Supplemental Online Appendix for "Trading Across Borders in Online Auctions"

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Appendix A: Methodological Challenges (Demand Side)

The summary of methodological challenges and of the heuristics of identification argument are reproduced from Krasnokutskaya, Song, and Tang (2018). Notice that in this discussion is restricted to the case of a single buyer group.

Recovering demand-side primitives presents several methodological challenges. To see this let us first consider an environment without transitory sellers (or where transitory sellers differ only in their observable attributes, x). In this setting we only need to focus on recovering the quality levels (fixed effects) of permanent sellers. Recall that in a traditional discrete choice setting fixed effects associated with different alternatives are identified from the observed probabilities that a given alternative is chosen, conditional on the choice set. In our setting, choice sets are buyer-specific since the sellers' participation varies across auctions. Due to the large numbers of sellers and buyers conditional choice probabilities cannot be precisely estimated. To get a sense of the magnitudes consider that the number of permanent sellers present in the market for a given type of work is around 300 to 500 whereas only 2 or 3 permanent sellers participate in any given auction. This means that the number of possible choice sets is at least $C^3_{300} = \frac{300!}{3!297!} = \frac{300*299*298}{3} = 8,910,200$ which exceeds the number of projects we have in our dataset. In fact, the highest number of projects sharing the same set of participating permanent sellers in our data is five. One way to deal with this issue would be to consider probabilities that aggregate

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over buyers' choice sets, such as:

$$\Pr(j \text{ wins}|j \in A_l) = \sum\nolimits_{a:j\in a} \Pr(j \text{ wins}|A_l = a) \Pr(A_l = a, j \in A_l), \text{ and}$$
$$\Pr(j \text{ wins}|B_l = b, j \in A_l) = \sum\nolimits_{a:j\in a} \Pr(j \text{ wins}|B_l = b, A_l = a) \Pr(A_l = a, j \in A_l),$$

where the sum above is over the choice sets a that contain j. However, such approach has not yet been explored in the literature and its properties are unknown. Specifically, it is unclear whether such moments could be used to identify seller-specific fixed effects and the distribution of buyers' tastes.[1] Further, even if this mechanism worked in theory, it is not certain that it would perform well in practice given that the weighting probabilities used in the aggregation above, $\Pr(A_l = a, j \in A_l)$, are very small.

Let us now return to the realistic setting where transitory sellers differ in their qualities and these qualities are observable to buyers. In contrast to permanent sellers we cannot use transitory sellers' identities as proxies for their quality. Thus, the information which underlies buyer's choice is not observed in the data. Instead, a researcher has to deal with a mixture problem where the probability distribution over the transitory sellers' qualities depends on these sellers' bids and observable attributes. More specifically, suppose the support of a transitory seller's quality $Q_{h,l}$ is $\{\bar{q}_1, \bar{q}_2\}$ and let x and b be the vectors of observable attributes and bids characterizing the entrants in the auction respectively. Then, the probability that the buyer chooses a permanent seller j while his choice set includes a single transitory seller h, $\Pr(j \text{ wins}|B_l = b, x)$, is a mixture of the following form:

$$\sum\nolimits_{s=1,2} \Pr(j \text{ wins}|Q_{h,l} = \bar{q}_s, B_l = b, x) \Pr(Q_{h,l} = \bar{q}_s|B_l = b, x).$$

The mixing weights $\Pr(Q_{h,l} = \bar{q}_s|B_l = b, x)$ are unknown and correlated with the conditional choice probability through the bid vector b and attributes vector x.

One might attempt to deal with this problem by solving for $\Pr(Q_{h,l} = \bar{q}_s|B_l = b, x)$ from the model within the estimation routine for a given vector of parameter values. However, solving one such bidding and participation game would take a long time and solutions can be very fragile if the parameter values are far from the truth. Further, a large number of sellers and projects result in a huge number of possible choice sets for which the problem would have to be solved. These issues combined make this approach computationally infeasible. Alternatively, one may adopt an ad hoc functional form assumption for the mixing distributions and attempt to recover them jointly with other primitives of the model. It is doubtful that separate identification of

