Sophisticated Bidders in Beauty-Contest Auctions

Online Appendix

Stefano Galavotti* Luigi Moretti† Paola Valbonesi‡

Abstract

This Online Appendix contains additional material that complements the paper "Sophisticated Bidders in Beauty-Contest Auctions". In particular: Section A and B contain the proofs of Proposition 1 and 2, respectively; Section C presents the results of a simulation exercise that support the theoretical predictions of the Cognitive Hierarchy model; Section D contains additional descriptive and inferential evidence that is discussed in the empirical part of the paper.

*Department of Economics and Management, University of Padova, Via del Santo 33, 35123 Padova, Italy. Email: stefano.galavotti@unipd.it.

†Centre d'Economie de la Sorbonne, Université Paris 1 Panthéon-Sorbonne, 106-112 Bvd de l'Hôpital, 75013 Paris, France. Email: luigi.moretti@univ-paris1.fr.

‡Department of Economics and Management, University of Padova, Via del Santo 33, 35123 Padova, Italy; Higher School of Economics, National Research University, Moscow-Perm, Russia. Email: paola.valbonesi@unipd.it.

A Proof of Proposition 1

Before proving Proposition 1, for convenience, we restate (in more detail) the assumptions of the model and we prove two lemmas that are used to demonstrate the Proposition.

The model. A single contract is auctioned off through an AB or ABL auction. There are n risk neutral firms that participate in the auction. Firm i's cost of completing the job is given by $c_i(x_1, x_2, \ldots, x_n)$, where x_i is a cost signal privately observed by firm i (x_i is the *type* of firm i). We sometimes write compactly x_{-i} to denote the vector of signals of all firms other than i. We assume that firm i's cost is separable in her own and other firms' signals and linear in x_i, that $c_i(x_i, x_{-i}) = a_i x_i + \Gamma_i(x_{-i})$, with $a_i > 0$, $\partial\Gamma_i/\partial x_j \geq 0$, for all $i, j \neq i$. Firm i's signal is distributed according to a cumulative distribution function $F_i(x_i)$, with full support $[\underline{x}_i, \overline{x}_i]$ and density $f_i(x_i)$. Signals are independent. The cost functions as well as the signals' distributios are common knowledge. Firms submit sealed bids formulated as percentage discounts over the reserve price R. We restrict our attention to situations in which all firms always participate in the auction, because they find it worthwhile to do so. Without this restriction, one should take into account the possibility that a firm may decide not to participate for some cost signal realizations: this would complicate the analysis but would not change the results qualitatively. Moreover, this restriction rules out the possibility of non serious bids.

Now, let $d_i \in [0, 1]$ denote firm i's bid (discount). The expected profit of firm i, type x_i, when she participates and bids d_i and the other firms follow the strategies δ_{-i} is:

$$\pi_i(x_i, d_i, \delta_{-i}) = [(1 - d_i)R - C_i(x_i, d_i, \delta_{-i})] \, \mathrm{PW}_i(d_i, \delta_{-i}),$$

where $\mathrm{PW}_i(d_i, \delta_{-i})$ is the probability that firm i wins when she bids d_i and the other firms follow the strategies δ_{-i}, and $C_i(x_i, d_i, \delta_{-i})$ is the expected cost of firm i when her signal is x_i, she bids d_i and the other firms follow the strategies δ_{-i}, conditional on the fact that i wins the auction. In symbols,

$$C_i(x_i, d_i, \delta_{-i}) = a_i x_i + E_{-i}\left[\Gamma_i(x_{-i}) \,|\, i \text{ wins when strategies are } (d_i, \delta_{-i})\right].$$

In the AB auction, the winning bid is the bid closest *from below* to $A2$. In the ABL auction, the winning bid is the bid closest *from above* to W, provided that this bid does not exceed $A2$. If no bid satisfies this requirement, the winning bid will be the one equal, if there is one, or closest from below to W. In both auctions, if all firms submit the same bid, the contract is assigned randomly. Similarly, if two or more firms make the same winning bid, the winner is chosen randomly among them.

We first show that, for both auctions, whatever the strategies of the others, a firm has always the possibility of placing a bid that gives her a strictly positive probability of winning the auction. Since this result is pretty intuitive, we omit the proof.

LEMMA 1. *Consider firm i and denote by δ_{-i} the bidding strategies of the other firms. Then, for any δ_{-i}, there exists d_i such that* $\mathrm{PW}_i(d_i, \delta_{-i}) > 0$.

This result, together with the restriction of full participation, implies that, in equilibrium, all firms will have a strictly positive probability of winning the auction.

We now show that, for both auctions, equilibrium bids are monotone.

LEMMA 2. *Let $\delta = (\delta_1, \delta_2, \ldots, \delta_n)$ be a (Bayes-) Nash equilibrium of either auction formats. Then, for all i, $\delta_i(x_i)$ is weakly decreasing.*

Proof. Consider firm i, and let $d_i = \delta_i(x_i)$, $d_i' = \delta_i(x_i')$, be her equilibrium bids when her signals are x_i and x_i', respectively, with $x_i < x_i'$. Notice first that, in equilibrium, the probability of winning the auction must be weakly decreasing in types. In fact, since d_i and d'_i are equilibrium bids, it must be true that:

$$[(1-d_i)R - C_i(x_i, d_i, \delta_{-i})]\ \mathrm{PW}_i(d_i, \delta_{-i}) \geq [(1-d'_i)R - C_i(x_i, d'_i, \delta_{-i})]\ \mathrm{PW}_i(d'_i, \delta_{-i}), \quad (1)$$

and that

$$[(1-d'_i)R - C_i(x'_i, d'_i, \delta_{-i})]\ \mathrm{PW}_i(d'_i, \delta_{-i}) \geq [(1-d_i)R - C_i(x'_i, d_i, \delta_{-i})]\ \mathrm{PW}_i(d_i, \delta_{-i}). \quad (2)$$

Summing them up, we obtain

$$[C_i(x'_i, d_i, \delta_{-i}) - C_i(x_i, d_i, \delta_{-i})]\, \mathrm{PW}_i(d_i, \delta_{-i}) \geq [C_i(x'_i, d'_i, \delta_{-i}) - C_i(x_i, d'_i, \delta_{-i})]\, \mathrm{PW}_i(d'_i, \delta_{-i}).$$

Notice that $C_i(x'_i, d, \delta_{-i}) - C_i(x_i, d, \delta_{-i}) = a_i(x'_i - x_i) > 0$, for all d. Hence, we obtain

$$\mathrm{PW}_i(d_i, \delta_{-i}) \geq \mathrm{PW}_i(d'_i, \delta_{-i}),$$

i.e., in equilibrium, the probability of winning the auction is weakly decreasing in types.

We now show that the equilibrium bidding function $\delta_i(x_i)$ must be weakly decreasing. Now, suppose, by contradiction, that there exists x_i, x'_i, with $x_i < x'_i$ and $d_i < d'_i$. Notice that, because, in equilibrium, $\mathrm{PW}_i(d_i, \delta_{-i}) > 0$ and $\mathrm{PW}_i(d'_i, \delta_{-i}) > 0$, the LHS and the RHS of (1) are strictly positive and the LHS of (2) is weakly positive. Hence, multiplying (1) by (2), we get

$$[(1-d_i)R - C_i(x_i, d_i, \delta_{-i})]\, [(1-d'_i)R - C_i(x'_i, d'_i, \delta_{-i})] \geq$$

$$[(1-d'_i)R - C_i(x_i, d'_i, \delta_{-i})]\, [(1-d_i)R - C_i(x'_i, d_i, \delta_{-i})]\,,$$

and, after some manipulation,

$$R\,[1 - d'_i + C_i(x'_i, d'_i, \delta_{-i})]\, [C_i(x'_i, d_i, \delta_{-i}) - C_i(x_i, d_i, \delta_{-i})] \geq$$

$$R\,[1 - d_i + C_i(x'_i, d_i, \delta_{-i})]\, [C_i(x'_i, d'_i, \delta_{-i}) - C_i(x_i, d'_i, \delta_{-i})]\,.$$

Now, since $C_i(x'_i, d, \delta_{-i}) - C_i(x_i, d, \delta_{-i}) = a_i(x'_i - x_i) > 0$, for all d, the inequality above reduces to

$$C_i(x'_i, d'_i, \delta_{-i}) - C_i(x'_i, d_i, \delta_{-i}) \geq d'_i - d_i > 0.$$

Hence, we get $C_i(x'_i, d'_i, \delta_{-i}) > C_i(x'_i, d_i, \delta_{-i})$; but this implies

$$[(1-d'_i)R - C_i(x'_i, d'_i, \delta_{-i})]\ \mathrm{PW}_i(d'_i, \delta_{-i}) < [(1-d_i)R - C_i(x'_i, d_i, \delta_{-i})]\ \mathrm{PW}_i(d_i, \delta_{-i}),$$

which contradicts (2). □

From the monotonicity property above, we can derive more precise predictions on the (Bayes-) Nash equilibria in the two formats. Let's start with the AB auction.

Proposition 1-*(i)*. *In the AB auction, there is a unique equilibrium in which all firms submit a* 0*-discount (irrespective of their signals), i.e., for all* i, $\delta_i(x_i) = 0$, *for all* x_i.

Proof. The proof proceeds in three steps.

Step 1. In equilibrium, for all i, there must exist $\hat{x}_i > \underline{x}_i$ such that $\delta_i(x_i) = \bar{d}$, for all $x_i \in [\underline{x}_i, \hat{x}_i)$, there must be a strictly positive probability that all firms make the same discount $\bar{d}$ (where $\bar{d}$ is the largest conceivable discount in equilibrium, see Lemma 2). Suppose not. Let $\bar{d}_i = \max_{x_i} \delta_i(x_i)$ be the largest bid of firm i (from Lemma 2, we know that $\bar{d}_i = \delta_i(\underline{x}_i)$) and let $\bar{d} = \max_i \bar{d}_i$ be the maximum conceivable bid in equilibrium. Notice that a firm that bids $\bar{d}$ can win if and only if all other firms bid $\bar{d}$. However, under our hypothesis, there exists at least one firm that, with probability one, bids less than $\bar{d}$. Hence, at least one of the firms that bid $\bar{d}$ has a zero probability of winning the auction, but this cannot occur in equilibrium. Hence, we have reached a contradiction.

