Electricity and Firm Productivity: A General Equilibrium Approach

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Online Appendix

A. Solution to the Analytic Version of the Model

Proof of Proposition 1: For firms with and without generator access, the result that capital demand is increasing in the probability of grid power follows directly from the first order condition for capital. We focus instead on understanding the cutoff, $v^{\star}(R_t)$, below which firms with generator access purchase enough generator capital to operate at full capacity during an outage.

The firm always idles generator capital when the power is on because the price of grid electricity is zero, and this is lower than the marginal cost of fuel to operate the generator (which equals unity). Whether the firm idles productive capital when the power is off depends critically on whether the constraint that $k_t^S \leq k_t^M$ binds. We show that this constraint binds for all values of v less than the cutoff, $v^{*}(R_t)$, which we will characterize below.

The modern firm's first order conditions for k_t^S, and k_t^M are:

$$k_t^S: \quad \eta(1-v_t)A^M z^{1-\eta}(k_t^S)^{\eta-1} = R_t + 1 - v + \theta \tag{1}$$

$$k_t^M: \quad \eta v A^M z^{1-\eta}(k_t^M)^{\eta-1} + \theta = R_t, \tag{2}$$

and the complementary slackness condition requires that:

$$\theta\left(k_t^M - k_t^S\right) = 0. \tag{3}$$

When the constraint does not bind, the solutions are:

$$k_t^M = z\left(\frac{v\eta A^M}{R_t}\right)^{\frac{1}{1-\eta}} \quad \text{and} \quad k_t^S = z\left[\frac{(1-v)\eta A^M}{R_t+1-v}\right]^{\frac{1}{1-\eta}}. \tag{4}$$

The constraint does not bind as long as $k_t^M > k_t^S$. Substituting in the expressions for k_t^S and k_t^M from equation (4) implies that the constraint will not bind as long as,

$$\frac{v}{R_t} - \frac{1-v}{R_t+1-v} > 0. \tag{5}$$

The left-hand side of equation (5) is increasing in v. Define $v^{*}(R_t)$ to be the value of v that makes (5) hold with equality given R_t. When $v < v^{*}(R_t)$, the constraint binds and the firm rents enough generator capital to operate at full capacity when the power is off. As a result, there are zero idle resources during instants without power. When $v \geq v^{*}(R_t)$, the constraint does not bind and the firm rents less generator capital that what is required to operate her productive capital. In this case, she must idle some productive capital when the power is off. Power outages are entirely responsible for the existence of idle resources. When there are no power outages, $v = 1$, and equation (4) implies that generator capital equals zero. Zero generator capital combined with zero instants without power results in zero idle resources. □

Modern sector entry: We show that the difference in expected profits, $E(\pi^M(z)) - \pi^T(z)$ is increasing in z. The difference in expected profits for a given entrepreneur with productivity z equals:

$$E(\pi^M(z)) - \pi^T(z) = \gamma\pi^M(1,z) + (1-\gamma)\pi^M(0,z) - \pi^T(z)$$

Substituting in the relationship that profits equal fraction $1-\eta$ of output yields:

$$E(\pi^M(z)) - \pi^T(z) = (1-\eta)[\gamma y_t^M(1,z) + (1-\gamma)y_t^M(0,z) - y_t^T(z)] \tag{6}$$

where,

$$y^T(z) = z\left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}}, \quad y^M(0,z) = zvA^M\left(\frac{vA^M\eta}{R}\right)^{\frac{\eta}{1-\eta}} \quad \text{and} \tag{7}$$

$$y^M(1,z) = \begin{cases} zA^M\left(\frac{A^M\eta}{2R+1-v}\right)^{\frac{\eta}{1-\eta}} & : v \leq v^{\star}(R) \\ z\left[vA^M\left(\frac{vA^M\eta}{R}\right)^{\frac{\eta}{1-\eta}} + (1-v)\left(\frac{(1-v)\eta A^M}{R+1-v_a}\right)^{\frac{\eta}{1-\eta}}\right] & : v > v^{\star}(R). \end{cases} \tag{8}$$

Substituting equations (7) - (8) into equation (6) shows that $E(\pi^M(z)) - \pi^T(z)$ is increasing in z, since, by assumption, $A^M > \left[(1-\gamma)v^{1/(1-\eta)}\right]^{-1/(1-\eta)}$.

Proof of Proposition 2: The household's first order condition for capital implies that the steady-state rental rate is:

$$R = \frac{1}{\beta} + \delta - 1, \tag{9}$$

and hence the steady-state price of capital does not depend on the probability of grid power. It follows that an increase in v does not affect profits for a traditional firm. Therefore, we only need to show that expected profits for a potential modern entrant with a given z are increasing in v. By the envelope theorem:

$$\frac{\partial E(\pi^M(z))}{\partial v} = \gamma[y_1^M(1,z) - y_0^M(1,z) + f_0(1,z)] + (1-\gamma) y_1^M(0,z). \tag{10}$$

Note that the expectation is taken over x, the indicator variable which determines if the firm has generator access. If $v < v^\star$, then firms with generator access operate at full capacity when the power is off, implying that $y_0^M(1,z) = y_1^M(1,z)$. When $v > v^\star$, $y_1^M(1,z) > y_0^M(1,z)$. In either case, equation (10) is positive, implying that $E(\pi^M(z))$ is increasing in v. □

Proof of Proposition 3: We define steady state A to be the steady state of the economy when the probability of grid power is $v_a < 1$. We use this steady state as a baseline from which to calculate the short-run partial-equilibrium and long-run general-equilibrium effects of eliminating outages. The first order conditions from the traditional firm's profit maximization problem imply that capital demand for a traditional firm with productivity, z in steady state A equals:

$$k_a^T(z) = z\left(\frac{\eta}{R}\right)^{\frac{1}{1-\eta}}. \tag{11}$$

Similarly, the first order conditions from the modern firm's profit maximization problem imply that productive capital demand for a modern firm with generator access $x \in \{0,1\}$ and productivity z in steady state A equals:

$$k_a^M(0,z) = z\left(\frac{v_a \eta A^M}{R}\right)^{\frac{1}{1-\eta}} \quad \text{and} \quad k_a^M(1,z) = \begin{cases} z\left(\frac{\eta A^M}{2R+1-v_a}\right)^{\frac{1}{1-\eta}} & : v_a \leq v^\star(R) \\ z\left(\frac{v_a \eta A^M}{R}\right)^{\frac{1}{1-\eta}} & : v_a > v^\star(R). \end{cases} \tag{12}$$

