\chi^{*} = \infty$);*
3) *Export prices are set in the producer's currency (PCP), or prices are flexible.*

PROOF:

Condition 1 implies that $\pi^{IX} = \pi^{VP} = 0$ and $\rho = 1$. In this case the transition matrices in A.59 and A.61 are simply

$$\Omega^{IX} = \Omega^{VP} = \begin{bmatrix} \Omega & 0 \\ 0 & \Omega \end{bmatrix} \tag{A.63}$$

Let $\left\{\Psi\left(s^t\right)\right\}_{s^t \in (S)^t, t \geq 0}$ denote an equilibrium process before the implementation of the new policy, i.e. when s_t is governed $\{S, \Omega\}$. Assume without loss of generality that the new policy is implemented at $t^* = 0$.

Neutrality of IX

Let $\{\mu_t^{IX}\}_{t \geq 0}$ be a sequence of function that map histories in which IX is implemented into a histories in which IX is not implemented: i.e. $\forall \tilde{s}^t = (\tilde{s}_0, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^{t+1}$, $\mu_t\left(\tilde{s}^t\right) = s^t = (s_0, ..., s_t) \in (S)^{t+1}$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if\ \tilde{s}_i \in S \\ \left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_i) & if\ \tilde{s}_i \in S^{IX} \end{cases}$$

where σ_δ^{IX} is as defined in A.60.

Consider now a process $\left\{\tilde{\Psi}^{IX}\left(s^t\right)\right\}_{s^t \in \left(\tilde{S}\right)^t, t \geq 0}$ with an unanticipated permanent IX such that, for each element $\tilde{\kappa}^{IX}$ of $\tilde{\Psi}^{IX}$, other than the nominal exchange rate, $\tilde{\varepsilon}_t^{IX}$,and home currency producer prices of foreign exporters, $\tilde{P}_{X^*t}^{IX}(i)$, we have

$$\tilde{\kappa}^{IX}\left(\tilde{s}^t\right) = \kappa\left(\mu_t^{IX}\left(\tilde{s}^t\right)\right) \qquad \forall \tilde{s}^t \in \left(GP^{IX}\right)^t, \quad \forall t \geq 0 \tag{A.64}$$

where κ is the corresponding element of the equilibrium process Ψ without IX. For ease of notation in what follows, for any $\tilde{s}^t = (\tilde{s}_0, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^{t+1}$, we let $\tilde{\kappa}_t^{IX} = \tilde{\kappa}^{IX}\left(\tilde{s}^t\right)$ and $\kappa_t = \kappa\left(\mu_t^{IX}\left(\tilde{s}^t\right)\right)$.

The nominal exchange rate and the home currency producer prices of foreign exporters are $\forall \tilde{s}^t = (\tilde{s}_0, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^{t+1}$

$$\tilde{\varepsilon}_t^{IX} = \begin{cases} \varepsilon_t & if\ \tilde{s}_t \in S \\ \frac{\varepsilon_t}{1+\delta} & if\ \tilde{s}_t \in S^{IX} \end{cases} \tag{A.65}$$

$$\tilde{P}_{X^*t}^{IX}(i) = \begin{cases} P_{X^*t}(i) & if\ s_t \in S \\ \frac{1}{1+\delta} P_{X^*t}(i) & if\ s_t \in S^{IX} \end{cases} \tag{A.66}$$

We want to show that $\left\{\tilde{\Psi}^{IX}\left(s^t\right)\right\}_{s^t \in \left(GP^{IX}\right)^{t+1}, t \geq 0}$ is an equilibrium.

We first show that $\tilde{\Psi}^{IX}\left(s^t\right)$ satisfies all of the equations directly affected by the tariffs and export subsidy change when $\tilde{s}_t \in S^{IX}$. These equations are the laws of one price $(A.42a)-(A.43a)$, the tax pass-through equations $(A.37)-(A.36)$, and the balance of payment equilibrium $(A.56)$. Considering the law of one price for domestic goods at an history $\tilde{s}^t$ such that $\tilde{s}_t \in S^{IX}$ and letting $\left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_t) \in S$ we see that

$$\tilde{P}_{H,t}^{*IX}(i) = P_{H,t}^{*}(i) = P_{H,t}(i)\frac{1+\tau_t^{m*}}{1+\sigma_t^x}\frac{1}{\varepsilon_t} \tag{A.67}$$

$$= \tilde{P}_{H,t}^{IX}(i)\frac{1+\tau_t^{m*}}{(1+\tilde{\sigma}_t^x)}\frac{1}{\tilde{\varepsilon}_t^{IX}} \tag{A.68}$$

where the first and third equalities follow from $(A.64)$, $(A.65)$ and $(A.60)$ and the second from the fact that Ψ is an equilibrium. An analogous arguemt holds for $(A.43a)$ and $(A.37)$.

Consider now the balance of payment equilibrium which, under condition 2 is

$$\tilde{B}_{Ft}^{IX} = \tilde{B}_{Ft-1}^{IX}\tilde{R}_{t-1}^{*IX} + \frac{\tilde{P}_{Pt}^{IX}}{(1+\tilde{\varsigma}_t^x)\,\tilde{\varepsilon}_t^{IX}}\left[\tilde{Y}_{Ht}^{*IX} - (1+\tilde{\varsigma}_t^x)\,\tilde{\varepsilon}_t^{IX}\frac{\tilde{P}_{Pt}^{*IX}}{\tilde{P}_{Pt}^{IX}}\tilde{Y}_{Ft}^{IX}\right]$$

to see that this is satisfied, let again $\left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_t) = s_t \in S$ to get

$$\begin{aligned}\tilde{B}_{Ft}^{IX} &= B_{Ft} = B_{Ft-1}R_{t-1}^{*} + \frac{P_{Pt}}{(1+\varsigma_t^x)\,\varepsilon_t}\left[Y_{Ht}^{*} - (1+\varsigma_t^x)\,\varepsilon_t\frac{P_{Pt}^{*}}{P_{Pt}}Y_{Ft}\right]\\ &= \tilde{B}_{Ft-1}^{IX}\tilde{R}_{t-1}^{*IX} + \frac{\tilde{P}_{Pt}^{IX}}{(1+\tilde{\varsigma}_t^x)\,\tilde{\varepsilon}_t^{IX}}\left[\tilde{Y}_{Ht}^{*IX} - (1+\tilde{\varsigma}_t^x)\,\tilde{\varepsilon}_t^{IX}\frac{\tilde{P}_{Pt}^{*IX}}{\tilde{P}_{Pt}^{IX}}\tilde{Y}_{Ft}^{IX}\right]\end{aligned}$$

where the first and third equality follow from $(A.64)$ $(A.65)$ and $(A.60)$ and the second from the fact that Ψ is an equilibrium.

We then need to check that the adjustment of the nominal exchange rate and local currency producer prices of exports in $(A.65)-(A.66)$ does not induce violations in other equilibrium equations. Under PCP $\tilde{P}_{Xt}^{IX*}(i)$ and $\tilde{P}_{X^*t}^{IX}(i)$ only affect $(A.37)$ and $(A.36)$, i.e. they are definitions. The exchange rate ε_t affects optimal holdings of foreign bonds $(A.31)$ and an analogous condition abroad. As long as $\pi^{IX} = 0$ and $\rho = 1$ we have that $\forall s^t \in \left(GP^{IX}\right)^t$, if $s^{t+1} \in \left(GP^{IX}\right)^t$ has positive probability, $\Pr\left\{s^{t+1}\left|s^t\right.\right\} > 0$, the appreciation is identical across equilibria:

$$\frac{\tilde{\varepsilon}_{t+1}}{\tilde{\varepsilon}_t} = \frac{\varepsilon_{t+1}}{\varepsilon_t}$$

and since these conditions only depend on exchange rate appreciation they are satisfied.

