Online Appendix of “Searching for Service”

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1 Alternative Consumer Search Strategies

To simplify the exposition, we assume that $c_S = c_N = c$. We consider the case with $p_S^* - p_N^* > w(c - v_S) - w(c)$ first, and the case with $p_S^* - p_N^* \leq 0$ next.

First Case with $p_S^* - p_N^* > w(c - v_S) - w(c)$

First notice that under monopolistic competition with infinite number of non-service providers, if initially, a consumer prefers to visit a non-service provider than a service provider, the consumer will never visit a service provider, and thus this is not an equilibrium.

Next, consider the potential equilibrium with M_N non-service providers and infinite number of service providers. Given infinite number of service providers, we need to impose $F = 0$; otherwise, an individual service provider has no incentive to provide service.

Under $p_S^* - p_N^* > w(c - v_S) - w(c)$, consumers will search among non-service providers first and only after they have visited all non-service providers they will visit service providers if they decide to continue to search. We are going to prove the following claims:

1. The equilibrium requirement that non-service providers have no incentive to deviate by providing service imposes a lower bound on δ. As M_N is sufficiently large, the lower bound goes to zero. This means that for sufficiently large M_N, given any $\delta > 0$, non-service providers have no incentive to deviate by providing service.

2. The equilibrium requirement that service providers have no incentive to deviate by providing service imposes an upper bound on δ. As M_N is sufficiently large, the upper bound goes to zero. This means that for

sufficiently large M_N, given any $\delta > 0$, service providers always have incentives to deviate by not providing service.

3. The consumer search strategy imposes a lower bound on δ. As M_N is sufficiently large, the lower bound is finite and positive.

Proof. • First consider a non-service provider i who does not provide service and charges price p. Its demand function is,

$$\begin{aligned} D_N(p) = \alpha_N & \sum_{n=0}^{M_N-1} G(w(c))^n \left[1 - G(w(c) - p_N^* + p)\right] \\ & + \int_{w(c-v_S)-p_S^*+p}^{w(c)-p_N^*+p} G(v + p_N^* - p)^{M_N-1} g(v) dv \\ & + \int_{w(c)-p_S^*+p}^{w(c-v_S)-p_S^*+p} G(v + p_N^* - p)^{M_N-1} G(v + p_S^* - p) g(v) dv, \end{aligned}$$

where the first term on the righthand side of the equation above represents the sum of probabilities that a consumer who, after visiting n non-service providers, visits firm i, discovers $v_i - p \geq w(c) - p_N^*$, and decides to stop searching and make a purchase; the second term represents a consumer who has visited all M_N non-service providers and decides not to continue to search service providers and return to make a purchase from firm i; the third term represents a consumer who has visited all M_N non-service providers as well as one service provider and decides to stop searching and return to make a purchase from firm i.

If the non-service provider deviates by providing service and charges price p, its demand function is,

$$\begin{aligned} \widetilde{D}_N(p) = \alpha_N & \sum_{n=0}^{M_N-1} G(w(c))^n \left[1 - G(w(c) - p_N^* + p)\right] \\ & + \int_{w(c)-p_S^*+p}^{w(c)-p_N^*+p} G(v + p_N^* - p)^{M_N-1} g(v) dv. \end{aligned}$$

Notice that,

$$\begin{aligned} \widetilde{D}_N(p) - D_N(p) &= \int_{w(c)-p_S^*+p}^{w(c-v_S)-p_S^*+p} G(v + p_N^* - p)^{M_N-1} \left[1 - G(v + p_S^* - p)\right] g(v) dv \\ &\geq 0. \end{aligned}$$

This implies that by deviating to provide service, a non-service provider can increase demand. Therefore, the cost of service provision, δ has to be sufficiently large to ensure the non-service provider has no incentive to deviate.

It is easy to show that as $M_N \to \infty$, $D_N(p) \to \alpha_N[1-G(w(c)-p_N^*+p)]/[1-G(w(c))]$, $\widetilde{D}_N(p) - D_N(p) \to 0$ and $(\widetilde{D}_N(p) - D_N(p))/D_N(p) \to 0$. This implies that given any $\delta > 0$, as M_N is sufficiently large, for any p,

$$\delta > \frac{\widetilde{D}_N(p) - D_N(p)}{\widetilde{D}_N(p)} p, \text{ or equivalently, } pD(p) > (p-\delta)\widetilde{D}_N(p).$$

Therefore, for M_N sufficiently large, it is not profitable for a non-service provider to deviate by providing service.