Demand for generator capital by a modern firm with generator access and productivity z equals:

$$k_a^S(1,z) = \begin{cases} z\left(\frac{\eta A^M}{2R+1-v_a}\right)^{\frac{1}{1-\eta}} & : v_a \leq v^\star(R) \\ z\left(\frac{(1-v_a)\eta A^M}{R+1-v_a}\right)^{\frac{1}{1-\eta}} & : v_a > v^\star(R). \end{cases} \tag{13}$$

Let $z_a^\star$ denote the equilibrium productivity cutoff in steady state A. To calculate aggregate output, we substitute the above expressions for capital demand in each sector into the respective production functions and integrate over the distribution of entrepreneurs. Aggregate output in the traditional, Y_a^T, and modern, Y_a^M, sectors equals:

$$Y_a^T = \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_1^{z_a^\star} z\,dG(z) \quad \text{and} \tag{14}$$

$$Y_a^M = \begin{cases} \left[\gamma A^M \left(\frac{A^M \eta}{2R+1-v_a}\right)^{\frac{\eta}{1-\eta}} + (1-\gamma) v_a A^M \left(\frac{v_a \eta A^M}{R}\right)^{\frac{\eta}{1-\eta}}\right] \int_{z_a^\star}^{\infty} z\,dG(z) & : v_a \leq v^\star(R) \\ \left[v_a A^M \left(\frac{v_a A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} + \gamma (1-v_a) A^M \left(\frac{(1-v_a)\eta A^M}{R+1-v_a}\right)^{\frac{\eta}{1-\eta}}\right] \int_{z_a^\star}^{\infty} z\,dG(z) & : v_a > v^\star(R). \end{cases} \tag{15}$$

Aggregate output, Y_a, equals the sum of aggregate output in the traditional and modern sectors: $Y_a = Y_a^T + Y_a^M$.

We compute the short-run partial-equilibrium effect of eliminating outages. Define $\tilde{Y}$ to equal aggregate output when $v = 1$, but the productivity cutoff equals its value when $v = v_a$, $z_a^\star$, and the demands for capital equal their values when $v = v_a$ (equations (11) - (13)):

$$\tilde{Y} = Y_a^T + \begin{cases} \left[\gamma A^M \left(\frac{A^M \eta}{2R+1-v_a}\right)^{\frac{\eta}{1-\eta}} + (1-\gamma) A^M \left(\frac{v_a \eta A^M}{R}\right)^{\frac{\eta}{1-\eta}}\right] \int_{z_a^\star}^{\infty} z\,dG(z) & : v_a \leq v^\star(R) \\ \left[A^M \left(\frac{v_a A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}}\right] \int_{z_a^\star}^{\infty} z\,dG(z) & : v_a > v^\star(R). \end{cases} \tag{16}$$

The short-run partial-equilibrium effect of eliminating outages equals $\tilde{Y} - Y_a$. This represents the difference between output when firms choose their scale and sector but experience no outages ex-post and actual output.

To compute the long-run general-equilibrium effect of eliminating outages, we define steady state B to be the steady state of the economy when $v = 1$. Let $z_b^\star$ be the equilibrium productivity cutoff in steady state B. Aggregate traditional and modern output in steady state B equals:

$$Y_b^T = \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_1^{z_b^\star} z\,dG(z) \quad \text{and} \quad Y_b^M = A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_{z_b^\star}^{\infty} z\,dG(z).$$

Aggregate output in steady state B equals $Y_b = Y_b^T + Y_b^M$. The long-run general-equilibrium effect of eliminating outages equals $Y_b - Y_a$. This represents the difference between output in the steady state with no outages and output in the steady state with outages when the probability of grid power equals v_a.

To demonstrate that the long-run general-equilibrium effect of eliminating outages exceeds the short-run partial-equilibrium effect, we must show that $Y_b - Y_a > \tilde{Y} - Y_a$. First, note that by Proposition 2, the productivity cutoff in steady state B is less than the productivity cutoff in steady state A: $z_b^\star < z_a^\star$. Then, since $A^M > \left[(1-\gamma)v^{1/(1-\eta)}\right]^{-1/(1-\eta)} > 1$, it follows that:

$$Y_b > \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_1^{z_a^\star} z\,dG(z) + A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_{z_a^\star}^{\infty} z\,dG(z). \tag{17}$$

We show that the right-hand-side of equation (17) is larger than the value of $\tilde{Y}$ defined in equation (16). First, observe that the first term on the right-hand-side of equation (17) equals Y_a^T. Second, one can show that the second term on the right-hand-side of equation (17) always exceeds the second term in equation (16). Therefore, it follows that $Y_b > \tilde{Y}$ which implies that $Y_b - Y_a > \tilde{Y} - Y_a$. □

Derivation of equation (7) in the main text. Using the notation from the Proof of Proposition 3, $Y^{LRGE} - Y^{SRPE} = Y_b - \tilde{Y}$. Focusing first on an economy with $v > v^\star$ we have:

$$\begin{aligned}
&Y_b - \tilde{Y} \\
&= \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_1^{z_b^\star} z\,dG(z) + A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_{z_b^\star}^{\infty} z\,dG(z) - \left[\left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_1^{z_b^\star} z\,dG(z) + A^M \left(\frac{v_a A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} \int_{z_a^\star}^{\infty} z\,dG(z)\right] \\
&= \left[A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} - A^M \left(\frac{v_a A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}}\right] \int_{z_a^\star}^{\infty} z\,dG(z) + \left[A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} - \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}}\right] \int_{z_b^\star}^{z_a^\star} z\,dG(z) \\
&= \underbrace{\left[A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} - A^M \left(\frac{v_a A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}}\right] E(z|z > z_a^\star)(1-G(z_a^\star))}_{\text{Firm expansion}} + \underbrace{\left[A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} - \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}}\right] E(z|z_b^\star < z < z_a^\star)(G(z_a^\star) - G(z_b^\star))}_{\text{Firm entry}}.
\end{aligned}$$