Neutrality of VP

Let $\{\mu_t^{VP}\}_{t\geq 0}$ be a sequence of function that map histories in which VP is implemented into a histories in which VP is not implemented: i.e. $\forall \tilde{s}^t = (\tilde{s}_0, ..., \tilde{s}_t) \in \left(GP^{VP}\right)^{t+1}$, $\mu_t\left(\tilde{s}^t\right) = s^t = (s_0, ..., s_t) \in (S)^{t+1}$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if\ \tilde{s}_i \in S \\ \left(\sigma_\delta^{VP}\right)^{-1}(\tilde{s}_i) & if\ \tilde{s}_i \in S^{VP} \end{cases}$$

where σ_δ^{VP} is as defined in A.60.

Consider the process $\left\{\tilde{\Psi}^{VP}\left(s^t\right)\right\}_{s^t\in(GP^{VP})^t, t\geq 0}$ with an unanticipated permanent VP implementation such that, for each element $\tilde{\kappa}^{VP}$ of $\tilde{\Psi}^{VP}$, *other than domestic prices* $\left(\tilde{P}_{Ht}^{VP}(i), \tilde{P}_{Ft}^{VP}(i), \tilde{P}_t^{VP}(i)\right)$ *and wages* $\left(\tilde{\bar{w}}_t^{VP}(i), \tilde{w}_t^{VP}(i), \tilde{W}_t^{VP}\right)$ *and the associated price indexes* $\left(\tilde{P}_{Ht}^{VP}, \tilde{P}_{Ft}^{VP}, \tilde{P}_t^{VP}\right)$,

$$\tilde{\kappa}^{VP}\left(\tilde{s}^t\right) = \kappa\left(\mu_t^{VP}\left(\tilde{s}^t\right)\right) \qquad \forall \tilde{s}^t \in \left(\tilde{S}^{VP}\right)^t, \quad \forall t \geq 0 \tag{A.69}$$

where κ is the corresponding element of the equilibrium process Ψ without VP.

Prices and wages satisfy $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{VP}\right)^t$

$$\frac{\tilde{P}_{H,t}^{VP}(i)}{P_{H,t}(i)} = \frac{\tilde{P}_{F,t}^{VP}(i)}{P_{F,t}(i)} = \frac{\tilde{P}_t^{VP}(i)}{P_t(i)} = \begin{cases} 1 & if\ \tilde{s}_t \in S \\ (1+\delta) & if\ \tilde{s}_t \in S^{VP} \end{cases} \tag{A.70}$$

$$\frac{\tilde{\bar{w}}_t^{VP}(i)}{\bar{w}_t(i)} = \frac{\tilde{w}_t^{VP}(i)}{\tilde{w}_t(i)} = \frac{\tilde{W}_t^{VP}}{W_t} = \begin{cases} 1 & if\ \tilde{s}_t \in S \\ (1+\delta) & if\ \tilde{s}_t \in S^{VP} \end{cases} \tag{A.71}$$

We want to show that $\left\{\tilde{\Psi}^{VP}\left(s^t\right)\right\}_{s^t\in(GP^{VP})^t, t\geq 0}$ is an equilibrium, which given $(A.70)$ and the fact that ε_t is unaffected also implies that the real echange rate appreciates by δ.

As discussed in section II of the paper, VP instruments directly effect the two equations determining the labor market equilibrium and the dynamic Euler equations for consumption. Consider the optimality condition for the price of the domestic good at home at an history $\tilde{s}^t \in \left(GP^{VP}\right)^t$ such that $\tilde{s}_t \in S^{VP}$:

$$\begin{aligned}\bar{P}^{VP}_{Pt}(i) &= \bar{P}_{Pt}(i) = (1-\varsigma^p_t) E_t \sum_{s\geq t} \tilde{\Lambda}_{t,s}(i) \frac{(1-\varsigma^p_s)}{(1-\varsigma^p_t)} \frac{\gamma}{\gamma-1} \frac{W_s}{\alpha A_s N_s^{\alpha-1}(i)} \\ &= (1-\tilde{\varsigma}^p_t) E_t \sum_{s\geq t} \tilde{\Lambda}^{VP}_{t,s}(i) \frac{(1-\tilde{\varsigma}^p_s)}{(1-\tilde{\varsigma}^p_t)} \frac{\gamma}{\gamma-1} \frac{W^{VP}_s}{\alpha A_s \left(N^{VP}_s\right)^{\alpha-1}(i)} \end{aligned} \tag{A.72}$$

where the first equality follows from A.69, the second from the fact that Ψ is an equilibrium and the third from A.62 and A.71 together with the fact that with $\rho = 1$, we have $\frac{\tilde{P}^{VP}_s}{P_s} = \frac{\tilde{W}^{VP}_s}{W_s} = \frac{\left(1-\varsigma^p_s\right)}{\left(1-\tilde{\varsigma}^p_s\right)} = 1+\delta$ w.p. 1. Notice that the permanent effect on consumer price inflation is need to ensure that $\tilde{\Lambda}^{VP}_{t,s} = \tilde{\Lambda}^{VP}_{t,s}$ state by state, as can be seen by equation (28) in the paper.

With flexible wages, optimal labor supply is also satisfied since real wages are unaffected:

$$\frac{\left[\tilde{n}^{VP}_t(i)\right]^\eta}{\tilde{C}^{VP-\sigma}_t} \frac{\gamma_n}{(\gamma_n-1)} - \frac{\tilde{\bar{w}}^{VP}_t(i)}{\tilde{P}^{VP}_t} = \frac{\left[n_t(i)\right]^\eta}{C^{-\sigma}_t} \frac{\gamma_n}{(\gamma_n-1)} - \frac{\bar{w}_t(i)}{P_t} = 0$$

Morevoer, since the transition from $s_{t-1} \in S$ to $s_t \in S^{VP}$ is unanticipated, the different inflation dynamic ex post does not affect optimal bond holdings ex ante. On the other hand since the policy is permanent, future inflation is unaffected by its implementation as is clear from $(A.70)$

A4. Proof of Proposition 2

We start by giving a definition of a permanent unexpected appreciation of the nominal exchange rate.

DEFINITION 4: *Assume that s_t is governed by $\{S, \Omega\}$ from $t = 0, ..., t^*$. A currency devaluation of size δ is described by $\{GP^\epsilon, \Omega^\epsilon, \sigma^\epsilon_\delta\}$ with $GP^\epsilon = S \cup S^\epsilon$ where the new set of states is*

$$S^\epsilon = \left\{ \tilde{s} = \left(\tau^m, \varsigma^x, \tau^v, \varsigma^p, \tilde{\bar{\epsilon}}, \tau^{m*}, \varsigma^{x*}\right) \middle| \begin{array}{c} \frac{\tilde{\bar{\epsilon}}}{\bar{\epsilon}} = 1+\delta \\ \exists s = \left(\tau^m, \varsigma^x, \tau^v, \varsigma^p, \bar{\epsilon}, \tau^{m*}, \varsigma^{x*}\right) \in S \end{array} \right\},$$

the transition matrix is

$$\Omega^\epsilon = \begin{bmatrix} (1-\pi^\epsilon)\,\Omega & \pi^\epsilon \Omega \\ (1-\rho)\,\Omega & \rho\Omega \end{bmatrix}$$

and the function describing the unanticipated transition to VP is given by

$$\sigma_{\delta}^{\epsilon}(s) = (\tau^{m}, \varsigma^{x}, \tau^{v}, \varsigma^{p}, \tilde{\tilde{\epsilon}}, \tau^{m*}, \varsigma^{x*})$$

$$s.t. \tag{A.73}$$

$$\frac{\tilde{\tilde{\epsilon}}}{\tilde{\epsilon}} = 1 + \delta.$$

PROPOSITION 2: *In a fixed exchange rate regime ($\varphi_{\varepsilon} = \infty$), under assumptions 1.- 3. of Proposition 1, an IX policy of size δ has the same allocative effects as a once-and-for-all unexpected currency devaluation of size δ. A VP policy of the same size $\frac{\delta}{1+\delta}$ has no effect on the allocation but causes the real exchange rate to appreciate by δ.*

PROOF:

The fact that VP is still neutral even under fixed exchange rates is a straightforward consequence of the proof of Proposition 2. Since under flexible exchange rates VP is neutral and the nominal exchang rate is unaffected by its implementation, it follows that even if monetary policy targets a given fixed exchange rate the policy still remains neutral.

