

# Informational Cycles in Search Markets

## Online Appendix

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### APPENDIX A

Appendix A contains the proofs of the Propositions in the main part of the paper.

#### A1. Benchmarks

##### PROOF OF PROPOSITION 1:

Here buyers know the state. For each state, I derive the conditions under which a young buyer prefers accepting a low offer to rejecting it.

A young buyer's continuation value in state  $\theta$  is

$$(A1) \quad V^\theta = \delta[v_L + \mu^\theta(v_H - v_L)].$$

The buyer accepts any offer when old. She gets  $v_L$  for sure and the extra value  $v_H - v_L$  only if she gets a high offer. The probability of getting an offer is equal to the fraction of these offers because of random matching. She prefers continuing to accepting a low offer if  $v_L < V^\theta$ . Rearranging gives that she prefers to continue in the good state if  $\bar{\pi} < 1$ , where  $\bar{\pi}$  is defined in (1). ■

##### PROOF OF PROPOSITION 2:

Here, a buyer does not know the state and learns about it only from her own experience, i.e., from the offers that she gets. I derive the conditions under which a young buyer prefers accepting a low offer to rejecting it.

Consider any period  $t$ . For a young buyer with posterior belief  $\pi'$ , the expected value of continuing and accepting either offer when old is

$$(A2) \quad V(\pi') = \delta [v_L + \pi' \mu^G(v_H - v_L)].$$

She gets utility  $v_L$  for sure and the extra value  $v_H - v_L$  only if she gets a high offer. She gets a high offer with probability  $\mu^G$  only if the state is good. The young buyer optimally rejects a low offer if  $V(\pi') > v_L$ , or if  $\pi' > \bar{\pi}$ , where the cutoff belief  $\bar{\pi}$  is the same as in the known-state benchmark, defined in (1).

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Let  $\pi(v_L)$  denote a young buyer's posterior belief that the state is good after getting a low offer. Her posterior odds are

$$\frac{\pi(v_L)}{1 - \pi(v_L)} = \omega \frac{P(v_L|G)}{P(v_L|B)} = \omega(1 - \mu^G),$$

where  $\omega := \frac{\pi}{1-\pi}$  denotes the prior odds. I focus on the posterior odds throughout because the odds contain the same information as the posterior belief but are easier to interpret. The posterior odds are lower than the prior odds because getting a low offer makes the buyer more pessimistic about the state.

A young buyer who gets a low offer optimally rejects it and continues if  $V(\pi' = \pi(v_L)) > v_L$  or, equivalently, if  $\pi(v_L) > \bar{\pi}$ , which is the condition in the Proposition if rearranged. The argument holds both if the economy starts at  $t_1 = 1$  and at  $t_1 = -\infty$  because a buyer's optimal decision only depends on the distribution of offers and that is given. ■

*A2. Buyers do not know the state and learn from their own experience and the others' trades*

#### PROOF OF PROPOSITION 3:

(i) Steady state 0: A young buyer who gets a low offer accepts the offer.

If buyers use this strategy, a randomly drawn buyer trades with probability one in both states. This makes the trade signal uninformative. But if the signal is uninformative, then the equilibrium exists for the same parameter values as in a market where buyers learn only from their own experience (see Proposition 2).

(ii) Steady state 1: A young buyer who gets a low offer accepts the offer after observing no trade and rejects it after observing a trade.

I first derive the equilibrium objects (amounts of old buyers and trading probabilities) assuming that the strategy profile constitutes an equilibrium and then derive the conditions under which no buyer has an incentive to deviate. Let  $O_t^\theta$  denote the amount of old buyers and  $\tau_t^\theta$  the probability of a randomly drawn buyer trading at date  $t$  in state  $\theta$ . The amounts of buyers are measured at the start of a period, after entry.

Consider any period  $t$  and state  $\theta$ . Given the above strategy, the probability of a trade at  $t$  in state  $\theta$  is

$$(A3) \quad \tau_t^\theta = 1 - \frac{(1 - \mu^\theta)\tau_{t-1}^\theta}{1 + O_t^\theta}.$$

The only buyers who do not trade are young buyers who got a low offer and observed a trade ( $T_{t-1}$ ). The probability of getting a low offer is equal to the

fraction of these offers (because of random matching) and the probability of observing a trade is the probability that a randomly drawn buyer traded at  $t - 1$ . The total amount of buyers is the sum of the amounts of young and old buyers.

The young buyers who become old, i.e., buyers who are carried over to  $t + 1$ , are young buyers who get low offers and see a trade. So the amount of old buyers at  $t + 1$  in state  $\theta$  is

$$(A4) \quad O_{t+1}^\theta = (1 - \mu^\theta)\tau_{t-1}^\theta.$$

Imposing the steady state condition that  $x_t = x_{t+1}$  for all endogenous variables  $x$  and solving equations (A3) and (A4) for  $\theta = B, G$  gives that  $\tau^\theta$  solves  $(\tau^\theta)^2(1 - \mu^\theta) = 1 - \tau^\theta$ . A trade is good news, as required, because  $\tau^G > \tau^B$  (and  $\tau^B > \frac{1}{2}$ ). The trade probability is explicitly

$$\tau^\theta = \frac{\sqrt{5 - 4\mu^\theta} - 1}{2(1 - \mu^\theta)}.$$

The proposed strategy is optimal if no young buyer wants to deviate. At  $t$ , the posterior odds of a young buyer who gets a low offer and observes no trade  $N_{t-1}$  are

$$(A5) \quad \frac{\pi(v_L, N_{t-1})}{1 - \pi(v_L, N_{t-1})} = \omega \frac{P(v_L|G) P(N_{t-1}|G)}{P(v_L|B) P(N_{t-1}|B)} = \omega \frac{1 - \mu^G}{1 - \mu^B} \frac{1 - \tau_{t-1}^G}{1 - \tau_{t-1}^B}.$$

The posterior odds of a young buyer who gets a low offer and observes a trade  $T_{t-1}$  are

$$(A6) \quad \frac{\pi(v_L, T_{t-1})}{1 - \pi(v_L, T_{t-1})} = \omega \frac{P(v_L|G) P(T_{t-1}|G)}{P(v_L|B) P(T_{t-1}|B)} = \omega \frac{1 - \mu^G}{1 - \mu^B} \frac{\tau_{t-1}^G}{\tau_{t-1}^B}.$$

For a young buyer with belief  $\pi'$ , the value of continuing and accepting either offer when old is given by (A2). Thus, the cutoff belief is still  $\bar{\pi}$  as defined in (1) and the proposed equilibrium strategy is optimal if  $\pi(v_L, N) < \bar{\pi} < \pi(v_L, T)$ , which, if rearranged, give the exact conditions in the Proposition. A trade being good news guarantees that the conditions are satisfied for an open set of parameter values.

(iii) Steady state 2: A young buyer who gets a low offer rejects it and continues.

I go through the same two steps as in (ii): I derive first the equilibrium objects given the strategy profile and then the conditions under which no buyer deviates. According to the above strategy, all young buyers who get a low offer continue at

$t$  so the probability of a trade at  $t$  in state  $\theta = B, G$  is

$$(A7) \quad \tau_t^\theta = 1 - \frac{1 - \mu^\theta}{1 + O_t^\theta},$$

and the amount of old buyers at  $t + 1$  is

$$(A8) \quad O_{t+1}^\theta = 1 - \mu^\theta.$$

Imposing the steady state condition gives  $\tau^\theta = (2 - \mu^\theta)^{-1}$ , so  $\tau^G > \tau^B$ .

The strategy is optimal if the most pessimistic young buyer (one who gets a low offer and observes no trade) does not want to deviate and accept  $v_L$ . Her posterior odds are given by (A5), where the trade probabilities are  $\tau^\theta = (2 - \mu^\theta)^{-1}$  for  $\theta = B, G$ . Thus, steady state 2 exists if  $\bar{\pi} < \pi(v_L, N)$ , which, once rearranged, gives the exact condition in the Proposition.

Finally, I prove by contradiction that a trade cannot be bad news in a pure-strategy steady-state equilibrium. Since in the above steady states a trade is (weakly) good news, I have to consider only one candidate steady state: where young buyers who get a low offer reject it after observing no trade and accept it after observing a trade. The derivation is analogous to that of steady state 1 (where a trade event is replaced by a no trade event) so I am brief.

Given the proposed strategy, the buyers who become old are young buyers who get a low offer and see no trade so the amount of old buyers is  $O^\theta = (1 - \mu^\theta)(1 - \tau^\theta)$  in state  $\theta$ . The probability of a trade in state  $\theta$  is  $\tau^\theta = 1 - \frac{(1 - \mu^\theta)(1 - \tau^\theta)}{1 + O^\theta}$ . Combining the equations gives  $\tau^G = 1$  and  $\tau^B = 1$ . But a trade is bad news only if  $\tau^G < \tau^B$ , a contradiction. ■

#### PROOF OF PROPOSITIONS 4 AND 7:

I provide here the proof for the the general case where the fraction of high offers in the bad state is  $\mu^B \in [0, \mu^G)$  for brevity (Proposition 4 is a special case where  $\mu^B = 0$ ).

In the cyclical equilibrium, in an odd period  $2t - 1$  (for any integer  $t$ ) buyers behave as in steady state 2: no young buyer accepts a low offer. I assume that a trade at  $2t - 1$  ( $T_{2t-1}$ ), which is observed at  $2t$ , is good news. In an even period  $2t$  buyers behave as in steady state 1: the young buyers who observe bad news about the state (i.e., no trade  $N_{2t-1}$ ), accept a low offer. I assume that both a trade and no trade at  $2t$  are relatively uninformative (so that all young buyers at  $2t + 1$  optimally reject a low offer). Let  $O_t^\theta$  denote the amount of old buyers and  $\tau_t^\theta$  the probability of a randomly drawn buyer trading in period  $t$  in state  $\theta$ .