Similarly, for an economy with $v < v^\star$, we have:

$$\begin{aligned}
Y_b - \tilde{Y} = &\underbrace{\left[A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} - \gamma A^M \left(\frac{A^M \eta}{2R + 1 - v_a}\right)^{\frac{\eta}{1-\eta}} - (1-\gamma) v_a A^M \left(\frac{v_a \eta A^M}{R}\right)^{\frac{\eta}{1-\eta}}\right] E(z|z > z_a^\star)(1-G(z_a^\star))}_{\text{Firm expansion}} \\
&+ \underbrace{\left[A^M \left(\frac{A^M \eta}{R}\right)^{\frac{\eta}{1-\eta}} - \left(\frac{\eta}{R}\right)^{\frac{\eta}{1-\eta}}\right] E(z|z_b^\star < z < z_a^\star)(G(z_a^\star) - G(z_b^\star))}_{\text{Firm entry}}.
\end{aligned}$$

Definition of a competitive equilibrium: We define a sequence-of-markets equilibrium for this economy. We denote whether a modern firm has generator access with indicator variable $x \in \{0,1\}$, where $x=1$ denotes generator access and $x=0$ denotes no access. A *sequence-of-markets equilibrium* consists of: a sequence of rental rates of capital, $\{R_t\}_{t=0}^{\infty}$; productivity cutoffs, $\{z_t^\star\}_{t=0}^{\infty}$; household allocations, $\{C_t, K_t\}_{t=0}^{\infty}$; and entrepreneurial allocations $\{k_t^T(z), k_t^S(1,z), k_t^M(x,z), e_{1t}^G(x,z), f_{0t}(1,z)\}_{t=0}^{\infty}$ for all $x \in \{0,1\}$ and for all $z \in [1,\infty]$ such that:

1. Given prices, all entrepreneurs with productivity $z > z_t^\star$ choose to be modern. Cutoff $z_t^\star$ is the productivity value such that $E(\pi_t^M) - \Omega = \pi_t^T$.

2. Given prices, traditional-sector allocations solve the profit maximization problem for all firms in the traditional sector and modern-sector allocations solve the profit maximization problem for all firms in the modern sector.

3. Given prices and firm profits, household allocations maximize lifetime utility subject to the budget constraint:

$$C_t + K_{t+1} = R_t K_t + (1-\delta)K_t + \int_1^{z_t^\star} \pi_t^T(z)dG(z) + \int_{z_t^\star}^{\infty} (\gamma \pi_t^M(1,z) + (1-\gamma)\pi_t^M(0,z) - \Omega)dG(z)$$

   and the non-negativity constraints, $C_t \geq 0$, and $K_t \geq 0$, where $\pi_t^T(z)$ are the profits of the traditional firms with productivity z, $\pi_t^M(x,z)$ are the profits of the modern firms with generator access, x, and productivity z.

4. The market for capital clears:

$$K_t = \int_1^{z_t^\star} k_t^T(z)dG(z) + \int_{z_t^\star}^{\infty} (\gamma k_t^M(1,z) + (1-\gamma)k_t^M(0,z) + \gamma k_t^S(1,z))dG(z). \tag{18}$$

We are interested primarily in how eliminating outages effects the economy in the long-run. For this it is useful to define a *steady state competitive equilibrium*, which consists of a constant rental rate, R; productivity cutoff $z^\star$; household allocations, $\{C, K\}$; and entrepreneurial allocations $\{k^T(z), k^S(1,z), k^M(x,z), e_1^G(x,z), f_0(1,z)\}$ for all $x \in \{0,1\}$ and for all $z \in [1,\infty]$ such that conditions (1)-(4) hold and all variables are constant from one period to the next.

B. Quantitative Version of the Model

Definition of a competitive equilibrium. We define a *sequence-of-markets* equilibrium for this economy as sequences of prices $\{W_t, R_t\}_{t=0}^{\infty}$, grid-power probabilities, $\{\nu_t\}_{t=0}^{\infty}$, fraction of entrepreneurs with political connectedness q and managerial ability z in the traditional sector, $J^T(q,z)$, in the modern sector without generator access, $J^{M,N}(q,z)$ and in the modern sector with generator access, $J^{M,S}(q,z)$, allocations for the households $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ and allocations for firms with political connectedness q and managerial ability z:
$\{n_t^T(q,z), n_t^M(q,z), k_t^T(q,z), k_t^S(q,z), k_t^M(q,z), e_{1t}^G(q,z), f_{0t}(q,z)\}_{t=0}^{\infty}$ for all $z \in [1,\infty]$ and $q \in \{0,1\}$ such that:

1. Given prices, the fraction of entrepreneurs that operate the traditional technology, the modern technology without a generator and the modern technology with a generator satisfy the optimality condition defined in equation (10) in the main text.

2. Given prices, traditional-sector allocations solve the profit maximization problem for all entrepreneurs in the traditional sector and modern-sector allocations solve the profit maximization problem for all entrepreneurs in the modern sector.

3. Given prices and entrepreneurial profits, household allocations maximize lifetime utility subject to the budget constraint:

$$\begin{aligned} C_t + K_{t+1} = Q_t + W_t + R_t K_t + (1-\delta)K_t \\ + \int_1^{\infty} \left[\pi_t^T(0,z)J^T(0,z) + \pi_t^T(1,z)J^T(1,z)\right] dG(z) \\ + \int_1^{\infty} \left[(\pi_t^{M,N}(0,z) - A\Omega)J^{M,N}(0,z) + (\pi_t^{M,N}(1,z) - A\Omega)J^{M,N}(1,z)\right] dG(z) \\ + \int_1^{\infty} \left[(\pi_t^{M,S}(0,z) - A\Omega)J^{M,S}(0,z) + (\pi_t^{M,S}(1,z) - A\Omega)J^{M,S}(1,z)\right] dG(z) \end{aligned} \tag{19}$$

and the non-negativity constraints, $C_t \geq 0$, and $K_t \geq 0$.