Turning to the equivalence between a currency devaluation and IX, let $\{\mu_t^{\epsilon}\}_{t\geq 0}$ be a sequence of functions that map histories in which IX is implemented into histories in which a currnecy devaluation is implemented instead: i.e. $\forall \tilde{s}^t = (\tilde{s}_0, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^{t+1}$, $\mu_t^{\epsilon}\left(\tilde{s}^t\right) = s^t = (s_0, ..., s_t) \in \left(GP^{\epsilon}\right)^{t+1}$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if \ \tilde{s}_i \in S \\ \sigma_{\delta}^{\varepsilon}\left(\left(\sigma_{\delta}^{IX}\right)^{-1}(\tilde{s}_i)\right) & if \ \tilde{s}_i \in S^{IX} \end{cases}$$

where σ_{δ}^{IX} is as defined in A.60 and $\sigma_{\delta}^{\epsilon}$ is as defined in A.73.

Let $\left\{\Psi^{\varepsilon}\left(s^t\right)\right\}_{s^t\in(S^T)^t, t\geq 0}$ denote an equilibrium process under $\{GP^{\varepsilon}, \Omega^{\varepsilon}, \sigma_{\delta}^{\epsilon}\}$ and consider now the process $\left\{\tilde{\Psi}^{IX}\left(s^t\right)\right\}_{s^t\in(GP^{IX})^t, t\geq 0}$ with an unanticipated permanent IX such that, for each element $\tilde{\kappa}^{IX}$ of $\tilde{\Psi}^{IX}$, *apart from the nominal exchange rate*, we have

$$\tilde{\kappa}^{IX}\left(\tilde{s}^t\right) = \kappa^{\varepsilon}\left(\mu_t^{\varepsilon}\left(\tilde{s}^t\right)\right) \qquad \forall \tilde{s}^t \in \left(GP^{IX}\right)^t, \ \ \forall t \geq 0 \tag{A.74}$$

where κ^{ε} is the corresponding element of the equilibrium process Ψ^{ε}.

The exchange rate satisfies $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^t$

$$\tilde{\varepsilon}_t^{IX} = \begin{cases} \varepsilon_t^{\varepsilon} & if \ \tilde{s}_t \in S \\ \frac{\varepsilon_t^{\varepsilon}}{1+\delta} & if \ \tilde{s}_t \in S^{IX} \end{cases} \tag{A.75}$$

To show that $\left\{\tilde{\Psi}^{IX}\left(s^t\right)\right\}_{s^t\in\left(GP^{IX}\right)^t,t\geq 0}$ is an equilibrium we can follow the same steps as in the proof Proposition 1.

At $\tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(GP^{IX}\right)^t$ such that $\tilde{s}_t \in S^{IX}$, the laws of one price and the balance of payment equilibrium equations are satisfied since

$$\frac{\tilde{\varepsilon}_t^{\epsilon}}{\tilde{\varepsilon}_t^{IX}} = \frac{(1+\tilde{\sigma}_t^x)}{(1+\sigma_t^x)} = \frac{(1+\tilde{\tau}_t^m)}{(1+\tau_t^m)}$$

and the only other equations in which the exchange rate appears only depend on its expected appreciation which is the same in the two processes.

A5. Reversal of IX policies and retaliation

We have asserted that the IX policy with reversal considered in the text has very similar effects to an IX policy subject to possible retaliation, meaning in the latter case that agents expect that the foreign government may retaliate in kind sometime in the future. Here we make this argument formally.

First we introduce a new variable, T_t^I, that measures international transfers from the foreign to the home country. The introduction of these transfers allows us to measure the distance between the allocations under reversal and under retaliation in a very simple way. The only equilibrium equation that is modified by the introduction of this transfer is the balance of payment equation A.56 which becomes

(A.76)
$$B_{Ft}-\frac{B_{Ht}^*}{\varepsilon_t} = B_{Ft-1}R_{t-1}^*-\frac{B_{Ht-1}^*}{\varepsilon_t}R_{t-1}+\frac{P_{Pt}}{(1+\varsigma_t^x)\,\varepsilon_t}\left[Y_{Ht}^*-\frac{(1+\varsigma_t^x)}{(1+\varsigma_t^{x*})}\varepsilon_t\frac{P_{Pt}^*}{P_{Pt}}Y_{Ft}\right]+T_t^I.$$

Consider an IX policy subject to policy reversal and characterized by $\left\{S^T,\Omega^T\right\}$ where $S^T = \left\{s^{NT}, s^{IX}\right\}$. In state $\left(s^{NT}\right)$ no country levies any taxes and in the second state $\left(s^{IX}\right)$ the home country unilaterally raises import tariffs and export subsidies by the same amount δ. The transition matrix is

$$\Omega^T = \begin{bmatrix} 1 & 0 \\ 1-\rho & \rho \end{bmatrix} \tag{A.77}$$

Consider also an IX policy that triggers retaliation and characterized by $\left\{S^R,\Omega^R\right\}$, where $S^R = \left\{S^T, s^{TW}\right\}$. S^T includes the same two states as described above but in s^{TW} the foreign country retaliates with a symmetric policy (i.e. $\tau_t^m = \varsigma_t^x = \tau_t^{m*} = \varsigma_t^{x*} = \delta$). In this case the transition probability matrix is:

$$\Omega^R = \begin{bmatrix} 1 & 0 & 0 \\ (1-\pi)(1-\rho) & \rho & \pi(1-\rho) \\ 1-\varphi & 0 & \varphi \end{bmatrix} \tag{A.78}$$

Lemma 1 *If export prices are set in producer currency, a unilateral implementation of IX with policy reversal, i.e. s_t governed by $\left\{S^T, \Omega^T\right\}$, implements the same equilibrium allocation as a unilateral implementation of IX that triggers retaliation,* i.e. *s_t governed by $\left\{S^R, \Omega^R\right\}$, coupled with international transfers that satisfy:*

$$T^I_{t_1} = -\frac{\delta}{1+\delta}\left[B_{F,t_1-1}R^*_{t_1-1}\varepsilon_{t_1} + B_{H,t_1-1}R_{t_1-1}\right]$$

$$T^I_{t_2} = \delta\left[B_{F,t_2-1}R^*_{t_2-1}\varepsilon_{t_2} + B_{H,t_2-1}\frac{R_{t_2-1}}{\pi_{t_2}}\right]$$

where t_1 is the first time the economy transits to the retaliation state s^{TW} and $t_2 > t_1$ is the first time it leaves the retaliation state s^{TW}.

The intuition of this lemma can be easily understood by considering the special case of a permanent transition to a trade war regime starting from balanced trade. In this case, $T^I_{t_1} = 0$ and $T^I_{t_2}$ never occurs so that Lemma 1 implies that the effects of starting a trade war are identical to the effects of abolishing all tariffs and subsidies in both countries. The reason can be easily understood by inspecting equation $(A.43a)$,where export subsidies in the foreign country exactly offset import tariffs in the home country, and, symmetrically, equation $(A.42a)$.

When the home country has a positive net foreign asset position, however, a transition to a trade war regime will not be equivalent to a transition to a state with no taxes. Given that a positive net foreign asset position implies that the home country is expected to run trade deficits in the future, import tariff revenues will exceed export subsidy expenditures, implying a positive wealth effect and an associated appreciation of the home currency. Symmetrically, the foreign economy will suffer wealth losses from its implementation of IX. Consequently, a transfer of resources that corrects this international wealth redistribution is needed to implement the same allocation under policy reversal and retaliation. Under our assumption of balanced trade in the long run, however, the economic effects of these transfers are of second order.

Proof. Let $\left\{\Psi\left(s^t\right)\right\}_{s^t\in(S^T)^t, t\geq 0}$ be an equilibrium with no international transfers and no retaliation, i.e. $T^I\left(s^t\right) = 0\ \forall s^t \in \left(S^T\right)^t$.