STEP 2. $\bar{d} = 0$. To see this, notice that a firm bidding $\bar{d}$ wins if and only if all other firms bid $\bar{d}$ as well, and in this case every firm will win with probability $1/n$. However, a downward deviation would be profitable in any case: by making a lower bid, any firm will win with probability one when all other participating firms bid $\bar{d}$ (moreover, with a lower discount). The incentive to make a lower bid does not bite only when a lower bid is not allowed, only when $\bar{d} = 0$.

STEP 3. For all i, $\hat{x}_i = \overline{x}_i$. This is an immediate consequence of the fact that equilibrium bidding functions are weakly decreasing.

□

Consider now the ABL auction. In this case there is a multiplicity of equilibria. This discrepancy with respect to the AB format is not much due to the different way in which the winning threshold is computed, but rather to the fact that in ABL the winning bid is the one closest from above (rather than below) to the winning threshold, provided this bid does not exceed $A2$.

PROPOSITION 1-*(ii)*. *In the ABL auction, there exists a continuum of equilibria in which all firms make the same discount d (irrespective of their signals), for all i, $\delta_i(x_i) = d$ for all x_i, where d is such that $\pi_i(x_i, d, \delta_{-i}) > 0$ for all i and for all x_i.*

Proof. If all firms make the same bid d, whatever their signal is, every firm will have a $1/n$ chance of winning. If firm i (of any type) makes a bid larger than d, then $A2$ will necessarily be equal to d and firm i will have a zero probability of winning as her bid exceeds $A2$. If instead firm i (of any type) makes a bid below d, then W will necessarily be equal to d and the winner will be one of the other firms. Again, the probability of winning of firm i will fall to zero. Therefore, d is the only bid that guarantees a strictly positive probability of winning.

□

Beyond the flat equilibria described above, the ABL auction may possibly have other equilibria. In any case, these equilibria display a very large degree of pooling on the maximum discount. The next propositions formalizes this idea.

PROPOSITION 1-*(iii)* - FIRST STATEMENT. *Consider any equilibrium of the ABL auction: let $\bar{d}$ denote the highest conceivable bid in equilibrium, $\bar{d} = \max_i \delta_i(\underline{x}_i)$; let K be the set of firms that bid $\bar{d}$ with strictly positive probability and let k denote the cardinality of K. Then, in any equilibrium, $k \geq n - \tilde{n}$.*

Proof. The proof proceeds by showing that if $k < n - \tilde{n}$, any firm $\in K$ has a profitable (downward) deviation.

- $k \leq \tilde{n}$. In this case, any firm $i \in K$ that bids $\bar{d}$ would have a zero probability of winning the auction ($A2$ will necessarily be lower than $\bar{d}$, hence $\bar{d}$ cannot be a winning bid); but this cannot occur in equilibrium.
- $\tilde{n} < k < n - \tilde{n}$. Consider any firm $i \in K$ with signal $\underline{x}_i$. This firm bids $\bar{d}$ and can win the auction if and only if $A2 = \bar{d}$ and the winning threshold W is greater than or equal to the largest bid lower than $\bar{d}$. If this occurs, the winner will be chosen randomly from those firms that bid $\bar{d}$. Hence, the expected profit of firm i, type $\underline{x}_i$ is

  $$\pi_i(\underline{x}_i, \bar{d}, \delta_{-i}) =$$
  $$\sum_{j=\tilde{n}}^{k-1} \frac{(1-\bar{d})R - C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)}{j+1} \Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j),$$

  where J denotes the number of firms in K, beyond firm i, that do bid $\bar{d}$.

  Consider now what happens when firm i, type $\underline{x}_i$ bids slightly less than $\bar{d}$. In this case, her expected profit would at least be

  $$\pi_i(\underline{x}_i, \bar{d} - \varepsilon, \delta_{-i}) \geq$$

$$\sum_{j=\tilde{n}}^{k-1}[(1-\bar{d}+\varepsilon)R - C_i(\underline{x}_i, \bar{d}-\varepsilon, \delta_{-i}|J=j)]\Pr(\bar{d}-\varepsilon \text{ is winning bid}|J=j)\Pr(J=j).$$

In order for $\bar{d}$ to be the equilibrium bid of firm i, type $\underline{x}_i$, it must hold that $\pi_i(\underline{x}_i, \bar{d}, \delta_{-i}) \geq \pi_i(\underline{x}_i, \bar{d}-\varepsilon, \delta_{-i})$, for all $\varepsilon > 0$. In the limit,[1] this implies that

$$\sum_{j=\tilde{n}}^{k-1} -\frac{j}{j+1}[(1-\bar{d})R - C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j) \geq 0. \quad (3)$$

Notice that $\Pr(\bar{d}$ is winning bid$|J=j)$ must be strictly greater than zero for at least some j (if not, firm i, type $\underline{x}_i$, would have a zero probability of winning and would rather deviate downward or not participate). Hence, if the term between square brackets in (3) is positive for all j (notice that individual rationality implies that at least one of these terms must be strictly positive), then the inequality above cannot be satisfied. However, consider the possibility that the term between square brackets in (3) is positive for some j and strictly negative for the others. Notice that, because all bidding functions are weakly decreasing, $C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)$ must be weakly decreasing in j. Hence, there must be some $\hat{n}$ such that the term between square brackets is strictly negative for $\tilde{n} \leq j \leq \hat{n}$, and positive for $\hat{n} < j \leq k-1$. In light of this, inequality (3) can be written as

$$\sum_{j=\tilde{n}}^{\hat{n}} \frac{j}{j+1}[C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j) - (1-\bar{d})R]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j) \geq$$
$$\sum_{j=\hat{n}+1}^{k-1} \frac{j}{j+1}[(1-\bar{d})R - C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j).$$

Notice that the LHS of the inequality above (which now contains only strictly positive terms) is necessarily lower than

$$\sum_{j=\tilde{n}}^{\hat{n}} \frac{\hat{n}}{j+1}[C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j) - (1-\bar{d})R]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j),$$

and the RHS is necessarily strictly greater than

$$\sum_{j=\hat{n}+1}^{k-1} \frac{\hat{n}}{j+1}[(1-\bar{d})R - C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)]\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j).$$

But this would imply that $\pi_i(\underline{x}_i, \bar{d}, \delta_{-i}) < 0$, which contradicts the fact that this is an equilibrium.

□

Proposition 1-*(iii)* - Second statement. *In any equilibrium of the ABL auction in which there is at least one firm $i \in K$ such that $\mathrm{PW}_i(\bar{d}, \delta_{-i}) \geq \mathrm{PW}_i(\bar{d}-\varepsilon, \delta_{-i})$ for $\varepsilon \to 0^+$, the probability that at least $n-\tilde{n}-1$ firms do bid $\bar{d}$ must be larger than $\sum_{j=n-\tilde{n}-1}^{k-1} r^j / \sum_{j=0}^{k-1} r^j$, where r solves $\sum_{j=1}^{k-(n-\tilde{n}-1)} r^j = T$, where $T = (n-\tilde{n})(n-\tilde{n}-2)/(n-\tilde{n}-1)$.*

Proof. Consider firm i, type $\underline{x}_i$. This firm bids $\bar{d}$ and wins the auction with probability

$$\mathrm{PW}_i(\bar{d}, \delta_{-i}) = \sum_{j=\tilde{n}}^{n-\tilde{n}-3} \frac{\Pr(\bar{d} \text{ is winning bid}|J=j)\Pr(J=j)}{j+1} + \sum_{j=n-\tilde{n}-2}^{k-1} \frac{\Pr(J=j)}{j+1},$$

[1] Notice that, for $j \leq n-\tilde{n}-1$, when $\varepsilon \to 0$, $\Pr(\bar{d}-\varepsilon$ is winning bid$|J=j) \to \Pr(\bar{d}$ is winning bid$|J=j)$, and $C_i(\underline{x}_i, \bar{d}-\varepsilon, \delta_{-i}|J=j) \to C_i(\underline{x}_i, \bar{d}, \delta_{-i}|J=j)$.

where J is the number of firms in K that do bid $\bar{d}$ (beyond firm i itself). Notice that, when $J \geq n - \tilde{n} - 2$, the winning threshold W is necessarily equal to $\bar{d}$. Suppose that firm i, type $\underline{x}_i$, bids slightly less than $\bar{d}$. Her probability of winning the auction would at least be

$$\mathrm{PW}_i(\bar{d} - \varepsilon, \delta_{-i}) \geq \sum_{j=\tilde{n}}^{n-\tilde{n}-3} \Pr(\bar{d} - \varepsilon \text{ is winning bid}|J = j)\Pr(J = j)$$

$$+\Pr(\bar{d} - \varepsilon \text{ is winning bid}|J = n - \tilde{n} - 2)\Pr(J = n - \tilde{n} - 2).$$

Notice that, when $J > n - \tilde{n} - 2$, W will be equal to $\bar{d}$ and $\bar{d} - \varepsilon$ cannot be a winning bid.

By assumption, for sufficiently small ε, it must be $\mathrm{PW}_i(\bar{d}, \delta_{-i}) \geq \mathrm{PW}_i(\bar{d}-\varepsilon, \delta_{-i})$. In the limit, this inequality becomes

$$\sum_{j=\tilde{n}}^{n-\tilde{n}-3} \frac{\Pr(\bar{d} \text{ is winning bid}|J = j)\Pr(J = j)}{j+1} + \sum_{j=n-\tilde{n}-2}^{k-1} \frac{\Pr(J = j)}{j+1} \geq$$

$$\sum_{j=\tilde{n}}^{n-\tilde{n}-3} \Pr(\bar{d} \text{ is winning bid}|J = j)\Pr(J = j) + \Pr(J = n - \tilde{n} - 2),$$

or, equivalently,

$$\sum_{j=n-\tilde{n}-1}^{k-1} \frac{\Pr(J = j)}{j+1} - \frac{n - \tilde{n} - 2}{n - \tilde{n} - 1}\Pr(J = n-\tilde{n}-2) \geq \sum_{j=\tilde{n}}^{n-\tilde{n}-3} \frac{j}{j+1}\Pr(\bar{d} \text{ is winning bid}|J = j)\Pr(J = j). \quad (4)$$

Notice that the RHS of the (4) is positive. Hence, in order for (4) to be satisfied, it must necessarily hold that

$$\sum_{j=n-\tilde{n}-1}^{k-1} \frac{\Pr(J = j)}{j+1} \geq \frac{n - \tilde{n} - 2}{n - \tilde{n} - 1}\Pr(J = n - \tilde{n} - 2). \quad (5)$$

Notice that the LHS of the (4) is lower than

$$\sum_{j=n-\tilde{n}-1}^{k-1} \frac{\Pr(J = j)}{n - \tilde{n}}.$$

Hence, in order for (5) to be satisfied, it must necessarily hold that

$$\sum_{j=n-\tilde{n}-1}^{k-1} \Pr(J = j) \geq \frac{(n - \tilde{n})(n - \tilde{n} - 2)}{n - \tilde{n} - 1}\Pr(J = n - \tilde{n} - 2). \quad (6)$$

Our goal is to find a lower bound to $\sum_{j=n-\tilde{n}-1}^{k-1} \Pr(J = j)$, knowing that (6) must necessarily hold.