4. The markets for capital and labor clear:

$$K_t = \int_1^\infty \left[k_t^T(0,z)J^T(0,z) + k^{M,N}(0,z)J^{M,N}(0,z) + k^{M,S}(0,z)J^{M,S}(0,z)\right] dG(z) \quad (20)$$
$$+ \int_1^\infty \left[k_t^T(1,z)J^T(1,z) + k^{M,N}(1,z)J^{M,N}(1,z) + k^{M,S}(1,z)J^{M,S}(1,z)\right] dG(z)$$
$$+ K^G$$

$$N_t = \int_1^\infty \left[n_t^T(0,z)J^T(0,z) + n^{M,N}(0,z)J^{M,N}(0,z) + n^{M,S}(0,z)J^{M,S}(0,z)\right] dG(z) \quad (21)$$
$$+ \int_1^\infty \left[n_t^T(1,z)J^T(1,z) + n^{M,N}(1,z)J^{M,N}(1,z) + n^{M,S}(1,z)J^{M,S}(1,z)\right] dG(z)$$

5. The probability of grid power is such that the rationed grid electricity demand equals the supply (equation (15) in the main text).

C. Calibration

Table C.1 reports the model and empirical values of the moments used for the baseline calibration. Table C.2 reports the effect of a one percent increase in each parameter on the values of each of the eight moments. We order the table so that the parameter's primary moments are on and near the diagonal of the matrix. In all cases, changes in the parameter values meaningfully affect the parameter's primary moments.

Table C.1: Model Fit

Moment	Model	Target
(variable cost of self-generation)/(grid electricity price)	4.33	4.33
(average cost of self-generation)/(grid electricity price)	5.51	5.51
Fraction of self-generated electricity	0.59	0.59
Modern electricity share	0.07	0.07
Fraction of modern labor	0.63	0.63
Fraction of modern entrepreneurs	0.30	0.30
Fraction of modern entrepreneurs with a generator	0.82	0.82
Fraction of modern entrepreneurs without outages	0.20	0.20

Note: This table reports the empirical and model values of the moments used to calibrate the parameters in Table **??** for the baseline economy. We compute the modern electricity share in the model in a counterfactual steady state in which the probability of grid power equals 0.93.

Table C.2: Elasticities of Moments to Parameters

	$\frac{P_S}{P_G}$	Avg cost$\left(\frac{self}{grid}\right)$	$\frac{Self}{Grid}$	$\frac{L_m}{L}$	$\frac{N_m}{N}$	$\frac{N_{m,gen}}{N_m}$	$\frac{N_{m,q1}}{N_m}$	elec share
χ	-1.0	-0.8	0.3	0.4	0.4	0.1	-0.5	0.0
A^S	-1.0	-1.0	0.4	0.6	0.6	0.2	-0.8	0.4
P^G	-1.0	-1.0	-3.7	1.5	1.4	0.3	-2.1	0.0
λ	-0.0	0.0	-0.4	-0.3	0.0	0.0	0.2	0.0
Ω	-0.0	0.0	-0.5	-0.2	-1.0	0.1	0.6	-0.0
ζ	-0.0	0.0	0.1	0.1	0.8	-0.3	-0.8	0.0
ρ	-0.0	0.0	0.6	-0.1	-0.1	-0.1	1.2	0.0
μ	-0.0	0.0	-0.7	1.3	1.1	0.3	-1.3	-0.8

Note: Each row reports the percent change in the eight moments (from their values in the baseline calibration) from a one percent increase in the parameter value. The primary moments that discipline each parameter are on or near the diagonal of the matrix.

We re-calibrate P^G, χ, A^S, A^T, ζ, and ρ for each country in our study. We discipline ζ to match the country-specific fraction of modern firms that experience outages that have access to a generator. We pin down ρ to match the country-specific fraction of modern firms that report zero outages. We choose χ and A^S for each country to match the ratios of the average and marginal cost of self-generated electricity relative to grid electricity. We choose the regulated grid electricity price in each country, P^G, to match the country-specific fraction of electricity that firms with generators produce themselves. We determine the country-specific values of A^T to match the ratio of output per worker in the specific country relative to its value in Nigeria. Table C.3 reports the values of these country-specific targets and Table C.4 reports calibrated parameter values. The calibrated model in each country matches the corresponding targets out to four decimal places. We describe the data sources and the construction of these targets below.

Table C.3: Country Specific Targets

Country	AC^{self}/AC^{grid}	MC^{self}/MC^{grid}	E^s/E	Y/Y_{NGA}	$N_{m,q=1}/N_m$	$N_{m,gen}/N_{m,q=0}$
Ghana	3.66	3.00	0.21	0.90	0.06	0.75
Nigeria	5.51	4.33	0.59	1.00	0.20	0.82
Tanzania	3.22	2.70	0.28	0.38	0.13	0.52
Uganda	2.47	2.04	0.21	0.32	0.24	0.80

Note: This table reports the empirical values of the country-specific targets for each of the four countries in our study. The targets are: (1) the average cost of self-generated electricity relative to grid electricity, (2) the marginal cost of self-generated electricity relative to grid electricity, (3) self-generated electricity relative to total electricity among firms with generators (4) output per worker in the specific country relative to output per worker in Nigeria (5) fraction of modern entrepreneurs that do not experience outages and (6) the fraction of modern entrepreneurs that experience outages that have a generator.