Consider now the process $\left\{\tilde{\Psi}\left(s^t\right)\right\}_{s^t\in(S^R)^t, t\geq 0}$ such that, for each element $\tilde{\kappa}$ of $\tilde{\Psi}$, *other than bond holdings and local currency producer prices of exports*, we have

$$\tilde{\kappa}\left(s^t\right) = \kappa\left(\mu_t\left(s^t\right)\right) \qquad \forall s^t \in \left(S^R\right)^t, \quad \forall t \geq 0 \tag{A.79}$$

where κ is the corresponding element of the equilibrium process Ψ without trade wars and function μ_t maps all histories in which a trade war occurs into a history in which no taxes are levied: that is $\forall s^t = (s_1, ..., s_t) \in \left(S^R\right)^t$, $\mu_t\left(s^t\right) = \tilde{s}^t =$

$(\tilde{s}_1, ..., \tilde{s}_t) \in \left(S^T\right)^t$ where $\forall i \geq 1$

$$\tilde{s}_i = \begin{cases} s_i & if \ s_i \neq s^{TW} \\ s^{NT} & if \ s_i = s^{TW} \end{cases}.$$

For ease of notation in what follows, for any $s^t = (s_1, ..., s_t) \in \left(S^R\right)^t$, we let $\tilde{\kappa}_t = \tilde{\kappa}\left(s^t\right)$ and $\kappa_t = \kappa\left(\mu\left(s^t\right)\right)$.

Bond holdings and local currency producer prices of exports satisfy $\forall s^t = (s_1, ..., s_t) \in \left(S^R\right)^t$

$$\text{(A.80)} \qquad \frac{\tilde{B}_{F,t}}{B_{F,t}} = \frac{\tilde{B}_{H,t}}{B_{H,t}} = \begin{cases} 1 & if \ s_t \neq s^{TW} \\ \frac{1}{1+\delta} & if \ s_t = s^{TW} \end{cases}$$

$$\text{(A.81)} \qquad \frac{\tilde{P}_{X^*t}}{P_{X^*t}} = \frac{\tilde{P}^*_{Xt}}{P^*_{Xt}} = \begin{cases} 1 & if \ s_t \neq s^{TW} \\ \frac{1}{1+\delta} & if \ s_t = s^{TW} \end{cases}$$

We want to show that $\left\{\tilde{\Psi}\left(s^t\right)\right\}_{s^t \in (S^R)^t, t \geq 0}$ is an equilibrium when international transfers satisfy

(A.82)

$$\tilde{T}^I\left(s^t\right) = \begin{cases} 0 & if \ s_{t-1} \neq s^{TW} \text{ and } s_t \neq s^{TW} \\ -\frac{\delta}{1+\delta}\left[\tilde{B}_{F,t-1}\tilde{R}^*_{t-1}\tilde{\varepsilon}_t + \tilde{B}_{H,t-1}\tilde{R}_{t-1}\right] & if \ s_{t-1} \neq s^{TW} \text{ and } s_t = s^{TW} \\ \frac{\delta}{1+\delta}\left[\tilde{B}_{F,t-1}\tilde{R}^*_{t-1}\tilde{\varepsilon}_t + \tilde{B}_{H,t-1}\tilde{R}_{t-1}\right] & if \ s_{t-1} = s^{TW} \text{ and } s_t \neq s^{TW} \end{cases}.$$

It is straightforward to check that if Ψ_t is an equilibrium then $\tilde{\Psi}_t$ satisfies all equilibrium equations other than $(A.56)$. When $s_t = s^{TW}$ the only conditions that need to be checked are the laws of one price $(A.42a) - (A.43a)$ and the tax pass-through equations $(A.37) - (A.36)$ which are satisfied under $(A.81)$. All the other equations are clearly satisfied by construction of $\tilde{\Psi}$, and by the fact that the probability of leaving the unilateral IX state is the same in $(A.77)$ and $(A.78)$.

Consider now the balance of payment equilibrium $(A.56)$ which we rewrite as follows

$$\tilde{A}_t = \tilde{A}_{t-1}\tilde{r}^a_t + N\tilde{X}_t + \tilde{T}^I_t$$

where

$$\tilde{A}_{t-1} = \tilde{B}_{F,t-1}\tilde{\varepsilon}_{t-1} + \tilde{B}_{ht-1}$$

$$r_t^a = \frac{\left[\tilde{B}_{F,t-1}\tilde{R}^*_{t-1}\tilde{\varepsilon}_t + \tilde{B}_{ht-1}\tilde{R}_{t-1}\right]}{\tilde{A}_{t-1}}$$

$$N\tilde{X}_t = \varepsilon_t \frac{P^*_{Ht}}{1+\tau_t^{m*}}\frac{s^*}{s}Y^*_{Ht} - \frac{(1-\tau_t^v)\,P_{Ft}}{(1+\tau_t^m)}Y_{Ft}$$

Take any history $\tilde{s}^\infty = (\tilde{s}_1, ..., \tilde{s}_t, ...) \in \left(S^R\right)^\infty$ such that $s_i = s^{TW}$ $\exists i$. Let t_1 and t_2 satisfy $s_{t_1} = s^{TW}$, $s_{t_1-1} \neq s^{TW}$, $s_{t_2} \neq s^{TW}$, $s_{t_2-1} = s^{TW}$. At t_1 we have

$$\begin{aligned} \tilde{A}_{t_1} &= \frac{A_{t_1}}{1+\delta} \\ &= \frac{A_{t_1-1}r^a_{t_1} + NX_{t_1}}{1+\delta} \\ &= A_{t_1-1}r^a_{t_1} + \frac{NX_{t_1}}{1+\delta} - \frac{\delta}{1+\delta}A_{t_1-1}r^a_{t_1} \\ &= \tilde{A}_{t_1-1}\tilde{r}^a_{t_1} + N\tilde{X}_{t_1} + \tilde{T}^I_{t_1} \end{aligned} \tag{A.83}$$

where, the first follows from $(A.80)$ given $s_{t_1} = s^{TW}$; the second from the fact that Ψ is an equilibrium; and the last follows from the fact that $(A.80)$ imply $A_{t_1-1}r^a_{t_1} = \tilde{A}_{t_1-1}\tilde{r}^a_{t_1}$ given $s_{t_1-1} \neq s^{TW}$ together with the fact that $s_{t_1} = s^{TW}$ implies $N\tilde{X}_{t_1} = \frac{NX_{t_1}}{1+\delta}$ and that $\tilde{T}^I_{t_1}$ is given by $(A.82)$.

As long as the trade war is in place $(A.80)$ readily imply that $\forall s$ and $t_1 < s < t_2$

$$\begin{aligned} \tilde{A}_s &= \frac{A_s}{1+\delta} \\ &= \tilde{A}_{s-1}\tilde{r}^a_s + N\tilde{X}_s \end{aligned} \tag{A.84}$$

And when it ends, at t_2, we have

$$\begin{aligned} \tilde{A}_{t_2} &= A_{t_2} \\ &= A_{t_2-1}r^a_{t_2} + NX_{t_2} \\ &= \frac{A_{t_2-1}r^a_{t_2}}{1+\delta} + NX_{t_2} + \frac{\delta}{1+\delta}A_{t_2-1}r^a_{t_2} \\ &= \tilde{A}_{t_2-1}\tilde{r}^a_{t_2} + N\tilde{X}_{t_2} + \tilde{T}^I_{t_2} \end{aligned} \tag{A.85}$$

where we are using again $(A.80)$ as in $(A.83)$.

A6. Anticipation Effects of IX

While we have shown that IX policies may boost output if their implementation is a surprise, the anticipation that such policies may be implemented sometime in the future can have immediate contractionary effects. The importance of anticipation effects was recognized by Krugman (1982) in a setting in which agents were certain about the future implementation date, but is useful to revisit in our Markov-switching framework given that it provides a convenient way of capturing uncertainty about the implementation date. In this vein, Figure A.1 shows the response of the economy when agents learn that IX policies will be introduced in the future, but are unsure about the timing. Specifically, as long as IX policies are not implemented, agents believe that there is a 10 percent chance that IX policies will be implemented in the subsequent period (i.e., $a = 0.10$), and that – once implemented – the policies will not be reversed ($\rho = 1.0$).

The anticipation effects of IX policies work through an exchange rate channel: The expectation that the exchange rate must appreciate in the long-run causes the exchange rate to appreciate in the near-term, when agents first come to believe that IX policies will eventually be implemented (first panel). The stronger currency leads to a decline in competitiveness for domestic firms, a drop in exports, and an output contraction.