[1] The invertibility argument underlying the standard approach (the most well-known exposition can be found in Berry, Levinsohn, and Pakes (1995)) does not apply to these moments because the probability of observing a given choice set, $\Pr(A_l = a, j \in A_l)$, depends in the model on the qualities of potential (and, in the consequence, actual) sellers. So further insight is necessary on how to achieve invertability in this context. For example, it is possible that the inversion could be made to work if we use empirical probabilities of observing different choice sets in the expression on the righthand side. It also has to be established that such moments allow us to exploit the exogenous variation present in the data to recover the distribution of buyers' tastes.

these components can be established formally. In practice, such approach has been shown to perform poorly.[2,3]

Because of the reasons outlined above, we adopt an identification strategy and a two-step estimation approach proposed in Krasnokutskaya, Song, and Tang (2018). Under this methodology, in the first step permanent sellers characterized by a common vector of x-attributes are classified into groups of equal quality. Such a grouping, once constructed, facilitates recovery of the model's primitives in several ways. First, the task of recovering permanent sellers' qualities is reduced since we now only need to recover the quality levels associated with different groups rather than the quality level for every permanent seller. Further, buyers' choice sets may now be represented in terms of participating sellers' (x, q)-group memberships for permanent sellers and x-group memberships for transitory sellers, rather than in terms of their identities. Thus, representing sellers in terms of group memberships offers a natural way for the aggregation of buyers' choice sets and thus facilitates recovery of the distributions of the utility coefficients and buyers' outside options. Finally, such representation permits imposing the restriction that the distributions of permanent and transitory sellers qualities have the same support. The latter feature allows us to separate identification of the payoffs associated with various bundles of (x, q) attributes from the identification of the mixing probabilities. A formal identification argument can be found in Krasnokutskaya, Song, and Tang (2018).

Heuristics for Identification

Classification into Quality Groups. Consider sellers i and j with $x_i = x_j$ who participate in two auctions that are *ex ante* identical (i.e., the project characteristics and the set of competitors are the same, and both i and j are in the set of potential bidders) and submit equal bids. Under such circumstances a seller with a higher value of q has a higher chance of winning. This ranking of winning probabilities is preserved after aggregating over possible sets of competitors, as long as the chance of encountering any given set of competitors is the same for both sellers. This condition holds if, for example, the pool from which competitors are drawn does not include either i or j. Specifically, for any pair of sellers i and j such that $x_i = x_j$, define:

$$r_{i,j}(b) \equiv \Pr(i \,\text{wins} \mid B_{i,l} = b,\ i \in A_l,\ j \notin A_l,\ i, j \in N_l) \tag{1}$$

[2]See Heckman and Singer (1984) for details.

[3]A researcher may also consider an approach proposed by Kasahara and Shimotsu (2009) in the context of a dynamic discrete choice model. However, the model considered by these authors does not readily map into our environment so the applicability of this method, even if possible, is far from obvious.

where A_l denotes the set of entrants.[4] Then

$$r_{i,j}(b) > r_{j,i}(b) \text{ if and only if } q_i > q_j,$$
$$r_{i,j}(b) < r_{j,i}(b) \text{ if and only if } q_i < q_j \text{ and}$$
$$r_{i,j}(b) = r_{j,i}(b) \text{ if and only if } q_i = q_j.$$

For a formal statement of results and conditions, see Proposition 1 in Krasnokutskaya, Song, and Tang (2018). As long as the conditional winning probabilities defined above are identified from data, we can use them to order sellers i and j with respect to their qualities. By implementing such comparison for every pair of permanent sellers within x-group, the quality ranking of the sellers within this group can be recovered.[5] This identifies the quality group structure.

A Simple Example. Let us see how to identify the rest of the model primitives, given the quality classification of the permanent sellers recovered above. Consider a simple setting with two groups of sellers defined by observable characteristics $\bar{x}_1$ and $\bar{x}_2$. Each group is further partitioned into two unobservable subgroups based on quality levels $\bar{q}_1(\bar{x}_1), \bar{q}_2(\bar{x}_1)$ and $\bar{q}_1(\bar{x}_2), \bar{q}_2(\bar{x}_2)$ respectively. Some sellers are permanent and others transitory. For simplicity, assume the components in the buyers' weights are mutually independent, which is relaxed in the formal identification results which could be found in Krasnokutskaya, Song, and Tang (2018).