Table C.4: Country Specific Parameters

Country	Price of grid electricity: P^G	Self-gen Leontief: χ	Self-gen TFP: A^S	Trad. TFP: A^T	Gumbel scale: ζ	Fraction with no outages: ρ
Ghana	0.13	1.73	1.54	0.87	0.25	0.04
Nigeria	0.11	2.14	1.02	1.00	0.24	0.08
Tanzania	0.11	1.52	2.26	0.49	1.60	0.13
Uganda	0.10	1.66	2.97	0.43	0.08	0.13

Note: This table reports the values of the country-specific parameters in the model.

We calculate the average and variable costs of self-generation in each country. All cross-country variation in these cost estimates comes from variation in diesel fuel prices; we assume that the capital and maintenance costs of self-generation are the same for all countries. We use estimates from the World Bank Technical Assessment (World Bank, 2007) to calculate the capital cost of a typical generator. The Technical Assessment reports that a 100 kW diesel generator would cost 640 in year 2005 dollars per kW (Table A13.4), and last 20 years with a capacity factor of 0.8 (Table A13.2)[1]. Using the US GDP deflator to adjust for inflation between 2005 and 2014, and an interest rate of 10 percent implies that the annual capital cost per kWh equals 1.3 cents in 2014 dollars. Table A13.5 in World Bank (2007) reports that the total maintenance cost of the generator is 5 cents per kwh in 2005 dollars. Again using the US GDP deflator to adjust for inflation implies that the maintenance cost is 5.9 cents per kWh in 2014 dollars.

We calculate the variable cost of self-generated electricity for the 100 kilowatt diesel generator with 30 percent efficiency. We use the 2014 price of diesel fuel in each country from the World

[1]The capacity factor is the ratio of the generator's actual generation to its maximum potential generation. It reflects the amount of time the generator is in use.

Development Indicators (series EP.PMP.DESL.CD). To convert dollars per liter of diesel fuel into dollars per kWh, we convert liters of diesel fuel into BTUs, and then convert BTUs into kWhs, adjusting for the 30 percent efficiency of the generator.

The average cost per kWh equals the sum of the capital, maintenance and variable costs per kWh. We use a capacity factor of 0.8 when we construct the average cost ratio in the model to ensure that it is consistent with the data. Table C.5 reports the average price per liter of diesel fuel and the variable and average costs of self-generated electricity in each country.

Table C.5: Self-Generation Costs

Country	Diesel Price ($/ltr)	Variable cost ($/kWh)	Average cost ($/kWh)
Ghana	1.03	0.32	0.40
Nigeria	0.84	0.26	0.34
Tanzania	1.20	0.38	0.45
Uganda	1.11	0.35	0.42

Note: This first column of this table reports the the price of diesel fuel in 2014 from the World Development Indicators. The second and third columns report the authors' calculations of the variable and average cost of self-generated electricity. All values are in year 2014 dollars.

We use the micro data from the WES to compute the following targets in each country: (1) the fraction of modern firms that experience outages that have access to a generator, (2) the fraction of electricity firms with a generator generate themselves, and (3) the fraction of modern firms that do not experience outages. We drop all observations for which the firm size is less than 10. The reported targets in each country are the mean values from the WES.

To calibrate TFP in each country, we compare output per worker in the specific country to output-per worker in Nigeria. Data on output per worker is from the Penn World Tables. We use output per worker in each country in the same year as the WES for that country. The survey year is 2014 in Nigeria and 2013 in Ghana, Tanzania, and Uganda.

The baseline calibration measures the size of the modern sector from the 2017 Nigerian National Survey of Micro, Small, and Medium Enterprises (SMEDA, 2017). The survey includes a representative sample of micro enterprises (those with less than 10 employees) and small (between 10 and 49 employees) and medium enterprises (between 50 and 199 employees). There is no information on large enterprises (those with greater than 199 employees). The survey has two modules: (1) the micro enterprises are covered in the National Integrated Survey of Households and (2) the small and medium enterprises are covered in National Integrated

Survey of Establishments. The survey covers all major sectors of the Nigerian economy, all geographic areas, and includes both formal and informal firms. We do not have access to the raw data from the survey. We take all information from the report, SMEDA (2017), assembled by the Small and Medium Enterprises Development Agency of Nigeria. The report compiles two sets of aggregate statistics, one set for micro enterprises and the second set for small and medium enterprises.

We use the survey results to compute the fraction of firms and workers that use electricity. The survey reports the average number of hours an enterprise operates with an alternative source of power. Firms that report zero use of alternative power "have little-to-nil need for [any] power supply" (SMEDA (2017), page 33). Since these firms do not use electricity in the production process, they correspond to the traditional sector in our model. Using this definition, approximately 70 percent of micro enterprises are traditional and 6 percent of small and medium enterprises are traditional. To calculate the fraction of traditional enterprises relative to total enterprises, we divide the number of traditional enterprises by the total number of micro, small, and medium enterprises. This calculation assumes that the number of large enterprises equals zero. Micro enterprises are so prevalent that any reasonable assumption about the number of large enterprises does not have meaningful effects on the fraction of traditional enterprises. In particular, micro, small and medium enterprises account for 76.5 percent of total employment in the Nigerian economy, implying that large enterprises employ at most 183,232,27 people. Since each large enterprise must employ 200 or more people, there can be at most 183,232,27/200 = 91,616 large enterprises. If we assume that there are 91,616 large enterprises, instead of zero, and all large enterprises are modern, then the fraction of traditional enterprises decreases from 0.703 to 0.701.

We calculate the fraction of total workers that are traditional. Approximately, 76.5 percent of the workers in Nigeria are employed by micro, small, and medium enterprises (SMEDA, 2017). We assume that the remaining 23.5 percent of workers are employed by large firms in the modern sector. To divide the workers at micro, small and medium enterprises into the traditional and modern sectors, we need information on how firm-size varies between the modern and the traditional firms. While this information is not available in the Nigerian survey, we use the fact that 70.4 percent of micro enterprises and 6 percent of small and medium enterprises are traditional, and we assume that each traditional micro enterprise employs one person and each traditional small and medium enterprise employs 10 people.