A7. Trade in home currency bonds

The neutrality result presented in Proposition 1 requires the strong condition that asset market incompleteness takes the form of no international trade in home currency denominated bonds. To understand the role of this restriction, note that the implementation of IX induces changes in two different components of households wealth. First, the IX policy generates fiscal revenues whenever the home country has a trade deficit since in this case revenues from tariffs exceed subsidies to exporters. The wealth increase associated with a permanent IX policy of size δ, $G_t^F(\delta)$, is then given by the present discounted value of the fiscal revenues it generates

$$\begin{aligned} G_t^F(\delta) &= E_t \sum_{i \geq 0} \left(\prod_{j=1}^{i} \frac{\pi_{t,t+j}^*}{R_{t+j}^*} \right) \frac{\delta}{1+\delta} \left(\frac{P_{Ft+i}}{P_{t+j}} Y_{Ft+i} - Q_{t+i}(0) \frac{P_{Ht+i}^*}{P_{t+j}^*} Y_{Ht+i}^* \right) \\ &= \frac{\delta}{1+\delta} \left[Q_t(0) \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*} - \frac{B_{Ht-1}^*}{P_{t-1}} \frac{R_{t-1}}{\pi_t} \right] \end{aligned} \tag{A.86}$$

where the second equality uses the fact that in equilibrium the present discounted value of future trade deficits is equal to the net foreign asset position of the home country, that is, the difference between home country holdings of foreign bonds $\left[Q_t(0) \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*} \right]$ and foreign country holdings of home bonds $\left[\frac{B_{Ht-1}^*}{P_{t-1}} \frac{R_{t-1}}{\pi_t} \right]$.

Second, the exchange rate appreciation decreases the value of home holdings of foreign bonds. Denote with $L_t^B(\delta)$ the losses on foreign bond holdings under an appreciation of size δ, then

$$L_t^B(\delta) = [Q_t(\delta) - Q_t(0)] \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*} = -\frac{\delta}{1+\delta} Q_t(0) \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*} \tag{A.87}$$

Equations $(A.86)$ and $(A.87)$ imply:

$$L_t^B(\delta) = G_t^F(\delta) + \frac{\delta}{1+\delta} \frac{B_{Ht-1}^*}{P_{t-1}} \frac{R_{t-1}}{\pi_t}. \tag{A.88}$$

Expression $(A.88)$ summarizes the wealth effects associated with IX policies. When there is no international trading of bonds denominated in home currency $(B_{Ht}^* = 0)$, as required in Proposition 1, wealth gains through higher fiscal revenues $G_t^F(\delta)$ are exactly offset by the wealth losses induced by lower valuations of foreign holdings $L_t^B(\delta)$, thus preserving neutrality of IX policies. In contrast, when the home country borrows in home currency bonds $\left(B_{Ht-1}^* > 0\right)$ and invests in foreign currency bonds $(B_{Ft-1} > 0)$, it acquires a leveraged exposure to foreign exchange variations and the sensitivity of wealth in the home country to an exchange rate appreciation is bigger than its net foreign asset position. Consequently, given an unchanged path for future trade deficits, an exchange rate appreciation of the same size of the policy reduces wealth in the home country as the increase in fiscal revenues is not large enough to offset the capital losses on foreign bonds holdings implied by equation $(A.88)$. These wealth losses induce households to reduce their savings and, in equilibrium, the exchange rate appreciates less while the trade balance increases.

Figure A.2 shows the response of the economy to a permanent unilateral IX policy when the home country has a leveraged exposure to exchange rate fluctuations. In particular, this experiment assumes that in the initial state international trade is balanced but countries hold offsetting positions in domestic and foreign currency denominated bonds $\left(\text{i.e. } B_{F_{-1}} = B_{H-1}^* > 0\right)$ scaled to be twice as large as the value of annual GDP. As anticipated in our previous discussion, when foreign holdings of home currency denominated bonds are positive the implementation of a permanent IX lowers households wealth, consumption, and savings, thus dampening the appreciation of the exchange rate (solid lines). As a result, the home country runs a permanently positive trade balance to pay interest on its negative net foreign asset position. For comparison, we also plot the response of the baseline economy when there is no international trade in domestic currency bonds, as required in Proposition 1, and a permanent IX policy is neutral (dashed lines).

A8. Departing from Producer's Currency Pricing

We conclude this section with a brief discussion on the requirement of producer's currency pricing (PCP) in Proposition 1 to deliver neutrality of IX policies. We follow the literature and compare the transmission of policies under PCP, local currency pricing (LCP), and dominant currency pricing (DCP).[1]

Figure A.3 compares the effects of an IX policy under PCP (dotted lines), LCP (solid lines), and DCP (dashed lines), assuming that all other conditions in Proposition 1 are satisfied. As discussed before, under PCP international relative prices are insulated by the immediate appreciation of the exchange rate and the allocation is unaffected. In contrast, when foreign exporters prices are sticky in the currency of the home country the IX policy has allocative effects: Imports contract, inflation jumps, and output experiences a very small boost.

The source of non-neutrality, both for LCP and DCP, is the asymmetric pass-through of tariff changes and exchange rate movements to import prices. As shown by the expression for the price of imported goods in the home country

$$P_{Ft} = (1 + \tau_t^m)\, P_{X_t^*} \tag{A.89}$$

changes in import tariffs are fully passed through to import prices (P_{Ft}) whereas movements in the exchange rate only pass-through gradually as foreign exporters adjust their prices in the home market $\left(P_{X_t^*}\right)$ infrequently under our Calvo pricing assumption. Hence, the rise in import prices reduces the demand for imported varieties and boosts output through import-substitution channels. The effects under DCP are nearly identical to the effects under LCP. The only difference is that with full exchange rate pass-through, home exports become more expensive causing exports to contract sligthly and, accordingly, output to expand less.

A9. Model fit as a function of passthrough and reversal

Figure A.4 graphically describe the objective function $\mathcal{O}(\Theta)$ optimized in equation (64) in the paper. For convenience we report the optimization here:

$$\Theta^* = \arg\max \mathcal{O}(\Theta) = \arg\max - [M_D - M_M(\Theta)]' \, [M_D - M_M(\Theta)] \tag{A.90}$$

Two key results emerge. First, the objective function is maximized at the point $\Theta^* = [0.6;\ 0.93]$. These values suggest that, in order to account for the price increase and output decline observed in the data, the model requires a significant fraction of firms passing VAT changes through to consumer prices ($\mu = 0.6$) and a positive probability of policy reversal ($1 - \rho = 0.07$). While the large estimated share of firms passing through VAT changes to prices is obtained purely from aggregate data, it is also in line with the heterogenous pricing response across sectors documented in Bundesbank (2007). For instance, the pass-through during the first quarter of 2007 was full in the automotive sector but muted in the retail

[1]For a discussion of transmission under PCP and LCP see, for instance, Devereux and Engel (2002). In our two-country model, under DCP the home country adopts PCP and the foreign country adopts LCP.

sector. Similarly, an expected duration of the policy of about eight years, as implied by our estimated value of ρ, is consistent with reasonable assumptions about the likelihood of political turnover and its implications for the evolution of fiscal policy.[2]

Second, the limiting assumption of permanent policy changes ($1-\rho = 0$) and all prices sticky inclusive of VATs ($\mu = 0$), typically adopted in the fiscal devaluation literature, appears to be strongly rejected by the data. As shown in Figure A.4, the fit of the model declines sharply as Θ takes these limiting values, with the objective function $\mathcal{O}$ reaching its lowest value of negative 12, compared with a value of negative 2 at the optimum. In addition, the objective function features a strongly nonlinear behavior. There is a large flat region around the optimum suggesting that several values of (μ, ρ) close to Θ^* deliver similar responses for output and inflation. The fit of the model then deteriorates very quickly when approaching extremely low values in either dimension. Both low levels for the proportion of firms that fully pass-through VAT changes (i.e. around $\mu = .2$ or lower) and very low levels for the probability of reversal (i.e. around $\rho = .02$ or lower) seem to be at odds with the time-series data for German GDP and inflation.