I first derive the equilibrium objects  $O_{2t-1}^\theta, O_{2t}^\theta, \tau_{2t-1}^\theta$  and  $\tau_{2t}^\theta$  given the proposed strategy and show that an odd-period trade is good news. I then derive the conditions under which no buyer has an incentive to deviate from the strategy. Finally, I show that the volume of trading fluctuates more in the bad than in the good state (and also the probability of trading if  $\mu^B = 0$ ).

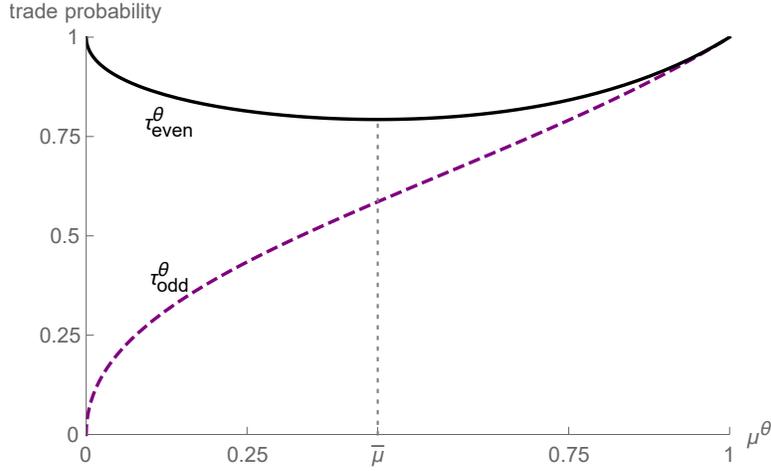


Figure A1: The proposed equilibrium probabilities of trading in odd periods (purple dashed) and in even periods (black).

Consider an odd period  $2t - 1$ . Since at  $2t - 1$  buyers behave as in steady state 2, the equations for the probability of a trade at  $2t - 1$  and the amount of old buyers at  $2t$  are given by equations (A7) and (A8) respectively (where  $2t - 1$  replaces  $t$ ).

Consider an even period  $2t$ . Since at  $2t$  buyers behave as in steady state 1, the equations for the probability of a trade at  $2t$  and the amount of old buyers at  $2t + 1$  are given by equations (A3) and (A4) respectively (where  $2t$  replaces  $t$ ).

To complete this step, I impose the condition that the cycle is two periods long, i.e., that for  $t' = 2t - 1, 2t$  and all endogenous variables  $x, x_{t'+2} = x_{t'}$ . I denote the proposed equilibrium values of the endogenous variables with subscripts “odd” and “even”. The probabilities of trading solve  $(\tau_{odd}^\theta)^2(1 - \mu^\theta) = \mu^\theta(1 - \tau_{odd}^\theta)$  and  $\tau_{even}^\theta = 1 - (2 - \mu^\theta)^{-1}(1 - \mu^\theta)\tau_{odd}^\theta$  for  $\theta = B, G$ , which give, explicitly

$$(A9) \quad \tau_{odd}^\theta = \frac{\sqrt{\mu^\theta(4 - 3\mu^\theta)} - \mu^\theta}{2(1 - \mu^\theta)},$$

and

$$(A10) \quad \tau_{even}^\theta = 1 - \frac{\sqrt{\mu^\theta(4 - 3\mu^\theta)} - \mu^\theta}{2(2 - \mu^\theta)}.$$

The probability of trading in an odd period,  $\tau_{odd}^\theta$ , strictly increases in  $\mu^\theta$  while the probability of trading in an even period,  $\tau_{even}^\theta$ , is convex in  $\mu^\theta$ . It is easy to check that  $\tau_{even}^\theta > \tau_{odd}^\theta$  for all  $\mu^\theta$ . Figure A1 depicts equations (A9) and (A10).

A trade in an odd period is always good news,  $\tau_{odd}^G > \tau_{odd}^B$ , because  $\tau_{odd}^\theta$  strictly increases in  $\mu^\theta$ . Two sufficient conditions for a trade in an even period to be bad news,  $\tau_{even}^G < \tau_{even}^B$ , are (i)  $\mu^B = 0$ , and (ii)  $\mu^G \leq \bar{\mu}$ , where  $\bar{\mu} := \frac{2}{7}(3 - \sqrt{2})$  minimises  $\tau_{even}^\theta$ . Broadly, a trade in an even period is bad news also if  $\mu^B$  is close to zero and  $\mu^G$  is not too close to one (see Figure A1).

Next I determine the parameter values for which the proposed strategy is optimal for young buyers. For a young buyer with belief  $\pi'$ , the value of continuing and accepting either offer when old is given by

$$V(\pi') = \delta \{v_L + [\pi' \mu^G + (1 - \pi') \mu^B](v_H - v_L)\}.$$

So the cutoff belief  $\bar{\pi}$  that makes a young buyer just indifferent between accepting and rejecting a low offer is

$$\bar{\pi} := (\mu^G - \mu^B)^{-1} \left[ \frac{(1 - \delta)v_L}{\delta(v_H - v_L)} - \mu^B \right].$$

I make the following assumption:

ASSUMPTION 2:  $0 < \bar{\pi} < 1$ .

The assumption ensures that, if the state is known, a young buyer's optimal behaviour depends on the true state: she optimally accepts a low offer if the state is bad and rejects it if the state is good.

At period  $t' = 2t - 1, 2t$ , a young buyer's posterior odds after getting a low offer and observing signal outcome  $N_{t'-1}$  are given by equation (A5) and after signal outcome  $T_{t'-1}$  by equation (A6) where  $t = t'$ . The proposed strategy constitutes an equilibrium if a young buyer who gets a low offer in an odd period optimally continues regardless of the signal outcome, i.e., if  $\bar{\pi} < \pi(v_L, T_{even}), \pi(v_L, N_{even})$ , and in an even period optimally continues after a trade, but accepts the offer after no trade, i.e., if  $\pi(v_L, N_{odd}) < \bar{\pi} < \pi(v_L, T_{odd})$ . Because a trade in an odd period is good news, the strategy is optimal if  $\pi(v_L, N_{odd}) < \bar{\pi} < \pi(v_L, T_{even}), \pi(v_L, N_{even})$ , which, once rearranged, are the conditions in the Proposition. The conditions are satisfied for a range of parameter values. Note that if a trade in an even period is bad news, then the strategy is optimal if  $\pi(v_L, N_{odd}) < \bar{\pi} < \pi(v_L, T_{even})$ .

Finally, I show that the volume of trading fluctuates more in the bad than in the good state. In an odd period, the volume of trading is

$$Vol_{odd}^\theta = O_{odd}^\theta + \mu^\theta = (1 - \mu^\theta)\tau_{odd}^\theta + \mu^\theta = 1 - (1 - \mu^\theta)(1 - \tau_{odd}^\theta).$$

In an even period, the volume is

$$Vol_{even}^\theta = O_{even}^\theta + \mu^\theta + (1 - \mu^\theta)(1 - \tau_{odd}^\theta) = 1 + (1 - \mu^\theta)(1 - \tau_{odd}^\theta),$$

with  $Vol_{even}^\theta - Vol_{odd}^\theta > 0$  and  $Vol_{even}^\theta - Vol_{odd}^\theta = 2$  for all  $\mu^\theta$ . The fluctuations in

trading volume are larger in the bad state than in the good state, or,  $Vol_{even}^B > Vol_{even}^G > Vol_{odd}^G > Vol_{odd}^B$ , because  $\mu^G > \mu^B$  and  $\tau_{odd}^G > \tau_{odd}^B$ .

A sufficient condition for the trade probability to fluctuate more in the bad than in the good state is that a trade in an even period is bad news. If this is the case, then together,  $\tau_{odd}^G > \tau_{odd}^B$ ,  $\tau_{even}^G < \tau_{even}^B$ , and  $\tau_{even}^\theta > \tau_{odd}^\theta$  for all  $\mu^\theta$  imply that  $\tau_{even}^B > \tau_{even}^G > \tau_{odd}^G > \tau_{odd}^B$ . ■

### A3. Efficiency

#### PROOF OF PROPOSITION 5:

The inefficiency of an equilibrium can be measured by its shortfall from the complete-information benchmark,  $\Delta W$ . The shortfall measures the cost of a young buyer taking the “wrong” action with respect to the low offer in the equilibrium as compared to the complete-information benchmark: of accepting the low offer if the state is good and of rejecting it if the state is bad.<sup>1</sup> The shortfall in an equilibrium using strategy  $\sigma$  is

$$\begin{aligned} \Delta W_{eq} &:= P(G)P(\text{gets offer } v_L|G)P(\text{accepts}|v_L, \sigma, G)(V^G - v_L) \\ &\quad + P(B)P(\text{gets offer } v_L|B)P(\text{rejects}|v_L, \sigma, B)(v_L - V^B) = \\ &= \pi(1 - \mu^G)P(\text{posterior} < \bar{\pi}|v_L, \sigma, G)(V^G - v_L) + (1 - \pi)P(\text{posterior} > \bar{\pi}|v_L, \sigma, B)(v_L - V^B), \end{aligned}$$

where  $V^\theta$ , defined in (A1), is the discounted value of accepting any offer tomorrow.

In steady state 2, a young buyer never takes the wrong action if the state is good and always takes the wrong action if the state is bad because her posterior always exceeds the critical belief  $\bar{\pi}$ . Thus,

$$\Delta W_{ss2} = (1 - \pi)(v_L - V^B).$$

In steady state 1, a young buyer takes the wrong action with respect to the low offer if the state is good and she observes no trade or if the state is bad and she observes a trade. The shortfall is

$$\Delta W_{ss1} = \pi(1 - \mu^G)(1 - \tau^G)(V^G - v_L) + (1 - \pi)\tau^B(v_L - V^B),$$

where  $\tau^\theta$  solves  $(\tau^\theta)^2(1 - \mu^\theta) = 1 - \tau^\theta$ .