We use the US GDP deflator to convert all monetary values to 2014 dollars.

D. Additional Results From the Quantitative Model

Table D.1: Equilibrium Values of Macro-Aggregates in the Initial Steady State

	Ghana	Nigeria	Tanzania	Uganda
Grid-electricity price: P^G	0.13	0.11	0.11	0.10
Grid-electricity capital: K^G	0.61	0.35	0.37	0.28
Grid-electricity supply: E^G	0.71	0.48	0.50	0.41
Fraction of modern labor: N^M	0.78	0.63	0.74	0.61
Fraction of modern entrepreneurs: Q^M	0.41	0.30	0.64	0.23
Probability of grid power: ν	0.79	0.41	0.72	0.79

Note: This table reports the equilibrium values of a number of variables in the initial steady state in each country. While the units of the grid electricity price, capital, and supply are not meaningful independent of the model, the comparison of the different values across countries is meaningful.

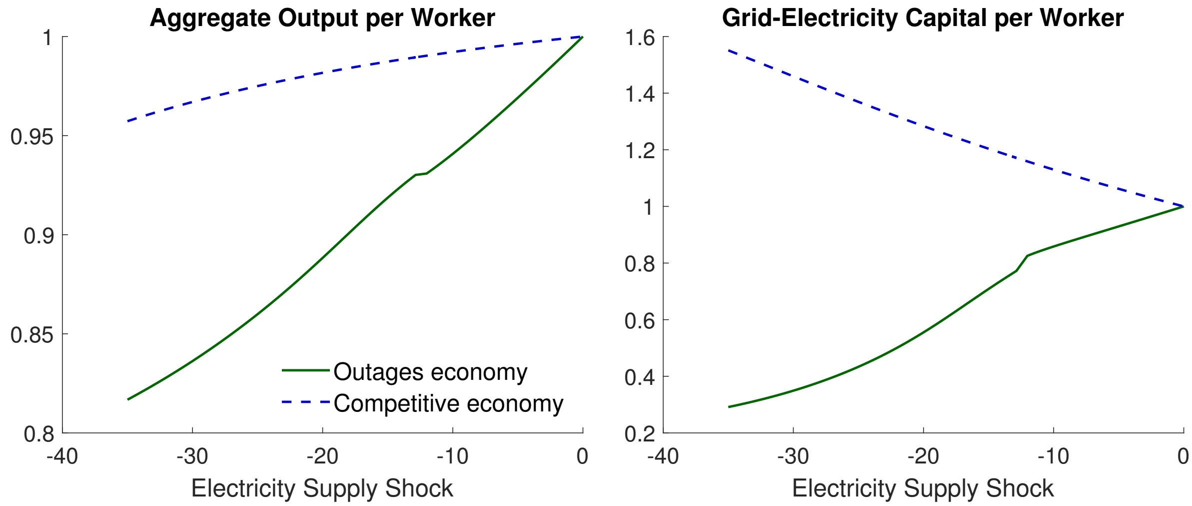
E. Extensions of the Quantitative Model

Insights on weak links, aggregate productivity, and electricity taxes. The regulated electricity price and resulting outages in our model are fundamentally different from other causes of low electricity supply, such as low productivity in electricity production or a tax on electricity producers. Insights from the previous literature (Jones, 2011) imply that if output from a low-productivity or taxed sector is sufficiently scarce, then its price will rise to attract more inputs, thus raising output in that sector. In our context, with electricity rationed through power outages, these forces do not operate in the same way. The electricity price is regulated to be artificially low, which prevents resources from reallocating to the electricity sector and, as a result, outages can more severely constrain aggregate output.

To illustrate the how the effects of power outages differ from low productivity in the electricity sector, Figure E.1 plots the responses of output per worker and grid electricity capital per worker to an electricity supply shock in our outages economy (solid green line) and in a competitive economy in which the grid electricity price adjusts to clear the market (dashed blue line). The right-most point in both panels corresponds to the no-outages steady state for Nigeria. Moving from right to left along the outages-economy line, we reduce the price of grid electricity so that total electricity supply (grid plus self-generated) in steady state decreases by the amount on the x-axis, causing outages to become more frequent. The left-most point corresponds to the initial steady state in the Nigerian economy.[2]

[2] In a small region around $\nu^\star$, there are two different steady states that generate the same amount of grid electricity supply, one in which $\nu < \nu^\star$ and relatively more electricity comes from generators and one in which $\nu > \nu^\star$ and relatively more electricity comes from the grid. As a convention, we plot only the steady state with

Figure E.1: Competitive Economy versus Outages Economy



Note: This figure plots the responses of output per worker and grid electricity capital per worker in Nigeria to an electricity supply shock in our outages economy (solid green line) and in a competitive economy (dashed blue line). The right-most point on the solid green line in both panels corresponds to the counterfactual no-outages steady state for Nigeria and the left-most point corresponds to the initial steady state. Moving from right to left along the outages-economy (solid green) line, we reduce the price of grid electricity so that total electricity supply (grid plus self-generated) in steady state decreases by the amount on the x-axis. Moving from right to left along the dashed blue line, we reduce productivity in grid electricity production, A^G, by the percentage on the x-axis.

Moving from right to left along the dashed blue line, we conduct a similar exercise for the competitive economy by reducing productivity in grid electricity production, A^G, by the percentage on the x-axis. This exercise results in higher grid electricity prices, but no outages, because the price endogenously adjusts to clear the grid electricity market.