A10. Government Expenditure in Germany in 2007

Our key assumption in the quantitative analysis of section IV of the paper is that the VP is responsible for the differential macroeconomic behavior of Germany with respect to other euro-area countries over the 2006Q3-2007Q4 period. A possible objection to this assumption is that government expenditure grew less in Germany during this period than in the rest of Europe. Here we address this point by showing that the quantitative relevance of government expenditure dynamics in this period appears to be in fact negligible.

To calibrate the size of government shocks we assume that government expenditure is constant in the Euro area and in Germany it follows an AR(1) process given by:

$$g_t = \rho^g g_{t-1} + \varepsilon_t^g \tag{A.91}$$

Given the observed behavior of government expenditure in the euro area and in Germany, we can use equation (A.91) to back out a time series for government expenditure shocks, under the assumption that $\rho^g = .95$.

Figure A.7 compares our baseline model responses to the model responses where government shocks are added on top of our VP shocks. Overall, this experiment

[2] As noted in D'Acunto, Hoang and Weber (2016), at the time there was severe disagreement between the two main parties on the benefit of the VP policy and, thus, uncertainty about the duration of the policy in case of a change in government.

suggests that the observed pattern of government expenditure in Germany did not have a material impact on the German economy.

A11. Data Sources and Calculation for the Quantitative Section "2007 Fiscal Devaluation In Germany"

Macroeconomic data for Germany and the euro area (EA) are from Haver (EU and Germany Database). Mnemonics and details about the construction of the series are provided below.

Germany. Consumption is real private final consumption (J134PCT) and investment is real gross fixed capital formation (J134IFT). Net exports to the euro area are the difference between nominal goods exports to the euro area (DESIXEZ) and nominal goods imports form the euro area (DESIMEZ). We construct real GDP as the sum of nominal private consumption (J134PCN) divided by the consumption deflator (J134PCP), nominal gross fixed capital formation (J134IFN) divided by its deflator (J134IFP), plus nominal exports to the euro area (DESIXEZ) divided by the export deflator (J134EXPP) minus nominal import from the euro area (DESIMEZ) divided by the import deflator (J134IMPP). Consumer price inflation is the four-quarter change in the price level of the core HICP series, which excludes energy, food, alcohol, and tobacco (H134HOEF). Wage inflation is the four-quarter change in the series "Total Labor Cost" (S134LTBN). Labor input is total hours worked from the National Accounts (DEBNHT).

EA ex-Germany. Variables are constructed by subtracting the nominal German counterparts from the EA nominal data and then deflating the resulting series using the adjusted NIPA deflators. Specifically, consumption is EA nominal private final consumption (J025PCN) less Germany's nominal private final consumption (J134PCN) divided by the EA ex-Germany consumption deflator. Investment is EA nominal gross fixed capital formation (J025IFN) less Germany's nominal gross fixed capital formation (J134IFT) divided by the EA ex-Germany investment deflator (J025IFP). Real GDP is consumption plus investment less Germany's real net exports to the euro area. The inflation series is the four-quarter change in the price level of the EA ex-Germany core HICP series, which excludes energy, food, alcohol, and tobacco (H023HOEF), and the corresponding series for Germany. We use the HICP weights of Germany in total EA HICP (P134BE11) to construct the EA ex-Germany series. Wage inflation constructed as the (weighted) difference between the four-quarter change in the series "EA: Total Labor Cost" (S025 LTBN) and the corresponding series for Germany. We use the HICP weights of Germany in total EA HICP (P134BE11) to construct the EA ex-Germany series. Labor input is total hours worked from the National Accounts (J025OETE).

Fiscal data. Data on social security contributions are from the OECD Tax - Tax Wedge Database obtained through Haver (OECD Government Statistics Database). Data for Germany refer to the average social security tax rate as a percent of total labor costs for workers with income equal to the average wage and include both employer (A132ME2) and employee (A132MS2) taxes. The aggregate for the EA ex-Germany is constructed as a GDP-weighted average the average social security tax rates of Belgium (A124ME2, A124MS2), France (A132ME2, A132MS2), Italy (A134ME2, A134MS2), the Netherlands (A138ME2, A138MS2), and Spain (A184ME2, A184MS2). Data on VAT tax rates refer to the standard VAT rate for Germany and for the EA ex-Germany as in European Commission (2019), "VAT rates applied in the member States of the European Union". On a GDP basis, the countries of Belgium, France, Italy, the Netherlands, and Spain altogether account for about 85 percent of the EA ex-Germany region.

A12. Computation

We start from a model

$$E_t \left\{ F\left(X_{t+1}, X_t, X_{t-1}, \varepsilon_t, MS_t, MS_{t+1}\right)\right\} = 0 \tag{A.92}$$

where $MS_t \in \{1, ..., n_s\}$ is the Markov state determining the regime. Regimes evolve according to the transistion probability TP

$$TP\left(i, j\right) = \Pr\left\{MS_{t+1} = j \left| MS_t = i\right.\right\}.$$

Let

$$x_t^i = X_t - \bar{X}_i$$

where $\bar{X}_i$ is the steady state under regime i.

We can linearize the system of equations in $(A.92)$ to obtain

$$SS_i + M_i^b x_{t-1}^i + M_i^s x_t^i + E_i M_{ij}^f x_{t+1}^i + M_{ij}^{\varepsilon} \varepsilon_t = 0$$

where

$$SS_i = E_i \left\{ F\left(\bar{X}_i, \bar{X}_i, \bar{X}_i, 0, MS_i, MS_j\right)\right\}$$

$$M_i^b = E_i \left\{ \frac{\partial F\left(\bar{X}_i, \bar{X}_i, \bar{X}_i, 0, MS_i, MS_j\right)}{\partial X_{t-1}} \right\}$$

$$M_i^s = E_i \left\{ \frac{\partial F\left(\bar{X}_i, \bar{X}_i, \bar{X}_i, 0, MS_i, MS_j\right)}{\partial X_t} \right\}$$

$$M_i^{\varepsilon} = E_i \left\{ \frac{\partial F\left(\bar{X}_i, \bar{X}_i, \bar{X}_i, 0, MS_i, MS_j\right)}{\partial \varepsilon_t} \right\}$$

$$M_{ij}^f = \frac{\partial F\left(\bar{X}_i, \bar{X}_i, \bar{X}_i, 0, MS_i, MS_j\right)}{\partial X_{t+1}}$$

Notice that in general it is possible that $F\left(\bar{X}_i, \bar{X}_i, \bar{X}_i, 0, MS_i, MS_j\right) \neq 0$ when

$MS_i \neq MS_j$ and it is the case in our setup in shich future tariffs enter the optimal pricing condition.

Now let

$$x_t = X_t - \bar{X}$$

where $\bar{X}$ is the steady stae in a reference regime

(A.93)

$$SS_t + M_t^b (x_{t-1} + D_t) + M_t^s (x_t + D_t) + E_t \left\{ M_{t,t+1}^f (x_{t+1} + D_t) \right\} + M_t^\varepsilon \varepsilon_t = 0$$

where

$$D_t = \bar{X} - \bar{X}_i \text{ when } MS_t = i$$

$$M_t^b = M_i^b \text{ when } MS_t = i$$

and so on. Now I can premultiply everything by $(M_t^s)^{-1}$ and rearrange to get

$$(x_t + D_t) = -(M_t^s)^{-1} SS_t + E_t \{A_{t,t+1} (x_{t+1} + D_t)\} + B_t (x_{t-1} + D_t) + C_t \varepsilon_t$$

where

$$A_{t,t+1} = -(M_t^s)^{-1} M_{t,t+1}^f$$

$$B_t = -(M_t^s)^{-1} M_t^b$$

$$C_t = -(M_t^s)^{-1} M_t^\varepsilon$$

So we can write

$$x_t = F_t + E_t \{A_{t,t+1} x_{t+1}\} + B_t x_{t-1} + C_t \varepsilon_t \tag{A.94}$$

where

$$F_t = E_t \{(A_{t,t+1} + B_t - I_n)\} D_t - (M_t^s)^{-1} SS_t \tag{A.95}$$

We look for a policy functions of this system given by

$$x_t = \varphi_t + \Omega_t x_{t-1} + \Gamma_t \varepsilon_t \tag{A.96}$$

Using $(A.96)$ into $(A.94)$ we get

$$x_t = F_t + E_t \{A_{t,t+1} (\varphi_{t+1} + \Omega_{t+1} x_t)\} + B_t x_{t-1} + C_t \varepsilon_t$$

$$x_t = (I_{n_v} - E_t \{A_{t,t+1} \Omega_{t+1}\})^{-1} [F_t + E_t \{A_{t,t+1} \varphi_{t+1}\} + B_t x_{t-1} + C_t \varepsilon_t]$$