Moreover, as $M_N \to \infty$, $D_N(p) \to \alpha_N[1-G(w(c)-p_N^*+p)]/[1-G(w(c))]$, we have that the non-service provider's equilibrium price,

$$p_N^* \to \frac{1-G(w(c))}{g(w(c))}, \text{ as } M_N \to \infty.$$

• Next, consider a service provider who provides service and charges price p. Its demand function is,

$$\begin{aligned} D_S(p) = \alpha_S & \int_{w(c)-p_S^*+p}^{w(c-v_S)-p_S^*+p} G(v-p+p_N^*)^{M_N} g(v)dv \\ & + \alpha_S G(w(c-v_S)-p_S^*+p_N^*)^{M_N} \left[1-G(w(c-v_S)-p_S^*+p)\right] \\ & + \alpha_S G(w(c)-p_S^*+p_N^*)^{M_N} \sum_{n=1}^{\infty} G(w(c))^n \left[1-G(w(c)-p_S^*+p)\right], \end{aligned}$$

where the first and second terms on the righthand side of the equation above come from consumers who visit the service provider and make a purchase right after visiting all non-service providers; the third terms represents the sum of probabilities that a consumer who, after visiting all non-service providers as well as n service providers, visits the service provider and make a purchase.

If the service provider deviates by not providing service and charges price p, its demand function is,

$$\begin{aligned} \widetilde{D}_S(p) = \alpha_S & \int_{w(c)-p_S^*+p}^{w(c-v_S)-p_S^*+p} G(v-p+p_N^*)^{M_N} G(v-p+p_S^*) g(v)dv \\ & + \alpha_S G(w(c-v_S)-p_S^*+p_N^*)^{M_N} \left[1-G(w(c-v_S)-p_S^*+p)\right] \\ & + \alpha_S G(w(c)-p_S^*+p_N^*)^{M_N} \sum_{n=1}^{\infty} G(w(c))^n \left[1-G(w(c)-p_S^*+p)\right]. \end{aligned}$$

Notice that,

$$
\begin{aligned}
D_S(p) - \widetilde{D}_S(p) &= \alpha_S \int_{w(c)-p_S^*+p}^{w(c-v_S)-p_S^*+p} G(v-p+p_N^*)^{M_N} \left[1 - G(v-p+p_S^*)\right] g(v)dv \\
&\geq 0.
\end{aligned}
$$

This implies that by deviating to not provide service, a service provider's demand decreases. The cost of service provision, δ has to be sufficiently small enough to ensure the service provider has no incentive to deviate.

It is easy to show that as $M_N \to \infty$, both $D_S(p)$ and $D_S(p) - \widetilde{D}_S(p)$ go to 0, and furthermore, $[D_S(p) - \widetilde{D}_S(p)]/D_S(p) \to 0$. Following the same argument above, we can show that given any $\delta > 0$, as M_N is sufficiently large, we have $(p-\delta)D_S(p) < p\widetilde{D}_S(p)$. Therefore, for M_N sufficiently large, it is always profitable for a service provider to deviate by not providing service.

Moreover, by solving the first-order optimality condition, $(p-\delta)D_S'(p) + D_S(p) = 0$, we can show that the service provider's equilibrium price

$$
p_S^* \to \delta + \frac{1 - G(w(c-v_S))}{g(w(c-v_S))}, \text{ as } M_N \to \infty.
$$

- Lastly, the consumer's search strategy implies that,

$$
p_S^* - p_N^* > w(c - v_S) - w(c).
$$

Based on the expressions of p_N^* and p_S^*, the above inequality implies that,

$$
\delta > \left(w(c-v_S) - \frac{1 - G(w(c-v_S))}{g(w(c-v_S))} \right) - \left(w(c) - \frac{1 - G(w(c))}{g(w(c))} \right).
$$

Notice that $w(\cdot)$ is a decreasing function, and $[1 - G(\cdot)]/g(\cdot)$ is a decreasing function due to logconcavity of $1 - G(\cdot)$. This implies that for $v_S > 0$, the righthand side of the inequality above is positive. That is, the consumer search strategy imposes a lower bound on δ. ■

Second Case with $p_S^* - p_N^* \leq 0$

We consider the potential equilibrium with M_S service providers and infinite number of non-service providers. Under $p_S^* - p_N^* \leq 0$, consumers will search among service providers first and only after they have visited all service providers they will visit non-service providers if they decide to continue to search. We are going to prove that for any $\delta > 0$, this is not an equilibrium.