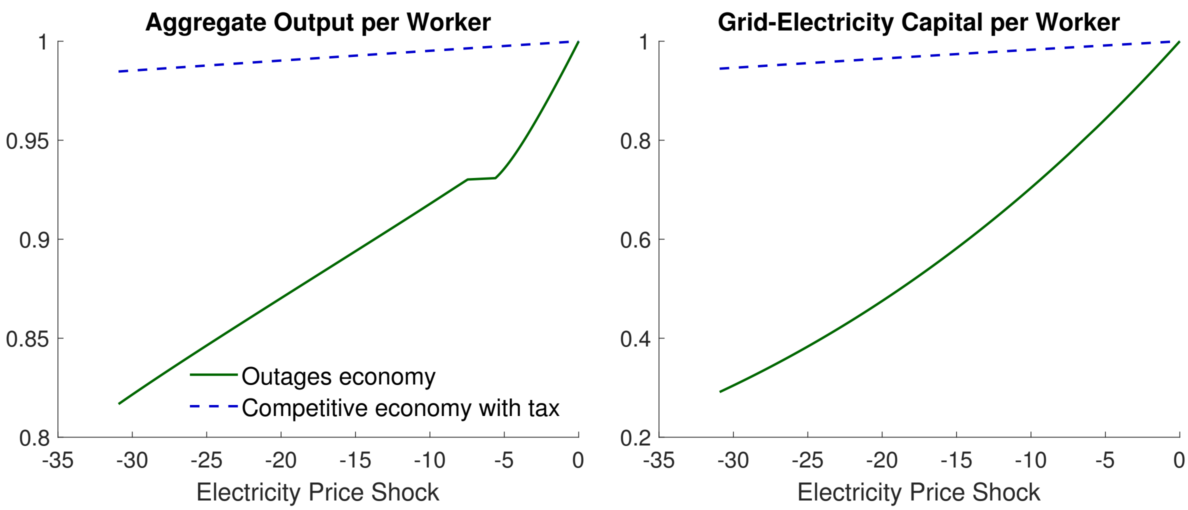
The electricity-supply shocks lead to larger decreases in output per worker in the outages economy than in the competitive economy; moving from right to left in the left panel of Figure E.1, the solid green line falls farther below the dashed blue line. Indeed, at the initial steady state (left-most point on the x-axis), output per worker falls by almost 20 percent in the outages economy but by less than 5 percent in the competitive economy. The reason for the difference is that in the competitive economy, the endogenous increase in the grid electricity price attracts more capital to the grid electricity sector, which substitutes for the low productivity and thus alleviates the constraints on aggregate output. In contrast, in the outages economy, the artificially low grid electricity price causes capital to leave the grid electricity sector, reducing supply and creating outages. The right panel of Figure E.1 illustrates these opposite responses; moving from right to left, grid-capital per worker increases in the competitive economy (dashed

$\nu < \nu^\star$ when there are two steady states. Plotting instead only the steady state with $\nu > \nu^\star$ when there are two steady states does not meaningfully change the graph.

blue line) and decreases in the outages economy (solid green line).

We can apply the same intuition to understand how outages are different from a tax on electricity suppliers. Figure E.2 plots the results from an experiment analogous to the one described above, except with a tax on electricity producers. The endogenous increase in the electricity price in response to the tax means that less capital leaves the electricity sector in the taxed economy, compared to the outages economy. As a result, the decline in output per worker in the taxed economy is much smaller than in the outages economy.

Figure E.2: Competitive Economy With a Tax versus Outages Economy



Note: This figure plots the responses of output per worker and grid electricity capital per worker in Nigeria to an electricity price shock in our outages economy (solid green line) and in a competitive economy with a tax (dashed blue line). The right-most point on the solid green line in both panels corresponds to the counterfactual no-outages steady state for Nigeria and the left-most point corresponds to the initial steady state. Moving from right to left along the outages-economy (solid green) line, we reduce the price of grid electricity by the amount on the x-axis. Moving from right to left along the dashed blue line, we introduce a tax on electricity suppliers, that would reduce the price electricity suppliers receive by the amount on the x-axis if electricity prices could not adjust.

In sum, the distortions in the electricity sector, caused by artificially depressed prices, fundamentally differ from those caused by low sector productivity or a tax in an otherwise competitive economy. Competitive forces can alleviate the consequences of tax distortions and low productivity by raising the price to attract more resources to the affected sector. But competitive forces cannot alleviate the consequences of depressed prices since, by design, the price cannot adjust to reflect the true scarcity of the input. Instead, the depressed prices discourage investment in electricity production, resulting in shortages. Thus, as long as electricity prices are artificially low, the electricity sector is likely to severely constrain aggregate output.

Generator fixed cost. We assume that the firm must pay an additional fixed cost, Υ, to operate a generator. The inclusion of the generator fixed cost creates economies of scale in the cost of generator capacity, consistent with the findings in Allcott et al. (2016). We consider four different possible values of the fixed cost, ranging from 0.05 to 0.2. We re-calibrate the model for Nigeria for each value of the fixed cost. For reference, in the re-calibrations, the generator fixed cost ranges from 11 percent of the modern entry cost, Ω, when $\Upsilon = 0.05$, to 125 percent of the modern entry cost when $\Upsilon = 0.2$. We and recalculate the short-run partial-equilibrium and long-run general-equilibrium effects of eliminating outages, as before. Table E.1 reports the results. Focusing on the third row, for all values of the fixed cost, including the baseline value of zero, the long-run general-equilibrium effect is substantially larger than the short-run partial-equilibrium effect.

Table E.1: Sensivity to Generator Fixed Cost: Nigeria

	Generator fixed cost				
	0	0.05	0.1	0.15	0.2
Short-run partial-equilibrium effect	5.8	5.9	6.0	6.1	6.1
Long-run general-equilibrium effect	22.4	23.2	24.4	26.0	28.5
Ratio	3.9	3.9	4.1	4.3	4.7

Note: This table reports the short-run partial-equilibrium (first row), long-run general-equilibrium (second row) percent increase in aggregate output between the initial and no-outages steady states in the Nigerian economy. The third row reports the ratio of the long-run general equilibrium to the short-run partial equilibrium. The five columns correspond to different values of the generator fixed cost, with zero corresponds to the baseline calibration in the main text.

Capital versus TFP decomposition. The first row of Table E.2 reports the short-run partial equilibrium effect from Figure 6 in the main text. The second row measures the impact of increased capital. Beginning from the short-run partial equilibrium, we exogenously increase modern and traditional capital by the percent that total productive firm capital ($K^M + K^T$) increases between the initial and no-outages steady states. Comparing the first and second rows reveals that increased capital results in a 50-100 percent larger increase in output per worker than predicted by the short-run partial equilibrium.