So the policy functions satisfy

$$\varphi_t = (\Theta_t)^{-1} F_t + (\Theta_t)^{-1} E_t \{A_{t,t+1} \varphi_{t+1}\} \tag{A.97}$$

$$\Omega_t = (\Theta_t)^{-1} B_t \tag{A.98}$$

$$\Gamma_t = (\Theta_t)^{-1} C_t \tag{A.99}$$

$$\Theta_t = I - E_t \{A_{t,t+1}\Omega_{t+1}\} \tag{A.100}$$

Notice that the solution for Ω_t and Γ_t does not depend on the constant correctin F_t so these can be computed independently given knowledge of $A_{t,t+1}$, B_t, and C_t. On the other hand to find φ_t notice that we can rewrite $(A.97)$ as

$$\varphi = \xi + \mathbf{R}^M \varphi$$

where

$$\underset{n_v \cdot n_s \times 1}{\varphi} = [\varphi_1 .... \varphi_{n_s}]$$

$$\underset{n_v \cdot n_s \times 1}{\xi} = \left[(\Theta_1)^{-1} F_1 ..... (\Theta_{n_s})^{-1} F_{n_s}\right]$$

$$\underset{n_v \cdot n_s \times n_v \cdot n_s}{\mathbf{R}^M} = \begin{bmatrix} (\Theta_1)^{-1} A_{1,1} TP(1,1) & ... & (\Theta_1)^{-1} A_{1,n_s} TP(1,n_s) \\ ... & ... & ... \\ (\Theta_{n_s})^{-1} A_{n_s,1} TP(n_s,1) & ... & (\Theta_{n_s})^{-1} A_{n_s,n_s} TP(n_s,n_s) \end{bmatrix}$$

the solution is given by

$$\varphi = \left(I - \mathbf{R}^M\right)^{-1} \xi$$

To find Ω_t and Γ_t use the following algorithm:
Let $F^1_{t,t+1} = A_{t,t+1}, \Omega^1_t = B_t, \Gamma^1_t = C_t$;
Assume $F^{k-1}_{t,t+1}, \Omega^{k-1}_t, \Gamma^{k-1}_t$ have been defined. Let

$$\Theta^{k-1}_t = I_n - E_t \left\{A_{t,t+1}\Omega^{k-1}_{t+1}\right\}$$

then we update

$$F^k_{t,t+1} = \left(\Theta^{k-1}_t\right)^{-1} A_{t,t+1}$$

$$\Omega^k_t = \left(\Theta^{k-1}_t\right)^{-1} B_t$$

$$\Gamma^k_t = \left(\Theta^{k-1}_t\right)^{-1} C_t$$

The solution of the system is

$$x_t = \varphi_t + \Omega^{\infty}_t x_{t-1} + \Gamma^{\infty}_t \varepsilon_t$$

Figure A.1. Macroeconomic Effects of an Anticipated Permanent IX

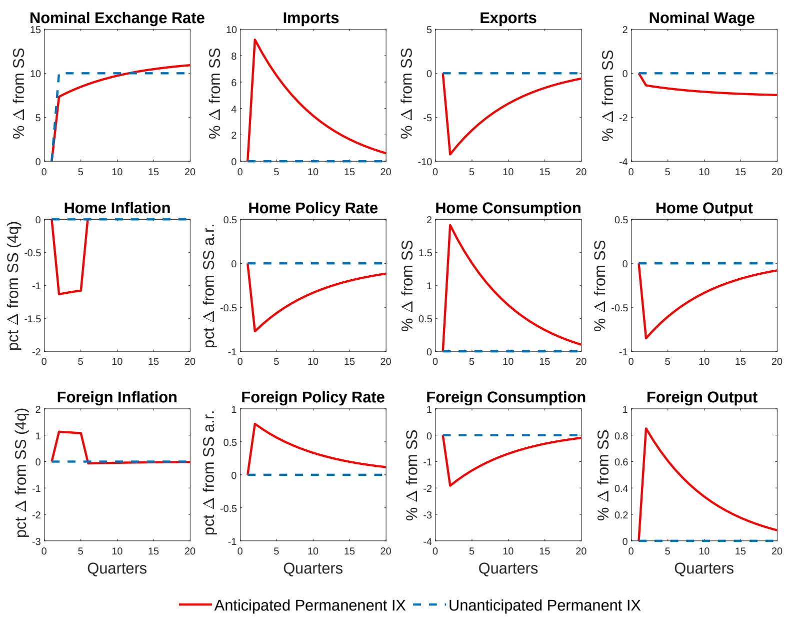
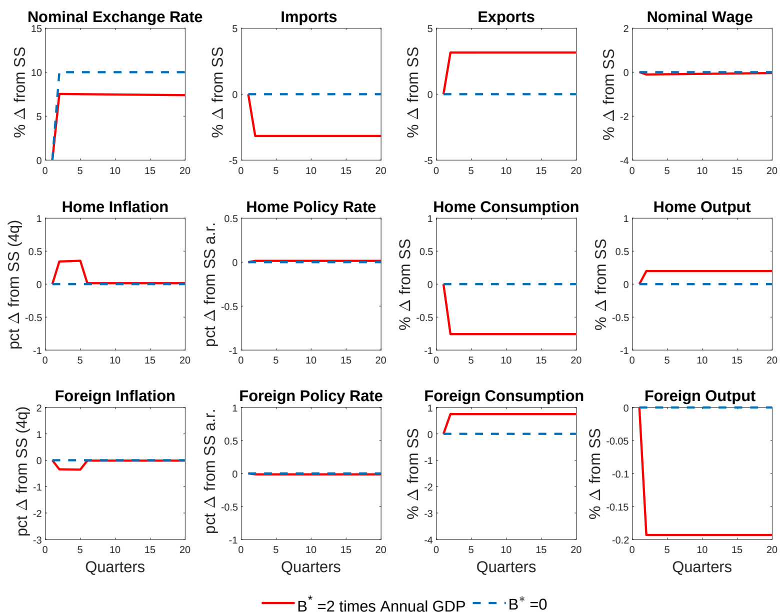
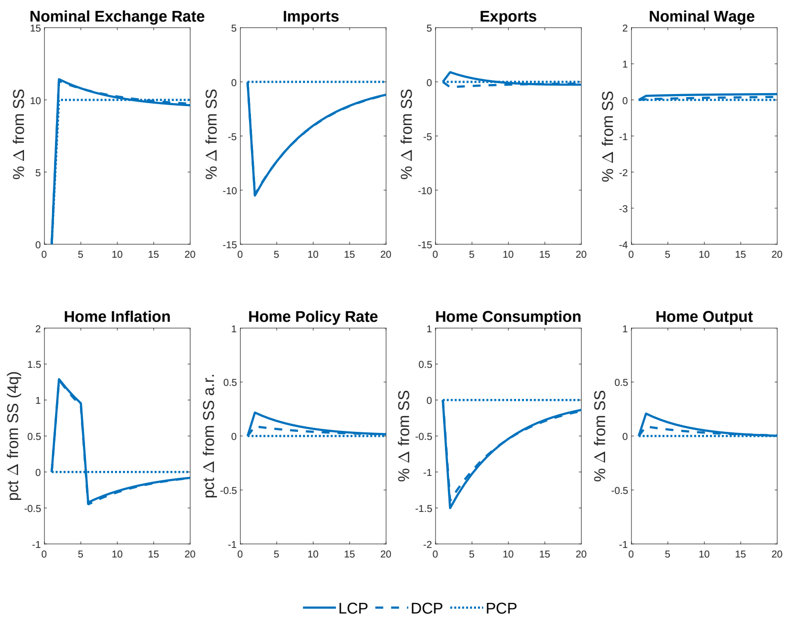


Figure A.2. Permanent IX with Foreign Holdings of Home Currency Bonds



Note: In both experiments we assume that prices are sticky, wages are flexible, and the exchange rate is flexible. The solid line shows the case in which, in the initial state, the home country has offsetting bond holdings in domestic and foreign currency equal to two times annual GDP. The dashed line is the case in which countries hold no bonds in the initial state. The figure shows the (expected) path of each variable after the policy is implemented and given that it is (expected to be) permanent.