Suppose there is a large number of sellers in each observable group defined by x. The number of choice sets defined in terms of specific identities of sellers is large. Nevertheless, this number can be drastically reduced if choice sets are defined in terms of quality groups instead of specific identities.[6] Such a definition of choice sets is feasible only after we use the argument above to classify sellers into groups based on unobserved quality levels.

In what follows, we discuss how to identify the payoff structure (specifically the distributions of ϵ_l, α_l, β_l and quality levels associated with each quality group, $\bar{q}_k(x_m)$ where $k = 1, 2$ and $m = 1, 2$). After that, we identify the remaining model primitives (such as F_{U_0} and $F_{Q^t|B_l^t}$).

The Distribution of Payoffs. To identify components in the buyers' payoffs, we exploit how a buyers' decisions vary with the choice set (defined in terms of (x, q)-groups of active permanent

[4] One could use an alternative, similar index conditional on $i \in A_l$, $j \in A_l$, $B_{i,l} = B_{j,l} = b$. In our data, for many pairs of bidders, there is only a small number of auctions where both i and j participate and submit similar bids. Hence the estimation of such an alternative index is much more problematic than that of $r_{i,j}(b)$ defined above. In this paper we do not pursue such an alternative strategy.

[5] Intuitively, if comparisons for all pairs of permanent sellers are available, we can always split a given x-group into two subgroups where the first subgroup consists of the sellers with the lowest quality among all the sellers in the x-group and the second subgroup consists of the remaining sellers. Then we split this second subgroup similarly into two further subgroups so that the first consists of the lowest quality sellers within this second subgroup and the other consists of the rest of the sellers. By continuing this process, we can identify the quality group structure.

[6] Suppose there are 100 sellers. Then the number of choice sets that consist of three sellers is $C_{100}^3 = 100!/97!3! = 100 * 99 * 98/6 = 161,700$. However, if we define choice sets in terms of groups, we can reduce the number of distinct choice sets to $4^3 = 64$.

bidders). Let us first demonstrate how to identify the distribution of a stochastic component ϵ_l. For this, we focus on auctions which attract at least two permanent sellers with the same observed characteristics $x_i = x_j$ and the unobserved quality $q_i = q_j$. We allow for the presence of a transitory seller h. The payoffs from different options are:

$$
\begin{aligned}
&U_{i,l} = \alpha_l q_i + \beta_l x_i - B_{i,l} + \epsilon_{i,l} \; ; \; U_{j,l} = \alpha_l q_j + \beta_l x_j - B_{j,l} + \epsilon_{j,l}; \\
&U_{h,l} = \alpha_l Q_{h,l} + \beta_l x_1 - B_{h,l} + \epsilon_{h,l} \text{ and } U_{0,l}.
\end{aligned}
$$

The scale of α_l and the quality levels can not be jointly identified; hence we normalize $\mathbf{E}[\alpha_l] = 1$.

When $B_{i,l} = -t_2$ and $B_{j,l} = t_1 - t_2$ for some t_1, t_2, the buyer chooses seller i with probability

$$
\begin{aligned}
& \Pr(i \text{ wins in auction } l | B_{i,l} = -t_2, B_{j,l} = t_1 - t_2, B_{h,l} = b_h) \\
= \; & \Pr(\epsilon_{j,l} - \epsilon_{i,l} \leq t_1 \text{ and } Y_{i,l}(x_h) - \epsilon_{i,l} \leq t_2 | B_{i,l} = -t_2, B_{j,l} = t_1 - t_2, \; B_{h,l} = b_h) \\
= \; & \Pr(\epsilon_{j,l} - \epsilon_{i,l} \leq t_1 \text{ and } Y_{i,l}(x_h) - \epsilon_{i,l} \leq t_2 | B_{h,l} = b_h) \equiv F(t_1, t_2 | b_h),
\end{aligned}
$$

where we let $Y_{0,l}(x_h) \equiv \max\{\alpha_l Q_{h,l} + \beta_l x_h - B_{h,l} + \epsilon_{h,l}, U_{0,l}\}$ and $Y_{i,l}(x_h) \equiv Y_{0,l}(x_h) - \alpha_l q_i(x_i) - \beta_l x_i$. The last equality follows because under our model assumptions the bids are independent across sellers and independent of buyers' tastes, and because $(\epsilon_{i,l}, \epsilon_{j,l}, Y_{i,l}(x_h), B_{h,l})$ are independent of $(B_{i,l}, B_{j,l})$.