The third row of Table E.2 measures the impact of higher productivity by exogenously increasing productivity for a portion of traditional output. Specifically, we calculate the decrease in traditional output between the initial and no-outages steady states. We reallocate this traditional output to the modern sector by raising its productivity by factor $1 + \phi = 1.4$. For example, in Nigeria, eliminating outages reduces traditional output by approximately 70 percent. Nigerian output after the exogenous productivity increase equals $0.3\hat{Y}^T + 1.4 \times 0.7 \times \hat{Y}^T + \hat{Y}^M$, where 'hat' denotes the output from row 2 of Table E.2. Comparing the second and third rows of Table E.2 reveals that the increased-productivity mechanism results in a 50-110 percent larger increase in output per worker than predicted by the short-run partial equilibrium combined with exogenously higher capital.

The fourth row of Table E.2 reports the long-run general-equilibrium effect. The long-run general-equilibrium effect is similar to the short-run partial-equilibrium effect combined with the exogenous increases in capital and productivity (row 3 of Table E.2). This similarity implies that the increased availability of grid power (e.g., the short-run partial-equilibrium effect) and the increases in capital and productivity are largely responsible for the gains in output per worker from eliminating outages.

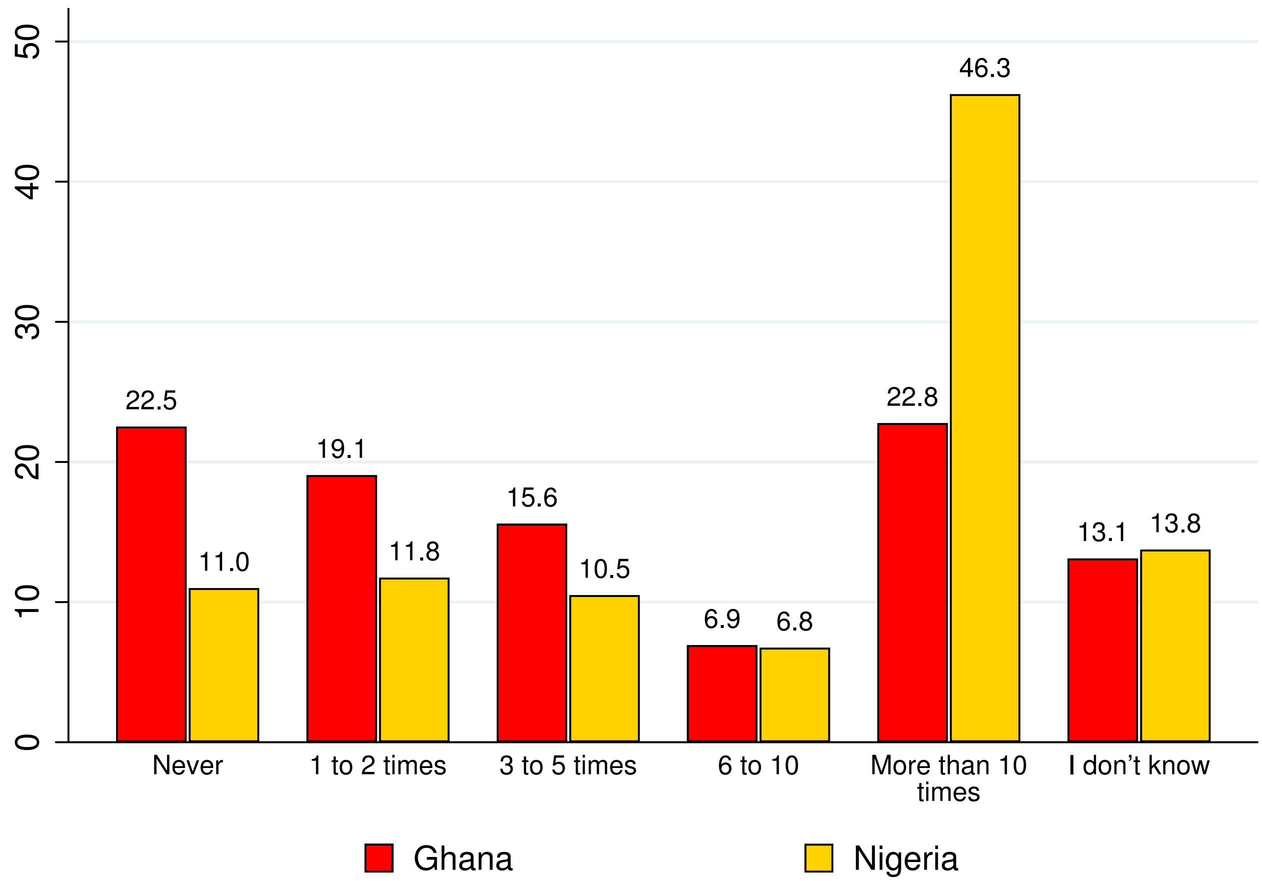
Table E.2: Effects of Higher Capital and Productivity
(percent increase in output per worker from the initial steady state)

	Ghana	Nigeria	Tanzania	Uganda
Short-run partial equilibrium	4.3	5.8	7.3	2.6
Add exogenous capital increase	7.6	11.4	11.6	4.3
Add exogenous productivity increase	11.6	21.7	15.9	9.1
Long-run general equilibrium	11.3	22.4	17.0	8.8

Note: This table reports the increase in output per worker from the no-outages steady state in the short-run partial equilibrium (row 1), the short-run partial equilibrium plus an exogenous increase in capital (row 2), the short-run partial equilibrium plus an exogenous increase in capital and productivity (row 3) and the long-run general equilibrium (row 4).

F. Firm Surveys

Figure F.1: Frequency of Power Outages of Surveyed Firms



Note: This figure reports the frequency of power outages in the previous year among the surveyed firms in Ghana (red) and Nigeria (yellow).

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