Notice that the number J of firms that bid $\bar{d}$ (beyond firm i) is the number of successes in $k - 1$ independent trials, where the probability of success in the l-th trial is $p_l = F_l(\hat{x}_l)$; hence, J is a random variable with Poisson binomial distribution. Now, denote by r_j the ratio $\frac{\Pr(J=j)}{\Pr(J=j-1)}$. Inequality (6) can be rewritten as

$$r_{n-\tilde{n}-1} \geq \frac{T}{1 + \sum_{j=n-\tilde{n}}^{k-1} \prod_{i=n-\tilde{n}}^{j} r_i}, \quad (7)$$

where $T = (n - \tilde{n})(n - \tilde{n} - 2)/(n - \tilde{n} - 1)$.

It can easily be shown that, if $r_j = t$, then $r_{j+1} > t$, i.e., r_j is increasing in j. Hence, we have the following constraints:

$$r_{k-1} > r_{k-2} > \ldots > r_1. \quad (8)$$

Finally, it must be that $\sum_{j=0}^{k-1} \Pr(J = j) = 1$, which can be rewritten as

$$\Pr(J = n - \tilde{n} - 1) = \frac{\prod_{i=1}^{n-\tilde{n}-1} r_i}{1 + \sum_{j=1}^{k-1} \prod_{i=1}^{j} r_i}. \tag{9}$$

Our objective is to find a lower bound to $\sum_{j=n-\tilde{n}-1}^{k-1} \Pr(J = j)$, i.e., we want to solve

$$\inf_{\{r_i\}} \quad \Pr(J = n - \tilde{n} - 1) \left[1 + \sum_{j=n-\tilde{n}}^{k-1} \prod_{i=n-\tilde{n}}^{j} r_i \right]$$

under the constraints (7), (8), (9).

The solution to the above problem is no greater than the solution to the problem

$$\inf_{\{r_i\}} \quad \Pr(J = n - \tilde{n} - 1) \left[1 + \sum_{j=n-\tilde{n}}^{k-1} \prod_{i=n-\tilde{n}}^{j} r_i \right]$$

under the constraints (7), (9) and under the constraint

$$r_{k-1} \geq r_{k-2} \geq \ldots \geq r_1. \tag{10}$$

(We are replacing (8) with a looser constraint). It's easy to show, that, in the solution to the above problem all constraints (7) and (10) are binding. Hence, the objective function is minimized at

$$r_1 = r_2 = \ldots = r_k - 1 = r, \quad \text{with} \quad \sum_{j=1}^{k-(n-\tilde{n}-1)} r^j = T,$$

and the minimum is $\sum_{j=n-\tilde{n}-1}^{k-1} r^j / \sum_{j=0}^{k-1} r^j$.

□

B Proof of Proposition 2

Proposition 2 can be easily obtained as a corollary of the following two lemmas that precisely characterize the asymptotic (optimal) behavior of level-k firms, $k \geq 1$.

Lemma 3

(i) Consider the AB auction. Let $\delta_k^{(n)}(x)$ be the bidding strategy of a level-k firm, type x, for $k \geq 1$, when there are n firms and the other firms' levels range from 0 to $k-1$ (and the proportion of level-j firms is $p_j / \sum_{i=0}^{k-1} p_i$). Then, as $n \to \infty$, $\delta_k^{(n)}(x) \to \overline{A2}_{k-1}$ for all x, where:

- $\overline{A2}_0 = \mathbb{E}[d_0 | \overline{A1}_0 < d_0 < d_{[90]}]$;
- for $j \geq 1$, $\overline{A2}_j = (p_0 \mathbb{E}[d_0 | \overline{A1}_j < d_0 < d_{[90]}] + \sum_{i=1}^{j} p_i \overline{A2}_{i-1} \mathbb{1}_{[\overline{A2}_{i-1} > \overline{A1}_j]}) / (p_0 + \sum_{i=1}^{j} p_i \mathbb{1}_{[\overline{A2}_{i-1} > \overline{A1}_j]})$;
- $\overline{A1}_0 = \mathbb{E}[d_0 | d_{[10]} < d_0 < d_{[90]}]$;
- for $j \geq 1$, $\overline{A1}_j = (p_0 \overline{A1}_0 + \sum_{i=1}^{j} p_i \overline{A2}_{i-1}) / (\sum_{i=0}^{j} p_i)$;
- $d_{[10]}$ and $d_{[90]}$ are the 10-th and 90-th percentile of $G_0(d)$, $G_0(d_{[10]}) = 0.1$ and $G_0(d_{[90]}) = 0.9$.

(ii) Consider the ABL auction. Let $\delta_k^{(n)}(x)$ be the bidding strategy of a level-k firm, for $k \geq 1$, when there are n firms and the other firms' levels range from 0 to $k-1$ (and the proportion of level-j firms is $p_j / \sum_{i=0}^{k-1} p_i$). Then, as $n \to \infty$, $\delta_k^{(n)}(x) \to \overline{A3}_{k-1}$ for all x, where:

- for $j \geq 1$, $\overline{A3}_j = (\overline{A2}_j + d_{[10]})/2$;
- $\overline{A2}_0 = \mathbb{E}[d_0 | \overline{A1}_0 < d_0 < d_{[90]}]$;
- for $j \geq 1$, $\overline{A2}_j = (p_0 \mathbb{E}[d_0 | \overline{A1}_j < d_0 < d_{[90]}] + \sum_{i=1}^{j} p_i \overline{A3}_{i-1} \mathbb{1}_{[\overline{A3}_{i-1} > \overline{A1}_j]})/(p_0 + \sum_{i=1}^{j} p_i \mathbb{1}_{[\overline{A3}_{i-1} > \overline{A1}_j]})$;
- $\overline{A1}_0 = \mathbb{E}[d_0 | d_{[10]} < d_0 < d_{[90]}]$;
- for $j \geq 1$, $\overline{A1}_j = (p_0 \overline{A1}_0 + \sum_{i=1}^{j} p_i \overline{A3}_{i-1})/(\sum_{i=0}^{j} p_i)$;
- $d_{[10]}$ and $d_{[90]}$ are the 10-th and 90-th percentile of $G_0(d)$, $G_0(d_{[10]}) = 0.1$ and $G_0(d_{[90]}) = 0.9$.

Proof.

(i) Consider the AB auction. Let $A1_{k-1}$ and $A2_{k-1}$ be the value of $A1$ and $A2$ when firms' levels range from 0 to $k-1$ (with frequencies $(p_0/\sum_{i=0}^{k-1} p_i,\ \ldots,\ p_{k-1}/\sum_{i=0}^{k-1} p_i)$), level-0 firms bid according to $G_0(d)$ and level-j firms, $0 < j \leq k-1$, bid their best responses to their own beliefs. Consider a level-1 firm first. In order to choose her optimal bid, a level-1 firm has to compute the probability distribution of the winning threshold $A2_0$, which in turn depends on $A1_0$. Now, $A1_0 = \sum_{j=\tilde{n}+1}^{n-\tilde{n}} d_0^{(j)}/(n-2\tilde{n})$, where $d_0^{(j)}$ is the j-th lowest bid by the level-0 firms. Let Y_i, $i = 1, \ldots n$ be a sequence of i.i.d. random variables with distribution $G_Y(y) = G_0(y | d_{[10]} < d_0 < d_{[90]})$. The crucial thing to show is that, when $n \to \infty$, $A1_0$ converges almost surely to $\overline{A1}_0 = \mathbb{E}[Y]$. To see this, notice first that, by the strong law of large numbers, $\sum_{j=1}^{n} d_0^{(j)}/n \xrightarrow{a.s.} \mathbb{E}[d_0]$, and, consequently, $d_0^{(\tilde{n})} \xrightarrow{a.s.} d_{[10]}$, $d_0^{(n-\tilde{n}+1)} \xrightarrow{a.s.} d_{[90]}$. Now, let $m_1 = \min\{m \in 1, \ldots, n : d_0^{(m)} > d_{[10]}\}$ and $m_n = \max\{m \in 1, \ldots, n : d_0^{(m)} < d_{[90]}\}$. Notice that $\sum_{j=m_1}^{m_2} d_0^{(j)}/(m_2 - m_1 + 1)$ converges almost surely to $\mathbb{E}[Y]$ (because the random variables $d_0^{(l)}$, with $l \in [m_1, m_2]$, have the same distributions as the Y_i's). Given this, in order to show that $A1_0 \xrightarrow{a.s.} \mathbb{E}[Y]$, it is sufficient to show that the difference $A1_0 - \sum_{j=m_1}^{m_2} d_0^{(j)}/(m_2 - m_1 + 1)$ converges almost surely to 0. Now, this difference can be written as

$$A1_0 - \frac{\sum_{j=m_1}^{m_2} d_0^{(j)}}{n-2\tilde{n}} + \frac{\sum_{j=m_1}^{m_2} d_0^{(j)}}{n-2\tilde{n}} - \frac{\sum_{j=m_1}^{m_2} d_0^{(j)}}{m_2 - m_1 + 1}. \tag{11}$$

Notice that, since $d_0 \in [\underline{d}, \overline{d}] \subseteq [0, 1]$, the first two addends in (11) are certainly no greater than

$$\frac{|m_1 - \tilde{n}| + |m_2 - (n - \tilde{n}) + 1|}{n - 2\tilde{n}},$$

and this term goes to 0 almost surely. The last two terms in (11) can be written as

$$\frac{\sum_{j=m_1}^{m_2} d_0^{(j)}}{m_2 - m_1 + 1} \left(\frac{m_2 - m_1 + 1}{n - 2\tilde{n}} - 1 \right).$$

Notice that the first fraction converges to $\mathbb{E}[Y]$, and that $(m_2 - m_1 + 1)/(n - 2\tilde{n})$ goes to 1. Hence, expression (11) converges to 0 almost surely.