The winning probability on the lefthand side is directly identifiable from the data. Hence the joint distribution F on the righthand side is identified. Since $\epsilon_{j,l}$, $\epsilon_{i,l}$ and $(Y_{i,l}(x_h), B_{h,l})$ are independent, the conditional distribution of $Y_{i,l}(x_h)$ given $B_{h,l} = b_h$ and the distributions of $\epsilon_{j,l}$ and $\epsilon_{i,l}$ are identified up to a location normalization if the support of $(B_{i,l}, B_{j,l})$ is large enough.[7] This intuition continues to hold when the conditional winning probabilities are aggregated over distinct choice sets that include two permanent sellers from the same $(x,\, q)$-group.

The quality levels $\bar{q}_1(\bar{x}_1), \bar{q}_1(\bar{x}_2), \bar{q}_1(\bar{x}_2), \bar{q}_2(\bar{x}_2)$ and the distributions of α_l and β_l are identified similarly. Specifically, we identify the distribution of $\alpha_l(\bar{q}_1(x) - \bar{q}_2(x))$ by applying similar arguments to the subset of auctions with choice sets consisting of permanent sellers i and j from the same observable group x but different quality groups, $\bar{q}_1(x)$ and $\bar{q}_2(x)$, a transitory seller h and the outside option. The mean of the distribution $\alpha_l(\bar{q}_1(x) - \bar{q}_2(x))$ identifies $\bar{q}_1(x) - \bar{q}_2(x)$ under the normalization $\mathbf{E}[\alpha_l] = 1$. Further, we consider the subset of auctions with choice sets consisting of permanent sellers i and j from the lowest quality from the observable groups, $\bar{x}_1$ and $\bar{x}_2$, a transitory seller h from $\bar{x}_2$ and the outside option. Then we can identify the distribution of β_l under an additional normalization that $\bar{q}_1(\bar{x}_1) = \bar{q}_1(\bar{x}_2) = 0$, i.e., the lowest quality in each observable group is normalized to zero. This restriction on the lowest quality levels can be relaxed if β_l is a fixed constant parameter (equal to β) rather than a random variable. In this case it is enough to normalize the lowest quality level for a single observable group, e.g., $\bar{q}_1(\bar{x}_1)$,

[7] This is a consequence of the Kotlarski Theorem. See Rao (1992) for details. The formal support requirements are stated in Section 5.2.2 and discussed in the Web Supplement to the paper.

whereas $\bar{q}_1(\bar{x}_2) - \bar{q}_1(\bar{x}_1) = \bar{q}_1(\bar{x}_2)$ is identified with the rest of the model.

Outside Option and Transitory Sellers' Quality Distribution. We now explain how to identify the distribution of payoffs from the outside option $U_{0,l}$, and the conditional quality distribution $\Pr(Q_{h,l} = \bar{q}_k(x_h)|B_{h,l} = b_h, X_h = x_h)$ for transitory sellers. The latter depends on sellers' bidding strategies in equilibrium.

Consider auctions with one permanent bidder and one transitory bidder, and assume that $U_{0,l}$, β_l and α_l are independent. (The formal results in **?** allow for the correlation between $U_{0,l}$ and α_l.) Recall that $(\bar{q}_1(\bar{x}_1), \bar{q}_1(\bar{x}_2))$ and the distributions of α_l and β_l and that of $Y_{1,l}(x_h) = Y_{0,l}(x_h) - \alpha_l \bar{q}_1(\bar{x}_1) - \beta_l \bar{x}_1$ conditional on $B_{h,l} = b_h$ are identified in the previous step. Hence the conditional distribution of $Y_{0,l}(x_h)$, which is the maximum of the payoff from the outside option and that from the transitory seller, is also identified.