In the cyclical equilibrium, a buyer who is born in an odd period behaves exactly like a buyer in steady state 2 and a buyer who is born in an even period behaves like a buyer in steady state 1 (although the trade probabilities differ). So in the

<sup>1</sup>Alternatively, the shortfall can be calculated as the difference the maximum possible welfare  $W_{max} := \pi[\mu^G v_H + (1 - \mu^G)V^G] + (1 - \pi)[\mu^B v_H + (1 - \mu^B)v_L]$ , and welfare  $W$  in the equilibrium in question. Then the shortfall is simply  $\Delta W := W_{max} - W$ .

cyclical equilibrium the shortfall is

$$\Delta W_c = \frac{1}{2} [\pi(1 - \mu^G)(1 - \tau_{odd}^G)(V^G - v_L) + (1 - \pi)(1 + \tau_{odd}^B)(v_L - V^B)],$$

where  $\tau_{odd}^G$  solves  $(\tau_{odd}^G)^2(1 - \mu^G) = \mu^G(1 - \tau_{odd}^G)$  and  $\tau_{odd}^B = 0$ .

The shortfall is larger in steady state 2 than in the cyclical equilibrium if  $\Delta W_{ss2} > \Delta W_c$ , or, equivalently, if

$$(A11) \quad \frac{\bar{\pi}}{1 - \bar{\pi}} \frac{1}{1 - \mu^G} \frac{1}{1 - \tau_{odd}^G} > \frac{\pi}{1 - \pi},$$

where I have used the fact that  $\frac{\bar{\pi}}{1 - \bar{\pi}} = \frac{v_L - V^B}{V^G - v_L}$ . Condition (A11) holds because it must be satisfied for the cyclical equilibrium to exist (see Proposition 4).

The shortfall is larger in steady state 1 than in the cyclical equilibrium if  $\Delta W_{ss1} > \Delta W_c$  or,

$$(A12) \quad \pi(1 - \mu^G)(V^G - v_L)(1 - 2\tau^G + \tau_{odd}^G) + (1 - \pi)(v_L - V^B)(2\tau^B - 1) > 0.$$

Since  $\tau^B > \frac{1}{2}$ , a sufficient condition for the inequality to hold is that  $1 - 2\tau^G + \tau_{odd}^G > 0$ : then on average across two periods, a young buyer takes the wrong action with respect to a low offer less often in the cyclical equilibrium (than in steady state 1) both in the good and bad state. Finding an interior root to  $1 - 2\tau^G + \tau_{odd}^G = 0$  is equivalent to finding the root of  $1 - 12\mu^G + 9(\mu^G)^2 = 0$ . The unique interior root is  $\hat{\mu} := \frac{2 - \sqrt{3}}{3}$ . The expression  $1 - 2\tau^G + \tau_{odd}^G$  is positive for  $\mu^G > \hat{\mu}$ . Thus, a sufficient condition for the cyclical equilibrium to be more efficient than steady state 1 is  $\mu^G > \hat{\mu}$ .

If  $\mu^G < \hat{\mu}$ , then in the good state, on average across two consecutive periods, a young buyer takes the wrong action with respect to a low offer more often in the cyclical equilibrium. But in expectation the cyclical equilibrium is more efficient because the cost of a good-state mistake must be small for the cyclical equilibrium to exist. If  $\mu^G < \hat{\mu}$ , condition (A12) can be rewritten as

$$\frac{\bar{\pi}}{1 - \bar{\pi}} \frac{1}{1 - \mu^G} \frac{2\tau^B - 1}{2\tau^G - 1 - \tau_{odd}^G} > \frac{\pi}{1 - \pi}.$$

I show that this conditions holds if the two equilibria coexist because the LHS of this inequality is larger than the LHS of (A11) if  $\mu^G < \hat{\mu}$ . The LHS of this inequality is larger than the LHS of (A11) if

$$\tau_{odd}^G(1 - \tau^B) > \tau^G - \tau^B.$$

Recall that in steady state 1, the trade probabilities solve  $(\tau^\theta)^2(1 - \mu^\theta) = 1 - \tau^\theta$ .

Then I can rewrite the last inequality as

$$(\tau^G)^2(1 - \mu^G) > (\tau^B)^2(1 - \tau_{odd}^G).$$

As  $\tau^G > \tau^B$ , it is sufficient to show that  $\tau_{odd}^G > \mu^G$ . Given that  $\tau_{odd}^G$  solves  $\mu^G \tau_{odd}^G + (1 - \mu^G)(\tau_{odd}^G)^2 = \mu^G$ , and  $\tau_{odd}^G < 1$ , it must be that  $\tau_{odd}^G > \mu^G$ . In sum, the cyclical equilibrium is more efficient than steady state 1. ■

#### A4. A market that starts at $t_1 = 1$

#### PROOF OF PROPOSITION 6:

The proof for part (i) is separate and for parts (ii) and (iii) is joint. Let  $\bar{\omega} := \frac{\bar{\pi}}{1 - \bar{\pi}}$  denote the critical odds, where  $\bar{\pi}$  is defined in (1).

- (i)  $\omega < \bar{\omega} \frac{1}{1 - \mu^G}$ : all buyers trade in their entry period. Steady state 0 is reached at  $t = 1$ .

Assume that at  $t = 1$ , a young buyer who gets a low offer optimally accepts it (i.e., that  $\omega < \bar{\omega} \frac{1}{1 - \mu^G}$ ). But then there are no old buyers at  $t = 2$ , exactly as at  $t = 1$ . Hence, all young buyers at  $t = 2$  trade and likewise in the following periods. In other words, steady state 0 is reached at  $t = 1$ .

- (ii)  $\omega > \bar{\omega} \frac{1}{1 - \mu^G} \frac{1}{1 - \tau_1^G}$ , where  $\tau_1^G = \mu^G$ : at all  $t$ , young buyers who get a low offer continue. Steady state 2 is reached at  $t = 2$ .
- (iii)  $\bar{\omega} \frac{1}{1 - \mu^G} \frac{1}{\tau_{even}^G} < \omega < \bar{\omega} \frac{1}{1 - \mu^G} \frac{1}{1 - \tau_1^G}$ , where  $\tau_{even}^G = 1 - (2 - \mu^G)^{-1}(1 - \mu^G)\tau_{odd}^G$ ,  $\tau_{odd}^G$  solves  $(\tau_{odd}^G)^2(1 - \mu^G) = \mu^G(1 - \tau_{odd}^G)$ , and  $\tau_1^G = \mu^G$ : convergence to cycles where a young buyer who gets a low offer in an odd period continues and in an even period accepts the offer if she observes no trade. The cyclical equilibrium is reached in the limit as  $t \rightarrow \infty$ .

I start the market off at  $t_1 = 1$  and show that it converges to the two equilibria in the specified times.

At  $t = 1$ , I assume that a young buyer who gets a low offer optimally continues (i.e., that  $\omega > \bar{\omega} \frac{1}{1 - \mu^G}$ ). Then  $\tau_1^\theta = \mu^\theta$  so a trade at  $t = 1$  is good news.

Consider  $t = 2$ . The amount of old buyers is  $O_2^\theta = 1 - \mu^\theta$ . At  $t = 2$  a young buyer who gets a low offer and sees good news (a trade), is more optimistic about the state than a young buyer who got a low offer at  $t = 1$ . Since at  $t = 1$  the young buyer optimally continued, the more optimistic young buyer at  $t = 2$  optimally continues, too. The same argument holds for all subsequent periods: a young buyer who gets a low offer and sees good news optimally continues. At  $t = 2$ , a young buyer who gets a low offer and sees bad news (no trade) optimally either continues or accepts the offer. I consider both cases in turn.

(a) Assume that a young buyer who gets a low offer and sees bad news at  $t = 2$  optimally continues. I show that the necessary and sufficient condition for this is  $\omega > \bar{\omega} (1 - \mu^G)^{-2}$ .

If at  $t = 2$  the pessimistic young buyers continue, only old buyers accept low offers at  $t = 2$  and the trade probability in state  $\theta$  is given by (A7) where  $t = 2$ . The solution is  $\tau_2^\theta = (2 - \mu^\theta)^{-1}$ . Thus, a trade at  $t = 2$  is good news, but not as good news as at  $t = 1$ . The pessimistic young buyers do not want to deviate at  $t = 2$  if  $\pi(v_L, N_1) > \bar{\pi}$ . The posterior odds are given by (A5) where  $t = 2$ , explicitly,  $\frac{\pi(v_L, N_1)}{1 - \pi(v_L, N_1)} = \omega (1 - \mu^G)^2$ . The inequality  $\pi(v_L, N_1) > \bar{\pi}$  is thus equivalent to  $\omega > \bar{\omega} (1 - \mu^G)^{-2}$ . Since all young buyers who get a low offer reject it and continue, the amount of old buyers at  $t = 3$  is  $O_3^\theta = 1 - \mu^\theta$ .

Consider  $t = 3$  and recall that a trade at  $t = 2$  is good news as  $\tau_2^G > \tau_2^B$ . At  $t = 3$ , the pessimistic young buyers' posterior odds are given by (A5) where  $t = 3$ . The odds are explicitly  $\frac{\pi(v_L, N_2)}{1 - \pi(v_L, N_2)} = \omega(1 - \mu^G) \frac{2}{2 - \mu^G}$ , which are higher than for the pessimistic young buyers at  $t = 2$ . Thus, at  $t = 3$  the pessimistic young buyers continue, the trade probabilities are exactly like at  $t = 2$ , and steady state 2 is reached at  $t = 2$ . The condition that ensures convergence to steady state 2 is  $\omega > \bar{\omega} (1 - \mu^G)^{-2}$ .