In a similar way, one can show that $A2_0$ converges almost surely to $\overline{A2}_0 = \mathbb{E}[d_0 | \overline{A1}_0 < d_0 < d_{[90]}]$. Moreover, notice that, because $G_0(d)$ has full support, when n grows to infinity, for all $\varepsilon > 0$, $\Pr(d_0 \in (\overline{A2}_0 - \varepsilon, \overline{A2}_0)) \to 1$. Hence, as n increases, to get a positive chance of winning, a level-1 has to make a bid which is closer and closer to the expected value (from her viewpoint) of the winning threshold $A2$, $\delta_1^{(n)}(x) \to \overline{A2}_0$, for all x.

Consider now a level-2 firm. From her point of view, the winning threshold is $A2_1$, which, in turn, depends on $A1_1$. Reasoning in the same way as before, and given that level-1 firms' bids tend to $\overline{A2}_0$, one show that $A1_1$ converges almost surely to $\overline{A1}_1 = (p_0\overline{A1}_0 + p_1\overline{A2}_0)/(p_1+p_2)$, and $A2_1$ converges almost surely to $\overline{A2}_1 = (p_0\mathbb{E}[d_0|\overline{A1}_1 < d_0 < d_{[90]}] + p_1\overline{A2}_0)/(p_0+p_1)$. As n increases, to get a positive chance of winning, a level-2 has to make a bid which is closer and closer to the expected value (from her viewpoint) of the winning threshold $A2$, $\delta_2^{(n)}(x) \to \overline{A2}_1$, for all x.

Proceeding recursively, it is easy to show that, for all $k \geq 1$, $A1_k$ converges almost surely to $\overline{A1}_k = (p_0\overline{A1}_0 + \sum_{i=1}^k p_i\overline{A2}_{i-1})/(\sum_{i=0}^k p_i)$, and $A2_k$ converges almost surely to

$$\overline{A2}_k = \frac{p_0\mathbb{E}[d_0|\overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=1}^k p_i\overline{A2}_{i-1}\mathbb{1}_{[\overline{A2}_{i-1} > \overline{A1}_k]}}{p_0 + \sum_{i=1}^k p_i\mathbb{1}_{[\overline{A2}_{i-1} > \overline{A1}_k]}}.$$

Hence, $\delta_k^{(n)}(x) \to \overline{A2}_{k-1}$ for all x.

(ii) Apart from minor differences, the proof is the same for the ABL auction. Just one point is worth mentioning: from the point of view of a level-k firm, when n grows to infinity, the interval from which the winning threshold is drawn converges to $[\overline{A3}_{k-1}, \overline{A2}_{k-1}]$. Now, since every number in this interval has the same probability of being extracted, a level-k firm will bid closer and closer to the lowest value of this interval, $\delta_k^{(n)}(x) \to \overline{A3}_{k-1}$.

□

Lemma 4

(i) In the AB auction, $\overline{A2}_{k-1} < \overline{A2}_k$*, for all* $k \geq 1$.

(ii) In the ABL auction: if $\overline{A1}_0 < (d_{[10]} + \overline{A2}_0)/2$*, then* $\overline{A2}_{k-1} < \overline{A2}_k$ *for all* $k \geq 1$*; if* $\overline{A1}_0 > (d_{[10]} + \overline{A2}_0)/2$*, then* $\overline{A2}_{k-1} > \overline{A2}_k$ *for all* $k \geq 1$*; if* $\overline{A1}_0 = (d_{[10]} + \overline{A2}_0)/2$*, then* $\overline{A2}_{k-1} = \overline{A2}_k$ *for all* $k \geq 1$.

Proof.

(i) Notice first that, by construction, for all k, $\overline{A1}_k < \overline{A2}_k$: in fact, $\overline{A2}_k$ is a weighted average of numbers that are strictly greater than $\overline{A2}_k$. Second, for all $k \geq 1$, $\overline{A1}_{k-1} < \overline{A1}_k < \overline{A2}_{k-1}$: for $k = 1$, this is fairly obvious; for $k > 1$, notice that, since $\overline{A1}_{k-1} = (p_0\overline{A1}_0 + \sum_{i=1}^{k-1} p_i\overline{A2}_{i-1})/\sum_{i=0}^{k-1} p_i$, we have that

$$\sum_{i=0}^{k-1} p_i\overline{A1}_{k-1} = p_0\overline{A1}_0 + \sum_{i=1}^{k-1} p_i\overline{A2}_{i-1}.$$

Using this and substituting into the expression for $\overline{A1}_k$, we get

$$\overline{A1}_k = \frac{\sum_{i=0}^{k-1} p_i\overline{A1}_{k-1} + p_k\overline{A2}_{k-1}}{\sum_{i=0}^k p_i}.$$

Hence, $\overline{A1}_k$ is a weighted average of $\overline{A1}_{k-1}$ and $\overline{A2}_{k-1}$, but since $\overline{A1}_{k-1} < \overline{A2}_{k-1}$, it must be $\overline{A1}_{k-1} < \overline{A1}_k < \overline{A2}_{k-1}$.

We now show, by induction, that, if $\overline{A2}_{j-1} < \overline{A2}_j$ for all $j \leq k$, then $\overline{A2}_k < \overline{A2}_{k+1}$. So, assume $\overline{A2}_{j-1} < \overline{A2}_j$ for all $j \leq k$, $k \geq 1$; let $s = \min j = 0, \ldots, k-1|\overline{A1}_k < \overline{A2}_j$ and let

$t = \min j = 0, \ldots, k | \overline{A1}_{k+1} < \overline{A2}_j$. Notice that, necessarily, it must be $s \leq t$; when $s < t$, we have

$$\begin{aligned}\overline{A2}_k =& \frac{p_0 \mathbb{E}[d_0 | \overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=s+1}^{k} p_i \overline{A2}_{i-1}}{p_0 + \sum_{i=s+1}^{k} p_i} \\ =& \frac{p_0 \mathbb{E}[d_0 | \overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=s+1}^{t} p_i \overline{A2}_{i-1} + \sum_{i=t+1}^{k} p_i \overline{A2}_{i-1}}{p_0 + \sum_{i=s+1}^{t} p_i + \sum_{i=t+1}^{k} p_i}.\end{aligned}$$

Hence,

$$(p_0 + \sum_{i=s+1}^{t} p_i + \sum_{i=t+1}^{k} p_i)\overline{A2}_k - \sum_{i=s+1}^{t} p_i \overline{A2}_{i-1} = p_0 \mathbb{E}[d_0 | \overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=t+1}^{k} p_i \overline{A2}_{i-1}. \quad (12)$$

Now, notice that, since $\overline{A1}_{k+1} > \overline{A1}_k$, it must be

$$\begin{aligned}\overline{A2}_{k+1} =& \frac{p_0 \mathbb{E}[d_0 | \overline{A1}_{k+1} < d_0 < d_{[90]}] + \sum_{i=t+1}^{k+1} p_i \overline{A2}_{i-1}}{p_0 + \sum_{i=t+1}^{k+1} p_i} \\ >& \frac{p_0 \mathbb{E}[d_0 | \overline{A1}_k < d_0 < d_{[90]}] + \sum_{i=t+1}^{k+1} p_i \overline{A2}_{i-1}}{p_0 + \sum_{i=t+1}^{k+1} p_i}.\end{aligned}$$

Using (12), the last inequality becomes

$$\begin{aligned}\overline{A2}_{k+1} >& \frac{(p_0 + \sum_{i=s+1}^{t} p_i + \sum_{i=t+1}^{k} p_i)\overline{A2}_k - \sum_{i=s+1}^{t} p_i \overline{A2}_{i-1} + p_{k+1}\overline{A2}_k}{p_0 + \sum_{i=t+1}^{k+1} p_i} \\ =& \frac{(p_0 + \sum_{i=t+1}^{k} p_i + p_{k+1})\overline{A2}_k + \sum_{i=s+1}^{t} p_i (\overline{A2}_k \overline{A2}_{i-1})}{p_0 + \sum_{i=t+1}^{k+1} p_i} \\ =& \overline{A2}_k + \frac{\sum_{i=s+1}^{t} p_i (\overline{A2}_k \overline{A2}_{i-1})}{p_0 + \sum_{i=t+1}^{k+1} p_i} \\ \geq& \overline{A2}_k.\end{aligned}$$

When $s = t$, the whole derivation above goes through with the only difference that all terms involving $\sum_{i=s+1}^{t}$ are absent. To complete the proof, we have to show that $\overline{A2}_0 < \overline{A2}_1$, which is fairly obvious, since $\overline{A2}_1 = (p_0 \mathbb{E}[d_0 | \overline{A1}_1 < d_0 < d_{[90]}] + p_1 \overline{A2}_0)/(p_0 + p_1)$, is a wighted average of $\overline{A2}_0$ and a number ($\mathbb{E}[d_0 | \overline{A1}_1 < d_0 < d_{[90]}]$) strictly greater than $\overline{A2}_0$.

(ii) The proof for the ABL auction follows exactly the same procedure as the previous one, but with one caveat: if $\overline{A1}_0 < \overline{A3}_0$, we have that the sequence of $\overline{A3}_k$'s is strictly increasing; if, instead, $\overline{A1}_0 > \overline{A3}_0$, the sequence of $\overline{A3}_k$'s is strictly decreasing (in the proof, all inequalities are reversed); of course, it is in principle possible that $\overline{A1}_0 = \overline{A3}_0$, in which case the sequence of $\overline{A3}_k$'s is constant. (Typically, we expect $\overline{A1}_0 > \overline{A3}_0$: in fact, $\overline{A3}_0$ is the average between $d_{[10]}$ and $\overline{A2}_0$, and the latter is no greater than $d_{[90]}$; hence, if G_0 is symmetric, $\overline{A3}_0$ is necessarily below the mean of $G_0(d)$. To have $\overline{A1}_0 \leq \overline{A3}_0$, $G_0(d)$ must be heavily skewed.)

□

The previous result immediately implies Proposition 2, that, for convenience, is reported below.

Proposition 2. *In the AB auction, in the limit, the (expected) distance of a firm's bid from A2 is strictly decreasing in her level of sophistication. In the ABL auction, in the limit, the (expected) distance of a firm's bid from A3 is strictly decreasing in her level of sophistication.*[2]

[2] To be precise, this proposition holds only when $\overline{A1}_0 \neq \overline{A3}_0$; when $\overline{A1}_0 \neq \overline{A3}_0$, the (expected) distance of a firm's bid from $A2$ is constant in her level of sophistication.