Next, let us now argue that knowledge of the conditional distributions of $Y_{0,l}(x_h)$ helps to identify the distribution of the outside option and the conditional distribution of a transitory seller's quality. With $U_{0,l}$, β_l and α_l independent of each other, the payoff to the outside option $(U_{0,l})$ and the payoff to the transitory seller $(U_{h,l})$ are independent. Hence, for each fixed pair of numbers (y_0, b_h),

$$\Pr(Y_{0,l}(x_h) \leq y_0|B_{h,l} = b_h, X_h = x_h) = \Pr(U_{0,l} \leq y_0) \Pr(U_{h,l}(x_h) \leq y_0|B_{h,l} = b_h, X_h = x_h). \quad (2)$$

This leads to two equations, one with $x_h = \bar{x}_1$ and the other with $x_h = \bar{x}_2$. After rearranging terms in these equations, we have:

$$\begin{aligned} &g_1(y_0; b_h) \Pr(Q_{h,l} = \bar{q}_1(\bar{x}_2)|B_{h,l} = b_h, X_h = \bar{x}_2) \\ &-g_2(y_0; b_h) \Pr(Q_{h,l} = \bar{q}_1(\bar{x}_1)|B_{h,l} = b_h, X_h = \bar{x}_1) = g_3(y_0; b_h), \end{aligned}$$

where for each b_h, $g_s(y_0; b_h)$, $s = 1, 2, 3$, are known functions of y_0, and $\Pr(Q_{h,l} = \bar{q}_1(\bar{x}_j)|B_{h,l} = b_h, X_h = \bar{x}_j)$, $j = 1, 2$, are unknown probabilities to be recovered.[8] These probabilities are over-identified since we have infinitely many linear equations associated with different values of y_0. Once these probabilities are identified, the distribution of the payoff from the outside option $U_{0,l}$ is identified from (2).

[8]Specifically, $g_1(y_0; b_h) = \Pr(Y_0(\bar{x}_1) \leq y_0|b_h, \bar{x}_1)[J_1(b_h, \bar{x}_2) - J_2(b_h, \bar{x}_2)]$,
$g_2(y_0; b_h) = \Pr(Y_0(\bar{x}_2) \leq y_0|b_h, \bar{x}_2)[J_1(b_h, \bar{x}_1) - J_2(b_h, \bar{x}_1)]$,
$g_3(y_0; b_h) = \Pr(Y_0(\bar{x}_2) \leq y_0|b_h, \bar{x}_2)J_2(b_h, \bar{x}_1) - \Pr(Y_0(\bar{x}_1) \leq y_0|b_h, \bar{x}_1)J_2(b_h, \bar{x}_2)$,
where $J_k(b, x)$ denotes $\Pr(U_h(x) \leq y_0|B_h = b_h, x, Q_h = \bar{q}_k(x))$ for $k = 1, 2$. The functions J_k, the conditional distribution of Y_0, and therefore $g_s(y_0; b)$ for $s = 1, 2, 3$, are recovered from the previous steps. See Section B2 in the Web Supplement to Krasnokutskaya, Song, and Tang (2018) for details.

Appendix B: Numerical Algorithm

In this appendix we summarize the numerical algorithm used to solve for bidding strategies in the paper. Our numerical strategy combines insights from projection methods[9]and the numerical approach developed in Marshall, Meurer, Richard, and Stromquist (1994).

We maintain the following notations:

1. K is the number of different seller groups.

2. $N_{k;o_b}$ is the vector of numbers of potential bidders from group k; $n_{k;o_b}$ is the number of actual bidders from group k.

3. $\kappa(j)$ is a group membership of bidder j; $\bar{q}_{\kappa(j)}$ is the quality level corresponding to the group of bidder j.

4. $(\alpha_l, \epsilon_l, U_{0,l})$ summarize buyer's l utility components so that the utility a buyer l derives from seller j is given by
$$u_{j,l} = \alpha_l q_{\kappa(j)} - b_{l,j} + \epsilon_{l,j},$$
where $b_{l,j}$ is the bid seller j submits in the auction of buyer l; $U_{0,l}$ represents the outside option of buyer l.

5. $F_k^C(.)$ represent the distributions of the project and entry costs for group k.

To simplify the presentation we summarize the method for the case when (a) all sellers are permanent; (b) $\sigma_\epsilon = 1$; (c) α and U_0 are independent.

The algorithm describes how to solve for a type-specific equilibrium bidding strategies $\{\beta_k(.|o_b)\}_{k=1,\dots,K}$.