Figure A.3. Permanent IX: LCP, DCP and PCP



Note: In all the experiments we assume that prices are sticky, wages are flexible, and the exchange rate is flexible. The solid line shows the case in which both domestic and foreign exporters adopt LCP. The dashed line shows the case in which domestic exporters adopt PCP and foreign exporters adopt LCP. The dotted line shows the case in which both foreign and domestic exporters adopt PCP. The figure shows the (expected) path of each variable after the policy is implemented and given that it is (expected to be) permanent.

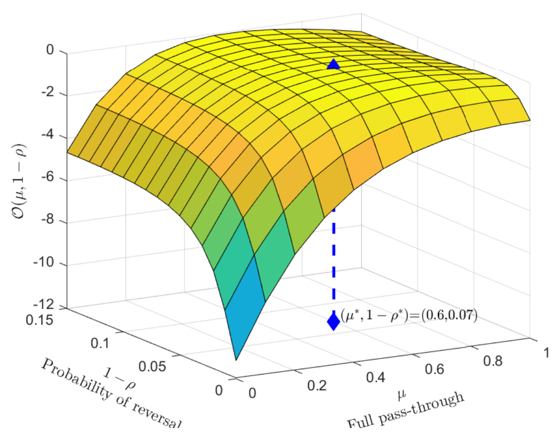


Figure A.4. Policy Reversal, Pass-Through, and Distance between German Data and Model

Note: The figure plots the objective function in (A.90), that is the negative of the squared distance between the model implied time series on inflation and output and the observed realizations. The blue diamonds show the optimal point on the surface and its projection in the (x, y) plane which reports an optimal value of $\mu^* = 0.6$ and $1 - \rho^* = 0.07$.

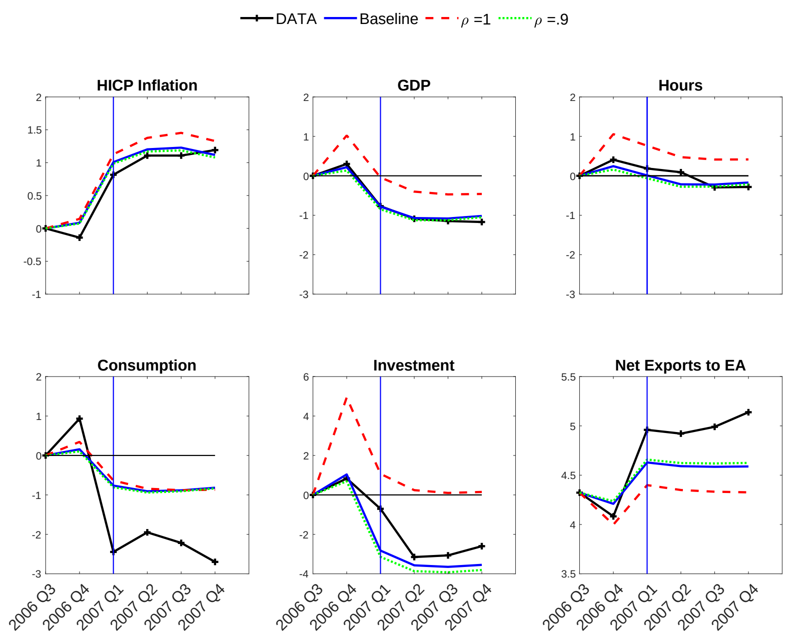


FIGURE A.5. THE ROLE OF EXPECTATIONS ABOUT POLICY REVERSAL

Note: The data line (black crossed) and the baseline (blue solid) are as in figure 6 of the paper. The dashed red line assumes permanent VP. The dotted green line assumes that while VP remains in place throughout 2007, it is expected to be abandoned with a 0.1 probability in the following quarter. All experiments assume $\mu = \mu^{*}$.

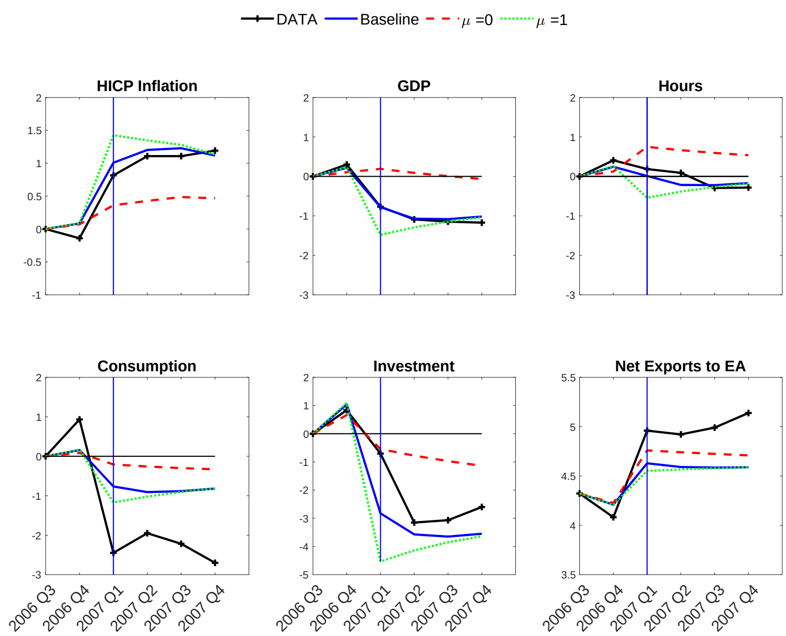
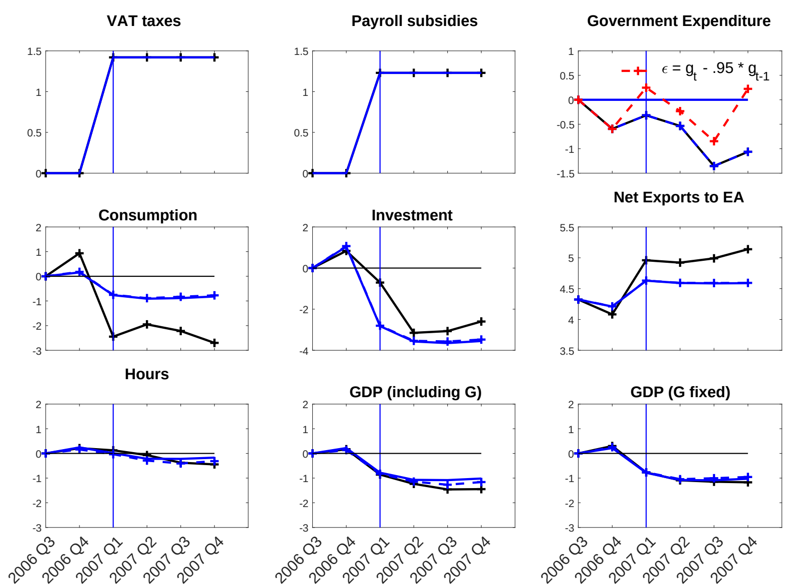


Figure A.6. The role of the tax pass-through

Note: The data line (black crossed) and the baseline (blue solid) are as in figure 6 of the paper. The dashed red line assumes incomplete pass-through for all firms. The dotted green line assumes complete pass-through for all firms. All experiments assume $\rho = \rho^*$.

FIGURE A.7. VP WITH AND WITHOUT GOVERNMENT EXPENDITURE SHOCKS



NOTE: The data line (black crossed) and the baseline (blue solid) are as in figure 6 of the paper. The dashed blue line includes the effects of government spending shocks. All experiments assume $\mu = \mu^*$ and $\rho = \rho^*$.

*

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