(b) Assume now that a young buyer who gets a low offer and sees bad news at  $t = 2$ , no trade, accepts the offer. We know from Part (a) that the necessary and sufficient condition for this to be optimal is that

$$(A13) \quad \omega < \bar{\omega} (1 - \mu^G)^{-2}.$$

I show that if this condition holds, the market converges to the cyclical equilibrium for an open set of parameter values. Convergence to the long-run value is immediate in the bad state, but not in the good state.

I show that the trade probabilities converge to the trading probabilities of the cyclical equilibrium (in Proposition 4). If that is the case, a necessary and sufficient condition for the market to converge to the cyclical equilibrium is that no buyer wants to deviate, i.e., that the most optimistic (pessimistic) of the young buyers who at any  $t$  is supposed to accept (reject) a low offer optimally does so.

First, I show for all  $t \geq 1$  that a trade that takes place in an odd period  $2t + 1$  is good news for any  $\mu^G$  because  $\tau_1^G > \tau_1^B = 0$ . In particular, I show that  $\tau_1^G > \tau_1^B$  implies that  $\frac{\tau_{2t+1}^G}{\tau_{2t+1}^B} > 1$  for any  $t \geq 1$ . Equations (A4) and (A7) together imply that in an odd period  $2t + 1$ , the trade probability can be written as

$$(A14) \quad \tau_{2t+1}^\theta = 1 - (1 - \mu^\theta)[1 + (1 - \mu^\theta)\tau_{2t-1}^\theta]^{-1}.$$

Thus, in any odd period  $\tau_{2t+1}^B = 0$  and  $\tau_{2t+1}^G > \tau_{2t+1}^B$ . From (A3) and (A8) it then also follows that  $\tau_{2t}^B = 1$  for all  $t$ . So I only need to show that the good-state

trade probabilities converge.

Second, I show that  $\tau_{2t+1}^G$  and  $\tau_{2t}^G$  as sequences in  $t$  converge respectively to  $\tau_{odd}^G$  and  $\tau_{even}^G$ , the trade probabilities of the cyclical equilibrium in a market without a starting date. From (A14) it follows that  $\tau_{2t+1}^G \geq \tau_{2t-1}^G$  if  $\mu^G(1 - \tau_{2t-1}^G) \geq (1 - \mu^G)(\tau_{2t-1}^G)^2$ , which holds with strict inequality for  $\tau_{2t-1}^G = \tau_1^G$  and with equality for  $\tau_{2t-1}^G = \tau_{odd}^G$ . So we know that  $\tau_3^G > \tau_1^G$ . But this implies that  $\tau_{2t+1}^G > \tau_{2t-1}^G$  for all  $t$  because  $\tau_{2t+1}^G$  increases in  $\tau_{2t-1}^G$ :  $\frac{\partial \tau_{2t+1}^G}{\partial \tau_{2t-1}^G} = (1 - \mu^G)^2 [1 + (1 - \mu^G)\tau_{2t-1}^G]^{-2} > 0$ . Also,  $\tau_{2t+1}^G/\tau_{2t-1}^G$  decreases in  $\tau_{2t-1}^G$ :

$$\frac{\partial}{\partial \tau_{2t-1}^G} \frac{\tau_{2t+1}^G}{\tau_{2t-1}^G} \propto -2\tau_{2t-1}^G \mu^G (1 - \mu^G) - \mu^G - (\tau_{2t-1}^G)^2 (1 - \mu^G)^2 < 0.$$

Thus, the sequence  $\tau_{2t+1}^G$  converges in  $t$  to  $\tau_{odd}^G$ . Note that from (A3) and (A8) it follows that for any even period  $2t$ ,

$$\tau_{2t}^G = 1 - (1 - \mu^G)(2 - \mu^G)^{-1} \tau_{2t-1}^G,$$

and we know that in the cyclical equilibrium in a market without a starting date,  $\tau_{even}^G = 1 - (1 - \mu^G)(2 - \mu^G)^{-1} \tau_{odd}^G$ . Thus,  $\tau_{2t}^G$  converges to  $\tau_{even}^G$ . Also,  $\frac{\tau_{2t}^G}{\tau_{2t}^B} < 1$  for all  $t$  because  $\tau_{2t}^B = 1$  and  $\tau_{odd}^G \in (0, 1)$ .

Finally, I derive the conditions under which the most optimistic (pessimistic) of the young buyers who along the path is supposed to accept (reject) the low offer optimally does so. I need that a young buyer who gets a low offer and sees no trade of any odd period wants to accept the offer. The most optimistic of these buyers is the one who sees no trade of the first period because  $1 - \tau_{2t-1}^G > 1 - \tau_{2t+1}^G$  and  $1 - \tau_{2t+1}^B = 1$  for all  $t$ . So all young buyers who get a low offer and see no trade of an odd period want to accept the offer if the condition  $\frac{\pi(v_L, N_1)}{1 - \pi(v_L, N_1)} \leq \bar{\omega}$ , or, equivalently, (A13), holds.

I also need that a young buyer who gets a low offer and sees a trade of any even period wants to continue. But because  $\tau_{2t}^B = 1$ ,  $\tau_{2t}^G$  decreases in  $\tau_{2t-1}^G$ , and  $\tau_{2t-1}^G < \tau_{2t+1}^G$ , a buyer is the more pessimistic the later-period trade she sees. Thus, all young buyers who get a low offer and see a trade of an even period want to continue if the limit condition  $\frac{\pi(v_L, T_{even})}{1 - \pi(v_L, T_{even})} = \omega(1 - \mu^G)\tau_{even}^G \geq \bar{\omega}$  holds, or, equivalently, if

$$\omega \geq \bar{\omega} \frac{1}{1 - \mu^G} \frac{1}{\tau_{even}^G}.$$

This concludes the proof. ■

## APPENDIX B

Appendix B contains details on the extensions presented in Section VI.

B1. Positive fraction of high offers in bad state

Consider the main model, except that a positive fraction of offers are good also in the bad state:  $\mu^B \in [0, \mu^G)$ .

PROPOSITION 7: Consider a strategy whereby a young buyer who gets a low offer  $v_L$

- (i) in an odd period, rejects the offer, and
- (ii) in an even period, accepts the offer after observing no trade and rejects the offer after observing a trade.

The necessary and sufficient conditions for the strategy profile to be an equilibrium are that

- 1) a trade in an odd period is good news ( $\tau_{odd}^G > \tau_{odd}^B$ ).
- 2) no buyer wants to deviate, i.e., that

$$\frac{\bar{\pi}}{1 - \bar{\pi}} \frac{1 - \mu^B}{1 - \mu^G} \frac{\tau_{even}^B}{\tau_{even}^G}, \frac{\bar{\pi}}{1 - \bar{\pi}} \frac{1 - \mu^B}{1 - \mu^G} \frac{1 - \tau_{even}^B}{1 - \tau_{even}^G} < \frac{\pi}{1 - \pi} < \frac{\bar{\pi}}{1 - \bar{\pi}} \frac{1 - \mu^B}{1 - \mu^G} \frac{1 - \tau_{odd}^B}{1 - \tau_{odd}^G},$$

where the probabilities of trading are  $\tau_{odd}^\theta = \frac{\sqrt{(4-3\mu^\theta)\mu^\theta - \mu^\theta}}{2(1-\mu^\theta)}$  and  $\tau_{even}^\theta = 1 - (2 - \mu^\theta)^{-1}(1 - \mu^\theta)\tau_{odd}^\theta$  for  $\theta = B, G$ .

In the cyclical equilibrium, the volume of trading fluctuates more in the bad state than in the good state.

One sufficient condition for the equilibrium to exist for an open set of parameter values is that  $\mu^G < \bar{\mu} := \frac{2}{7}(3 - \sqrt{2})$ . Another is that  $\mu^B = 0$ .

PROOF:

See the proof on p. 4. ■

In general, a trade in an even period does not have to be bad news about the state for the cyclical equilibrium to exist. However, the trade signal from an even period needs to be sufficiently uninformative so that young buyers in an odd period optimally ignore its outcome.

The first sufficient condition guarantees that a trade is bad news in an even period:  $\tau_{even}^\theta$  decreases in  $\mu^\theta$  for all  $\mu^\theta < \bar{\mu}$ . But both sufficient conditions are much stronger than the necessary and sufficient conditions. Broadly, the cyclical equilibrium also exists for an open set of parameter values if  $\mu^B$  is close to zero and  $\mu^G$  is not too close to one (Figure A1 on p. 1 depicts  $\tau_{odd}^\theta$  and  $\tau_{even}^\theta$ ).

PROPOSITION 8: In the region of the parameter space where the cyclical equilibrium coexists with

- (i) the steady state where a young buyer rejects a low offer  $v_L$  (steady state 2), the cyclical equilibrium is more efficient.
- (ii) the steady state where a young buyer rejects a low offer  $v_L$  only after a trade (steady state 1), two sufficient conditions for the cyclical equilibrium to be more efficient are  $\mu^B \leq \hat{\mu} = \frac{2-\sqrt{3}}{3}$  and  $\mu^G \leq \check{\mu} = \frac{5-\sqrt{10}}{6}$ .