Notice that in this setting the seller observes the set of sellers, N_{o_b}, who decided to participated in the auctions of type o_b. These sellers are then randomly allocated across auctions of type o_b. Thus, the set of actual competitors that a given seller faces remains uncertain. However, he has no control over the probability of encountering a specific set of competitors (that is the probability of seller i being in the auction with a set of actual competitors A, $\Pr(A)$, does not depend on seller i's strategic choices).

1. Seller chooses a bidding strategy to maximize the following objective function:
$$(b - c_i)\sum_A \Pr(U_0 \leq \alpha q_{\kappa(i)} - b + \epsilon_i \text{ and } \alpha q_{\kappa(j)} - B_j + \epsilon_j \leq \alpha q_{\kappa(i)} - b + \epsilon_i \,\forall\, j \in A)\Pr(A)$$
which can be rewritten as
$$(b - c_i)\sum_A \int_\alpha \int_{\epsilon_i} F_{U_0}(q_{\kappa(i)} - \alpha b + \epsilon_i) \times$$
$$\prod_{j \neq i} \int_{c_j} F_\epsilon(\Delta_{i,j} q - \alpha(b - \beta_{\kappa(j)}(c_j)) + \epsilon_i) dF_{\kappa(j)}(c_j) dF_\epsilon(\epsilon_i) dF_\alpha(\alpha)\Pr(A).$$

[9]See, for example, Bajari (2001).

Here summation is over all possible sets of entrants, A, and $\Pr(A)$ reflects the probability that a given set of entrants is realized.

2. Further, for the derivation simplicity we rewrite sellers' strategies in terms of 'offers', $\omega_i = \mu_\alpha q_{\kappa(i)} - b_i$ that are functions of a seller's surplus, $s_i = \mu_\alpha q_{\kappa(i)} - c_i$. That is, the seller's strategy is now given by $\gamma_k : S_k \to R$. This is just a simple re-parametrization of the seller's problem which now becomes

$$(s_i - \omega_i) \sum_A [\int_\alpha \int_{\epsilon_i} F_{U_0}(\omega_i + \alpha_0 q_{\kappa(i)} + \epsilon_i) \times$$
$$\prod_j \int_{s_j} F_\epsilon(\omega_i - \gamma_{\kappa(j)}(s_j) + \alpha_0(q_{\kappa(i)} - q_{\kappa(j)}) + \epsilon_i) dF_{\kappa(j)}(s_j) dF_{\epsilon_i}(\epsilon_i) dF_\alpha(\alpha_0)] \Pr(A) \text{ with } \alpha_0 = \alpha - \mu_\alpha.$$

3. Then, the first order conditions for this problem are given by

$$\sum_A [- \int_\alpha \int_{\epsilon_i} F_{U_0}(\omega_i + \alpha_0 q_{\kappa(i)} + \epsilon_i) \prod_j \int_{s_j} F_\epsilon(\omega_i - \gamma_{\kappa(j)}(s_j) + \alpha_0 \Delta_{i,j} q + \epsilon_i) dF_{\kappa(j)}(s_j) dF_{\epsilon_i}(\epsilon_i) dF_{\alpha_0}(\alpha_0) +$$
$$(s_i - \omega_i) \int_{\alpha_0} \int_{\epsilon_i} \left\{ f_{U_0}(\omega_i + \alpha_0 q_{\kappa(i)} + \epsilon_i) \prod_j \int_{s_j} F_\epsilon(\omega_i - \gamma_{\kappa(j)}(s_j) + \alpha_0 \Delta_{ij} q + \epsilon_i) dF_{\kappa(j)}(s_j) + \right.$$
$$F_{U_0}(\omega_i + \alpha_0 q_{\kappa(i)} + \epsilon_i) \sum_{j_1=1}^{L} \left(\int_{s_{j_1}} f_\epsilon(\omega_i - \gamma_{\kappa(j_1)}(s_{j_1}) + \alpha_0 \Delta_{ij} q + \epsilon_i) dF_{\kappa(j_1)}(s_{j_1}) \times \right.$$
$$\left. \left. \prod_{j \neq j_1} \int_{s_{j_1}} F_\epsilon(\omega_i - \gamma_{\kappa(j)}(s_j) + \alpha_0 \Delta_{ij} q + \epsilon_i) dF_{\kappa(j)}(s_j) \right) \right\} dF_{\epsilon_i}(\epsilon_i) dF_\alpha(\alpha_0)] \Pr(A) = 0.$$