Proof. Take the AB auction. If we denote by k^{max} the highest level of sophistication in the population of firms, then: if k^{max} is finite, the expected value of $A2$, when $n \to \infty$, is simply $\overline{A2}_{k^{max}+1}$; if not, the expected value of $A2$, when $n \to \infty$, is $\lim_{k\to\infty} \overline{A2}_k$. In any case, $\overline{A2}_k < \mathbb{E}[A2]$, for all k. This, together with the fact that the sequence of $\overline{A2}_k$'s is strictly increasing, implies that the distance between $\overline{A2}_k$ (which is the optimal bid of a level-$k+1$ firm) and $\mathbb{E}[A2]$ is strictly decreasing in k.
For the ABL auction, the proof is analogous.

□

C Numerical simulations

In this section, we present the results of some simulation exercises from a CH model of bidding behavior in AB and ABL. The purpose of this exercise is twofold: on the one hand, it shows that the main prediction of the CH model – the distance of a firm's bid from $A2$ in AB, from $A3$ in ABL, is strictly decreasing in her level of sophistication – does not hold only asymptotically (as was proved in Proposition 2), but also for finite n; on the other hand, it provides support to the additional empirical evidence presented in Subsection IV.C.

The simulations are run under the following assumptions and parametrization: we fix the reserve price to 100 and assume that firms' costs are private and independently and identically distributed according to a uniform distribution on the interval $[\underline{c} = 50, \bar{c} = 70]$, with increments of 0.2. We assume that firms' levels of sophistication range from 0 to 2[3] and that they are distributed according to a truncated Poisson with parameter λ.[4] Level-0 firms are assumed to draw their bids from a uniform distribution over the interval $[0, 0.3]$. This assumption is roughly consistent with our evidence (the minimum and maximum discounts observed in our sample are 0 and 0.421 in AB and 0.016 and 0.317 in ABL) and ensures that level-0 firms will never play dominated strategies.[5] Level-1 firms choose their bids to maximize their expected payoffs under the belief that all other firms are level-0, while level-2 firms choose their bids to maximize their expected payoffs under the belief that other firms are a mixture of level-0 and level-1. Given the behavior of level-0, level-1 and level-2 firms, we compute the expected value of $A2$ (for the AB auction) or $A3$ (for the ABL auction), and, for each level, the expected value and the variance (in square brackets) of the distance between their bids and $A2$ or $A3$. Since our objective is to check the consistency of the results of the simulations with real data, we must allow for errors. Hence, the distance from $A2$ or $A3$ is computed supposing that level-1 and level-2 firms' bids are subject to logistic errors: every bid is played with positive probability but the probability that a level-l firm ($l = 1, 2$) with cost c bids $\hat{d}$ is $\exp(\eta\Pi_l(\hat{d}; c))/\sum_d \exp(\eta\Pi_l(d; c))$, where $\Pi_l(d; c)$ is the expected payoff of a level-l firm when her cost is c and she bids d, and where η denotes the error parameter (with $\eta = 0$ meaning random behavior and $\eta \to \infty$ meaning no errors). We also computed the truly optimal bid, i.e., the bid that would maximize the expected payoff of a firm who has fully correct beliefs about the behavior of other firms. Proposition 2 showed that, when $n \to \infty$, this truly optimal bid converges to $A2$ in AB, to $A3$ in ABL, but for finite n, it may be different. Hence, it is important to verify whether $A2$ and $A3$ are indeed good proxies for the optimal bid. The results of the simulations are reported in Tables C1-C6, for different values of the parameter of the distribution

[3] We consider only level-1 and level-2 firms because experimental evidence has shown that the majority of subjects performs no more than 2 levels of iteration (see, e.g., Crawford, Costa-Gomes and Iriberri 2013).

[4] This is the usual assumption adopted in this literature for the distribution of levels. The parameter λ is the expected value (and also the variance) of the distribution. Hence, a higher λ means that firms are, on average, more sophisticated.

[5] In this sense, level-0 firms have at least a minimum degree of rationality. Their random behavior could be interpreted as the consequence of a total absence of any precise beliefs about the behavior of others. The assumption that level-0 players do not play dominated strategies represents a small departure from the standard CH-literature. However, we believe that this represents a reasonable assumption in real world applications: all firms, also the most naive ones, should easily realize that it is not a good idea to offer a discount that would not allow it to cover the cost of realizing the work. In a similar vain, Goldfarb and Xiao (2011), in their estimated CH-model of entry decisions by firms, endow level-0 firms with a minimum degree of rationality.

of levels ($\lambda = 0.5, 1, 2$), of the number of firms ($n = 25, 50, 100$) and of the parameter of the error distribution ($\eta = 0.5, 1, 2$).

Table C1 – Simulation results for the AB auction with $\eta = 0.5$.

n	λ	*A2*	*distance from A2*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	5.2 [0.9]	4.2 [0.6]	20.4	8.5 [2.4]	5.3 [0.9]	4.3 [0.6]
	1	20.1	8.4 [2.4]	5.2 [0.9]	4.0 [0.5]	19.5	8.2 [2.3]	5.1 [0.9]	3.6 [0.4]
	2	19.8	8.3 [2.3]	5.1 [0.9]	1.6 [0.1]	19.5	8.2 [2.3]	5.1 [0.9]	1.3 [0.1]
50	0.5	21.0	8.8 [2.6]	6.7 [1.5]	5.9 [1.2]	20.1-21.3	8.5 [2.4]	6.6 [1.4]	5.8 [1.1]
	1	20.7	8.6 [2.5]	6.6 [1.5]	5.9 [1.2]	20.4	8.5 [2.4]	6.6 [1.4]	5.8 [1.1]
	2	20.6	8.6 [2.5]	6.6 [1.5]	2.6 [0.2]	20.4	8.5 [2.4]	6.6 [1.4]	2.4 [0.2]
100	0.5	21.0	8.8 [2.6]	7.7 [2.0]	7.1 [1.7]	20.4	8.5 [2.4]	7.5 [1.9]	7.0 [1.6]
	1	20.7	8.6 [2.5]	7.6 [1.93]	7.5 [1.88]	20.4	8.5 [2.4]	7.5 [1.9]	7.4 [1.8]
	2	20.6	8.6 [2.5]	7.6 [1.9]	4.2 [0.6]	20.4	8.5 [2.4]	7.5 [1.9]	4.1 [0.5]

Table C2 – Simulation results for the AB auction with $\eta = 1$.

n	λ	*A2*	*distance from A2*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	2.7 [0.2]	1.1 [0.0]	20.4	8.5 [2.4]	2.7 [0.2]	1.2 [0.0]
	1	20.1	8.4 [2.4]	2.6 [0.2]	1.5 [0.1]	19.5	8.2 [2.3]	2.6 [0.2]	0.9 [0.0]
	2	19.8	8.3 [2.3]	2.6 [0.2]	0.6 [0.0]	19.5	8.2 [2.3]	2.6 [0.2]	0.3 [0.0]
50	0.5	21.0	8.8 [2.6]	4.2 [0.6]	2.1 [0.1]	20.1-21.3	8.5 [2.4]	4.1 [0.6]	2.4 [0.2]
	1	20.7	8.6 [2.5]	4.1 [0.6]	2.4 [0.2]	20.4	8.5 [2.4]	4.0 [0.5]	2.2 [0.2]
	2	20.6	8.6 [2.5]	4.1 [0.6]	0.5 [0.0]	20.4	8.5 [2.4]	4.0 [0.5]	0.3 [0.0]
100	0.5	21.0	8.8 [2.6]	6.0 [1.2]	3.3 [0.4]	20.4	8.5 [2.4]	5.9 [1.2]	3.6 [0.4]
	1	20.7	8.6 [2.5]	6.0 [1.2]	4.7 [0.7]	20.4	8.5 [2.4]	5.9 [1.2]	4.5 [0.7]
	2	20.6	8.6 [2.5]	6.0 [1.2]	0.9 [0.0]	20.4	8.5 [2.4]	5.9 [1.2]	0.7 [0.0]

Table C3 – Simulation results for the AB auction with $\eta = 2$.

n	λ	*A2*	*distance from A2*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	20.3	8.5 [2.4]	1.1 [0.03]	0.2 [0.00]	20.4	8.5 [2.4]	1.1 [0.04]	0.3 [0.00]
	1	20.1	8.4 [2.4]	1.0 [0.03]	0.8 [0.02]	19.5	8.2 [2.3]	1.0 [0.03]	0.3 [0.00]
	2	19.8	8.3 [2.3]	1.0 [0.03]	0.4 [0.00]	19.5	8.2 [2.3]	1.0 [0.03]	0.1 [0.00]
50	0.5	21.0	8.8 [2.6]	1.5 [0.08]	0.2 [0.00]	20.1-21.3	8.5 [2.4]	1.5 [0.07]	0.9 [0.03]
	1	20.7	8.6 [2.5]	1.4 [0.06]	0.5 [0.01]	20.4	8.5 [2.4]	1.4 [0.06]	0.3 [0.00]
	2	20.6	8.6 [2.5]	1.4 [0.06]	0.3 [0.00]	20.4	8.5 [2.4]	1.4 [0.06]	0.0 [0.00]
100	0.5	21.0	8.8 [2.6]	2.8 [0.26]	0.5 [0.01]	20.4	8.5 [2.4]	2.8 [0.26]	1.0 [0.03]
	1	20.7	8.6 [2.5]	2.7 [0.25]	1.2 [0.05]	20.4	8.5 [2.4]	2.8 [0.26]	1.0 [0.03]
	2	20.6	8.6 [2.5]	2.7 [0.25]	0.2 [0.00]	20.4	8.5 [2.4]	2.8 [0.26]	0.0 [0.00]

Table C4 – Simulation results for the ABL auction with $\eta = 0.5$.

n	λ	*A3*	*distance from A3*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	5.6 [1.1]	4.5 [0.7]	15.0	7.6 [1.9]	5.0 [0.8]	4.1 [0.5]
	1	14.5	7.6 [1.9]	5.2 [0.9]	3.2 [0.4]	15.3	7.6 [1.9]	4.9 [0.8]	3.1 [0.3]
	2	15.6	7.6 [1.9]	4.9 [0.8]	2.1 [0.1]	15.3	7.6 [1.9]	4.9 [0.8]	2.0 [0.1]
50	0.5	12.7	7.8 [2.0]	6.8 [1.6]	6.0 [1.2]	13.5	7.6 [1.9]	6.6 [1.4]	5.7 [1.1]
	1	13.5	7.7 [2.0]	6.6 [1.4]	5.0 [0.8]	15.0	7.6 [1.9]	6.3 [1.3]	4.7 [0.7]
	2	15.3	7.6 [1.9]	6.2 [1.3]	3.3 [0.4]	15.3	7.6 [1.9]	6.2 [1.3]	3.3 [0.4]
100	0.5	12.7	7.8 [2.0]	7.3 [1.8]	6.9 [1.6]	15.6	7.6 [1.9]	6.9 [1.6]	6.6 [1.5]
	1	13.1	7.7 [2.0]	7.2 [1.7]	6.5 [1.4]	14.1	7.6 [1.9]	7.0 [1.6]	6.4 [1.4]
	2	14.3	7.6 [1.9]	7.0 [1.6]	5.4 [1.0]	14.1	7.6 [1.9]	7.0 [1.6]	5.4 [1.0]