PROOF:

The cyclical equilibrium is more efficient than a steady state if the expected cost of a young buyer's mistake with respect to a low offer (or, shortfall of the equilibrium from the complete-info benchmark) is smaller. The shortfall in the cyclical equilibrium is

$$\Delta W_c = \pi(1 - \mu^G) \frac{1 - \tau_{odd}^G}{2} (V^G - v_L) + (1 - \pi)(1 - \mu^B) \frac{1 + \tau_{odd}^B}{2} (v_L - V^B),$$

in steady state 2 is

$$\Delta W_{ss2} = (1 - \pi)(1 - \mu^B)(v_L - V^B),$$

and in steady state 1 is

$$\Delta W_{ss1} = \pi(1 - \mu^G)(1 - \tau^G)(V^G - v_L) + (1 - \pi)(1 - \mu^B)\tau^B(v_L - V^B).$$

The shortfall is smaller in the cyclical equilibrium than in steady state 2 if

$$\omega_L(1 - \tau_{odd}^G) < \bar{\omega}(1 - \tau_{odd}^B),$$

where I have denoted  $\omega_L := \omega \frac{1 - \mu^G}{1 - \mu^B}$  and used that  $\bar{\omega} = \frac{v_L - V^B}{V^G - v_L}$ . But this inequality has to hold for the cyclical equilibrium to exist.

The shortfall is smaller in the cyclical equilibrium than in steady state 1 if

$$(B1) \quad \bar{\omega} (1 - 2\tau^B + \tau_{odd}^B) < \omega_L (1 - 2\tau^G + \tau_{odd}^G).$$

I derive two sufficient conditions for this inequality to hold. After the proof, I show numerically that another sufficient condition holds for all parameter values.

Note first that  $\bar{\omega} \leq \omega_L$  must hold for the cyclical equilibrium to exist because two of the necessary conditions are  $\bar{\omega} \leq \omega_L \frac{1 - \tau_{even}^G}{1 - \tau_{even}^B}$ ,  $\omega_L \frac{\tau_{even}^G}{\tau_{even}^B}$  and either  $\frac{1 - \tau_{even}^G}{1 - \tau_{even}^B} \leq 1$  or  $\frac{\tau_{even}^G}{\tau_{even}^B} \leq 1$ . Since  $\bar{\omega} \leq \omega_L$ , a sufficient condition for equation (B1) to hold is that  $1 - 2\tau^B + \tau_{odd}^B < 1 - 2\tau^G + \tau_{odd}^G$ . I first derive the condition under which the LHS of this inequality is negative and the RHS is positive. Using the explicit solutions for  $\tau^\theta$  and  $\tau_{odd}^\theta$ , and rearranging,  $1 - 2\tau^\theta + \tau_{odd}^\theta = 0$  can be shown to be equivalent to  $1 - 12\mu^\theta + 9(\mu^\theta)^2 = 0$  which has a unique solution in  $(0, 1)$ :

$\hat{\mu} = \frac{2-\sqrt{3}}{3}$ . Since  $\lim_{\mu^\theta \rightarrow 0} 1 - 2\tau^\theta + \tau_{odd}^\theta < 0$  and  $\lim_{\mu^\theta \rightarrow 1} 1 - 2\tau^\theta + \tau_{odd}^\theta = 0$ , we know that  $1 - 2\tau^B + \tau_{odd}^B < 1 - 2\tau^G + \tau_{odd}^G$  holds for sure if  $\mu^B < \hat{\mu}$ .

Now I derive a sufficient condition for  $1 - 2\tau^\theta + \tau_{odd}^\theta$  to increase in  $\mu^\theta$ . The expression  $-2(\tau^\theta)' + (\tau_{odd}^\theta)' > 0$  is equivalent to

$$\frac{1}{2}(1 - \mu^\theta)^{-2}[1 - 2A + 4(1 - \mu^\theta)A^{-1} + B + (1 - \mu^\theta)(2 - 3\mu^\theta)B^{-1}] > 0,$$

where I let  $A := \sqrt{5 - 4\mu^\theta}$  and  $B := \sqrt{(4 - 3\mu^\theta)\mu^\theta}$ . Note that  $A > 0$  and  $B > 0$  for all  $\mu^\theta > 0$ . Multiplying both sides of the inequality by  $2(1 - \mu^\theta)^2 AB$  and rearranging gives that  $-2(\tau^\theta)' + (\tau_{odd}^\theta)' > 0$  is equivalent to

$$C := AB - 2A^2B + 4(1 - \mu^\theta)B + AB^2 + (1 - \mu^\theta)(2 - 3\mu^\theta)A > 0.$$

I show that the condition holds if  $\mu^\theta < \check{\mu} := \frac{5-\sqrt{10}}{6}$ . Substituting  $A^2 = 5 - 4\mu^\theta$  and  $B^2 = (4 - 3\mu^\theta)\mu^\theta$  into the inequality and rearranging gives that  $C > 0$  is equivalent to

$$B[A - 2(1 - \mu^\theta)] + (2 - \mu^\theta)(A - 2B) > 0.$$

As  $A > 2(1 - \mu^\theta)$  for all  $\mu^\theta$  and  $A > 2B$  for  $\mu^\theta < \check{\mu} = \frac{5-\sqrt{10}}{6}$ , a sufficient condition for  $1 - 2\tau^\theta + \tau_{odd}^\theta$  to increase in  $\mu^\theta$  is that  $\mu^\theta \leq \check{\mu}$ . ■

The efficiency comparison between steady state 2 and the cyclical equilibrium is the same as in the main part of the paper. As compared to steady state 1, for some parameter values, across two periods young buyers make a certain mistake with respect to the low offer (either accept it if the state is good or reject it if the state is bad) more often in the cyclical equilibrium. But the cost of this mistake must be small enough for the cyclical equilibrium to exist, which makes the cyclical equilibrium more efficient than steady state 1. Both types of mistake are made with positive probability in the cyclical equilibrium and for any buyer to be ex ante willing to make either mistake, its cost must be low enough. The cyclical equilibrium is, as a result, on average across the states more efficient than steady state 1.

My numerical results suggest that the cyclical equilibrium is more efficient than steady state 1 for all parameter values, but I have not been able to show the general result analytically. The numerical condition that I plot in Figure B1 shows that  $(1 - \tau_{even}^\theta)^{-1}(1 - 2\tau^\theta + \tau_{odd}^\theta)$  increases in  $\mu^\theta$ . This is a sufficient condition for the cyclical equilibrium to be more efficient than steady state 1 when they coexist because  $\bar{\omega} < \omega_L \frac{1 - \tau_{even}^G}{1 - \tau_{even}^B}$  must be satisfied for the cyclical equilibrium to exist. Thus, if  $\omega_L \frac{1 - \tau_{even}^G}{1 - \tau_{even}^B} (1 - 2\tau^B + \tau_{odd}^B) < \omega_L (1 - 2\tau^G + \tau_{odd}^G)$  holds, then equation (B1) holds for sure. Unfortunately, I am not able to prove this inequality analytically. To show that the derivative  $\frac{\partial}{\partial \mu^\theta} (1 - \tau_{even}^\theta)^{-1}(1 - 2\tau^\theta + \tau_{odd}^\theta)$  is positive, all terms of the derivative seem to be crucial.

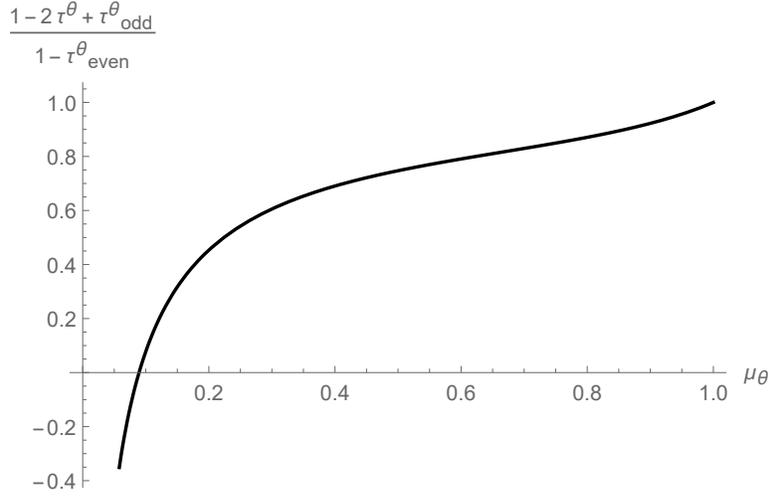


Figure B1: A sufficient condition for the cyclical equilibrium to be more efficient than steady state 1 is satisfied if the curve,  $(1 - \tau_{\text{even}}^\theta)^{-1}(1 - 2\tau^\theta + \tau_{\text{odd}}^\theta)$ , increases everywhere.

### B2. Only value of an accepted offer is observed

Consider the same model as in the main part of the paper with the modification that the signal that a buyer observes tells her the value of the offer that one randomly drawn buyer accepted yesterday. Let the fraction of accepted offers that are low in period  $t$  and state  $\theta$  be denoted  $\alpha_t^\theta$ . I denote the posterior odds of a period- $t$  young buyer who gets a low offer by  $\frac{\pi(L_{t-1})}{1-\pi(L_{t-1})} := \omega_L \frac{\alpha_{t-1}^G}{\alpha_{t-1}^B}$  if she observes that a low offer was accepted at  $t-1$  and by  $\frac{\pi(H_{t-1})}{1-\pi(H_{t-1})} := \omega_L \frac{1-\alpha_{t-1}^G}{1-\alpha_{t-1}^B}$ , if she observes that a high offer was accepted at  $t-1$  for  $t = \text{odd}, \text{even}$ , where  $\omega_L := \frac{\pi}{1-\pi} \frac{1-\mu^G}{1-\mu^B}$ . Since  $\mu^B = 0$ , seeing that a high offer was accepted reveals the good state so that  $\pi(H_{t-1}) = 1$  and  $\alpha_t^B = 1$  for all  $t$ . I construct a cyclical equilibrium and show that when it coexists with a steady state, the cyclical equilibrium is more efficient.

The following steady states exist in this model.