4. Following Marshall, Meurer, Richard, and Stromquist (1994), we divide the support of S_k into small intervals. Further, we approximate each of the $\gamma_k(.)$ functions by a polynomial of $(s_k - \tilde{s}_k^l)$ where $\tilde{s}_k^l$ is a centroid of l's interval on the support of S_k. Specifically, we assume that $\gamma_k(s) = \sum_{p=0}^{\infty} a_{k,p}^{(l)} (s - \tilde{s}_k^l)^p$ on interval l. We also use their technique for representation of the nonlinear function of a bidding strategy in the form of a polynomial of $(s - \tilde{s}_k^l)$ (see the non-uniform case). We use spline approximation of the estimated densities to obtain coefficients in the polynomial expansion of the outside functions.[10]

5. The polynomial expansion of first order conditions discussed in point (4) is summarized by a set of coefficients in front of the polynomial terms. To obtain the $n-$th order approximation of the offer function we set the first n coefficients of the first order conditions expansion to zero and solve for a set of $\{a_{k,p}^{(l)}\}$ coefficients that satisfy this restriction. This part of our algorithm is borrowed from the projection methods. We deviate from the algorithm in Marshall, Meurer, Richard, and Stromquist (1994) at this point because the expressions for the coefficients in a first-order-conditions representation are non-linear functions of $\{a_{k,p}^{(l)}\}$

[10]The exact expression of the polynomial expansion of the first order conditions is 12 pages long and is available from the authors upon request.

and thus we were unable to obtain an iterative expression similar to that in Marshall, Meurer, Richard, and Stromquist (1994).

6. The set of coefficients is obtained for a given set of starting points (boundary conditions). Once the set of coefficients is obtained, we compute the approximation error associated with a solution obtained under such starting point. Unlike a standard auction model, the multi-attribute auction model does not have a singularity on either end because of ϵ and integration over ϵ. On the other hand, we do not have such a precise knowledge of a boundary conditions as we have in the standard case. For this reason we do not target an objective function which reflects the fit of the numerical solution at the boundary. Instead, we compute an error for each subinterval and then average it over the intervals. We incorporate this error into an iterative mechanism which searches for an optimal boundary conditions. The search stops once the targeted precision is reached. In this our approach resembles the projection method.

7. We have verified that this algorithm converges to a vector of equilibrium bidding functions in the case when all the relevant distributions are uniform.

Appendix C: Additional Tables

Table 1: Estimated Quality Groups by Supplier Covariates

Country Group	Average Score	Total Number of Suppliers	$Q = L$	$Q = M$	$Q = H$
North America	low	12	4	8	
			(6)	(10)	
North America	medium	13	4	9	
			(6)	(11)	
North America	high	17	12	5	
			(13)	(6)	
Eastern Europe	low	18	6	12	
			(8)	(14)	
Eastern Europe	medium	52	33	12	7
			(37)	(14)	(9)
Eastern Europe	high	83	6	65	12
			(7)	(69)	(15)
East Asia	low	91	62	18	11
			(68)	(22)	(13)
East Asia	medium	66	6	53	7
			(8)	(57)	(9)
East Asia	high	58	50	8	
			(53)	(11)	

This table is reproduced from Krasnokutskaya, Song, and Tang (2018). It shows the estimated group structure and a consistently-selected number of groups for each cell determined by covariate values. Column 3 indicates the total number of the suppliers in each cell. Columns 4-6 report the size of the estimated quality group. The size of the corresponding confidence set with 90% coverage is reported in parenthesis. Note that the confidence set with the level (1 - α) for a given quality group is defined to be a random set whose probability of containing this quality group is ensured to be asymptotically bounded from below by (1-α).