Table C5 – Simulation results for the ABL auction with $\eta = 1$.

n	λ	*A3*	*distance from A3*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	4.2 [0.6]	2.3 [0.2]	15.0	7.6 [1.9]	3.1 [0.3]	1.5 [0.1]
	1	14.5	7.6 [1.9]	3.4 [0.4]	1.2 [0.0]	15.3	7.6 [1.9]	3.0 [0.3]	0.9 [0.0]
	2	15.6	7.6 [1.9]	2.8 [0.3]	0.5 [0.0]	15.3	7.6 [1.9]	3.0 [0.3]	0.5 [0.0]
50	0.5	12.7	7.8 [2.0]	5.8 [1.1]	4.1 [0.5]	13.5	7.6 [1.9]	5.4 [1.0]	3.6 [0.4]
	1	13.5	7.7 [2.0]	5.4 [1.0]	2.6 [0.2]	15.0	7.6 [1.9]	4.8 [0.8]	2.0 [0.1]
	2	15.3	7.6 [1.9]	4.8 [0.8]	0.9 [0.0]	15.3	7.6 [1.9]	4.8 [0.8]	0.9 [0.0]
100	0.5	12.7	7.8 [2.0]	6.8 [1.5]	6.0 [1.2]	15.6	7.6 [1.9]	6.2 [1.3]	5.5 [1.0]
	1	13.1	7.7 [2.0]	6.6 [1.4]	5.2 [0.9]	14.1	7.6 [1.9]	6.3 [1.3]	4.9 [0.8]
	2	14.3	7.6 [1.9]	6.3 [1.3]	3.0 [0.3]	14.1	7.6 [1.9]	6.3 [1.3]	3.1 [0.3]

Table C6 – Simulation results for the ABL auction with $\eta = 2$.

n	λ	*A3*	*distance from A3*			*opt. bid*	*distance from opt. bid*		
			level 0	*level 1*	*level 2*		*level 0*	*level 1*	*level 2*
25	0.5	13.5	7.7 [2.0]	3.3 [0.4]	1.7 [0.1]	15.0	7.6 [1.9]	1.9 [0.1]	0.5 [0.0]
	1	14.5	7.6 [1.9]	2.4 [0.2]	0.9 [0.0]	15.3	7.6 [1.9]	1.7 [0.1]	0.4 [0.0]
	2	15.6	7.6 [1.9]	1.5 [0.1]	0.2 [0.0]	15.3	7.6 [1.9]	1.7 [0.1]	0.3 [0.0]
50	0.5	12.7	7.8 [2.0]	4.5 [0.7]	2.3 [0.2]	13.5	7.6 [1.9]	3.9 [0.5]	1.7 [0.1]
	1	13.5	7.7 [2.0]	3.9 [0.5]	1.6 [0.1]	15.0	7.6 [1.9]	2.9 [0.3]	0.6 [0.0]
	2	15.3	7.6 [1.9]	2.8 [0.3]	0.3 [0.0]	15.3	7.6 [1.9]	2.7 [0.3]	0.3 [0.0]
100	0.5	12.7	7.8 [2.0]	5.7 [1.1]	4.2 [0.6]	15.6	7.6 [1.9]	4.6 [0.7]	3.4 [0.4]
	1	13.1	7.7 [2.0]	5.4 [1.0]	3.0 [0.3]	14.1	7.6 [1.9]	5.0 [0.8]	2.6 [0.2]
	2	14.3	7.6 [1.9]	4.9 [0.8]	0.9 [0.0]	14.1	7.6 [1.9]	5.0 [0.8]	1.0 [0.0]

Looking at the results of these numerical simulations, we detect some regularities, that we summarize below.

(a) For all values of n, λ, and η, the optimal bid (i.e., the bid that maximizes the expected payoff of a firm that has fully correct beliefs about the behavior of all other firms) is essentially unaffected by the private cost. In fact, of the 54 possible combinations of parameters considered, there are only two cases in which the optimal bid is not constant in the private cost: in AB, with $\eta = 0.5$, $n = 50$, $\lambda = 0.5$ and in AB, with $\eta = 1$, $n = 50$, $\lambda = 0.5$. Moreover, in these two cases, the range of the optimal bidding function is pretty narrow (about 1 percent). This supports the intuition that, in these auctions, costs do not matter much for bidding.

(b) For all values of n, λ, and η, the optimal bid is extremely close to the expected value of $A2$ in AB, of $A3$ in ABL. This supports the intuition that, in these auctions, $A2$ and $A3$ are good proxies for the optimal bid, even when n is finite.

(c) For all values of n, λ, and η, the distance of a firm's bid from the expected value of $A2$ in AB, of $A3$ in ABL, is decreasing in her level of sophistication. Hence, the main theoretical prediction of the asymptotic CH model (Proposition 2) seem to hold also when n is finite.

(d) For given n, λ, and η, level-1 and level-2 firms' bids are, on average, lower in ABL than in AB. This fact is consistent with the empirical evidence discussed in Subsection IV.C.

(e) In either auction, for all values of n, λ, and η, the variance of the distance from $A2$ or $A3$ is decreasing in the sophistication level of the firm. This fact is consistent with the empirical evidence discussed in Subsection IV.C.

(f) For given λ and η, the optimal bid and the expected value of $A2$ are increasing in n in AB, the optimal bid and the expected value of $A3$ are decreasing in n in ABL. This fact is consistent with the empirical evidence discussed in Subsection IV.C.

D Additional empirical evidence

This Section presents additional empirical evidence, both descriptive and inferential, recalled and commented in Sections III and IV of the paper. In particular:

- Figures D1, D2 and D3 report descriptive evidence about the distribution of the sophistication index over time and by firm size;
- Table D1, columns (1) and (5), show that our main empirical result does not change when we amend our baseline model (equation (2) in the paper) including in the estimation those firms with a sophistication index equal to 0 (replacing $\log(BidderSoph)$ with $\log(1+BidderSoph)$);
- Table D1, columns (2) and (6), show that our main empirical result does not change when we amend our baseline model (equation (2) in the paper) adopting a log-linear specification instead of a log-log one;
- Table D1, columns (3) and (7), show that our main empirical result does not change when we amend our baseline model (equation (2) in the paper) adding the number of bidders as a control variable;
- Table D1, columns (4) and (8), show that our main empirical result does not change when we amend our baseline model (equation (2) in the paper) replacing auction controls with auction-fixed effects;
- Table D3, columns (1)-(4), show, for the AB auctions, that our main empirical result does not change when we adopt a two-step Heckman model to control for selection bias problems;
- Table D3, columns (5)-(10), show, for the AB auctions, that our main empirical result does not change when the sophistication index is category-specific: when a firm participates in auction j, only her performances in past auctions of the same format and of the same category of work as j are considered in the computation of her sophistication level;
- Table D2, columns (1)-(3), show, for the AB sample, the estimation results when firm- and firm-year-fixed effects are not included in the models discussed in Subsection III.E;
- Table D2, columns (4)-(7), show, for the AB sample, the estimation results when firm-year-fixed effects are replaced by firm-fixed effects in the models discussed in Subsection III.E;
- Table D2, column (8), shows, for the AB sample, additional estimation results with firm-year-fixed effects in the model discussed in Subsection III.E;

- Table D4 shows, for the ABL sample, the estimation results of the models discussed in Subsection III.E;
- Table D5 shows that our main empirical result does not change when we amend our baseline model (equation (2) in the paper) replacing firm- or firm-year-fixed effects with firm-semester or firm-category of work- or firm-category of work-semester- or firm-category of work-year-fixed effects;
- Table D6 shows the estimation results of a model in which the dependent variable is the level of bids;
- Table D7 shows the estimation results of quantile regression models in which the dependent variable is the level of bids;
- Table D8 shows that our main empirical result does not change when we control for potential cartels in AB;
- Table D9 shows that our main empirical result does not change when we control for potential cartels in ABL;
- Table D10, columns (1) and (2), show that bids are, on average, lower in ABL than in AB;
- Table D10, column (3), shows that $A2$ in AB increases with the number of participating firms;
- Table D10, column (4), shows that $A3$ in ABL decreases with the number of participating firms;
- Table D10, columns (5)-(10), show that the standard deviation of the average distance between bids and $A2$ [$A3$] in an AB [ABL] auction is decreasing in the average sophistication level of the firms participating in that auction;
- Table D11 shows that our main empirical result does not change when we use some potential instruments to proxy bidders' sophistication.

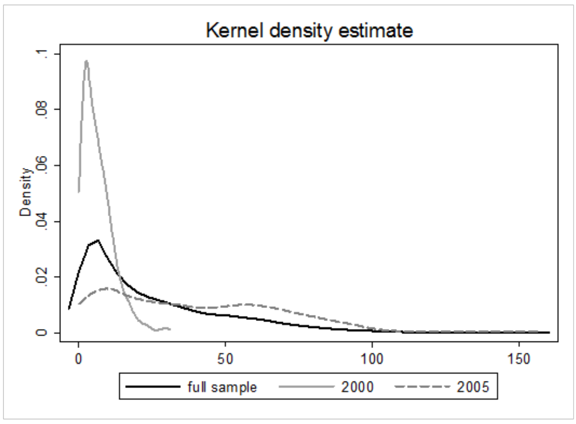

Figure D1 – Distribution of the sophistication index in AB.