PROPOSITION 9: *A strategy whereby a young buyer who gets a low offer  $v_L$*

(i) *accepts the offer after observing that  $v_L$  was accepted and rejects the offer after observing that  $v_H$  was accepted (steady state 1) is an equilibrium if  $\frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1}{\alpha^G}$ , where  $\alpha^G = \frac{(1-\mu^G)^2}{1-\mu^G(1-\mu^G)}$ .*

(ii) *rejects the offer (steady state 2) is an equilibrium if  $\frac{\bar{\pi}}{1-\bar{\pi}} \frac{1}{(1-\mu^G)^3} < \frac{\pi}{1-\pi}$ .*

PROOF:

I derive the conditions under which the two strategy profiles constitute an equilibrium.

(ii) Steady state 2: If all young buyers reject the low offer, all young buyers become old:  $O^\theta = 1 - \mu^\theta$ . The probability that a randomly drawn accepted offer was low is equal to  $\alpha^\theta = (1 - \mu^\theta)O^\theta = (1 - \mu^\theta)^2$  because the total amount of accepted offers in a steady state is one. Then  $\alpha^B = 1 > \alpha^G$ .

The strategy is optimal for young buyers if even the pessimistic young buyer wants to reject the low offer, i.e., if

$$\omega_L \alpha^G = \omega_L (1 - \mu^G)^2 > \bar{\omega}.$$

The equilibrium exists for an open set of parameter values.

(i) Steady state 1: Only young buyers who see that  $v_H$  was accepted reject a low offer and become old:  $O^\theta = (1 - \mu^\theta)(1 - \alpha^\theta)$ . The probability that a randomly drawn accepted offer was low is equal  $\alpha^\theta = (1 - \mu^\theta)(O^\theta + \alpha^\theta) = (1 - \mu^\theta)[(1 - \mu^\theta)(1 - \alpha^\theta) + \alpha^\theta]$ . Solving for  $\alpha^\theta$  gives  $\alpha^\theta = \frac{(1 - \mu^\theta)^2}{1 - \mu^\theta(1 - \mu^\theta)}$  so that  $\alpha^B = 1 > \alpha^G$ .

The strategy is optimal for young buyers if the pessimistic young buyer wants to accept the low offer and the optimistic one to reject it, i.e., if  $\omega_L \alpha^G < \bar{\omega} < +\infty$ . The equilibrium exists for an open set of parameter values. ■

I construct a cyclical equilibrium and show that when it coexists with a steady state, the cyclical equilibrium is more efficient.

PROPOSITION 10: *A strategy whereby a young buyer who gets a low offer  $v_L$*

*(i) in an odd period, rejects the offer, and*

*(ii) in an even period, accepts the offer after observing a trade at  $v_L$  and rejects the offer after observing a trade at  $v_H$ ,*

*is an equilibrium for an open set of parameter values.*

*In the cyclical equilibrium, the volume of trading fluctuates more in the bad state than in the good state.*

*The cyclical equilibrium is more efficient than the coexistent steady state.*

PROOF:

If young buyers use the above strategy, then the amount of old buyers at the start of an even period is  $O_{even}^\theta = 1 - \mu^\theta$ , and at the start of an odd period is  $O_{odd}^\theta = (1 - \mu^\theta)(1 - \alpha_{odd}^\theta)$ . In an odd period, the fraction of the accepted offers that are low is

$$\alpha_{odd}^\theta = \frac{(1 - \mu^\theta)O_{odd}^\theta}{O_{odd}^\theta + \mu^\theta},$$

because in an odd period only old buyers accept low offers and the total number of offers accepted is the number of low and high offers accepted. In an even period,

the fraction of the accepted offers that are low is

$$\alpha_{even}^\theta = \frac{(1 - \mu^\theta)(O_{even}^\theta + \alpha_{odd}^\theta)}{O_{even}^\theta + \mu^\theta + (1 - \mu^\theta)\alpha_{odd}^\theta},$$

because in an even period low offers are accepted by old buyers and by young buyers who observe that an accepted offer from yesterday was low. The total number of offers accepted is the number of low and high offers accepted. Note that  $\alpha_{odd}^B = \alpha_{even}^B = 1$ .

I show that an open set of parameter values exists for which no buyer wants to deviate, or, the following inequalities hold:

$$\frac{\pi(H_{even})}{1 - \pi(H_{even})}, \frac{\pi(L_{even})}{1 - \pi(L_{even})}, \frac{\pi(H_{odd})}{1 - \pi(H_{odd})} > \frac{\bar{\pi}}{1 - \bar{\pi}} > \frac{\pi(L_{odd})}{1 - \pi(L_{odd})}.$$

Since observing that a high offer was accepted reveals the good state, a sufficient condition for an open set of parameter values to exist such that these inequalities hold is that  $\frac{\pi(L_{even})}{1 - \pi(L_{even})} = \omega_L \alpha_{even}^G > \frac{\pi(L_{odd})}{1 - \pi(L_{odd})} = \omega_L \alpha_{odd}^G$ , or,  $\alpha_{even}^G > \alpha_{odd}^G$ .

I show that  $\alpha_{even}^\theta > \alpha_{odd}^\theta$  for all  $\mu^\theta > 0$ . I first rewrite  $\alpha_{odd}^\theta$  as

$$\alpha_{odd}^\theta = \frac{(1 - \mu^\theta)^2(1 - \alpha_{odd}^\theta)}{1 - (1 - \mu^\theta)\alpha_{odd}^\theta},$$

and  $\alpha_{even}^\theta$  as

$$\alpha_{even}^\theta = \frac{(1 - \mu^\theta)(1 - \mu^\theta + \alpha_{odd}^\theta)}{1 + (1 - \mu^\theta)\alpha_{odd}^\theta}.$$

Then I plug the expressions on the RHSs into the inequality and rearrange to get that  $\alpha_{even}^\theta > \alpha_{odd}^\theta$  is equivalent to

$$\mu^\theta(1 - \mu^\theta) + 1 - (1 - \mu^\theta)^2 > \mu^\theta(1 - \mu^\theta)\alpha_{odd}^\theta,$$

which holds for all  $\mu^\theta > 0$ .

I now show that this cyclical equilibrium never coexists with steady state 1. Note that  $\alpha_{ss1}^\theta = \alpha_{even}^\theta|_{\alpha_{odd}^\theta = \alpha_{ss2}^\theta}$ . Since  $\alpha_{even}^\theta$  increases in  $\alpha_{odd}^\theta$  and  $\alpha_{odd}^\theta < \alpha_{ss2}^\theta$ , then  $\alpha_{ss1}^\theta > \alpha_{even}^\theta$ . Then for  $\mu^B = 0$ , the cyclical equilibrium and steady state 1 do not coexist.<sup>2</sup>

The cyclical equilibrium is more efficient than steady state 2 if they coexist.<sup>3</sup>

<sup>2</sup>My numerical results suggest that this holds for general  $\mu^B$ .

<sup>3</sup>I can show this result, and that the cyclical equilibrium is also more efficient than steady state 0 if they coexist, for general  $\mu^B$ .

The shortfall in steady state 2 is

$$\Delta W_{ss2} = (1 - \pi)(v_L - V^B),$$

and in the cyclical equilibrium is

$$\Delta W_c = \pi(1 - \mu^G)(V^G - v_L) \frac{\alpha_{odd}^G}{2} + (1 - \pi)(v_L - V^B) \frac{1}{2}.$$

The cyclical equilibrium is more efficient if its shortfall is smaller, or if

$$\omega_L \alpha_{odd}^G < \bar{\omega},$$

which must be satisfied for the cyclical equilibrium to exist. ■

### B3. Signals on past trading volume(s)

I show here, for general  $\mu^B$ , that if buyers have access to an exogenous noisy binary signal about past trading volume(s), then all fluctuating equilibria are more efficient than the coexistent steady states.<sup>4</sup> I first show that only steady state 0 (where young buyers who get the low offer accept it regardless of the observed signal outcome) and steady state 2 (where young buyers who get the low offer reject it regardless of the observed signal outcome) exist. I then show that if these steady states coexist with equilibria that feature fluctuations in trading volume, the equilibria with fluctuations are more efficient than the steady states.

Let buyers observe an outcome of a signal which generates one with some probability that is a function of past trading volume(s) and zero otherwise:  $P(s_t = 1|\theta) = p(\text{Vol}_{t-1}^\theta, \dots, \text{Vol}_{t-k}^\theta)$  and  $P(s_t = 0|\theta) = 1 - p(\text{Vol}_{t-1}^\theta, \dots, \text{Vol}_{t-k}^\theta)$ , for some finite positive  $k$ , where  $\text{Vol}_t$  is the trading volume in period  $t$  and  $p(\cdot)$  is a function that is independent of  $\theta$  and  $p(\cdot) \in (0, 1)$ .

**PROPOSITION 11:** *If buyers learn about the state from an exogenous noisy binary signal about past trading volume(s), then all fluctuating equilibria are more efficient than the coexistent steady states.*

**PROOF:**

First, I show that in the only steady states that exist in such a model, the signal outcome is ignored. Because the amount on entering buyers is independent of the state  $\theta$ , in a steady state the volume of trading is the same in all periods

<sup>4</sup>For the sake of (relative) brevity, I do not provide here the calculations that prove the existence of a fluctuating equilibrium for a concrete example from this class of signals. The simplest signal of the class is a noisy signal about yesterday's trading volume, with exogenous precision  $p(\text{Vol}_{t-1})$ , increasing  $p$ . I have explicit calculations showing that an open set of parameter values supports an equilibrium with fluctuations in the volume of trading if the precision function satisfies certain conditions (which is approximately that  $p$  is concave).

and both states. Thus, the signal about past trading volume(s) is uninformative and is optimally ignored by young buyers.<sup>5</sup> So buyers face the same situation as when they do not have access to the signal. If the buyers' prior is low enough ( $\frac{\pi}{1-\pi} \frac{1-\mu^G}{1-\mu^B} < \frac{\bar{\pi}}{1-\bar{\pi}}$ ), a young buyer accepts a low offer in the unique steady state (steady state 0). If the buyers' prior is high enough ( $\frac{\pi}{1-\pi} \frac{1-\mu^G}{1-\mu^B} > \frac{\bar{\pi}}{1-\bar{\pi}}$ ), a young buyer rejects a low offer in the unique steady state (steady state 2). These are the only two steady states that (generically) exist when buyers have access to a signal about past trading volume(s).