Table 2: Sellers' Quality Levels

Country	Score	Quality	Specifications		
			(I)	(II)	(III)
North America	Low	1	0.173 (0.301)	-0.018 (0.31)	0.061 (0.302)
North America	Low	3	0.611*** (0.030)	0.538*** (0.031)	0.578*** (0.033)
North America	Medium	1	0.311 (0.517)	0.116 (0.221)	0.123 (0.221)
North America	Medium	3	0.729*** (0.152)	0.575*** (0.101)	0.604*** (0.111)
North America	High	1	0.124*** (0.051)	0.143*** (0.043)	0.127*** (0.051)
North America	High	3	0.606*** (0.211)	0.619*** (0.204)	0.636*** (0.211)
Eastern Europe	Low	1	-0.121 (0.051**)	-0.097*** (0.041)	-0.011** (0.052)
Eastern Europe	Low	2	0.429*** (0.133)	0.405*** (0.131)	0.405*** (0.123)
Eastern Europe	Medium	1	0.121*** (0.101)	0.162 (0.102)	0.117 (0.101)
Eastern Europe	Medium	2	0.416*** (0.022)	0.408*** (0.021)	0.423*** (0.021)
Eastern Europe	Medium	3	0.704*** (0.061)	0.757*** (0.063)	0.707*** (0.066)
Eastern Europe	High	1	-0.240*** (0.111)	-0.063 (0.090)	-0.012 (0.071*)
Eastern Europe	High	2	0.433*** (0.011)	0.401*** (0.010)	0.427*** (0.021)
Eastern Europe	High	3	0.753*** (0.121)	0.782*** (0.113)	0.745*** (0.131)
South and East Asia	Low	1	normalized to 0		
South and East Asia	Low	2	0.366*** (0.211)	0.235** (0.132)	0.323*** (0.142)
South and East Asia	Low	3	0.707*** (0.215)	0.572*** (0.101)	0.674*** (0.102)
South and East Asia	Medium	1	-0.310 (0.221)	0.041 (0.211)	-0.054 (0.321)
South and East Asia	Medium	2	0.369*** (0.021)	0.325*** (0.032)	0.319*** (0.024)
South and East Asia	Medium	3	0.765*** (0.032)	0.675*** (0.043)	0.703*** (0.033)
South and East Asia	High	2	0.293*** (0.171)	0.282*** (0.144)	0.307*** (0.123)
South and East Asia	High	3	0.672*** (0.031)	0.681*** (0.033)	0.618*** (0.031)

This table reports the estimates of quality levels for various quality groups that correspond to the three specifications we consider.

Table 3: Parameters of the Distribution of Buyers' Tastes

Parameters	Specifications		
	(I)	(II)	(III)
Standard Deviations of Unobservables:			
log (σ_ϵ)	-0.458*** (0.021)	-0.293*** (0.023)	-0.496*** (0.021)
log (σ_{v_0})	-0.221*** (0.101)	-0.245*** (0.100)	-0.336*** (0.111)
log (σ_α)	-0.490*** (0.010)	-0.368*** (0.011)	-1.310*** (0.010)
Mean of Price Sensitivity:			
Constant	1	1	1
United Kingdom		0.188*** (0.076)	0.177*** (0.071)
Western Europe		-0.017 (0.021)	-0.229*** (0.043)
Southern Europe		-0.220* (0.121)	-0.176 (0.132)
Eastern Europe		-0.090*** (0.112)	0.212** (0.111)
Australia		0.120*** (0.030)	0.161*** (0.044)
South and East Asia		0.307*** (0.021)	0.299*** (0.025)
Mean of an Outside Option:			
Constant	-1.221*** (0.322)	-1.264*** (0.432)	-1.66*** (0.443)
United Kingdom			-0.992*** (0.341)
Western Europe			-0.519*** (0.213)
Southern Europe			-1.745 (1.261)
Eastern Europe			0.576*** (0.221)
Australia			0.321*** (0.101)
South and East Asia			0.771*** (0.042)

This table reports the estimated parameters of the distributions of buyers' utility coefficients and outside options.

Table 4: Expected Entry Costs Conditional on Participation

	Sellers		
Buyer Country	North America	Eastern Europe	SE Asia
North America	0.132	0.120	0.099
UK	0.111	0.064	0.039
Western Europe	0.105	0.071	0.031
Eastern Europe	0.130	0.063	0.020
Australia	0.129	0.073	0.046
SE Asia	0.096	0.013	0.019

This table reports the average entry costs conditional on participation for various country pairs. The numbers reported in this table reflect equilibrium outcomes and, thus, are not directly informative about any specific primitive. We include them here to illustrate the magnitude of entry costs incurred in this market.

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