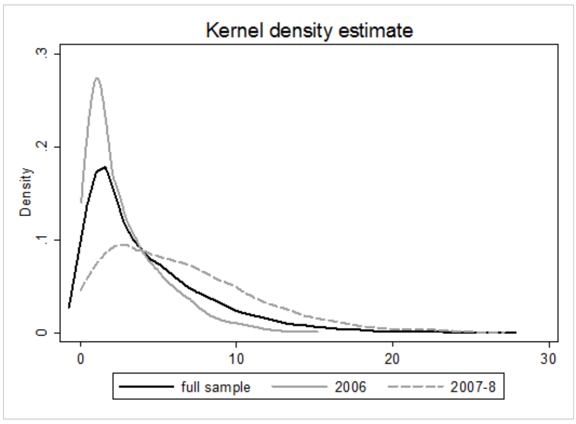

Figure D2 – Distribution of the sophistication index in ABL.

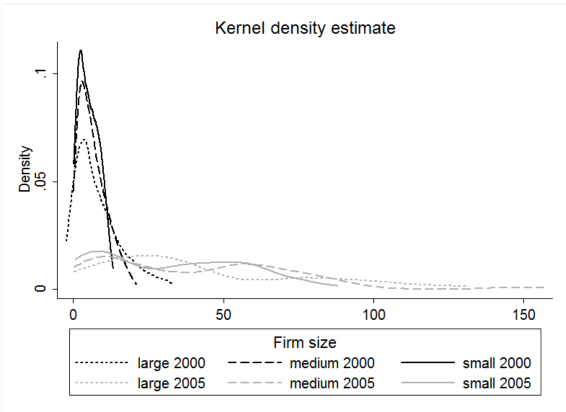

Figure D3 – Distribution of the sophistication index in AB by firm size.

Table D1 – Robustness checks on the baseline model specification.

Dependent variable:	log $\|Distance\|$							
Auction format	AB	AB	AB	AB	ABL	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log(1+$BidderSoph$)	-0.240***				-0.568***			
	(0.048)				(0.083)			
$BidderSoph$		-0.020***				-0.113***		
		(0.003)				(0.021)		
log($BidderSoph$)			-0.241***	-0.140***			-0.440***	-0.293***
			(0.041)	(0.023)			(0.072)	(0.050)
Auction controls	YES	YES	YES	NO	YES	YES	YES	NO
Firm controls	NO	NO	NO	YES	NO	NO	NO	YES
Firm-year FE	YES	YES	YES	NO	YES	YES	YES	NO
Auction-FE	NO	NO	NO	YES	NO	NO	NO	YES
Firm-Auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,965	8,965	8,573	8,924	1,591	1,591	1,266	1,501
R-squared	0.361	0.361	0.356	0.379	0.524	0.516	0.506	0.361

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Auction controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction, and, in columns (3) and (7), the number of bidders. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D2 – Learning dynamics: further results for AB auctions.

Dependent variable	log $\|Distance\|$							
Auction format	AB							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log($PastPart$)	-0.139***			-0.212***	-0.220***			-0.366***
	(0.024)			(0.041)	(0.041)			(0.044)
log($PastPerf$)					0.126			
					(0.130)			
log($1 + PastWins$)		-0.075	-0.027			0.217***	0.192**	
		(0.056)	(0.056)			(0.079)	(0.076)	
log($1 + PastDefeats$)		-0.141***				-0.310***		
		(0.030)				(0.048)		
log($BidderSoph$)			-0.166***				-0.186***	
			(0.025)				(0.039)	
Auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	YES	NO	NO	NO	NO	NO
Firm-FE	NO	NO	NO	YES	YES	YES	YES	NO
Firm-year-FE	NO	NO	NO	NO	NO	NO	NO	YES
Firm-Auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	8,927	8,927	8,927	8,838	8,838	8,838	8,838	8,573
R-squared	0.188	0.188	0.192	0.267	0.267	0.269	0.267	0.354

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Auction controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D3 – AB auctions: selection bias problems (two-step Heckman model) and category-specific sophistication index.

Dependent variable:	log $\lvert Distance\rvert$	Pr.Part.	log $\lvert Distance\rvert$	Pr.Part.	log $\lvert Distance\rvert$					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
log($BidderSoph$)	-0.158***		-0.464***	0.233***						
	(0.026)		(0.181)	(0.010)						
log($TimeToBid$)		0.034**		0.101**						
		(0.014)		(0.042)						
log($BidderSophInCat$)					-0.128***		-0.150***		-0.229***	
					(0.029)		(0.042)		(0.052)	
log($1 + BidderSophInCat$)						-0.179***		-0.254***		-0.388***
						(0.032)		(0.051)		(0.063)
Auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm-controls	YES	YES	YES	YES	YES	YES	NO	NO	NO	NO
Firm-FE	NO	NO	NO	NO	NO	NO	YES	YES	NO	NO
Firm-year FE	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES
Firm-Auction controls	NO	NO	NO	NO	YES	YES	YES	YES	YES	YES
Observations	3,877	13,517	13,517	13,517	3,658	3,982	3,624	3,910	3,505	3,727
R-squared	0.190	0.047			0.190	0.199	0.263	0.277	0.366	0.378

In columns (1)-(2) and (5)-(10) OLS estimates with robust standard errors clustered at firm-level in parentheses.
Auction controls include: the auction's reserve price, the expected duration of the work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
The analysis focuses on AB auctions for roadworks because they represent the largest share of projects in our data (87 auctions). OLS regression in column (1) shows the coefficient of $BidderSoph$ estimated on the subsample of roadworks. The potential market for roadworks is defined as those firms that, according to our dataset, bid at least once for roadworks in a given year. As an exogenous instrument that is related to the probability of firms' participation but has an influence only on the cost of participation, we use $TimeToBid$ (column (2)), which is the length of time between the date in which the project is advertised and when the bid letting occurs (this instrument is also used by Gil and Marion 2013, and Moretti and Valbonesi 2015). The hypothesis is that the longer the time between the beginning of project's publicity and the deadline for bid's submission, the lower the cost borne by firms to prepare their bids. Our data show that there is variability in terms of auctions' advertise lead time, with an average of 28.6 days (and a standard deviation of 11.4 days). In columns (3) and (4), the outcome and the selection equation of a two-step Heckman selection model are reported.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D4 – Learning dynamics: results for ABL auctions.

Dependent variable	log $\|Distance\|$											
Auction format	ABL											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\log(PastPart)$	-0.272***	-0.237***			-0.558***	-0.522***			-0.645***	-0.626***		
	(0.058)	(0.050)			(0.073)	(0.074)			(0.085)	(0.093)		
$\log(PastPerf)$		-0.792***				-0.295*				-0.136		
		(0.074)				(0.165)				(0.278)		
$\log(1 + PastWins)$			0.124	0.323**			0.198	0.347			-0.042	0.122
			(0.159)	(0.153)			(0.253)	(0.241)			(0.314)	(0.289)
$\log(1 + PastDefeats)$			-0.345***				-0.754***				-0.907***	
			(0.077)				(0.097)				(0.117)	
$\log(BidderSoph)$				-0.395***				-0.477***				-0.525***
				(0.043)				(0.064)				(0.074)
Auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	YES	YES	NO	NO	NO	NO	NO	NO	NO	NO
Firm-FE	NO	NO	NO	NO	YES	YES	YES	YES	NO	NO	NO	NO
Firm-year-FE	NO	NO	NO	NO	NO	NO	NO	NO	YES	YES	YES	YES
Firm-Auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Observations	1,501	1,501	1,501	1,501	1,410	1,410	1,410	1,410	1,266	1,266	1,266	1,266
R-squared	0.244	0.296	0.243	0.280	0.459	0.460	0.458	0.460	0.499	0.500	0.501	0.498

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Auction controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction.
Firm controls include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D5 – Further results: firm-category of work-fixed effects and firm-semester fixed effects.

Dependent variable			log $\|Distance\|$			
Auction format	AB	AB	ABL	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)
log($BidderSoph$)	-0.166***	-0.124**	-0.089	-0.476***	-0.507***	0.132
	(0.040)	(0.052)	(0.117)	(0.076)	(0.102)	(0.164)
Auction controls	YES	YES	YES	YES	YES	YES
Firm-semester-FE	NO	NO	YES	NO	NO	NO
Firm-category of work-FE	YES	NO	NO	YES	NO	NO
Firm-category of work-semester-FE	NO	YES	NO	NO	NO	YES
Firm-category of work-year-FE	NO	NO	NO	NO	YES	NO
Firm-Auction controls	YES	YES	YES	YES	YES	YES
Observations	8,642	7,463	1,154	1,287	1,053	838
R-squared	0.303	0.478	0.580	0.528	0.584	0.691

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Auction controls include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog. Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D6 – Sophistication and bid levels: overall sample.

Dependent variable	log($Discount$)	$Discount$	$Discount$	log($Discount$)	$Discount$	log($Discount$)	$Discount$
Auction format	AB	AB	AB	ABL	ABL	ABL	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
log($BidderSoph$)	0.006	0.083	-0.247*	-0.014	-0.367	-0.015	-0.390
	(0.010)	(0.117)	(0.142)	(0.022)	(0.239)	(0.024)	(0.260)
Auction controls	YES	YES	YES	YES	YES	YES	YES
Firm-FE	YES	YES	NO	YES	YES	NO	NO
Firm-year-FE	NO	NO	YES	NO	NO	YES	YES
Firm-Auction controls	YES	YES	YES	YES	YES	YES	YES
Observations	8,838	8,838	8,573	1,410	1,410	1,266	1,266
R-squared	0.348	0.530	0.590	0.525	0.678	0.583	0.715

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
Auction controls include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog. Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D7 – Sophistication and bid levels: quantile regressions.

Quantile (th)	5	10	20	30	40	50	60	70	80	90	95
Auction format						AB					
Dependent variable						log($Discount$)					
log($BidderSoph$)	0.040***	0.028***	0.011***	0.007***	0.004***	0.002*	0.001	0.001	-0.000	-0.003***	-0.008***
	(0.008)	(0.006)	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
Dependent variable						$Discount$					
log($BidderSoph$)	0.635***	0.421***	0.193***	0.125***	0.074***	0.036**	0.023	0.013	-0.004	-0.061***	-0.166***
	(0.110)	(0.090)	(0.047)	(0.027)	(0.023)	(0.018)	(0.017)	(0.015)	(0.016)	(0.018)	(0.037)
Observations	8,927	8,927	8,927	8,927	8,927	8,927	8,927	8,927	8,927	8,927	8,927
Auction format						ABL					
Dependent variable						log($Discount$)					
log($BidderSoph$)	0.065***	0.054***	0.026***	0.013***	0.004	0.000	-0.008	-0.022***	-0.051***	-0.081***	-0.091***
	(0.018)	(0.016)	(0.008)	(0.005)	(0.004)	(0.004)	(0.005)	(0.008)	(0.013)	(0.005)	(0.011)
Dependent variable						$Discount$					
log($BidderSoph$)	0.599***	0.537***	0.295***	0.153***	0.053	0.002	-0.101	-0.297***	-0.764***	-1.330***	-1.615***
	(0.160)	(0.144)	(0.099)	(0.058)	(0.047)	(0.047)	(0.069)	(0.110)	(0.188)	(0.092)	(0.161)
Observations	1,501	1,501	1,501	1,501	1,501	1,501	1,501	1,501	1,501	1,501	1,501
Auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm-Auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES

Robust standard errors clustered at firm-level in parentheses.
Auction controls include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D8 – Controlling for potential cartels in AB auctions.