Proof. • A service provider's demand function is,

$$D_S(p) = \alpha_S \sum_{n=0}^{M_S-1} G(w(c))^n \left[1 - G(w(c) - p_S^* + p)\right] + \int_{w(c)-p_N^*+p}^{w(c)-p_S^*+p} G(v - p + p_S^*)^{M_S-1} g(v) dv. \tag{1}$$

The equilibrium price p_S^* satisfies that,

$$p_S^* = \delta - \frac{D_S(p_S^*)}{D_S'(p_S^*)}. \tag{2}$$

If the firm deviates to not providing service and charging price p. We have that,

$$\begin{aligned}
\widetilde{D}_S(p) &= \alpha_S \left[1 - G(w(c - v_S) - p_S^* + p)\right] \\
&+ \alpha_S \int_{w(c)-p_S^*+p}^{w(c-v_S)-p_S^*+p} G(v - p + p_S^*) g(v) dv \\
&+ \alpha_S \sum_{n=1}^{M_S-1} G(w(c))^n \left[1 - G(w(c) - p_S^* + p)\right] \\
&+ \int_{w(c)-p_N^*+p}^{w(c)-p_S^*+p} G(v - p + p_S^*)^{M_S-1} g(v) dv.
\end{aligned}$$

Then, we have that

$$D_S(p) - \widetilde{D}_S(p) = \alpha_S \int_{w(c)-p_S^*+p}^{w(c-v_S)-p_S^*+p} \left[1 - G(v - p + p_S^*)\right] g(v) dv \geq 0.$$

Therefore, by deviating to not provide service, a service provider suffers from a demand loss. Following the same line of proof in Proposition 3 in the main text, we can show that when δ is below a threshold, it is not profitable for a service provider to deviate by not providing service.

• Now, consider a non-service provider. Its demand function is,

$$D_N(p) = \alpha_N G(w(c) - p_N^* + p_S^*)^{M_S} \frac{1 - G(w(c) - p_N^* + p)}{1 - G(w(c))}.$$

The equilibrium price is then,

$$p_N^* = \frac{1 - G(w(c))}{g(w(c))}. \tag{3}$$

It is straightforward to show that the non-service provider has no incentive to deviate by providing service.

• Lastly, we examine consumers' search strategy. Let's first prove that $D_S(p)$ in equation (1) is a log-concave function. In fact,

$$\begin{aligned}
D_S(p) = {} & \alpha_S \sum_{n=0}^{M_S-1} G(w(c))^n \left[1 - G(w(c) - p_S^* + p)\right] \\
& + \int_{w(c)-p_N^*+p_S^*}^{w(c)} G(v)^{M_S-1} g(v + p - p_S^*) dv \\
& = \alpha_S \sum_{n=0}^{M_S-1} G(w(c))^n \left[1 - G(w(c) - p_S^* + p)\right] \\
& + G(w(c))^{M_S-1} G(w(c) + p - p_S^*) \\
& - G(w(c) + p_S^* - p_N^*)^{M_S-1} G(w(c) + p - p_N^*) \\
& - (M_S - 1) \int_{w(c)-p_N^*+p_S^*}^{w(c)} G(v + p - p_S^*) G(v)^{M_S-2} dv \\
& = \left(\frac{1}{M_S} \sum_{n=0}^{M_S-1} G(w(c))^n - G(w(c))^{M_S-1} \right) \left[1 - G(w(c) - p_S^* + p)\right] \\
& + G(w(c) + p_S^* - p_N^*)^{M_S-1} [1 - G(w(c) + p - p_N^*)] \\
& + (M_S - 1) \int_{w(c)-p_N^*+p_S^*}^{w(c)} G(v)^{M_S-2} [1 - G(v + p - p_S^*)] dv \\
& + G(w(c))^{M_S-1} - G(w(c) + p_S^* - p_N^*)^{M_S-1} \\
& - (M_S - 1) \int_{w(c)-p_N^*+p_S^*}^{w(c)} G(v)^{M_S-2} dv,
\end{aligned}$$

where the first equation is due to the change of argument and the second equation is due to integration by parts. Notice that $1 - G(\cdot)$ is log-concave, and thus $1 - G(w(c) - p_S^* + p)$, $1 - G(w(c) + p - p_N^*)$, and $1 - G(v + p - p_S^*)$ are all log-concave in p. By Prekopa-Leindler inequality, $D_S(p)$, as a linear combination of these log-concave functions, is also log-concave (Lynch 1999).

Similarly to the proof of Proposition 3, we define $\Delta p(\delta) \equiv p_S^* - p_N^*$, where p_S^* and p_N^* are given by equations (2) and (3). By taking derivatives, we have that,

$$\Delta p'(\delta) = \left(2 - \frac{D_S(p_S^*)D_S''(p_S^*)}{D_S'(p_S^*)^2}\right)^{-1} > 0.$$

where the inequality above is due to log-concavity of $D_S(p)$. Therefore, $\Delta p(\delta)$ strictly increasing with δ. Moreover, one can verify that,

$$\Delta p(0) = 0.$$

This implies that the consumer search strategy requirement that $p_S^* - p_N^* \leq 0$ is equivalent to $\delta \leq 0$. This implies that for any $\delta > 0$, the equilibrium does no exist. ■

References

James D Lynch. On conditions for mixtures of increasing failure rate distributions to have an increasing failure rate. *Probability in the Engineering and Informational Sciences*, 13(1):33–36, 1999.