The inefficiency of an equilibrium can be measured by its shortfall from the complete-information benchmark,  $\Delta W$ . The shortfall measures the cost of a young buyer taking the "wrong" action with respect to the low offer in the equilibrium as compared to the complete-information benchmark: of accepting the low offer if the state is good and of rejecting it if the state is bad. The shortfall in steady state 0 is

$$\Delta W_{ss0} = \pi(1 - \mu^G)(V^G - v_L).$$

A young buyer takes the wrong action with respect to a low offer in steady state 0 with probability one if the state is good and never if the state is bad. The shortfall in steady state 2 is

$$\Delta W_{ss2} = (1 - \pi)(1 - \mu^B)(v_L - V^B),$$

because a young buyer takes the wrong action with respect to a low offer in steady state 2 with probability one if the state is bad and never if the state is good.

Now let us suppose that a Markovian fluctuating equilibrium exists where a young buyer who gets a low offer

- (i) in periods  $t \in \mathcal{T}_+$ , rejects it regardless of the signal outcome,
- (ii) in periods  $t \in \mathcal{T}_-$ , accepts it regardless of the signal outcome,
- (iii) in periods  $t \in \mathcal{T}_1$ , rejects it after observing a one and accepts it after observing a zero, and
- (iv) in the remaining periods, periods  $t \in \mathcal{T}_0$ , rejects it after observing a zero and accepts it after observing a one.

In the below arguments, any one of the sets  $\mathcal{T}_+$ ,  $\mathcal{T}_-$ ,  $\mathcal{T}_1$  or  $\mathcal{T}_0$  can be empty, but at least two must be nonempty for the equilibrium to feature fluctuations. Let the set of the signal's precisions in the respective periods be denoted by  $\mathcal{P}_+$ ,  $\mathcal{P}_-$ ,  $\mathcal{P}_1$  and  $\mathcal{P}_0$ , and  $p_+$ ,  $p_-$ ,  $p_1$  and  $p_0$  denote generic elements from the respective sets.

<sup>5</sup>Note that this means that, if signals are about past trading volume(s), no steady state exists that is like the one I call steady state 1 in my main model.

The described behaviour is optimal for the young buyers if, for all  $p_+^\theta \in \mathcal{P}_+^\theta$ ,

$$\omega_L \frac{p_+^G}{p_+^B}, \omega_L \frac{1-p_+^G}{1-p_+^B} > \bar{\omega};$$

for all  $p_-^\theta \in \mathcal{P}_-^\theta$ ,

$$\bar{\omega} > \omega_L \frac{p_-^G}{p_-^B}, \omega_L \frac{1-p_-^G}{1-p_-^B};$$

for all  $p_1^\theta \in \mathcal{P}_1^\theta$ ,

$$\omega_L \frac{p_1^G}{p_1^B} > \bar{\omega} > \omega_L \frac{1-p_1^G}{1-p_1^B};$$

and, for all  $p_0^\theta \in \mathcal{P}_0^\theta$ ,

$$\omega_L \frac{1-p_0^G}{1-p_0^B} > \bar{\omega} > \omega_L \frac{p_0^G}{p_0^B},$$

where I have denoted  $\omega_L := \frac{\pi}{1-\pi} \frac{1-\mu^G}{1-\mu^B}$  and  $\bar{\omega} := \frac{\pi}{1-\pi}$  for brevity.

This equilibrium coexists only with steady state 2 if  $\mathcal{T}_+$  is nonempty (because either  $\frac{p_+^G}{p_+^B} < 1$  or  $\frac{1-p_+^G}{1-p_+^B} < 1$ ) and only with steady state 0 if  $\mathcal{T}_-$  is nonempty (because either  $\frac{p_-^G}{p_-^B} > 1$  or  $\frac{1-p_-^G}{1-p_-^B} > 1$ ).

To show that this fluctuating equilibrium is more efficient than both steady states, I show that the welfare shortfall of the fluctuating equilibrium is weakly smaller in all  $t$  and strictly smaller in some  $t$  than in the steady states. Let the shortfall in the fluctuating equilibrium in periods  $t \in \mathcal{T}_x$  be denoted  $\Delta W_f^x$ . Then the shortfall in  $t \in \mathcal{T}_+$  is

$$\Delta W_f^+ = (1-\pi)(1-\mu^B)(v_L - V^B);$$

in  $t \in \mathcal{T}_-$  is

$$\Delta W_f^- = \pi(1-\mu^G)(V^G - v_L);$$

in  $t \in \mathcal{T}_1$  is

$$\Delta W_f^1 = \pi(1-\mu^G)(V^G - v_L)(1-p_1^G) + (1-\pi)(1-\mu^B)(v_L - V^B)p_1^B;$$

and in  $t \in \mathcal{T}_0$  is

$$\Delta W_f^0 = \pi(1-\mu^G)(V^G - v_L)p_0^G + (1-\pi)(1-\mu^B)(v_L - V^B)(1-p_0^B),$$

because the inefficient action is to accept the low offer if the state is good and to reject it if the state is bad.

I show that if the fluctuating equilibrium exists, it is weakly more efficient than

steady state 0 in all periods and strictly more efficient in some periods. In all  $t \in \mathcal{T}_-$ ,  $\Delta W_f^- = \Delta W_{ss0}$ . In all  $t \in \mathcal{T}_1$ , showing that  $\Delta W_f^1 < \Delta W_{ss0}$  is equivalent to showing that

$$\omega_L(1 - p_1^G) + \bar{\omega}p_1^B < \omega_L,$$

where I have used that  $\bar{\omega} = \frac{v_L - V^B}{V^G - v_L}$ . But this condition has to hold for the fluctuating equilibrium to exist if  $\mathcal{T}_1$  is nonempty. In all  $t \in \mathcal{T}_0$ , showing that  $\Delta W_f^0 < \Delta W_{ss0}$  is equivalent to showing that

$$\omega_L p_0^G + \bar{\omega}(1 - p_0^B) < \omega_L,$$

which has to hold for the fluctuating equilibrium to exist if  $\mathcal{T}_0$  is nonempty. Since at least two of the sets  $\mathcal{T}_-$ ,  $\mathcal{T}_1$  and  $\mathcal{T}_0$  must be nonempty for the fluctuating equilibrium to coexist with steady state 0, the fluctuating equilibrium is strictly more efficient than steady state 0.

I show by an analogous argument that the fluctuating equilibrium is weakly more efficient than steady state 2 in all periods and strictly more efficient in some periods. In all  $t \in \mathcal{T}_+$ ,  $\Delta W_f^+ = \Delta W_{ss2}$ . In all  $t \in \mathcal{T}_1$ , showing that  $\Delta W_f^1 < \Delta W_{ss2}$  is equivalent to showing that

$$\omega_L(1 - p_1^G) + \bar{\omega}p_1^B < \bar{\omega}.$$

This condition has to hold for the fluctuating equilibrium to exist if  $\mathcal{T}_1$  is nonempty. In all  $t \in \mathcal{T}_0$ , showing that  $\Delta W_f^0 < \Delta W_{ss2}$  is equivalent to showing that

$$\omega_L p_0^G + \bar{\omega}(1 - p_0^B) < \bar{\omega},$$

which has to hold for the fluctuating equilibrium to exist if  $\mathcal{T}_0$  is nonempty. Since at least two of the sets  $\mathcal{T}_+$ ,  $\mathcal{T}_1$  and  $\mathcal{T}_0$  must be nonempty for the fluctuating equilibrium to coexist with steady state 2, the fluctuating equilibrium is strictly more efficient than steady state 2 when they coexist. ■

#### B4. Long-lived buyers

Consider a model where a mass one of buyers enters in each period and each buyer can live for ever, but survives till the next period with a fixed probability  $\delta \in (0, 1)$ . The survival probability replaces the discount factor and ensures the existence of a steady-state equilibrium. A buyer observes in each period of life whether a randomly drawn buyer traded in the previous period or did not trade. That is, the buyer can tell trades apart from exits due to the exogenous destruction rate. In this version of the model, some buyers learn the state if the state is good so the equilibrium strategy must specify their behaviour. A cyclical equilibrium is sustained by an open set of parameter values. I let  $\mu^G = \frac{1}{2}$  for brevity in this subsection, but the assumption is not necessary for the result.