Dependent variable	log $\|Distance\|$						
Auction format	AB						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\log(BidderSoph)$	-0.163***	-0.163***	-0.193***	-0.189***	-0.282***	-0.153***	-0.198***
	(0.023)	(0.023)	(0.037)	(0.037)	(0.042)	(0.031)	(0.053)
$\log(GroupMembers)$	-0.169***	0.031	-0.246***	-0.208*	-0.252*		
	(0.032)	(0.068)	(0.046)	(0.109)	(0.132)		
$\log(GroupMembers)$ $\times ShareGroupMembers$		-0.416***		-0.797***	-0.526*		
		(0.120)		(0.248)	(0.274)		
$ShareGroupMembers$		0.153		1.425**	1.019		
		(0.095)		(0.551)	(0.663)		
Auction controls	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	NO	NO	NO	YES	NO
Firm-FE	NO	NO	YES	YES	NO	NO	YES
Firm-year-FE	NO	NO	NO	NO	YES	NO	NO
Firm-Auction controls	YES	YES	YES	YES	YES	YES	YES
Observations	8,037	8,037	7,993	7,993	7,794	3,861	3,834
R-squared	0.202	0.203	0.263	0.264	0.343	0.181	0.300

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
ShareGroupMembers is the fraction of the members of a potential cartel actually bidding in that auction. In columns (6)-(7), samples include only firms with no connection with any other firm participating in that auction. *Auction controls* include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D9 – Controlling for potential cartels in ABL auctions.

Dependent variable	log $\|Distance\|$								
Auction format	ABL								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\log(BidderSoph)$	-0.369***	-0.356***	-0.430***	-0.428***	-0.489***	-0.484***	-0.270***	-0.365***	-0.421***
	(0.049)	(0.051)	(0.065)	(0.066)	(0.073)	(0.074)	(0.071)	(0.100)	(0.113)
$\log(GroupMembers)$	0.029	0.301**	-0.141	0.157	-0.170	0.105			
	(0.056)	(0.120)	(0.121)	(0.434)	(0.139)	(0.560)			
$\log(GroupMembers)$ $\times ShareGroupMembers$		-0.528**		-0.458		-0.466			
		(0.243)		(0.443)		(0.510)			
$ShareGroupMembers$		0.315		0.100		0.157			
		(0.263)		(1.355)		(1.727)			
Auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	YES	NO	NO	NO	NO	YES	NO	NO
Firm-FE	NO	NO	YES	YES	NO	NO	NO	YES	NO
Firm-year-FE	NO	NO	NO	NO	YES	YES	NO	NO	YES
Firm-Auction controls	YES	YES	YES	YES	YES	YES	YES	YES	YES
Observations	1,316	1,316	1,258	1,258	1,131	1,131	578	543	472
R-squared	0.264	0.268	0.439	0.440	0.479	0.480	0.240	0.570	0.635

OLS estimations. Robust standard errors clustered at firm-level in parentheses.
ShareGroupMembers is the fraction of the members of a potential cartel actually bidding in that auction. In columns (7)-(9), samples include only firms with no connection with any other firm participating in that auction. *Auction controls* include: the auction's reserve price, the expected duration of the work,dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D10 – Additional empirical edvidence.

Dependent variable:	*Normalized Discount*	*Normalized Discount*	*A2*	*A3*	*Auction St.Dev.*	*Auction St.Dev.*	*Firm St.Dev.*	*Firm St.Dev.*	*Firm St.Dev.*	*Firm St.Dev.*
Auction format	AB+ABL	AB+ABL	AB	ABL	AB	ABL	AB	ABL	AB	ABL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ABL	-0.321***	-0.216***								
	(0.024)	(0.019)								
log($No.Participants$)			0.463**	-0.712**						
			(0.221)	(0.285)						
log($MeanBidderSoph$)					-0.310*	-0.734**				
					(0.162)	(0.165)				
log($RollingMeanBidderSoph$)							-0.292***	-0.604***	-0.644***	-0.943***
							(0.047)	(0.168)	(0.113)	(0.225)
Auction controls	YES	YES	YES	YES	YES	YES	NO	NO	NO	NO
Firm controls	YES	NO	NO	NO	NO	NO	YES	YES	NO	NO
Firm-fixed effects	NO	YES	NO	NO	NO	NO	NO	NO	YES	YES
Firm-Auction controls	YES	YES	NO	NO	NO	NO	NO	NO	NO	NO
Observations	10,428	9,647	232	28	228	26	7,273	656	7,273	656
R-squared	0.188	0.248	0.848	0.993	0.246	0.902	0.212	0.225	0.380	0.601

In columns (1)-(2) and (7)-(10): OLS estimations and robust standard errors clustered at firm-level in parentheses.
In columns (3)-(6), an IRLS estimator is used to account for the influence of outliers (given the small samples).
In columns (1) and (2), the dependent variable is the (min-max rescaled) discount offered by firms. ABL is a dummy variable which takes value 1 (0) if the auction is ABL (AB). Though not reported, the sophistication index is included among the covariates.
In columns (3) and (4) the dependent variable is the (auction-specific) reference point.
In columns (5) and (6), the dependent variable is the standard deviation of the (absolute value of the standardized) distance of bids from the reference point.
In columns (7)-(10), the dependent variable is the rolling standard deviation of the (absolute value of the standardized) distance of bids from the reference point in the last five auctions.
$MeanBidderSoph$ is the average of the sophistication index across firms in the auction.
$RollingMeanBidderSoph$ is the rolling average of the sophistication index of the firm in the last five auctions.
Auction controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work, dummy variables for the year of the auction. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
In columns (7)-(10) time dummies taking the value 1 for the mean year of the bidders' last five auctions are included.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

Table D11 – Further controls: 2SLS estimations.

Dependent variable	log($BidderSoph$)	log($BidderSoph$)	log $\|Distance\|$	log $\|Distance\|$	log($BidderSoph$)	log($BidderSoph$)	log $\|Distance\|$	log $\|Distance\|$
Auction format	AB+ABL	AB+ABL	AB+ABL	AB+ABL	AB+ABL	AB+ABL	AB+ABL	AB+ABL
	stage 1	stage 1	stage 2	stage 2	stage 1	stage 1	stage 2	stage 2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log($BidderSoph$)			-0.196***	-0.237***			-0.162***	-0.182***
			(0.023)	(0.024)			(0.024)	(0.029)
ABL	-0.534**	-0.527**			-0.535***	-0.612***		
	(0.226)	(0.213)			(0.165)	(0.154)		
log($BidderSoph00-04$)	0.786***							
	(0.038)							
$ABL \times$ log($BidderSoph00-04$)	-0.688***	-0.705***						
	(0.063)	(0.060)						
log($BidderSoph00-03$)					0.705***			
					(0.041)			
$ABL \times$ log($BidderSoph00-03$)					-0.585***	-0.582***		
					(0.050)	(0.047)		
Auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Firm controls	YES	NO	YES	NO	YES	NO	YES	NO
Firm-fixed effects	NO	YES	NO	YES	NO	YES	NO	YES
Firm-Auction controls	YES	YES	YES	YES	YES	YES	YES	YES
Observations	1,888	1,888	1,888	1,888	2,954	2,954	2,954	2,954
R-squared	0.888	0.882	0.102	0.106	0.831	0.815	0.060	0.062
Hansen-J test			0.256	0.285			0.722	0.490

2SLS estimations; Robust standard errors clustered at firm-level in parentheses.
Auction controls include: the auction's reserve price, the expected duration of the work, dummy variables for the type of work. *Firm controls* include: dummy variables for the size of the firm, and the distance between the firm and the CA. *Firm-Auction controls* include: a dummy variable for the firm's subcontracting position (mandatory or optional), and a measure of the firm's backlog.
In columns (1)-(4), the analysis is limited to the period 2005-2006, i.e., the last year of adoption of the AB format and the first year of adoption of the ABL format. In columns (5)-(8), the analysis is limited to the period 2004-2007, i.e., the last two years of adoption of the AB format and the first two years of adoption of the ABL format. Focusing on such a short period of time, we can reduce omitted variable problems and exploit the 2006 variation in the auction format, which, it is worth recalling, took place only in Valle d'Aosta. We consider three potential instruments for the sophistication index, namely: *(i)* the dummy variable ABL (which takes value equal 1 for ABL and 0 for AB) to capture the average overall change in the firms' sophistication level determined by the change of format; *(ii)* for each firm, the value of her sophistication index *BidderSoph00-04* [*BidderSoph00-03*] at the end of 2004 [2003], a proxy for the firm-fixed level of sophistication (relative to the others) acquired out-of-sample; *(iii)* the interaction between the previous two variables, which accounts for possibly differential effects that the new ABL format had on firms' current sophistication across firms with different levels of past (and out-of-sample) sophistication.
Inference: (***) = $p < 0.01$, (**) = $p < 0.05$, (*) = $p < 0.1$.

References

[1] **Crawford, Vincent P., Miguel A. Costa-Gomes, and Nagore Iriberri.** 2013. "Structural Models of Nonequilibrium Strategic Thinking: Theory, Evidence, and Applications." *Journal of Economic Literature* 51 (1): 5-62.

[2] **Gil, Ricard, and Justin Marion.** 2013. "Self-Enforcing Agreements and Relational Contracting: Evidence from California Highway Procurement." *Journal of Law, Economics, and Organization* 29 (2): 239-277.

[3] **Goldfarb, Avi, and Mo Xiao.** 2011. "Who Thinks About the Competition? Managerial Ability and Strategic Entry in US Local Telephone Markets." *American Economic Review* 101 (7): 3130-3161.

[4] **Moretti, Luigi, and Paola Valbonesi.** 2015. "Firms' Qualifications and Subcontracting in Public Procurement: An Empirical Investigation." *Journal of Law, Economics, and Organization* 31 (3): 568-598.