PROPOSITION 12: *Sufficient conditions for a strategy whereby a buyer who gets a low offer  $v_L$ ,*

- (i) *and knows that the state is good, rejects the offer,*
- (ii) *and does not know the state,*
  - *in odd periods, rejects the offer and*
  - *in even periods, rejects the offer after observing a trade and accepts the offer after observing no trade,*

*to be an equilibrium are that  $\frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{\tau_{even}^B}{\tau_{even}^G} < \frac{\pi}{1-\pi} < \frac{\bar{\pi}}{1-\bar{\pi}} \frac{1-\mu^B}{1-\mu^G} \frac{1-\tau_{odd}^B}{1-\tau_{odd}^G}$ , where the probabilities of trading are  $\tau_{odd}^B = 0$ ,  $\tau_{even}^B = 1$ ,  $\tau_{odd}^G = 1/2$ , and*

$$\tau_{even}^G = \frac{1}{2\delta^3} \left( 16 + 6\delta - 2\delta^2 + \delta^3 - \sqrt{256 + 192\delta - 28\delta^2 - 40\delta^3 + 4\delta^5 + \delta^6} \right).$$

*In the cyclical equilibrium, a trade in an odd period is good news and a trade in an even period is bad news. The volume of trading fluctuates more in the bad than in the good state.*

PROOF:

I construct an equilibrium where a buyer who gets a low offer

- (i) rejects it if her posterior belief is  $\pi' = 1$ ,
- (ii) in any odd period  $2t + 1$ , rejects it if her posterior belief is  $\pi' < 1$  (regardless of whether she sees  $N_{2t}$  or  $T_{2t}$ ), and
- (iii) in any even period  $2t$ , rejects it after observing  $T_{2t-1}$ , and accepts it after observing  $N_{2t-1}$  if her posterior belief is  $\pi' < 1$ .

I first derive the probability of trading and the amounts of buyers given the above strategy and then the conditions under which no buyer has an incentive to deviate. Let the mass of “uninformed” buyers (i.e., buyers who do not know that the state is good, with posterior  $\pi' < 1$ ) be denoted by  $M_t^\theta$  and the total mass of buyers by  $N_t^\theta$  at  $t$  and in state  $\theta$  as measured at the start of  $t$ , after entry.

The probability of trading at an odd period  $2t - 1$  is  $\tau_{2t-1}^\theta = \mu^\theta$ , because all buyers accept a high offer and no buyer accepts a low offer. A trade at  $2t - 1$  is good news because  $\tau_{2t-1}^G > \tau_{2t-1}^B$ . The probability of trading at  $2t$  is

$$\tau_{2t}^\theta = \mu^\theta + (1 - \mu^\theta) \frac{M_{2t}^\theta}{N_{2t}^\theta} (1 - \tau_{2t-1}^\theta),$$

because all buyers accept a high offer and a buyer accepts a low offer if she is uncertain of the state and observes no trade. The probabilities at  $2t$  are, more

explicitly,  $\tau_{2t}^G = \frac{1}{2} \left( 1 + \frac{M_{2t}^G}{N_{2t}^G} \right)$  and  $\tau_{2t}^B = \frac{M_{2t}^B}{N_{2t}^B} = 1$ , where the last equality follows from the fact that no buyer can know that the state is good if the state is in fact bad. A trade at  $2t$  is bad news because  $\tau_{2t}^G < 1 = \tau_{2t}^B$ . Note that if the cycles are two periods long, then  $\tau_{2t}^G = \tau_{2t-2}^G$  and observing no trade at  $2t-1$  ( $N_{2t-2}$ ) or a trade at  $2t$  ( $T_{2t-1}$ ), reveal the good state (so the equilibrium strategy has to specify what buyers who know that the state is good do).

Now I derive  $M_{2t}^G$  and  $N_{2t}^G$  to get a closed-form solution for  $\tau_{2t}^G$ . What are the flows into these masses of buyers? First, consider period  $2t-1$ . How many buyers who start at  $2t-1$  as uninformed,  $M_{2t-1}^G$ , reach  $2t$  as uninformed? An uninformed buyer does not learn the state at  $2t-1$  if she gets a low offer and observes a trade ( $T_{2t-2}$ ). These buyers continue and they reach  $2t$  with probability  $\delta$ . In addition to them, the buyers who enter at  $2t$  start off as uninformed. Thus,

$$M_{2t}^G = M_{2t-1}^G(1 - \mu^G)\tau_{2t-2}^G\delta + 1.$$

Consider an even period  $2t$ . How many buyers who start at  $2t$  as uninformed,  $M_{2t}^G$ , reach period  $2t+1$  as uninformed? An uninformed buyer does not learn the state at  $2t$  if she gets a low offer and observes no trade ( $N_{2t-1}$ ). But all of these buyers accept the offer according to the proposed strategy so no uninformed buyers are carried over to  $2t+1$  from  $2t$ . All buyers who enter at  $2t+1$  start off as uninformed. Thus,  $M_{2t+1}^G = 1$ .

How many buyers who start at  $2t-1$  as informed,  $N_{2t-1}^G - M_{2t-1}^G$ , reach period  $2t$  as informed? All informed buyers remain informed, but some of them exit: only those reach period  $2t$  who get a low offer and survive. Buyers who start  $2t-1$  off as uninformed, in the amount  $M_{2t-1}^G$ , reach period  $2t$  as informed if they become informed, don't exit at  $2t-1$ , and survive. They become informed if they get a high offer or observe  $N_{2t-2}$ . They continue if they get a low offer, regardless of the signal outcome. Thus, the amount of informed buyers at  $2t$  is

$$N_{2t}^G - M_{2t}^G = (N_{2t-1}^G - M_{2t-1}^G)(1 - \mu^G)\delta + M_{2t-1}^G(1 - \mu^G)(1 - \tau_{2t-2}^G)\delta.$$

Finally, how many buyers who start at  $2t$  as informed,  $N_{2t}^G - M_{2t}^G$ , reach period  $2t+1$  as informed? All informed buyers remain informed, but only those reach  $2t+1$  who at  $2t$  get a low offer and survive. Buyers who start  $2t$  off as uninformed,  $M_{2t}^G$ , reach period  $2t+1$  as informed if they get a low offer, observe  $T_{2t-1}$ , and survive. Thus, the amount of informed buyers at  $2t+1$  is

$$N_{2t+1}^G - M_{2t+1}^G = (N_{2t}^G - M_{2t}^G)(1 - \mu^G)\delta + M_{2t}^G(1 - \mu^G)\tau_{2t-1}^G\delta.$$

Combining these equations and imposing that  $x_{t'+2} = x_{t'}$  for  $t' = 2t, 2t-1$  and

all endogenous variables  $x$ , gives a solution

$$\tau_{even}^G = \frac{1}{2\delta^3}(16 + 6\delta - 2\delta^2 + \delta^3 - \sqrt{256 + 192\delta - 28\delta^2 - 40\delta^3 + 4\delta^5 + \delta^6}),$$

which decreases in  $\delta$  and is in the interval  $[\frac{21-\sqrt{385}}{2} \approx 0.69, \frac{3}{4}]$  for all  $\delta \in (0, 1)$ . This implies that the volume of trading fluctuates more in the bad state than the good because  $Vol_{odd}^\theta + Vol_{even}^\theta = 2$  for  $\theta = B, G$ , and  $\tau_{odd}^B = 0$ ,  $\tau_{even}^B = 1$  while  $\tau_{odd}^G, \tau_{even}^G \in (0, 1)$ .

Finally, I derive the conditions under which the proposed strategy is optimal. For a buyer with belief  $\pi'$ , the value of continuing for one more period and then accepting either offer is given by equation (A2) so the critical belief is again  $\bar{\pi}$  as defined in (1) where  $\mu^G = \frac{1}{2}$ .

Let the beliefs of a buyer who has seen  $h$  of  $T_{odd}$ ,  $i$  of  $N_{odd}$ ,  $j$  of  $T_{even}$ , and  $k$  of  $N_{even}$ , be  $\pi(h, i, j, k)$ . Since odd and even periods alternate, it must be that  $h + i \in \{j + k - 1, j + k, j + k + 1\}$ . A sufficient condition for the proposed strategy to be optimal is that a buyer who is supposed to continue according to the strategy wants to continue for at least one period and that a buyer who is supposed to accept a low offer according to the strategy prefers accepting the offer to continuing for one more period. Then the strategy is optimal if the following three sets of conditions hold:

- (i) buyers who get a low offer and know that the state is good prefer to continue:  $\pi(h, i, j, k) > \bar{\pi}$  for all  $h, k \geq 1$ ,
- (ii) buyers who get a low offer, do not know the state, and have not seen  $N_{odd}$  prefer to continue:  $\pi(0, 0, j, 0) > \bar{\pi}$  for all  $j$  (i.e., for  $j = 0, 1$ ), and
- (iii) buyers who get a low offer, do not know the state and have seen at least one  $N_{odd}$  prefer to accept the offer:  $\pi(0, i, j, 0) < \bar{\pi}$  for all  $i \geq 1$  and all  $j$ .

The set of conditions in (i) is satisfied as  $\pi(h, i, j, k) = 1$  for all  $h, k \geq 1$ . Of the conditions in set (ii), the stricter is for the more pessimistic buyer, i.e., for  $j = 1$  since  $T_{even}$  is bad news. The stricter condition,  $\pi(0, 0, 1, 0) > \bar{\pi}$ , can be written as

$$\frac{\pi(0, 0, 1, 0)}{1 - \pi(0, 0, 1, 0)} = \omega \frac{1 - \mu^G \tau_{even}^G}{1 - \mu^B \tau_{even}^B} = \frac{\omega \tau_{even}^G}{2} > \frac{\bar{\pi}}{1 - \bar{\pi}}.$$

Of the conditions in set (iii), the strictest is for the most optimistic buyer, i.e., for  $i = 1$  and  $j = 0$  because both  $N_{odd}$  and  $T_{even}$  are bad news. The strictest condition,  $\pi(0, 1, 0, 0) < \bar{\pi}$ , can be written as

$$\frac{\pi(0, 1, 0, 0)}{1 - \pi(0, 1, 0, 0)} = \omega \frac{1 - \mu^G}{1 - \mu^B} \frac{1 - \tau_{odd}^G}{1 - \tau_{odd}^B} = \frac{\omega}{4} < \frac{\bar{\pi}}{1 - \bar{\pi}}.$$

The conditions can be satisfied simultaneously because  $\frac{1}{4} < \frac{\tau_{even}^G}{2}$ . I rearrange the

two inequalities to get the exact conditions in the Proposition. ■